Panel Smooth Transition Regression Models*

Andrés González†

Banco de la República de Colombia
Stockholm School of Economics

Timo Teräsvirta

Department of Economic Statistics
Stockholm School of Economics

Dick van Dijk

Econometric Institute
Erasmus University Rotterdam

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Abstract

We develop a non-dynamic panel smooth transition regression model with fixed individual effects. The model is useful for describing heterogenous panels, with regression coefficients that vary across individuals and over time. Heterogeneity is allowed for by assuming that these coefficients are continuous functions of an observable variable through a bounded function of this variable and fluctuate between a limited number (often two) of "extreme regimes". The model can be viewed as a generalization of the threshold panel model of Hansen (1999). We extend the modelling strategy for univariate smooth transition regression models to the panel context. This comprises of model specification based on homogeneity tests, parameter estimation, and diagnostic checking, including tests for parameter constancy and no remaining nonlinearity. The new model is applied to describe firms' investment decisions in the presence of capital market imperfections.

Keywords: financial constraints; heterogenous panel; investment; misspecification test; nonlinear modelling panel data; smooth transition models.

JEL Classification Codes: C12, C23, C52, G31, G32.

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†Department of Economic Statistics, Stockholm School of Economics, Box 6501, SE-113 83 Stockholm, Sweden, `email: Andres.Gonzalez@hhs.se` (corresponding author)

1 Introduction

In regression models for panel data it is typically assumed that the heterogeneity in the data can be captured completely by means of (fixed or random) individual effects and time effects, such that the coefficients of the observed explanatory variables are identical for all observations. In many empirical applications, however, this poolability assumption may be violated or at least may be questionable. For example, there is a sizable literature documenting that, due to capital market imperfections such as information asymmetry between borrowers and lenders, investment decisions of individual firms depend on financial variables such as cash flow, see Hubbard (1998) for a review. The sensitivity of investment to cash flow often is found to vary across firms according to the severity of the information asymmetry problem or their investment opportunities. In particular, external finance may be limited mainly for firms facing high agency costs due to information asymmetry or for firms with limited profitable investment opportunities. For such constrained firms, investment will depend on the availability of internal finance to a much larger extent than for unconstrained firms.

Various panel data models that allow regression coefficients to vary over time and across cross-sectional units have been developed, see Hsiao (2003, Chapter 6) for an overview. These include random coefficients models and models with coefficients that are functions of other exogenous variables. A specific example of the latter type of parameter heterogeneity is the panel threshold regression (PTR) model developed by Hansen (1999). In this model, regression coefficients can take on a small number of different values, depending on the value of another observable variable. Interpreted differently, the observations in the panel are divided into a small number of homogenous groups or 'regimes', with different coefficients in different regimes.

A feature that makes the PTR model quite appealing is that individuals are not restricted to remain in the same group for all time periods if the so-called threshold variable that is used for grouping the observations is time-varying. In the empirical example of firms' investment decisions given above, it is likely that information costs and investment opportunities change over time, such that firms switch between constrained and unconstrained regimes. On the other hand, the PTR model implies that the different groups of observations can be clearly distinguished from each other based on the value of the threshold variable alone, with sharp 'borders' or thresholds separating the groups. In practice, this may not always be feasible though. In this paper we consider a generalization of the PTR model that relaxes this restriction in Hansen's (1999) original proposal. In particular, we develop a panel smooth transition regression (PSTR) model, which has essentially the same features as the PTR model but allows the regression coefficients to change gradually when moving from one group to another.

The paper is organized as follows. Section 2 introduces the panel smooth transition regression model, focusing on interpretation of the model structure and on its relation to the PTR model of Hansen (1999). Section 3 develops a model building procedure for PSTR models, including model specification, parameter estimation and diagnostic checking. The modelling cycle is an extension of the procedure that is available for smooth transition regression models for a single cross-section or time series, see Teräsvirta (1998) and van Dijk, Teräsvirta, and Franses (2002), among others. As part of the specification stage we develop a novel Lagrange Multiplier (LM) test of parameter homogeneity. Although the test is designed specifically against the PSTR alternative, it has wider applicability as a general test of poolability of the data, see also Baltagi (2005, Section 4.1). Similarly, we develop a test of parameter constancy in PSTR models as part of the evaluation stage, which also is applicable in other panel models. Section 4 considers the small sample properties of the different test statistics involved in the modelling cycle by means of Monte Carlo simulation. Special attention is given here to the issue of cross-sectional heteroskedasticity and the consequences thereof for the performance of the tests. Section 5 contains an empirical application of the proposed methodology to the problem of individual firms' investment decisions in the presence of credit market imperfections. Finally, Section 6 concludes.

2 Panel smooth transition regression model

The Panel Smooth Transition Regression (PSTR) model is a fixed effects model with exogenous regressors. The model can be interpreted in two different ways. First, it may be thought of as a linear heterogenous panel model with coefficients that vary across individuals and over time. Heterogeneity in the regression coefficients is allowed for by assuming that these coefficients are continuous functions of an observable variable through a bounded function of this variable, called the transition function, and fluctuate between a limited number (often two) of "extreme regimes". As the transition variable is individual-specific and time-varying, the regression coefficients for each of the individuals in the panel are changing over time. Second, the PSTR model can simply be considered as a nonlinear homogenous panel model. The latter interpretation is in fact common in the context of single-equation smooth transition regression (STR) or univariate smooth transition autoregressive (STAR) models, see Teräsvirta (1994, 1998). Given the current context, we prefer the first interpretation.

The basic PSTR model with two extreme regimes is defined as

$$y_{it} = \mu_i + \beta_0' x_{it} + \beta_1' x_{it} g(q_{it}; \gamma, c) + u_{it} \quad (1)$$

for $i = 1, \ldots, N$, and $t = 1, \ldots, T$, where N and T denote the cross-section and time dimensions of the panel, respectively. The dependent variable y_{it} is a scalar, x_{it} is a k-dimensional vector of time-varying exogenous variables, μ_i represents the fixed individual effect, and u_{it} are the errors. Transition function $g(q_{it}; \gamma, c)$ is a continuous function of the observable variable q_{it} and is normalized to be bounded between 0 and 1, and these extreme values are associated with regression coefficients β_0 and $\beta_0 + \beta_1$. More generally, the value of q_{it} determines the value of $g(q_{it}; \gamma, c)$ and thus the effective regression coefficients $\beta_0 + \beta_1 g(q_{it}; \gamma, c)$ for individual i at time t. We follow Granger and Teräsvirta (1993), Teräsvirta (1994) and Jansen and Teräsvirta (1996) by using the logistic specification

$$g(q_{it}; \gamma, c) = \left(1 + \exp\left(-\gamma \prod_{j=1}^{m} (q_{it} - c_j)\right)\right)^{-1} \quad \text{with } \gamma > 0 \text{ and } c_1 \leq c_2 \leq \ldots \leq c_m \quad (2)$$

where $c = (c_1, \ldots, c_m)'$ is an m-dimensional vector of location parameters and the slope parameter γ determines the smoothness of the transitions. The restrictions $\gamma > 0$ and $c_1 \leq \ldots \leq c_m$ are imposed for identification purposes. In practice it is usually sufficient to consider $m = 1$ or $m = 2$, as these values allow for commonly encountered types of variation in the parameters. For $m = 1$, the model implies that the two extreme regimes are associated with low and high values of q_{it} with a single monotonic transition of the coefficients from β_0 to $\beta_0 + \beta_1$ as q_{it} increases, where the change is centred around c_1. When $\gamma \to \infty$, $g(q_{it}; \gamma, c)$ becomes an indicator function $\mathtt{I}[q_{it} > c_1]$, defined as $\mathtt{I}[A] = 1$ when the event A occurs and 0 otherwise. In that case the PSTR model in (1) reduces to the two-regime panel threshold model of Hansen (1999). For $m = 2$, the transition function has its minimum at $(c_1 + c_2)/2$ and attains the value 1 both at low and high values of q_{it}. When $\gamma \to \infty$, the model becomes a three-regime threshold model whose outer regimes are identical and different from the middle regime. In general, when $m > 1$ and $\gamma \to \infty$, the number of distinct regimes remains two, with the transition function switching back and forth between zero and one at $c_1, \ldots, c_m$. Finally, for any value of m the transition function (2) becomes constant when $\gamma \to 0$, in which case the model collapses into a homogenous or linear panel regression model with fixed effects.

A generalization of the PSTR model to allow for more than two different regimes is the additive model

$$y_{it} = \mu_i + \beta_0' x_{it} + \sum_{j=1}^{r} \beta_j' x_{it} g_j(q_{it}^{(j)}; \gamma_j, c_j) + u_{it} \tag{3}$$

where the transition functions $g_j(q_{it}^{(j)}; \gamma_j, c_j)$, $j = 1, \ldots, r$, are of the logistic type (2). If $m = 1$, $q_{it}^{(j)} = q_{it}$, and $\gamma_j \to \infty$ for all $j = 1, \ldots, r$, the model in (3) becomes a PTR model with $r + 1$ regimes. Consequently, the additive PSTR model can be viewed as a generalization of the multiple regime panel threshold model in Hansen (1999). Additionally, when the largest model that one is willing to consider is a two-regime PSTR model (1) with $r = 1$ and $m = 1$ or $m = 2$, model (3) plays an important role in the evaluation of the estimated model. In particular, the multiple regime model (3) is an obvious alternative in diagnostic tests of no remaining heterogeneity. Evaluation of PSTR models will be discussed in Section 3.3.2.

3 Building panel smooth transition regression models

Application of nonlinear models such as the panel smooth transition regression model requires a careful and systematic modelling strategy. The modelling cycle that is available for smooth transition regression (STR) models for a single time series y_t, $t = 1, \ldots, T$, or potentially also for a single cross-section y_i, $i, \ldots, N$, can be readily extended to panel STR models. The STR model building procedure consists of specification, estimation and evaluation stages. Specification includes testing homogeneity, selecting the transition variable q_{it} and, if homogeneity is rejected, determining the appropriate form of the transition function, that is, choosing the proper value of m in (2). Nonlinear least squares is used for parameter estimation. At the evaluation stage the estimated model is subjected to misspecification tests to check whether it provides an adequate description of the data. The null hypotheses to be tested at this stage include parameter constancy, no remaining heterogeneity and no autocorrelation in the errors. Finally, one also has to choose the number of regimes in the panel, which means selecting r in model (3). In the following subsections we discuss these elements in more detail, see also Teräsvirta (1998) and van Dijk, Teräsvirta, and Franses (2002), among others.

3.1 Model specification: testing homogeneity

The initial specification stage of the modelling cycle essentially consists of testing homogeneity against the PSTR alternative. This is important for both statistical and economic reasons. Statistically, the PSTR model is not identified if the data-generating process is homogenous, and a homogeneity test is necessary to avoid the estimation of unidentified models. From an economics point of view, such a test may be useful for testing a certain proposition from economic theory, such as identical sensitivity of investment to variables such as cash flow for all firms in a sample.

The PSTR model (1) with (2) can be reduced to a homogenous model by imposing either $\mathsf{H}_0 : \gamma = 0$ or $\mathsf{H}_0' : \beta_1 = 0$. The associated tests are nonstandard because under either null hypothesis the PSTR model contains unidentified nuisance parameters. In particular, the location parameters c are not identified under both null hypotheses, while this also is the case for β_1 under H_0 and for γ under H_0'. The problem of hypothesis testing in the presence of unidentified nuisance parameters was first studied by Davies (1977, 1987). Luukkonen, Saikkonen, and Teräsvirta (1988), Andrews and Ploberger (1994) and Hansen (1996) proposed alternative solutions in the time series context. We follow Luukkonen, Saikkonen, and Teräsvirta (1988) and test homogeneity using the null hypothesis $\mathsf{H}_0 : \gamma = 0$. To circumvent the identification problem we replace $g(q_{it}; \gamma, c)$ in (1) by its first-order Taylor expansion around $\gamma = 0$. After reparameterization, this leads to the auxiliary regression

$$y_{it} = \mu_i + \beta_0'^* x_{it} + \beta_1'^* x_{it} q_{it} + \ldots + \beta_m'^* x_{it} q_{it}^m + u_{it}^* \tag{4}$$

where the parameter vectors $\beta_1^*, \ldots, \beta_m^*$ are multiples of γ and $u_{it}^* = u_{it} + R_m \beta_1' x_{it}$, where R_m is the remainder of the Taylor expansion. Consequently, testing $\mathsf{H}_0 : \gamma = 0$ in (1) is equivalent to testing the null hypothesis $\mathsf{H}_0^* : \beta_1^* = \ldots = \beta_m^* = 0$ in (4). Note that under the null hypothesis $\{u_{it}^*\} = \{u_{it}\}$, so the Taylor series approximation does not affect the asymptotic distribution theory. This null hypothesis may be conveniently tested by an LM test. In order to define the LM statistic, we write (4) in matrix notation as follows:

$$y = D_\mu \mu + X\beta + W\beta^* + u^* \tag{5}$$

where $y = (y_1', \ldots, y_N')'$ with $y_i = (y_{i1}, \ldots, y_{iT})'$, $i = 1, \ldots, N$, $D_\mu = (I_N \otimes \iota_T)$ where I_N is the identity matrix of dimension N and ι_T a $(T \times 1)$ vector of ones, and $\mu = (\mu_1, \ldots, \mu_N)'$. Moreover, $X = (X_1', \ldots, X_N')$ where $X_i = (x_{i1}', \ldots, x_{iT}')'$, $W = (W_1', \ldots, W_N')'$ with $W_i = (w_{i1}', \ldots, w_{iT}')'$ and $w_{it} = (x_{it}' q_{it}, \ldots, x_{it}' q_{it}^m)'$, $\beta = \beta_0^*$ and $\beta^* = (\beta_1^{*\prime}, \ldots, \beta_m^{*\prime})'$. Finally,

$u^* = (u_1'^*, \ldots, u_N'^*)'$ is a $(TN \times 1)$ vector with $u_i^* = (u_{i1}^*, \ldots, u_{iT}^*)'$. The LM test statistic has the form

$$\mathrm{LM}_\chi = \hat{u}^{0\prime}\tilde{W}\hat{\Sigma}^{-1}\tilde{W}'\hat{u}^0 \tag{6}$$

where $\hat{u}^0 = (\hat{u}_1^{0\prime}, \ldots, \hat{u}_N^{0\prime})'$ is the vector of residuals obtained under the null hypothesis, $\tilde{W} = M_\mu W$ where $M_\mu = I_{NT} - D_\mu(D_\mu' D_\mu)^{-1}D_\mu'$ is the standard within-transformation matrix. Furthermore, $\hat{\Sigma}$ is any consistent estimator of the appropriate covariance matrix. When the errors are homoskedastic and identically distributed across time and individuals $\hat{\Sigma}$ is given by

$$\hat{\Sigma}^{\mathrm{ST}} = \hat{\sigma}^2(\tilde{W}'\tilde{W} - \tilde{W}'\tilde{X}(\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{W}) \tag{7}$$

where $\tilde{X} = M_\mu X$, and $\hat{\sigma}^2$ is the estimated error variance under the null. When the errors are heteroskedastic or autocorrelated, $\hat{\Sigma}$ is given by

$$\hat{\Sigma}^{\mathrm{HAC}} = [-\tilde{W}'\tilde{X}(\tilde{X}'\tilde{X})^{-1} : I_l]\hat{\triangle}[-\tilde{W}'\tilde{X}(\tilde{X}'\tilde{X})^{-1} : I_l]' \tag{8}$$

where I_l is the identity matrix of dimension $l = \dim(W) - \dim(X) = k(m-1)$, and

$$\hat{\triangle} = \sum_{i=1}^{N} \tilde{Z}_i'\hat{u}_i^0\hat{u}_i^{0\prime}\tilde{Z}_i$$

with $\tilde{Z}_i = M_\mu Z_i$, where $Z_i = [X_i, W_i]$, $i = 1, \ldots, N$. The estimator (8) is consistent for fixed T as $N \to \infty$, see Arellano (1987) for details. Under the null hypothesis the LM_χ statistic (6) is asymptotically distributed as $\chi^2(mk)$, while the F-version $\mathrm{LM_F} = \mathrm{LM}_\chi/mk$ has an approximate $\mathrm{F}(mk, TN - N - m(k+1))$ distribution.

Two remarks concerning the homogeneity test are in order. First, the test can be used for selecting the appropriate transition variable q_{it} in the PSTR model. In this case, the test is carried out for a set of 'candidate' transition variables and the variable that gives rise to the strongest rejection of linearity (if any) is chosen as the transition variable. Second, the homogeneity test can also be used for determining the appropriate order m of the logistic transition function in (2). Granger and Teräsvirta (1993) and Teräsvirta (1994) proposed a sequence of tests for choosing between $m = 1$ and $m = 2$. Applied to the present situation this testing sequence reads as follows: Using the auxiliary regression (4) with $m = 3$, test the null hypothesis $\mathsf{H}_0^* : \beta_3^* = \beta_2^* = \beta_1^* = 0$. If it is rejected, test $\mathsf{H}_{03}^* : \beta_3^* = 0$, $\mathsf{H}_{02}^* : \beta_2^* = 0|\beta_3^* = 0$ and $\mathsf{H}_{01}^* : \beta_1^* = 0|\beta_3^* = \beta_2^* = 0$. Select $m = 2$ if the rejection of H_{02}^* is the strongest one, otherwise select $m = 1$. For the reasoning behind this rule, see Teräsvirta (1994).

3.2 Parameter estimation

Estimating the parameters $\theta = (\beta_0', \beta_1', \gamma, c')'$ in the PSTR model (1) is a relatively straightforward application of the fixed effects estimator and nonlinear least squares (NLS). We first eliminate the individual effects μ_i by removing individual-specific means and then apply NLS to the transformed data.

While eliminating fixed effects using the within transformation is standard in linear panel data models, the PSTR model calls for a more careful treatment. Rewrite model (1) as follows:

$$y_{it} = \mu_i + \beta' x_{it}(\gamma, c) + u_{it} \tag{9}$$

where $x_{it}(\gamma, c) = (x_{it}', x_{it}' g(q_{it}; \gamma, c))'$ and $\beta = (\beta_0', \beta_1')'$. Subtracting individual means from (9) yields

$$\tilde{y}_{it} = \beta' \tilde{x}_{it}(\gamma, c) + \tilde{u}_{it} \tag{10}$$

where $\tilde{y}_{it} = y_{it} - \bar{y}_i$, $\tilde{x}_{it}(\gamma, c) = (x_{it}' - \bar{x}_i', x_{it}' g(q_{it}; \gamma, c) - \bar{w}_i'(\gamma, c))'$, $\tilde{u}_{it} = u_{it} - \bar{u}_i$, and $\bar{y}_i$, $\bar{x}_i$, $\bar{w}_i$ and $\bar{u}_i$ are individual means, with $\bar{w}_i(\gamma, c) \equiv T^{-1} \sum_{t=1}^{T} x_{it} g(q_{it}; \gamma, c)$. Consequently, the transformed vector $\tilde{x}_{it}(\gamma, c)$ in (10) depends on γ and c through both the levels and the individual means. For this reason, $\tilde{x}_{it}(\gamma, c)$ needs to be recomputed at each iteration in the NLS optimization.

From (10) it is seen that the PSTR model is linear in β conditional on γ and c. Thus, we apply NLS to determine the values of these parameters that minimize the concentrated sum of squared errors

$$Q^c(\gamma, c) = \sum_{i=1}^{N} \sum_{t=1}^{T} \left(\tilde{y}_{it} - \hat{\beta}(\gamma, c') \tilde{x}_{it}(\gamma, c) \right)^2 \tag{11}$$

where $\hat{\beta}(\gamma, c)$ is obtained from (10) by ordinary least squares at each iteration in the nonlinear optimization. In case the errors u_{it} in (9) are normally distributed, this estimation procedure is equivalent to maximum likelihood, where the likelihood function is first concentrated with respect to the fixed effects μ_i. An appendix that is available upon request considers the properties of the ML estimator in full detail, including a formal proof of its consistency and asymptotic normality.

A practical issue that deserves special attention in the estimation of the PSTR model is the selection of starting values. For the smooth transition model, it is often suggested that sensible starting values can be obtained by means of a grid search across the parameters in the transition function $g(q_{it}; \gamma, c)$. This suggestion is based on the fact that (10) is linear in β when γ and c are fixed. Hence, the concentrated sum of squared residuals (11) can be computed easily for an array ("grid") of values for γ and c such that $\gamma > 0$, and $c_{j,\min} > \min_{i,t}\{q_{it}\}$ and $c_{j,\max} < \max_{i,t}\{q_{it}\}$, $j = 1, \ldots, m$, and the values minimizing $Q^c(\gamma, c)$ can be used as starting values of the nonlinear optimization algorithm. In this paper, we apply simulated annealing instead of a grid search for this purpose. The $(\gamma, c)-$space is then sampled more densely than in the case of a grid search, which improves the quality of the starting values. For practical implementation of simulated annealing, see, for example, Goffe, Ferrier, and Rogers (1994) and Brooks and Morgan (1995).

3.3 Model evaluation

Evaluation of an estimated PSTR model is an essential part of the model building procedure. In this section we consider two misspecification tests for this purpose. Specifically, we adapt the tests of parameter constancy over time and of no remaining nonlinearity developed by Eitrheim and Teräsvirta (1996) for univariate STAR models to fit the present panel framework, where we interpret the latter as a test of no remaining heterogeneity. We do not consider a panel version of their test of no error autocorrelation, because Baltagi and Li (1995) have already derived such a test for panel models. We discuss an alternative use of the test of no remaining heterogeneity as a specification test for determining the number of regimes in the PSTR model.

3.3.1 Testing the hypothesis of parameter constancy

Testing parameter constancy in panel data models has not received as much attention as it has in the time series literature. A possible explanation is that in many applications the time dimension T is relatively small, which makes the assumption of parameter constancy a less interesting hypothesis to test. However, as the number of empirical panel data sets with relatively large T increases testing parameter constancy becomes important. Even though we develop a test specifically for PSTR models, it can after minor modifications be applied to other fixed effects models.

Our alternative to parameter constancy is that the parameters in (1) change smoothly over time. The model under the alternative may be called the Time Varying Panel Smooth Transition Regression (TV-PSTR) model and is defined as follows:

$$y_{it} = \mu_i + (\beta_{10}' x_{it} + \beta_{11}' x_{it} g(q_{it}; \gamma_1, c_1)) + f(t; \gamma_2, c_2)(\beta_{20}' x_{it} + \beta_{21}' x_{it} g(q_{it}; \gamma_1, c_1)) + u_{it} \quad (12)$$

where $g(q_{it}; \gamma_1, c_1)$ is defined in (2) and $f(t; \gamma_2, c_2)$ is another transition function. Model (12) has the same structure as the time-varying smooth transition autoregressive (TV-

STAR) model discussed in Lundbergh, Teräsvirta, and van Dijk (2003). We may also write (12) as

$$y_{it} = \mu_i + (\beta_{10} + \beta_{20} f(t;\gamma_2, c_2))' x_{it} + (\beta_{11} + \beta_{21} f(t;\gamma_2, c_2))' x_{it} g(q_{it};\gamma_1, c_1) + u_{it} \qquad (13)$$

to explicitly show the deterministic character of time-variation in the parameters of the model.

The TV-PSTR model accommodates various alternatives to parameter constancy depending on the definition of $f(t;\gamma_2, c_2)$. This function has the form

$$f(t;\gamma_2, c_2) = \left(1 + \exp\left(-\gamma_2 \prod_{j=1}^{h} (t - c_{2j})\right)\right)^{-1} \qquad (14)$$

where $c_2 = (c_{21}, \ldots, c_{2h})'$ is an h-dimensional vector of location parameters with $c_{21} \leq c_{22} \leq \ldots \leq c_{2h}$, and $\gamma_2 > 0$ is the slope parameter. This is identical to $g(q_{it};\gamma, c)$ as defined in (2) with $q_{it} = t$. Thus, when setting $h = 1$ the TV-PSTR model allows for a single monotonic change, while the change is symmetric around $(c_{21}+c_{22})/2$ in case $h = 2$. The smoothness of the change is controlled by γ_2. When $\gamma_2 \to \infty$, $f(t;\gamma_2, c_2)$ becomes an indicator function $\mathtt{I}[t > c_{21}]$ in case $h = 1$ and $1 - \mathtt{I}[c_{21} < t \leq c_{22}]$ in case $h = 2$. This means that (14) also accommodates instantaneous structural breaks.

When $\gamma_2 = 0$ in (14), the function $f(t;\gamma_2, c_2)$ equals 1/2 for all t, so (12) has constant parameters and $\mathsf{H}_0 : \gamma_2 = 0$ can be chosen to be the null hypothesis of parameter constancy. When it holds, the parameters β_{20}, β_{21} and c_2 in (12) are not identified. The solution to this identification problem is the same as the one proposed in Section 3.1: to replace $f(t;\gamma_2, c_2)$ by its first-order Taylor expansion around $\gamma_2 = 0$. After rearranging terms this yields the following auxiliary regression:

$$\begin{aligned} y_{it} = \mu_i + \beta_{10}^{*\prime} x_{it} + \beta_1^{*\prime} x_{it} t + \beta_2^{*\prime} x_{it} t^2 + \ldots + \beta_h^{*\prime} x_{it} t^h \\ + \left(\beta_{20}^{*\prime} x_{it} + \beta_{h+1}^{*\prime} x_{it} t + \ldots + \beta_{2h}^{*\prime} x_{it} t^h\right) g\left(q_{it};\gamma_1, c_1\right) + u_{it}^* \end{aligned} \qquad (15)$$

where $u_{it}^* = u_{it} + R\left(t, \gamma_2, c_2\right)$ and $R\left(t, \gamma_2, c_2\right)$ is the remainder term. In (15), the parameter vectors β_j^* for $j = 1, 2, \ldots, h, h+1, \ldots, 2h$ are multiples of γ_2, such that the null hypothesis $\mathsf{H}_0 : \gamma_2 = 0$ in (12) can be reformulated as $\mathsf{H}_0^* : \beta_j^* = 0$ for $j = 1, 2, \ldots, h, h+1, \ldots, 2h$ in the auxiliary regression. Under H_0^* $\{u_{it}^*\} = \{u_{it}\}$, so the Taylor series approximation does not affect the asymptotic distribution theory. The χ^2- and F-versions of the LM-type test can be computed as in (6) defining $w_{it}' = (x_{it}', x_{it}' g(q_{it}, \hat{\gamma}_1, \hat{c}_1)) \otimes s_t'$ with $s_t = (t, \ldots, t^h)'$ and replacing $\tilde{X}$ in (7) and (8) by $\tilde{V} = M_\mu V$, where $V = (V_1', \ldots, V_N')'$ with $V_i = (v_{i1}', \ldots, v_{iT}')'$

and $v_{it} = (x'_{it}, x'_{it}g(q_{it}, \hat{\gamma}_1, \hat{c}_1), (\partial\hat{g}/\partial\gamma_1)x'_{it}\hat{\beta}_2, (\partial\hat{g}/\partial c'_1)x'_{it}\hat{\beta}_2)'$. Under the null hypothesis, LM_χ is asymptotically distributed as $\chi^2(2hk)$ and $\mathrm{LM}_\mathrm{F} = \mathrm{LM}_\chi/2hk$ is approximately distributed as $F(2hk, TN-N-2k(h+1)-(m+1))$. When the null model is a homogeneous fixed effects model ($\beta_{11} \equiv \beta_{21} \equiv 0$ in (12)), (15) collapses into a parameter constancy test in this model.

Eitrheim and Teräsvirta (1996) pointed out potential numerical problems in the computation of the test of parameter constancy (as well as the test of no remaining heterogeneity to be discussed below). In particular, when the estimate of γ_1 in the model under the null hypothesis is relatively large, such that the transition between regimes occurs rapidly, the partial derivatives of $g(q_{it}; \gamma_1, c_1)$ with respect to γ_1 and c_1 evaluated at the estimates under the null are equal to zero for almost all observations. As a result, the moment matrix of $\tilde{V}$ becomes near-singular such that the LM test cannot be reliably computed. However, the contribution of the terms involving these partial derivatives to the test statistic is negligible at large values for γ_1. They can simply be omitted from the auxiliary regression without influencing the empirical size (or power) of the test statistic. If this is done, the degrees of freedom in the F-tests have to be modified accordingly. Scale differences between the variables $x_{it}t$, $x_{it}t^2$, …, may also cause numerical problems, but they are overcome simply by standardization.

3.3.2 Testing the hypothesis of no remaining heterogeneity

The assumption that a two-regime PSTR model (1) with (2) adequately captures the heterogeneity in a panel data set can be tested in various ways. In the PSTR framework it is a natural idea to consider an additive PSTR model (3) with $r = 2$, or three regimes, as an alternative. Thus,

$$y_{it} = \mu_i + \beta'_0 x_{it} + \beta'_1 x_{it} g_1(q^{(1)}_{it}; \gamma_1, c_1) + \beta'_2 x_{it} g_2(q^{(2)}_{it}; \gamma_2, c_2) + u_{it} \qquad (16)$$

where the transition variables $q^{(1)}_{it}$ and $q^{(2)}_{it}$ can but need not be the same. The null hypothesis of no remaining heterogeneity in an estimated two-regime PSTR model can be formulated as $\mathsf{H}_0 : \gamma_2 = 0$ in (16). This testing problem is again complicated by the presence of unidentified nuisance parameters under the null hypothesis. As before, the identification problem is circumvented by replacing $g_2(q^{(2)}_{it}; \gamma_2, c_2)$ by a Taylor expansion around $\gamma_2 = 0$. Choosing a first-order Taylor approximation leads to the auxiliary regression

$$y_{it} = \mu_i + \beta^{*\prime}_0 x_{it} + \beta'_1 x_{it} g_1(q^{(1)}_{it}; \hat{\gamma}_1, \hat{c}_1) + \beta^{*\prime}_{21} x_{it} q^{(2)}_{it} + \ldots + \beta^{*\prime}_{2m} x_{it} q^{(2)m}_{it} + u^*_{it} \qquad (17)$$

where $\hat{\gamma}_1$ and $\hat{c}_1$ are estimates under the null hypothesis. The hypothesis of no remaining heterogeneity can then be restated as $\mathsf{H}_0^* : \beta_{21}^* = \ldots = \beta_{2m}^* = 0$. If $\beta_1 \equiv 0$ in (17), the resulting test collapses into the homogeneity test discussed in Section 3.1.

In order to compute the LM test statistic defined in (6) and its F-version we set $w_{it} = (x'_{it}q_{it}^{(2)}, \ldots, x'_{it}q_{it}^{(2)m})'$ and again replace $\tilde{X}$ in (7) and (8) by $\tilde{V}$, where in this case $v_{it} = (x'_{it}, x'_{it}g(q_{it}^{(1)}, \hat{\gamma}, \hat{c}_1), (\partial\hat{g}/\partial\gamma)x'_{it}\hat{\beta}_1, (\partial\hat{g}/\partial c'_1)x'_{it}\hat{\beta}_1)'$. When H_0^* holds, the LM_χ statistic has an asymptotic $\chi^2(mk)$ distribution, whereas LM_F is approximately distributed as $F(mk, TN - N - 2 - k(m+2))$.

3.3.3 Determining the number of regimes

The tests of parameter constancy and of no remaining heterogeneity can be generalized to serve as misspecification tests in an additive PSTR model of the form (3) for any value of r. The purpose of the test of no remaining heterogeneity is actually twofold. It is a misspecification test, but it is also a useful tool for determining the number of transitions in the model. The following sequential procedure may be used for this purpose:

1. Estimate a linear (homogenous) model and test homogeneity at a predetermined significance level α.

2. If homogeneity is rejected, estimate a two-regime PSTR model.

3. Test the hypothesis of no remaining heterogeneity for this model. If it is rejected at significance level $\tau\alpha$, $0 < \tau < 1$, estimate an additive PSTR model with $r = 2$. The purpose of reducing the significance level by a factor τ is to avoid excessively large models.

4. Continue until the first acceptance of the hypothesis of no remaining heterogeneity.

4 Size and power simulations

The small sample properties of the different LM tests developed in Section 3 are studied by means of Monte Carlo experiments. In the simulations we do not only consider different combinations of N and T but also investigate the effect of a particular form of cross-sectional heteroskedasticity on the size and power of the tests.

The design of the Monte Carlo experiments is as follows. The number of replications equals 10,000 throughout. Each experiment is carried out for all possible combinations of $N = 20, 40, 80, 160$ and $T = 5, 10, 20$. The $(2 + r) \times 1$ vector of exogenous regressors and transition variables $\left(x_{it}', q_{it}^{(1)}, \ldots, q_{it}^{(r)}, \right)'$ is generated independently for each individual from the following VAR(1) model:

$$\begin{pmatrix} x_{it} \\ q_{it}^{(1)} \\ \vdots \\ q_{it}^{(r)} \end{pmatrix} = \kappa + \Theta \begin{pmatrix} x_{it-1} \\ q_{it-1}^{(1)} \\ \vdots \\ q_{it-1}^{(r)} \end{pmatrix} + \varepsilon_{it} \tag{18}$$

where $\kappa = (0.2, 0.2, 2.45, \ldots, 2.45)'$ and $\Theta = \text{diag}(0.5, 0.4, 0.3, \ldots, 0.3)$. The error ε_{it} is drawn from a $N(0, \Sigma_\varepsilon)$ distribution where $\Sigma_\varepsilon = DRD$, $D = \sqrt{0.3}I_{2+r}$ and $R = [r_{ij}]$ with $r_{ii} = 1$ and $r_{ij} = 1/3$, $i \neq j$, $i, j = 1, \ldots, 2 + r$. This generates both serial and contemporaneous correlation between the regressors and the transition variables. The endogenous variable y_{it} is generated from the additive PSTR model

$$y_{it} = \mu_i + \beta_{i0}' x_{it} + \sum_{j=1}^{r} \beta_j' x_{it} g(q_{it}^{(j)}; \gamma_j, c_j) + u_{it} \tag{19}$$

where $\mu_i = \sigma_\mu e_i$ with $\sigma_\mu = 10$, and both e_i and u_{it} are i.i.d. standard normal. The values of r, m, and $(\gamma_j, c_j')'$ vary from one experiment to another. We consider two definitions of β_{i0}. In the first case, referred to as homoskedasticity, $\beta_{i0} = \beta_0 = (1, 1)'$ for all individuals i. In the second case, $\beta_{i0} = \beta_0 + \nu_i$, where $\nu_i \sim N(0, I_2)$. This results in heteroskedastic errors in the auxiliary regressions, the degree of heteroskedasticity being positively related to the regressors x_{it}.

Homogeneity test

In order to investigate the empirical size of the homogeneity test as discussed in Section 3.1 we generate samples from a homogenous panel with fixed effects ($r = 0$ in (19)). Results for both the homoskedastic and heteroskedastic cases appear in panels (a) and (b) of Table 1. The table contains rejection frequencies of the null hypothesis for both the standard LM_F test (indicated by ST) and the robust test (HAC) based on the nominal significance level of 5%. We compute the test statistics for $m^* = 1, 2, 3$, where m^* is the order of the auxiliary regression (4). The reason for reporting results for the F-version of the LM test only is that according to previous studies it has better size properties in small samples than the asymptotic χ^2-based statistic.

In the homoskedastic case the empirical size of the standard test is close to the nominal significance level for all sample sizes and choices of m^*. The test can be considerably oversized, however, when the DGP has heteroskedastic errors. This size distortion does not vanish with increasing N and T. For instance, when $N = 160$ and $T = 20$ the empirical size of the standard test still is around 20%. Results are rather different for the robust test statistic. Panel (b) of Table 1 shows that the test remains well-behaved in the presence of heteroskedastic errors. For small values of N and T it can be undersized, but when N increases the empirical size quickly approaches the nominal significance level.

In the power experiments, we generate samples from the PSTR model (19) with $r = 1$ and with either a monotonically increasing ($m = 1$) or symmetric ($m = 2$) transition function (2). In both cases, we set $\beta_1 = (0.7, 0.7)'$. Finally, the parameters in the transition function are set equal to $c_1 = 3.5$ when $m = 1$ and $c_1 = (3.0, 4.0)$ when $m = 2$, and $\gamma_1 = 4$ in both cases. Table 2 displays the empirical power of the tests based on the auxiliary regression (4) with $m^* = 1, 2, 3$. The order m^* may affect the power of the test. For instance, if $m^* < m$, the order of the exponent in (2), this is likely to cause a power loss compared to the choice $m^* = m$. As before, we show results for the homoskedastic and heteroskedastic cases in separate panels.

Several interesting conclusions emerge. First, as a general observation, the power of the test is highest when $m^* = m$, as may be expected. Second, from the results for the DGP with $m = 2$, it is seen that the decline in power compared to the test with $m^* = 2$ is much larger when $m^* = 1$ than it is when $m^* = 3$. Similarly, the power of the test with $m^* = 2$ or 3 for the DGP with $m = 1$ remains reasonable. Hence, it seems advisable not to be conservative when choosing the maximum order of the auxiliary regression (4). Third, for the homoskedastic DGP the standard test outperforms the robust test for small panels, although the difference in power quite rapidly becomes small as N and T increase. The results for the heteroskedastic DGP are not comparable because of the positive size distortion of the standard test. Fourth, the power of the robust test is lower in the presence of heteroskedasticity than without it. Fifth, the robust test has low power in small panels ($N = 20$ or $T = 5$), in particular when $m^* > 1$. This is due to the negative size distortion of this test that is visible in Table 1. The power improves drastically, however, when the number of individuals or time periods in the panel increases.

Parameter constancy test

In order to gauge the size properties of the parameter constancy test we generate samples from the PSTR model (19) with constant parameters, setting $r = 1$, $\beta_1 = (1, 1)$, $m = 1$, $\gamma_1 = 3$ and $c_1 = 3.5$. Table 3 contains the results for the $\mathrm{LM_F}$ test based on the auxiliary regression (15), where we set the maximum power of t, denoted by h^*, equal to 1,2 or 3. The results closely correspond with those obtained before for the homogeneity test. When the errors are homoskedastic, the standard F-test has size close to the nominal significance level. The robust tests are undersized in small panels, in particular for $h^* = 2$ and 3, although this effect already becomes quite small when $NT > 400$. With heteroskedastic errors, the standard test is oversized, and, as may be expected, this size distortion increases with increasing T (and N). The robust test remains generally well-behaved, although it is somewhat undersized even here. In fact, its empirical size as a function of N and T closely resembles the results obtained with homoskedastic errors.

In the power simulations we consider two TV-PSTR models of the type (12). In the first model, we allow a single permanent structural change in the coefficients centered around the middle of the sample by setting $h = 1$ and $c_2 = 0.5T$. The second model contains a 'temporary' structural change which is obtained by setting $h = 2$, $c_2 = (0.3T, 0.7T)$. In both cases $\gamma_2 = 4$ in (14), $(\beta_{10}', \beta_{11}')' = (1, 1, 1, 1)$ and $(\beta_{20}', \beta_{21}')' = 0.7(\beta_{10}', \beta_{11}')'$. Hence, we assume that the transition occurs at the same time for all N individuals and that the change in the parameters is the same for all of them. This design is consistent with the alternative hypothesis although it excludes other interesting options such as the situation that only a certain fraction of individuals experiences the change in parameters. If this is the case, the power of the test may be affected and, in particular, it may have very low power if this fraction is small.

The results in Table 4 largely correspond with those obtained for the homogeneity test. The conclusions from Table 2 are equally valid here. In addition, the parameter constancy test has much higher power against permanent structural change ($h = 1$) than against temporary change ($h = 2$). This observation does not generalize, however, because the power of the parameter constancy test crucially depends on both the timing and the magnitude of the structural change.

Test of no remaining heterogeneity

We examine the size properties of the test of no remaining heterogeneity by generating panels from (19) setting $r = 1$, $m = 1$, $\gamma_1 = 4$, $c_1 = 3.5$ and $(\beta_0', \beta_1')' = (1, 1, 1, 1)'$. We apply the $\mathrm{LM_F}$ test based on (17) with $m^* = 1, 2, 3$ using either the original transition variable $q_{it}^{(1)}$ or another variable $q_{it}^{(2)}$ in the second transition function. The results appear in Table 5. Even here, the robust test is somewhat undersized in small samples, independent of the presence of heteroskedasticity. Heteroskedasticity again causes positive size distortion in the standard test, which increases with the cross-section and time dimensions of the panel. Note that the size distortion occurs for both choices of the second transition variable. Thus, in large panels, the standard test may quite likely indicate the presence of (additional) heterogeneity because of the presence of heteroskedasticity. This suggests caution when applying the test for determining the number of transitions in the multiple PSTR model.

We consider the power properties of the test of no remaining heterogeneity under different circumstances. First, we generate panels from (19) with $r = 2$, $q_{it}^{(1)} = q_{it}^{(2)}$, $m_1 = m_2 = 1$, $\gamma_1 = \gamma_2 = 8$, $c_1 = 3$, and $c_2 = 4$. The regression coefficients are set equal to $\beta_0 = (1, 1)'$, $\beta_1 = (0.7, 0.7)$, and $\beta_2 = \beta_1$ or $\beta_2 = -\beta_1$. In the first case, the heterogeneity is monotonic in $q_{it}^{(1)}$, in the sense that the effective regression coefficients are monotonically increasing functions of the transition variable as they change from β_0 to $\beta_0 + \beta_1$ to $\beta_0 + 2\beta_1$ as q_{it} increases. In the second case, the coefficients in the lower regime ($q_{it} \ll c_1$) and in the upper regime ($q_{it} \gg c_2$) are the same. In both cases, we estimate a PSTR model with $r = 1$ and $m = 1$ and then apply the test of no remaining heterogeneity using the correct transition variable. Note that the above DGP resembles a PSTR model with $r = 1$ and $m = 2$. This in fact is the second DGP we consider, with all parameters defined as before. Also in this case, we estimate a PSTR model with $r = 1$ and $m = 1$, and examine whether the test of no remaining heterogeneity is able to detect the misspecification of the form of the heterogeneity (that is, of the order of the logistic function). Third, we employ a PSTR model (19) specified as in the first DGP above with $\beta_2 = \beta_1$, but with $q_{it}^{(1)} \neq q_{it}^{(2)}$. For these panels, we estimate a PSTR model with $r = 1$ and $m = 1$ using $q_{it}^{(1)}$ as transition variable and consider whether the test can detect the remaining heterogeneity that is a function of $q_{it}^{(2)}$.

Results for the first DGP are shown in Table 6. Most results obtained for the homogeneity test continue to hold for the test of no remaining heterogeneity. In addition, we observe that monotonic heterogeneity ($\beta_2 = \beta_1$) is more difficult to detect than non-monotonic parameter variation ($\beta_2 = -\beta_1$). This is probably due to the fact that in the first case a PSTR model with $r = 1$ and $m = 1$ can provide a reasonable approximation to the true form of the heterogeneity, whereas in the second case it cannot. The results for the second DGP, in columns 3-8 of Table 7, suggest that misspecification of the logistic transition function is picked up quite well by the test of no remaining heterogeneity. The remaining columns of Table 7 contain the results for the third DGP. They indicate that the test has no problem in detecting remaining heterogeneity when it is a function of another variable $q_{it}^{(2)}$.

5 Investment and capital market imperfections

In the presence of capital market imperfections, firms' investment decisions are not independent of financial factors such as cash flow and leverage. First, asymmetric information between borrowers and lenders concerning the quality of available investment opportunities generates agency costs that result in outside investors demanding a premium on newly issued debt or equity. This creates a 'pecking order' or 'financing hierarchy' with internal funds having a cost advantage relative to external capital. Hence, investment will be positively related to the availability of internal sources of finance, as measured by cash flow, for example. Second, high leverage reduces firms' ability to finance growth through a liquidity effect, such that firms with valuable investment opportunities should choose lower leverage. Therefore, one may expect a negative relationship between future investment and leverage or 'debt overhang'.

The impact of these capital market imperfections and the severity of the resulting problems varies across firms and over time, depending on the degree of informational asymmetry and growth opportunities, among others. For firms with low information costs or ample growth opportunities, internal and external finance are (close to) perfect substitutes and investment decisions are (close to) independent of their financial structure. In contrast, firms with high information costs and limited growth opportunities face much higher costs of external finance or may even be rationed in their access to external funds, resulting in greater sensitivity of investment to cash flow. Similarly, capital structure theory suggests a disciplinary role of debt, in the sense that leverage restricts managers of firms with poor growth opportunities from investing when they should not. Thus, leverage should mainly affect such firms and have much less effect on investment for firms with valuable growth opportunities that are recognized by the market.

A substantial number of empirical studies examine the effects of capital market imperfections on investment, see Fazzari, Hubbard, and Petersen (1988), Whited (1992), Bond and Meghir (1994), Carpenter, Fazzari, and Petersen (1994), Gilchrist and Himmelberg (1995), Lang, Ofek, and Stulz (1996), Hsiao and Tahmiscioglu (1997), Hu and Schiantarelli (1998), Moyen (2004), and references cited therein. Schiantarelli (1996) and Hubbard (1998) provide surveys of this literature. Most studies are conducted in the context of the Q theory of investment, adding measures of cash flow or leverage to empirical models that relate investment to Tobin's Q. In perfect capital and output markets, Tobin's Q, defined as the market valuation of capital relative to its replacement value, is a sufficient statistic for investment. A significant positive coefficient on cash flow, for example, then can be interpreted as evidence in favour of the relevance of financing constraints.

To examine whether the effects of financing constraints or other capital market imperfections depend on financial factors, firms are typically divided into groups of 'constrained' and 'unconstrained' firms based on a variable that measures the degree of information asymmetry such as the dividend pay-out ratio, size, age, the presence of a bond rating, and the debt ratio, or based on a variable that measures growth opportunities such as Tobin's Q. This approach potentially suffers from several limitations. First, the distinction between 'constrained' and 'unconstrained' firms is often based on an arbitrary threshold level of the variable that is used to split the sample. Second, in most studies, the composition of these groups is fixed for the complete sample period, in the sense that firms are not allowed to switch groups over time. In this section, we apply the PSTR model to alleviate these shortcomings.

Following Hansen (1999), we use a balanced panel of 565 US firms observed for the years 1973–1987, extracted from the data set used by Hall and Hall (1993). For each firm i and year t, we obtain the ratios of investment to assets (I_{it}), Tobin's Q or total market value to assets (Q_{it}), long-term debt to assets (D_{it}), cash flow to assets (CF_{it}) and sales to assets (S_{it}). We delete five firms from the original sample because they have aberrant values for some of these variables.

We begin modelling the firms' investment behaviour by estimating a homogenous panel data model for I_{it} with lagged Q, sales, debt and cash-flow as regressors. The lagged sales to assets ratio can be interpreted as a proxy for future demand for a firm's output and therefore is included as an additional control for a firm's future profit opportunities, following Hsiao and Tahmiscioglu (1997) and Hu and Schiantarelli (1998). In addition, we include a set of year dummies to capture macroeconomic effects on investment. We then apply the LM test of homogeneity developed in Section 3.1, using lagged Q and lagged debt as transition variables, again following Hu and Schiantarelli (1998). We only test homogeneity of the coefficients of lagged Q, sales, debt and cash-flow, assuming that the macroeconomic effects on investment do not differ across firms. Restricting coefficients of some variables to be constant in the PSTR model has no effect on the distribution theory. Results for the F-version of the standard and robust tests for m=1,2, and 3 are shown in Table 9. Homogeneity is rejected for both choices of the transition variable, although the p-values of the tests with Tobin's Q are considerably smaller. Next we apply the sequence of tests discussed at the end of Section 3.1 to determine the order m of the logistic function. The results of the specification test sequence, shown in Table 10, point at $m = 1$ as the strongest rejection does not occur for H_{02}^*. This is the case for both the standard and robust tests when Tobin's Q is used as transition variable. Thus we proceed with estimating the following PSTR model:

$$
\begin{aligned}
I_{it} = \mu_i + \delta' d_t + \beta_{01} Q_{it-1} + \beta_{02} S_{it-1} + \beta_{03} D_{it-1} + \beta_{04} CF_{it-1} \\
+ \left(\beta_{11} Q_{it-1} + \beta_{12} S_{it-1} + \beta_{13} D_{it-1} + \beta_{14} CF_{it-1}\right) g(Q_{it-1}; \gamma, c) + u_{it} \quad (20)
\end{aligned}
$$

where d_t denotes the vector of year dummies, and

$$
g(Q_{it-1}; \gamma, c) = \left(1 + \exp\left(-\gamma(Q_{it-1} - c)\right)\right)^{-1}, \quad \text{with } \gamma > 0. \quad (21)
$$

Before discussing the estimation results in detail, we examine the adequacy of the two-regime PSTR model by applying the misspecification tests of parameter constancy and of no remaining heterogeneity. The results in Table 11 suggest that according to the standard tests, the model does not completely capture the heterogeneity in regression coefficients across firms, while some indication of time-variation in the parameters is found as well. In contrast, based on the robust test no evidence whatsoever is found for remaining heterogeneity, while parameter constancy cannot be rejected either. Given the simulation evidence presented in Section 4, the small p-values of the standard tests are likely caused by neglecting cross-sectional heteroskedasticity, which renders the tests unreliable. Hence, based on the robust test we conclude that the two-regime model is adequate.

Parameter estimates appear in Tables 12 and 13, together with conventional standard errors and heteroskedasticity-consistent standard errors. To facilitate interpretation, in Table 12 we report estimates of β_{0j} and $\beta_{0j} + \beta_{1j}$, for $j = 1, \ldots, 4$, corresponding to regression coefficients in the regimes associated with $g(Q_{it-1}; \gamma, c) = 0$ and 1, respectively. The estimates of γ and c are such that the transition from the lower regime associated with small values of Tobin's Q to the upper regime with large values of Q is smooth but relatively rapid. This is seen from Figure 1, in which the transition function is plotted against Tobin's Q with each circle representing an observation. A clear majority of observations lie in either one of the extreme regimes, but there is also a number of them located in-between.

Table 14 provides some rough insight into the distribution of firms across regimes: firm i is assigned into the low (high) regime in year t when $g(Q_{it-1}; \gamma, c) < (>) 0.5$. For all years, a large majority is classified into the low regime. Together with the point estimate $\hat{c} = 1.51$ this shows that the model identifies firms with excellent growth opportunities, signalled by their rather high Q values, as a separate group that is distinct from firms with bad or moderate growth opportunities. Also shown in Table 14 are the percentage of firms moving from one regime to the other in each year. On average almost 10% of all firms switches regimes in a given year, clearly illustrating the relevance of not constraining firms to remain in the same group over time.

Turning to the estimated regression coefficients, it is seen that the estimate of the coefficient on lagged debt is negative and significant for low Q firms, while it is insignificantly different from zero for high Q firms. This is consistent with the findings of Lang, Ofek, and Stulz (1996) that leverage matters for investment only for firms with poor growth opportunities or firms with growth opportunities that are not recognized by the market. The coefficient on lagged cash flow is positive and significant for both groups of firms, although it is considerably smaller for high Q firms. This corroborates previous findings that internal finance is relevant for investment mainly for financially constrained firms. We also find that the coefficient on Tobin's Q is positive and significant for low Q firms and positive but much smaller for high Q firms. Hence, firms with poor growth prospects respond more strongly to changes in their investment opportunities than the other companies. This goes against the results of Hu and Schiantarelli (1998), who document the opposite pattern, but then their regime classification is based on multiple indicators including variables that measure the degree of information asymmetry and financial constraints. Our findings, however, are in line with the results of Barnett and Sakellaris (1998), who report evidence for a nonlinear relationship between investment and Tobin's Q similar to our findings. Theoretically this can be explained by the presence of fixed costs and (partial) irreversibility of investment, see also Nilsen and Schiantarelli (2003).

Table 13 shows the estimates of the coefficients of the year dummies in the PSTR model. These are to be interpreted relative to a value of zero for 1974, the first year in the effective sample period. It is seen that there remains some variation in investment over time beyond what is explained by the included regressors. In particular, the coefficient estimates strongly suggest the presence of macroeconomic effects, as the average level of investment closely follows the business cycle and growth cycle, being lower in 1975-1977, in 1982-1983, and in 1987 than during other years.

Finally, we acknowledge that our analysis is subject to caveats, including the possibility that cash flow and leverage contain useful information about growth opportunities not captured by Tobin's Q and the possibility of measurement error in Q. Both of these may lead to spurious effects of cash flow and leverage on investment, as discussed at length in Gilchrist and Himmelberg (1995), Erickson and Whited (2000), Gomes (2001), and

Hennessy (2004), among others. A thorough analysis of these issues, however, is beyond the scope of this paper.

6 Conclusions

In this paper we have developed the panel smooth transition regression model, which incorporates heterogeneity by allowing regression coefficients to vary as a function of an exogenous variable and fluctuate between a limited number (often two) of "extreme regimes". As the transition variable is individual-specific and time-varying, the regression coefficients for each of the individuals in the panel are changing over time. The model is a generalization of the panel threshold regression model of Hansen (1999) in the sense that our new model allows coefficients to change smoothly when moving from one regime to another. Our approach includes a modelling cycle for the PSTR model, containing tests of homogeneity, of parameter constancy and of no remaining nonlinearity. Monte Carlo experiments demonstrate that these statistics behave satisfactorily even in panels with small N and T, although the standard tests should be applied with caution given that they are affected considerably by cross-sectional heteroskedasticity. An application to firms' investment behaviour aptly demonstrates the usefulness of the model.

The PSTR model as considered in this paper has fixed effects and exogenous regressors. Obviously, models with random effects and with lagged dependent variables are interesting alternatives. In addition, a model allowing for multiple variables entering the transition function might be relevant in practice and hence worthwhile considering. In Hu and Schiantarelli (1998), for example, several factors including Q, firm size and leverage jointly determine the classification of firms into regimes with different characteristics of investment behavior. Investigation of these extensions of the model are left for future research.

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Table 1: Empirical size of homogeneity test

		$T = 5$		$T = 10$		$T = 20$	
N	m^*	ST	HAC	ST	HAC	ST	HAC
Panel (a): Homoskedasticity							
20	1	5.3	3.2	5.2	3.5	4.7	3.6
	2	5.2	1.1	5.3	1.6	4.8	1.9
	3	5.4	0.3	5.1	0.6	5.3	0.8
40	1	5.4	4.8	5.0	4.1	4.8	4.5
	2	5.3	2.5	4.9	2.8	4.8	3.4
	3	5.2	1.7	4.9	1.7	4.9	2.8
80	1	5.4	4.6	5.1	4.7	4.7	4.7
	2	5.2	3.6	5.3	4.0	5.2	4.0
	3	5.4	2.7	5.0	3.2	4.9	3.4
160	1	4.6	4.5	5.2	4.9	5.1	5.0
	2	5.0	3.8	4.6	4.1	5.2	4.9
	3	5.0	3.3	5.0	3.6	5.7	4.5
Panel (b): Heteroskedasticity							
20	1	9.7	3.1	12.1	3.3	14.8	3.7
	2	10.5	1.0	12.8	1.8	16.7	1.9
	3	10.3	0.3	13.5	0.5	18.3	0.9
40	1	9.8	3.9	12.3	4.0	15.2	4.4
	2	10.7	2.5	13.8	2.9	18.0	3.3
	3	11.0	1.6	14.9	2.1	19.8	2.3
80	1	10.2	4.7	12.9	4.6	14.8	4.4
	2	11.3	3.6	15.6	3.9	19.0	3.9
	3	12.4	2.3	16.9	3.0	21.0	3.3
160	1	10.4	4.9	13.2	4.7	15.7	4.9
	2	11.6	3.9	15.4	3.8	19.7	4.7
	3	12.8	3.2	17.1	3.4	22.4	4.1

Note: Rejection frequencies of the standard (ST) and robust (HAC) versions of the LM_F test of homogeneity based on (4) at 5% nominal significance level. Panels are generated according to the model (18) and (19) with $r = 0$ and homoskedastic (panel (a)) or heteroskedastic errors (panel (b)). The table is based on 10,000 replications.

Table 2: Empirical power of the homogeneity test

		$m=1$						$m=2$					
		$T=5$		$T=10$		$T=20$		$T=5$		$T=10$		$T=20$	
	m^*	ST	HAC	ST	HAC	ST	HAC	ST	HAC	ST	HAC	ST	HAC
Panel (a): Homoskedasticity													
20	1	12.8	7.4	26.2	16.1	51.5	37.1	10.1	4.0	14.3	6.1	23.8	10.7
	2	10.6	1.9	20.2	5.9	41.3	17.6	24.3	2.9	51.5	12.3	86.2	40.6
	3	9.5	0.4	17.6	2.2	37.6	6.8	21.5	0.7	48.0	4.3	83.9	18.7
40	1	23.3	17.7	48.0	41.0	82.7	76.9	13.3	7.1	22.3	12.3	39.1	24.4
	2	17.9	8.7	37.9	25.0	73.7	60.2	46.1	19.1	83.6	58.4	99.3	94.6
	3	15.8	5.5	34.3	16.8	70.2	49.0	42.1	12.0	80.5	46.6	99.1	90.6
80	1	41.9	38.0	79.6	76.7	98.9	98.6	19.7	12.2	37.5	25.4	66.3	51.2
	2	33.2	25.4	70.1	62.8	97.8	95.8	77.2	59.9	98.9	96.3	100.0	100.0
	3	29.7	19.2	66.7	54.8	96.6	93.3	74.0	51.9	98.6	95.2	100.0	100.0
160	1	73.2	71.2	98.0	97.7	100.0	100.0	32.3	23.3	62.9	50.4	91.4	85.0
	2	62.0	57.8	95.9	94.9	100.0	100.0	97.7	95.1	100.0	100.0	100.0	100.0
	3	58.0	50.5	95.3	93.1	100.0	100.0	97.3	93.8	100.0	100.0	100.0	100.0
Panel (b): Heteroskedasticity													
20	1	15.2	5.2	24.7	9.3	39.8	16.5	12.9	3.8	17.7	4.8	25.1	6.9
	2	14.6	1.4	23.3	3.2	38.0	7.7	22.5	1.8	40.9	6.4	67.3	18.6
	3	13.7	0.4	23.0	1.1	37.8	3.0	20.8	0.5	40.6	2.3	66.8	7.6
40	1	21.9	10.5	38.1	19.1	59.3	35.1	14.8	5.6	23.2	7.8	33.5	12.6
	2	19.5	5.5	34.2	10.7	56.0	22.9	37.4	10.5	65.1	28.6	89.3	60.8
	3	18.7	3.5	33.1	7.5	56.3	17.6	35.7	6.5	63.0	20.7	89.2	51.5
80	1	33.4	20.1	58.0	38.0	84.0	64.8	19.8	8.8	31.7	13.6	49.3	23.5
	2	29.5	13.1	53.8	27.6	80.7	53.3	60.5	34.1	88.6	67.2	99.1	93.6
	3	28.1	9.6	52.5	22.0	80.7	47.1	58.1	27.1	88.7	62.4	99.3	92.1
160	1	54.4	38.9	83.0	67.0	97.8	92.3	28.2	14.3	47.8	26.0	71.6	45.4
	2	48.1	29.2	79.1	56.1	96.8	87.0	87.8	71.5	99.3	95.4	100.0	99.9
	3	46.8	24.9	78.8	51.6	97.1	85.2	86.7	67.2	99.2	95.0	100.0	99.9

Note: Rejection frequencies of the standard (ST) and robust (HAC) versions of the LM_F test of homogeneity based on (4) at 5% nominal significance level. Panels are generated according to the model (18) and (19) with $r=1$ and homoskedastic (panel (a)) or heteroskedastic errors (panel (b)). The table is based on 10,000 replications.

Table 3: Empirical size of test of parameter constancy

		$T=5$		$T=10$		$T=20$	
N	h	ST	HAC	ST	HAC	ST	HAC
Panel (a): Homoskedasticity							
20	1	4.5	1.4	3.8	2.0	4.9	2.3
	2	4.2	0.0	3.9	0.2	4.4	0.3
	3	4.2	0.0	4.0	0.0	4.5	0.0
40	1	4.3	3.2	4.5	3.6	4.9	4.4
	2	4.1	1.6	4.3	2.5	4.5	2.5
	3	3.4	0.6	4.1	0.8	4.4	1.5
80	1	4.2	3.9	4.5	4.2	4.8	4.6
	2	4.1	2.9	4.8	3.9	4.8	4.0
	3	4.1	2.8	4.7	2.9	5.2	3.4
160	1	4.8	4.8	4.8	4.6	5.3	5.5
	2	5.1	4.1	4.8	4.5	5.1	4.8
	3	5.0	4.1	4.5	3.8	5.3	4.5
Panel (b): Heteroskedasticity							
20	1	7.4	1.6	11.2	2.2	14.1	2.2
	2	7.1	0.1	11.8	0.2	16.8	0.3
	3	6.2	0.0	12.4	0.0	19.2	0.0
40	1	8.1	3.2	13.1	3.6	16.3	3.8
	2	7.7	1.8	14.0	2.1	20.3	2.5
	3	7.0	0.7	15.3	1.1	22.7	1.2
80	1	8.9	4.3	13.3	4.9	15.8	4.0
	2	9.1	3.3	15.5	3.7	20.0	3.6
	3	8.9	2.5	16.3	2.8	23.8	3.2
160	1	10.2	5.0	13.4	4.4	16.5	5.2
	2	10.4	4.7	16.5	4.2	20.5	4.4
	3	9.9	4.1	17.6	3.9	24.4	4.4

Note: Rejection frequencies of the standard (ST) and robust (HAC) versions of the LM_F test of parameter constancy based on (15) at 5% nominal significance level. Panels are generated according to the model (18) and (19) with $r = 1$ and homoskedastic (panel (a)) or heteroskedastic errors (panel (b)). The table is based on 10,000 replications.

Table 4: Empirical power of the test of parameter constancy

		$h=1$						$h=2$					
		$T=5$		$T=10$		$T=20$		$T=5$		$T=10$		$T=20$	
	h	ST	HAC	ST	HAC	ST	HAC	ST	HAC	ST	HAC	ST	HAC
Panel (a): Homoskedasticity													
20	1	59.3	24.5	87.8	63.7	98.6	93.1	16.0	6.0	19.5	10.5	24.6	13.6
	2	54.4	0.6	86.2	8.9	98.4	38.8	19.3	0.3	32.7	1.8	49.0	6.8
	3	49.9	0.0	84.3	0.0	98.3	0.0	20.3	0.0	33.0	0.0	51.1	0.0
40	1	83.1	75.0	95.9	93.6	99.7	99.5	27.5	23.6	32.8	28.4	36.9	31.2
	2	82.9	56.8	96.3	89.0	99.8	98.9	38.6	21.8	56.1	38.8	69.9	56.3
	3	82.3	31.0	96.4	75.9	99.8	97.4	40.4	11.5	59.1	28.0	74.8	48.4
80	1	92.3	91.1	98.8	98.4	100.0	99.9	43.6	42.0	48.3	46.8	50.6	48.7
	2	93.7	90.8	99.0	98.4	100.0	100.0	62.2	56.0	73.9	70.6	83.5	81.2
	3	94.9	89.2	99.2	98.2	100.0	99.9	67.2	53.6	79.2	72.7	88.5	84.6
160	1	96.3	96.2	99.2	99.1	100.0	100.0	58.2	58.2	62.7	62.1	63.0	62.1
	2	98.1	97.8	99.6	99.6	100.0	100.0	79.4	77.4	87.5	87.3	91.8	91.6
	3	98.7	98.3	99.8	99.8	100.0	100.0	85.2	82.2	92.1	91.3	95.4	95.2
Panel (b): Heteroskedasticity													
20	1	49.1	14.2	78.9	40.5	94.8	74.4	14.7	3.4	19.5	4.9	23.6	5.3
	2	42.0	0.2	75.6	3.2	94.8	16.2	17.4	0.1	29.1	0.5	43.9	1.5
	3	36.6	0.0	72.1	0.0	94.7	0.0	15.9	0.0	29.1	0.0	46.6	0.0
40	1	75.5	60.2	92.3	83.2	98.9	96.0	24.9	14.3	29.9	15.0	33.2	14.5
	2	73.3	36.8	92.6	68.9	99.0	92.6	31.5	10.5	46.3	17.8	62.1	26.8
	3	70.7	16.0	92.4	47.4	98.9	82.9	31.2	4.0	49.5	9.9	67.4	16.2
80	1	90.2	85.5	97.3	94.8	99.8	99.1	36.2	27.6	40.8	27.1	45.5	27.4
	2	90.2	82.2	97.9	94.7	99.9	99.2	50.9	35.8	65.6	45.0	77.3	57.6
	3	90.7	77.0	98.1	93.8	99.9	98.9	53.3	31.1	71.0	42.9	83.2	56.7
160	1	95.1	92.9	98.8	98.0	99.9	99.8	50.8	43.4	55.1	43.4	57.6	42.7
	2	96.6	95.0	99.3	98.4	100.0	99.9	70.8	62.2	81.5	71.3	87.8	78.1
	3	96.9	95.2	99.6	99.0	100.0	99.9	75.5	64.3	86.3	75.0	91.8	81.6

Note: Rejection frequencies of the standard (ST) and robust (HAC) versions of the LM_F test of parameter constancy based on (15) at 5% nominal significance level. Panels are generated according to the model (18) and (12) with homoskedastic (panel (a)) or heteroskedastic errors (panel (b)). The table is based on 10,000 replications.

Table 5: Empirical size of the test of no remaining heterogeneity

		Same transition variable						Different transition variable					
		$T=5$		$T=10$		$T=20$		$T=5$		$T=10$		$T=20$	
	h	ST	HAC	ST	HAC	ST	HAC	ST	HAC	ST	HAC	ST	HAC
Panel (a): Homoskedasticity													
40	1	3.9	3.2	4.7	3.8	4.8	4.2	5.4	4.5	4.5	4.3	5.4	4.3
	2	4.1	2.0	4.3	2.4	4.7	2.7	5.3	2.8	4.8	3.3	4.9	3.2
	3	4.0	1.4	4.5	1.5	4.9	1.8	5.4	1.9	5.3	2.3	4.9	2.4
80	1	4.5	3.6	5.0	4.6	4.9	4.4	5.1	4.9	4.9	4.6	4.9	4.5
	2	4.3	2.8	5.2	3.8	4.7	3.3	5.3	4.0	5.4	4.2	4.8	3.8
	3	4.5	2.2	5.0	3.0	4.8	2.9	5.0	3.0	5.9	3.4	4.6	3.3
160	1	4.6	4.0	5.0	4.6	5.1	5.1	4.7	4.8	5.0	5.0	4.8	4.6
	2	5.0	3.8	4.5	3.7	4.8	4.0	5.0	4.2	4.7	4.4	5.1	4.7
	3	5.1	3.5	4.7	3.2	5.0	3.3	4.7	3.8	4.9	3.8	5.1	4.1
Panel (b): Heteroskedasticity													
40	1	4.6	2.6	8.5	3.2	11.0	3.8	10.4	4.7	14.0	4.0	16.5	4.6
	2	6.5	1.7	13.1	2.3	18.1	2.2	11.7	3.1	16.3	2.9	20.6	3.9
	3	7.2	1.1	14.6	2.0	19.9	2.1	11.9	1.7	17.4	1.9	22.9	2.5
80	1	7.9	3.8	11.1	4.4	12.8	4.5	10.4	4.9	14.0	4.8	17.9	5.3
	2	10.4	2.9	16.9	3.1	22.7	3.5	12.5	3.3	18.4	4.1	21.2	3.7
	3	11.9	2.0	18.4	2.6	23.4	2.8	13.0	2.4	20.3	2.7	25.3	3.1
160	1	9.2	4.1	11.5	5.1	13.0	4.9	11.2	4.6	14.6	4.5	17.0	4.8
	2	14.2	3.0	19.3	3.6	25.1	4.4	13.6	4.1	18.9	4.7	22.3	4.2
	3	14.8	2.9	20.6	3.5	26.8	4.1	15.2	3.4	21.6	4.2	25.3&3.7	

hline

Note: Rejection frequencies of the standard (ST) and robust (HAC) versions of the LM_F test of no remaining heterogeneity based on (17) at 5% nominal significance level. Panels are generated according to the model (18) and (19) with homoskedastic (panel (a)) or heteroskedastic errors (panel (b)). The table is based on 10,000 replications.

Table 6: Empirical power of the test of no remaining heterogeneity

		$\beta_2 = \beta_1$						$\beta_2 = -\beta_1$					
		$T = 5$		$T = 10$		$T = 20$		$T = 5$		$T = 10$		$T = 20$	
	h	ST	HAC	ST	HAC	ST	HAC	ST	HAC	ST	HAC	ST	HAC
Panel (a): Homoskedasticity													
20	1	4.1	2.3	6.5	4.2	12.5	7.4	12.4	6.2	29.4	16.9	67.7	49.3
	2	4.5	0.9	8.7	2.8	15.9	5.2	10.5	2.3	26.4	7.4	66.4	28.6
	3	4.2	0.3	8.2	0.9	16.5	2.5	8.2	0.5	22.4	2.8	60.8	11.5
40	1	6.1	4.1	11.4	8.3	20.0	15.2	26.0	19.6	63.5	53.6	95.6	92.4
	2	8.0	3.7	14.0	8.4	26.9	17.1	22.8	11.5	62.4	41.1	97.3	90.1
	3	7.2	2.3	14.8	6.8	29.4	15.3	19.4	6.1	56.9	27.6	96.0	79.4
80	1	11.3	8.5	18.9	16.2	36.0	33.1	57.2	49.7	95.0	92.5	100.0	100.0
	2	14.0	9.4	24.7	18.7	45.6	39.8	54.5	42.2	96.8	92.5	100.0	100.0
	3	14.2	8.1	27.2	19.2	55.3	46.4	48.7	30.9	95.3	86.4	100.0	100.0
160	1	17.4	14.1	33.2	32.4	61.9	61.4	91.6	88.9	99.9	99.9	100.0	100.0
	2	22.5	18.4	43.4	39.9	74.2	71.8	93.6	88.7	100.0	100.0	100.0	100.0
	3	25.5	19.1	52.8	47.6	87.2	85.2	91.2	83.9	100.0	100.0	100.0	100.0
Panel (b): Heteroskedasticity													
20	1	5.2	2.5	7.4	3.1	11.8	3.4	10.2	4.3	19.2	7.7	32.9	12.5
	2	5.7	0.9	10.1	1.8	15.9	2.2	9.1	1.4	18.5	3.3	33.0	6.3
	3	5.9	0.3	9.5	0.8	17.5	1.1	8.3	0.4	17.3	1.3	31.4	2.3
40	1	6.4	3.1	11.1	4.7	18.4	6.7	16.0	8.5	32.5	16.8	57.9	32.1
	2	7.8	2.1	14.5	3.7	24.3	6.5	15.3	4.7	32.8	11.1	59.8	24.6
	3	8.0	1.3	15.4	2.3	27.1	4.8	13.1	2.5	31.1	7.5	57.6	16.8
80	1	9.8	4.0	16.9	7.4	25.0	11.0	30.3	19.1	58.9	39.0	87.3	69.0
	2	12.3	4.1	22.5	7.6	33.8	12.4	30.2	14.2	60.3	32.6	90.1	65.6
	3	12.7	3.3	24.6	6.2	40.0	12.9	27.6	9.6	57.0	25.6	88.7	57.5
160	1	16.1	8.3	23.0	12.3	34.7	19.0	55.2	40.9	88.9	76.3	99.5	96.3
	2	20.4	8.5	31.6	14.0	47.9	23.2	56.5	36.9	92.0	76.1	99.7	97.6
	3	23.0	8.0	37.3	15.0	57.8	26.9	52.6	29.7	90.5	69.5	99.8	96.5

Note: Rejection frequencies of the standard (ST) and robust (HAC) versions of the $\mathrm{LM_F}$ test of no remaining heterogeneity based on (17) at 5% nominal significance level. Panels are generated according to the model (18) and (19) with $r = 2$, $q_{it}^{(1)} = q_{it}^{(2)}$, $m_1 = m_2 = 1$ and homoskedastic (panel (a)) or heteroskedastic errors (panel (b)). The table is based on 10,000 replications.

Table 7: Empirical power of the test of no remaining heterogeneity

		Misspecified m						Different transition variable					
		$T = 5$		$T = 10$		$T = 20$		$T = 5$		$T = 10$		$T = 20$	
	h	ST	HAC	ST	HAC	ST	HAC	ST	HAC	ST	HAC	ST	HAC
Panel (a): Homoskedasticity													
20	1	20.6	9.4	52.7	30.4	89.8	73.3	59.2	39.2	95.0	85.5	100.0	99.7
	2	17.0	2.8	49.9	13.0	91.0	48.5	49.3	13.2	91.5	56.1	100.0	95.7
	3	13.8	0.6	44.9	4.1	89.5	20.4	46.3	3.7	91.4	26.4	100.0	79.1
40	1	44.7	32.0	88.2	77.9	99.8	99.1	90.7	85.2	100.0	99.9	100.0	100.0
	2	41.0	18.9	89.7	67.3	100.0	99.0	85.0	68.5	99.8	99.3	100.0	100.0
	3	36.3	11.0	87.4	50.8	100.0	95.7	85.3	55.5	99.9	97.8	100.0	100.0
80	1	83.6	74.8	99.8	99.4	100.0	100.0	99.9	99.7	100.0	100.0	100.0	100.0
	2	84.1	67.2	100.0	99.4	100.0	100.0	99.6	99.2	100.0	100.0	100.0	100.0
	3	80.4	54.3	99.9	98.2	100.0	100.0	99.8	98.6	100.0	100.0	100.0	100.0
160	1	99.3	98.7	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	2	99.8	99.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	3	99.7	98.1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Panel (b): Heteroskedasticity													
20	1	16.6	7.5	35.5	17.4	65.5	39.5	38.3	19.4	70.6	42.5	92.3	72.3
	2	14.5	2.2	34.4	7.0	67.6	22.0	32.8	6.0	66.2	19.7	91.5	49.9
	3	12.4	0.3	31.5	2.3	64.5	9.0	32.0	1.3	66.7	7.7	92.4	28.9
40	1	31.0	18.4	63.5	44.5	93.6	81.1	64.5	46.8	91.7	77.1	99.5	96.2
	2	28.7	11.4	63.7	32.7	94.9	74.3	59.6	31.2	89.9	65.3	99.3	92.1
	3	26.2	7.3	61.2	22.4	94.1	62.6	59.9	21.7	90.9	55.3	99.6	89.6
80	1	58.5	44.8	93.0	83.8	99.8	98.9	90.3	80.3	99.5	97.1	100.0	99.9
	2	58.4	36.9	95.0	81.5	99.9	99.2	87.7	70.6	99.6	95.9	100.0	99.9
	3	53.8	27.6	94.3	74.8	99.9	98.3	88.8	66.1	99.6	94.9	100.0	99.9
160	1	91.9	85.0	99.8	99.2	100.0	100.0	99.6	97.8	100.0	100.0	100.0	100.0
	2	93.6	84.4	100.0	99.6	100.0	100.0	99.0	96.5	100.0	100.0	100.0	100.0
	3	92.5	78.2	100.0	99.3	100.0	100.0	99.5	96.7	100.0	100.0	100.0	100.0

Note: Rejection frequencies of the standard (ST) and robust (HAC) versions of the $\mathrm{LM_F}$ test of no remaining heterogeneity based on (17) at 5% nominal significance level. Panels are generated according to the model (18) and (19) with $r = 1$ and $m = 2$ ('Misspecified m) or with $r = 2$, $m_1 = m_2 = 1$ and $q_{it}^{(1)} \neq q_{it}^{(2)}$ ('Different transition variable') and with homoskedastic (panel (a)) or heteroskedastic errors (panel (b)). The table is based on 10,000 replications.

Table 8: Summary statistics

	Mean	St.Dev	Percentile 10	25	50	75	90
I_{it}	0.088	0.059	0.031	0.049	0.076	0.112	0.158
Q_{it-1}	1.053	1.201	0.224	0.370	0.670	1.286	2.281
S_{it-1}	1.843	0.949	0.899	1.271	1.696	2.225	2.835
CF_{it-1}	0.241	0.197	0.055	0.124	0.215	0.319	0.447
D_{it-1}	0.233	0.207	0.007	0.090	0.206	0.319	0.471

Table 9: Homogeneity tests

	ST		HAC	
m	Test	p-value	Test	p-value
Transition variable Q_{it-1}				
1	29.5	2×10^{-24}	7.51	5×10^{-6}
2	25.9	9×10^{-40}	6.88	5×10^{-9}
3	23.3	1×10^{-51}	6.38	2×10^{-11}
Transition variable D_{it-1}				
1	8.8	4×10^{-7}	3.43	8.3×10^{-3}
2	10.2	3×10^{-14}	2.73	5.2×10^{-3}
3	7.0	9×10^{-13}	2.02	0.019

Table 10: Sequence of homogeneity tests for selecting m

	ST		HAC	
	Test	p-value	Test	p-value
Transition variable Q_{it-1}				
$\mathsf{H}_{03}^*: \beta_3^* = 0$	17.62	2×10^{-14}	6.15	6×10^{-5}
$\mathsf{H}_{02}^*: \beta_2^* = 0 \mid \beta_3^* = 0$	21.93	5×10^{-18}	5.53	2×10^{-4}
$\mathsf{H}_{01}^*: \beta_1^* = 0 \mid \beta_3^* = \beta_2^* = 0$	29.46	2×10^{-24}	7.51	5×10^{-6}
Transition variable D_{it-1}				
$\mathsf{H}_{03}^*: \beta_3^* = 0$	0.66	0.618	0.33	0.859
$\mathsf{H}_{02}^*: \beta_2^* = 0 \mid \beta_3^* = 0$	11.48	3×10^{-9}	2.74	0.027
$\mathsf{H}_{01}^*: \beta_1^* = 0 \mid \beta_3^* = \beta_2^* = 0$	8.79	4×10^{-7}	3.43	8.3×10^{-3}

Table 11: Misspecification tests

m/h	ST Test	ST p-value	HAC Test	HAC p-value
No remaining heterogeneity				
Transition variable Q_{it-1}				
1	2.34	0.05	0.55	0.70
2	1.45	0.17	0.32	0.96
3	2.29	0.01	0.73	0.72
Transition variable D_{it-1}				
1	2.30	0.06	1.05	0.38
2	2.29	0.02	1.15	0.32
3	1.95	0.03	0.94	0.50
Parameter Constancy				
1	1.13	0.34	0.69	0.70
2	1.45	0.11	0.65	0.85
3	2.75	0.00	1.03	0.43

Table 12: Estimation results of two-regime PSTR model

	Coefficient estimate	ST	HAC
β_{0j} ($\times 10^2$)			
Q_{it-1}	2.82	0.16	0.17
S_{it-1}	0.37	0.06	0.05
D_{it-1}	-2.27	0.24	0.26
CF_{it-1}	6.18	0.51	0.53
$\beta_{0j} + \beta_{1j}$ ($\times 10^2$)			
Q_{it-1}	0.74	0.07	0.10
S_{it-1}	1.49	0.10	0.12
D_{it-1}	0.18	0.48	0.87
CF_{it-1}	4.14	0.48	0.67
γ	118.77	190.16	247.02
c	1.51	0.01	0.02

Table 13: Coefficient estimates of year dummies

	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
Coefficient/10^2	-0.52	-0.80	-0.53	0.08	0.32	0.69	0.17	-0.74	-1.35	0.18	0.62	0.25	-0.44
ST	0.21	0.22	0.22	0.22	0.22	0.21	0.21	0.21	0.21	0.22	0.22	0.22	0.23
HAC	0.19	0.21	0.21	0.21	0.20	0.23	0.22	0.20	0.20	0.24	0.27	0.26	0.25

Table 14: Regime statistics

	Percentage of firms in upper regime	Percentage of firms switching from lower to upper regime	Percentage of firms switching from upper to lower regime
1974	20.9	-	-
1975	11.3	0.0	9.6
1976	15.0	4.5	0.7
1977	15.2	2.0	1.8
1978	13.9	3.0	4.3
1979	14.1	3.6	3.4
1980	15.2	4.6	3.6
1981	18.6	5.5	2.1
1982	16.4	3.2	5.4
1983	20.5	7.3	3.2
1984	29.5	11.1	2.1
1985	22.0	2.5	10.0
1986	28.6	9.5	2.9
1987	33.2	8.9	4.3
Avg.	19.6	5.1	4.1

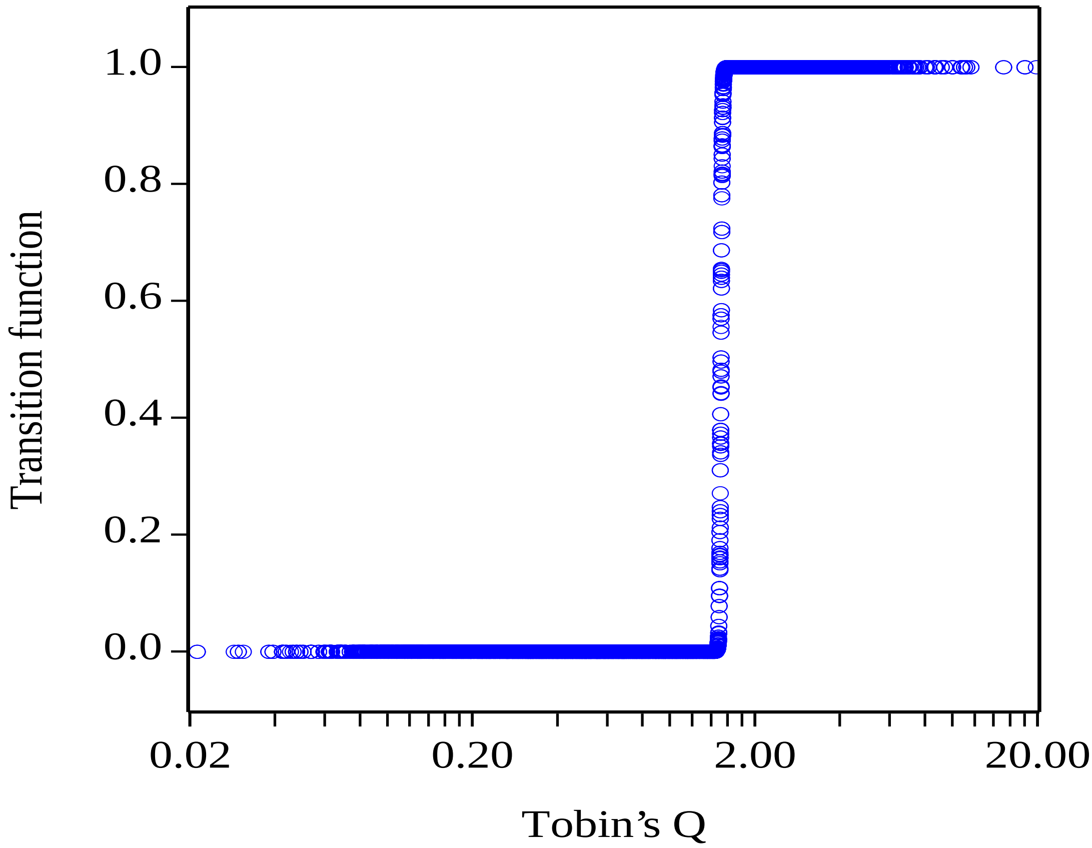

Figure 1: Estimated transition function (21) of the PSTR model (20). Each circle represents an observation.