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Financial Crisis in Emerging Markets and the Optimal Bailout Policy*

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Abstract

This paper develops a framework for analyzing optimal government bailout policy in a dynamic stochastic general equilibrium model where financial crises are exogenous. Important elements of the model are that private borrowers only internalize part of the social cost of foreign borrowing in the emerging market and that the private sector is illiquid in the event of a crisis. The distinguishing feature of our paper is that it addresses the optimal bailout policy in an environment where there are both costs and benefits of bailouts, and where bailout guarantees potentially distort investment decisions in the private sector. We show that it is always optimal to commit to a bailout policy that only partially protects investment against inefficient liquidation, both in a centralized economy and a market economy. Due to overinvestment in the market economy, the government's optimal level of bailout guarantees is lower than in the social optimum. Further, we show that, in contrast to a social planner, the government in the market economy should optimally bail out a smaller fraction of private investments when the probability of a crisis is higher.

JEL Classification Codes: F34, F40

Key Words: financial crisis; government bailout; emerging markets

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1 Introduction

A wide range of emerging market economies have experienced financial crises in recent decades. In the wake of these events, governments of such countries as Chile, Argentina, Mexico, Korea and Indonesia have spent large shares of their GDP, sometimes more than 30 percent, on saving financial systems in distress.[1] In light of the large costs involved, investigating the macroeconomic role and efficiency of such rescue packages must be of prime concern. The economic profession is divided on the role of bailouts in financial crises in emerging markets. On the one hand, governments should limit the provision of guarantees to avoid distorting investment incentives in the private sector, on the other hand, they should stand ready to provide liquidity in times of crisis to minimize the negative consequences of financial panics and maturity mismatch. What advice should we give to policy makers in emerging markets? What is the optimal bailout policy in response to a financial crisis?

In this paper, we analyze optimal bailout policy under commitment in a dynamic stochastic general equilibrium model where financial crises are exogenous. Important elements of the model are that private borrowers only internalize part of the social cost of foreign borrowing in the emerging market and that the private sector is illiquid in the event of a crisis. We model the strategic interaction between the government and the private sector, assuming the government to be benevolent in the sense of maximizing consumer utility.

The distinguishing feature of our paper is that it addresses the optimal bailout policy in an environment where there are both costs and benefits of bailouts, and where bailout guarantees potentially distort investment decisions in the private sector. In the model, the cost of bailouts arises because bailouts lead to more volatile government consumption, while the benefit of bailouts is that they help avoid inefficient liquidation of investments in the private sector. The cost is aggravated by the distortion of private investment incentives which arises from bailout guarantees in the model.

To examine the optimal bailout policy, we consider a range of alternative bailout policies and construct equilibrium with commitment and a Markov structure. We show that in both a centralized economy and a market economy where the government is restricted to providing bailouts for free, it is always optimal to commit to a bailout policy that only partially protects investment against inefficient liquidation. Due to overinvestment in the decentralized economy, the government's optimal level of bailout guarantees is lower than in the social optimum. Further, we show that, in contrast to a social planner, the government in the decentralized economy should optimally bail out a smaller fraction of private investments when the probability of a crisis increases.

[1] See, for example, Dziobek and Pazarbasioglu (1997), Kaminsky and Reinhart (1999) and Eichengreen and Rose (1997).

Previous work on government bailouts has included some of the different elements that we merge into a unified framework in our model. Gale and Vives (2002) model a moral hazard cost of bailouts by making the private managerial effort depend on the size of bailout guarantees. Freixas (1999) pursues a cost-benefit analysis to characterize the optimal bailout policy of the Lender of Last Resort. Mundaca (2001) develops a game theoretic setting for the interaction between the government and the market to address the optimal bailout policy, whereas Schneider and Tornell (2000) investigate the effects of government bailouts on market behavior in an infinite horizon setting. The general structure of our model has been inspired by Cole and Kehoe (2000), who address the issue of optimal government debt policies and self-fulfilling debt crises.

Our paper is related to the strand of research arguing that bailout guarantees are a 'bad policy', in that the provision of these leads to distortions in investment incentives. According to the theoretical work of Corsetti et al. (1998a) and Burnside et al. (2003), bailout guarantees induce moral hazard by providing insurance against future crises to the private sector. Market participants willingly take on excessive risk, which leads to over-investment, excessive external borrowing or unhedged foreign loans. Empirical support for the 'bad policy' argument has been provided by Corsetti et al. (1998b) and Dooley and Shin (2000).

Our model also captures the arguments of the literature on maturity mismatch and financial panics, which holds a more positive view on government bailouts. The provision of emergency liquidity can help avoid inefficient liquidation of investments in times of crisis, as argued by Chang and Velasco (2001) and Allen and Gale (2000). We believe potential illiquidity to be a characteristic feature of producers in emerging markets, who are often forced to borrow abroad at short maturities. Empirical work by Radelet and Sachs (1998) and Rodrik and Velasco (1999) has found evidence supporting this view.

The next section of the paper lays out the model. In Section 3, we define an equilibrium which takes into account the strategic incentives of a government that must commit to a level of bailout guarantees in the first period of the model and adhere to this in all subsequent periods. In section 4, we analyze the market response to bailout guarantees. Section 5 presents the social planner's solution of the model and Section 6 analyzes the government's optimal bailout policy. In Section 7, we show that the formal analysis and the policy conclusions of the paper remain unchanged if the commitment assumptions in the model are relaxed. Section 8 provides a numerical example to illustrate the model, while concluding remarks and suggestions for future research are presented in Section 9.

2 The Model

We model an emerging market as a small economy opening up to the international capital market in period $t = 0$. The economy is inhabited by a continuum of consumers and a government. The consumers receive an endowment of a consumption good in each period. Additional consumption goods can be produced with borrowed foreign capital as the only input. In the first period, the government must commit to a bailout policy, to which it must adhere in all subsequent periods.[2]

2.1 The International Capital Market

A continuum of risk neutral agents act in an environment of perfect competition on the international capital market. This implies that the expected return on any one-period loan must equal $1/\beta$, where β is the universal and subjective discount rate.

In every period, the international capital market offers one-period loans to the consumers in the emerging economy. We assume all international loans in some periods to be recalled before having reached full maturity. Define ζ_t to be an exogenous random variable realized at the beginning of each period and following the process

$$\zeta_t = \begin{cases} 0 & \text{with prob. } (1-\pi) \\ 1 & \text{with prob. } \pi \end{cases} \quad , \; t > 0 \tag{1}$$

$$\zeta_0 = 0, \tag{2}$$

where $\pi \in [0, 1)$ is an exogenous parameter.

If $\zeta_t = 1$, all international loans are recalled before they have reached full maturity in period t, which is what we define as a financial crisis in the emerging market. Repayments of international loans are requested after full maturity if $\zeta_t = 0$. Note that a financial crisis in the model economy occurs with the exogenous probability π in every period.

2.2 The Consumers

There is a continuum with measure one of identical and infinitely lived consumers, who consume, invest, borrow from abroad, and pay lump-sum taxes. The individual consumer's utility function is

$$E \sum_{t=0}^{\infty} \beta^t (c_t + v(g_t)),$$

where c_t is private consumption and g_t is government consumption. We assume v to be twice continuously differentiable, strictly concave, monotonically increasing and that $v(0) = -\infty$.

If the probability of crisis is zero (i.e. if $\pi = 0$), the individual consumer is subject to

[2] The formal analysis and the policy conclusions of the paper would remain unchanged if the assumption that the government must commit to its bailout policy in the first period were relaxed to allow for one-period commitment. In Section 7, we discuss the commitment assumption further.

the following budget and investment constraints

$$c_t + k_{t+1} + R_t b_t + Q_t \leq f(k_t) + b_{t+1} + \omega \tag{3}$$

$$k_{t+1} = b_{t+1}. \tag{4}$$

Here, b_{t+1} is a foreign loan to be repaid in period $t+1$. The investment constraint in (4) specifies that international loans can only be used to augment the capital stock, k_{t+1}, and that the capital used in production must be borrowed from abroad. R_t is the gross interest rate on foreign loans, Q_t is a lump-sum tax, and $\omega > 0$ is an endowment received in each period. We assume the production function to be of the functional form

$$f(k) = Ak^a, \ \ \alpha < 1. \tag{5}$$

The consumer is endowed with $k_0 = 0$ and $b_0 = 0$ in period $t = 0$. We assume that capital depreciates fully after one period.

When the probability of crisis is positive and a financial crisis occurs in period t, repayment of international loans must be made before production takes place. We assume that in this case, international lenders can liquidate the capital stock, k_t, with a linear return of $1/\beta$. This simplifying assumption implies that the interest rate on international loans in the model economy is constant and equal to the world interest rate, $R_t = 1/\beta$ $\forall t$.[3]

When the probability of a crisis is positive, the individual consumer's budget constraint will depend on government policy, which is the reason why we now turn to describing the government.

2.3 The Government

The government is benevolent in the sense of its objective being to maximize the consumers' utility. The government is the only strategic agent in the model, and when making its decisions, it takes into account the effects of these decisions on the level of the aggregate capital stock K_t, the aggregate level of international private debt B_t, government revenue, and the level of private consumption.

We define a bailout policy x as the fraction of international liabilities the government provides to international lenders in the event of a crisis.

In period $t = 0$, the government can commit to any bailout policy $x \in [0, 1]$, to which it must subsequently adhere forever. Choosing $x = 0$ corresponds to a policy of *No Bailout*, which implies that the government never provides any resources for repaying

[3] Previous versions of this paper included costs of liquidation so that the return to liquidating capital was smaller than unity. The assumption that capital can be liquidated with a return of $1/\beta$ greatly facilitates our analytical investigation, without affecting the policy conclusions of the paper.

international lenders in the event of a crisis. Committing to $x = 1$ corresponds to a policy of *Full Bailout*, by which the government provides RB_t to repay international loans in a crisis.

The consumers must pay an exogenous price, $P \geq 0$, for the bailouts provided by the government. For a bailout policy x, the government will spend xRB_t on bailouts in a financial crisis and receive xPB_t from consumers later in the same period. A price of $P = 1/\beta$ thus implies that the consumers repay exactly what the government spends on bailouts. In the model, the price of bailouts will affect the investment decision of the individual consumer. When analyzing the government's optimal bailout policy in Section 6, the price P will therefore play an important role for the outcomes of the model.

In the first period, the government commits to a bailout policy x, and in every period, it chooses the size of the lump-sum tax, $Q_t \geq 0$, after observing the realization of the crisis variable, ζ_t. The timing of events in the model is such that in every period, the government must spend resources before receiving income in that period. When spending resources on government consumption and bailouts in period t, the government's budget constraint is

$$g_t + \zeta_t xRB_t = Q_{t-1} + \zeta_{t-1} PxB_{t-1}, \tag{6}$$

where $g_t \geq 0 \ \forall t$. The left-hand side of equation (6) shows that resources are spent on government consumption in every period. If there is a financial crisis in the period, the government also spends resources on pursuing bailouts according to the bailout policy, x. The first term on the right-hand side of equation (6) says that in every period, the lump-sum tax levied in the previous period contributes to government resources in the current period. Similarly, the second term states that any repayment of bailouts received in period $t - 1$ contributes to the government resources in period t. Note that we have assumed that the government must run a balanced budget. The government is subject to a revenue lag, but has no other possibility of saving resources to build a buffer against future bailout costs. For expositional reasons, it is useful to define government revenues in period t as

$$T_t = \begin{cases} Q_t, & \text{if } \zeta_t = 0 \\ Q_t + PxB_t, & \text{if } \zeta_t = 1 \end{cases}, \ t \geq 0. \tag{7}$$

Because consumers are competitive, we need to distinguish between the individual decisions k_{t+1} and b_{t+1} and the aggregate values K_{t+1} and B_{t+1}. Consumers should not think that their individual actions affect the aggregate state in the next period, thereby affecting prices or the government's actions. In equilibrium, because all consumers are identical, $k_{t+1} = K_{t+1}$ and $b_{t+1} = B_{t+1}$.[4]

Since a fraction of the individual's capital stock must sometimes be liquidated in the model, we need to enhance the notation we have used so far. Let k_{t+1} denote the

[4]The initial endowments are such that $k_0 = K_0$ and $b_0 = B_0$.

consumers' choice in period t and let κ_{t+1} denote the part of the capital stock actually used in production in period $t+1$. The law of motion for κ_{t+1} depends on the bailout policy in the following way:

$$\kappa_{t+1} = \kappa(x, k_{t+1}, \zeta_{t+1}) = \begin{cases} k_{t+1} & \text{if } \zeta_{t+1} = 0 \\ xk_{t+1} & \text{if } \zeta_{t+1} = 1 \end{cases} . \tag{8}$$

The first line of equation (8) simply states that if a crisis does not occur in period $t+1$, there is no liquidation. The second line says that for a given bailout policy x, consumers can keep xk_{t+1} in production if a crisis occurs in period $t+1$.

2.4 The Timing

The timing of actions within period $t = 0$ differs from subsequent periods, since the government chooses its bailout policy only in the first period. In subsequent periods, the actions within a period depend on whether a crisis occurs; the government pursues bailouts and investments are liquidated only if a crisis occurs in that period. The timing of actions within a period is the following:

1. The variable ζ_t is realized and the aggregate state is $S_t = (K_t, B_t, T_{t-1}, \zeta_t)$.
2. If $t = 0$, the government commits to a bailout policy x.

If a crisis does not occur, $\zeta_t = 0$,

3. The government provides g_t and decides Q_t.
4. Production takes place, the endowment is realized and taxes are paid.
5. Each consumer repays his international loans, amounting to Rb_t.
6. Each consumer chooses c_t, k_{t+1} and b_{t+1}, taking x, P and Q_t as given.

If a crisis occurs, $\zeta_t = 1$,

3. Each consumer is asked to repay Rb_t early.
4. The government provides g_t, decides Q_t, and spends xRB_t on bailouts.
5. Each consumer must liquidate part of his invested capital, $(1-x)k_t$, and use it to repay part of his international loans. Since capital can be liquidated with a return of $1/\beta = R$, and since $k_t = b_t$, the repayment is worth $(1-x)Rb_t$ to the international lenders.
6. Using the part of the capital which is left in production, xk_t, production takes place, the endowment is realized, taxes are paid and bailouts are repaid to the government with the amount PxB_t.

7. Each consumer chooses c_t, k_{t+1} and b_{t+1}, taking x, P and Q_t as given.

An important feature of the timing is that the government in a period must spend resources on government consumption and bailouts *before* it receives any revenues in that period. The government carries over any revenues from one period to the next. If loans need to be repaid early, the government disposes of revenues from the previous period and can therefore bail out the consumers, who are illiquid at the beginning of each period.

3 Equilibrium in period $t = 0$

Within each period, the aggregate state $S_t = (K_t, B_t, T_{t-1}, \zeta_t)$, the bailout policy x, the government's choice of Q_t, the consumers' choices c_t, k_{t+1} and b_{t+1}, the interest rate on international loans R and the price of bailouts P determine the equilibrium. Since we want to analyze the government's optimal bailout policy, we will focus on the equilibrium in period $t = 0$, which is the only period where x is a choice variable for the government.[5]

To define a recursive equilibrium, we first present the individual consumer's problem, which takes x and Q_t as given. Next, we present the government's problem, which takes into account that the consumer's choices will depend on the bailout policy x and the lump-sum tax, Q_t.

The solution of an agent's maximization problem is given by the value function providing the maximum attainable value of the agent's utility function given his state, and by policy functions providing the maximizing choices of the agent's choice variables in the current period, given his state. In equilibrium, agents solve their own problems by correctly predicting other agents' policies.

When an individual consumer acts, he knows the bailout policy x, the size of the lump-sum tax Q_t, the aggregate state S_t, his individual levels of k_t and b_t, the interest rate R and the price of bailouts, P. To save on notation, let $h_t = (k_t, b_t, S_t)$. The individual consumer's value function is defined by the following functional equation:

$$V^C(x, h_t) = \max_{\{c_t, k_{t+1}, b_{t+1}\}} \left\{c_t + v(g_t) + \beta E_t V^C(x, h_{t+1})\right\} \tag{9}$$

$$\begin{aligned} s.t \quad c_t + k_{t+1} + (1 - \zeta_t)Rb_t + \zeta_t P x b_t + Q_t &\leq f(\kappa_t) + b_{t+1} + \omega \\ k_{t+1} &= b_{t+1} \\ k_{t+1} &\geq 0 \end{aligned}$$

[5] In section 7, we show that the optimal bailout policy in this equilibrium is identical to the optimal policy in a model where the government in every period must commit to a bailout policy for the next period.

$$
\begin{aligned}
\zeta_{t+1} &= \begin{cases} 0 & \text{with prob. } (1-\pi) \\ 1 & \text{with prob. } \pi \end{cases} \\
Q_t &= Q(x, S_t) \\
K_{t+1} &= K(x, S_t) \\
B_{t+1} &= B(x, S_t) \\
\kappa_t &= \kappa\left(x, k_t, \zeta_t\right) \ \forall t \\
T_t &= Q_t + \zeta_t P x B_t \\
& \quad x, h_t \text{ given,}
\end{aligned}
$$

where the functions $Q(.)$, $K(.)$ and $B(.)$ will subsequently be defined and where $\kappa(.)$ was defined in equation (8). The consumer's policy functions are $c(x, h_t)$, $k(x, h_t)$ and $b(x, h_t)$.

When the government chooses its bailout policy x in period $t = 0$, it knows the initial aggregate state $S_0 = (K_0, B_0, T_{-1}, \zeta_0)$, the interest rate R and the price of bailouts P. The government realizes that it can affect the consumers' decisions through its choices of x and Q_t. When the government in any period chooses Q_t, it also knows the bailout policy x. Let $H_t = (K_t, B_t, S_t)$. The government's value function in period $t = 0$ is defined by the following functional equation:

$$
V_0^G\left(S_0\right) = \max_{\{x, Q_0\}} \left\{ c\left(x, H_0\right) + v\left(g_0\right) + \beta E_0 V^G\left(x, S_1\right) \right\} \tag{10}
$$

where

$$
V^G\left(x, S_t\right) = \max_{\{Q_t\}} \left\{ c\left(x, H_t\right) + v\left(g_t\right) + \beta E_t V^G\left(x, S_{t+1}\right) \right\}, \ t > 0
$$

$$
\begin{aligned}
s.t \qquad g_t + \zeta_t x R B_t &= T_{t-1} \\
g_t &\geq 0 \\
x &\in [0, 1] \\
\zeta_{t+1} &= \begin{cases} 0 & \text{with prob. } (1-\pi) \\ 1 & \text{with prob. } \pi \end{cases} \\
K_{t+1} &= K(x, S_t) \\
B_{t+1} &= B(x, S_t) \\
T_t &= Q_t + \zeta_t P x B_t \\
& \quad S_0 \text{ given.}
\end{aligned}
$$

The government's policy function for the lump-sum tax is $Q(x, S_t)$, and the set of optimal bailout policies is X^*.

Having developed these concepts, we can now define an equilibrium for the first period in our model economy.

Definition of equilibrium in period $t = 0$. An equilibrium in period $t = 0$ is a list of value functions V^C for individual consumers and V_0^G for the government; policy functions $c(x, h_t)$, $k(x, h_t)$ and $b(x, h_t)$ for the consumers; a policy function $Q(x, S_t)$ and an optimal bailout policy $x^* \in X^*$ for the government; an interest rate R; a price of bailouts P; and laws of motion for the aggregate state variables, $K(x, S_t)$, and $B(x, S_t)$, such that the following conditions hold.

1. Given R, P, x and $Q(x, S_t)$, V^C is the value function for the solution to the consumer's problem and $c(x, h_t)$, $k(x, h_t)$ and $b(x, h_t)$ are the maximizing choices.

2. Given R, P, $c(x, h_t)$, $k(x, h_t)$ and $b(x, h_t)$, V_0^G is the value function for the solution to the government's problem, and x^* and $Q(x, S_t)$ are the maximizing choices.

3. $K(x, S_t) = k(x, H_t)$ and $B(x, S_t) = b(x, H_t)$.

4 The Market Response to Bailouts

To characterize the equilibrium in period $t = 0$, we start by presenting the individual consumer's optimal behavior in response to a given bailout policy, a given lump-sum tax and a given price of bailouts. Then, we characterize the government's optimal choice for lump-sum taxation, still for a given bailout policy.

To find the consumer's optimal response to a given bailout policy, we can use the investment constraint in equation (4) to substitute for b_{t+1} in the maximization program presented in (9). The fact that $f'(0) = \infty$ ensures that the non-negativity constraint on borrowed capital never binds in equilibrium. Using the consumer's budget constraint to substitute for c_t, the form of the function $\kappa(.)$ and the fact that $R = 1/\beta$, we obtain the following optimality condition for the consumer's investment decision:

$$(1 - \pi) f'(k_{t+1}) + \pi x f'(x k_{t+1}) = (1 - \pi)\frac{1}{\beta} + \pi P x. \tag{11}$$

The details of the derivation are presented in appendix A. The left-hand side of (11) contains the expected marginal return on investment, which is the weighted sum of the marginal product of borrowed capital in a period without and with a crisis, respectively. The right-hand side represents the consumer's expected marginal cost of capital. Note that for crisis periods, the consumer need only consider the marginal return and cost of the part of the capital stock that will be left in production after government bailouts, $x k_{t+1}$.

In equation (11), we see that the optimal level of borrowed capital is independent of the bailout policy x, if the probability of crisis is zero. This is natural, since bailouts never occur if crises never happen. Since our interest lies in analyzing the optimal bailout policy in an environment where crises can occur, we will focus on equilibria with a positive probability of crisis in the rest of the paper.

Note that for a given $\pi > 0$, the consumer's optimal investment decision will only depend on the bailout policy x, not on the individual consumer's state h_t. In equilibrium, the consumer therefore makes the same investment decisions in every period. To simplify the exposition, we will henceforth denote the consumer's policy function for borrowed capital as $k(x)$.

Since the individual consumer is risk neutral, the optimal rule for consumption is simply to consume whatever resources are left in each period, once investments, tax payments, loan repayments or bailout repayments have been made.

To find the government's optimal lump-sum tax, Q_t, for a given bailout policy x, we first use equation (7) to recast the government's value function (10) as a maximization problem in terms of government revenues. This can be done, since we know from equation (11) that the consumer's choice of k_{t+1} is independent of the lump-sum tax. Note also that, since $v(0) = -\infty$, the non-negativity constraint on government consumption never binds in equilibrium.

Using the fact that $k_{t+1} = K_{t+1}$ and $b_{t+1} = B_{t+1}$, the investment constraint in equation (4) further enables us to substitute for B_{t+1} in the maximization program presented in equation (10). Substituting for $R = 1/\beta$, the optimality condition for government revenues, T_t, is given by

$$(1-\pi)\, v'\left(T_t\right) + \pi v'\left(T_t - \frac{xk(x)}{\beta}\right) = \frac{1}{\beta}. \tag{12}$$

The details of the derivation are presented in appendix A. Note that equation (12) implies that the equilibrium level of T_t is constant in the model, since the optimal level of revenues only depends on x and the consumer's (constant) equilibrium investment decision. For simplicity, we henceforth denote the optimal level of government revenues for a given bailout policy by $T(x)$. The optimality condition in equation (12) can be understood by remembering that government revenues can only be used for government consumption with a one-period lag. The government optimally equates the discounted expected marginal utility from public consumption tomorrow to the marginal utility from private consumption today which, due to linearity, is constant and equal to 1 (in equation (12), we have divided both sides by β).

Given the constant level of $T(x)$, the optimal lump-sum tax, $Q(x, S_t)$, can be obtained as a residual from equation (7). For a given bailout policy x, the optimal tax is smaller in periods of financial crisis, since the government in such periods receives additional revenues from the repayments of bailouts.

The fact that utility is linear in private consumption implies that the optimal level of private consumption, $c(x, H_t)$, may be negative in response to a particular bailout policy x and the associated level of government revenues. If the lump-sum tax exceeds the endowment, ω, consumption in period $t = 0$ will, for example, be negative. Negative consumption can be avoided in the model if the endowment in each period is sufficiently large.

5 The Social Planner Solution

Before proceeding to analyze the government's optimal bailout policy, it is instructive to consider the solution of a centralized economy where a social planner maximizes consumer utility, subject to the resource constraints of the economy. This solution will serve as a useful benchmark in interpreting the decentralized equilibrium defined in section 3.

To facilitate comparisons between the social optimum and the market economy, we assume that the social planner must also commit to a bailout policy x at the beginning of period $t = 0$ and adhere to it in all subsequent periods.[6] Apart from choosing x in period $t = 0$, the social planner will act at two points in time within each period. At the beginning of a period, the social planner knows S_t and delivers g_t, subject to the budget constraint

$$g_t = T_{t-1} - \zeta_t x R B_t, \tag{13}$$

where $R = 1/\beta$ as in the decentralized equilibrium, and T_{t-1} is the amount of resources transferred from the consumers to the public sector in period $t - 1$.

Later in the period, the social planner makes decisions for c_t, K_{t+1}, B_{t+1} and T_t. The value function when the social planner acts for the second time in a period is defined by the following functional equation

$$V^{SP}(x, S_t) = \max_{\{c_t, K_{t+1}, B_{t+1}, T_t\}} \left\{ c_t + v(g_t) + \beta E_t V^{SP}(x, S_{t+1}) \right\} \tag{14}$$

$$s.t \quad c_t + K_{t+1} + (1 - \zeta_t) R B_t + T_t \leq f\left(\kappa(x, K_t, \zeta_t)\right) + B_{t+1} + \omega$$

$$\begin{aligned} g_t &= T_{t-1} - \zeta_t x R B_t \\ g_t &\geq 0 \\ K_{t+1} &= B_{t+1} \\ K_{t+1} &\geq 0 \\ \zeta_{t+1} &= \begin{cases} 0 & \text{with prob. } (1-\pi) \\ 1 & \text{with prob. } (\pi) \end{cases} \\ & x, S_t \ \text{ given,} \end{aligned} \tag{15}$$

[6] The social planner's problem can be formulated without defining bailout policies or assuming commitment, as discussed in Section 7.

where the function $\kappa(.)$ was defined in equation (8). The policy functions associated with V^{SP} are $c_{SP}(x, S_t)$, $K_{SP}(x, S_t)$, $B_{SP}(x, S_t)$ and $T_{SP}(x, S_t)$.

The value function for the centralized problem in period $t = 0$ can now be specified as

$$V_0^{SP}(S_0) = \max_{x \in [0,1]} \left\{ c_{SP}(x, S_0) + v(T_{-1}) + \beta E_0 V^{SP}(x, S_1) \right\} \qquad (16)$$
$$S_0 \text{ given.}$$

The set of optimal bailout policies is X_{SP}^*.

For a given probability of crisis, π, and a given bailout policy, x, the socially optimal choices of c_t, K_{t+1}, B_{t+1} and T_t can be found by using the budget constraints for private and government consumption to substitute for c_t and g_t, and the investment constraint in (15) to substitute for B_{t+1} in the maximization program in equation (14). Since $v(0) = -\infty$ and $f'(0) = \infty$, the non-negativity constraints will never bind at the social optimum. Substituting for $R = 1/\beta$, the optimality conditions for K_{t+1} and T_t are, respectively, given by

$$(1-\pi) f'(K_{t+1}) + \pi x f'(x K_{t+1}) = (1-\pi)\frac{1}{\beta} + \pi \frac{x}{\beta} v'\left(T_t - \frac{x K_{t+1}}{\beta}\right) \qquad (17)$$

$$(1-\pi) v'(T_t) + \pi v'\left(T_t - \frac{x K_{t+1}}{\beta}\right) = \frac{1}{\beta}. \qquad (18)$$

A more detailed derivation is presented in appendix A. Note that the first-order conditions in (17) and (18) for a given bailout policy are independent of the aggregate state variables. For a given bailout policy, x, the optimal investment and transfer decisions in the centralized economy will therefore be constant over time. To simplify the exposition, we henceforth denote the social planner's policy functions for borrowed capital and the transfer as $K_{SP}(x)$, and $T_{SP}(x)$.

We now proceed to analyze the optimal bailout policy in the centralized model economy. Since the investment decisions and the optimal tax are state independent for a given bailout policy, the economy can be in one of two states only after the initial period. Depending on the realization of the random variable ζ_t, the economy is either in a crisis or in a period of no crisis. The stationary nature of the equilibrium considerably simplifies the recursive value functions. Letting superscripts n and cr denote consumption in periods of no crisis and crisis, respectively, the value function of the centralized economy in period $t = 0$ in equation (16) can be written as

$$V_0^{SP}(S_0) = \max_{x\in[0,1]} \left\{c_0(x) + v(T_{-1}) + \beta E_0 V^{SP}(x, S_1)\right\} \tag{19}$$

where

$$E_0 V^{SP}(x, S_1) = \frac{1}{1-\beta}\Big((1-\pi)\left[c^n(x) + v(g^n(x))\right] + \pi\left[c^{cr}(x) + v(g^{cr}(x))\right]\Big)$$

$$\begin{array}{llcl} s.t. & c_0(x) & = & \omega - T_{SP}(x) \\ & c^n(x) & = & f(K_{SP}(x)) + \omega - RK_{SP}(x) - T_{SP}(x) \\ & g^n(x) & = & T_{SP}(x) \\ & c^{cr}(x) & = & f(xK_{SP}(x)) + \omega - T_{SP}(x) \\ & g^{cr}(x) & = & T_{SP}(x) - xRK_{SP}(x), \end{array}$$

where we have used the investment constraint in equation (15) to substitute for $B_{SP}(x)$.

In the centralized economy, we can use the Envelope Theorem to find the optimal bailout policy since the policy rules $K_{SP}(x)$ and $T_{SP}(x)$ attain the social optimum for any given bailout policy, x. Substituting for private and government consumption in the value function, the derivative w.r.t. x of the expected utility function in equation (19) is given by

$$\frac{d}{dx}\left\{c_0(x) + v(T_{-1}) + \beta E_0 V^{SP}(x, S_1)\right\} = \tag{20}$$

$$\frac{\beta}{1-\beta}\pi K_{SP}(x)\left[f'(xK_{SP}(x)) - \frac{1}{\beta}v'\left(T_{SP}(x) - \frac{xK_{SP}(x)}{\beta}\right)\right],$$

where we have used the equilibrium value of $R = 1/\beta$.

Proposition 1 *For any positive crisis probability, the optimal bailout policy in the centralized economy lies in the interior of the policy space and only partially protects investment against liquidation. Formally, $X_{SP}^* \subset (0, 1)$. Furthermore, for any positive crisis probability, the social planner's optimal bailout policy is unique.*

Proof. See appendix B. ■

According to Proposition 1, the optimality condition for the bailout policy in the centralized economy can be written as

$$f'(x_{SP}^* K_{SP}(x_{SP}^*)) = \frac{1}{\beta}v'\left(T_{SP}(x_{SP}^*) - \frac{x_{SP}^* K_{SP}(x_{SP}^*)}{\beta}\right), \tag{21}$$

where x_{SP}^* is the unique optimal bailout policy, and $K_{SP}(x_{SP}^*)$ and $T_{SP}(x_{SP}^*)$ are determined by the optimality conditions for borrowed capital and the transfer in equations

(17) and (18). Equation (21) tells us that the optimal bailout policy in the centralized economy must trade off the benefit of bailouts against the cost they incur. The left hand side is associated with the benefit, i.e. that bailouts help to avoid inefficient liquidation of investment in crises. The right hand side of equation (21) is associated with the cost, i.e. that bailouts cause government consumption to fluctuate, since resources that are spent on bailouts cannot be used for government consumption.

Proposition 1 tells us that at $x = 0$, the benefit of reducing inefficient liquidation outweighs the cost of increased volatility in government consumption. At $x = 1$, the volatility of government consumption should instead be reduced at the expense of some inefficient liquidation.

The model also enables us to analyze how the optimal bailout policy varies across countries with different probabilities of experiencing a crisis. The social planner's optimality conditions in equations (17), (18) and (21) implicitly define the optimal bailout policy, x^*_{SP}, as a function of the crisis probability, π.

Proposition 2 *For a higher probability of crisis, the social planner should optimally commit to bailing out a larger fraction of the borrowed capital in the economy. Formally, for any π_1 and π_2, such that $\pi_1 < \pi_2$, it is the case that $x^*_{SP}(\pi_2) > x^*_{SP}(\pi_1)$.*

Proof. See appendix B. ■

The result in proposition 2 can be understood by jointly considering the three optimality conditions in equations (17), (18) and (21). We start by noting that in the social optimum, for any probability of crisis, the social planner always chooses the same level of borrowed capital, at which the marginal product of capital in periods of no crisis is equal to the world interest rate, $1/\beta$.[7] Cost-benefit considerations for capital in crisis periods can be ignored, since in the social optimum, the levels of bailouts and transfers are chosen so that the marginal product of borrowed capital always equals its marginal cost in a crisis according to equation (21).

When there is an increase in the probability of a crisis, *ceteris paribus*, the social planner will, according to equation (18), need to transfer more resources to the public sector to guard against low government consumption in crisis periods. The social planner must further trade off the benefit of bailouts against their costs according to equation (21). For a higher level of public resources in a crisis, the social planner should optimally spend more resources on both the provision of public consumption and protection against liquidation. For a higher probability of crisis, the social planner therefore transfers more to the public sector *and* spends more resources on bailouts.[8]

[7] This can be seen by plugging equation (21) into equation (17) and evaluating at x^*_{SP}.

[8] Note that with the level of borrowed capital being constant, equation (21) implies that a higher value of T must be associated with a higher value of x^*_{SP}.

6 The government's optimal bailout policy

The government's value function in period $t = 0$, given by (10), can be written on exactly the same form as the social planner's value function in equation (19). The only differences stem from the decision rules for borrowed capital and the government revenues entering the value function

$$V_0^G(S_0) = \max_{x \in [0,1]} \left\{ c_0(x) + v(T_{-1}) + \beta E_0 V^G(x, S_1) \right\} \tag{22}$$

where

$$E_0 V^G(x, S_1) = \frac{1}{1-\beta} \Big((1-\pi) \left[c^n(x) + v(g^n(x)) \right] + \pi \left[c^{cr}(x) + v(g^{cr}(x)) \right] \Big)$$

$$\begin{aligned}
s.t. \qquad c_0(x) &= \omega - T(x) \\
c^n(x) &= f(k(x)) + \omega - Rk(x) - T(x) \\
g^n(x) &= T(x) \\
c^{cr}(x) &= f(xk(x)) + \omega - T(x) \\
g^{cr}(x) &= T(x) - xRk(x),
\end{aligned}$$

The optimal bailout policy can be derived from the government value function in (22). The Envelope Theorem does not hold in the decentralized economy, since the consumer's decision rule for borrowed capital, $k(x)$, is not socially optimal. In appendix A, we show that the derivative of the government's expected utility function in (22) w.r.t. x is given by

$$\begin{aligned}
&\frac{d}{dx} \left\{ c_0(x) + v(T_{-1}) + \beta E_0 V^G(x, S_1) \right\} = \\
&\qquad \frac{\beta}{1-\beta} \pi \left(k(x) \left[f'(xk(x)) - \frac{1}{\beta} v' \left(T(x) - \frac{xk(x)}{\beta} \right) \right] + xk'(x) D(x) \right),
\end{aligned} \tag{23}$$

where we have used the equilibrium value of $R = 1/\beta$, and $D(x)$ is defined as

$$D(x) = P - \frac{1}{\beta} v' \left(T(x) - \frac{xk(x)}{\beta} \right). \tag{24}$$

6.1 Optimally priced bailouts

Comparing the optimality conditions in the decentralized economy for borrowed capital and government revenues in equations (11) and (12) with the corresponding conditions in the social planner's solution in equations (17) and (18), the only difference is one term in the condition for borrowed capital. While the price of borrowed capital actually used in production for the atomistic consumers in crisis periods is P, the social planner takes into account that loans through bailouts reduce government consumption in a crisis.

If the price of bailouts, P, is equal to

$$P^* = \frac{1}{\beta} v' \left(T_{SP}(x_{SP}^*) - \frac{x_{SP}^* K_{SP}(x_{SP}^*)}{\beta} \right), \tag{25}$$

the optimality conditions for borrowed capital in equations (11) and (17) coincide for $x = x_{SP}^*$, which implies that $k\left(x_{SP}^*\right) = K_{SP}(x_{SP}^*)$ and $T\left(x_{SP}^*\right) = T_{SP}(x_{SP}^*)$. When $P = P^*$, the derivative of the expected utility w.r.t. x therefore coincides in the decentralized and centralized economies for $x = x_{SP}^*$, which can be seen by comparing equations (23) and (20). The government's optimal bailout policy is to set $x = x_{SP}^*$, which enables the government to achieve the social optimum since P^* is the price of bailouts that makes the atomistic consumers internalize the costs associated with bailouts in the social optimum.

6.2 Suboptimally priced bailouts

For any positive probability of crisis, equation (18) implies that

$$v' \left(T_{SP}(x) - \frac{x K_{SP}(x)}{\beta} \right) > \frac{1}{\beta}, \tag{26}$$

whenever $x > 0$. Given equation (25), we can therefore conclude that $P^* > 1/\beta^2$ for any positive probability of crisis. The price of bailouts associated with the social optimum is thus higher for individual consumers, for a loan with a maturity of less than a period, than the one-period world interest rate.

It is interesting to analyze economies where the price of bailouts is lower than P^*, since the available empirical evidence suggests that governments in emerging markets actually lose resources when trying to help the financial system in a crisis (Dziobek and Pazarbasioglu (1997), Kaminsky and Reinhart (1999) and Eichengreen and Rose (1997)).

If the price of bailouts is such that $P \leq 1/\beta^2$, the consumers will not internalize the full social costs of using risky borrowed capital in production. For any positive level of bailout guarantees, this will lead the consumers to choose a level of borrowed capital that is too high compared to the social optimum. This is formally stated in proposition 3.[9]

Proposition 3 *For a given positive probability of crisis and* $P \leq 1/\beta^2$*,* $k(x) \geq K_{SP}(x)$*, with equality iff* $x = 0$*.*

Proof. See appendix B. ■

[9] We omit the analysis of the cases when P lies between $1/\beta^2$ and P^*, in order to focus on prices of bailouts that are empirically relevant. For $1/\beta^2 < P < P^*$, propositions 3 and 4 do not hold, but it is the case that $k(x_{SP}^*) > K_{SP}(x_{SP}^*)$ and $T(x_{SP}^*) > T_{SP}(x_{SP}^*)$.

From the atomistic consumer's perspective, a bailout guarantee at a price lower than $1/\beta^2$ increases the expected return on borrowed capital, without appropriately increasing the perceived cost of taking loans. Only for $x = 0$, i.e. a policy of *No Bailout*, do the investment allocations in the centralized and decentralized solutions coincide. This can easily be seen by comparing the consumer's optimality condition for borrowed capital in equation (11) with the social first-order condition in equation (17).

A consequence of the distorted investment decisions of the individual consumer is that to optimally smooth government consumption for a given bailout policy x, the government must set the lump-sum tax so that government revenues are higher than in the centralized solution.

Proposition 4 *For a given positive probability of crisis and $P \leq 1/\beta^2$, $T(x) \geq T_{SP}(x)$, with equality iff $x = 0$.*

Proof. See appendix B. ■

The extreme case of suboptimally priced bailouts is a model economy where bailouts are provided free of charge, so that their price is $P = 0$. In such an environment, the level of borrowed capital will be monotonically increasing in the level of bailout guarantees, since for a higher level of bailouts, the consumer can keep a larger part of his borrowed capital in production during a crisis, without having to pay for it.

Proposition 5 *For a given positive probability of crisis and $P = 0$, the optimal level of borrowed capital, $k(x)$, and the optimal level of government revenues, $T(x)$, are increasing in the level of bailout guarantees. Formally, $k'(x) > 0$ and $T'(x) > 0$.*

Proof. See appendix B. ■

The result concerning government revenues in proposition 5 can be understood by considering that the government needs more resources if it is to pursue a higher level of bailouts. The fact that the individual consumer takes larger loans for a higher level of bailouts further increases the need for resources in order to smooth government consumption.

When the government is restricted to providing bailouts for free, the policy considerations differ from the case when bailouts are optimally priced. Free bailouts induce consumers to choose higher levels of borrowed capital, which aggravates the social cost of providing the bailout guarantees in the first place. However, Proposition 6 states that in such an environment, the government should still provide a positive level of bailout guarantees.

Proposition 6 *For any positive crisis probability and $P = 0$, the government should optimally choose a bailout policy in the interior of the policy space, and should thereby* *only partially protect private investment against liquidation. Formally,* $X^* \subset (0,1)$*. Furthermore, for any positive crisis probability, the government's optimal bailout policy is unique.*

Proof. See appendix B. ■

According to Proposition 6, the government should optimally set the derivative in equation (23) equal to zero, which implies that its optimality condition for the bailout policy can be written as

$$f'\left(x^* k(x^*)\right) = \frac{1}{\beta} v'\left(T(x^*) - \frac{x^* k(x^*)}{\beta}\right)\left[1 + \frac{x^* k'(x^*)}{k(x^*)}\right], \tag{27}$$

where we have substituted for $P = 0$ and $D(x)$ in equation (23) and x^* is the optimal bailout policy. The right-hand side of equation (27) captures the fact that the cost of bailouts is larger in the decentralized economy with free bailouts than in the centralized economy. Compared to the optimality condition in the centralized economy in equation (21), the right-hand side of equation (27) contains an additional cost term associated with the distortion of investment decisions. In addition to reducing government consumption as in the centralized economy, bailout guarantees induce the individual consumers to choose suboptimally high levels of borrowed capital, which aggravates the volatility of government consumption in the decentralized economy. The left-hand side of equation (27) shows that when bailouts are provided for free, their social benefit is the same as in the centralized economy, i.e. bailouts help avoiding inefficient liquidation in a crisis.

Proposition 6 tells us that, as in the centralized economy, the government chooses a bailout policy in between the extremes of *No Bailout* and *Full Bailout*, to optimally weigh the benefits against the costs of bailouts. Uniqueness of the optimal bailout policy is ascertained by the fact that the benefit on the left-hand side of equation (27) is decreasing in x, while the cost on the right-hand side increases with the bailout policy.

In an environment where bailouts must be provided for free, the government must choose a level of bailout guarantees addressing the problem of consumers' overinvestment. The distorted investment incentives thus imply that the government's optimal bailout policy must deviate from the bailout policy in the social optimum.

Proposition 7 *For any positive crisis probability, the optimal level of bailout guarantees is lower in the decentralized economy with* $P = 0$ *than in the centralized economy. Formally,* $x^* < x^*_{SP}$*.*

Proof. See appendix B. ■

The reason for the result in Proposition 7 is that the cost associated with bailouts is higher in the decentralized economy, whereas the benefit of bailouts is the same as in the centralized economy. The distortion leads to larger fluctuations in government consumption for a given bailout policy. Since the marginal benefit of reducing inefficient liquidation decreases with the level of bailouts, the government must choose a lower bailout policy to make the benefit equal the cost.

Just as in the centralized economy, the model enables us to analyze how the optimal bailout policy varies across countries with different probabilities of experiencing a crisis. When bailouts are provided for free, the optimality condition in equation (27) implicitly defines the optimal bailout policy, x^*, as a function of the crisis probability, π, since the decision rules $k(x)$ and $T(x)$ depend on π.

Proposition 8 *For a higher probability of crisis and $P = 0$, the government should optimally commit to bailing out a smaller fraction of the borrowed capital in the economy. Formally, for any π_1 and π_2 such that $\pi_1 < \pi_2$, it is the case that $x^*(\pi_1) > x^*(\pi_2)$.*

Proof. See appendix B. ■

The result in proposition 8 stands in stark contrast to the centralized economy, where a higher probability of crisis implied a higher optimal level of bailouts. When bailouts are provided for free, the consumer chooses a higher level of borrowed capital when the probability of crisis is higher, for a given bailout policy x. From the atomistic consumer's perspective, a higher probability of crisis increases the expected net return to borrowed capital, since it is more likely that he gets to keep part of the return of the investment, without having to pay for it. For a given bailout policy, a higher level of borrowed capital decreases the social benefit and increases the social cost of bailouts. For a higher probability of crisis, the government must therefore choose a lower level of bailout guarantees to make the benefit equal the cost.

7 Relaxing the commitment assumption in the model

The formal analysis and the policy conclusions of the paper would remain unchanged if in each period, the government were allowed to commit to a bailout policy for the subsequent period. In the centralized economy, the analysis and the results are robust to relaxing the commitment assumption altogether and allowing the social planner to reconsider the level of bailouts, once a crisis has occurred.

As noted in Section 5, the social planner's problem can be formulated without bailout policies and policy functions for a given level of bailouts. In a centralized economy without the artificial assumption of commitment, the social planner would simply invest foreign capital in production until the marginal product equalled $1/\beta$, and transfer the optimal amount of resources to the public sector in the next period, knowing how much resources he would want to spend on productive capital and government consumption in the event of a crisis. In the actual event of a crisis in the next period, the social planner would not want to act differently than what he foresaw one period ago.

In the market economy, the government's optimal bailout policy depends on the consumer's policy function for the level of borrowed capital, which is state independent as noted in the discussion of equation (11). In each period, the government would therefore optimally choose the same level of bailout guarantees for the subsequent period. This level would be identical to the optimal bailout under commitment in period $t = 0$, since the relative magnitudes of the costs and benefits associated with bailouts would be unchanged. In such an environment, the equilibrium for period $t = 0$, which we defined in Section 3, would be replaced by a definition of equilibrium under one-period commitment, which would be valid for any period t.

8 A numerical example

In this section, we present a numerical example to illustrate the model outcomes for the centralized economy and an economy where bailouts are provided for free. This example is not intended to reveal any new results on the optimal bailout policy, but rather to convey a sense of the relative magnitudes of the variables in the model for a reasonable parameterization. The graphs presented in figures 1 and 2 quantify the effects of bailouts that have so far only been analytically investigated in the paper.

We assume the utility function for government consumption to be $v(g) = \gamma \ln(g)$, where γ represents the relative weight of government consumption in the consumers' utility. As stated in Section 2, the production function is assumed to be of Cobb-Douglas form, $f(k) = Ak^{\alpha}$.

The parameter values used in the numerical example are presented in Table 1. We assume the length of a period to be one year. Using a standard value in the literature, we set $\beta = 0.95$, which implies a world interest rate of about 5 percent. Gollin (2002) shows the capital income share, α, to roughly equal 1/3 for a large number of countries around the world. The TFP parameter, A, is normalized to unity.

The value for the domestic endowment, ω, should ideally be obtained by matching the model's ratio of borrowed capital to total output, $k/(Ak^{\alpha} + \omega)$, with the ratio of short-term foreign debt to GDP in the data. Rodrik and Velasco (1999) consider 16 episodes of financial crisis emerging markets between 1990 and 1998, and find that the average ratio of capital outflows to GDP was 0.09. Interpreting this observed ratio in terms of our model is problematic, however, since the level of borrowed capital in the model depends on the probability of a crisis, π, and the bailout policy, x. However, as shown in section 4, the level of borrowed capital in the model is independent of the bailout policy, if the crisis probability is zero, which might be the case for the largest economies in the industrialized world. Assuming the ratio of short-term foreign debt to GDP to be

Parameter	Value	Description
P	0	Price of bailouts in the decentralized economy
β	0.95	Discount factor
α	1/3	Capital income share
A	1	Productivity
ω	3.00	Domestic endowment of consumption goods
γ	1.05	Relative weight of government consumption in utility
π	$[0, 1/2]$	Probability of crisis

Table 1: Parameter values used in the numerical example

lower in these economies than in emerging markets, we set the ratio of short-term debt to GDP to 0.05,

$$\frac{k}{\omega + Ak^{\alpha}} = 0.05, \tag{28}$$

in a country where $\pi = 0$. This enables us to find a value of ω, since when $\pi = 0$,

$$k = (A\beta\alpha)^{\frac{1}{1-\alpha}}, \tag{29}$$

according to equation (11).

The value for γ is also chosen for a country where $\pi = 0$. In this case, the size of the equilibrium government revenues can be expressed as

$$T = \beta\gamma, \tag{30}$$

according to equation (12). The value of γ is set so that the ratio of government revenues to total output in the model equals the average ratio of government spending to GDP in the G7 countries between 1990 and 1992. According to Rodrik (1998), the G7 average is 0.28, so that

$$\frac{T}{Ak^{\alpha} + \omega} = 0.28, \tag{31}$$

and

$$\gamma = \frac{0.28}{\beta}\left(Ak^{\alpha} + \omega\right). \tag{32}$$

When presenting the solutions to the model, we only consider crisis probabilities up to 1/2, since it is hard to imagine countries for which the crisis probability would exceed 1/2 for a prolonged period of time.

For a given probability of crisis, figure 1.a shows how the consumer's optimal choice of borrowed capital varies with the level of bailout guarantees in the decentralized economy with free bailouts. In line with proposition 5, we see that the level of borrowed capital is increasing in the level of bailout guarantees. As stated in proposition 3, we also see that, for a given $\pi > 0$ and a given $x > 0$, the level of borrowed capital is higher than in figure 1.b, which presents the level of borrowed capital in the centralized economy.

Interestingly, figure 1.b reveals that the social planner optimally decreases the level of foreign capital for higher levels of bailouts to reduce the social cost of taking loans.

Analogously, figure 1.c shows that in the decentralized economy, the optimal level of government revenues increases in the level of bailout guarantees for a given probability of crisis, which is in line with proposition 5. Comparing figures 1.c and 1.d, we also see that, for a given $\pi > 0$ and a given $x > 0$, the level of government revenues is higher in the market economy than in the centralized economy.

For a given probability of crisis, figure 2.a shows the expected utility attained by the government by committing to different levels of bailout guarantees in the environment where bailouts must be provided for free. At least for higher levels of π, we can see that Proposition 6 holds, since the expected utility function reaches a unique maximum in the interior of the policy space. Single peakedness and interior solutions are harder to discern in figure 2.b, which shows the expected utility function in the centralized economy. However, figure 2.c numerically verifies proposition 1, by showing that for all crisis probabilities, the socially optimal bailout policy is indeed unique and interior in our numerical example. The same figure also illustrates propositions 2,7 and 8. We see that the socially optimal bailout policy is increasing in the probability of crisis, that the optimal bailout policy in the decentralized economy is lower than in the social optimum for any positive crisis probability, and that the optimal bailout policy in the decentralized economy is decreasing in the probability of crisis. In figure 2.c, we cannot plot the optimal bailout policies for $\pi = 0$, since the bailout policy is irrelevant if crises never occur.

Finally, in figure 2.d, we compare the expected welfare attained in period $t = 0$ under the optimal bailout policy in the decentralized economy to the social optimum. We see that the value attainable to the government in the environment of free bailouts lies below the social optimum for positive probabilities of crisis, and that the economy is worse off for a higher probability of crisis.

For all crisis probabilities considered in the numerical example, private consumption is positive in the entire policy space.

9 Concluding remarks

The model presented in this paper provides a framework for analyzing the effects of bailout policies in a general equilibrium environment including both benefits and costs of bailouts. Considering both aspects of bailout guarantees, the model provides a beginning to bridging the gap between the two strands in the literature treating bailouts as a 'good' or a 'bad' policy.

We showed that committing to a partial bailout of borrowed capital is always socially optimal in the centralized economy. The extreme policy of *No Bailout* is inferior, since the benefit of reducing the inefficient liquidation associated with such a policy outweighs the cost of increased volatility in government consumption. Similarly, a policy of *Full Bailout* can be improved on by reducing the associated volatility of government consumption at the expense of some inefficient liquidation.

In the centralized economy, it was further shown that the social planner optimally chooses a higher level of bailout guarantees and public sector revenues in countries with a higher probability of crisis. For a higher probability of crisis, the social planner provides more protection against inefficient liquidation and provides more resources to the public sector to guard against low government consumption in crisis periods, at the expense of private sector consumption.

In a decentralized economy, the conclusions of the model depend on the price of bailouts that the government charges the private sector. Empirical evidence indicates that the economically relevant price of bailouts in a decentralized economy is below the price associated with the social optimum in the model. With a sub-optimally low price of bailouts, the decentralized model economy exhibits higher levels of foreign borrowing than what is socially optimal, since the private sector can enjoy the benefits of foreign borrowing without paying its full social cost. In such an environment, bailout guarantees lead to overinvestment in the emerging market.

In the extreme case when bailouts must be provided for free, we showed that, analogously to the centralized economy, the government should commit to a bailout policy only partially protecting private investment against liquidation. The optimal level of bailout guarantees in such an environment will always be lower than in the social optimum, however. Due to the investment distortions induced by bailouts, a given bailout policy leads to larger fluctuations in government consumption in the decentralized economy. In equilibrium, the government must counter the higher cost of bailouts by choosing a lower level of bailout guarantees.

We also showed that, in contrast to the results for the centralized economy, the government optimally bails out a smaller fraction of private investments for a higher probability of crisis. When crises occur more frequently, the government finds it optimal to expose the private sector more to the negative effects of crises, to make the consumers internalize more of the social costs of foreign borrowing.

In this paper, we have restricted our analysis to the optimal bailout policy under commitment and an exogenous price of bailouts. If the government could choose the price of bailouts, the endogenous price would be P^*, which is the price of bailouts associated with the social optimum. In such a model, the optimal bailout policy would be time consistent. It would be an interesting topic for future research to investigate what the government should optimally do if it cannot commit nor control the price of bailouts. We believe that a version of our model incorporating a reputational mechanism for the government could provide an appropriate framework for starting to analyze the optimal credible bailout policy.

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A Derivations of first-order conditions

A.1 Derivation of the consumer's first-order condition with respect to k_{t+1}

Using the investment constraint in equation (4) to substitute for b_{t+1}, the consumer's budget constraint to substitute for c_t, the form of the function $\kappa(.)$ and the fact that $R = 1/\beta$, the value function in equation (9) can be written as

$$V^C(x, h_t) = \max_{k_{t+1}} \Big\{ f(\kappa_t) + \omega - (1-\zeta_t)\frac{1}{\beta} b_t - \zeta_t P \kappa_t - Q_t + v(g_t) + \tag{33}$$
$$+\beta(1-\pi)\left(f(k_{t+1}) + \omega - \frac{1}{\beta}k_{t+1} - Q(x, S_{t+1,0}) + v(g_{t+1,0}) + \beta E_{t+1}\left[V^C(x, h_{t+2})|\zeta_{t+1} = 0\right]\right)$$
$$+\beta\pi\left(f(xk_{t+1}) + \omega - Pxk_{t+1} - Q(x, S_{t+1,1}) + v(g_{t+1,1}) + \beta E_{t+1}\left[V^C(x, h_{t+2})|\zeta_{t+1} = 1\right]\right) \Big\}$$
$$s.t. \qquad k_{t+1} \geq 0,$$

where $S_{t+1,0} = (K_{t+1}, B_{t+1}, T_t, 0)$, $S_{t+1,1} = (K_{t+1}, B_{t+1}, T_t, 1)$, $g_{t+1,0} = T_t$ and $g_{t+1,1} = T_t - \frac{x}{\beta}B_{t+1}$. The first-order condition of (33) with respect to k_{t+1} is given by equation (11).

A.2 Derivation of the government's first-order condition with respect to T_t

Using equation (7), we recast the government's problem in equation (10) as a maximization with respect to bailouts and government revenues. Using the fact that $k_{t+1} = K_{t+1}$ and $b_{t+1} = B_{t+1}$, the investment constraint in equation (4) enables us to substitute b_{t+1} and B_{t+1} with $k(x)$. Employing the consumer's optimal consumption rule, the government's budget constraint and substituting for $R = 1/\beta$, the government's second value function in equation (10) can be written as

$$V^G(x, S_t) = \max_{T_t} \Big\{ f(\kappa(x, K_t, \zeta_t)) + \omega - (1-\zeta_t)\frac{1}{\beta}B_t - T_t + v(g_t) \tag{34}$$
$$+\beta(1-\pi)\left(f(k(x)) + \omega - \frac{1}{\beta}k(x) - T_{t+1} + v(T_t) + \beta E_{t+1}\left[V^G(x, S_{t+2})|\zeta_{t+1} = 0\right]\right)$$
$$+\beta\pi\left(f(xk(x)) + \omega - T_{t+1} + v\left(T_t - \frac{xk(x)}{\beta}\right) + \beta E_{t+1}\left[V^G(x, S_{t+2})|\zeta_{t+1} = 1\right]\right)\Big\}$$
$$s.t. \qquad T_t - \frac{xk(x)}{\beta} \geq 0.$$

The first-order condition of (34) with respect to T_t is given by equation (12).

A.3 Derivation of the Social Planner's first-order conditions with respect to K_{t+1} and T_t

Using the budget constraint for private and government consumption to substitute for c_t and g_t, the investment constraint to substitute for B_{t+1}, the form of the function $\kappa(.)$ and the fact that $R = 1/\beta$, the social planner's value function in equation (14) can be written as

$$
\begin{aligned}
V^{SP}(x, S_t) = \max_{\{K_{t+1}, T_t\}} \Bigg\{ & f\left(\kappa(x, K_t, \zeta_t)\right) + \omega - (1-\zeta_t)\frac{1}{\beta}B_t - T_t + v(g_t) \qquad (35) \\
&+\beta(1-\pi)\left(f\left(K_{t+1}\right) + \omega - \frac{1}{\beta}K_{t+1} - T_{t+1} + v(T_t) + \beta E_{t+1}\left[V^{SP}(x, S_{t+2})|\zeta_{t+1} = 0\right]\right) \\
&+\beta\pi\left(f\left(xK_{t+1}\right) + \omega - T_{t+1} + v\left(T_t - \frac{xK_{t+1}}{\beta}\right) + \beta E_{t+1}\left[V^{SP}(x, S_{t+2})|\zeta_{t+1} = 1\right]\right)\Bigg\} \\
&s.t. \qquad K_{t+1}, T_t - \frac{xK_{t+1}}{\beta} \geq 0.
\end{aligned}
$$

The first-order conditions of (35) with respect to K_{t+1} and T_t are given by equations (17) and (18).

A.4 Derivation of the government's first-order condition with respect to x

Using the Implicit Function Theorem, it can be shown that equations (11) and (12) implicitly define the policy functions $k(x)$ and $T(x)$ as continuously differentiable in x. This, in turn, implies that the government's expected utility function in (22) is continuously differentiable in x. Since x must be an element of the compact set $[0, 1]$, we know by the Weierstrass Theorem that the government's set of optimal bailout policies, X^*, is non-empty and compact.

Substituting for private and government consumption in (22), and differentiating the government's expected utility function w.r.t. x, we obtain

$$
\begin{aligned}
&\frac{d}{dx}\left\{c_0(x) + v\left(T_{-1}\right) + \beta E_0 V^G(x, S_1)\right\} = \\
&\quad -T'(x) + \frac{\beta(1-\pi)}{1-\beta}\left[f'\left(k(x)\right)k'(x) - \frac{1}{\beta}k'(x) - T'(x) + v'(T(x))T'(x)\right] \\
&\quad +\frac{\beta\pi}{1-\beta}\left[f'\left(xk(x)\right)\left[k(x) + xk'(x)\right] - T'(x)\right] \qquad (36) \\
&\quad +\frac{\beta\pi}{1-\beta}v'\left(T(x) - \frac{xk(x)}{\beta}\right)\left[T'(x) - \frac{k(x)}{\beta} - \frac{xk'(x)}{\beta}\right].
\end{aligned}
$$

Rearranging terms and adding and subtracting $\frac{\beta}{1-\beta}k'(x)\pi Px$, we obtain

$$\frac{d}{dx}\left\{c_0(x)+v\left(T_{-1}\right)+\beta E_0 V^G(x,S_1)\right\}=$$

$$\frac{\beta k'(x)}{1-\beta}\left[\underbrace{(1-\pi)\,f'\left(k(x)\right)+\pi x f'\left(xk(x)\right)-\frac{(1-\pi)}{\beta}-\pi Px}_{C_1(x)}+\pi Px-\frac{\pi x}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right)\right]$$

$$+\frac{\beta T'(x)}{1-\beta}\underbrace{\left[(1-\pi)\,v'\left(T(x)\right)+\pi v'\left(T(x)-\frac{xk(x)}{\beta}\right)-\frac{1}{\beta}\right]}_{C_2(x)}$$

$$+\frac{\beta\pi k(x)}{1-\beta}\left[f'\left(xk(x)\right)-\frac{1}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right)\right]. \qquad (37)$$

Now, $C_1(x)=0$ by equation (11) and $C_2(x)=0$ by equation (12), which implies that

$$\frac{d}{dx}\left\{c_0(x)+v\left(T_{-1}\right)+\beta E_0 V^G(x,S_1)\right\}= \qquad (38)$$

$$=\frac{\beta}{1-\beta}\pi\left(k(x)\left[f'\left(xk(x)\right)-\frac{1}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right)\right]+xk'(x)D(x)\right),$$

where $D(x)$ is defined as

$$D(x)=P-\frac{1}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right).$$

B Proofs of propositions

B.1 Proof of Proposition 1

Using the Implicit Function Theorem, it can be shown that equations (17) and (18) implicitly define the policy functions $K_{SP}(x)$ and $T_{SP}(x)$ as continuously differentiable in x. This, in turn, implies that the expected utility function in equation (19) is continuously differentiable in x.

The derivative of the expected utility function w.r.t. x can be found by applying the Envelope Theorem, since the policy functions, $K_{SP}(x)$ and $T_{SP}(x)$, have been defined as the socially optimal choices for each given bailout policy x.

To see that a policy of *No Bailout* is never optimal in the social planner's solution, consider the limit of the derivative of the expected utility function in equation (20),

$$\lim_{x\to 0}\left\{\frac{\beta}{1-\beta}\pi K_{SP}(x)\left[f'\left(xK_{SP}(x)\right)-\frac{1}{\beta}v'\left(T_{SP}(x)-\frac{xK_{SP}(x)}{\beta}\right)\right]\right\}=$$
$$\frac{\beta}{1-\beta}\pi K_{SP}(0)\left[f'\left(0\right)-\frac{1}{\beta}v'\left(T_{SP}(0)\right)\right]. \tag{39}$$

The limit value in equation (39) is clearly positive when $\pi > 0$, since $K_{SP}(0) > 0$, $f'(0) = \infty$ and $v'(T_{SP}(0)) = 1/\beta$ by equation (18). By continuity of the expected utility function, we can conclude that a policy of *No Bailout* is never optimal. Formally, $0 \notin X_{SP}^*$.

To see that a policy of *Full Bailout* is never optimal in the social planner's solution, consider the limit

$$\lim_{x\to 1}\left\{\frac{\beta}{1-\beta}\pi K_{SP}(x)\left[f'\left(xK_{SP}(x)\right)-\frac{1}{\beta}v'\left(T_{SP}(x)-\frac{xK_{SP}(x)}{\beta}\right)\right]\right\}=$$
$$\frac{\beta}{1-\beta}\pi K_{SP}(1)\left[\frac{(1-\pi)}{\beta}-\frac{(1-\pi)}{\beta}v'\left(T_{SP}(1)-\frac{K_{SP}(1)}{\beta}\right)\right], \tag{40}$$

where we have used the fact that

$$f'\left(K_{SP}(1)\right)=(1-\pi)\frac{1}{\beta}+\pi v'\left(T_{SP}(1)-\frac{K_{SP}(1)}{\beta}\right)\frac{1}{\beta}, \tag{41}$$

according to equation (17). The derivative of the expected utility function in equation (20) is negative in the upper limit when $\pi > 0$, since by equation (18),

$$v'\left(T_{SP}(x)-\frac{x}{\beta}K_{SP}(x)\right)\geq\frac{1}{\beta}.$$

By continuity of the expected utility function, we can therefore conclude that a policy of *Full Bailout* is never optimal in the social planner's solution. Formally, $1 \notin X_{SP}^*$.

To prove that the social planner's optimal bailout policy is unique, first note that in equilibrium, the equilibrium values of $K_{SP}(x), T_{SP}(x)$ and X_{SP}^* are determined by a system of the three following equations

$$(1-\pi)\,f'\left(K_{SP}(x_{SP}^*)\right)+\pi x_{SP}^* f'\left(x_{SP}^*K_{SP}(x_{SP}^*)\right)=$$
$$(1-\pi)\frac{1}{\beta}+\pi\frac{x_{SP}^*}{\beta}v'\left(T_{SP}(x_{SP}^*)-\frac{x_{SP}^*K_{SP}(x_{SP}^*)}{\beta}\right) \tag{42a}$$

$$(1-\pi)\,v'\left(T_{SP}(x_{SP}^*)\right)+\pi v'\left(T_{SP}(x_{SP}^*)-\frac{x_{SP}^*K_{SP}(x_{SP}^*)}{\beta}\right)=\frac{1}{\beta} \tag{42b}$$

$$f'\left(x_{SP}^*K_{SP}(x_{SP}^*)\right)=\frac{1}{\beta}v'\left(T_{SP}(x_{SP}^*)-\frac{x_{SP}^*K_{SP}(x_{SP}^*)}{\beta}\right), \tag{42c}$$

where equation (42c) holds because the optimal policy must be interior. Substituting (42c) into (42a) leads to

$$\pi x_{SP}^{*} f'\left(x_{SP}^{*} K_{SP}(x_{SP}^{*})\right)=(1-\pi) f'\left(K_{SP}(x_{SP}^{*})\right)+\pi x_{SP}^{*} f'\left(x_{SP}^{*} K_{SP}(x_{SP}^{*})\right)-(1-\pi) \frac{1}{\beta},$$

which implies that

$$\begin{aligned} f'\left(K_{SP}(x_{SP}^{*})\right) & =\frac{1}{\beta} \\ K_{SP}(x_{SP}^{*}) & =f'^{-1}\left(\frac{1}{\beta}\right). \end{aligned} \tag{43}$$

Thus, in equilibrium, the social planner's optimal level of investments is constant. We denote it by K_{SP}^{*}.

Next, by expressing $T_{SP}(x_{SP}^{*})$ from (42c), we obtain

$$T_{SP}(x_{SP}^{*})=w\left(\beta f'\left(x_{SP}^{*} K_{SP}^{*}\right)\right)+\frac{x_{SP}^{*} K_{SP}^{*}}{\beta}, \tag{44}$$

where $w=(v')^{-1}$. The value of x_{SP}^{*} can then be determined from equation (42b). After substituting the expression for $T_{SP}(x_{SP}^{*})$ into (42b) and rearranging it, we obtain

$$(1-\pi) v'\left(w\left(\beta f'\left(x_{SP}^{*} K_{SP}^{*}\right)\right)+\frac{x_{SP}^{*} K_{SP}^{*}}{\beta}\right)=\frac{1}{\beta}-\pi \beta f'\left(x_{SP}^{*} K_{SP}^{*}\right). \tag{45}$$

For uniqueness of x_{SP}^{*}, it needs to be demonstrated that the right-hand side of (45) is increasing in x, while the left-hand side of (45) is decreasing in x. We have that

$$\frac{\partial}{\partial x}\left\{\frac{1}{\beta}-\pi \beta f'\left(x K_{SP}^{*}\right)\right\}=-\pi \beta f''(x K_{SP}^{*}) K_{SP}^{*},$$

which is positive since $f''(.)<0$, and

$$\begin{gathered} \frac{\partial}{\partial x}\left\{(1-\pi) v'\left(w\left(\beta f'\left(x K_{SP}^{*}\right)\right)+\frac{x K_{SP}^{*}}{\beta}\right)\right\}= \\ (1-\pi) v''\left(w\left(\beta f'\left(x K_{SP}^{*}\right)\right)+\frac{x K_{SP}^{*}}{\beta}\right)\left(w'(\beta f'\left(x K_{SP}^{*}\right)) \beta f''(x K_{SP}^{*}) K_{SP}^{*}+\frac{K_{SP}^{*}}{\beta}\right), \end{gathered}$$

which is negative, since $v''(.)<0, w'(.)<0$ and $f''(.)<0$.■

B.2 Proof of Proposition 2

According to equation (42b), the socially optimal bailout policy satisfies

$$(1-\pi) v'\left(T_{SP}(x_{SP}^{*})\right)+\pi v'\left(T_{SP}(x_{SP}^{*})-\frac{x_{SP}^{*} K_{SP}^{*}}{\beta}\right)=\frac{1}{\beta}, \tag{46}$$

where K^*_{SP} is given by (43) and $T_{SP}(x^*_{SP})$ is given by (44). Implicitly differentiating (46) w.r.t. π, and rearranging, we obtain

$$\frac{\partial x^*_{SP}}{\partial \pi} = \frac{v'\left(T_{SP}(x^*_{SP})\right) - v'\left(T_{SP}(x^*_{SP}) - \frac{x^*_{SP}K^*_{SP}}{\beta}\right)}{\left(1-\pi\right) v''\left(T_{SP}(x^*_{SP})\right) T'_{SP}(x^*_{SP}) + \pi v''\left(T_{SP}(x^*_{SP}) - \frac{x^*_{SP}K^*_{SP}}{\beta}\right)\left(T'_{SP}(x^*_{SP}) - \frac{K^*_{SP}}{\beta}\right)}. \tag{47}$$

Next, according to equation (44), the derivative of $T_{SP}(x^*_{SP})$ w.r.t. x^*_{SP} is

$$T'_{SP}(x^*_{SP}) = \frac{K^*_{SP}}{\beta} + w'(\beta f'\left(x^*_{SP}K^*_{SP}\right))\beta f''(x^*_{SP}K^*_{SP})K^*_{SP}, \tag{48}$$

which is positive, since $w'(.) < 0$ and $f''(.) < 0$. We conclude that $\frac{\partial x^*_{SP}}{\partial \pi} > 0$, since

$$v'\left(T_{SP}(x^*_{SP})\right) - v'\left(T_{SP}(x^*_{SP}) - \frac{x^*_{SP}K_{SP}}{\beta}\right) < 0,$$

$v''(.) < 0$, $T'_{SP}(x^*_{SP}) > 0$ and since $T'_{SP}(x^*_{SP}) - \frac{K^*_{SP}}{\beta} > 0$, according to equation (48).■

B.3 Proof of Proposition 3

For any given bailout policy, x, and any given probability of crisis, $\pi > 0$, compare the first-order conditions for borrowed capital in the decentralized and centralized economies, respectively

$$\left(1-\pi\right) f'\left(k(x)\right) + \pi x f'\left(xk(x)\right) = (1-\pi)\frac{1}{\beta} + \pi x P \tag{49}$$

$$\left(1-\pi\right) f'\left(K_{SP}(x)\right) + \pi x f'\left(xK_{SP}(x)\right) = \left(1-\pi\right)\frac{1}{\beta} + \pi x \frac{1}{\beta} v'\left(T_t - \frac{xK_{SP}(x)}{\beta}\right). \tag{50}$$

For $P \leq 1/\beta^2$, the left-hand side of equation (49) must be smaller than the left-hand side of equation (50) if $x > 0$. This, together with the concavity of the production function, implies that $k(x) \geq K_{SP}(x)$, with equality if and only if $x = 0$.■

B.4 Proof of Proposition 4

For any given bailout policy, x, and any given probability of crisis, $\pi > 0$, compare the first-order conditions for the lump-sum tax in the decentralized and centralized economies, respectively

$$\left(1-\pi\right) v'\left(T(x)\right) + \pi v'\left(T(x) - \frac{xk(x)}{\beta}\right) = \frac{1}{\beta} \tag{51}$$

$$\left(1-\pi\right) v'\left(T_{SP}(x)\right) + \pi v'\left(T_{SP}(x) - \frac{xK_{SP}(x)}{\beta}\right) = \frac{1}{\beta}. \tag{52}$$

By Proposition 3, $k(x) \geq K_{SP}(x)$, when $P \leq 1/\beta^2$, with equality iff $x = 0$. Together with strict concavity of the utility function $v(.)$, this implies that $T(x) \geq T_{SP}(x)$, with equality if and only if $x = 0$.■

B.5 Proof of Proposition 5

With $P = 0$, the optimality condition for investments in the decentralized economy in equation (11) becomes

$$(1-\pi) f'(k_{t+1}) + \pi x f'(x k_{t+1}) = (1-\pi)\frac{1}{\beta}. \tag{53}$$

Implicitly differentiating (53) w.r.t. x, and rearranging, we obtain

$$k'(x) = -\frac{\pi \left[f'(xk(x)) + xk(x) f''(xk(x)) \right]}{(1-\pi) f''(k(x)) + x^2 \pi f''(xk(x))}, \tag{54}$$

which is positive when $\pi > 0$, since $f''(.) < 0$ and $f'(i) + i f''(i) > 0$ for any Cobb-Douglas production function.

Implicitly differentiating the first-order condition in equation (12) w.r.t. x, and rearranging, we obtain

$$T'(x) = \frac{\pi v''\left(T(x) - \frac{x}{\beta}k(x)\right)\left[k(x) + xk'(x)\right]}{\beta\left[(1-\pi)v''(T(x)) + \pi v''\left(T(x) - \frac{x}{\beta}k(x)\right)\right]}, \tag{55}$$

which is positive when $\pi > 0$, since $v''(.) < 0$ and $k'(x) > 0$ according to equation (54).■

B.6 Proof of Proposition 6

To see that the government never finds it optimal to choose a policy of *No Bailout*, consider the limit of (23) when $P = 0$ and x approaches zero,

$$\lim_{x \to 0} \left\{ \frac{\beta}{1-\beta} \pi \left(k(x) \left[f'(xk(x)) - \frac{1}{\beta} v'\left(T(x) - \frac{xk(x)}{\beta} \right) \right] + xk'(x) D(x) \right) \right\} = \frac{\beta}{1-\beta} \pi k(0) \left[f'(0) - \frac{1}{\beta} v'(T(0)) \right]. \tag{56}$$

The limit value in equation (56) is clearly positive when $\pi > 0$, since $k(0) > 0$, $f'(0) = \infty$ and $v'(T(0)) = 1/\beta$ by equation (12). By continuity of the objective function, we can conclude that the government never finds a policy of *No Bailout* optimal. Formally, $0 \notin X^*$.

To see that the government never finds it optimal to choose a policy of *Full Bailout* when $P = 0$, consider the limit

$$\lim_{x \to 1} \left\{ \frac{\beta}{1-\beta}\pi \left(k(x) \left[f'(xk(x)) - \frac{1}{\beta} v' \left(T(x) - \frac{xk(x)}{\beta} \right) \right] + xk'(x)D(x) \right) \right\} =$$
$$\frac{\beta}{1-\beta}\pi \left(k(1) \left[(1-\pi)\frac{1}{\beta} - \frac{1}{\beta} v' \left(T(1) - \frac{k(1)}{\beta} \right) \right] - k'(1)v' \left(T(1) - \frac{k(1)}{\beta} \right) \frac{1}{\beta} \right), \quad (57)$$

where we have used the fact that $f'(k(1)) = (1-\pi)/\beta$ according to equation (11) when $P = 0$. The derivative of the expected utility function is negative in the upper limit when $\pi > 0$, since $k'(.) > 0$ according to proposition 5, $v'(.) > 0$ and since, by equation (12),

$$v' \left(T(x) - \frac{x}{\beta} k(x) \right) \geq \frac{1}{\beta}.$$

By continuity of the objective function, we can therefore conclude that the government never finds a policy of *Full Bailout* optimal. Formally, $1 \notin X^*$.

The fact that the optimal bailout policy must lie in the interior of the policy space implies that any optimal bailout policy satisfies

$$f'(x^* k(x^*)) = \frac{1}{\beta} v' \left(T(x^*) - \frac{x^* k(x^*)}{\beta} \right) \left[1 + \frac{x^* k'(x^*)}{k(x^*)} \right], \quad (58)$$

where we have substituted $P = 0$ into equation (23) and rearranged under the condition that the derivative w.r.t. x equals zero.

To prove that the optimal bailout policy is unique, we now proceed to show that the left-hand side of equation (58) is a decreasing function of x, and that the right-hand side is an increasing function of x.

The derivative of the left-hand side of equation (58) is

$$\frac{d}{dx} \left\{ f'(xk(x)) \right\} = f''(xk(x) \left[k(x) + xk'(x) \right], \quad (59)$$

which is negative, since $f''(.) < 0$ and $k'(x) > 0$ according to equation (54).

Let $p(x)$ denote the second factor on the right-hand side of equation (58). The derivative of $p(x)$ is

$$\begin{aligned} p'(x) &= \frac{d}{dx} \left\{ v' \left(T(x) - \frac{xk(x)}{\beta} \right) \right\} \quad (60) \\ &= v'' \left(T(x) - \frac{xk(x)}{\beta} \right) \left[T'(x) - \frac{k(x)}{\beta} - \frac{xk'(x)}{\beta} \right], \end{aligned}$$

which is positive, since $v''(.)$ is negative and since

$$T'(x) < \frac{1}{\beta} \left[k(x) + xk'(x) \right], \quad (61)$$

according to equation (55).

For any production function of the form $f(k) = Ak^{\alpha}$, $\alpha < 1$, the optimality condition in equation (53) implies that the consumer's policy function for borrowed capital is

$$k(x) = \left[\frac{A\alpha\beta}{(1-\pi)}\right]^{\frac{1}{1-\alpha}} \left[1 - \pi + \pi x^{\alpha}\right]^{\frac{1}{1-\alpha}}, \tag{62}$$

in an environment where $P = 0$. This, in turn, implies that the third factor on the right-hand side of equation (58), which we denote $q(x)$, can be written as

$$\begin{aligned} q(x) &= 1 + \frac{xk'(x)}{k(x)} \\ &= 1 + \frac{\alpha}{1-\alpha}\left[1 - \pi + \pi x^{\alpha}\right]^{-1} \pi x^{\alpha}. \end{aligned} \tag{63}$$

The derivative of $q(x)$ is

$$q'(x) = \frac{\alpha^2}{(1-\alpha)} \frac{\pi (1-\pi) x^{\alpha-1}}{(1 - \pi + \pi x^{\alpha})^2}, \tag{64}$$

which is positive.

The derivative of the right-hand side of equation (58) can now be expressed as

$$\frac{d}{dx}\left\{\frac{1}{\beta}p(x)q(x)\right\} = \frac{1}{\beta}\left[p'(x)q(x) + p(x)q'(x)\right], \tag{65}$$

which is positive, since $p'(x) > 0$, $q'(x) > 0$ and $v'(.) > 0$.■

B.7 Proof of Proposition 7

To construct the proof, we start by noting that in both the decentralized and the centralized economies, the equilibrium level of resources transferred from the private to the public sector can be written as a function of the optimal level of borrowed capital. According to the first-order conditions in equations (12) and (18), $T(x) = j(k(x))$ and $T_{SP}(x) = j(K_{SP}(x))$. For a given x, the partial derivative of $j(.)$ w.r.t. k is given by

$$\frac{\partial j(k)}{\partial k} = \frac{x\pi v''\left(j(k) - \frac{xk}{\beta}\right)}{\beta\left[(1-\pi)v''(j(k)) + \pi v''\left(j(k) - \frac{xk}{\beta}\right)\right]}. \tag{66}$$

Now, consider the bailout policy x^*_{SP}, which according to Proposition 1 must be such that

$$f'\left(x^*_{SP}K_{SP}(x^*_{SP})\right) - \frac{1}{\beta}v'\left(T_{SP}(x^*_{SP}) - \frac{x^*_{SP}K_{SP}(x^*_{SP})}{\beta}\right) = 0. \tag{67}$$

Using the fact that when $P = 0$ and $\pi > 0$, by Proposition 4, $k(x) > K_{SP}(x)$ for $x > 0$, we next proceed to show that evaluated at any such bailout policy x^*_{SP}, the derivative of the government's expected utility function in (23) is negative, i.e. that

$$k(x)\left[f'\left(xk(x)\right)-\frac{1}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right)\right]+xk'(x)\left[P-\frac{1}{\beta}v'\left(T(x)-\frac{xk(x)}{\beta}\right)\right]<0, \tag{68}$$

for $x=x^*_{SP}$.

Notice that, when $P=0$, the last term on the left-hand side in inequality (68) is negative, since $k'(x)>0$ according to proposition 5 and since $v'(.)>0$. The derivative of the government's expected utility function will therefore be negative whenever the term in the first square brackets in (68) is negative.

Let this term be denoted by Z, so that

$$Z(k(x))=f'\left(xk(x)\right)-\frac{1}{\beta}v'\left(j(k(x))-\frac{xk(x)}{\beta}\right). \tag{69}$$

For a given x, the partial derivative of $Z(.)$ w.r.t. k is given by

$$\frac{\partial Z(k)}{\partial k}=f''(xk)x-\frac{1}{\beta}v''\left(j(k)-\frac{xk}{\beta}\right)\left(\frac{\partial j(k)}{\partial k}-\frac{x}{\beta}\right), \tag{70}$$

which is negative, since $f''(.)<0$, $v''(.)<0$ and since, by equation (66),

$$\frac{\partial j(k)}{\partial k}<\frac{x}{\beta}.$$

Since $Z(K_{SP}(x^*_{SP}))=0$ and $k(x^*_{SP})>K_{SP}(x^*_{SP})$, the fact that the partial derivative of $Z(.)$ w.r.t. k is negative enables us to conclude that $Z(k(x^*_{SP}))<0$ and hence, that the derivative of the government's expected utility function is negative at x^*_{SP}.

The fact that the government's expected utility function is continuously differentiable and has a positive slope as x approaches zero, together with the result that there is a unique bailout policy which satisfies the first-order condition in equation (27), allow us to conclude that it must be the case that $x^*<x^*_{SP}$.■

B.8 Proof of Proposition 8

We start by showing that for a given bailout policy x and $P=0$, the level of borrowed capital, $k(x)$, is an increasing function of π. For a given bailout policy, the partial derivative of $k(x)$ w.r.t. to π is given by

$$\begin{aligned} \frac{\partial k(x)}{\partial \pi} &= \frac{\partial}{\partial \pi}\left\{\left[\frac{A\alpha\beta}{(1-\pi)}\right]^{\frac{1}{1-\alpha}}\left[(1-\pi)+\pi x^{\alpha}\right]^{\frac{1}{1-\alpha}}\right\} \\ &= \frac{1}{(1-\alpha)}\left[\frac{A\alpha\beta}{(1-\pi)}\right]^{\frac{1}{1-\alpha}}\left[(1-\pi)+\pi x^{\alpha}\right]^{\frac{1}{1-\alpha}}\left[\frac{1}{(1-\pi)}-\frac{(1-x^{\alpha})}{[(1-\pi)+\pi x^{\alpha}]}\right], \end{aligned} \tag{71}$$

which is positive.

To complete the proof, we next show that for a given bailout policy, the left-hand side of equation (58) is a decreasing function of π and the right-hand side is an increasing function of π.

For a given bailout policy x, the partial derivative of the left-hand side of equation (58) w.r.t. π is given by

$$\frac{\partial}{\partial \pi}\left\{f'\left(xk(x)\right)\right\} = f''(xk(x))x\frac{\partial k(x)}{\partial \pi}, \tag{72}$$

which is negative, since $f''(.) < 0$. Put differently, the social benefit associated with a given bailout level shifts downwards for a higher π.

For a given bailout policy x, the partial derivative of the right-hand side of equation (58) w.r.t. π is given by

$$\frac{\partial}{\partial \pi}\left\{\frac{1}{\beta}p(x)q(x)\right\} = \frac{1}{\beta}\left[\frac{\partial p(x)}{\partial \pi}q(x)+p(x)\frac{\partial q(x)}{\partial \pi}\right], \tag{73}$$

where $p(x)$ and $q(x)$ were defined in equations (60) and (63), respectively. To see that the derivative in equation (73) is positive, first consider

$$\frac{\partial p(x)}{\partial \pi} = v''\left(T(x)-\frac{xk(x)}{\beta}\right)\left[\frac{\partial j(k)}{\partial k}-\frac{x}{\beta}\right]\frac{\partial k(x)}{\partial \pi}, \tag{74}$$

which is positive, since $v''(.) < 0$ and since, by equation (66),

$$\frac{\partial j(k)}{\partial k} < \frac{x}{\beta}.$$

Second, consider the partial derivative of $q(x)$ w.r.t. π,

$$\frac{\partial q(x)}{\partial \pi} = \frac{\alpha x^{\alpha}}{(1-\alpha)}\left[(1-\pi)+\pi x^{\alpha}\right]^{-2}, \tag{75}$$

which is also positive. Put differently, the social cost associated with a given bailout level shifts upwards for a higher π.

The fact that for a given bailout policy, x, the benefit of bailouts becomes lower and the cost becomes higher when π increases, implies that the government must choose a lower level of bailout guarantees for a higher probability of crisis. As shown in the proof of Proposition 6, the reason is that for a given probability of crisis, a lower level of bailout guarantees increases the marginal benefit and reduces the marginal cost of bailouts. Formally, we have shown that for any π_1 and π_2 such that $\pi_1 < \pi_2$, it is the case that $x^*(\pi_1) > x^*(\pi_2)$.■

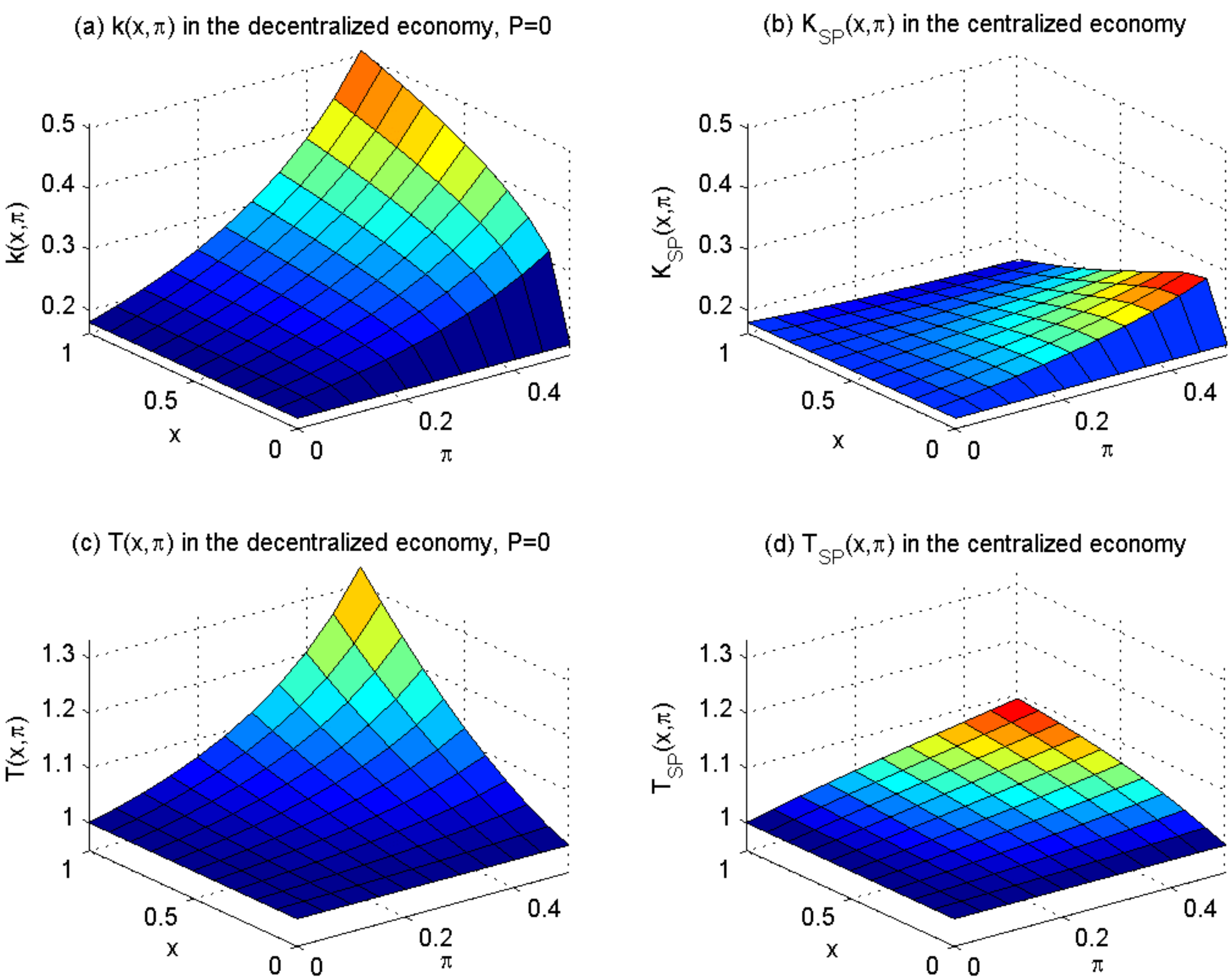

Figure 1: The optimal decision rules for the level of borrowed capital and government revenues in the decentralized and centralized economies

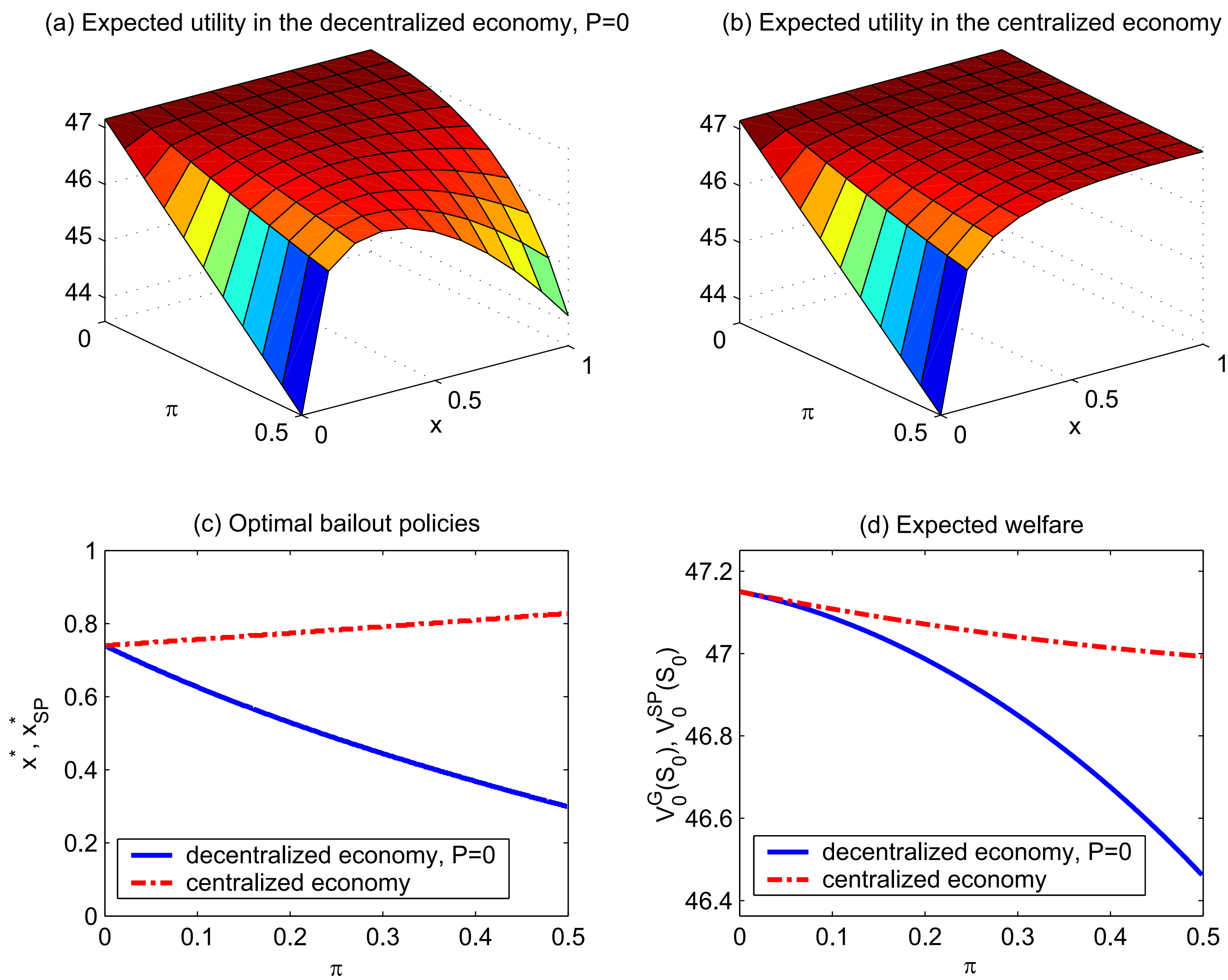

Figure 2: Expected utility functions, optimal bailout policies and expected welfare in the decentralized and centralized economies