Organizational Structure as the Channeling of Boundedly Rational Pre-play Communication*

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Abstract

We model organizational decision making as costless pre-play communication. Decision making is called authoritarian if only one player is allowed to speak and consensual if all players are allowed to speak. Players are assumed to have limited cognitive capacity and we characterize their behavior under each decision making regime for two different cognitive hierarchy models. Our results suggest that authoritarian decision making is optimal when players have conflicting preferences over the set of Nash equilibrium outcomes, whereas consensual decision making is optimal when players have congruent preferences over this set. The intuition is that authoritarian decision making avoids conflict, but sometimes creates insufficient mutual trust to implement socially optimal outcomes.

Keywords: Organizational decision making, coordination games, communication, cognitive hierarchy models.
JEL codes: C72, L20, M21, M54.

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1 Introduction

"A body with two heads is in the social as in the animal sphere a monster, and has difficulty in surviving." (Henri Fayol 1916/2001)

"What gives unity to organized societies, however, as to all organisms, is the spontaneous consensus of parts. Such is the internal solidarity which not only is as indispensible as the regulative actions of higher centres, but which also is their necessary condition, for they do no more than translate it into another language and, so to speak, consecrate it." (Emile Durkheim 1893/1933)

Participation, teambuilding, and consensual decision making are positively laden management concepts. Yet, the principle of participation stands in contrast to other classic principles such as unity of direction and command, epitomized by famous leaders and espoused by Fayol (1916/2001) and other early management thinkers. Empirical studies show that the magnitude of participation varies substantially both across countries and across firms, and suggest that no practice is universally superior.[1] The challenge is therefore to identify the costs and benefits associated with each mechanism.

In this paper we use a model of games between boundedly rational players to analyze the optimal choice of coordination mechanism. Specifically, we show that the optimal structure of pre-play communication in a complete information game with multiple equilibria depends on the nature of the game: Roughly, command is optimal in games of conflicting interests, because it resolves ambiguity as to which equilibrium will be played, and consent is optimal in games of common interests because communication brings mutual assurance that the best equilibrium will be played. The practical implication is that consensual decision making might be expected when (i) the production technology displays strong complementarities due to specialization of tasks; (ii) several agents have discretion over their work; and (iii) the implementation of decisions is carried out within a compressed time frame. Under these circumstances, mutual trust is a prerequisite for successful cooperation and all communication channels should be open.

Our findings echo old and intuitive insights from Burns and Stalker (1961) and other non–mathematical management writers. Informally, the general idea that communication fosters mutual trust also appears in several other parts of social science – see for example Moscovici and Doise (1994) for a broad picture, and Schelling (1966, Chapter 7) for a discussion of the value of bilateral communication in preventing and limiting wars. However,

[1] Surveying several thousand managers, Myers, Kakabadse, McMahon and Spony (1995) find that there are four distinct management styles in Europe, and that these to a considerable extent are national. At the two extremes, Swedish and Finnish managers emphasize consensus, whereas French managers emphasize command structures. See also Hofstede (1991) for a more elaborate discussion of national cultures. Management practices display much variation also within geographical regions, and it is sometimes difficult to disentangle cultural effects from effects due to technology.

we believe that ours is the first model to formally demonstrate a benefit of multilateral communication over unilateral communication in one-shot complete information games.

A superficial explanation for the previous neglect of the problem that we study is that organizational economists have emphasized the role of *environmental uncertainty.* Under environmental uncertainty, people often hold private information about the state of the world. Organizational design then matters because it affects the aggregation of information, as in team theory, the cost of information revelation, as in mechanism design theory, or the efficient acquisition and use of information by decision makers, as in incomplete contracts theory; see Dewatripont (2006) for an overview. However, the considerable progress in all these areas hardly explains why organizational economists have neglected the conceptually simpler problem of *strategic uncertainty*, where people hold private information only about their own plans.[2]

A deeper explanation for the lack of attention to strategic uncertainty is that the existing game theoretic analysis of costless communication – the cheap talk literature – has failed to establish a generally accepted solution concept. After a promising start, where cheap talk arguments were formally developed by Farrell (1988), Myerson (1989) and Rabin (1990, 1994), and used to analyze core questions in industrial organization, such as market entry (Farrell 1987) and standardization processes (Farrell and Saloner 1988), the literature came under attack from theorists (Aumann 1990) as well as experimentalists (Cooper, DeJong, Forsythe and Ross 1992). Put simply, the proposed cheap talk models seemed convincing in games with conflicting interests, where they also tracked experimental data well (Cooper, DeJong, Forsythe and Ross 1989), but were less convincing in some games with common interests. Much of the criticism focussed on the Stag Hunt game, a version of which is depicted in Figure 1.

	H(igh)	L(ow)
H(igh)	$9, 9$	$0, 8$
L(ow)	$8, 0$	$7, 7$

Figure 1. Stag Hunt

The two players both prefer the (H, H) equilibrium to the (L, L) equilibrium, but without communication many theories predict the (L, L) equilibrium instead, since L is considerably less risky than H in case the player is uncertain about what the opponent will do. The Stag Hunt game is therefore a classic illustration of coordination failure due to lack of mutual trust. The proposal of Farrell (1988) implied that one–way communication would

[2]Strategic uncertainty has received more attention in the literature on organizational behavior. Yet, even that literature has tended to focus on conflicts of interest rather than lack of trust as the main obstacle to ex post implementation of decisions. Indeed, Scott's (2003) authoritative text continues to see bureaucracy as the optimal mechanism for computing and implementing decisions whenever the organization's participants agree both about means and ends (page 304), citing Thompson and Tuden (1959).

suffice to solve the problem, because sending the message of "H" would convince the receiver that the sender intends playing H, and hence the best response is for the receiver to play H as well. The theoretical problem, as pointed out by Aumann (1990), is that even a sender who has decided to play L has an incentive to induce the opponent to play H. Therefore, a pessimistic receiver should not be affected by the sender's message. The empirical problem, as pointed out by Cooper, DeJong, Forsythe and Ross (1992), is that one–way communication is not sufficient to guarantee coordination on the efficient equilibrium. Another intriguing finding of the study is that two–way communication entails substantially more coordination.

A satisfactory theory of communication in games needs to preserve the predictive power of the existing theory in games with conflicting interests, while better predicting the outcomes in games with common interests. Our approach deals with Aumann's critique by relaxing the assumption that players are completely rational. More specifically, we assume that players are only capable of limited strategic thinking and underestimate the cognitive capacity of their opponents. Models of this kind have been provided for normal form games by, e.g., Nagel (1995), Stahl and Wilson (1995), Costa-Gomes, Crawford and Broseta (2001) and Camerer, Ho and Chong (2004). These models are straightforwardly extended to the simple extensive form games that we analyze. We find that, for realistic levels of rationality, the models fit the experimental data of Cooper et al. (1992). However, we caution that a recent related experimental study by Burton, Loomes and Sefton (2005) arrive at a different conclusion regarding the contrast between one–way and two–way communication, so the model's explanatory power is still an open issue.[3]

A noteworthy theoretical finding is that two–way communication selects the efficient outcome in Stag Hunt and similar games when players' rationality tends to infinity. We are also able to extend all our main results to more general classes of games than the 2x2 games which have been the focus of experimental attention so far. Generalization is important both because it provides ample scope for out–of–sample tests of our approach and because credible analysis of many applications require models with a richer set of strategies.

Before providing the complete argument, it is useful to present the main ingredients of it in the Stag Hunt with two-way communication. *Example a*: Consider an otherwise rational player who believes that the opponent randomizes between the two actions available, but is completely honest and always plays according to his or her message. This player will listen to the opponent's message and react to it, playing L if and only if the opponent sent the message "L". The player's option to send an own message does not matter in this example. *Example b*: Consider an otherwise rational player who believes the opponent to be completely gullible, i.e., that the opponent will always believe one's own message and respond optimally to it. In this case, the player will send the message

[3] Other related experimental work includes Charness (2000), Clark, Kay and Sefton (2001), Duffy and Feltovich (2002), and Blume and Ortmann (2006).

"H" and play H in the expectation that the opponent will disregard his or her own message. Note that the receiver's assumed gullibility here promotes honesty by the sender; this is an important property of the Stag Hunt game. Depending on the player's belief about the opponent, the two examples illustrate that it can be better to be either receiver (as in example a) or sender (as in example b). Crucially, in neither of the two extreme cases considered here would a player strictly prefer to close a communication channel. As our analysis will show, the conclusion continues to hold as players get more sophisticated. Open information channels are generally optimal in Stag Hunt and related games of common interests. On the other hand, it is always optimal to close some communication channels in games with conflicting interests.

We are not the first to suggest that studies of strategic communication will benefit from taking bounded rationality into account. Crawford (2003, page 134) observes that the cheap talk theory has been lagging behind the public's intuition, and argues that the rationality assumption is at fault.[4] In particular, Crawford proposes that *deceptive* communication is best understood as a result of bounded rationality. Deception in complete information zero-sum games requires some people to assume others to be boundedly rational. We propose that an analogous claim is true for *honest* communication in other complete information games. Some honest talk is based on the assumption that the listener may be gullible, and some careful listening is based on the premise that the speaker may be honest.[5]

Observe that we take for granted that players have access to a common language. That is, we take an eductive approach to communication. A substantial fraction of the literature on cheap talk starts from the presumption that messages are not inherently meaningful; instead, messages may or may not acquire meaning in equilibrium – where equilibrium is typically depicted, implicitly or explicitly, as a steady state of an evolutionary process of random matches between boundedly rational players (see, for example, Matsui 1991, Wärneryd 1991, Kim and Sobel 1995, and Banerjee and Weibull 2000). The eductive and evolutionary approaches are complementary, and our assumption of bounded rationality closes part of the gap between them. However, while the evolutionary approach can explain how language emerges in "old" games, the eductive approach asks how an existing language will be used in "new" games. Within the evolutionary cheap talk literature, we are only aware of one piece of work that emphasizes the distinction between one-way and two-way communication. In a paper quite closely related to ours, Blume (1998) proves that two-way communication can be superior to one-way communication in games with strategic risk, such as Stag Hunt. Interestingly, Blume's result requires that messages have some small a priori information content. For example, players may have a slight preference

[4]See also Cai and Wang (2006).

[5]There are several differences between our approach and Crawford (2003) and Cai and Wang (2006). Crawford (2003) considers one-sided communication in 2x2 zero-sum games, whereas Cai and Wang (2006) studies one-sided communication in two-player strategic information transmission games of incomplete information.

for playing HH if both players sent the message "H" and the expected payoffs to playing H and L are otherwise equal. As Blume notes, his assumption amounts to assuming some small amount of gullibility on the part of receivers. In our eductive model, a grain of honesty of senders is instead what drives the superiority of two-way communication in the Stag Hunt game.[6] Taken together, Blume's results and ours suggest that players who face strategically risky situations ought to engage in multilateral communication both in order to develop an efficient language in the long run and in order to ensure efficient communication in the face of new short-run challenges.

In the next section we discuss some underlying assumptions and present the notation that is used throughout the paper. Section 3 presents our arguments within a simplified cognitive hierarchy model; Section 4 offers an analysis based on the full-fledged model of Camerer et al. (2004). The final section concludes.

2 Preliminaries

Camerer et al. (2004) present a one-parameter model with a hierarchy of player types. At the lowest level, there are zero-step thinkers, who are completely nonstrategic and randomize uniformly over available strategies. At the next level, a one-step thinker acts optimally given the belief that the opponents are zero-step thinkers. Each layer of higher step thinkers act optimally given the belief that all opponents constitute a mixture of lower-step thinkers. The behavior of zero-step thinkers does not necessarily reflect actual behavior of a significant fraction of the player population. It is a heuristic for what higher-step players tend to believe about the behavior of the most primitive players. Since we merely utilize the model, without adding any major components, we refer to Camerer et al. (2004) and to Costa-Gomes and Crawford (2006) for a discussion of the strengths and weaknesses of the cognitive hierarchy model.

As we study games with pre-play communication, we have to make two additional assumptions. The first assumption concerns the communication strategies of the least sophisticated players. For simplicity, we initially assume that zero-step thinkers randomize uniformly over the messages available and then play accordingly irrespective of any message received from opponents.[7] (Alternative assumptions are explored below.) With one-way communication the assumption implies that zero-step senders randomize uniformly between messages and then play according to the message sent, while receivers randomize uniformly over the actions available. These assumptions are broadly line with

[6] The distinction between our form of honesty and Blume's for of gullibility is not so great, however. It is the (somewhat excessive) belief of advanced players in the existence of honest players that ultimately generates our result.

[7] See Blume, DeJong, Kim and Sprinkle (2001), Cai and Wang (2006) and Duffy and Feltovich (2006) for experimental evidence that senders in cheap talk games to large extent are truthful.

Crawford (2003) and Cai and Wang (2006).[8]

Since the model by Camerer et al. (2004) and other similar models are developed for normal form games, we also need to make additional assumptions concerning how players update their beliefs about their opponents' type. We shall discuss this modelling choice as it arrives.

2.1 Notation

Let G denote some finite n-person, complete information normal form game. We refer to G as an *action game* and the strategies available in G as *actions*. Let $N = \{1, 2, ..., n\}$ denote the set of players of G, and let A_i denote the (finite) set of pure actions available to player $i \in N$. The payoff to player i is given by a von Neumann-Morgenstern utility function $\pi_i : \times_{i \in N} A_i \to \mathbb{R}$.

Let $\Gamma_1(G)$ denote the action game G preceded by one-way communication. That is, in $\Gamma_1(G)$ exactly one of the players is allowed to send a message $m_i \in M_i$ before the action game G is played. Nature decides with equal probability which of the players will act as sender. Initially, we assume that the set of messages has the same number of elements as the set of actions available to each player. Assuming that players share a common language, we adopt the convention $M_i = A_i$. Since the message is observed before the action game is played, the actions chosen by the receivers can be made conditional on the received message. A strategy s_i for a player i of the full game $\Gamma_1(G)$ prescribes what to do in the sender role as well as in the receiver role. Thus, the strategy is given by a message $m_i \in M_i$ and an action $a_i \in A_i$ for the sender role and a mapping $f_i : \times_{j \in N} M_j \to A_i$ for the receiver role. A pure strategy s_i of $\Gamma_1(G)$ is called *truthful* if $a_i = m_i$, i.e., if a player acting as a sender sends a message that indicates what action he will play.

Let $\Gamma_2(G)$ denote the action game G preceded by two-way communication. That is, in $\Gamma_2(G)$ all the players are allowed to send a message $m_i \in M_i$ before the action game G is played. Since messages are observed before the action game G is played, the actions chosen can be made conditional on messages sent. A strategy s_i for player i of the full game is therefore given by a message $m_i \in M_i$ and a mapping $f_i : \times_{j \in N} M_j \to A_i$. A pure strategy s_i of $\Gamma_2(G)$ is called *truthful* if $f_i(m_i, m_{-i}) = m_i$ for all $m_i \in M_i$ and all $m_{-i} \in M_{-i}$, i.e., if a player sends a message and plays an action consistent with that message irrespective of any messages received.

For some of the proofs it will be useful to define the best-reply correspondence in terms

[8] These authors assume that zero-step receivers are credulous – in the sense that they believe that senders are truthful and best respond given this belief. Thus, our one-step receivers are equivalent to their zero-step receivers. Although it might seem natural for the case with one-sided communication to assume that zero-step senders are truthful and zero-step receivers are credulous, it leaves open the question how zero-step thinkers behave with two-sided communication (since a player cannot be both credulous and truthful). This could be handled by assuming that zero-step thinkers are truthful and credulous with certain probabilites. We consider this possibility in Appendix 1.

of actions of the action game. The best response of player i given the action a_{-i} of the opponents is given by

$$BR_i(a_{-i}) = \arg\max_{a_i \in A_i} \pi_i(a_i, a_{-i}).$$

With some abuse of notation, let $BR_i(m_{-i})$ denote a best response to received messages given the belief that all other players' messages are truthful.

Most of the analysis will concern two-player games in which each player has (the same) two available actions in the action game. The two actions are labeled H and L. As a convention, we denote actions by capital letters and messages by the corresponding small letters. Under one-way communication, we write a pure strategy of player i (given the received message m_j) as

$$s_i = \langle m_i, a_i, f_i(m_j = h), f_i(m_j = l) \rangle.$$

For example, $s_1 = \langle h, H, L, L \rangle$ means that player 1 sends the message h and takes the action H if he is the sender, while playing L whenever acting as receiver. Under two-way communication, a strategy consists of a message to send and what to play conditional on received messages. A pure strategy of player i (given the message m_j sent by player j) can thus be written

$$s_i = \langle m_i, f_i(m_j = h), f_i(m_j = l) \rangle.$$

For example, $s_1 = \langle h, H, L \rangle$ means that player 1 sends the message h, but plays according to the received message (i.e., plays H if player 2 sends message h and L if player 2 plays message l).

We also allow mixed strategies. For example, $\frac{1}{2}\langle h, H, H \rangle, \frac{1}{2}\langle l, H, H \rangle$ means that the player always plays the action H but randomize uniformly between the two messages h and l.

Observe that we neglect unused strategy components. For example, we do not specify what action a player would take in the counterfactual case when he sends another message than the message specified by his strategy. The reason is that in the models we consider, interesting counterfactuals cannot arise. First, the model does not permit implementation mistakes. Second, there always exist (at least in the players' imagination) zero-step thinkers, and these send all messages with positive probability. Thus, no player is ever completely surprised by the opponent's message.

3 Simple Cognitive Hierarchy (SCH) Model

In this section we demonstrate our main argument within a simplified version of Camerer et al. (2004); call it the simple cognitive hierarchy (SCH) model – in contrast to the Poisson cognitive hierarchy (PCH) model that complies fully with the original model. (The SCH model is similar to earlier cognitive hierarchy models by Nagel 1995, Stahl and

Wilson 1995 and Costa-Gomes et al. 2001.) We mainly focus on the two-player Stag Hunt and Battle of the Sexes, but also generalize these results to broader classes of games.

The simplicity of the SCH model is largely due to the assumption that any $k \geq 1$ step thinker believes that all opponents are level $k-1$ thinkers. The assumption implies that players are often indifferent between strategies, and, in addition, that there is nothing to pin down k-step thinkers' beliefs when they receive a message that $k-1$ step thinkers would never send. For convenience we assume that whenever a $k \geq 2$ step thinker is indifferent between strategies or receives a message that a $k-1$ player would not send, he takes into account the possibility that the opponent might be a $k-2$ step thinker, proceeding to lower levels if the player continues to be indifferent or receive a message only lower-level thinkers would send. If this procedure still leaves a player indifferent between strategies, we assume that he randomizes, albeit not necessarily uniformly, over the remaining strategies. The assumptions underlying the SCH model corresponds closely to the PCH model if the average number of thinking steps (τ) is high and k sufficiently small, since this implies that a k-step thinker will believe that almost all opponents are of level $k-1$.

As will become clear, players who perform at least two thinking steps often, but not always, behave alike. Therefore, players who perform at least two thinking steps are called *advanced thinkers.* Players who are not advanced, i.e., zero-step and one-step thinkers, are called *primitive thinkers.*

3.1 Stag Hunt

Consider the Stag Hunt game in Figure 2.

	H(igh)	L(ow)
H(igh)	c, c	$0, b$
L(ow)	$b, 0$	a, a

Figure 2. Stag Hunt

Let $0 < a \leq b < c$ and $c < a + b$, so that HH is the Pareto dominant equilibrium and LL is the risk dominant equilibrium (see Harsanyi and Selten 1988).

With no possibility to communicate, the simple cognitive hierarchy model implies that one-step and all advanced ($k \geq 2$) thinkers play L. With two-way communication, advanced thinkers play H instead. With one-way communication, actions alternate between even and odd thinking steps.

To understand the difference in behavior between one-way and two-way communication, consider first the behavior of zero-step thinkers. With two-way communication, zero-step thinkers always play according to their own sent message, while with one-way communication, they play randomly as receivers. Next, consider the one-step thinkers.

With two-way communication one-step thinkers play according to the received message. Thus, two-step thinkers will send a message indicating that they will play H and then play H (since one-step thinkers always best respond to received messages). All higher-step thinkers will consequently play H as well. If there are many high-level thinkers playing the game, players are therefore likely to coordinate on the Pareto dominant equilibrium. With one-way communication, on the other hand, one-step thinkers acting as senders always play L (since zero-step thinkers do not respond to messages), which in turn implies that two-step thinkers acting as receivers will play L as well. Similarly, one-step thinkers acting as receivers will play according to received messages, which implies that two-step thinkers will send and play H. With one-way communication, behavior thus alternates between even and odd thinking steps – even-step thinkers play as two-step thinkers while odd-step thinkers play as three-step thinkers.

Proposition 1 *In the SCH model, the actions in Stag Hunt taken by advanced thinkers are as follows: (i) Under two-way communication, all players play H. (ii) Under one-way communication, even-step thinkers play L and odd-step thinkers play H.*

Proof. First consider one-way communication. Then zero-step thinkers will play $\frac{1}{2}\langle h, H, H, H\rangle, \frac{1}{2}\langle l, L, L, L\rangle$ and one-step thinkers $p_1\langle h, L, H, L\rangle, (1-p_1)\langle l, L, H, L\rangle$ with $p_1 \in (0,1)$. This in turn implies that two-step thinkers will play $\langle h, H, L, L\rangle$. Three-step thinkers will play $\langle h, L, H, L\rangle$ if $b > a$ and $p_3\langle h, L, H, L\rangle, (1-p_3)\langle l, L, H, L\rangle$ with $p_3 \in (0,1)$ if $b = a$. The behavior of higher-step thinkers will alternate between even and odd thinking steps – even-step thinkers play as two-step thinkers while odd-step thinkers play as three-step thinkers.

With two-way communication one-step thinkers will play $p_1\langle h, H, L\rangle, (1-p_1)\langle l, H, L\rangle$ with $p_1 \in (0,1)$. This in turn implies that all higher-step thinkers will play $\langle h, H, H\rangle$. ■

An immediate consequence of Proposition 1 is that, for any given distribution of types, two-way communication results in more coordination on the Pareto dominant equilibrium than does one-way communication.

Note that under two-way communication, advanced thinkers play H irrespective of received messages. The reason is that if a two-step thinker receives a message indicating that L will be played, he believes that the message comes from a one-step thinker that plays according to received messages. This is at odds with the idea that communication creates reassurance that H will be played when both players indicate that they intend to play H. This is partly an artefact of this simplified model – in the PCH two-step thinkers' behavior depends on the average number of thinking steps.

The result that two-way communication leads advanced thinkers to play H holds also if zero-step thinkers are not always truthful. Let p denote the probability that a zero-step thinker is truthful. Then it is straightforward to show that behavior with two-way communication will be identical to Proposition 1 whenever p satisfies

$$p > \frac{a - (c - b)}{a + (c - b)}.$$

The assumptions on parameters imply that the right hand side will always be between zero and one. For example, in the Stag Hunt shown in Figure 1, the truthfulness of zero-step thinkers must be above $7 - (9 - 8) / 7 + (9 - 8) = 3/4$ for two-way communication to lead to the Pareto dominant equilibrium.

3.2 Battle of the Sexes

Consider now instead the two-player Battle of the Sexes game in Figure 3.

	L(ow)	H(igh)
H(igh)	b, a	$0, 0$
L(ow)	$0, 0$	a, b

Figure 3. Battle of the Sexes

For this game to be a Battle of the Sexes, and for the labels to make sense, we require that $0 < a < b$.

Recall that zero-step thinkers randomize uniformly. Without communication, one-step thinkers will therefore play H and as a result two-step thinkers will play L. Generally, play alternates between even and odd thinking steps, resulting in little coordination.

Now consider the possibility to communicate. With two-way communication, one-step thinkers will send random messages and play whatever the zero-step thinkers indicated that they will play. This in turn implies that the two-step thinkers will send and play H, which in turn implies that three-step thinkers will send and play L. Similar to the case without communication, behavior will alternate between thinking steps, resulting in little overall coordination. With one-way communication matters are quite different. One-step thinkers still play according to the received message in one-way communication, but they play H as senders. Advanced thinkers therefore play L as receivers and send and play H as senders. This implies that whenever advanced thinkers meet, there will be perfect coordination. Intuitively, the asymmetric roles at the communication stage admit coordination on the efficient but asymmetric outcomes – receivers always have to give in and play their least preferred option.

Proposition 2 *In the SCH model, advanced thinkers take the following actions in the Battle of the Sexes: (i) Under two-way communication, two-step thinkers play H, three-step thinkers best respond to received messages and the behavior of higher-step thinkers cycles over four thinking steps. (ii) Under one-way communication, senders play H and receivers play L.*

Proof. First consider one-way communication. Zero-step thinkers play $\frac{1}{2}\langle h,H,H,H\rangle$, $\frac{1}{2}\langle l,L,L,L\rangle$ and one-step thinkers $p_1\langle h,H,L,H\rangle, (1-p_1)\langle l,H,L,H\rangle$ with $p_1 \in (0,1)$. All advanced thinkers play $\langle h,H,L,L\rangle$. With two-way communication, zero-step thinkers play $\frac{1}{2}\langle h,H,H\rangle, \frac{1}{2}\langle l,L,L\rangle$ and one-step thinkers play $p_1\langle h,L,H\rangle, (1-p_1)\langle l,L,H\rangle$ with $p_1 \in (0,1)$. Two-step thinkers play $\langle h,H,H\rangle$ while three-step thinkers play $\langle h,L,H\rangle$. Four-step thinkers play $\langle h,H,H\rangle$, whereas five-step thinkers play $\langle l,L,L\rangle$. Six-step thinkers play $\langle l,L,H\rangle$ and seven-step thinkers play $\langle h,L,H\rangle$. Eight-step and higher-step thinkers play as $k-4$ step thinkers, so that behavior alternates in cycles of four thinking steps. ■

Note that the truthfulness of zero-step thinkers is important for this result. If the messages of zero-step senders were to convey no information, one-step receivers would always play H and will not respond to sent messages. In order for one-step receivers to respond to messages, the probability of truthfulness of zero-step thinkers must satisfy $p > (b-a)/(a+b)$. Hence, for sufficiently high degree of truthfulness, one-way communication will work according to Proposition 2. Note that the required degree of truthfulness is increasing in the difference $b-a$, suggesting that one-way communication would work less well if equilibrium outcomes are very unequal.

3.3 Multiplayer Generalizations

From the preceding sections it is tempting to draw the conclusion that two-way communication results in coordination on the Pareto dominant equilibrium when players have common interests, whereas one-way communication works fine when players have conflicting interests in the sense of having different "favorite" equilibria. In Appendix 2 we show that this conjecture is true, with some additional qualifications, for a broad class of two-player games. In this section we discuss two classes of n-player games that demonstrate the relative advantages of two-way versus one-way communication.

3.3.1 Two-way communication

In order to find a class of games for which two-way communication induces advanced thinkers to play the Pareto dominant equilibrium, let us recall three pieces of terminology. A *common interest game* is a finite normal form game with one Pareto dominant equilibrium such that this equilibrium gives strictly higher payoffs to all players than all other outcomes of the game. We restrict attention to games with a finite number of actions and we can therefore without generality assign integers $\{1,2,...,\overline{a}_i\}$ to each player's actions. A normal form game G has *strategic complementarities* if best responses are increasing in the opponents' actions, i.e., if $a_{-i} \geq a'_{-i}$ implies $BR_i(a_{-i}) \geq BR_i(a'_{-i})$. Finally, the game G has *positive spillovers* if the payoff increases in the opponents' actions, i.e., if $a_{-i} \geq a'_{-i}$ implies $\pi_i(a_i,a_{-i}) \geq \pi_i(a_i,a'_{-i})$. If a common interest game has positive spillovers, then all players choosing their highest action is a Pareto dominant equilibrium. This fact is used in the following proof, so we state it as a separate lemma.

Lemma 1 *The Pareto dominant equilibrium of a common interest game with positive spillovers, is given by the action profile* $\overline{a} = (\overline{a}_1, \overline{a}_2, ..., \overline{a}_n)$.

Proof. Suppose the Pareto dominant equilibrium is some profile $a^* \neq \overline{a}$. Then at least one player has an action $a_i > a_i^*$ available that by positive spillovers gives the opponents the same or higher payoffs, contradicting the assumption that a^* is a Pareto dominant equilibrium of a common interest game with positive spillovers. ■

Using Lemma 1 it is straightforward to show that two-way communication will result in the Pareto dominant equilibrium.

Proposition 3 *Let G be an n-player common interest game with strategic complementarities and positive spillovers. Then two-way communication implies perfect coordination whenever all players are advanced thinkers.*

Proof. Zero-step thinkers randomize over truthful strategies, so one-step thinkers play $BR_i(m_{-i})$ and randomize messages. In particular, if $m_{-i} = \overline{a}_{-i}$, one-step thinkers play $\overline{a}_i$ since $\overline{a}$ is an equilibrium (by Lemma 1). A two-step thinker believes that the opponents are one-step thinkers that best-respond to messages. Since G has strategic complementarities, $BR_i(m_{-i})$ is non-decreasing in m_{-i}, and since there are positive spillovers, it is weakly dominant for a two-step thinker to send the message $m_i = \overline{a}_i$. From Lemma 1 we know that $\overline{a}$ gives higher payoff than all other outcomes of the game implying that a two-step thinker will send $\overline{a}_i$ and play $\overline{a}_i$ if $m_{-i} = \overline{a}_{-i}$. Higher-step thinkers will play in the same way and whenever advanced thinkers play the game, the outcome is $\overline{a}$. ■

To understand the logic of Proposition 3, consider the weak-link game. In a weak-link game, each player picks an integer from 1 to m. Payoffs are such that all players want to play the minimum of what the opponents play, but all players are better off if everybody chooses higher numbers. Any strategy profile in which all players choose the same number constitutes a Nash equilibrium, and the Pareto dominant equilibrium involves all players playing m. (For a more detailed exposition of the weak-link game see for example Camerer 2003, Chapter 7.) If the weak-link game is played with two-way communication, zero-step thinkers will randomize over all truthful strategies. One-step thinkers best respond by (sending random messages and) playing according to the minimum of the received messages. In particular, if all received messages are equal to m, then a one-step thinker plays m as well. Two-step thinkers believe they face $n-1$ one-step thinkers that send random messages and play the minimum of received messages. The best-response of two-step thinkers must therefore be also to play the minimum of all received messages, but to send the message m (since it might be the case that all other $n-1$ player sent m so that it is optimal for two-step thinkers to do so too). Since all two-step thinkers send the message m and play the minimum of the received messages, three-step thinkers best-respond using the same strategy. Hence, as long as only advanced thinkers play the game, there will be perfect coordination on the Pareto dominant equilibrium, whereas this will typically not be the case if some primitive thinkers play the game.

This reasoning and the proof of Proposition 3 rely on the fact that zero-step thinkers are believed to be truthful. In order for the argument to work, truthfulness must be high enough to induce one-step thinkers to play the Pareto dominant equilibrium whenever he receives messages indicating that the opponents intend to play that equilibrium.

3.3.2 One-way communication

Here is a natural n-player extension of the Battle of the Sexes: Each player i chooses an integer $a_i \in \{1, 2, ..., n\}$. If all players pick different integers they get positive payoffs, and payoffs are increasing in a_i. If two or more players choose the same number, all players get the lowest possible payoff. A natural interpretation is that there are n workers and tasks, that each worker can undertake exactly one task, that each task has to be be performed for the project to succeed, and that the tasks differ in popularity. For future reference, let us call it the Battle of the Workers.

In the Battle of the Workers, there are $n!$ different pure strategy equilibria and there is little hope of attaining coordination as long as each player is confined to communicate a plan for the own action only. We therefore extend the message space. Let a message consist of a list of recommended actions, one for each player, i.e., $M = \times_{i \in N} A_i$. In order for communication to affect the outcome, we also assume that zero-step receivers respond obediently to received messages under one-way communication. Since zero-step receivers always act obediently, one-step senders assign action n to themselves and divide up the remaining $n - 1$ actions between the opponents. One-step receivers, believing that the other receivers are zero-step receivers, play obediently if the sender has assigned a unique number to each player. By induction, all advanced thinkers play in the same way. Hence one-way communication attains perfect coordination.

The argument relies on the assumption that zero-step receivers are believed to be perfectly obedient. However, this sufficient condition can often be relaxed. In Appendix 1 we derive a necessary and sufficient condition for one-way communication to sustain perfect coordination in the Battle of the Workers. Extending the message space and introducing the possibility that zero-step thinkers are obedient also affects the previous results. In Appendix 1 we therefore also provide a complete analysis of the Stag Hunt and Battle of the Sexes with the extended message space.

4 Poisson Cognitive Hierarchy (PCH) Model

A drawback of the SCH model is that beliefs become more and more unrealistic the more steps of thinking a player does. The Poisson cognitive hierarchy model of Camerer et al. (2004) does not have this undesirable implication. As a robustness check, let us also conduct our analysis within the PCH model.

Camerer et al. (2004) assume that the distribution of types is Poisson distributed, i.e., the proportion of k-step thinkers is given by

$$f(k) = \frac{e^{-\tau}\tau^k}{k!}.$$

A player of level k best responds given the belief that everybody else are of level 0 up to $k-1$. The conditional density function for the belief of a k-step thinker about the proportion of $l < k$ step thinkers is

$$g_k(l) = \frac{f(l)}{\sum_{h=0}^{k-1} f(h)}.$$

The PCH model is developed for normal form games only. In order to adapt the model to games with pre-play communication we must specify how beliefs are updated after messages have been received. As it turns out, our results are quite insensitive to the updating rule, so for reasons of familiarity we assume Bayesian updating.[9] For the games preceded by one round of communication, let $p_{ki}(m_i)$ denote the probability that player $i \in N$ sends the message m_i when the player thinks k steps (and is allowed to send a message). A k-step thinker's belief that the sender i is a $l < k$ step thinker conditional upon receiving the message m_i is

$$g_{ki}(l|m_i) = \frac{g_k(l)\, p_{li}(m_i)}{\sum_{h=0}^{k-1} g_k(h)\, p_{hi}(m_i)} = \frac{f(l)\, p_{li}(m_i)}{\sum_{h=0}^{k-1} f(h)\, p_{hi}(m_i)},$$

where the latter equality follows from the definition of $g_k(l)$. Note that if all $l < k$ step players randomize uniformly, then $g_{ki}(l|m_i) = g_k(l)$ for all i and m_i. It is also useful to note that the relative conditional proportion of two lower level types is the same for players of different level of rationality for a given received message m_i. For any l such that $1 < l < k-1$ the relative conditional proportion of l-step and $l+1$-step thinkers is (supposing $p_{li}(m_i) > 0$)

$$\frac{g_{ki}(l+1|m_i)}{g_{ki}(l|m_i)} = \frac{f(l+1)\, p_{(l+1)i}(m_i)}{\sum_{h=0}^{k-1} f(h)\, p_{hi}(m_i)} \Big/ \frac{f(l)\, p_{li}(m_i)}{\sum_{h=0}^{k-1} f(h)\, p_{hi}(m_i)} = \frac{\tau}{i+1} \frac{p_{(l+1)i}(m_i)}{p_{li}(m_i)}.$$

[9] Results are robust to updating assumptions because zero-step and one-step thinkers randomize uniformly over the messages available. Thus, two-step thinkers' updated beliefs coincide with their initial beliefs. Since Lemma 2 also applies when beliefs are not updated, our results are robust to the updating rule.

Retaining our previous assumption about the behavior of zero-step thinkers, the behavior of one-step thinkers will be practically identical in the SCH and PCH model (since in both models a one-step thinker believes that the opponent is a zero-step thinker). However, in the PCH model we adopt the tie-breaking rule of Camerer et al. (2004), so that players randomize uniformly over all strategies that give the same expected payoff.

A feature of the PCH model is that if a k-step thinker plays a pure equilibrium strategy of a two-player game with one round of pre-play communication, then all higher-step thinkers will play that strategy too (this result also holds if a k-step player is indifferent between strategies). Since this result will be used repeatedly we state it separately in Lemma 2.

Lemma 2 *Let G be a two-player normal form game. If k-step thinkers play mutual best responses of $\Gamma_1(G)$ or $\Gamma_2(G)$, then all higher-step thinkers will play these strategies too.*

Proof. Consider the case of two-way communication (the proof for one-way communication is analogous). Let the strategy played by k-step thinkers be denoted $s_i^* = \langle m_i^*, f_i^*(m_j)\rangle$ and $s_j^* = \left\langle m_j^*, f_j^*(m_i)\right\rangle$. Consider a k-step player i that received the message m_j. We know that $f_i^*(m_j)$ is the action that maximizes expected payoff conditional on receiving m_j given the belief that the opponent is a $l < k$ step thinker with probability

$$g_{kj}(l|m_j) = \frac{f(l)\, p_{lj}(m_j)}{\sum_{h=0}^{k-1} f(h)\, p_{hj}(m_j)}.$$

Similarly, a $k+1$ step thinking player i that receives the same message m_j best responds given the belief that the opponent is a $l < k+1$ step thinker with probability

$$g_{(k+1)j}(l|m_j) = \frac{f(l)\, p_{lj}(m_j)}{\sum_{h=0}^{k} f(h)\, p_{hj}(m_j)}.$$

Since $f_i^*(m_j)$ maximizes the expected payoff of a k-step thinker i and is a best response against a k-step opponent j, by linearity of expected payoffs it must be a best response also to the mixture of types a $k+1$ step player i believes to be facing (note that this argument does not extend to more than two players). Similarly, if a k-step thinker is indifferent between certain actions, a $k+1$ step thinker will be so too and both k-step and $k+1$ step thinkers will therefore randomize uniformly over these actions.

Now consider the communication stage of the game. If a k-step sender i plays s^*, then then m_i^* is a best response given the belief that the opponent is a $l < k$ step thinker with probability

$$g_k(l) = f(l) \Big/ \sum_{h=0}^{k-1} f(h).$$

Similarly a $k+1$ step sender believes that the opponent is a $l<k+1$ step thinker with probability

$$g_{k+1}(l)=f(l)/\sum_{h=0}^{k}f(h).$$

Since m_i^* maximizes the payoff of a k-step sender and is a best response against another k-step thinker it must be a best response also to the mixture of types a $k+1$ sender believes to be facing. If the best-response is not unique, both k and $k+1$ step senders randomize uniformly among these messages.

By induction this reasoning holds for all higher level players as well. ■

In the PCH model, behavior depends both on the payoffs of the game and on the average level of thinking steps, τ. Unless all advanced thinkers play the same strategy, we will focus on the behavior of two-step thinkers. This is enough to establish the qualitative conclusions about one-way and two-way communication.

4.1 Stag Hunt

Consider the Stag Hunt game of Figure 2. If there is no pre-play communication, one-step thinkers and all advanced thinkers play L in the Stag Hunt, implying coordination on the risk dominant equilibrium LL unless zero-step thinkers play.

To determine what happens under two-way communication turns out to be straightforward.

Proposition 4 *In the Stag Hunt with two-way communication, the PCH model predicts that advanced thinkers play $\langle h,H,L\rangle$ if $\tau<a/(c-b)$ and $\langle h,H,H\rangle$ if $\tau>a/(c-b)$.*

Proof. Zero-step thinkers play $\frac{1}{2}\langle h,H,H\rangle,\frac{1}{2}\langle l,L,L\rangle$ and one-step thinkers respond by playing $\frac{1}{2}\langle h,H,L\rangle,\frac{1}{2}\langle l,H,L\rangle$. Two-step thinkers either respond to messages or send and play H. Since zero-step and one-step thinkers send both messages with equal probabilities, $g_{2i}(l|m_i)=g_2(l)$. Two-step thinkers therefore prefer $\langle h,H,L\rangle$ over $\langle h,H,H\rangle$ whenever

$$g_2(0)\frac{1}{2}(a+c)+g_2(1)\frac{1}{2}(b+c)>g_2(0)\frac{1}{2}c+g_2(1)c,$$

which is equivalent to the condition $\tau<a/(c-b)$. Conversely, if $\tau>a/(c-b)$, two-step thinkers play $\langle h,H,H\rangle$.

Since $\langle h,H,L\rangle$ and $\langle h,H,H\rangle$ are both pure equilibrium strategies, by Lemma 2 all higher-step thinkers will play like the two-step thinkers. ■

As in the SCH model, one-step thinkers play according to received messages and send random messages. Proposition 4 shows that we observe the same outcome as in the simplified model also for higher-step thinkers whenever τ is sufficiently high. If τ is lower, however, higher-step thinkers believe there is enough zero-step thinkers around to make it worthwhile to play $\langle h, H, L\rangle$ rather than $\langle h, H, H\rangle$. For example, in the Stag Hunt depicted in Figure 1, the threshold is given by $\hat{\tau} = 7/(9-8) = 7$. Since τ is typically around 1.5 we would therefore expect that higher-step thinkers will play $\langle h, H, L\rangle$ in that game.[10] This in turn implies perfect coordination on the Pareto dominant equilibrium whenever advanced thinkers play the game. In addition, advanced thinkers will end up coordinating on one of the equilibria when they play against zero-step thinkers, whereas there will be coordination failure in half of the cases when they meet one-step thinkers (i.e., when they play $\langle l, H, L\rangle$).

Note also that Proposition 4 implies that as $\tau \to \infty$, there is perfect coordination on the Pareto dominant equilibrium. This contrasts with Aumann's (1990) argument that only the risk dominant equilibrium LL is self-enforcing. If we consider full rationality as a limiting case of the PCH model, the Pareto dominant equilibrium HH is the only self-enforcing equilibrium.

The case with one-way communication is slightly more complicated since advanced thinkers may play in different ways depending on payoffs. Proposition 5 states the model predictions for two-step thinkers.

Proposition 5 *In the Stag Hunt with one-way communication, the PCH model predicts the following behavior:*

(i) *If* $\tau < (a+b-c)/2(c-b)$ *and* $\tau > (c-b)/a$, *then advanced thinkers play* $\langle h, L, L, L\rangle$ *if* $b > a$ *and* $\frac{1}{2}\langle h, L, L, L\rangle, \frac{1}{2}\langle l, L, L, L\rangle$ *if* $b = a$.

(ii) *If* $\tau > (a+b-c)/2(c-b)$ *and* $\tau < (c-b)/a$, *then advanced thinkers play* $\langle h, H, H, L\rangle$.

(iii) *In the remaining two cases, two-step thinkers play either* $\langle h, L, H, L\rangle$, $\langle h, H, L, L\rangle$ *or* $\frac{1}{2}\langle h, L, H, L\rangle, \frac{1}{2}\langle l, L, H, L\rangle$ *depending on* τ *and the payoffs of the game.*

Proof. Zero-step thinkers play $\frac{1}{2}\langle h, H, H, H\rangle, \frac{1}{2}\langle l, L, L, L\rangle$ and one-step thinkers play $\frac{1}{2}\langle h, L, H, L\rangle, \frac{1}{2}\langle l, L, H, L\rangle$. It remains to work out the strategies of more sophisticated players. Senders have four strategies to consider: $\langle h, H\rangle, \langle h, L\rangle, \langle l, H\rangle$ and $\langle l, H\rangle$. The strategy $\langle l, H\rangle$ is dominated by $\langle h, H\rangle$, so we need not consider that strategy. Receiver strategies are $\langle H, H\rangle, \langle H, L\rangle, \langle L, H\rangle$ and $\langle L, L\rangle$, but $\langle L, H\rangle$ is dominated by $\langle L, L\rangle$. Again we note that since zero-step and one-step thinkers send both messages with equal probabilities, $g_{2i}(l|m_i) = g_2(l)$. The three relevant sender strategies of two-step thinkers give

[10] This estimate of τ comes from Camerer et al. (2004) and is based on the PCH without pre-play communication – results might of course be different if τ was estimated based on games with pre-play communication.

payoffs

$$\pi(\langle l, L\rangle) = g_2(0)\frac{1}{2}(a+b) + g_2(1)a,$$
$$\pi(\langle h, H\rangle) = g_2(0)\frac{1}{2}c + g_2(1)c,$$
$$\pi(\langle h, L\rangle) = g_2(0)\frac{1}{2}(a+b) + g_2(1)b.$$

Clearly, $\pi(\langle h, L\rangle) \geq \pi(\langle l, L\rangle)$ with strict inequality if $b > a$. The strategy $\langle h, H\rangle$ is preferred over $\langle h, L\rangle$ whenever

$$g_2(0)\frac{1}{2}c + g_2(1)c > g_2(0)\frac{1}{2}(a+b) + g_2(1)b,$$

which simplifies to $(c-b)/a > 1/(1+2\tau)$ or $\tau > (a+b-c)/2(c-b)$. Now consider the behavior of two-step receivers. Since $\langle H, H\rangle$ and $\langle L, H\rangle$ are dominated we note that $\langle H, L\rangle$ is preferred over $\langle L, L\rangle$ when

$$g_2(0)\frac{1}{2}(a+c) + g_2(1)\frac{1}{2}a > g_2(0)\frac{1}{2}(a+b) + g_2(1)a,$$

which simplifies to $\tau < (c-b)/a$. Since $c < a+b$, τ must be below 1 for this condition to be satisfied.

Of the strategies above, $\langle h, L, L, L\rangle$ and $\langle h, H, H, L\rangle$ are pure equilibrium strategies, so it is only in these cases we know that all higher-step thinkers will play that strategy too (according to Lemma 2). If $a = b$, two-step thinkers mix uniformly between strategies involving $\langle h, L\rangle$ and $\langle l, L\rangle$ instead of playing $\langle h, L\rangle$, which by Lemma 2 implies that all higher-step thinkers will play $\frac{1}{2}\langle h, L, L, L\rangle, \frac{1}{2}\langle l, L, L, L\rangle$. ■

As shown by Proposition 5, the behavior of higher-step thinkers depends heavily on payoffs and τ. For the game in Figure 1, $(c-b)/a = 1/7$, implying that advanced thinkers play $\langle h, L, L, L\rangle$ as long as $1/7 < \tau < 3$. If τ is in this range, we would observe coordination on the risk dominant equilibrium, but also a considerable amount of non-equilibrium play when advanced thinkers playing L fool one-step thinkers into playing H. In this case, Aumann's (1990) argument against the honesty presumption has full force.

Although behavior depends on payoffs and the distribution of player types, coordination on the Pareto dominant equilibrium is unambiguously higher with two-way than with one-way communication for a given τ.

Proposition 6 *In the PCH model, for a given τ, two-way communication leads to more coordination on the Pareto dominant outcome in the Stag Hunt than does one-way communication.*

Proof. In order to prove the result we must verify that two-way communication leads to more coordination on the Pareto dominant outcome for all combination of types than does one-way communication.

First consider when two advanced thinkers play the game. Proposition 4 then implies that there will be perfect coordination on the HH equilibrium under two-way communication.

Now consider when primitive thinkers play the game using the behavior of primitive thinkers specified in the proof of Proposition 4 and 5. When two zero-step thinkers meet, there will be 25 percent average coordination on HH under both one-way and two-way communication. If instead two one-step thinkers meet, two-way communication results in 25 percent coordination on HH, whereas one-way communication never results in the HH outcome being played. If one-step thinkers play against zero-step thinkers, two-way communication results in 50 percent coordination on HH, whereas one-way communication results in 25 percent coordination on HH. It is clear that two-way communication results in more coordination on the Pareto dominant outcome when only primitive thinkers play the game.

It remains to determine the outcome when advanced thinkers play against primitive thinkers. For two-way communication this is straightforward. From Proposition 4 we know that advanced thinkers play either $\langle h, H, L \rangle$ or $\langle h, H, H \rangle$. When advanced thinkers play $\langle h, H, L \rangle$, coordination is 50 percent on HH when playing against primitive thinkers. If they instead play $\langle h, H, H \rangle$, there is 50 percent coordination on the HH outcome when playing against zero-step thinkers and 100 percent when they play against one-step thinkers. Hence, if we can show that one-way communication never results in more than 50 percent coordination on the Pareto dominant outcome when advanced thinkers meet primitive thinkers, we have established the result. To see that this never can happen, note from the proof of Proposition 5 that zero-step thinkers play $\frac{1}{2} \langle h, H, H, H \rangle, \frac{1}{2} \langle l, L, L, L \rangle$. Clearly, no matter what strategy advanced thinkers play, they can never obtain average coordination on HH of more than 50 percent. Similarly, one-step thinkers play $\frac{1}{2} \langle h, L, H, L \rangle, \frac{1}{2} \langle l, L, H, L \rangle$. As senders, advanced thinkers can maximally achieve 100 percent average coordination when playing aginst one-step thinkers, but as receivers they can never achieve coordination on the HH outcome. Since players are randomly assigned to either the sender or receiver role, coordination on HH can never be higher than 50 percent on average.

Since types are Poisson distributed, there is a positive probability of all type combinations when the game is played. Since coordination on the Pareto dominant outcome under two-way communication is the same for some type combinations and higher for others, this implies that two-way communication results in more coordination on the Pareto dominant outcome for any given value of τ. ■

4.2 Battle of the Sexes

In the Battle of the Sexes, one-way communication is easiest to analyze. As in the SCH model, higher-step thinkers typically send and play their preferred action H as senders, whereas they give in and play L as receivers. Although exact behavior will depend on payoffs and τ, the qualitative conclusion is clear.

Proposition 7 *In the PCH model, whenever advanced thinkers play the game, one-way communication leads to more coordination on efficient outcomes in the Battle of the Sexes than does two-way communication. As $\tau \to \infty$, there is virtually always coordination on the sender's preferred Nash equilibrium outcome under one-way communication.*

Proof. First consider behavior under one-way communication. As in the SCH model, zero-step thinkers play $\frac{1}{2}\langle h, H, H, H\rangle, \frac{1}{2}\langle l, L, L, L\rangle$ and one-step thinkers $\frac{1}{2}\langle h, H, L, H\rangle$, $\frac{1}{2}\langle l, H, L, H\rangle$. As before, $g_{2i}(l|m_i) = g_2(l)$ for two-step receivers. A two-step receiver therefore prefers $\langle L, H\rangle$ over $\langle L, L\rangle$ whenever

$$g_2(0)\frac{1}{2}(a+b) + g_2(1)\frac{1}{2}a > g_2(0)\frac{1}{2}a + g_2(1)a,$$

which simplifies to $\tau < b/a$. Hence, for sufficiently high τ, two-step thinkers will play $\langle L, L\rangle$. Under the opposite inequality, two-step thinkers play $\langle L, H\rangle$. As senders, two-step thinkers face zero-step receivers that randomize and one-step receivers that play $\langle L, H\rangle$, so it is optimal to play $\langle h, H\rangle$. Hence, two-step thinkers play $\langle h, H, L, H\rangle$ if $\tau < b/a$ and $\langle h, H, L, L\rangle$ otherwise. Since both these strategies are pure equilibrium strategies, from Lemma 2 we have that all higher-step thinkers play these strategies too.

Now consider two-way communication. Zero-step thinkers play $\frac{1}{2}\langle h, H, H\rangle, \frac{1}{2}\langle l, L, L\rangle$ and one-step thinkers play $\frac{1}{2}\langle h, L, H\rangle, \frac{1}{2}\langle l, L, H\rangle$. Two-step thinkers face a trade-off between responding to messages and sending and playing H. The strategy $\langle h, H, H\rangle$ is preferred over $\langle h, L, H\rangle$ whenever (again noting that two-step players don't update their beliefs in the second stage of the game)

$$g_2(0)\frac{1}{2}b + g_2(1)b > g_2(0)\frac{1}{2}(a+b) + g_2(1)\frac{1}{2}b,$$

which simplifies to $\tau > a/b$. That is, two-step thinkers will play $\langle h, H, H\rangle$ if $\tau > a/b$ and $\langle h, L, H\rangle$ otherwise.

Since one-way communication implies lower coordination when one-step senders meet zero-step receivers, we can only state our conclusion for advanced thinkers. Whenever advanced thinkers play the game, one-way communication results in perfect coordination, while two-way communication does not. ■

4.3 Multiplayer Generalizations

In the PCH model it is difficult to derive results for more general classes of games since behavior depend on both payoffs and τ. However, as $\tau \rightarrow \infty$, the PCH model is very similar to the SCH model for low-step thinkers. If zero-step and one-step thinkers randomize messages uniformly, the PCH model implies that $g_2(1)/g_2(0) = \tau$. Therefore, as $\tau \rightarrow \infty$, two-step thinkers will behave as if they are certain that the opponents are one-step thinkers and only attach an infinitely small probability to the opponents being zero-step thinkers. This allows us to derive predictions for more general two-player games, but since Lemma 2 does not generally apply to games with more than two players it is difficult to predict behavior for general multiplayer games. Two generalizations for two-player games as $\tau \rightarrow \infty$ are provided in Appendix 2.

5 Concluding Remarks

We view organizational structure as a solution to the equilibrium selection problem. In a model with boundedly rational players, we have shown that an authoritarian structure with one-way communication improves coordination in games with conflicting interests such as the Battle of the Sexes, whereas a consensual structure with two-way communication is optimal in games with common interests such as the Stag Hunt. We have argued that the existence of boundedly rational players that communicate naively or truthfully may help to explain why how communication affects coordination. The fraction of such players need not be very large – and it can in fact be zero as long as there is some players that believe they exist (or some people that believe that people believe they exist, and so on *ad infinitum*). In fact, the model has some bite even in the limit when players' rationality goes to infinity. We prove that the unique outcome in the Stag Hunt is then for both players to say that they will play according to the efficient equilibrium, and to act accordingly. This result contrasts with Aumann's (1990) famous argument that only the inefficient equilibrium is self-enforcing when players are perfectly rational.

The assumption we make about the behavior of zero-step thinkers is made mainly on *a priori* grounds. The main ingredient required for communication to work is that zero-step thinkers send messages truly indicating what action they are going to take. We believe that our assumptions are plausible, but that is ultimately an empirical question. In order to develop a more realistic theory, we would need data about players' beliefs. Such data can be generated not only through survey questions, but also by measuring response times and information search (Costa-Gomes et al. 2001) or by using neuroimaging techniques (Bhatt and Camerer 2005).

Although we have emphasized the role of pre-play communication as a equilibrium selection device throughout the paper, the theory does not assume equilibrium play. It is therefore also applicable to situations in which players realistically fail to play a unique and efficient Nash equilibrium – such as the High Risk game devised by Gilbert (1990). Experimental results of Burton and Sefton (2004) confirms the prevalence of coordination failure in one-shot play of the High Risk game, but demonstrates that players learn to play the equilibrium after having played a number of practice rounds with the same opponent. Our theory predicts that two-way communication could induce the unique equilibrium outcome even in a one-shot game, at least if players are sufficiently sophisticated.

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Appendix 1: Extended Message Space in the SCH Model

Let us analyze the Stag Hunt, Battle of the Sexes and the Battle of the Workers with the extended message space. Let p denote the probability that a zero-step thinker is truthful and q the probability that the receiver is obedient. For two-way communication, consistency requires that $p + q \leq 1$, since a player cannot be truthful and obedient at the same time.

Extending the message space requires a generalization of our previous notation. The sender strategy with one-way communication is denoted

$$s_i = \left\langle m_i^i, m_i^j, a_i \right\rangle,$$

where m_i^i indicates the what action player i intends to take and m_i^j indicates what action player i wants player j to take. Since there are four different combinations of messages in the Stag Hunt and Battle of the Sexes, a receiver strategy consists of an action for each of these four possible message combinations. The receiver strategy is denoted

$$s_i = \langle f_i(h,h), f_i(h,l), f_i(l,h), f_i(l,l) \rangle.$$

With two-way communication, a strategy specifies what messages to send and what to do conditionally on the received messages. (As before we need not specify what a player does after sending a message that is not a part of his strategy.) Therefore a strategy is denoted

$$s_i = \left\langle m_i^i, m_i^j, f_i(h,h), f_i(h,l), f_i(l,h), f_i(l,l) \right\rangle.$$

Stag Hunt

One-way communication

Zero-step senders randomize over truthful strategies with probability p, i.e., they play

$$\frac{1}{4}\langle h,h,H\rangle, \frac{1}{4}\langle h,l,H\rangle, \frac{1}{4}\langle l,l,L\rangle, \frac{1}{4}\langle l,h,L\rangle$$

with probability p and randomize uniformly over all strategies with probability $(1-p)$. Zero-step receivers are obedient with probability q, i.e., they play $\langle H,L,H,L\rangle$ with probability q and randomizes uniformly with probability $(1-q)$. For simplicity, we assume that $b > a$ so that the only relevant one-step sender strategies give the payoffs

$$\pi(\langle h,h,H\rangle) = \pi(\langle l,h,H\rangle) = qc + (1-q)\frac{1}{2}c,$$
$$\pi(\langle h,h,L\rangle) = \pi(\langle l,h,L\rangle) = qb + (1-q)\frac{1}{2}(a+b).$$

A one-step sender plays the first of these pairs of strategies if

$$q > \frac{a-(c-b)}{a+(c-b)} \equiv A.$$

One-step receivers play either $\langle H, H, L, L\rangle$ or $\langle L, L, L, L\rangle$. They play $\langle H, H, L, L\rangle$ if

$$p\frac{1}{2}(a+c) + (1-p)\frac{1}{4}(a+b+c) > p\frac{1}{2}(a+b) + (1-p)\frac{1}{2}(a+b),$$

or equivalently if $p > A$. If $p < A$, they play $\langle L, L, L, L\rangle$. The two conditions for p and q yield four possible cases for the behavior of one-step thinkers, which in turn pin down the behavior of higher-step players. These results are summarized in the table below.

		Senders	Receivers
$p > A, q > A$	$k = 1$	$p_1 \langle h, h, H\rangle, (1-p_1)\langle l, h, H\rangle$ with $p_1 \in (0,1)$	$\langle H, H, L, L\rangle$
	$k \geq 2$	$\langle h, h, H\rangle$	$\langle H, H, H, L\rangle$
$p > A, q < A$	$k = 1$	$p_1 \langle h, h, L\rangle, (1-p_1)\langle l, h, L\rangle$ with, $p_1 \in (0,1)$	$\langle H, H, L, L\rangle$
	$k = 2$	$\langle h, h, H\rangle$	$\langle L, H, L, L\rangle$
	$k \geq 3$	$\langle h, l, H\rangle$	$\langle H, H, L, L\rangle$
$p < A, q > A$	$k = 1$	$p_1 \langle h, h, H\rangle, (1-p_1)\langle l, h, H\rangle$ with $p_1 \in (0,1)$	$\langle L, L, L, L\rangle$
	$k = 2$	$p_2 \langle h, h, L\rangle, (1-p_2)\langle l, h, L\rangle$ with $p_2 \in (0,1)$	$\langle H, L, H, L\rangle$
	$k \geq 3$	Alternates	Alternates
$p < A, q < A$	$k \geq 1$	$p_1 \langle h, h, L\rangle, (1-p_1)\langle l, h, L\rangle$ with, $p_1 \in (0,1)$	$\langle L, L, L, L\rangle$

Observe that one-way communication induces advanced thinkers to play H if both p and q exceed A. If both p and q are sufficiently low, however, communication becomes irrelevant; one-step and higher-step players then always play L. Remarkably, when truthfulness is high, but obedience is low, advanced thinkers still end up coordinating on the Pareto dominant equilibrium. The reason is that three-step senders mimic zero-step senders and send the message h, l which induces receivers to play H. There are at least two reasons why we might expect that this particular prediction is non-robust. First, higher-step thinkers would not play H if there is the slightest probability that a one-step thinker sends the message h, l. Second, suppose that there is a chance that zero-step thinkers realize (after having sent the message h, l) that it is unwise to play H when they have asked the opponent to play L. If a fraction of zero-step thinkers manage to think one more step after having sent messages, one-step thinkers may no longer play H upon seeing the message h, l. If so, the strategy $\langle h, l, H\rangle$ becomes unattractive for advanced thinkers.

Two-way communication

Zero-step players play truthfully with probability p and obediently with probability q. As long as $b > a$, it is optimal for one-step thinkers to propose that the opponent should play

H, whereas the message about his own play doesn't matter. A one-step player therefore faces a trade-off between playing H irrespective of the messages received, playing H if the opponent indicates that he will play H, and always playing L. The payoffs for these strategies are

$$\pi(\langle h,h,H,H,H,H\rangle) = \pi(\langle l,h,H,H,H,H\rangle) = p\frac{1}{2}c + qc + (1-p-q)\frac{1}{2}c,$$
$$\pi(\langle h,h,H,H,L,L\rangle) = \pi(\langle l,h,H,H,L,L\rangle) =$$
$$= p\frac{1}{2}(a+c) + q\frac{1}{2}(c+b) + (1-p-q)\frac{1}{4}(a+b+c),$$
$$\pi(\langle h,h,L,L,L,L\rangle) = \pi(\langle l,h,L,L,L,L\rangle) = p\frac{1}{2}(a+b) + qb + (1-p-q)\frac{1}{2}(a+b).$$

The strategies $\langle h,h,H,H,L,L\rangle$ and $\langle l,h,H,H,L,L\rangle$ are preferred over $\langle h,h,L,L,L,L\rangle$ and $\langle l,h,L,L,L,L\rangle$ whenever

$$p+q > \frac{a-(c-b)}{a+(c-b)} = A.$$

Similarly, $\langle h,h,H,H,H,H\rangle$ and $\langle l,h,H,H,H,H\rangle$ are preferred over $\langle h,h,H,H,L,L\rangle$ and $\langle l,h,H,H,L,L\rangle$ whenever

$$q-p > \frac{a-(c-b)}{a+(c-b)} = A.$$

Thus, if $p+q > A$ two-way communication induce the Pareto dominant equilibrium outcome whenever advanced thinkers meet. Note that the higher q is, the lower can p be, which is in line with the assurance motive of two-way communication discussed in the introduction. Finally, note that if $p+q=1$ then this condition is always fulfilled.

Battle of the Sexes

One-way communication

Zero-step senders randomize over truthful strategies with probability p, i.e., they play

$$\frac{1}{4}\langle h,h,H\rangle, \frac{1}{4}\langle h,l,H\rangle, \frac{1}{4}\langle l,l,L\rangle, \frac{1}{4}\langle l,h,L\rangle,$$

with probability p. Zero-step receivers are obedient, i.e., they play $\langle H,L,H,L\rangle$ with probability q. One-step senders thus play $p_1\langle h,h,H\rangle, (1-p_1)\langle l,h,H\rangle$ with $p_1 \in (0,1)$. One-step receivers face a trade-off between playing according to received messages or playing H. They play according to the received message if

$$p\frac{1}{2}(a+b) + (1-p)\frac{1}{4}(a+b) > p\frac{1}{2}b + (1-p)\frac{1}{2}b,$$

which simplifies to $p > (b-a)/(a+b)$. Since one-step senders play H irrespective of whether zero-step receivers are obedient or not, q does not affect the impact of one-way communication in the Battle of the Sexes.

Two-way communication

Zero-step players play truthfully with probability p and obey orders with probability q. One-step players face a trade-off between always playing H and responding to the senders message as it was truthful. They best-respond to messages if

$$p\frac{1}{2}(a+b) + q\frac{1}{2}b + (1-p-q)\frac{1}{4}(a+b) > p\frac{1}{2}b + qb + (1-p-q)\frac{1}{2}b,$$

which is equivalent to $p - q > (b-a)/(a+b)$. This condition can only hold if $p > q$. However, irrespective of the behavior of one-step thinkers, two-way communication entails little coordination.

Battle of the Workers

Let us derive a lower bound on the degree of zero-step thinkers' obedience that is required for one-way communication to induce loyalty by more advanced thinkers in the Battle of the Workers. For simplicity, assume throughout that zero-step thinkers are truthful. (Observe that the analysis applies only for $n \geq 3$. If $n = 2$ the game is equivalent to the Battle of the Sexes.)

For one-way communication to work, we must make sure that a one-step receiver plays the action that the sender intends. Let us assume that a one-step receiver has received a message allocating task n to the sender and each of the remaining $n-1$ tasks to separate receivers. Since zero-step thinkers are truthful, the best payoff a one-step receiver could hope for is $\pi(n-1)$, i.e., the equilibrium payoff when playing $n-1$. Since we are interested in deriving a lower bound on q, consider the problem of a one-step receiver who has been assigned the number 1 – so that the payoff difference between obedience and playing $n-1$ is as large as possible.

A one-step receiver believes that the other $n-2$ receivers are zero-step thinkers and consequently that they obey orders with probability q and play randomly with probability $(1-q)$. The probability that exactly x out of the opponents obey orders is

$$\binom{n-2}{x} q^x (1-q)^{n-2-x}.$$

If x out of $n-2$ receivers obey orders, the probability that the remaining receivers manage to pick one number each among the numbers that are not chosen by the $n-2-x$ receivers that are obedient is given by

$$(n-2-x)!\left(\frac{1}{n}\right)^{n-2-x}.$$

Using these probabilities we can derive the payoff to a one-step receiver of loyalty, i.e., playing 1

$$E[1] = \sum_{x=0}^{n-2} \binom{n-2}{x} q^x (1-q)^{n-2-x} (n-2-x)! \left(\frac{1}{n}\right)^{n-2-x} \pi(1)$$
$$= \pi(1)(n-2)! \sum_{x=0}^{n-2} \frac{q^x}{x!} \left(\frac{1-q}{n}\right)^{n-2-x}.$$

What is the expected payoff from picking $n-1$ instead of 1? For this strategy to result in positive payoff, it must be the case that the particular player that was assigned to $n-1$ does not obey orders. Given that one particular player does not obey orders, the probability that x players obey orders is

$$(1-q)\left[\binom{n-3}{x} q^x (1-q)^{n-3-x}\right].$$

Given that x players obey orders, the probability that the remaining $n-2-x$ players pick a unique number each is

$$(n-2-x)! \left(\frac{1}{n}\right)^{n-2-x},$$

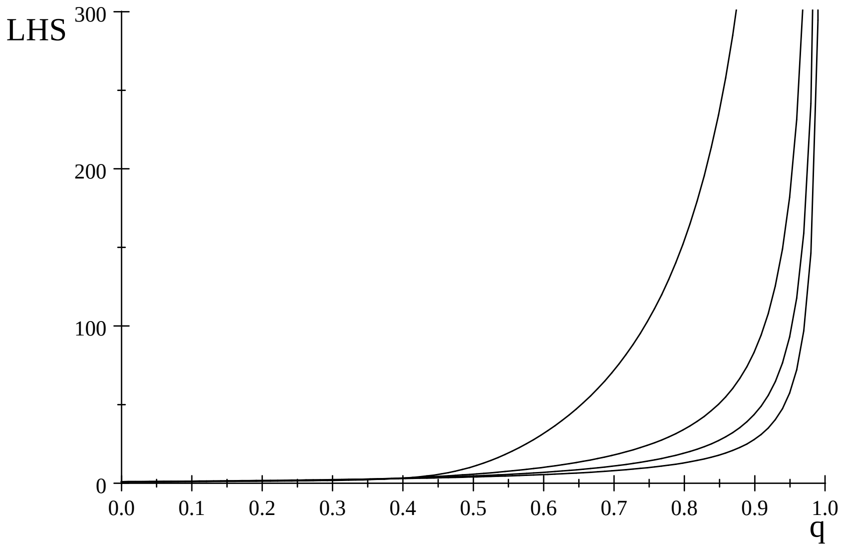
allowing us to calculate the expected payoff of picking $n-1$ as,

$$E[n-1] = \sum_{x=0}^{n-3} (1-q) \binom{n-3}{x} q^x (1-q)^{n-3-x} (n-2-x)! \left(\frac{1}{n}\right)^{n-2-x} \pi(n-1)$$
$$= \pi(n-1)(n-3)! \sum_{x=0}^{n-3} (n-2-x) \frac{q^x}{x!} \left(\frac{1-q}{n}\right)^{n-2-x}.$$

Thus, the expected payoff of obedience is higher than picking $n-1$ whenever

$$\frac{\sum_{x=0}^{n-2} (n-2) \frac{q^x}{x!} \left(\frac{1-q}{n}\right)^{n-2-x}}{\sum_{x=0}^{n-3} (n-2-x) \frac{q^x}{x!} \left(\frac{1-q}{n}\right)^{n-2-x}} > \frac{\pi(n-1)}{\pi(1)}.$$

It is difficult to solve this condition explicitly, but we can plot numerically the left hand side of the expression. The graph below plots the left hand side as a function of q for $n = \{3, 5, 10, 50\}$ where the leftmost graph correspond to $n = 50$ and the rightmost one to $n = 3$.



For example, one-way communication works fine if q is larger than 0.35 and $\pi(n-1)/\pi(1) = 2$. Thus, a relatively small degree of obedience among aero-step thinkers is sufficient to induce loyalty by one-step thinkers.

Appendix 2: Two-Player Generalizations

For two-player games it is possible to derive predictions for somewhat more general payoff structures. In order to do so, we introduce some additional terminology. First, we say that player i has a *favorite* equilibrium if there is a pure Nash equilibrium that gives that player strictly higher payoff than all other outcomes of the game. Secondly, we define an action $a_i^{S_i}$ in game G as *safe* for player i if it gives player i a strictly higher payoff than all other actions given that the opponent randomizes uniformly over his available actions.

SCH Model

The reason why two-way communication is so efficient in achieving coordination in the Stag Hunt is that two-step thinkers get the possibility to choose among equilibria when playing against one-step thinkers. If they choose a pure equilibrium action, then all higher-step players will play this strategy too. In order for this sort of argument to work, we must make sure that the Pareto dominant equilibrium gives higher payoffs than all other payoffs of the game – otherwise two-step thinkers will be tempted to play another action and send a message to get the one-step player to play the action that gives him the highest payoff. An equivalent way of formulating this requirement is that both players have the same favorite equilibrium, or that it is a common interest game. This result is stated in Proposition 8, but is a straightforward extension of Proposition 1.

Proposition 8 *Let G be a two-player common interest game. In the SCH model, advanced thinkers coordinate on the unique Pareto dominant equilibrium of G under two-way communication. Under one-way communication, advanced thinkers generally fail to coordinate on the Pareto dominant equilibrium.*

Proof. First consider two-way communication. Zero-step thinkers randomize over all truthful strategies, which implies that one-step thinkers send random messages but best respond to the received message, i.e., they play $BR_i(m_j)$. Since one-step thinkers best-respond to messages as if they are truthful, a two-step thinker can "pick" the best outcome among those actions that are best-responses of the opponent. The highest possible payoff to player i is by assumption achieved in the unique Pareto dominant equilibrium. The player can achieve this equilibrium by sending the message indicating that he will play the action corresponding to that equilibrium. Higher-step thinkers will play in the same way, i.e., they will play the truthful strategies involving the unique Pareto dominant equilibrium.

Now consider one-way communication. Zero-step senders play truthfully, whereas they respond randomly to messages. One-step receivers will therefore play $BR_i(m_j)$, whereas one-step senders will send any message and play the safe action $a_i^{S_i}$. Two-step senders will be able to "pick" their favorite outcome and therefore play the truthful strategy involving the Pareto dominant equilibrium. As receivers, they will best-respond to the safe action chosen by one-step senders, i.e., they will play $BR_i(a_j^{S_j})$. Unless the safe action profile $a_i^{S_i}, a_j^{S_j}$ happens to be the Pareto dominant equilibrium, it is clear that there will be more coordination with two-way communication than with one-communication (otherwise coordination will only be higher for two-way communication when zero-step and one-step players play the game). ■

Now consider games with multiple pure equilibria that cannot be Pareto ranked, but were each player has a favorite equilibrium. Following the logic of Proposition 2 regarding the Battle of the Sexes, it is tempting to conclude that one-way communication would result in more coordination than two-way communication. However, we also need to assume that the favorite equilibrium of each player coincide with the safe action of that player, i.e., the one that he plays if the opponent randomizes uniformly over available actions. The reason is that one-step senders play against zero-step receivers that respond randomly, which implies that one-step senders will play their safe action. Two-step receivers will therefore play the best-response to the safe action of the sender, which corresponds to the favorite equilibrium of the one-step senders only if the safe action is the same as the favorite equilibrium action. This result is stated in the next proposition.

Proposition 9 *Let G be a two-player game such that each player has a favorite equilibrium which coincides with the safe action of that player. Then coordination on the two favorite equilibria will be higher among advanced thinkers with one-way than with two-way* *communication in the SCH model. Furthermore, with one-way communication advanced thinkers will play so that the sender plays his favorite equilibrium and receivers play the favorite equilibrium of the sender.*

Proof. First consider one-way communication. Zero-step senders randomize over truthful strategies, whereas they respond randomly to messages. One step receivers will therefore play strategies $f_i(m_j) = BR_i(m_j)$, whereas one-step senders will send any message and play the safe action $a_i^{S_i}$. Two-step senders may thus "pick" their favorite equilibrium outcome. As receivers, two-step thinkers best-respond to the safe action chosen by one-step thinkers, i.e., they play $BR_i(a_j^{S_j})$, which corresponds to the favorite equilibrium of the opponent. Consequently, all higher-step thinkers will play in the same way – senders play the truthful strategy involving their favorite equilibrium outcome, while receivers play the favorite outcome of the opponent.

Now consider two-way communication. Zero-step thinkers randomize over all truthful strategies, which implies that one-step thinkers send random messages but best respond to the message sent by zero-step thinkers, i.e., they play strategies such that $f_i(m_i, m_j) = BR_i(m_j)$. Thus a two-step thinker plays the strategy involving his favorite equilibrium. Three-step thinkers play the favorite equilibrium of the opponent, and four-step thinkers again play their own favorite equilibrium. Behavior continues to alternate, leading to little coordination on either equilibrium. Clearly, for advanced thinkers, coordination will be higher with one-way than with two-way communication. ■

The conditions stated in Proposition 8 and 9 are sufficient but not necessary. For example, the result in Proposition 8 holds even if we add non-equilibrium payoffs that are higher than the payoffs in the Pareto dominant equilibrium, as long as the non-equilibrium outcome involves a dominated strategy for one of the players.

PCH Model

In the PCH model it is more difficult to obtain general results, since behavior depend on both payoffs and τ. However, as $\tau \to \infty$, there are analogs to Proposition 8 and 9 For example, it is straightforward to show that common interest games imply perfect coordination with two-way communication as $\tau \to \infty$. To say what happens with one-way communication as $\tau \to \infty$ is not straightforward, however, since we cannot make use of Lemma 2 to determine what higher-step players will do. Similarly, it is easy to show that one-way communication leads to perfect coordination as $\tau \to \infty$ for games where each player has a favorite equilibrium that coincides with his safe action, but it is more difficult to characterize the outcome of two-way communication as $\tau \to \infty$. The next two propositions summarize the perfect coordination results.

Proposition 10 *Let G be a two-player common interest game. Then, as $\tau \to \infty$, the PCH model implies that there is perfect coordination on the unique Pareto dominant equilibrium under two-way communication.*

Proof. Since zero-step and one-step players randomize messages uniformly, two-step thinkers do not update beliefs. Use the fact that $g_2(1)/g_2(0) = \tau$ to conclude that whenever $\tau \to \infty$ two-step thinkers behave as if they are certain that the opponent is a one-step thinker (and only attach an infinitely small probability to the opponent being a zero-step thinker). Therefore two-step thinkers behave in the same way as two-step thinkers in the SCH model unless they are indifferent between strategies. Following the proof of Proposition 8 we realize that two-step thinkers will play the truthful strategy associated with the Pareto dominant equilibrium. Since this is a pure equilibrium strategy, by Lemma 2 all higher-level thinkers will play that strategy too. ∎

Proposition 11 *Let G be a two-player normal form game in which each player have a favorite equilibrium which coincides with the safe action of that player. Then, as $\tau \to \infty$, the PCH model implies that there is perfect coordination on the sender's preferred equilibrium under one-way communication .*

Proof. Again we note that $g_2(1)/g_2(0) = \tau$ so that whenever $\tau \to \infty$ two-step thinkers behave as if they are certain that the opponent is a one-step thinker (and only attach an infinitely small probability to the opponent being a zero-step thinker). Unless two-step thinkers are indifferent between strategies, they will behave as two-step thinkers in the SCH model. For the same reason as in the proof of Proposition 9 two-step senders thus play the truthful strategy involving their safe action, whereas two-step receivers best-respond to the received message. Since this strategy is a best-response to itself, by Lemma 2 all higher-step players will play the same strategy – senders play the truthful strategy involving their favorite equilibrium, while receivers play the favorite equilibrium of the opponent. ∎