Error correction in DHSY

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Abstract

In this note, we consider the contradiction between the fact that the best fit for the UK consumption data in Davidson et al. (1978) is obtained using an equation with an intercept but without an error correction term, whereas the equation with error correction and without the intercept has better post-sample forecasting properties than the former equation. This contradiction is explained and the two equations reconciled in a nonlinear framework by applying a smooth transition regression model to the data.

Keywords: consumption equation, forecast failure, model misspecification testing, nonlinearity, smooth transition regression

JEL Classification Codes: C22

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1 Introduction

Davidson, Hendry, Srba and Yeo (1978), DHSY in short, is an influential article on modelling the consumption-income relationship, and it has inspired a large number of follow-up studies. It has contributed to the popularity of the concept of error correction, introduced by Phillips (1957), for which statistical justification arrived later in the form of cointegration; see Granger (1981). Nevertheless, the role of error correction in modelling the UK consumption 1958(2)-1970(4) in DHSY appears somewhat ambiguous. When the equation is estimated without the intercept, the error correction term, the logarithmic consumption/income ratio, has a significant coefficient estimate. When the same model contains an intercept, the error correction term is no longer significant at the 10% level, whereas the intercept is strongly significant. The authors are aware of this. They estimate a model without the intercept and another one with the intercept but without the error correction term. The latter equation yields better fit, but the former one is favoured for theoretical reasons and because of the fact that the former model has better out-of-sample forecasting performance of the two. DHSY note that the two equations "exhibit an interesting conflict between goodness of fit and parameter stability as criteria for model selection."

In this note we reconsider the situation in the light of the period 1958(2)-1970(4). In particular, we assume that the subsequent observations are completely unknown to us. We simply model the period 1958(2)-1970(4) as well as we can using the data for this period. The resulting model reconciles the model with an error correction term and the one with an intercept. This nonlinear model, however, while fitting the data very well, breaks down in the sense of Clements and Hendry (1998) when new information about the inflation rate becomes available in the early 1970's.

The note has the following structure. In Section 2 we introduce the data and the notation. The problem is defined in Section 3. In Section 4 we introduce our model, the smooth transition regression model. Section 5 contains the modelling results and some discussion. Section 6 concludes.

2 Data and notation

The data set used in the paper is the same as the one DHSY used. It has since been extended but, at the same time, some of the data definitions have been changed, so that the old and the revised data are not compatible. An essential feature is that the estimation period is the same as in DHSY: 1958(2) to 1970(4). The following notation is used. Lowercase letters represent logarithmic variables: $x_t = \ln X_t$. Variable C_t is the consumption of nondurable goods, Y_t is the disposable income, P_t is the consumption price index, and $D68_t$ is a dummy variable for a temporary switch in consumption between the first and the second quarter. The variables in the final consumption equation are the four-quarter differences $\Delta_4 c_t, \Delta_4 y_t, \Delta\Delta_4 y_t, \Delta_4 p_t, \Delta\Delta_4 p_t$, the error correction term $c_t - y_t$, and $D68_t$. Augmented Dickey-Fuller tests reject the unit root hypothesis at the 10% level of significance for all variables except $c_t - y_t$.

3 The problem

In order to illustrate the situation, we begin by estimating the DHSY equation (45) augmented by the intercept. It is based on four-quarter differences of the logarithmic series and has the following form:

$$\begin{aligned}\Delta_4 c_t &= \underset{(0.0048)}{0.014} + \underset{(0.049)}{0.37}\,\Delta_4 y_t - \underset{(0.047)}{0.17}\,\Delta\Delta_4 y_t - \underset{(0.070)}{0.21}\,\Delta_4 p_t - \underset{(0.13)}{0.16}\Delta\Delta_4 p_t \\ &\quad -\underset{(0.028)}{0.041}(c_{t-4} - y_{t-4}) + \underset{(0.0030)}{0.0074}\Delta_4 D68_t + \widehat{\varepsilon}_t \qquad (1)\end{aligned}$$

$$R^2 = 0.81, \quad \widehat{\sigma}_L = 0.0055 \quad pLJB = 0.66$$

$$pARCH(1) = 0.54 \quad pARCH(4) = 0.77 \quad pF_{\text{RESET}}(1,43) = 0.066$$
$$pLM_{\text{AC}}(1) = 0.56 \quad pLM_{\text{AC}}(4) = 0.81$$

where the figures in parentheses are estimated standard errors, $pLJB$ is the p-value of the Lomnicki-Jarque-Bera normality test, $pARCH(j)$ is the p-value of the LM test of no ARCH against ARCH of order j, $pLM_{\text{AC}}(j)$ is the p-value of the LM test of no error autocorrelation against autocorrelation of order j, $pF_{\text{RESET}}(1,43)$ is the p-value of the RESET with the quadratic term. It is seen from (1) that the intercept is highly significant, whereas the estimated coefficient of the error correction term $c_{t-4} - y_{t-4}$ is not. Removing it would be an obvious decision if a model selection criterion such as AIC or BIC were used as a guidance. However, DHSY omit the intercept instead for the reasons discussed in the Introduction. We shall take another look at the situation using a particular nonlinear model, the smooth transition regression (STR) model, as our tool. It turns out that with our approach we are able to reconcile the two competing alternatives.

It should be noted that Cook, Holly and Turner (1999) and Cook (2000) have also modelled the DHSY data using nonlinear (in variables, not in parameters) models. The aims of their analysis are, however, different from ours, and we do not consider their results in our discussion.

4 Smooth transition regression model

The STR model is nonlinear model capable of describing asymmetries and other types of nonlinearity in macroeconomic relationships; for a recent account, see

Teräsvirta (1998). It can be defined as follows:

$$\begin{aligned} y_t &= \boldsymbol{\phi}'\mathbf{z}_t + \boldsymbol{\theta}'\mathbf{z}_t G(\gamma, \mathbf{c}, s_t) + \varepsilon_t \\ &= \{\boldsymbol{\phi} + \boldsymbol{\theta} G(\gamma, \mathbf{c}, s_t)\}'\mathbf{z}_t + \varepsilon_t, t = 1, ..., T \end{aligned} \tag{2}$$

where $\mathbf{z}_t = (\mathbf{w}_t', \mathbf{x}_t')'$ with $\mathbf{w}_t = (1, y_{t-1}, ..., y_{t-p})'$, $\mathbf{x}_t = (x_{1t}, ..., x_{kt})', x_{jt}, j = 1, ..., k$, are exogenous to the parameters of interest, and $m = p + k + 1$. Furthermore, $\boldsymbol{\phi} = (\phi_0, \phi_1, ..., \phi_p)'$ and $\boldsymbol{\theta} = (\theta_1, ..., \theta_m)'$ are parameter vectors, $\mathbf{c} = (c_1, ..., c_K)'$ is a vector of location parameters, $c_1 \leq ... \leq c_K$, and $\varepsilon_t \sim$iid$(0, \sigma^2)$. Transition function $G(\gamma, \mathbf{c}, s_t)$ is a bounded function of s_t, continuous everywhere in the parameter space for any value of s_t. The last expression in (2) indicates that the model can be interpreted as a linear model with stochastic time-varying coefficients $\boldsymbol{\phi} + \boldsymbol{\theta} G(\gamma, \mathbf{c}, s_t)$. The logistic transition function has the general form

$$G(\gamma, \mathbf{c}, s_t) = (1 + \exp\{-\gamma \prod_{k=1}^{K} (s_t - c_k)\})^{-1}, \gamma > 0 \tag{3}$$

where $\gamma > 0$ is an identifying restriction. Equation (2) jointly with (3) defines the logistic STR (LSTR) model. The most common choices for K are $K = 1$ and $K = 2$. For $K = 1$, the parameters $\boldsymbol{\phi} + \boldsymbol{\theta} G(\gamma, \mathbf{c}, s_t)$ change monotonically as a function of s_t from $\boldsymbol{\phi}$ to $\boldsymbol{\phi} + \boldsymbol{\theta}$. For $K = 2$, they change symmetrically around the mid-point $(c_1 + c_2)/2$ where this logistic function attains its minimum value. Slope parameter γ controls the slope and c_1 and c_2 the location of the transition function. When $\gamma \to \infty$, (3) becomes a step function for $K = 1$ and a "double step" function for $K = 2$.

In this note, the transition variable s_t is an element of vector $\mathbf{z}_t$. As we have no theory for choosing s_t in advance, the choice will be data-driven and will be discussed in the next section.

5 Results

We follow the model selection strategy outlined in Teräsvirta (1998) in order to build our STR consumption equation for the DHSY data. Thus, we first test linearity of (1) against STR. This involves choosing $K = 1$ and approximating the transition function by a third-order Taylor expansion around $\gamma = 0$, see Teräsvirta (1998). The results can be found in Table 1. They show that linearity

Test statistic\Transition variable	$\Delta_4 y_t$	$\Delta\Delta_4 y_t$	$\Delta_4 p_t$	$\Delta\Delta_4 p_t$	$c_{t-4} - y_{t-4}$
F_{04} (3rd order Taylor expansion)	0.55	0.95	0.63	0.25	..
F_{02} (1st order Taylor expansion)	0.44	0.72	0.067	0.076	0.51

Table 1: p-values of linearity tests of equation (1) against STR

cannot be rejected at the conventional 5% significance level for any of the potential transitional variables. However, if linearity is tested directly against the logistic STR model ($K = 1$ and the first-order Taylor expansion) the test statistic when the inflation variable $\Delta_4 p_{t-4}$ is assumed to be the transition variable equals $p = 0.067$. Furthermore, if the latter test is performed using equation (45) in DHSY, the one without the intercept, as the null model, the corresponding p-value equals 0.0041. Therefore it seems fair to argue that equation (45) is misspecified and that the misspecification involves the intercept, the error correction component and the inflation rate. It should be noted that the asymptotic distribution theory for the tests is only applicable when the series are stationary. The error correction term appears not to be stationary, but due to short series it is unlikely that the results are much affected by that.

Based on the test results, we reject linearity and proceed with estimation of the appropriate STR model with $\Delta_4 p_t$ as the transition variable. After eliminating redundant variables and imposing appropriate parameter restrictions, the final STR model has the following form:

$$\begin{aligned}\Delta_4 c_t \;=\; & \underset{(0.038)}{0.34}\,\Delta_4 y_t - \underset{(0.034)}{0.15}\,\Delta\Delta_4 y_t - \underset{(0.078)}{0.11}\,\Delta_4 p_t - \underset{(0.20)}{0.35}\Delta\Delta_4 p_t - \underset{(0.027)}{0.18}\,(c_{t-4} - y_{t-4}) \\ & + \underset{(0.0025)}{0.0093} - \{\underset{(0.0038)}{0.016} - \underset{(0.20)}{0.35}\Delta\Delta_4 p_t - \underset{(0.027)}{0.18}\,(c_{t-4} - y_{t-4})\} \\ & \times[1 + \exp\{-\underset{(11.7)}{12.4}(\Delta_4 p_t - \underset{(0.0018)}{0.022})]^{-1} + \widehat{\varepsilon}_t \qquad (4)\end{aligned}$$

$$R^2 = 0.85, \quad \widehat{\sigma} = 0.0049 \quad \widehat{\sigma}_L/\widehat{\sigma} = 0.84 \quad pLJB = 0.68$$

$$\begin{aligned}& pARCH(1) = 0.94 \quad pARCH(4) = 0.69 \quad pF_{\text{RESET}}(1, 41) = 0.24 \\ & pLM_{\text{AC}}(1) = 0.62 \quad pLM_{\text{AC}}(4) = 0.60 \quad pF_{\text{encomp}}(3, 39) = 0.99\end{aligned}$$

where $pF_{\text{encomp}}(3, 39)$ is the p-value of the LM test of the restrictions on the intercept and the coefficients of $\Delta\Delta_4 p_t$ and $c_{t-4} - y_{t-4}$. $F_{\text{encomp}}(3, 39)$ is an F-statistic for testing the null hypothesis that (4) encompasses (1). This null hypothesis cannot be rejected, and no other test suggests model inadequacy either. Besides, the tests of no additive nonlinearity and parameter constancy (results not shown), see Teräsvirta (1998), do not contain evidence against (4). The residual standard deviation of the STR equation is only 84% of that of (1), which is a noticeable improvement.

Ericsson and MacKinnon (in press), using a longer observation period ending 1975(4), argued that seasonal dummy variables are required in the linear DHSY model. The coefficient estimates of seasonal dummies are not significant, however, when these variables are added to (4).

The estimated nonlinear error correction relationship equals

$$\begin{aligned}e_t \;=\; & -0.18(c_{t-4} - y_{t-4}) - \{0.016 - 0.18(c_{t-4} - y_{t-4})\} \\ & \times\{1 + \exp\{-12.4(\Delta_4 p_t - 0.022)\}^{-1} \qquad (5)\end{aligned}$$

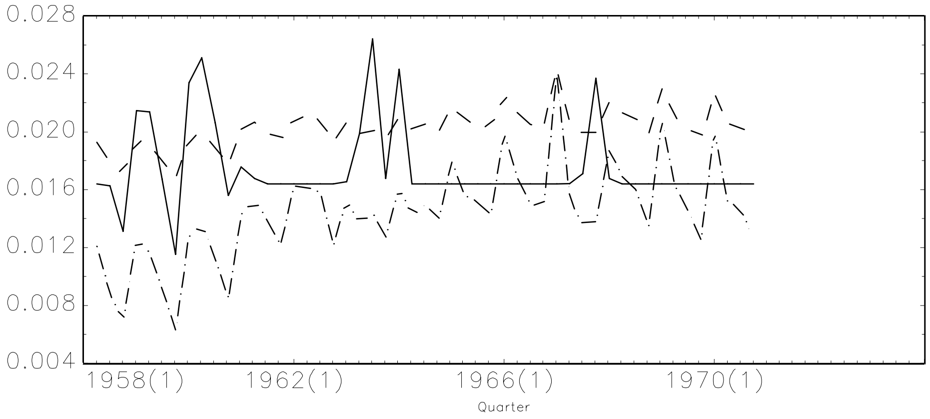

Figure 1: Error correction term (5) (solid line), the DHSY model (dashed-dotted line), and the linear model with intercept (dashed line) graphed over time

The restriction between the "linear" and "nonlinear" coefficient of $c_{t-4} - y_{t-4}$ is supported by the data. It is seen from (5) that the equilibrium error correction occurs only at low levels of the inflation. The graph of (5) appears in Figure 1. When inflation is sufficiently high, the intercept takes over, and the long run relationship between consumption and income ceases to exist. Figure 2 depicting the transition function as a function of the transition variable shows that a majority of quarters actually belong to the set in which no error correction takes place. The long-run relationship DHSY entertain is local in the sense that it is supported by the data only at periods of low inflation. This explains the presence of the intercept in (1), but retaining the error correction term in the linear equation also receives partial support from (4). Figure 1 shows that the error correction term in the equation of DHSY is trending, which explains the result of the unit root test mentioned in Section 2. On the other hand, the error correction of (4) is stationary so that the equation is a balanced one. The STR equation thus reconciles the two competing linear models in DHSY, the one with the intercept and no long-run relationship and the one with an error correction term but no intercept.

6 Final remarks

The STR model (4) represents what the data tell us about the relationship between consumption and inflation in the UK for 1958(2)-1970(4). Nevertheless, the model fails outside the estimation period in the sense that its forecast accuracy is inferior to that of the error correction model in DHSY. This appears contradictory as the inflation rate after the estimation period has been higher, not lower, than during it. One would therefore expect the nonlinear equation do

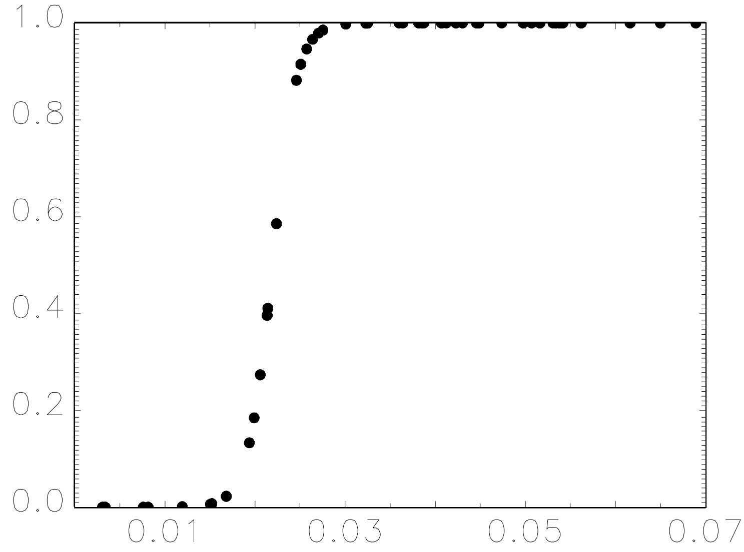

Figure 2: Transition function of model (4) graphed as a function of transition variable. Each dot represents an observation

better than the linear error correction one because according to the STR model the error correction is only operational when the inflation rate is sufficiently low.

The main reason for this outcome is that during the estimation period the inflation rate appears stationary. However, in the early 1970s there is a strong upsurge in inflation suggesting that the inflation rate is better modelled as an integrated variable. This is why equation (4) ceases to be a useful approximation to the data-generating process, and consequently it may be argued that it "breaks down" in the 1970s. The new information about the inflation rate can then be interpreted as a post-sample break in the model, as defined in Clements and Hendry (1998). It is quite conceivable, however, that the data-generating process remains unchanged for a period including the years 1971-1975. The problem is simply that the approximation to it that originally appeared fully plausible turns out to be inadequate when drastically new information about the dynamic behaviour of some of the time series included in the model becomes available. One cannot expect the DHSY equation to be an adequate description of the DGP either, but it happens to contain a linear combination of the three variables that in the light of later information appear cointegrated; see Hendry, Muellbauer and Murphy (1990). It thus has a chance of providing reasonable forecasts (in fact the decisive criterion for DHSY to choose this equation was its post-sample forecasting performance).

More generally, the results in this note suggest that when the time series initially available for macroeconometric modelling are short, the conclusions about the functional form of the model may change when more data become available. In other words, with more information, breaks are likely to occur. This may speak for parsimonious linear specifications. We have simply shown how fitting an STR model to the data from 1958(2)-1970(4) neatly solves the contradiction between the goodness of fit and parameter stability highlighted in DHSY.

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