Working Papers

Institute of Mathematical Economics
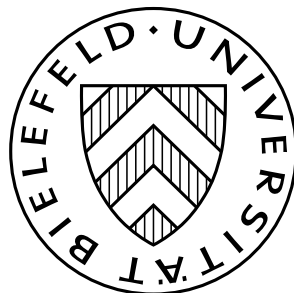
380

Brain Drain and Factor Complementarity

Patrice Pieretti and Benteng Zou

Brain Drain and Factor Complementarity [*]

Patrice Pieretti[†]   Benteng Zou[‡]

April 6, 2006

Abstract. In this paper we develop a neoclassical growth model that aggregates different types of labor skills from strict complementarity to perfect substitution. After having derived general balanced growth conditions and developed explicit growth paths for capital and aggregate labor force, the model serves to qualitatively study the effect of brain drain on income and wages of the source country.

Keywords: Brain drain, growth, complementary, migration.

JEL Classification: F22, J24, J61, E13.

1   Introduction

In recent years, migration of skilled workers (brain drain) from developing countries to OECD countries or from European countries to US has raised some anxiety about the

possible negative impact on source countries. Between 1990 and 2000, the stock of skilled immigrants in the OECD increased by 64 percent, and the rise was stronger for immigrants coming from less developed countries (Carrington and Detragiache,1998; Docquier, Lohert and Marfouk, 2005).

Brain drain is typically perceived as having a negative impact on the source country's welfare. The detrimental effect seems to be worst in developing countries where skilled manpower is already scarce. As a matter of fact, the highest brain drain rates relative to the educated labor force are observed in the Caribbean, Central America, and Western and Eastern Africa (Docquier and Marfouk, 2005). Beneficial effects to the source country induced by brain drain are however not excluded. Besides the positive impact of remittances sent home by emigrants it has recently been argued (Mountford,1997; Beine, Docquier and Rapoport, 2001) that brain drain may increase the incentive to acquire education in the source country.

The relevance of complementarities between skilled and unskilled labor in the understanding of the effect of outward flows of skilled people on home country output has been raised by some authors (Piketty, 1997; Beine, Docquier and Rapoport, 2001). In the same vein, Saint-Paul (2004) tries to calculate the impact on the per capita income of the source country of European expatriates flows in the USA, by assuming imperfect substitution between migrants and non migrants. This analysis is static in nature and is restricted to the case of unit substitution elasticity between the different labor types.

To our knowledge however, there is no literature that formally explores the role of elasticity of substitution between skilled and unskilled labor in the discussion about the impact of brain drain on the growth performance of the country of origin.

In this paper we try to fill that gap by developing a neoclassical growth model that aggregates different types of labor skills in order to grasp the full range of substitution degrees from strict complementarity to perfect substitution. After having derived general conditions for the existence of balanced growth paths and developed explicit growth dynamics for capital and aggregate labor force, the model serves to qualitatively study the effect of brain drain on income and wages of the source country. If prior to emigration the growth

2

rate of unskilled labor exceeds that of skilled workers, the model shows that in the short run, remittances from the emigrated improve per capita income in the country of emigration. In the long run however, emigration decreases per capita income if it aggravates the qualification gap between skilled and unskilled labor. It also appears that the more both labor kinds are complements in production the more emigration will eventually hurt per capita income in the home population.

The paper is organized as following. The growth model is developed in section 2, where explicit growth paths on capital and labor force accumulation are given. The theoretical and simulatively analysis of emigration effect on national income is shown in section 3. Section 4 conclude.

2   The model

2.1   Labor force

Assume that in a small open economy working population consists of unskilled workers (N, with growth rate $n \geq 0$) who are supposed internationally immobile and skilled workers (R) representing potential emigrants.

Denote the emigration flow by E and assume that the proportion of emigration to total skilled workers is represented by ξ ($\xi = E/R$)[1]. The natural growth rate of skilled labor is given by g. Furthermore, assume that the parameters n **and** g are also exogenously given. The law of motion of skilled labor is expressed as follows:

$$\dot{R} = gR - E.$$

It follows that the growth rate of skilled workers that do not emigrate is:

$$r = \frac{\dot{R}}{R} = g - \xi. \tag{1}$$

---

[1]Note that if $\xi < 0$, there is immigration

If we assume that emigration of skilled labor fosters investments in human capital (Beine, Docquier and Rapoport, 2001), we can refine equation (1) by imposing that g increases with the emigration flow $(g'(\xi) > 0)$. It follows then from equation (1) that a rise in ξ increases the growth rate of skilled labor (r) if $g'(\xi) > 1$.

The labor input is a synthetic measure of skilled and unskilled labor given by a CES function:

$$L = [bR^{-\beta} + (1-b)N^{-\beta}]^{-\frac{1}{\beta}}, \quad -1 < \beta < \infty, \tag{2}$$

where parameter $b \in (0,1)$ determines how the steady state (or balanced growth path) share of skilled and unskilled labor $(\kappa = \frac{R^*}{R^*+N^*})$ changes as the substitution parameter β varies between -1 and ∞ (see also de la Grandville, 1989). If we limit ourselves **to** the case where $\beta \neq 0$, that corresponds to non-unit labor force substitution. Following de la Grandville (1989, page 478) we normalize the parameter b of the CES labor index for given initial values of R_0 and N_0, and assume that $R_0 < N_0$. We obtain

$$b = \frac{\rho_0^{1+\beta}}{\rho_0^{1+\beta} + \mu_0},$$

where $\rho_0 = \frac{R_0}{N_0}$ and $\mu_0 = \frac{w_N(0)}{w_R(0)}$ is initial wage ratio of unskilled and skilled labor. It is easy to check that

$$b'(\beta) = \frac{\mu_0 \rho_0^{1+\beta} \ln(\rho_0)}{(\rho_0^{1+\beta} + \mu_0)^2} < 0.$$

Denote the elasticity of substitution between skilled labor R and unskilled labor N by $\sigma = \frac{1}{1+\beta}$. The labor force index L that combines R and N is able to grasp the full range of substitution degrees starting from strict complementarity $(\sigma = 0$ or $\beta \rightarrow \infty)$ to perfect substitution $(\sigma \rightarrow \infty$ or $\beta = -1)$. Note that in the second case both labor-types are additive since they are perfectly interchangeable.

We close this section by assuming that the total stock of emigrated people M is proportional to the total home country skilled worker R, with $M = \phi R$ $(\phi > 0)$.

4

2.2 Output, wages, and capital accumulation

National output derives from a classical constant returns production function :

$$Y = F(K, AL) = K^\alpha (AL)^{1-\alpha}, \tag{3}$$

where $0 < \alpha < 1$ is the share of capital in output and A is a technology parameter. Technological change is described by $A(t) = A(0)e^{g_A t}$, where $g_A \geq 0$ is an exogenous growth rate.

Profit maximization by competitive firms leads to following equilibrium wages for unskilled labor (w_N) and skilled labor (w_R) :

$$w_N = (1 - \alpha)(1 - b)F(K, AL) \left(\frac{N}{L}\right)^{-\beta} N^{-1}, \tag{4}$$

and

$$w_R = (1 - \alpha)bF(K, AL) \left(\frac{R}{L}\right)^{-\beta} R^{-1}. \tag{5}$$

It is easy to see that the wage ratio becomes:

$$\frac{w_R}{w_N} = \frac{b}{1-b}\left(\frac{N}{R}\right)^{1+\beta} = \frac{b}{1-b}\left(\frac{N(0)}{R(0)}\right)^{1+\beta} e^{(n-r)(1+\beta)t}, \tag{6}$$

where the last equality comes from the assumption of exogenous growth of unskilled and skilled labor.

Wages of actual emigrants are supposed to exceed those of potential emigrants in a fixed proportion $(1 + h)$. Assume that the emigrants repatriate a percentage θ of their wages to their home country:

$$\theta w_M \cdot M = \theta w_M \cdot \phi R = \theta(1 + h)w_R \cdot \phi R = \theta(1 + h)\phi(1 - \alpha)bF(K, AL) \left(\frac{R}{L}\right)^{-\beta}. \tag{7}$$

Following Solow's vein we assume that the saving rate s is positive and constant. Note that national income equals GDP $(F(K, AL))$ plus emigrant remittances $(\theta w_M \cdot M)$.

$$NI(t) = (F(K, AL) + \theta w_M \cdot M) = F(K, AL)\left[1 + \theta\phi b(1 + h)(1 - \alpha) \left(\frac{R}{L}\right)^{-\beta}\right]. \tag{8}$$

5

Capital accumulation is thus given by

$$
\begin{cases}
\dot{K} & = s\left(F(K, AL) + \theta w_M \cdot M\right) - \delta K \\[2ex]
& = sF(K, AL)\left[1 + \theta\phi b(1+h)(1-\alpha)\left(\dfrac{R}{L}\right)^{-\beta}\right] - \delta K, \\[2ex]
K(0) & \text{given,}
\end{cases}
\tag{9}
$$

where $\delta(>0)$ is the constant depreciation rate of capital.

2.3 Growth dynamics and balanced growth path

In this subsection, we are going to study the evolution of capital accumulation (9), labor force (2), wages ((4) and (5)), and the wage ratio (6) along transition and balanced growth paths. Note that balanced growth is reached when all growth rates of endogenous variables are constant. Let's denote the growth rate of variable X by g_X, and g_X^* its balanced growth rate.

The growth rate of capital g_K may be written as follows:

$$
g_K = s\frac{Y}{K}\left[1 + \theta\phi b(1+h)(1-\alpha)\left(\frac{R}{L}\right)^{-\beta}\right] - \delta.
\tag{10}
$$

This relation shows that the existence of a balanced growth path is not automatically verified even if output and capital grow at the same rate, because $\left(\frac{R}{L}\right)^{-\beta}$ is not necessary to be a constant. In order to get the conditions on existence of balanced growth paths, we set $a(t) = b\left(\frac{R}{L}\right)^{-\beta}$ and study the dynamics of $a(t)$ by following differential equation

$$
\dot{a}(t) = -\beta a(t)(r - g_L),
\tag{11}
$$

We find an explicit solution[2] that is given by:

$$
a(t) = \frac{1}{\frac{1-a(0)}{a(0)}e^{\beta(r-n)t} + 1}.
\tag{12}
$$

---

[2]The detail prove is given in Appendix.

On the other hand, we also have

$$\left(\frac{R(t)}{L(t)}\right)^{-\beta} = \left(\frac{R(0)}{L(0)}\right)^{-\beta} e^{-\beta \int_0^t (r - g_L(s)) ds}. \tag{13}$$

Combining the above two equations, we obtain the transitional growth rate of aggregated labor force

$$g_L(t) = \frac{(1 - a(0))(n - r)e^{\beta(r-n)t}}{[1 - a(0)]e^{\beta(r-n)t} + a(0)} + r. \tag{14}$$

Due to the above explicit growth path of labor force, it is possible to deduce the necessary and sufficient conditions for existence of balanced growth paths. Proofs are relegated to the appendix.

Proposition 1 *The necessary and sufficient conditions for the existence of balanced growth paths are*

$$g_K^* = g_Y^* = g_A + g_L^*, \quad and \quad \beta(r - g_L^*) = 0,$$

where

$$g_L^* = \begin{cases} br + (1 - b)n, & \beta = 0, \\ r(= n), & \beta \neq 0. \end{cases}$$

Or, if $(r - n)\beta < 0$, $g_L^* = r$; while if $(r - n)\beta > 0$, $g_L^* = n$.

Let us first study the case $\beta \neq 0$.

During transition, the growth rate of capital accumulation, obtained in appendix, is given by

$$g_K(t) = \left[e^{-\int_0^t G(s)ds}[g_K(0) + \delta] + (1 - \alpha)e^{-\int_0^t G(s)ds} \int_0^t e^{\int_0^\tau G(s)ds} d\tau\right]^{-1} - \delta, \tag{15}$$

where

$$G(t) = (1 - \alpha)(g_A + g_L(t) + \delta) + \frac{\theta\phi(1 + h)(1 - \alpha)\dot{a}(t)}{1 + \theta\phi(1 + h)(1 - \alpha)a(t)},,$$

$$g_K(0) = s\frac{Y(0)}{K(0)}[1 + \theta\phi(1 + h)(1 - \alpha)a(0)] - \delta.$$

After combining the above analysis with Proposition 1, we obtain:

7

Proposition 2 *Suppose $\beta \neq 0$.*

    a) During transition and if $r \neq n$, capital and aggregate labor growth rates are respectively given by (15) and (14) .

    Output growth is described by $g_Y = \alpha g_K + (1-\alpha)(g_A + g_L)$.

    b) During transition, wages of skilled and unskilled labor grow respectively at following rates

$$g_{w_R} = g_Y - \beta(r - g_L) - r, \quad g_{w_N} = g_Y - \beta(n - g_L) - n. \tag{16}$$

    c) If $r \neq n$, the growth rate of the wage ratio between skilled and unskilled workers is

$$g_{w_R/w_N} = (1 + \beta)(n - r). \tag{17}$$

    d) If $r = n$, the wage ratio between skilled and unskilled workers is constant and the wage gap will not change. Furthermore, along a balanced growth path, wages of skilled and unskilled labor grow at the same rate and are given by

$$g_{w_R}^* = g_{w_N}^* = g_Y^* - r. \tag{18}$$

Proposition a) gives a full description of the transitional dynamics of the economy when the elasticity of substitution between skilled and unskilled labor force **is** not unitary.

Proposition c) shows how emigration may hurt the wage structure in the source country. Assume that the skilled work force of an economy grows more slowly than the unskilled population, what is very likely for developing countries suffering from qualified labor scarcity. Since emigration pushes down r (see equation (1))and thus increases the growth rate gap $(n - r)$ it follows, according to equation (20), that the wage ratio between skilled and unskilled labor widens. It also appears that the wage growth asymmetry increases over-proportionally with the gap $(n - r)$ if skilled and unskilled labor are complements $(\beta > 1)$.

Proposition d) shows that along a balanced growth path induced by $r = n$, the wage rates of skilled and unskilled workers could decline (at the same growth rate) if labor grows faster than domestic output.

If we suppose that $\beta = 0$, the accumulated labor is given by the Cobb-Douglas form:

$$L(t) = R^b L^{1-b}.$$

Then it is easy to see that the growth rate of accumulated labor force is

$$g_L = br + (1-b)n = g_L^*. \tag{19}$$

After repeating the same analysis as above, we obtain following results, which is the case that labor force following unit-substitution, with Cobb-Douglas output function (of labor force).

Proposition 3 *Let $\beta = 0$.*

a) *During transition and if $r \neq n$, the growth rate of aggregate labor force is given by (19) and the growth rate of capital accumulation becomes*

$$g_K(t) = \frac{H(\widehat{g_K(0)} + \delta)}{(1-\alpha)(\widehat{g_K(0)} + \delta) + He^{-Ht}} - \delta, \tag{20}$$

where $H = (1-\alpha)(g_A + g_L + \delta)$, $\widehat{g_K(0)} = s\frac{Y(0)}{K(0)}[1 + \theta\phi(1+h)(1-\alpha)b] - \delta$. The growth rate of output is $g_Y(t) = \alpha g_K + (1-\alpha)(g_A + g_L)$.

b) *During transition, growth rates of skilled and unskilled labor are given by*

$$g_{w_R} = g_Y(t) - r, \quad g_{w_N} = g_Y(t) - n. \tag{21}$$

c) *If $r \neq n$, the growth rate of the wage ratio between skilled and unskilled worker is*

$$g_{w_R/w_N} = n - r. \tag{22}$$

d) *If $r = n$, the wage ratio between skilled and unskilled workers is constant over time and the gap will not change. Furthermore, along balanced growth path, the growth rates of skilled and unskill labor are the same and given by*

$$g_{w_R}^* = g_{w_N}^* = g_Y^* - r. \tag{23}$$

9

In this section, we developed explicit growth paths for capital and aggregate labor force, taking into account the full range of complementarity and substitutability between skilled and unskilled workers. This basic reflection will help us to study how brain drain may impact income and wages of the sending country.

3 Emigration effect on national income

In the following, we would like to study how emigration affects the national income of the sending country, which include theoretical analysis and some simulations.

¿From (8), we obtain that

$$g_{NI} - g_Y = -\beta(r - g_L)\left(1 - \frac{Y}{NI}\right), \tag{24}$$

where $\frac{Y}{NI}$ is always positive and smaller than one.

Equation (24) shows that emigration induces a gap between national income and domestic product growth rates if $\beta \neq 0$ (skilled and non skilled workers are not perfect substitutes) and if $r \neq g_L$. It also appears that this gap is increasing with the emigrant remittances relative to NI. Consequently, national income may grow at a slower rate than domestic product despite the fact that migrants' remittances are high. In particular, if we have $r < g_L$ and $-1 < \beta < 0$, it follows that $g_Y > g_{NI}$ and this growth gap is rising with $\frac{NI-Y}{NI}$. However, if skilled and non-skilled workers are complements ($\beta > 0$), the opposite conclusion must be drawn.

If $\beta = 0$, there is unit substitution between both labor forces and national income and domestic product grow at the same rate. The same conclusion holds if the economy moves along a balanced growth path where $\beta(r - g_L) = 0$.

We conclude the above analysis as a proposition.

Proposition 4   *a) $\beta > 0$.*

(a.1) If $r > g_L$, the growth rate of national income is lower than that of domestic output.

(a.2) If $r < g_L$, the growth rate of national income exceeds that of domestic output.

(a.3) if $r = g_L$, national income and domestic product grow at the same rate. Furthermore, the labor elasticity of substitution has no effect on the difference between national income and domestic output growth rates.

b) $-1 < \beta < 0$.

(b.1) If $r > g_L$, the growth rate of national income is higher than the growth rate of domestic output.

(b.2) If $r < g_L$, the growth rate of national income is lower than the growth rate of domestic output.

(b.3) If $r = g_L^*$, national income and domestic output grow at the same rate.

c) $\beta = 0$.

(c.1) National income and domestic product grow at the same rate during transition and along the balanced growth path.

We now turn to the study of the effect of elasticity of labor substitution and growing emigration flow on per capita national income (25) and low skilled workers' wages (26) of the source country. Following expressions are needed:

$$\frac{NI(t)}{N(t) + R(t)} = \frac{Y(0)e^{\int_0^t g_Y(s)ds}}{N + R} \left[1 + \theta\phi(1 + h)(1 - \alpha)a(t)\right]. \tag{25}$$

$$w_N(t) = (1 - \alpha)Y(0)e^{\int_0^t g_Y(s)ds}\frac{1 - a(t)}{N(t)}. \tag{26}$$

Since (25) and (26) are difficult to handle analytically, we shall run simulations that basically make use of equations (12), (14) and (15).

The purpose of this exercise is to show that emigration may eventually hurt welfare in the source country if it widens (or creates) the (an) imbalance between skilled and unskilled workers who are imperfect substitutes. For that purpose we compare the situation before and after emigration. To make things simple, skilled and unskilled populations are supposed to grow with the same rate $r = n$ if there is no emigration. A positive emigration flow then implies that $r < n$.
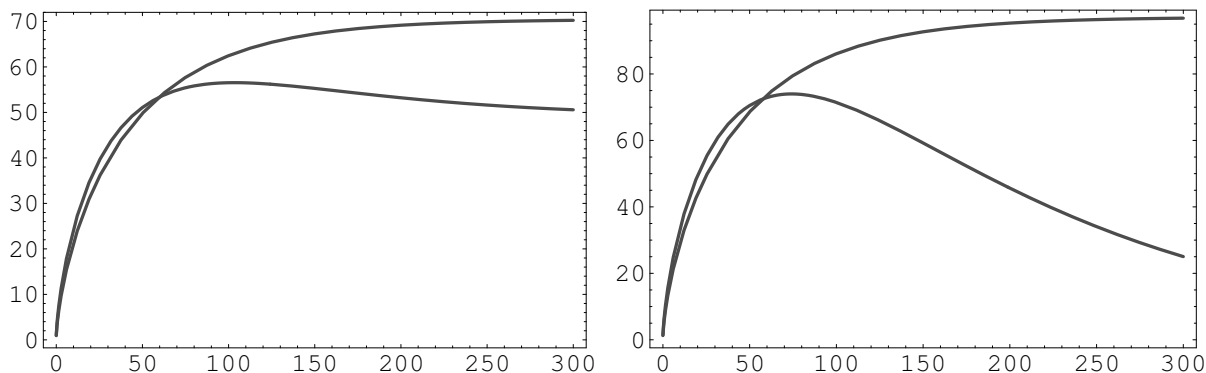

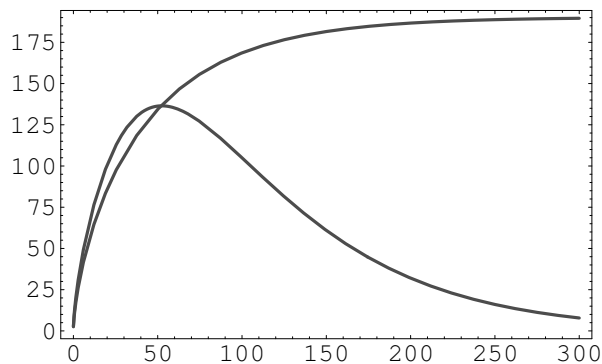Figure 1, Left: $\beta = -0.9$ (or $\sigma = 10$);   Right: $\beta = 0$ (or $\sigma = 1$).

Figure 2, $\beta = 3$ (or $\sigma = 0.25$).

Figures 1 and 2 (top lines) show that the profile of per capita income growth when $r = n$, is not altered (in shape) by changes in the elasticity of substitution between skilled and unskilled labor. Per capita income steadily rises until it reaches a balanced growth path.

Moreover, the simulations show that, before emigration $(r = n)$ the lower the elasticity of substitution between skilled and unskilled labor force, the higher per capita national income will be. This is the case during transition and balanced growth, which states that the positive effect of β over national income overcomes its negative effect.

If there is emigration $(n > r)$, the shape and the level of the per capita income path depends heavily on β. The bottom lines in Figures 1 and 2 show that in the short run, remittances from the emigrated improve per capita income in the country of emigration. At some point, per capita income reaches a peak and then begins to decrease steadily. Thus we see that in the long run, emigration becomes detrimental for the country of origin. It also appears that the lower labor elasticity the more emigration will hurt per capita income in the home population. This is the consequence of an increasing imbalance between skilled and non skilled populations (since $n > r$) resulting in a bottleneck effect that eventually impedes growth. This effect rises dramatically with the degree of labor complementarity as it appears in Figure 2. It should however be noticed that an initial gap between skilled and unskilled labor $(n > r)$ could be narrowed by emigration, if brain drain does sufficiently foster education (or more formally, if $g'(\xi) > 1$). In that case, emigration of skilled labor induces a net brain gain that takes the form of increased per capita income in the source country. It then may be shown that this positive effect is reinforced by the degree of complementarity between skilled and unskilled workers.

Saving rate $s = 0.2$	Capital share $\alpha = 0.4$
Initial marginal rate of substitution $\mu_0 = 0.4$	$\rho_0 = \frac{R_0}{N_0} = 0.3$
$1 + h = 4$	Wage repatriation rate $\theta = 0.5$
$\phi = \frac{M}{R} = 0.2$	$q = \theta\phi = 0.1$
$n = 0.03$	$\mu_0 = 0.4$
If $\xi = 0$, $r = n = 0.03$	If $\xi > 0$, $r < 0.03$

Table 1: Parameter values for the dynamic exercise

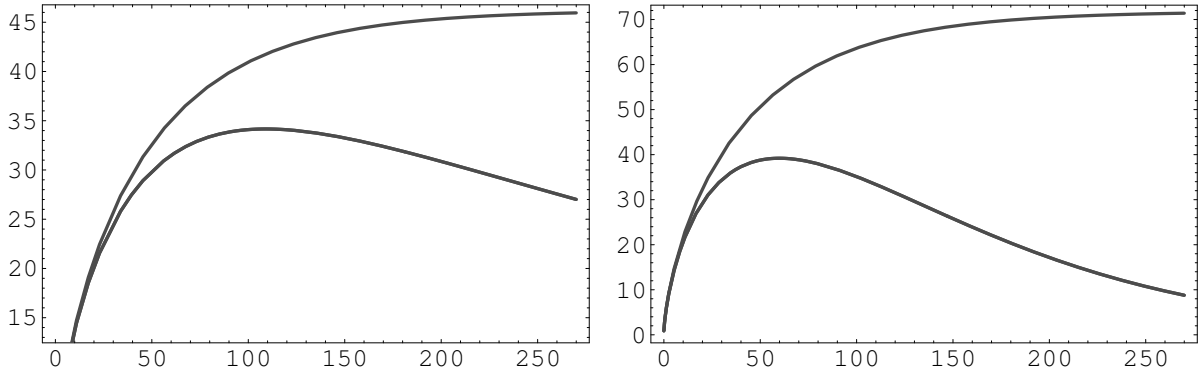The parameter values used in the simulations are reported in table 1.

Figure 3. Left: $\beta = 0$ (or $\sigma = 1$); Right: $\beta = 1.5$ (or $\sigma = 0.2$).

In Figure 3, we show the impact of labor substitution on non skilled wages. The top lines describe the evolution of wages accruing to unskilled labor when there is no emigration $(r = n)$. The wage profile of unskilled people rises steadily until reaching a balanced growth path. This is in accordance with Proposition 2, which states that wages of unskilled (and skilled) workers are increasing with domestic output if $r = n$ (for given r and n) along transitional and balanced growth paths. It follows that the larger the elasticity of substitution the lower the wages of both labor types will be since g_Y is decreasing with σ(or increasing with β).

When there is emigration $(n > r)$, Figure 3 shows that the wage rate of unskilled labor, after having reached a peak, declines steadily. It is also apparent that, the smaller the elasticity parameter σ the more rapid the wage drop will be. Consequently, emigration that aggravates the skills gap$(n - r > 0)$ induces wage losses that are the more harmful for the low skilled, the more they are complements for skilled labor.

4   Conclusion

The final aim of this paper was to study the impact of brain drain on the dynamics of income and wages in the sending country by explicitly taking into account the degree of substitutability between skilled (the potential emigrants) and unskilled workers. Laying

14

particular stress on intra labor complementarity was intended to focus on the effect of brain drain on productivity of the left behind. This impact depends crucially on the way emigration flow modify the ex ante imbalance between skilled and unskilled people living in the source country. As a matter of fact, our simulation exercises showed that negative spillovers appear if brain drain creates or aggravates the skills gap in the sending country and that these effects increase with rising complementarity between both labor skills.

Weak substitutability between different labor skills as a cause of adverse affects of brain drain have been stressed in the context of health systems in developing countries. For instance, emigration of health providers, induce important losses of part of the skill chain and may create substantial efficiency losses.

The simulations also show that negative spillovers on average productivity and per capita income of the source country that are caused by brain drain may even appear if skilled and unskilled workers are weak complements.

Furthermore, the model we developed does not exclude that emigration of talent may raise income growth in the sending country if it creates sufficient incentives to acquire more education. In this paper however, we did not endogenise the human capital formation induced by skill outflows. Such an extension of our model should be considered in a future paper.

5 Appendix

5.1 Proof of Proposition 1

The proof is arranged as follows. First, we check the necessary and sufficient conditions of the existence for a balanced growth path. Then we give further specifications about the existence of balanced growth.

Step 1.

Sufficiency is obvious. With $g_K^* = g_Y^*$ and $\beta(r - g_L^*) = 0$, all the endogenous variables

15

grow at constant rates, which is balanced growth by definition.

Now, we proof the necessity. Suppose there is a balanced growth path, where capital, output and aggregated labor grow at constant rates g_K^*, g_Y^*, and g_L^* respectively. Then we have:

$$Y(t) = \bar{Y}e^{g_Y^* t}, \ K(t) = \bar{K}e^{g_K^* t}, \ L(t) = \bar{L}e^{g_L^* t},$$

where $\bar{X}$ denotes the level of variable X along a balanced growth path.

Therefore, due to (15), we have the relation

$$g_k^* + \delta = s\frac{\bar{Y}}{\bar{K}}e^{(g_Y^* - g_K^*)t}\left[1 + \theta\phi b(1 + h)(1 - \alpha)\frac{R(0)}{\bar{L}}e^{-\beta(r - g_L^*)t}\right],$$

where, the left hand side is always constant. Hence the equality holds if and only if $g_Y^* = g_K^*$; and $\beta(r - g_L^*) = 0$, for, $t < \infty$, or $\beta(r - g_L^*)t = 0$, for, $t \to \infty$.

Step 2.

If $\beta = 0$, we have $L = R^b N^{1-b}$, then the growth rate of accumulated labor force is

$$g_L(t) = br + (1 - b)n = g_L^*.$$

Straightforward from the production function, and the fact that along balanced growth we have $g_Y^* = g_K^*$, it follows

$$g_Y^* = g_K^* = g_A + g_L^*.$$

If $\beta \neq 0$, we must have $g_L^* = r$, and $g_Y^* = g_K^*$.

Step 3. Now we study the other cases. Let $\beta \neq 0$.

During the dynamics, we straightforwardly have:

$$g_L'(t) = \frac{a(0)(1 - a(0))\beta(n - r)e^{\beta(r-n)t}}{[(1 - a(0))e^{\beta(r-n)t} + a(0)]^2}.$$

Case 1. $r < n$, $\beta > 0$.

The infinite time limit of aggregated labor growth yields:

$$\lim_{t \to \infty} g_L(t) = \lim_{t \to \infty} \frac{(1 - a(0))(n - r)e^{\beta(r-n)t}}{(1 - a(0))e^{\beta(r-n)t} + a(0)} + r = r.$$

16

Moreover, $g_L(0) = (1 - a(0))(n - r) + r > r$ and $g'_L(t) < 0$ demonstrate that the growth rate of aggregated labor force is decreasing over time and will reach the lower bound as time goes to infinity ($t \to \infty$).

Case 2. $r < n, -1 < \beta < 0$.

Applying the above reasoning, we obtain:

$$\lim_{t \to \infty} g_L(t) = n > r,$$

with $g_L(0) = (1 - a(0))(n - r) + r > r$ and $g'_L(t) > 0$.

Case 3. $r > n, \beta > 0$.

This is the same as Case 2.

Case 4. $r > n, -1 < \beta < 0$.

This is the same as same as Case 1. $\diamondsuit$

5.2   Proof of (12) and (14)

Suppose $\beta \neq 0$.

The difference between skilled and aggregated labor force is

$$\begin{aligned}
r - g_L &= \tfrac{\dot{R}}{R} - \tfrac{\dot{L}}{L} \\
&= \tfrac{\dot{R}}{R} - \left[br \left(\tfrac{R}{L} \right)^{-\beta} + (1 - b)n \left(\tfrac{N}{L} \right)^{-\beta} \right] \\
&= \tfrac{\dot{R}}{R} - \left[br \left(\tfrac{R}{L} \right)^{-\beta} + n - bn \left(\tfrac{R}{L} \right)^{-\beta} \right] \\
&= (r - n)\left(1 - b \left(\tfrac{R}{L} \right)^{-\beta} \right) \\
&= (r - n)(1 - a(t)).
\end{aligned}$$

Therefore the dynamics of $a(t)$ can be expressed as

$$\dot{a}(t) = -\beta(r - n)a(t)(1 - a(t)),$$

17

with given $a(0) = b\left(\frac{R(0)}{L(0)}\right)^{-\beta}$. With standard ordinary differential equation technique, (12) can be obtained.

Combining (12) and (13), it follows

$$a(0)e^{-\beta\left(\int_0^t (r-g_L(s))ds\right)} = \frac{1}{\left(\frac{1-a(0)}{a(0)}e^{\beta(r-n)t} + 1\right)}.$$

Taking logarithm

$$-\beta\left(\int_0^t (r - g_L(s))ds\right) = -\ln(a(0)) - \ln\left(\left(\frac{1-a(0)}{a(0)}e^{\beta(r-n)t} + 1\right)\right),$$

Derivative with respect to t on both sides and rearranging the terms yields (14). $\diamondsuit$

5.3 Proof of (15)

Suppose $\beta \neq 0$.

From the production function, we derive that

$$g_Y = \alpha g_K + (1-\alpha)(g_A + g_L),$$

and hence

$$g_Y - g_K = (\alpha - 1)g_K + (1-\alpha)(g_A + g_L).$$

Combining (10) and

$$Y(t) = Y(0)e^{\int_0^t g_Y(s)ds}, \quad K(t) = K(0)e^{\int_0^t g_K(s)ds},$$

we obtain for any $t \geq 0$,

$$\begin{aligned} g_K(t) &= s\frac{Y(0)}{K(0)}e^{\int_0^t [g_Y(s)-g_K(s)]ds}\left[1 + \theta\phi(1+h)(1-\alpha)a(t)\right] - \delta \\ &= s\frac{Y(0)}{K(0)}e^{\int_0^t (\alpha-1)g_K(s)ds}e^{\int_0^t (1-\alpha)(g_A+g_L(s))ds}\left[1 + \theta\phi(1+h)(1-\alpha)a(t)\right] - \delta, \end{aligned}$$

especially

$$g_K(0) = s\frac{Y(0)}{K(0)}\left[1 + \theta\phi(1+h)(1-\alpha)a(0)\right] - \delta.$$

18

Taking logarithm on both sides, it follows

$$
\begin{aligned}
\ln(g_K + \delta) \; = \; &\ln\left(s\frac{Y(0)}{K(0)}\right) + (\alpha - 1)\int_0^t g_K(s)ds + (1 - \alpha)\int_0^t \left(g_A + g_L(s)\right)ds \\
&+ \ln\left(1 + \theta\phi(1 + h)(1 - \alpha)a(t)\right).
\end{aligned}
$$

Derivative with respect to time t and rearrange the terms:

$$
\frac{\dot{g}_K}{g_K + \delta} = (\alpha - 1)g_K(t) + (1 - \alpha)\left(g_A + g_L(t)\right) + \frac{\theta\phi(1 + h)(1 - \alpha)\dot{a}(t)}{1 + \theta\phi(1 + h)(1 - \alpha)a(t)}.
$$

Let

$$
V(t) = g_K(t) + \delta, \quad V(0) = g_K(0) + \delta,
$$

and

$$
G(t) = -\delta(\alpha - 1) + (1 - \alpha)\left(g_A + g_L(t)\right) + \frac{\theta\phi(1 + h)(1 - \alpha)\dot{a}(t)}{1 + \theta\phi(1 + h)(1 - \alpha)a(t)},
$$

we have

$$
\frac{\dot{V}}{V} = (\alpha - 1)V + G(t),
$$

which is a Bernoulli differential equation, and its solution is given by

$$
V(t) = \left[e^{-\int_0^t G(s)ds}V(0) + (1 - \alpha)e^{-\int_0^t G(s)ds}\int_0^t e^{\int_0^s G(\tau)d\tau}ds\right]^{-1}.
$$

We finish the proof of (15). $\Diamond$

References

1. Barro R. J. and X. Sala-i-Martin, (1995), Economic Growth, McGraw-Hill. Inc.

2. Beine M., F. Docquier and H. Rapoport, (2001), Brain drain and economic growth: theory and evidence, Journal of Development Economics, 64, 275–289.

3. Bhagwati J. and K. Hamada, (1974), The Brain Drain, Integration of Markets for Professionals ans Unemployment: A theoretical analysis, Journal of Development Economics, 1, 19-42

4. Carrington W.J. et Detragiache E. (1998), How big is the brain drain ? IMF Working Paper, No 98.

5. de la Grandville O. (1989), In quest of the Slutsky Diamond, American Economic Review, 79, 468–481.

6. Docquier F., Lohest O. et Marfouk A. (2005), Brain drain in developing countries, IZA Discussion Paper n1668, July

7. Docquier, F. and A. Marfouk (2005), Measuring international migration by educational attainment in 1990-2000 - Release 1.1, The World Bank, Mimeo.

8. Klump R. and de la Grandville O. (2000), Economic growth and the elasticity of substitution: Two theorems and some suggestions, American Economic Review, 90, 282-291.

9. Mountford A., (1997), Can a brain drain be good for growth in the source economy?, Journal of Development Economics Vol. 53, 287-303

10. Mountford A.,(1997), Can a brain drain be good for growth in the source country?, Journal of Development Economics, 53, 237–303

11. Piketty, T., (1997), Immigration et justice sociale, Revue Economique 48(5), 1291-1309.

12. Docquier F., Lohest O. et Marfouk A., (2005), Brain drain in developing countries, IZA Discussion Paper n1668, July

13. Saint-Paul G., (2004), The brain Drain : Some experience from European Expatriates in the US , CEPR Discussion Paper n4680, October.