Working Papers
Institute of Mathematical Economics

383

September 2006

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http://www.wiwi.uni-bielefeld.de/~imw/Papers/showpaper.php?383
ISSN: 0931-6558

Procedural group identification*

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September 14, 2006

Abstract

In this paper we axiomatically characterize two recursive procedures for defining a social group. The first procedure starts with the

*This is a substantial revision of our CentER Discussion Paper 2003-10, Tilburg University, with the same title. D. Dimitrov gratefully acknowledges financial support from the German Research Foundation (DFG).

set of all individuals who are defined by everyone in the society as group members, while the starting point of the second procedure is the set of all individuals who define themselves as members of the social group. Both procedures expand these initial sets by adding individuals who are considered to be appropriate group members by someone in the corresponding initial set, and continue inductively until there is no possibility of expansion any more.

JEL Classification: D63, D71.
Keywords: consensus, liberalism, procedure, social identity

1 Introduction

The problem of group identification serves as a background in many social and economic contexts. For example, when one examines the political principle of self-determination of a newly formed country, one would like to define the extension of a given nationality. Or when a newly arrived person in Atlanta chooses where to live, the person is interested in finding out a residential neighborhood that would suit her: "Are they my kind of people? Do I belong to this neighborhood?" In all those contexts, it is typically assumed that there is a well-defined group of people who share some common values, beliefs, expectations, customs, jargon, or rituals. Consequently, questions like "how to define a social group" or "who belongs to the social group" arise. In very recent papers (see Billot (2003), Çengelci and Sanver (2006), Houy (2006a,b), Kasher and Rubinstein (1997), Miller (2006), Samet and Schmeidler (2003)) this problem has been related to formal models from social choice and voting theory.

Kasher's (1993) paper on collective identity can be considered as a first, non-formal attempt to look at the group identification problem as an aggregation task. In that paper the author views that each individual in a society has an opinion about every individual, including oneself, whether the latter is a member of a group to be formed[1]. The collective identity of the group to be formed is then determined by aggregating opinions of all individuals in the society. The formal link between Kasher's approach and the theory of aggregators mainly developed in economic theory was made by Kasher and Rubinstein (1997). For this purpose, they provide, among others, an axiomatic characterization of a "liberal" aggregator whereby the group consists of those and only those individuals who each of them views oneself a member of the group (see also Çengelci and Sanver (2006), Houy (2006a), Miller (2006), Samet and Schmeidler (2003), Sung and Dimitrov (2005)).

The purpose of this paper is to extend the study of the group identification problem by adding a *procedural* view in the analysis. This procedural view allows us to see a collective as "a family of groups, subcollectives, each with its own view of who is a member of the collective, its own sense of tradition and its own underlying conceptual realm, but each bearing some resemblance to the other ones" (Kasher (1993, p. 70)). More specifically, we axiomatically characterize two recursive procedures for determining "who is a member of a social group": a *consensus-start-respecting* procedure which is the one introduced by Kasher (1993) and a *liberal-start-respecting* procedure which adds a procedural view to the "liberal" aggregation of Kasher and Rubinstein (1997).

The structure of both procedures consists of two components: an initial set of individuals and a rule according to which new individuals are added to this initial set. As the names of the procedures suggest, the initial set of the first procedure consists of all individuals who are defined as group members

[1] An analysis of the question how the identity of an individual has been formed or what is the impact of identity on economic behavior is out of the scope of this paper. With respect to those questions the interested reader is referred to Sen (1999) and Akerlof and Kranton (2000), respectively.

by everyone in the society, while the initial set of the second procedure collects all individuals who define themselves as members of the social group. The extension rule for both procedures is the same: only those individuals who are considered to be appropriate group members by someone in the corresponding initial set are added. The application of this rule continues inductively until there is no possibility of expansion any more.[2]

An initial set can be interpreted for example as a set of society founders who choose new society members from a finite set of candidates (see Berga, Bergantiños, Massó, and Neme (2004)), and the extension rule (the voting rule) is "voting by quota one", i.e., it is enough for a candidate to receive one vote in order to be admitted (see Barberà, Maschler, and Shalev (2001)). In contrast to the cited papers, we study here the problem of group formation in a social choice setting and do not consider a predesignated set of society founders. We allow rather for the possibility that the views of all individuals in the society determine endogenously who is a society founder.

The rest of the paper is organized as follows. In Section 2, we present the basic notation and definitions. Sections 3 discusses the axioms that are necessary and sufficient to reach logically the consensus-start-respecting procedure and presents our characterization result. Section 4 is devoted to the corresponding axioms and characterization of the liberal-start-respecting procedure. We conclude in Section 5 with some final remarks.

[2] For an axiomatic characterization of the aggregator selecting the agents that are indirectly designated by all individuals in the society the reader is referred to Houy (2006b).

2 Basic notation and definitions

Let $N = \{1, \ldots, n\}$ denote the set of all individuals in the society and assume that $n \geq 2$. The set of all subsets of N is denoted by $P(N)$. Each individual $i \in N$ forms a set $G_i \subseteq N$ consisting of all society members that in the view of i have the social identity G. It may be noted that it is possible to have $G_i = \emptyset$ for some $i \in N$. For all $i \in N$, when $i \in G_i$, we also say that i considers himself as a G. A *profile* of views is an n-tuple of vectors $G = (G_1, \ldots, G_n)$ where $G_i \subseteq N$ for all $i \in N$. Let $\mathcal{G}$ be the set of all profiles of views, i.e., $\mathcal{G} = (P(N))^n$. A *collective identity function* (CIF) $F : \mathcal{G} \to P(N)$ assigns to each profile $G \in \mathcal{G}$ a set $F(G) \subseteq N$ of socially accepted group members. In what follows, we denote by $\mathcal{F}$ the set of all collective identity functions.

Kasher (1993) offers a *neutral* method for defining the collective identity, i.e., a method which is "... free of any commitment to some partisan view of the nature of the collective". This method is introduced as follows. For any $G \in \mathcal{G}$, let $K_0(G) = \{i \in N : i \in G_k \text{ for all } k \in N\}$. We define a CIF being *consensus-start-respecting*, to be denoted by $K(G)$, as follows: for each positive integer t, let $K_t(G) = K_{t-1}(G) \cup \{i \in N : i \in G_k \text{ for some } k \in K_{t-1}(G)\}$; and if for some $t \geq 0$, $K_t(G) = K_{t+1}(G)$, then $K(G) = K_t(G)$.

For each $G \in \mathcal{G}$ the procedure K starts with $K_0(G)$ which consists of all individuals who are viewed by everyone in the society as group members. Kasher (1993) calls the set $K_0(G)$ the "incontrovertible core" of a collective to be defined and he considers it as an initial approximation to an appropriate definition of the group identity. Notice that $K_0(G)$ does not reflect the differences in views of "who is a G" held by those who are unquestionably Gs. Because one is interested in a neutral aggregation rule, an "improved approximation" is needed. For each $G \in \mathcal{G}$, the CIF K now expands the set

$K_0(G)$ as follows. If, according to some individual $i \in K_0(G)$ an individual $k \in N$ is viewed as a G, then k should be a G collectively. By adding all such ks to $K_0(G)$, we obtain the set $K_1(G)$. We then repeat the above process with $K_1(G)$ by adding those individuals who are considered as Gs by some individual in $K_1(G)$ to $K_1(G)$ to obtain $K_2(G)$. Since n is finite, at a certain step t, we must have $K_t(G) = K_{t+1}(G)$, i.e., the set $K_t(G)$ can no longer be expanded. The intuition behind each step of the expansion is in line with Kasher's (1993) argument: every socially accepted G as being newly added brings a possibly unique new view of being a G collectively with him; since a collective identity function is supposed to aggregate those views, it must pay attention to this new individual's G-concept in order to cover the whole diversity of views in the society about the question "what does it mean to be a G?".

We turn now to the liberal-start-respecting procedure mentioned by Kasher and Rubinstein (1997). For any $G \in \mathcal{G}$, let $L_0(G) = \{i \in N : i \in G_i\}$. With the help of $L_0(G)$, we define a CIF being *liberal-start-respecting*, to be denoted by $L(G)$, as follows: for each positive integer t, let $L_t(G) = L_{t-1}(G) \cup \{i \in N : i \in G_k \text{ for some } k \in L_{t-1}(G)\}$; and if for some $t \geq 0$, $L_t(G) = L_{t+1}(G)$, then $L(G) = L_t(G)$.

Notice that the extension rule for L and K is the same (and so the intuition behind it), but the initial set is different: the liberal-start-respecting procedure starts with $L_0(G)$ which consists of all members of the society who view themselves as Gs. Thus, the set $L_0(G)$ reflects a weak notion of *self-determination*: if one considers oneself a member of G, then one should be a member of G collectively. Therefore, the procedure L reflects a strong liberal view of collective identity[3].

[3] See, for example, Çengelci and Sanver (2006), Houy (2006a), Kasher and Rubinstein

To illustrate the above procedures for defining collectively accepted group members, consider the following example. Let $N = \{1, 2, 3\}$ and consider the profile $G = (G_1, G_2, G_3)$ with $G_1 = \{1, 2\}, G_2 = \{2, 3\}$ and $G_3 = \{2\}$. Then, for this profile, $K_0 = \{2\}$, $K_1 = K_0 \cup \{3\} = \{2, 3\}$, $K_2 = K_1$. Therefore, the collectively accepted group members according to the consensus-start-respecting procedure are collected in the set $K = \{2, 3\}$. For the same profile G of individual views we have $L_0 = \{1, 2\}$, $L_1 = L_0 \cup \{3\} = \{1, 2, 3\}$, $L_2 = L_1$. Therefore, for the given profile of views, and as a result of the application of the liberal-start-respecting procedure, we have $L = \{1, 2, 3\}$.

It should be noted that, from their respective definitions, for all profiles $G \in \mathcal{G}$, $K(G) \subseteq L(G)$.

3 The consensus-start-respecting procedure

In this section we offer an axiomatic characterization of the Kasher's method for defining a social group. For that purpose we start with the following two axioms a CIF may satisfy.

- A CIF $F \in \mathcal{F}$ satisfies *consensus* (C) if for all $G \in \mathcal{G}$,
  - $[i \in G_k$ for every $k \in N]$ implies $[i \in F(G)]$, and
  - $[i \notin G_k$ for every $k \in N]$ implies $[i \notin F(G)]$.
- A CIF $F \in \mathcal{F}$ satisfies *irrelevance of an outsider's view* 1 (IOV1) if for all $G, G' \in \mathcal{G}$ and for all $i, j \in N$,

(1997), Miller (2006), Samet and Schmeidler (2003), Sung and Dimitrov (2005). If the determination of the membership of a social group is a personal matter, there is indeed some reason to call individuals in L_0 as liberals (see Sen (1970)).

- $G'_j = G_j \cup \{i\}$, and
- $G'_l = G_l$ for all $l \in N \setminus \{j\}$,

  imply
- $[j \notin F(G)$ and $i \notin G'_k$ for some $k \in N] \Rightarrow [i \in F(G)$ iff $i \in F(G')]$.

Consensus is used by Kasher and Rubinstein (1997) to reach logically their "liberal" CIF and, in fact, sounds very plausible when imposed as a requirement on a collective identity function. This axiom says that, if an individual is defined as a group member by everyone in the society, then this individual should be considered as a socially accepted group member; and, correspondingly, if no one defines this individual as a group member, then he or she should not deserve the social acceptance as a group member.

Irrelevance of an outsider's view 1 is in the spirit of the exclusive self-determination axiom introduced by Samet and Schmeidler (2003) and it basically stipulates that if someone is collectively defined as a non-G, then this person's view about any society member is not relevant in deciding his or her collective identity. Note however that there is one case, in which the view of an outsider cannot be deemed as irrelevant; this case corresponds to the situation in which everyone in the society except the outsider j considers i as a G, so that the change of j's view in favour of i is (via (C)) relevant for the social identification of i. As the reader can see, we exclude this case in (IOV1) by requiring that there is a $k \in N$ such that $i \notin G'_k$. It should also be noted that (IOV1) is weaker than the exclusive self-determination axiom used by Samet and Schmeidler (2003).

Consider now a profile $G \in \mathcal{G}$ and the group of socially accepted society members $F(G)$ generated by a CIF $F \in \mathcal{F}$ that satisfies both axioms. Our first result relates $F(G)$ with the result of the consensus-start-respecting procedure K at profile G. As it turns out, $K(G)$ acts as an upper bound for $F(G)$ at any profile $G \in \mathcal{G}$.

Proposition 1 *If a CIF $F \in \mathcal{F}$ satisfies (C) and (IOV1), then $F(G) \subseteq K(G)$ for all $G \in \mathcal{G}$.*

Proof. Let $F \in \mathcal{F}$ satisfy (C) and (IOV1). We start by observing that the claim "$F(G) \subseteq K(G)$ for all $G \in \mathcal{G}$" is equivalent to "for all $G \in \mathcal{G}$ and for all $i \in N : i \notin K(G)$ implies $i \notin F(G)$". Hence, we prove this equivalent claim by induction of $|G|$, where $|G| = |G_1| + |G_2| + \ldots + |G_n|$.

Basis Step: When $|G| = 0$, we have $G_l = \emptyset$ for all $l \in N$. Thus, $i \notin G_j$ for all $i, j \in N$. From (C), $F(G) = K(G) = \emptyset$.

Induction Step: Let g be a non-negative integer such that $g < n^2$. Assume that the claim holds for all $G \in \mathcal{G}$ with $|G| = g$, and we show that the claim holds for all $G \in \mathcal{G}$ with $|G| = g + 1$.

Let $G \in \mathcal{G}$ be such that $|G| = g+1$, and let $i \in N$ be such that $i \notin K(G)$. If $i \notin G_j$ for all $j \in N$, then from (C) we have $i \notin F(G)$. Suppose there exists $j \in N$ such that $i \in G_j$. By definition of K, from $i \notin K(G)$ and $i \in G_j$, it then follows that $j \notin K(G)$. Moreover, from $i \notin K(G)$, there also exists $k \in N$ such that $i \notin G_k$. Observe that $k \neq j$.

Let $G' \in \mathcal{G}$ be such that $G' = (G_1, \ldots G_{j-1}, G_j \setminus \{i\}, G_{j+1}, \ldots, G_n)$. By definition of K, we have $K(G') \subseteq K(G)$, which implies $i, j \notin K(G')$. Obviously, $|G'| = |G| - 1 = g$. By induction hypothesis, we have $F(G') \subseteq K(G')$. Thus, we have $i, j \notin F(G')$ from $i, j \notin K(G')$.

Notice that for the profile G' we have $i \notin G'_j$ and $i, j \notin F(G')$, and for the profile G we have $G_j = G'_j \cup \{i\}$, $i \notin G_k$ for some $k \neq j$ and $G_l = G'_l$ for all $l \in N \setminus \{j\}$. Hence, applying (IOV1) with G' and G in the roles of G and G', respectively, we conclude that $i \notin F(G)$. ■

In order to complete the characterization of K we have to show also the reverse inclusion to the one in Proposition 1. For that purpose, we introduce our third axiom.

- A CIF $F \in \mathcal{F}$ satisfies *equal treatment of insiders' views* (ETIV) if for all $G, G' \in \mathcal{G}$ and for all $i, j, k \in N$,
  - $i \in G_j$,
  - $G'_j = G_j \setminus \{i\}$, and $G'_k = G_k \cup \{i\}$
  - $G'_l = G_l$ for all $l \in N \setminus \{j, k\}$,

    imply
  - $[j \in F(G)$ and $k \in F(G')] \Rightarrow [i \in F(G)$ iff $i \in F(G')]$.

Equal treatment of insiders' views requires that if an individual i is considered to be an appropriate group member by an individual j, $i \in G_j$ in a given profile, and if in a new profile j does not consider i as an appropriate group member anymore but a third individual k does, and nothing else has changed, then, when j is a collectively accepted group member in the original profile and k is a collectively accepted member in the new profile, it must be true that i is a G collectively in the original profile if and only if i is a G collectively in the new profile. This axiom essentially requires that a CIF should treat the views of all the members who are considered to be Gs collectively equally.

Proposition 2 *If a CIF $F \in \mathcal{F}$ satisfies (C), (ETIV) and (IOV1), then $K(G) \subseteq F(G)$ for all $G \in \mathcal{G}$.*

Proof. Let $F \in \mathcal{F}$ satisfy (C), (ETIV) and (IOV1). Note first that the claim "$K(G) \subseteq F(G)$ for all $G \in \mathcal{G}$" is equivalent to "$K_t(G) \subseteq F(G)$ for all $G \in \mathcal{G}$

and for all non-negative integers $t \leq n$". We prove this equivalent claim by induction of $g = n^2 - |G|$ and t.

Basis Step $(g = 0)$: When $|G| = n^2$, we have $G_l = N$ for all $l \in N$. Thus, $i \in G_j$ for all $i, j \in N$, i.e., $F(G) = N$ follows from (C). Therefore, we have $K_t(G) \subseteq F(G)$ for all non-negative integers $t \leq n$.

Induction Step $(g \geq 0)$: Let g be a non-negative integer such that $g < n^2$. We assume that

$$K(G) \subseteq F(G) \text{ for all } G \in \mathcal{G} \text{ with } |G| = n^2 - g, \tag{IH1}$$

and show $K_t(G) \subseteq F(G)$ for all $G \in \mathcal{G}$ with $|G| = n^2 - g - 1$ and for all non-negative integers $t \leq n$.

Basis Step $(g \geq 0$ and $t = 0)$: From (C), we have $K_0(G) \subseteq F(G)$ for all $G \in \mathcal{G}$.

Induction Step $(g \geq 0$ and $t \geq 0)$: Let t be a non-negative integer such that $t < n$. We further assume that

$$K_t(G) \subseteq F(G) \text{ for all } G \in \mathcal{G} \text{ with } |G| = n^2 - g - 1, \tag{IH2}$$

and we show $K_{t+1}(G) \subseteq F(G)$ for all $G \in \mathcal{G}$ with $|G| = n^2 - g - 1$.

Let $G \in \mathcal{G}$ be such that $|G| = n^2 - g - 1$. If $K_{t+1}(G) \setminus K_t(G) = \emptyset$, then we have $K(G) = K_t(G) \subseteq F(G)$. Suppose there exists $i \in K_{t+1}(G) \setminus K_t(G)$. Then, by definition of K, there exists $j \in K_t(G)$ such that $i \in G_j$. Hence, we have $j \neq i$, and by induction hypothesis (IH2), we conclude that $j \in F(G)$. Moreover, since $i \notin K_0(G)$, there exists $k \in N$ such that $i \notin G_k$. Observe that $k \neq j$.

Let $H \in \mathcal{G}$ be such that $H = (G_1, \ldots, G_{k-1}, G_k \cup \{i\}, G_{k+1}, \ldots, G_n)$. We consider now the following two cases:

(1) Suppose $k \in F(H)$. Notice from $i \notin G_k$ that $|H| = |G| + 1 = n^2 - g$, and thus, by induction hypothesis (IH1), $K(H) \subseteq F(H)$. From Proposition 1, we have $F(H) \subseteq K(H)$, and thus, $F(H) = K(H)$. By definition of K, $K(G) \subseteq K(H)$, we have $i \in K_{t+1}(G) \subseteq K(G) \subseteq K(H) = F(H)$. Then, notice that for the profile H we have $i \in H_k$ and $i, k \in F(H)$, and for the profile G we have $i \in G_j$, $G_k = H_k \setminus \{i\}$, $G_l = H_l$ for all $l \in N \setminus \{k\}$, and $j \in F(G)$. Hence, applying (ETIV) with H, G, i, k, j in the roles of G, G', i, j, k, respectively, we conclude that $i \in F(G)$.

(2) Suppose $k \notin F(H)$. From $K(H) = F(H)$, we have $K(G) \subseteq K(H) = F(H)$, and thus, $k \notin K(G)$. From Proposition 1, we have $F(G) \subseteq K(G)$, and thus, $k \notin F(G)$. Moreover, from $i, j \in K_{t+1}(G) \subseteq K(G)$ and $k \notin K(G)$, we have $k \notin G_i$ and $k \notin G_j$. Let $H' \in \mathcal{G}$ be such that $H' = (G_1, \ldots, G_{i-1}, G_i \cup \{k\}, G_{k+1}, \ldots, G_n)$. From $k \notin G_i$, we have $|H'| = |G| + 1 = n^2 - g$. By induction hypothesis (IH1) and Proposition 1, we have $F(H') = K(H')$. From $i \in K(G) \subseteq K(H')$ and $k \in H'_i$, we have $k \in K(H')$, and from $F(H') = K(H')$, we have $k \in F(H')$. Notice that for the profile G we have $k \notin G_i$ and $k \notin F(G)$, and for the profile H' we have $H'_i = G_i \cup \{k\}$, $H'_l = G_l$ for all $l \in N \setminus \{i\}$, $k \in F(H')$, and from $j \neq i$, we have $k \notin G_j = H'_j$. Then, applying (IOV1) with G, H', k, i, j in the roles of G, G', i, j, k, respectively, we conclude that $i \in F(G)$. ■

Theorem 1 *A CIF $F \in \mathcal{F}$ satisfies (C), (ETIV) and (IOV1) if and only if $F = K$. Moreover, all three axioms are independent.*

Proof. It is easy to check that the consensus-start-respecting procedure satisfies (C), (ETIV) and (IOV1). The combination of Proposition 1 and Proposition 2 proves that, if a CIF $F \in \mathcal{F}$ satisfies (C), (ETIV) and (IOV1) then it is K. Hence, we need only to show that the axioms are tight. The proof consists of three examples, each of which satisfies exactly two of the three axioms.

($\neg$(C)) Let $F \in \mathcal{F}$ be such that $F(G) = \emptyset$ for all $G \in \mathcal{G}$. Clearly, this CIF satisfies all axioms but (C).

($\neg$(ETIV)) Consider the CIF $F \in \mathcal{F}$ with

$$F(G) = K_0(G) \cup \{i \in N : i \in G_j \text{ for all } j \in K_0(G)\}$$

for all $G \in \mathcal{G}$. The following example shows that this aggregator does not satisfy (ETIV). Let $N = \{1, 2, 3\}$, $G = (\{1, 2, 3\}, \{1\}, \{1\})$, and $G' = (\{1, 3\}, \{1\}, \{1, 2\})$. For these profiles of views we have $F(G) = \{1, 2, 3\}$ and $F(G') = \{1, 3\}$. In this case (ETIV) is violated because $1 \in F(G)$, $3 \in F(G')$, $2 \in G_1$, $G'_1 = G_1 \setminus \{2\}$, $G'_3 = G_3 \cup \{2\}$, but $2 \in F(G)$ and $2 \notin F(G')$.

($\neg$(IOV1)) Take the liberal-start-respecting procedure that defines the CIF $L \in \mathcal{F}$. If we set $j = i$ in the formulation of (IOV1) we immediately see that this axiom is violated. ■

4 The liberal-start-respecting procedure

For the axiomatic characterization of the liberal-start-respecting procedure defined in Section 2 we have first to modify the irrelevance of an outside's view 1 axiom.

- A CIF $F \in \mathcal{F}$ satisfies *irrelevance of an outsider's view* 2 (IOV2) if for all $G, G' \in \mathcal{G}$ and for all $i, j \in N$ with $i \neq j$,
  - $G'_j = G_j \cup \{i\}$, and

– $G'_l = G_l$ for all $l \in N \setminus \{j\}$,

imply

– $[j \notin F(G)] \Rightarrow [i \in F(G) \text{ iff } i \in F(G')]$.

This axiom basically says, like (IOV1), that if someone is collectively considered as a non-G, then this person's view about any society member is not relevant in deciding his or her collective identity. Recall that in the formulation of (IOV1) it was crucial to avoid the case, in which everyone in the society except the outsider j considers i as a G, so that the change of j's view in favour of i is relevant for the social identification of i (i.e., in this way a possible tension between (IOV1) and (C) was excluded). Notice that the liberal-start-respecting procedure does not satisfy (IOV1) because one's self-determination defines immediately one's social status: there is no consensus needed in this case. Therefore, in order to avoid the situation in which an individual becomes crucial for his own social determination (from being outsider to being insider) we require $i \neq j$ in the formulation of (IOV2).

As it turns out, the combination of (C) and (IOV2) plays a similar role for a CIF $F \in \mathcal{F}$ as the role of the combination of (C) and (IOV1): it produces an upper bound for F at any profile $G \in \mathcal{G}$. This upper bound is exactly the set of socially accepted group members at G according to the liberal-start-respecting procedure.

Proposition 3 *If a CIF $F \in \mathcal{F}$ satisfies (C) and (IOV2), then $F(G) \subseteq L(G)$ for all $G \in \mathcal{G}$.*

Proof. Let $F \in \mathcal{F}$ satisfy (C) and (IOV2). Observe again that the claim "$F(G) \subseteq L(G)$ for all $G \in \mathcal{G}$" is equivalent to "for all $G \in \mathcal{G}$ and for all $i \in N : i \notin L(G)$ implies $i \notin F(G)$". Hence, we prove this equivalent claim by induction of $|G|$, where $|G| = |G_1| + |G_2| + \ldots + |G_n|$.

Basis Step: When $|G| = 0$, we have $G_l = \emptyset$ for all $l \in N$. Thus, $i \notin G_j$ for all $i, j \in N$. From (C), $F(G) = L(G) = \emptyset$.

Induction Step: Let g be a non-negative integer such that $g < n^2$. Assume that the claim holds for all $G \in \mathcal{G}$ with $|G| = g$, and we show that the claim holds for all $G \in \mathcal{G}$ with $|G| = g + 1$.

Let $G \in \mathcal{G}$ be such that $|G| = g + 1$, and let $i \in N$ be such that $i \notin L(G)$. If $i \notin G_j$ for all $j \in N$, then from (C) we have $i \notin F(G)$. Suppose there exists $j \in N$ such that $i \in G_j$. Note that $j \neq i$. By definition of L, $i \notin L(G)$ and $i \in G_j$ imply $j \notin L(G)$.

Let $G' \in \mathcal{G}$ be such that $G' = (G_1, \ldots G_{j-1}, G_j \setminus \{i\}, G_{j+1}, \ldots, G_n)$. By definition of L, we have $L(G') \subseteq L(G)$, which implies $i, j \notin L(G')$. Obviously, $|G'| = |G| - 1 = g$. By induction hypothesis, we have $F(G') \subseteq L(G')$. Thus, we have $i, j \notin F(G')$ from $i, j \notin L(G')$.

Notice that for the profile G' we have $i, j \notin F(G')$, and for the profile G we have $G_j = G'_j \cup \{i\}$ and $G_l = G'_l$ for all $l \in N \setminus \{j\}$. By noticing that $i \neq j$ and applying (IOV2) with G' and G in the roles of G and G', respectively, we conclude that $i \notin F(G)$. ■

Finally, we introduce the following monotonicity requirement.

- A CIF $F \in \mathcal{F}$ satisfies *monotonicity* (MON) if for all $G, G' \in \mathcal{G}$,
  - $[G_k \subseteq G'_k$ for every $k \in N]$ implies $[F(G) \subseteq F(G')]$.

In this axiom, used also by Samet and Schmeidler (2003), profiles G and G' are considered such that every individual who deserves to be a group member according to someone in the profile G is defined as a group member by the same person also in G'. Then, (MON) requires that in this case every individual who is socially accepted in G is accepted in G' as well.

It turns out that combining (C), (MON), (ETIV) and (IOV2) results in the existence of a lower bound for a CIF $F \in \mathcal{F}$ at a given profile $G \in \mathcal{G}$; this lower bound is exactly the result of the liberal-start-respecting procedure at the same profile.

Proposition 4 *If a CIF $F \in \mathcal{F}$ satisfies (C), (MON), (ETIV) and (IOV2), then $L(G) \subseteq F(G)$ for all $G \in \mathcal{G}$.*

Proof. Let $F \in \mathcal{F}$ satisfy (C), (MON), (ETIV) and (IOV2). In the following, in order to prove $L(G) \subseteq F(G)$, we prove $L_t(G) \subseteq F(G)$ for all profiles $G \in \mathcal{G}$ by induction on t.

Basis Step: We first show $L_0(G) \subseteq F(G)$ for all $G \in \mathcal{G}$. Suppose to the contrary that there exists a profile $G \in \mathcal{G}$ such that $i \in L_0(G)$ but $i \notin F(G)$ for some $i \in N$. Notice then that, by $i \notin F(G)$ and (C), the set $Q := \{j \in N : i \notin G_j\}$ is nonempty; clearly, $i \notin Q$. Let $G' \in \mathcal{G}$ be a profile such that $G'_k = G_k \setminus Q$ for all $k \in N$. Then, by $i \notin F(G)$ and (MON), $i \notin F(G')$. By (C), $Q \cap F(G') = \emptyset$. Let $j \in Q$ and consider the profile $G'' \in \mathcal{G}$ where $G''_k = G'_k$ for all $k \in N \setminus \{j\}$, and $G''_j = G'_j \cup \{i\}$. By $i \notin F(G')$ and (IOV2), $i \notin F(G'')$. Notice again that, by (C), $Q \cap F(G'') = \emptyset$. By repeating the same argument $(|Q| - 1)$-times, we arrive at profile $G^* \in \mathcal{G}$ with $G^*_k = G'_k$ for all $k \in N \setminus Q$, and $G^*_k = G'_k \cup \{i\}$ for all $k \in Q$. Thus, by (IOV2), $i \notin F(G^*)$. Observe however that $i \in G^*_k$ for all $k \in Q$ and $i \in G'_k = G^*_k$ for all $k \in N \setminus Q$. By (C), $i \in F(G^*)$, a contradiction. We conclude that $L_0(G) \subseteq F(G)$ for all $G \in \mathcal{G}$ must hold.

Induction Step: Let t be a nonnegative integer. We assume that $L_t(G) \subseteq F(G)$ for all $G \in \mathcal{G}$ and show that $L_{t+1}(G) \subseteq F(G)$ for all $G \in \mathcal{G}$. Let $i \in L_{t+1}(G)$. If $i \in L_t(G)$, then $i \in F(G)$ from $L_t(G) \subseteq F(G)$. Assume therefore $i \notin L_t(G)$. From the definition of L, there exists $j \in L_t(G)$ such that $i \in G_j$.

Note that $j \in F(G)$, which follows from $j \in L_t(G) \subseteq F(G)$, and that $i \neq j$, which is due to $i \notin L_t(G)$ and $j \in L_t(G)$. Let $G' \in \mathcal{G}$ be a profile such that $G' = (G_1, \ldots, G_{i-1}, G_i \cup \{i\}, G_{i+1}, \ldots, G_{j-1}, G_j \setminus \{i\}, G_{j+1}, \ldots, G_n)$. From the definition of L_0, $i \in L_0(G')$. From $L_0(G') \subseteq F(G')$, it follows that $i \in F(G')$. Noting that $j \in F(G)$ and $i \in F(G')$, by (ETIV), we obtain $i \in F(G)$. Therefore, $L(G) \subseteq F(G)$ for all $G \in \mathcal{G}$. ■

Theorem 2 *A CIF $F \in \mathcal{F}$ satisfies (C), (MON), (ETIV) and (IOV2) if and only if $F = L$. Moreover, all four axioms are independent.*

Proof. It is easy to check that the liberal-start-respecting procedure satisfies (C), (MON), (ETIV) and (IOV2). The combination of Proposition 3 and Proposition 4 proves that, if a CIF $F \in \mathcal{F}$ satisfies (C), (MON), (ETIV) and (IOV2) then it is L. Hence, we need only to show that the axioms are tight. The proof consists of four examples, each of which satisfies exactly three of the four axioms.

($\neg$(C)) Let $F \in \mathcal{F}$ be such that $F(G) = N$ for all $G \in \mathcal{G}$. Clearly, this CIF satisfies all axioms but (C).

($\neg$(MON)) Let $N = \{1, 2\}$ and consider the CIF F defined as follows.

$$F(G) = \begin{cases} \emptyset & \text{if } G \in \{(\emptyset, \emptyset), (\emptyset, \{1\}), (\{2\}, \emptyset), (\{2\}, \{1\})\}, \\ \{1\} & \text{if } G \in \{(\{1\}, \{1,2\}), (\{1\}, \{1\}), (\{1\}, \emptyset)\}, \\ \{2\} & \text{if } G \in \{(\{2\}, \{2\}), (\emptyset, \{2\})\}, \\ \{1,2\} & \text{otherwise.} \end{cases}$$

This CIF satisfies all axioms except (MON). To see that (MON) is violated, take $G = (\{1\}, \{1,2\})$, $G' = (\{1\}, \{2\})$. According to the proposed aggregator we have $F(G) = \{1\}$ and $F(G') = \{1,2\}$. Notice that $G'_k = G_k$ for $k = 1, 3$, and $G'_2 \subset G_2$. Nevertheless, $F(G) \subset F(G')$.

($\neg$(ETIV)) Take Kasher and Rubinstein's "liberal" aggregator, i.e., consider $L_0 \in \mathcal{F}$ with $L_0(G) = \{i \in N : i \in G_i\}$ for all $G \in \mathcal{G}$. In order to see that L_0 violates (ETIV), take $j \in L_0(G)$, $i \notin L_0(G)$, $k \in L_0(G')$, and set $k = i$.

($\neg$(IOV2)) Let $F \in \mathcal{F}$ be defined as follows:

$$F(G) = \{i \in N : i \in G_j \text{ for some } j \in N\}$$

for all $G \in \mathcal{G}$. Clearly, this CIF does not satisfy (IOV2) because the change of an outsider's opinion in favour of i changes i's social status. ■

5 Conclusion

In this paper, we have axiomatically characterized the procedures that define the collective identity functions K and L in the framework proposed by Kasher and Rubinstein (1997).

The consensus-start-respecting procedure is characterized by consensus, irrelevance of an outsider's view 1, and equal treatment of insiders' views. The axioms (C) and (IOV1) guarantee that any CIF satisfying them selects only socially accepted group members that K would also select. The characterization of K is based on the following simple observation: given a profile $G \in \mathcal{G}$, the axiom (C) guarantees that, for any CIF $F \in \mathcal{F}$ that satisfies it, $F(G)$ contains $K_0(G)$. The application of (IOV1) gives the result that there is no individual in $K_1(G)$ whose social status is determined by someone outside of $K_0(G)$, and (ETIV) implies that only individuals in $K_0(G)$ are crucial for determining an individual's social status. The induction argument in the proofs completes the characterization.

A similar observation can be made with respect to the liberal-start-respecting procedure that is characterized by consensus, monotonicity, equal treatment of insiders' views, and irrelevance of an outsider's view 2. Here (MON) is crucial for guaranteeing, together with (C) and (IOV2), that, for any profile $G \in \mathcal{G}$ and any CIF $F \in \mathcal{F}$ satisfying these axioms, $F(G)$ contains $L_0(G)$. It may be noted that the roles an outsider plays in determining someone's social status in these two procedures are quite different: for K, an outsider's change of his opinion in favor of himself is inconsequential, while for L, an outsider who changes his opinion in favour of himself becomes crucial in determining his own social status.

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