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A Comparison of Electricity Market Designs in Networks

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Abstract

Two basic market designs are used for the trading of electricity in meshed electricity networks with transmission constraints. Analytical results show that, in two-node networks, a market design integrating transmission and energy markets reduces the ability of electricity generators to exercise market power, relative to a design with separated markets for transmission and energy designs. In multi-node networks, countervailing effects make an analytic analysis difficult.

We present a formulation of both market designs as an equilibrium problem with equilibrium constraints, and apply our model to a realistic node network for Belgium, Germany, France and the Netherlands. We find that the integrated market design results in lower prices.

1 Introduction

In electricity markets with frequently constrained transmission lines, these constraints must be explicitly addressed by the market design, in order to achieve efficient dispatch and appropriate locational signals for investment, and reduce the ability of strategic generators to game the system operator. We compare the two basic market designs that can be used to address transmission constraints:

- **Integrated Market Design:** The integrated market design corresponds to market designs such as market splitting, zonal pricing or nodal pricing, and is implemented in the North East of the US (PJM, NY ISO, New England ISO) and Scandinavia (Nordpool). A centralised system operator collects location-specific energy bids and then clears the market for the entire region according to a well-defined protocol. The allocation of transmission rights is implicit. This ensures that different locational markets are automatically arbitraged and the network is used efficiently.

- **Separated Market Design:** The separated market design is currently implemented between many Continental European markets. Physical transmission rights are auctioned and traded separately from the energy markets. Physical transmission rights are defined between areas, and traders must own these rights if they want to schedule a transmission between the areas.

In a competitive market without uncertainty, the integrated market approach will result in the same generation dispatch and prices as with an approach based on centrally-allocated physical transmission rights. This follows because Bohn, Caramanis and Schweppe (1983) showed that the methodology underlying integrated market prices (nodal prices) results in welfare-maximising dispatch, and Chao and Peck (1996) proved that, in a competitive environment without uncertainty, the physical-rights-based approach also results in a welfare-maximising dispatch.

Unfortunately, few electricity markets are perfectly competitive, and therefore we assess how both market designs perform if generators bid strategically.

Hobbs, Metzler and Pang (2003) showed that endogenous and exogenous arbitrage - corresponding to integrated and separated markets - are equivalent. However, their analysis is based on the following tacit assumption: strategic generators assume that transmission prices, defined as the price-differences between different nodes, do not change in response to their output decision.

As a result, the profit-maximisation condition of strategic generators (first order condition) assumes that additional output would induce traders to buy additional transmission rights (at the fixed price) and export some of the output to other nodes even if constraints are binding. Strategic generators decide on their output as though they were facing the demand-responsiveness of the unconstrained network, and will therefore exercise less market power than if they are aware of transmission constraints. This model approach therefore understates the exercise of market power.

Analytical models show that, in a two-node network and in meshed networks with market power at one node, integrating energy and transmission markets reduces prices and improves welfare (see section 3.2). This can be explained by the following effect: if transmission markets are separated from energy markets, the allocation of transmission capacity in the network to export to, or import from, specific regions is determined before the stage of the energy spot market. The bids of generators to energy spot markets no longer change the allocated transmission capacity. If transmission markets are integrated with the energy market, transmission rights are allocated after the energy bids are submitted. The net trade flows are responsive to output changes of generators, net demand elasticity is increased and generators exercise less market power.

In a three-node network, Cardell, Hitt and Hogan (1997) showed that, for an integrated market design with a strategic generator producing at two nodes, an increase of output by a generator at node A can lead to a decrease in the prices, and therefore revenues, of this generator at node B. This shows that the integrated market design can also provide incentives for generators to reduce output. In section 3.3, we construct a three-node example to show that the second effect dominates the first effect: integrating energy and transmission markets can, in theory, reduce welfare.

To assess the relative importance of these two countervailing effects in larger networks with realistic parametrisation, we implement both market designs formulated in a numerical model (see section 4). We chose the Benelux countries Belgium and the Netherlands, with a reduced representation of the neighbouring states, Germany and France.

The models presented in this paper are of interest from two different perspectives: from the economic perspective, regulators are interested in the optimal choice for market design. The ranking with regard to the behaviour in the presence of market power provides an argument in favour of the integrated market design.

From the mathematical perspective, both market design implementations are Equilibrium Problem with Equilibrium Constraints (EPEC), which are a recent field of research.

Cardell, Hitt and Hogan (1997) were the first to give an EPEC formulation of an integrated market design. Hobbs, Metzler and Pang (2000) calculate oligopolistic price equilibria, using the supply functions of conjectural variation. Strategic generators can decide either on slope or intersect of their bid functions for each location. The optimisation problem for each generator is a two-stage game, in which the generator anticipates the Independent Strategic Operator's (ISO) social welfare-maximising market clearing. Our integrated market model is a Cournot Game in accordance with Cardell, Hitt Hogan (1997), expanded by price-responsive fringe generators and implicit transmission quantities.

Separated market designs have so far been modelled as mixed complementarity problems (Hobbs et al. 2003). Due to the existence of price-responsive fringe, we expand the separated market model to an EPEC.

The search for an equilibrium is carried out by solving the generators' problems (which are Mathematical Programs with Equilibrium Constraints) sequentially, using the last bid of the competitors as the input parameter.

2 Notation, Market Designs and Simplifying Assumptions

In the following section, we introduce the notation for the mathematical formulation of the market design. Then we explain the features and differences in depth, and formulate the optimisation problems of the generators. Finally, we discuss the implications of additional features in real-world markets.

2.1 Notation

We summarise all indices, parameters, variables and shadow prices in table 1. The last column indicates whether the parameter/variable is used in the model for the integrated or separated market design.

The network has I nodes i with M links m between these nodes. We model J strategic generation companies j, which can control generation on one or several nodes of the network.

To allow for piecewise linear marginal cost-schemes, we split the cost-curves of generators in L sections l with linear cost-segments. The strategic generator j bids in location i in the cost-segment l the quantity $s_{i,j,l}$. The marginal cost-function for strategic generator j in i is $c'_j(s_{ijl}) = ca_{ijl} + cb_{ijl}s_{ijl},\ s_{ijl} \leq capg_{ijl}$. Likewise, to represent piecewise linear cost-schemes for competitive generation, their cost-curves in each location are divided into $K-1$ linear sections $k = 2, \ldots, K$.

The linear demand-curve for each node i is included as a negative competitive supply-curve, and is denoted by $k = 1$. The quantities of the demand and the competitive fringe at a fixed node are therefore q_{ik} with $c(q_{i1}, \ldots, q_{iK}) = \sum_l a_{ik} + b_{ik}q_{ik}$. For $k > 1$ we require $0 \leq q_{ik} \leq capd_{ik}$ and for $k = 1$ $0 \geq q_{i1} \geq capd_{i1}$.

Note that a_{i1} and b_{i1} the parameters of the linear demand function and the demand $q_{i,1}$ are always negative in our notation, while the fringe quantities $q_{i,k},\ k \geq 2$ are positive. We use an additional parameter σ_k to change the sign in the equations.

Table 1: Notation

Indices		
i	node	I,S
j	generator	I,S
l	step number strategic generator	I,S
k	step number net demand	I,S
m	line	I,S
Parameter		
ca_{ijl}	cost-curve intercept, node i, strategic generator j, section l	I,S
cb_{ijl}	cost-curve slope, node i, strategic generator j, section l	I,S
$capd_{ik}$	capacity limit fringe generator/demand node i	I,S
$capg_{ijl}$	capacity limit generator j, node i, section l	I,S
a_{ik}	bid curve intercept, node i, fringe generator/demand section k	I,S
b_{ik}	bid curve slope, node i, fringe generator/demand section k	I,S
$\gamma_{i,m}$	flow from node i over link m to swing bus (node 1)	I,S
$capk_m$	capacity link m	I,S
σ_k	$\sigma_1 = 1$ and $\sigma_k = -1$ for $k \neq 1$ for	I,S
	the different treatment of demand and competitive fringe.	
Variables		
$s_{i,j,l}$	quantities strategic generator j, node i, section l	I,S
$q_{i,k}$	quantity, consumer or fringe generator, node i.	I,S
t_i	export quantity, node i	S
Shadow Price	Constraint	
p	Conservation constraint in swing bus	I
p_i	Conservation constraint in each bus	S
$\lambda_{i,k}$	non-negative demand and fringe production	I,S
$\mu_{i,k}$	upper bound for fringe capacity.	I,S
$\rho_{i,m}$	capacity constraint on line k	I
$\delta_{i,m}$	capacity constraint on line k - inverse direction.	I

2.2 Market Designs

For each market design, we give a short description of the timing of the market. Subsequently, we translate this description into the optimisation problems that the different market participants must solve.

2.2.1 Separated Market Design

In period one, traders submit bids for transmission capacity to a transmission operator. As we assume competitive traders, the auction design need not be specified. Furthermore, it suffices to define transmission contracts to and from a swing bus, because any transmission in the network is a combination of two such contracts. The transmission operator issues transmission contracts to the traders to make most efficient use of the network.

In period two, traders, strategic generators, fringe generators and demand-side submit bids to the local energy market. Traders with transmission capacity can buy energy in the spot market at one node and sell it at a different node. Therefore, traders must submit bids to the exporting and importing spot markets simultaneously, and a trader failing at one end of the transaction is exposed to high imbalance fees. To avoid these imbalance fees, traders submit price-independent bids to both markets (Neuhoff, 2003). Traders do not receive additional information between period one and period two; therefore, we can assume that they will use all the transmission capacity they bought in period one to trade in period two.

Traders in period one correctly anticipate the spot market outcomes for period two. This is possible because of full information, no uncertainty and no mixed-strategy choices of strategic generators in the separated market design (Neuhoff, 2003), and the assumption of competitive traders. Competitive traders pay an amount for transmission contracts which will be the price-difference between the local spot markets. We can simplify the model by assuming that the transmission operator directly allocates transmission capacity between the spot markets in a way that makes most efficient use of the transmission capacity. If the bids that are submitted to the spot market are competitive, this results in a welfare-maximising use of the transmission capacity. Only the bids of competitive generation and demand are price-responsive, and both types of bids are competitive. Therefore, we assume that the transmission operator maximises social welfare taking bids by competitive generation and demand into account. (see also Smeers and Wei, 1999).

To represent the appropriate timing of the markets as the transmission auction happens before the energy spot market we must ensure that the allocation of transmission capacity by the transmission operator is not affected by changes in the bids of strategic generators. This can be achieved by treating the transmission operator on the same decision-level as the Cournot Game between strategic generators. This ensures that output-changes of strategic generators during their individual Cournot optimisation will not result in changes of the transmission allocation. Nevertheless, equilibrium is only reached once the transmission operator correctly anticipates the output-choice of strategic generators.

Due to the capacity constraints on the competitive generators, the net demand - which is demand minus competitive generation - is not differentiable at the points at which the capacity or non-negativity constraints become active. In the mathematical formulation, this is represented by mixed complementarity constraints. The resulting optimisation problems for the generators and the transmission operator are of the MPEC type. Furthermore, the non-negativity constraints of competitive generation imply that the net demand functions are not necessarily convex, and so the solution is not necessarily unique.

The optimisation problem of generator j then becomes:

$$\max_{s_{ijl}, q_{ik}, p_i, \lambda_{ik}, \gamma_{ik}} \sum_i \quad \sum_l (p_i s_{ijl}) - \sum_i \sum_l (ca_{ijl} + \tfrac{1}{2} cb_{ijl} s_{ijl}) s_{ijl} \tag{1}$$

$$\text{s.t.} \qquad \sum_{l,j} s_{i,j,l} + \sum_k q_{ik} - t_i = 0 \tag{2}$$

$$a_{ik} + b_{ik} q_{ik} + p_i + \sigma_k \lambda_{ik} - \sigma_k \mu_{ik} = 0 \tag{3}$$

$$\sigma_k q_{ik} \geq 0 \tag{4}$$

$$\lambda_{ik} \geq 0 \tag{5}$$

$$q_{i,k}\lambda_{ik} = 0 \tag{6}$$

$$-\sigma_k q_{ik} + \sigma_k capd_{ik} \geq 0 \tag{7}$$

$$\mu_{ik} \geq 0 \tag{8}$$

$$\mu_{ik}(-\sigma_k q_{ik} + \sigma_k Capd_{ik}) = 0 \tag{9}$$

$$s_{ijl} \geq 0 \tag{10}$$

$$-s_{ijl} + capg_{ijl} \geq 0 \tag{11}$$

The co-ordinated auction determines the export quantity t_i in order to maximise residual social welfare (expenditure on electricity equals generators' revenue plus transmission revenue and therefore cancels out):

$$\max_{t,q_i,p_i} \sum_{i,k} -\sigma_k \left(a_{i,k} + \frac{1}{2} b_{i,k} q_{ik} \right) q_{ik}, \tag{12}$$

subject to the energy balance in the network (13) and at each individual node (14). Constraints (15) are the market-clearing condition for competitive generation and demand with the non-negativity (16) to (18) and capacity constraints (19) to (21). Constraints (22) to (23) are the upper- and lower-line capacity constraints.

$$\text{s.t.} \qquad \textstyle\sum_i t_i = 0 \tag{13}$$

$$\textstyle\sum_{j,l} s_{ijl} + \sum_k q_{ik} - t_i = 0 \tag{14}$$

$$a_{ik} + b_{ik} q_{ik} + p_i + \sigma_k \lambda_{ik} - \sigma_k \mu_{ik} = 0 \tag{15}$$

$$\sigma_k q_{ik} \geq 0 \tag{16}$$

$$\lambda_{ik} \geq 0 \tag{17}$$

$$q_{i,k}\lambda_{ik} = 0 \tag{18}$$

$$-\sigma_k q_{ik} + \sigma_k capd_{ik} \geq 0 \tag{19}$$

$$\mu_{ik} \geq 0 \tag{20}$$

$$\mu_{ik}(-\sigma_k q_{ik} + \sigma_k Capd_{ik}) = 0 \tag{21}$$

$$Capk_m - \textstyle\sum_i \gamma_{m,i} t_i \geq 0 \tag{22}$$

$$\sum_i \gamma_{m,i} t_i + Capk_m \geq 0 \tag{23}$$

The resulting problem is of the EPEC type, since the ISO and strategic generators share the market-clearing conditions of demand and competitive fringe as common constraints.

2.2.2 Integrated Market Design

In the integrated market design, the timing is different. In the first stage, generators submit bids to the ISO. The ISO's objective, at the second stage, is to allocate transmission capacity, in order that the network be used optimally. This can be achieved by maximising social welfare, using demand and price responsive supply by competitive generators as variables. The optimisation problem of the ISO contains the energy-conservation constraint, the capacity constraints of the transmission lines and the capacity constraints for the competitive fringe:

$$\max_{q_{ik}} \quad \sum_i q_{ik}(a_{ik} + \tfrac{b_{ik}}{2} q_{ik}) \tag{24}$$

$$\text{s.t.} \quad \sum_i \left(\sum_j s_{ij} + q_{ik} \right) = 0 \qquad p \tag{25}$$

$$-capk_m \leq \sum_i \gamma_{m,i} \left(\sum_j s_{ijl} + \sum_k q_{ik} \right) \leq capk_m \qquad \rho_m, \delta_m \tag{26}$$

$$0 \geq \sigma_k q_{ik} \geq \sigma_k capg_{ik} \qquad \lambda_{ik}, \mu_{ik} \tag{27}$$

This is a quadratic optimisation problem with a unique solution: if $b_{ik} \neq 0$.
The KKT-stationarity conditions of the ISO's optimisation problem are:

$$(a_{ik} + b_{ik} q_{ik}) + p - \sum_m \rho_m \gamma_{m,i} + \sum_m \delta_m \gamma_{mi} + \sigma_k \lambda_{ik} - \sigma_k \mu_{ik} = 0 \tag{28}$$

$$\sum_i (\sum_{j,l} s_{ijl} + \sum_k q_{ik}) = 0 \tag{29}$$

$$0 \geq -Capk_m - \sum_i \gamma_{m,i} \left(\sum_{j,l} s_{ijl} + \sum_k q_{ik} \right) \perp \rho_m \geq 0 \tag{30}$$

$$0 \geq \left(\sum_i \gamma_{m,i} \left(\sum_{j,l} s_{ijl} + \sum_k q_i \right) \right) - Capk_m \perp \delta_m \geq 0 \tag{31}$$

$$\sigma_k q_{i,k} \geq 0 \tag{32}$$

$$\lambda_{ik} \geq 0 \tag{33}$$

$$q_{ik} \lambda_{ik} = 0 \tag{34}$$

$$-\sigma_k q_{ik} q_{ik} + \sigma_k capg_{ik} \geq 0 \tag{35}$$

$$\mu_{ik} \geq 0 \tag{36}$$

$$\left(-\sigma_k q_{ik} q_{ik} + \sigma_k capg_{ik}\right) \mu_{ik} = 0 \tag{37}$$

2.2.3 Formulation of the Leader's Problem (Generator)

In the leader problem for the strategic generators, the quantity bids of their fellow strategic generators are taken as fixed. Leaders anticipate the ISO response to their quantity bid. This is modelled by including the optimality conditions of the ISO's problem as constraints in the strategic generators' optimisation problem. The strategic generators maximise profits. Profits consist of electricity sales at his nodes of production minus production costs. The first 10 constraints are the KKT conditions of the ISO optimisation problem; the remaining two constraints are the non-negativity and capacity constraints of production.

$$\max_{s_{ijl}, q_{ik}, p, \rho_m, \delta_m, \lambda_{ik}, \mu_{ik}} \quad \sum_i \left(\left(p + \sum_m \gamma_{i,m} \left(-\rho_m + \delta_m\right)\right) \sum_l s_{ijl} - \sum_l \left(ca_{ijl} + \tfrac{1}{2} cb_{ijl} s_{ijl} \right) s_{ijl} \right) \tag{38}$$

$$\text{s.t} \quad \left(a_{ik} + b_{ik} q_{ik}\right) + p_1 - \sum_m \rho_m \gamma_{m,i} + \sum_m \delta_m \gamma_{mi} + \sigma_k \lambda_{ik} - \sigma_k \mu_{ik} = 0 \tag{39}$$

$$\sum_i \left(\sum_{j,l} s_{ijl} + \sum_k q_{ik}\right) = 0 \tag{40}$$

$$0 \geq -Capk_m - \sum_i \gamma_{m,i} \left(\sum_{j,l} s_{ijl} + \sum_k q_{ik} \right) \perp \rho_m \geq 0 \tag{41}$$

$$0 \geq \left(\sum_i \gamma_{m,i} \left(\sum_{j,l} s_{ijl} + \sum_k q_i \right) \right) - Capk_m \perp \delta_m \geq 0 \tag{42}$$

$$\sigma_k q_{i,k} \geq 0 \tag{43}$$

$$\lambda_{ik} \geq 0 \tag{44}$$

$$q_{ik} \lambda_{ik} = 0 \tag{45}$$

$$-\sigma_k q_{ik} + \sigma_k capg_{ik} \geq 0 \tag{46}$$

$$\mu_{ik} \geq 0 \tag{47}$$

$$\left(-\sigma_k q_{ik} + \sigma_k capg_{ik}\right) \mu_{ik} = 0 \tag{48}$$

$$s_{ijl} \geq 0 \tag{49}$$

$$s_{ijl} \leq capg_{ijl} \tag{50}$$

Note that electricity prices are calculated as nodal prices where the multiplier p of the energy-conservation constraint determines the price at the swing bus.

2.3 Institutional Assumptions

As with all models, we must abstract certain features from real-market designs.

The two major simplifications in our analysis are that we do not represent the market-power mitigating impact of, first, long-term energy and, second, long-term transmission contracts. It is easy to represent the impact of an exogenously-determined allocation of such contracts in our model, but it would require a more simplified representation of the transmission market if we were to represent an endogenous transmission or energy-contract allocation.

First, electricity generators sign long-term contracts, either in the form of explicit contracts with large customers or implicit contracts due to their vertical integration with the supply business. Exposure to the spot market is reduced thanks to these long-term contracts, and strategic generators face less incentive to exercise market power (Allaz and Vila, 1993). Not representing these contracts implies that we will observe higher prices than usually realised, but as we are interested in comparing the price-levels between two market designs, and both designs are equally affected, this does not impact on our analysis.

Second, electricity generators frequently acquire and own transmission contracts in order to hedge their transmission risk. The model with separated energy and transmission markets does include transmission contracts, but strategic generators are, in the model, excluded from participating in this contract market. Therefore, the model ignores the impact which contract ownership has on the bids of strategic generators (Joskow and Tirole, 2000) or the distortions to the contract auctions due to strategic bidding for contracts by generators (Gilbert et.al., 2004).

In reality, transmission contracts are not only part of separate energy and transmission market designs, but also part of integrated energy and transmission markets. They are equally required in an integrated market design, but this time as financial contracts, to allow market participants to hedge the base risk and to provide forward information for investment decisions. Ownership of financial transmission contracts distorts the dispatch decisions of strategic generators just as ownership of physical transmission contracts does, and bids for these contracts will likewise be affected. Therefore, we simplify both market designs in a symmetric way, by excluding generators from holding transmission contracts in either design.

A further aspect that impacts the exercise of market power is the regulatory threat. Monopolists anticipate regulatory intervention if they charge excessive prices and will therefore moderate their behaviour (for a modelling approach, see Neuhoff and Newbery, 2004).

One requirement for the application of our modelling approach is that physical transmission contracts are only traded at discrete points in time. In meshed networks, it is unlikely that any other design can be implemented, because any physical transmission right (point-to-point) is an aggregation of property rights to a multitude of potentially constrained transmission lines (flow-gate rights). All market participants must simultaneously state their willingness to pay for different transmission rights, to allow the system operator to efficiently aggregate the flow-gate rights into the transmission contracts.

Transmission contracts were traded continuously and simultaneously with energy contracts in the markets along the west coast of the US. However, this was only possible because physical transmission contracts directly corresponded to an access right to the constrained link of an almost linear system stretching from Canada down to California. Therefore no (re-) configuration of transmission contracts based on market information is required. This approach does not work in meshed networks such as in the US North-East or Continental Europe.

In Continental Europe, transmission rights for interconnections between different countries are still auctioned separately. This is known to be very inefficient, because only a fraction of physically-available transmission capacity can be provided to ensure that system security is preserved, irrespective of the loop flows created by other transmissions. This is the reason for recent efforts to combine these individual auctions. This paper was motivated by the question of what is the best way to combine these individual auctions.

Integrated markets can be more complex than those represented in our model. New York and New England have an integrated energy and transmission market that also includes marginal loss calculations. Furthermore, like PJM and Nordpool, they have a multi-settlement system with an integrated day-ahead and balancing market. The possibility of implementing these additional features in a consistent way can be seen as an additional benefit of an integrated market design. We believe that abstracting from these effects should in first order not distort this analysis, but we are excited to see, and are working on expanding our analysis to include question of, balancing markets, transmission contracts and forward contracting (see e.g. Kamat and Oren, 2003).

Also related is the debate on nodal versus zonal pricing. In separated energy and transmission markets as well as in integrated markets, the aggregation of individual nodes to zones either requires a more conservative definition of transmission capacity or increases the opportunity for generators to exercise market power (Harvey and Hogan, 2000); aggregation results in inefficient dispatch and causes generators to make incorrect location decisions. The comparison of integrated and separated markets in this paper is based on the same level of aggregation in both designs.

Note that, for the integrated energy and transmission markets, a shift to smaller zones or nodes is feasible. The only drawback is that generators are exposed to some basis risk if transmission contracts do not exactly match their scheduled flows. A design with separate energy and transmission rights is more affected. Liquidity is lost at each node and transmission contracts become more complex.

3 Illustrative Examples to Provide Economic Insight

In this section, we construct two stylised network examples, for which we calculate the results with a separate and integrated market design, to illustrate the countervailing effects.

3.1 A Three-Node Network

In this three-node network, only the transmission line between node 1 and 3 is constrained and of capacity *capk*. According to Kirchhofs Laws flows are split between all feasible paths proportional to the inverse of the resistance on these paths. Therefore, $\gamma_1 = \frac{2}{3}$ of the energy flowing from node 1 to node 3 crosses the constrained link. Likewise, $\gamma_2 = \frac{1}{3}$ of exports t_2 from node 2 to node 3.

For energy delivery from node 1 to node 3, the direct link is half the distance of the path via node 2. Therefore, physical laws imply that two-thirds of the energy pass along the direct path. The joint exports from 1 and 2 are constrained as follows:

$$\gamma_1 t_1 + \gamma_2 t_2 \leq capk. \tag{51}$$

Net production at each node i is q_i and positive for exports and negative for imports. The net-demand-curve is assumed to be linear:

$$p_i = -a_i - b_i q_i \quad \text{(Note that } a_i, b_i, q_i \leq 0.\text{)} \tag{52}$$

In example one (two), one strategic generator with output s_2 (s_1, s_2) is located at node 2 (nodes 1 and 2). Since there is only one strategic generator, we omit the index j. In this

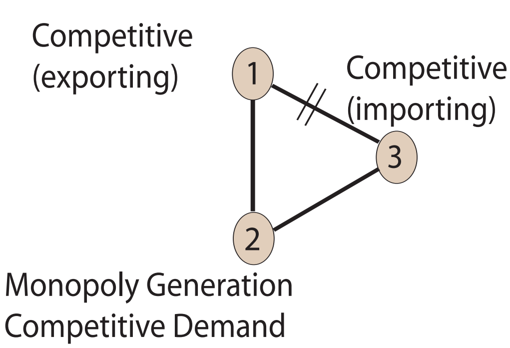

Figure 1: Example One: The monopolist has generation at node 2 only

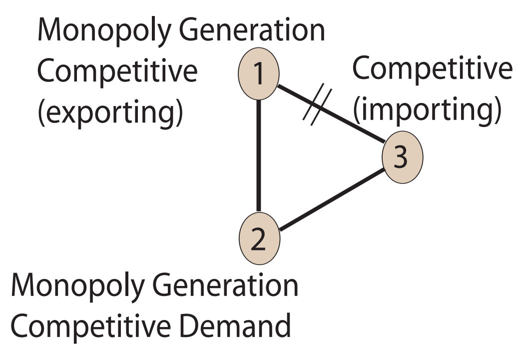

Figure 2: Example Two: The monopolist has generation at nodes 1 and 2

illustrative example, we also assume a linear cost-function and no fringe generators; therefore we also omit the indices l and k.

3.2 Example 1

The first example shows how, in a three-node network, the integration of energy and transmission markets can mitigate market power.

3.2.1 Separated Energy and Transmission Markets

In period one, traders submit bids for transmission capacity to the system operator. As argued in section 2.2.1, competitive traders can be represented by the welfare-maximising transmission operator, allocating transmission capacity $t_i(s2)$ and choosing output and demand quantities $q_i(s_2)$. We do not represent the output choice of the strategic generator in period two s_2 as an endogenous variable for the transmission operator because that would imply that the transmission operator allocates transmission capacity in order to influence the output choice of strategic generators. However, the competitive traders implementing the welfare-maximisation (53) do not take such considerations into account. We therefore set s_2 as fixed in (53) to (56) and calculate the equilibrium output choice of strategic generators in (57) to (59). The objective function of the transmission operator is:

$$W_r(s_2) = \max_{q_i(s_2),t_i(s_2)} \sum_i \int_q^0 p(q)dq = \max_{q_i,t_i} \sum_i \frac{b_i}{2} q_i^2 + a_i q_i. \tag{53}$$

subject to the network energy balance:

$$\sum_i t_i(s_2) = 0, \tag{54}$$

the local energy balance:

$$q_2(s_2) + s_2 = t_2(s_2), \quad q_i(s_2) = t_i(s_2), i = 1, 3 \tag{55}$$

and the transmission constraint (51).

For simplicity, we assume that the transmission constraint is binding, so we are able to use the equality sign in (51). We also assume that capacity and non-negativity constraints are satisfied. Using (55) to eliminate $t_i(s_2)$ in (54) and (51) $q_1(s_2)$ and $q_3(s_2)$ can be written as function of $q_2(s_2)$:

$$\begin{aligned} q_1(s_2) &= \frac{capk - \gamma_2 \left(q_2(s_2) + s_2\right)}{\gamma_1}, \\ q_3(s_2) &= \frac{-capk + \left(\gamma_2 - \gamma_1\right)\left(q_2(s_2) + s_2\right)}{\gamma_1}, \end{aligned}$$

allowing us to express the optimisation problem (53) as a function of the remaining choice variable $q_2(s_2)$ (note that all the a_i and b_i are negative). To simplify the calculations, we subsequently set parameter values $a_1 = b_3 = 0$:

$$W_r(s_2) = \max_{q_2(s_2)} \frac{b_1}{2}\left(\frac{capk - \gamma_2\left(q_2(s_2) + s_2\right)}{\gamma_1}\right)^2 + a_2 q_2(s_2) + \frac{b_2}{2} q_2^2(s_2) + a_3 \frac{-capk + \left(\gamma_2 - \gamma_1\right)\left(q_2(s_2) + s_2\right)}{\gamma_1}.$$

For $b_1 < 0$ this is a concave function in $q_2(s_2)$ with a unique maximum. The first order condition gives the optimal competitive output choice:

$$q_2(s_2) = \frac{\gamma_2 b_1 capk - \gamma_2^2 b_1 s_2 - \gamma_1^2 a_2 - a_3\left(\gamma_1\gamma_2 - \gamma_1^2\right)}{\gamma_2^2 b_1 + \gamma_1^2 b_2}.$$

Substituting back into (55) gives the transmission capacity allocated to exports from node 2 as a function of the expected output choice of the strategic generator:

$$t_2(s_2) = s_2 + q_2(s_2) = \frac{\gamma_2 b_1 capk + \gamma_1^2 b_2 s_2 - \gamma_1^2 a_2 - a_3\left(\gamma_1\gamma_2 - \gamma_1^2\right)}{\gamma_2^2 b_1 + \gamma_1^2 b_2}. \tag{56}$$

When the strategic generator bids to the energy spot market in period two, the transmission capacity t_2 allocated for exports from the node has already been decided in period one. The strategic generator takes t_i as fixed in his maximisation problem for the bid to the energy spot market in period two:

$$\Pi = \max_{s_2, q_2, p_2} p_2 s_2, \tag{57}$$

subject to the local energy balance (55) and the competitive bids of local net demand (52). The two constraints allow the expression of the two choice variables q_2 and p_2 as a function of s_2, in such a way that the optimisation problem reads:

$$\Pi = \max_{s_2} \left(b_2 \left(s_2 - t_2\right) - a_2\right) s_2, \tag{58}$$

and is solved:

$$s_2 = \frac{a_2}{2b_2} + \frac{t_2}{2}. \tag{59}$$

In equilibrium, traders, and therefore the transmission operator maximising (53), correctly anticipate the output choice s_2 of the strategic generator in the transmission auction. This implies that (56) and (59) must be simultaneously satisfied. This gives us the equilibrium output quantity of our generator:

$$s_2 = \frac{\gamma_2 b_1 capk + \gamma_2^2 \frac{b_1}{b_2} a_2 - \left(\gamma_1 \gamma_2 - \gamma_1^2\right) a_3}{2\gamma_2^2 b_1 + \gamma_1^2 b_2}. \tag{60}$$

3.2.2 Integrated Energy and Transmission Markets

The integrated energy and transmission market corresponds to a Stackelberg Game (Hobbs et al., 2000). Each leader (generator) continues to maximie his profit function (58) subject to local energy balance (55) and competitive local demand response (52). The difference is that t_i are no longer fixed but determined by the follower (system operator) as a function of the strategic output choice of the generator s_2.

We have already solved the followers' reaction function $t_2(s_2)$ and determined the optimal allocation of transmission rights as a function of s_2 in equation (56). Therefore, we need only substitute $t_2(s_2)$ for t_2 in the profit function of the strategic generator (58):

$$\Pi = \max_{s_2} \left(b_2 \left(s_2 - \frac{\gamma_2 b_1 capk + \gamma_1^2 b_2 s_2 - \gamma_1^2 a_2 - a_3 \left(\gamma_1 \gamma_2 - \gamma_1^2\right)}{\gamma_2^2 b_1 + \gamma_1^2 b_2} \right) - a_2 \right) s_2, \tag{61}$$

and calculate the first order conditions to obtain the optimal output choice s_2 :

$$s_2 = \frac{\gamma_2 b_1 capk + \gamma_2^2 \frac{b_1}{b_2} a_2 - \left(\gamma_1 \gamma_2 - \gamma_1^2\right) a_3}{2\gamma_2^2 b_1}. \tag{62}$$

The nominator in (62) and (60) is identical, but the denominator is larger in (60).

This shows that the production of the strategic generator is larger in the integrated market design. The generator no longer faces only local demand response, but also the response of the network. To calculate this demand slope, we differentiate the price at node 2, as expressed in the parentheses of equation (61), with respect to the output choice s_2 of the strategic generators:

$$-\frac{\partial p_2}{\partial s_2}|_{integrated} = -b_2 \frac{1}{1 + \frac{b_2 \gamma_1^2}{b_1 \gamma_2^2}} < -b_2 = -\frac{\partial p_2}{\partial s_2}|_{separated}$$

provided $b_1, b_2 < 0$. Integrating the energy and transmission markets implies that prices change less with output changes, or that effective demand is more responsive to price-changes. Higher effective demand elasticity is the main driver in mitigating market power and can be obtained at low cost by choosing the appropriate integrated market design.

Parameters	a_1	a_2	a_3	b_1	b_2	b_3	capk
Values	0	-8	-10	-2	-1	0	5
variables	s_2	q_1	q_2	q_3	p_1	p_2	p_3
Separated market design	8.25	3.25	0.25	-11.75	6.5	8.25	10
Integrated market design	16.5	0.5	-2.5	-14.5	1	5.5	10

The generators profit increases from 68.1 to 90.8 and the social welfare from 104.9 to 161.6 if we change from a separated to an integrated market design.

3.3 Example 2

The second example illustrates that the reverse effect is also possible, if one strategic generator is active in more than one node. We assume that the strategic generator is active at nodes 1 and 2 , see figure 2. The set-up is essentially the same as in example one, with the difference non-zero marginal costs of production at node 2. The results of a numerical calculation are:

Parameters	a_1	a_2	a_3	b_1	b_2	b_3	$capk$	cb_2
Values	0	-40	-80	-4	-1	-1	5	3
variables	s_1	s_2	q_1	q_2	q_3	p_1	p_2	p_3
Separated market design	1.86	9.52	1.86	-1.94	-12.35	7.42	38.07	68.71
Integrated market design	0.15	10.89	2.5	-1.18	-11.29	10	38.82	67.65

In this example, the price-increases at two nodes and decreases at one node if we change from the separated to the integrated market design. More importantly, while the profit of the strategic generator increases (from 240.2 to 246.3) the total welfare decreases from 772.3 to 768.2.

4 Set-up of the Comparison

The network consists of three nodes in the Netherlands, two nodes in Belgium and one node each in Germany and France, with generation and demand. Further intermediate nodes with neither demand nor generation are used to model the linearised DC network. The transmission constraints were summarised as 28 flow-gates. All flow-gates are characterised by an upper limit in MWh, and by power distribution functions characterising the amount of energy transmitted to each node from the reference node Germany, passing via the flow-gate.

For generation capacity, eight firms were considered as strategic generators, with production in one or several countries. Production in Belgium and the Netherlands is divided between the countries nodes according to the location of the generation plants. In Germany and France, production and demand are located at the national nodes D and F.

Country	Node	Generator
Germany		E.On, ENBW, RWE, Vattenfall, EdF
France		EdF, ENBW
Belgium	Merc	Electrabel
	Gram	Electrabel
Netherlands	Maas	Essent, Nuon, E.ON, Electrabel
	Krim	Essent, Nuon, E.ON, Electrabel
	Zwol	Electrabel

We assume that all these firms are bidding their entire output in the spot market, with the sole objective of maximising profit. As discussed in section 2.3, we ignore the impact of forward contracts and regulatory threat, and therefore model higher prices than those observed. The remaining generation plants, which are not allocated to one of the mentioned companies, are assumed to bid their marginal cost-curves into the spot market.

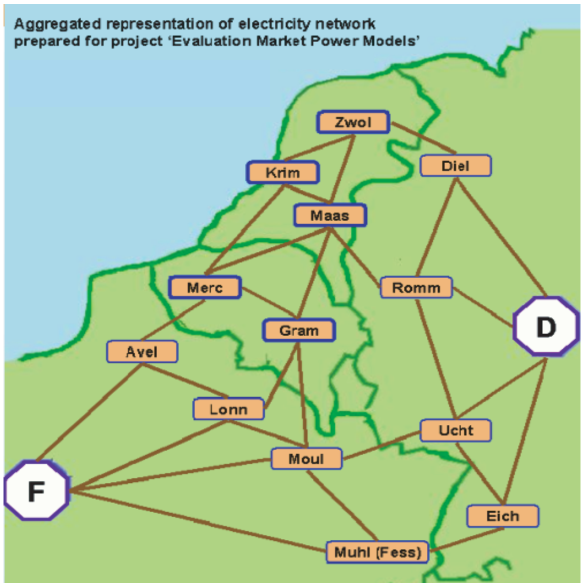

Figure 3: Network

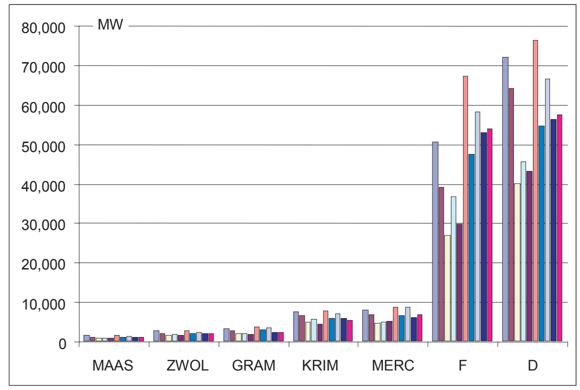

Figure 4: Load Scenarios

To create several different demand scenarios, empirical load data for the summer and winter peak are used and scaled with a scenario and location-specific random factor. With this method, we created ten different demand scenarios, five for summer and five for a winter generation structure. Figure 4 shows the demand levels at 30 Euro/MWh. We assume that demand is linear. To determine the slope, we assume that demand elasticity is 0.1 at the previously calculated demand for a price of 30 Euro/MWh.

All parameters can be found on the following website provided by ECN:
http://www.electricitymarkets.info/modelcomp/testdata.html
With these parameter values, we solve the equilibrium problems described in section two.

5 Numerical Issues

Both market designs lead to Equilibrium Problems with Equilibrium Constraints, which are a special case of a generalised Nash Game (GN). Calculating Nash equilibria for EPECs is difficult for several reasons: If solving for pure strategies, a solution does not necessarily exist (Oren, 1997). Borenstein, Bushnell and Stoft (2000) have calculated mixed strategy equilibria for a simple network. However, in complex games mixed strategy equilibria are difficult to calculate.

If pure strategy solutions exist, then they are usually not unique since the profit functions of the generators are nonlinear and non-differentiable. Additionally, Harker (1991) and Ehrenmann (2003) showed that GN-games can have non-isolated, multiple solutions (solution sets) which makes a local convergence analysis difficult.

We implement the different market designs in the modeling package GAMS and search for strategy tuples that fulfill the stationarity conditions for all generators using different starting points.

We follow the diagonalisation approach of Hobbs, Metzler and Pang (2000) which is similar to a Gauss-Seidel algorithm. We solve the optimisation problem of each generator sequentially holding the decisions of the other generators fixed and hope that the sequence converges (integrated market). In the separate market design, the co-ordinated auction becomes an additional step in the diagonalisation sequence. The optimisation problems of the generators are of the MPEC type. Since the linear independence constraint qualification (LICQ) is always violated for MPECs, they are already computationally challenging .

To solve the MPECs we used classical NLP solvers: Fletcher and Leyffer (2002) reported that they had successfully applied SQP methods on a non-linear program (NLP) reformulation of the original MPEC. In this NLP reformulation, they replaced the complementarity constraints

$$0 \leq f(z) \perp g(z) \geq 0$$

with

$$f(z) \geq 0, g(z) \geq 0, f(z)^\top g(z) \leq 0.$$

The advantage of such a formulation is, that the multiplier of the complementarity is sign constraint. Fletcher, Leyffer, Ralph and Scholtes examined the local convergence properties and showed that, under reasonable assumptions, SQP converges locally superlinearly near a strong stationary point for such a MPEC reformulation. Under the generic MPEC-LICQ (Scholtes and Stöhr, 2001), a B-stationary point (Scheel and Scholtes, 2000) of the original MPEC is a strong stationary point of the corresponding NLP. We used the standard SQP algorithms SNOPT (Gill, Murray and Saunders, 2002) and switched to PATHNLP (Dirkse and Ferris, 1995) to solve the reformulated MPECs after a fixed number of iterations. PATHNLP performed better because SNOPT stopped frequently at non-optimal intermediate solutions.

Each NLP has around 150 variables and 500 constraints (140 complementarity constraints). We initialise the problem by solving the ISO (follower) problem for fixed initial proportions of each strategic generators installed production capacity. We used levels of 20%, 30%, 40% ... 110%, 120% of the installed capacity of the first segment of each generator as starting points. Additional, we used the solution of the integrated market model as a starting point for the separated market model, and then again the solution of the separated market model as a starting point for the integrated market model. The computability of an equilibrium was sensitive to the choice of starting point.

6 Findings

For the separated market design, we always found solutions for all 10 scenarios. For the integrated market approach, the diagonalisation converged for six scenarios. For two scenarios the diagonalisation did not converge but created a sequence of prices with deviations of less than 10^{-5}%. In two case the sequence cycled with variations of up to 5%.

Also for several scenarios, we found several different equilibria and made the following observations:

- All solutions that we found for the integrated market led to lower prices in all nodes than in the separated design with one exception where the price increased in one node by 2% while the average prices fell by 28%.

- In the cases of cycling, all prices of the sequence generated for the integrated market were below the prices of the separated market.

The numeric results for scenario two for the two-market design are represented in Figure 6.

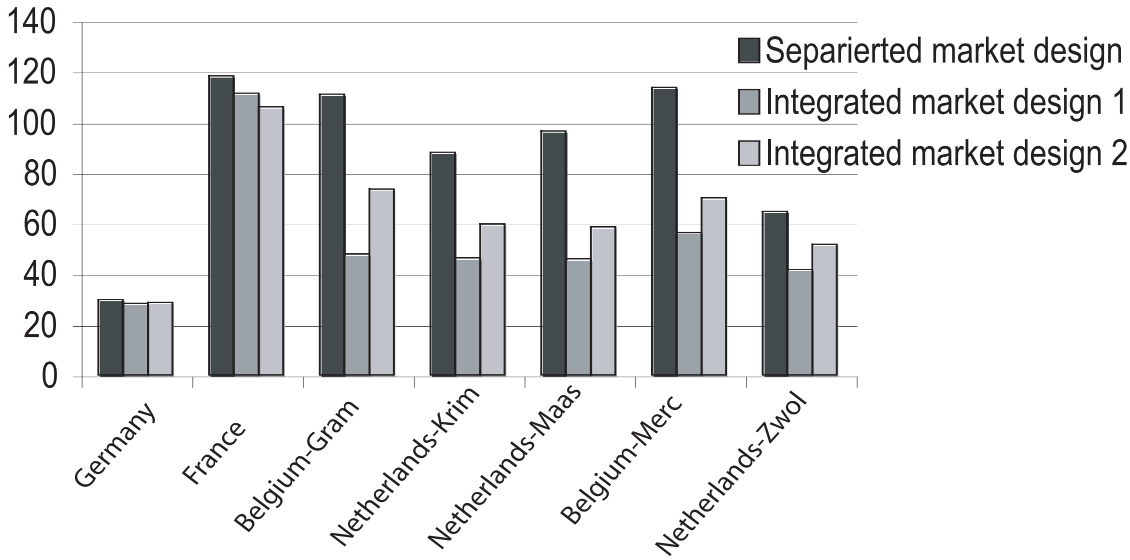

Figure 5: Price-Levels for the Separated and 2 equilibria of for the Integrated Market Design (Euro/MWh)

Here we found two equilibria for the integrated market design. The prices calculated for France are higher than in the other zones, which is due to EdF's monopoly position and binding import constraints from both Germany and Belgium. As discussed above, we did not model the fact that, in reality, a dominant national generator could not increase the price to the calculated level without triggering strong regulatory interference. By contrast, Germany has the lowest price-levels, since it has four strategic generators and a large share of competitive generation, which provides considerable responsiveness in net demand.

In our examples, prices are always higher in a separated market design. The lowest impact of integrating energy and transmission market is on price-levels in France, because transmission constraints are binding from both neighbouring countries represented in our model: Germany and Belgium. The flexibility of allocating transmission capacity provided by the integrated market design only allows limited readjustment and therefore only little additional demand responsiveness. By contrast, the nodes in the Benelux countries have significantly reduced prices because they are located in the middle of a meshed network.

Note that three separate zones represent the Netherlands and, in the separated market

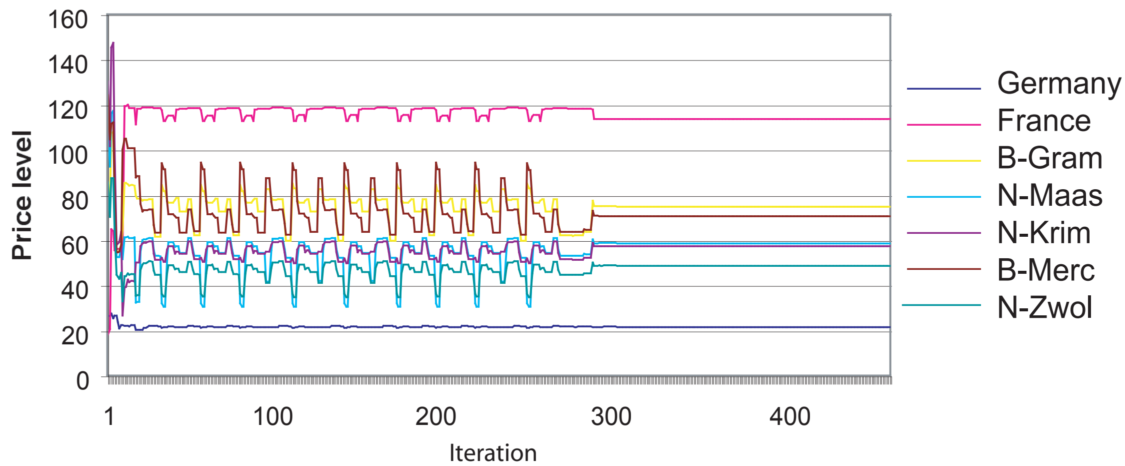

Figure 6: Convergence of the Diagonalisation

approach, generators compete in small markets. In the integrated market approach, the higher zonal resolution has less negative impact.

The output file with prices for all scenarios, quantities for the strategic and fringe generators can be found at:

http://www.econ.cam.ac.uk/electricity/rersearch/comparison/results_cam.xls

7 Conclusion

Two basic types of market design exist for the allocation of transmission capacity in meshed electricity networks. In the separated energy and transmission market, physical transmission rights are allocated in a co-ordinated auction. Trading electric energy between regions requires the ownership of transmission rights.

In the integrated energy and transmission market, energy is traded locally and a system operator schedules energy flows between regions. Financial transmission contracts allow for long-term hedging to facilitate trading between regions.

In competitive markets without uncertainty, both designs produce identical market outcomes. If generators act strategically, integration of energy and transmission markets effectively induces demand elasticity, since generators anticipate the impact of their bid on transmission. This should reduce the ability of strategic generators to exercise market power, and should therefore reduce prices. However, if companies own generation facilities at several nodes, integration also provides an incentive to increase the exercise of market power. The balance of these effects could not be determined analytically for realistic networks. We therefore implemented the two-market designs as an Equilibrium Problem with Equilibrium Constraints in the modelling package GAMS. We applied our models to data representing the Benelux situation. Comparing the resulting prices, we observed that the effect of importing net demand elasticity dominates and that prices were always lower in our test scenarios in the integrated market design. So far, we have only ensured that bidding strategies of generators are stationary points, but have not examined whether finite deviations are profitable.

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