# The Cost Efficiency of UK Debt Management: A Recursive Modelling Approach*

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**Abstract**

This paper presents an empirical analysis of the efficiency of the UK debt management authorities's (DMA) behaviour from a cost minimisation perspective over the period January 1985 to March 1995. During this period, the maturity structure of the government's bond portfolio was subject to frequent fine-tuning, aimed principally at lowering interest costs. We examine the efficiency of the DMA's behaviour from a cost minimisation perspective. Using a bi-variate version of the recursive modelling procedure applied to forecasting stock returns by Pesaran and Timmermann (1995, 2000), we show that bond returns are forecastable but the predictive power of macroeconomic variables is time dependent. We simulate the impact of adjusting the bond portfolio in response to our forecasts. The simulated average interest costs are lower than those resulting from the DMA's actual real-time behaviour. However, a substantial reduction in interest costs requires large monthly changes in the portfolio's maturity structure.

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## 1 Introduction

The maturity structure of the government's bond portfolio has not received the attention that it deserves. For the authorities assigned with the responsibility for debt management, however, the maturity structure is an incessant concern. Prior to the 1995 Debt Management Review (H.M. Treasury (1995)), the UK debt management authorities (DMA) frequently fine-tuned the maturity structure of the government's bond portfolio in an attempt to lower interest costs.[1]

In this paper, we examine the performance of the DMA over the period January 1985 to March 1995. We evaluate the cost efficiency of their behaviour, using a bi-variate version of the recursive modelling procedure applied to forecasting stock returns by Pesaran and Timmermann (1995, 2000). Our analysis is based on a "real time" simulation of the DMA's behaviour, using one-step ahead forecasts of the differences between holding period returns on bonds of different maturities. Hereafter, we refer to these differences in holding period returns as "return spreads". The forecasts exploit statistical patterns in the relationships between the return spreads and a variety of macroeconomic variables (factors). A base set of forecasting variables is established and, at the start of every time period (month), we search over these variables for the "best" system of forecasting equations. Model choice is based upon a pre-defined criterion (we examine a number of these); and selection utilises only information in the public domain. This procedure generates one-step ahead forecasts of the return spreads, which we use in our debt management simulations.

To our knowledge, this is the first application of the recursive modelling approach which examines the time-varying relationships between return spreads and business cycle indicators. Our approach contrasts with that adopted, for example, in Boothe and Reid's (1992) study of Canadian debt management, where they use recursive estimation of a given model rather than recursive modelling, and do not investigate the relevance of macroeconomic variables for forecasting of return spreads.

Applying the recursive modelling approach to forecasting return spreads each month, we find statistical and economic evidence of predictability using a common set of macroeconomic factors. For example, using the Akaike information criterion (AIC) for model selection, we predict correctly the maturity band with the minimum holding period return for 50% of the months in our evaluation period. We find that the predictive power of business cycle variables for return spreads is time dependent. Using the AIC, the variables selected most often include the change in the UK Treasury Bill rate, the yield spread between short and long US government debt, and the growth of Industrial Production.

[1] Pre-1997, the Bank of England was responsible for UK debt management, acting together with, and as an agent for H.M. Treasury. In 1997, the responsibility was transferred from the Bank of England to the UK Debt Management Office (an executive agency of H.M. Treasury). This shift in responsibility was part of the new Labour Government's attempt to separate responsibility for monetary and fiscal policies. For simplicity, we refer to the various government organisations involved as the "debt management authorities".

Using our recursively generated forecasts, we order bonds of different maturity according to their expected cost (holding period return) in each month. Based on this information, we simulate the impact of adjusting the portfolio's maturity structure (assuming that the DMA take bond prices as given). We find that the simulated average interest costs are lower than those resulting from the DMA's actual real-time behaviour. Unfortunately, a substantial reduction in interest costs requires large and frequent shifts in maturity structure. Constraining the monthly shifts in maturity structure to approximate the magnitude of the DMA's real-time behaviour, the simulated average interest costs are only slightly lower than those resulting from the DMA's real-time behaviour.

The rest of this paper is organised as follows. In the next Section, we review the literature and outline some theoretical considerations. In Section 3, we examine the UK debt management experience for evidence of the DMA's objectives and behaviour. The recursive modelling approach and its application to return spreads are discussed in Sections 4 and 5. Some general conclusions are drawn in the final section.

## 2 Literature review and theoretical considerations

In closed Ricardian macroeconomic models, the mix between taxes and debt is irrelevant (for example, see Wallace (1981), Liviatan (1983) and Chamley and Polemarchakis (1984)). In this type of model, the maturity structure of debt is also irrelevant. Even if interest costs differ by bond maturity, those costs do not constitute "net wealth". However, in reality, over 15% of UK bond sales are to foreign investors (H.M. Treasury (1995)). The following illustrative model demonstrates that if bonds are bought by non-UK investors, interest costs matter and cost minimisation can have beneficial macroeconomic effects.

A representative individual produces output from capital (and a fixed amount of labour) using the production function:

$$Y_t = f(K_t), \qquad f'(K_t) > 0, \quad f''(K_t) < 0, \tag{1}$$

where $Y_t$ and $K_t$ denote the output produced the capital stock at time $t$.

The DMA finances a stream of exogenous government expenditure, $\bar{G}_t$, by lump-sum taxes, $T_t$, and net issues of (zero-coupon, untaxed, one-period) bonds to domestic and foreign investors.[2,3] The DMA's budget constraint in each period is given by:

$$\bar{G}_t - T_t = P_t B_{t+1} - P_t B_t = P_t \Delta B_{t+1}, \tag{2}$$

where $B_{t+1}$ denotes the quantity of bonds maturing at time $t+1$ and $P_t$ denotes their price at time $t$, which

[2] The following analysis can be generalised to the non-zero coupon bond case.
[3] See Anderson et al (1996) for a discussion of bond taxation.

is assumed to be given.[4] It is further assumed that the debt stock, $P_t B_{t+1}$, is positive for all $t$ and that the DMA is solvent (namely, the transversality condition associated with (2) is satisfied). The holding period return (or interest rate) on holding a (one-period) bond from time $t$ to $t+1$, denoted $R_{t+1}$, is defined as:

$$R_{t+1} \;=\; (P_{t+1} - P_t)/P_t = \Delta P_{t+1}/P_t. \tag{3}$$

The individual's output is used for consumption, $C_t$, investment expenditures, (lump-sum) taxation, $T_t$, or domestic net purchases of bonds.[5] Hence, the dynamic evolution of the capital stock is given by:

$$\Delta K_{t+1} = K_{t+1} - K_t \;=\; f(K_t) - \delta K_t - C_t - T_t - (1-\gamma)P_t \Delta B_{t+1}, \qquad 0 < \gamma \leq 1, \qquad K_0 > 0, \tag{4}$$

where $\delta$ denotes the rate of depreciation and $\gamma$ denotes the exogenous proportion of bonds sold to foreign investors (at a fixed exchange rate).

The individual maximises (time-separable) utility over an infinite horizon:

$$U_t \;=\; \sum_{j=0}^{\infty} \beta^j u(C_{t+j}), \qquad u'(C_t) > 0, \quad u''(C_t) < 0, \tag{5}$$

where $\beta$ denotes the discount factor, $0 < \beta < 1$.[6]

Equation (4) can be rewritten using the budget identity, Equation (2), as:

$$C_t \;=\; f(K_t) - K_{t+1} + (1-\delta)K_t - \bar{G}_t + \gamma P_t \Delta B_{t+1}. \tag{6}$$

Note that if bonds are sold only to domestic residents, $\gamma = 0$, then the real economy is unaffected by interest costs. If bonds are sold to foreigners, $0 < \gamma \leq 1$, then interest costs matter.

The individual maximises utility, (5), by choosing $K_{t+1}$ and $B_{t+1}$, given Equation (6). The first order conditions are:

$$u'(C_t) \;=\; \beta u'(C_{t+1})\{f'(K_{t+1}) + (1-\delta)\}, \tag{7}$$

$$u'(C_t) \;=\; \beta u'(C_{t+1})(1 + R_{t+1}), \tag{8}$$

which yield

$$R_{t+1} = f'(K_{t+1}) - \delta.$$

Therefore:

$$\frac{\partial K_{t+1}}{\partial R_{t+1}} = \frac{1}{f''(K_{t+1})} < 0,$$

[4] Arguably, attempts by the DMA to implement large movements in maturity structure distort bond prices. We return to this issue in Sections 4 and 5.

[5] We assume that $C_t$ and $K_t$ are greater than zero for all $t$.

[6] The model can be generalised to allow utility to be a function of (exogenous) government expenditures, $\bar{G}_t$.

which establishes an inverse relationship between the level of capital stock and the rate of return on bonds, $R_{t+1}$. Furthermore, it is apparent from Equation (8) that the consumption path steepens as the interest rate increases, assuming that the interest rate differs from the subjective discount rate. Hence, interest cost minimisation has beneficial impacts on the capital stock and current (relative to future) consumption.

The model can be generalised to allow for many bonds, with different one-period interest rates, related to bond maturity. The DMA can stimulate current consumption and investment by altering the maturity structure of a given level of debt if bond returns vary with maturity in a manner that is predictable using macroeconomic indicators. Two prevailing theories of the term structure suggest that this condition holds in practice. The weak version of the expectations hypothesis suggests that predictability is caused by investors' risk preferences.[7] The segmented markets theory suggests that forecastability is the result of time dependent variations in transactions costs and limits on investor borrowing (Shleifer and Vishny (1997)).[8]

The above discussion makes clear that cost minimisation is a sensible objective for the DMA if foreign investors purchase bonds.[9] Of course, there is no reason to assume that only the level of interest costs matters—risk is important too. A number of researchers have noted that if markets are complete, debt maturity can be used to smooth consumption or income tax rates (for example, Barro (1979, 1995) and Bohn (1988, 1990)). Even if markets are incomplete, the DMA could smooth budget deficits (see Dale, Mongiardino and Quah (1997)).

The debt management literature has also focused on two other theoretical considerations. Aiyagri and McGrattan (1998) and Holmstrom and Tirole (1998) have examined the relationship between debt management and liquidity. Both firms and households face liquidity constraints (because of incomplete markets) and, in theory, debt management can be used to relax these constraints. Fischer (1983), Calvo and Guidotti (1992) and Favero, Missale and Primiceri (1999) have examined the implications of debt management for monetary credibility. Different types of debt can affect the monetary authorities' incentives to inflate.

Even if cost minimization is believed to be desirable, there is still the issue of its feasibility. In an "ideal" world where markets are efficient (at all times), expectations are formed rationally and there are no informational asymmetries, expected cost of debt servicing is pre-determined by the risk characteristics of different debt instruments, cost minimization can only be achieved at the expense of increased risk which will be difficult to justify from a risk-return trade off perspective. The best that can be done in these circumstances is to issue securities with the lowest expected risk premium, namely securities that have a

[7] Expected bond spreads are unpredictable only under risk neutrality (see, for example, Campbell, Lo and MacKinlay (1997, Ch.11)).

[8] Superior knowledge of the monetary feedback rule is also consistent with bond return forecastability. There is no evidence that the UK DMA utilised this approach.

[9] Tobin (1971) argued that interest cost minimisation can be beneficial if taxes are distortionary. Friedman (1960) criticised the cost minisation approach, arguing that the maturity structure is unimportant.

negative expected covariance with the return on labour and capital income. (See, for example, Missale (1999, Chs. 3 & 6).) However, whether markets are always efficient and expectations rational is an empirical issue; and an investigation that looks into the possibility of cost minimization for a given level of risk can therefore serve a useful purpose.

# 3 The UK debt management experience

## 3.1 Debt management objectives

Prior to the 1995 Debt Management Review, the UK's debt management policy objectives were set out in a lexicographic manner as:

> "(a) [debt management] must support and complement monetary policy in pursuit of the Government's objectives for inflation;
>
> (b) subject to (a), it should operate in a way which avoids distorting financial markets;
>
> (c) subject to (a) and (b), it should be conducted at least cost and risk."
>
> H.M. Treasury (1990) FSBR, p 23.

This description is strikingly vague. A subsequent H.M. Treasury publication, the Report of the Debt Management Review (1995), provided some insight into the DMA's interpretation of the objectives in practice. In particular, the Report suggested that the objective of market stability mattered only through its impacts on cost. It also suggested that debt management is little affected by monetary policy considerations.

The Report did not clarify the types of risks that concern the DMA. There was no mention of consumption (or income) tax rate smoothing, or deficit smoothing. Subsequent H.M. Treasury publications have been similarly silent on this issue. For example, the 1999-2000 Debt Management Report merely stressed the importance of "the Government's attitude to risks, both real and nominal" (p 4).

Neither the Report nor the FSBR mentioned the liquidity objective suggested by Aiyagri and McGrattan (1998) and Holmstrom and Tirole (1998).

Of the four candidate objectives for debt management implied by theoretical considerations—cost, risk, liquidity and monetary credibility—cost seems to have dominated the UK DMA's approach in practice:[10]

> "[M]y job is to minimise [interest] cost, by making sure that we manage our debt in an efficient and effective way"
>
> Angela Knight, Economic Secretary to the Treasury,
> H.M. Treasury (1995) p 1.

### 3.2 An Historical Overview

The (end-month) portfolio shares of short, medium and long maturity (non-index linked) bonds are shown in Figure 1 for the period from March 1980 to March 1995.[11,12] The maturity bands are consistent with the convention adopted by the Bank of England and H.M. Treasury: short $< 7$ years, medium $7 - 14$ years and long $\geq 15$ years.[13] (The Data Appendix contains further details of the construction of all variables used in this study, together with the data sources.) The sample means and standard deviations for the portfolio shares are summarised in Table 1.

Over the period from March 1980 to March 1995, the mean shares of short and medium maturity bonds are 44% and 38% of the government's portfolio, respectively. Long maturity bonds account for the remainder. It is clear from Figure 1 that these portfolio shares display a considerable degree of time variation. For example, prior to 1991, there is a clear movement away from long maturity bonds to medium and short maturity bonds. The share of the latter rises steadily from 39% in 1981 to 51% in 1991. Presumably, this change in portfolio maturity structure reflects the higher expected interest costs of long maturity bonds.

Figures 2a-2c show the (end-month) time variations in holding period returns for the three maturity bands; the mean and standard deviations of the holding returns are given in Table 1. In general, short bonds have the smallest mean returns and medium bonds have the highest mean returns; and the volatility of returns is increasing in bond maturity. Short and long bonds have the lowest return for 45% of the months in the evaluation period, January 1985 to March 1995.

[10] Comprehensive literature reviews are provided by Leong (1999) and Missale (1999).

[11] The market for index-linked bonds was extremely thin in the early part of our sample. Partly-paid and floating rate bonds were excluded, as were bonds with negligible amounts outstanding.

[12] The portfolio bond shares are calculated by market value.

[13] Unfortunately, the Bank of England does not provide data aggregated by alternative maturity bands or by duration.

# 4 Cost Efficiency of UK debt management

## 4.1 A Recursive Modelling Approach

In our analysis of the efficiency of UK debt management we are concerned with the cost-risk characteristics of the DMA's bond portfolio as it has evolved historically. For this purpose, we adopt the recursive modelling approach developed in Pesaran and Timmermann (1995, 2000) to evaluating the statistical and economic significance of the forecastability of return spreads. The analysis is based on the view that expected bond returns (and spreads) vary over the business cycle and can be (partly) forecast using business cycle indicators.[14] A review of the 1980-1994 issues of The Bank of England Quarterly Bulletin confirms that the DMA were aware of the relationships between bond returns and various business cycle indicators.[15] But, neither the Report of the Debt Management Review (H.M. Treasury (1995)) nor the relevant issues of The Bank of England Quarterly Bulletin indicate that the DMA used a particular model for these underlying relationships during our sample period. Although, it is widely acknowledged that, in principle, return spreads are predictable using macroeconomic indicators, to date there is no consensus regarding an optimal forecasting model. Given the possibility of technological change and switches in policy regimes, it is very unlikely that a given theoretical forecasting model could hold at all times. This suggests that a more pragmatic approach is needed.

We model the behaviour of the DMA, therefore, as searching recursively for a satisfactory empirical specification—among the many believed to be useful for forecasting bond returns—at each point in time. More specifically, we consider recursive models for:

$$X_{mt} = R_{mt} - R_{st} \qquad \text{and,} \qquad X_{lt} = R_{lt} - R_{st}, \qquad t = 1, 2, ..., T, \tag{9}$$

where $R_{st}$, $R_{mt}$ and $R_{lt}$ denote the holding period returns over the period $t-1$ to $t$ for short, medium and long maturity bonds, respectively.[16] Under recursive modelling, the vector of return spreads, $\mathbf{X}_t = (X_{mt}, X_{lt})'$, $t = t_0+1, t_0+2, ..., T$ is modelled at each point in time in terms of a base set of regressors contained in the publicly available information set, $\Omega_{t-1}$.[17] The recursive one-step ahead conditional forecasts of the two return spreads, $X_{mt}$ and $X_{lt}$, for $t = t_0+1, t_0+2, ..., T$ are used in the cost minimisation exercise. Recursive modelling extends recursive estimation, a frequently used technique, by allowing the prediction model to

[14] The idea that bond returns vary over the course of business cycle has a considerable tradition that can be traced back at least as far as Lutz (1940).

[15] The first reference to each macroeconomic variable used in our estimations is noted in the Data Appendix.

[16] The definition of bond spreads as relative to the holding period return for short bonds is arbitrary. Our results are robust to using either the medium or the long holding period return as the numeraire.

[17] The initial observations $t = 1, 2, ..., t_0$ are used to start the recursive process. The period $t = 1$ to $t = t_0$ is often referred to as the training period. To some extent, the choice of $t_0$ is arbitrary, but should exceed the total number of variables in the base set.

vary over time. At each point in time a forecasting model is chosen from a set of available models spanned by an *a priori* chosen base set, and this process is repeated for all the time periods, $t_0+1, t_0+2, ..., T$. Unlike recursive estimation, which simply updates the parameters of a given model, recursive modelling admits the possibility of model change and provides an *automated* search procedure. This automated approach reduces considerably—but does not eliminate—the "data mining" or "data snooping" problems highlighted recently by Sullivan, Timmermann, and White (1999).

We consider $2^k$ *a priori* linear models constructed from a base set of $k$ indicators.[18] A detailed review of the macroeconomic indicators discussed in the various issues of The Bank of England Quarterly Bulletin over the period 1980-1994 suggests the following macroeconomic variables:[19]

$$\begin{aligned}\mathbf{Z}_{1,t-1} &= \{\Delta TB_{t-1}, USSP_{t-1}, \Delta FT_{t-1}, \Delta ED_{t-1}, \Delta EM_{t-1}, \\ &\quad \Delta POIL_{t-1}, \Delta M0_{t-2}, \Delta RPIH_{t-2}, \Delta IP_{t-3}\},\end{aligned}$$

where $\Delta TB_{t-1}$ and $USSP_{t-1}$ denote the change in the UK Treasury Bill rate and the yield spread between long and short maturity US government bonds, respectively. The variables $\Delta FT_{t-1}$, $\Delta ED_{t-1}$, $\Delta EM_{t-1}$ and $\Delta POIL_{t-1}$ represent the growth rates of the Financial Times Index, the Sterling-US Dollar exchange rate, the Sterling-Deutschemark exchange rate and the spot price of oil, respectively. The variables $\Delta M0_{t-2}$, $\Delta RPIH_{t-2}$, and $\Delta IP_{t-3}$ represent the growth rates of the monetary base, the Retail Price Index (excluding housing) and Industrial Production, respectively.[20] For this last set of variables, we adopted the City analysts' convention of using 12-month rates of change. This helps minimise the impact of data revisions.[21] The macroeconomic indicators have different release dates into the public domain. Hence, each variable enters the model with a lag that reflects the availability of the most recent observation.[22]

In addition to these macroeconomic variables, the DMA could exploit functions of past bond prices when predicting return spreads. Hence, for the medium-short return spread, $X_{mt}$, we add the following variables to the base set:

$$\mathbf{Z}_{2,t-1} = \{X_{m,t-1}, \Delta R_{m,t-1}, \Delta R_{s,t-1}, (p_{m,t-1} - p_{s,t-1}), (p_{l,t-1} - p_{s,t-1}), \Delta GAP_{s,t-1}\} \tag{10}$$

where $\Delta R_{m,t-1}$ and $\Delta R_{s,t-1}$ denote the first difference of the holding period return on medium and short maturity bonds, respectively. The variables $(p_{m,t-1} - p_{s,t-1})$ and $(p_{l,t-1} - p_{l,t-1})$ are the differences between the (log) prices of medium and short, and long and short bonds, respectively. The variable $\Delta GAP_{s,t-1}$

[18] Non-linear terms can be included in the base set. But such an extension of the recursive modelling methodology increases the computational requirements significantly.

[19] The Data Appendix identifies the dates where each of the macroeconomic indicators were first mentioned.

[20] We experimented with including the housing component in the measure of retail price inflation. This made little difference to forecasting performance.

[21] The retail price index is generally not revised.

[22] The data sources are reported in the Data Appendix.

denotes the lagged first difference of the gap between the (log) price of short maturity bonds and the maximum (log) price of short bonds over the sample to date. That is, $GAP_{st} = Max\,(p_{s1}, p_{s2}, \ldots, p_{st}) - p_{st}$. This term captures non-linear effects similar to those in Beaudry and Koop (1993) and their extensions in Pesaran and Potter (1997).

For the long-short return spread, $X_{lt}$, we add the following variables to the base set, $\mathbf{Z}_{1,t-1}$:

$$\mathbf{Z}_{3,t-1} = \{X_{l,t-1}, \Delta R_{l,t-1}, \Delta R_{s,t-1}, (p_{m,t-1} - p_{s,t-1}), (p_{l,t-1} - p_{s,t-1}), \Delta GAP_{s,t-1}\} \tag{11}$$

where $\Delta R_{lt}$ is the first difference of the holding period return on long bonds.

Note that we use two (slightly) different base sets for predicting the two return spreads. We denote the base set for predicting $X_{mt}$ by

$$\begin{aligned} \mathbf{Z}_{m,t-1} &= \mathbf{Z}_{1,t-1} \cup \mathbf{Z}_{2,t-1} \\ &= \left\{ \begin{array}{c} X_{m,t-1}, \Delta R_{m,t-1}, \Delta R_{s,t-1}, \left(p_{m,t-1} - p_{s,t-1}\right), \left(p_{l,t-1} - p_{s,t-1}\right), \\ \Delta GAP_{s,t-1}, \Delta TB_{t-1}, USSP_{t-1}, \Delta FT_{t-1}, \Delta ED_{t-1}, \\ \Delta EM_{t-1}, \Delta POIL_{t-1}, \Delta M0_{t-2}, \Delta RPIH_{t-2}, \Delta IP_{t-3} \end{array} \right\} \end{aligned} \tag{12}$$

and the base set for predicting $X_{lt}$ by

$$\begin{aligned} \mathbf{Z}_{l,t-1} &= \mathbf{Z}_{1,t-1} \cup \mathbf{Z}_{3,t-1} \\ &= \left\{ \begin{array}{c} X_{l,t-1}, \Delta R_{l,t-1}, \Delta R_{s,t-1}, \left(p_{m,t-1} - p_{s,t-1}\right), \left(p_{l,t-1} - p_{s,t-1}\right), \\ \Delta GAP_{s,t-1}, \Delta TB_{t-1}, USSP_{t-1}, \Delta FT_{t-1}, \Delta ED_{t-1}, \\ \Delta EM_{t-1}, \Delta POIL_{t-1}, \Delta M0_{t-2}, \Delta RPIH_{t-2}, \Delta IP_{t-3} \end{array} \right\} \end{aligned} \tag{13}$$

In principle, the same base set could be used for predicting both return spreads. The slight differences between the two base sets is motivated by computational considerations.[23]

Our first step towards forecasting the two return spreads involves the identification of a preferred model among the many possible models that are implied by the different subsets of $\mathbf{Z}_{mt}$ and $\mathbf{Z}_{lt}$. Let $M_{imt}$ refer to model $i$ at time $t$ for predicting $X_{mt}$:

$$M_{imt} : X_{mt} = \alpha_{im} + \boldsymbol{\gamma}'_{im}\mathbf{z}_{im,t-1} + u_{imt}, \qquad \text{for } i = 1, 2, 3, ...., 2^{k_m}, \qquad t = 1, 2, ...., T \tag{14}$$

where $\mathbf{z}_{im,t-1}$ is a subset of the variables in $\mathbf{Z}_{m,t-1}$, $u_{imt}$ is a disturbance term, and $k_m = 15$. Each of the $2^{15} = 32,768$ models is identified by a $k_m \times 1$ vector of binary codes, $\mathbf{e}_{im}$, with a unity for an included variable and zero for an excluded variable. The unknown parameters, $\alpha_{im}$ and $\boldsymbol{\gamma}_{im}$, are estimated for each model by Ordinary Least Squares.

Similarly, models $M_{ilt}$, $i = 1, 2, 3, ...., 2^{k_l}$ are estimated for $X_{lt}$:

$$M_{ilt} : X_{lt} = \alpha_{il} + \boldsymbol{\gamma}'_{il}\mathbf{z}_{il,t-1} + u_{ilt}, \qquad \text{for } i = 1, 2, 3, ...., 2^{k_l}, \qquad t = 1, 2, ...., T, \tag{15}$$

[23] We experimented with adding the additional variables $X_{l,t-1}$ and $\Delta R_{l,t-1}$ ($X_{m,t-1}$ and $\Delta R_{m,t-1}$ ) to $\mathbf{Z}_{m,t-1}$ ($\mathbf{Z}_{l,t-1}$), but obtained similar results.

where $\mathbf{z}_{i,l,t-1} \in \mathbf{Z}_{l,t-1}$. Each model $M_{ilt}$ is identified by a $k_l \times 1$ vector of binary code, $\mathbf{e}_{il}$. Notice that in our application $k_l = k_m = 15$.

In each period, we estimate $2 \times 32,768$ models ($32,768$ models for $X_{mt}$ and $32,768$ models for $X_{lt}$). We choose the optimal models, $M_{*mt}$ and $M_{*lt}$ for each period $t = t_0 + 1,\ t_0 + 2, ..., T$, using one of a number of standard model selection criteria: Theil's (1958) $\bar{R}^2$, Akaike's (1973) information criterion (AIC), Schwarz's (1978) Bayesian criterion (SBC) and Hannan and Quinn's (1979) criterion (HQC). Each of these criteria offers the researcher a different trade-off between parsimony and fit. Throughout our sample, the two base sets of regressors, $\mathbf{Z}_{m,t-1}$ and $\mathbf{Z}_{l,t-1}$, are unchanged. However, the optimal subsets of these potential regressors, $\mathbf{z}_{*mt}$ and $\mathbf{z}_{*lt}$ are time dependent.

For each equation, estimation of our optimal model generates parameter estimates and an implied subset of forecasting variables, $\mathbf{z}_{*mt}$ and $\mathbf{z}_{*lt}$, respectively. These are used to generate point forecasts of $X_{m,t+1}$ and $X_{l,t+1}$, which are denoted $\hat{X}_{m,t+1}$ and $\hat{X}_{l,t+1}$, respectively. Given these recursive forecasts the following orderings over the three holding period returns can be derived:

$$\begin{aligned}
\hat{X}_{l,t+1} > \hat{X}_{m,t+1} \text{ and } \hat{X}_{m,t+1} > 0 \quad &\Longrightarrow \quad \hat{R}_{s,t+1} < \hat{R}_{m,t+1} < \hat{R}_{l,t+1}, \\
\hat{X}_{m,t+1} > \hat{X}_{l,t+1} \text{ and } \hat{X}_{l,t+1} > 0 \quad &\Longrightarrow \quad \hat{R}_{s,t+1} < \hat{R}_{l,t+1} < \hat{R}_{m,t+1}, \\
\hat{X}_{m,t+1} < \hat{X}_{l,t+1} \text{ and } \hat{X}_{l,t+1} < 0 \quad &\Longrightarrow \quad \hat{R}_{m,t+1} < \hat{R}_{l,t+1} < \hat{R}_{s,t+1}, \\
\hat{X}_{m,t+1} < 0 \text{ and } \hat{X}_{l,t+1} > 0 \quad &\Longrightarrow \quad \hat{R}_{m,t+1} < \hat{R}_{s,t+1} < \hat{R}_{l,t+1}, \\
\hat{X}_{l,t+1} < \hat{X}_{m,t+1} \text{ and } \hat{X}_{m,t+1} < 0 \quad &\Longrightarrow \quad \hat{R}_{l,t+1} < \hat{R}_{m,t+1} < \hat{R}_{s,t+1}, \\
\text{and } \hat{X}_{l,t+1} < 0 \text{ and } \hat{X}_{m,t+1} > 0 \quad &\Longrightarrow \quad \hat{R}_{l,t+1} < \hat{R}_{s,t+1} < \hat{R}_{m,t+1}.
\end{aligned}$$

### 4.2 Portfolio simulations

We use our orderings of maturity bands by expected cost to simulate the impact of adjusting the DMA's bond portfolio in each period (assuming that the DMA take bond prices as given). Clearly, if cost minimisation is the only debt management objective and transaction cost are negligible, then the DMA would shift the entire portfolio into the predicted lowest cost maturity band. That is, the DMA would buy back the part of the debt portfolio which is in the predicted high cost bands and then issue the equivalent amount into the predicted lowest cost maturity band.[24] We refer to this process of repurchasing and selling as a "debt switch".

In Section 3, however, we argued that although the DMA focuses primarily on cost minimisation, other objectives such as orderly financial markets and risk are important. It is clear that large shifts in maturity structure conflict with these other debt management objectives and may distort bond prices (which we

[24] We assume such transactions takes place at the start of each period at the previous period's close of business prices.

assume as given in our analysis). Furthermore, the transactions costs involved in large debt switches could be large, but are extremely difficult to measure.

With this in mind, we constrain the maximum debt switch as follows. We define a variable $\alpha$ such that $0 \leq \alpha \leq 1$. The maximum debt switch allowed (at the start of) each period is defined as $\alpha$ times the market value of the bond portfolio (at the end) of the previous period. We assume that bonds are always repurchased from the highest expected cost maturity band. But if the maximum debt switch exceeds the market value of the bonds in the highest expected cost maturity band, then additional bonds are repurchased from the next highest expected cost maturity band. (And, trivially, if the sum of the market values in the expected highest and second highest cost bands is exceeded, then additional bonds are repurchased from the expected lowest cost maturity band.)

Two features of this approach are worthy of further reflection. First, bond repurchases are sequential, with the highest expected cost being repurchased first. An alternative approach would be to conduct such repurchases non-sequentially. Second, bond sales are always into the lowest expected cost maturity band, rather than into all maturities. Both of these features are consistent with the cost minimisation objective but may conflict with the other debt management objectives outlined above.

Note that in addition to any debt switch, the DMA must sell (buy) sufficient bonds to just satisfy the budget identity, Equation (2)—so that the budget deficit (surplus) is financed. We assume that the deficit (surplus) is financed by selling (buying) into the lowest (highest) expected cost maturity band.

We simulate portfolios for a number of values of $\alpha$, over the evaluation period. We calculate the average interest costs from our simulated portfolios in each month (weighted by share of market value) and compare them with those resulting from the DMA's real-time behaviour.

# 5 Results

Recall that our forecasts are the result of a recursive, two-step procedure for each time period. In the first stage, we select an optimal model (for each selection criterion). In the second stage, we use that model to generate one-step ahead forecasts of return spreads. Using a training period from March 1980 to December 1984, we generate monthly forecasts based on recursively selected models for the evaluation period, January 1985 to March 1995 (see (14) and (15)).

## 5.1 Forecast performance

Figures 3a-3b show the recursive standard errors from the models selected recursively using the four model selection criteria over our evaluation period. The estimated standard errors of the recursively selected equations show a slight downward trend over the period, except in two periods: the first occurs in mid-1986—coinciding with the Big Bang (see Bank of England Quarterly Bulletin (March 1986, p 71-73)), but prior to the equity market crash of October 1987. The second occurs around 1990, reflecting perhaps the increased uncertainties surrounding the UK's position in the ERM. The standard errors for the medium-short return spreads, $X_{mt}$, are systematically lower than those of the long-short return spreads, $X_{lt}$. This result is robust to the choice of the model selection criteria and indicates that $X_{lt}$ is more difficult to forecast than $X_{mt}$.

The forecasts for the two return spreads for each of the four model selection criteria are shown in Figures 4a-4h. The one-step ahead directional information embodied in these forecasts is reflected in the ability of the optimal models to identify the lowest and highest cost maturity bands. This information is summarised in Table 2. Using the HQC for model selection, the lowest cost maturity band is selected correctly for 54% of the months in the evaluation period. The corresponding figures for the $\bar{R}^2$ criterion, the AIC, and the SBC are 50%, 50% and 46%, respectively. The highest cost maturity band is selected correctly in $40\% - 45\%$ of the sample.

We interpret these statistics as a preliminary indication that our forecasts contain useful information for analysing market timing. To formally test this hypothesis, we use Pesaran and Timmermann's (1994) generalised Henriksson-Merton (1981) non-parametric test of market timing. Recall that the orderings of maturity bands are based on our forecasts of return spreads. With three maturity bands, there are six possible orderings set out at the end of section 4.1. We refer to these in Table 3 as "predicted orderings". There are six corresponding orderings of the actual data; we refer to these as "realised orderings". For each selection criterion, there are 36 possible pairs of predicted and realised orderings. Each pair is represented by a cell in the contingency tables given in Tables 3a and 3b. The numbers in each cell give the frequencies and the figures in brackets show the proportion of the sample in which the designated event occurred in our evaluation period, January 1985 to March 1995.

The null hypothesis of the generlised Henriksson-Merton test advanced in Pesaran and Timmermann (1994) in the case of the $6 \times 6$ contingency table can be written as:

$$H_0: \quad \sum_{c=1}^{6}(\pi_{cc} - \pi_{c0}\pi_{0c}) = 0 \tag{16}$$

where $\pi_{cc}$ denotes the joint probability that the ordering in category $c$ is correctly predicted and $\pi_{c0}\pi_{0c}$ is the probability of this joint event on the assumption that processes generating the predicted and realized outcomes are independently distributed. $\pi_{c0}$ is the marginal probability of the realized ordering in category $c$, and $\pi_{0c}$ is the associated marginal probability of the predicted ordering. The null hypothesis (16) focuses on the sum of the diagonal elements of the contingency table and is less restrictive than the null hypothesis that the predicted and the realized orderings are independently distributed. It is implied by the independence hypothesis but does not require it. As it is argued in Pesaran and Timmermann (1994) for market timing purposes the off-diagonal elements of the contingency table is of secondary importance. In the present application, it is also worth noting the cases where the holding returns rise with the bond maturity ($R_{lt} > R_{mt} > R_{st}$) and the reverse case where the holding period returns fall with the bond maturity ($R_{st} > R_{mt} > R_{lt}$) account for 77% of the realised orderings. The contributions of the other orderings to the market timing test statistic is negligible.

Using the cell frequencies in Table 3, we calculated Pesaran and Timmermann's (1994) test statistics for different model selection criteria. These are displayed in the last column of Table 2, and are asymptotically distributed as $N(0, 1)$. Using a one-tailed test, the null hypothesis is rejected at the 5% significance level, for all selection criteria.

Having established that the forecasts are useful for market timing, we consider the inclusion frequency of the various regressors from the base set and the sign of their respective coefficients. Recall that, in theory, the recursively selected models need not include any of the variables in the base sets $\mathbf{Z}_{m,t-1}$ and $\mathbf{Z}_{l,t-1}$ (defined by (12) and (13)). In practice, we find that a number of variables are always included in the forecasting models, regardless of the selection criterion used. Figures 5a-5j show the estimated coefficients of selected regressors for the evaluation period January 1985 to March 1995, using the AIC and the SBC for model selection. (The results based on the $\bar{R}^2$ criterion are very similar to the AIC case; the results using the HQC lie between the AIC and SBC cases.) Figures 6a-6b show the number of regressors included over the evaluation period again using the AIC and the SBC. Both model selection procedures indicate fewer regressors than the 16 in the base set (15 plus the constant); with the SBC indicating fewer regressors than the AIC in almost every period. The maximum number of regressors included is 9 for the $X_{mt}$ models and 10 for the $X_{lt}$ models; but for most of the period, AIC selects 6-7 regressors and SBC selects 3-5 regressors which reflects the larger penalty imposed by the SBC criterion on the inclusion of the additional regressors.

The variables $X_{m,t-1}, X_{l,t-1}, \Delta R_{s,t-1}$, $\Delta FT_{t-1}$ and $\Delta R_{m,t-1}$ are rarely selected. To save space in Figure 5 we show the coefficients of the 5 most commonly included regressors. The coefficients of the relative price variables, $(p_{m,t-1} - p_{s,t-1})$ and $(p_{l,t-1} - p_{s,t-1})$, when selected, are negative throughout the period, suggesting an error correcting adjustment between return spreads and the associated price differentials.[25] The estimated coefficients of the non-linear variable, $\Delta GAP_{s,t-1}$, (when selected) are also systematically negative, for both equations and over the whole evaluation period. This indicates a faster downward adjustment to the return spreads when the change in the price of the short maturity bond falls below its previous peak. Changes in the UK Treasury Bill rate, $\Delta TB_{t-1}$, have a systematically positive effect on return spreads. The

[25] Notice that when a variable is not selected its coefficient is set to zero.

sign of these coefficients is consistent with market practitioners' perceptions that increases in the Treasury Bill rate increase the risk premia on holding long and medium bonds over short bonds. The yield spread between long and short maturity US government debt, $USSP_{t-1}$, also has an unambiguously positive effect on return spreads. This suggests a positive correlation between changes in risk premia in the US and the UK bond markets (with a lag). Finally, Figures 5i and 5j show a systematic positive effect of changes in industrial production, $\Delta IP_{t-3}$, on UK return spreads. This result is consistent with the view that output expansions are associated with increasing uncertainty over short-term interest rates and, hence, increasing risk premia. Overall, the estimated coefficients have remarkably consistent signs across the two return spread equations and over time.

### 5.2 Simulated portfolios

Given our recursively generated forecasts and the associated maturity band orderings, we carry out a number of debt management simulation exercises over the evaluation period, January 1985 to March 1995. We alter the maturity composition of the DMA's bond portfolio to minimise expected interest costs, subject to the maximum debt switch constraint—parameterised by $\alpha$.[26] Recall that this restriction is motivated by the influence of non-cost related debt management objectives (such as risk and market stability).[27] For each value of $\alpha$, we calculate the average interest costs and their variability and compare them with those resulting from the DMA's actual real-time behaviour.

It is apparent from Table 4 that the DMA's real-time behavior has been inefficient from a purely cost minimisation perspective. For $\alpha = 0.1$—a maximum monthly debt switch of 10% of the total bond portfolio market value—the average monthly interest costs resulting from the models selected by the $\bar{R}^2$ criterion, the AIC, the HQC, and the SBC are lower than those resulting from the DMA's real-time behavior. At annualised rates, these average interest rates range from 9.1% (HQC) to 9.3% (SBC)—approximately 1.8 percentage points less than the DMA's real-time annualised rate of 11.0%. This implies a cost saving of around *£1.8* billion per year at 1990 prices (evaluated at the bond portfolio market value of *£99* billion in February 1990). The amount of switching involved is substantial, however: a 10% switch is approximately equal to *£10* billion per month at 1990 prices.

A monthly debt switch of 1.0%, $\alpha = 0.01$, seems more realistic: a *£1.1* billion repurchase (or "reverse auction") per month at 1990 prices.[28] In this case, the HQC and the SBC generate the lowest interest

[26] For further details see Section 4.2.
[27] For further details see Section 3.1.
[28] Pre-1986, the DMA's actual real-time debt repurchases took place primarily through the calling of double-dated bonds, conversions, and taps. Since 1996, the DMA have used "switch auctions" which allow bond holders to bid to switch an existing specified bond into another specified bond (usually a benchmark stock). In 1999, the size of these auctions was around *£1* billion at current prices—roughly the same size as current conventional auctions. The size of conventional auctions in 1990 was

costs. The annualised average monthly interest rate from adopting these criteria for model selection is approximately 10.6% per month. The cost saving over the DMA's real-time behaviour is modest: an annualised rate of about 0.4 percentage points or *£(1990)0.4* billion per year. In the 0% switching case, the annualized saving falls to 0.1 − 0.2 percentage points or *£(1990)0.1 − 0.2* billion per year.

Figure 7 shows the recursive average interest costs for each period, for $\alpha = 1$ and $\alpha = 0.01$. For illustrative purposes, the figures show the simulated interest costs resulting from the AIC and SBC model selection criteria, together with those resulting from the DMA's real-time behaviour. In general, interest cost savings over the DMA's behavior are possible; but, the savings are small for realistic switching constraints.

Figure 8 shows the recursive standard deviations of the average monthly interest costs resulting from the simulated portfolios and the DMA's real-time behaviour. Interestingly, for the value of $\alpha = 1$, the interest rates generated by the simulated portfolios are more variable than those implied by the DMA's real-time behaviour. However, in the case where $\alpha = 0.01$, the *AIC* model delivers a modest fall in both interest rate variability and average interest rate.

Figure 9 shows the portfolio bond shares resulting from the various model selection criteria for $\alpha = 0.01$. The simulated portfolio's are very different from the DMA's real-time portfolio shown in Figure 1. In general, the simulated portfolios include a much greater proportion of short bonds. Towards the end of the evaluation period, the shares of medium and long bonds are close to zero.

The simulated bond portfolio shares highlight an interesting feature of our results. Most debt managers would regard a portfolio that comprises mostly short bonds as "risky"; but in the SBC case, the standard deviation of average interest costs over the period from January 1985 to March 1995 is smaller than for the DMA's real-time portfolio (see Figure 8b). Of course, the puzzle can be reconciled if one considers measures of risk other than the standard errors. However, to date the UK debt management objectives provide little guidance on what these measures might be (see Section 3.1).

## 6 Conclusions

We have provided an empirical analysis of the efficiency of the DMA's behaviour from a cost minimisation perspective. Adopting a recursive modelling approach, we generated one-step ahead forecasts of return spreads. We used the resulting implied orderings of the bond maturity bands to carry out debt management simulations. These involved adjusting the maturity structure of the simulated bond portfolios in response to expected costs. We found that the average interest costs resulting from the simulated portfolios were lower than those resulting from the DMA's actual real-time behaviour. However, a substantial reduction in

around *£1.2* billion at 1990 prices.

interest costs required large monthly changes in the portfolio's maturity structure which may have important undesirable risk and liquidity implications. But the recursive results show that cost savings would have been possible even if we constrain the changes in the public debt portfolio not to exceed 1% of total debt outstanding. Given the size of the debt even small reductions in the average interest rates on public debt can have large financial implications.

The recursive modelling strategy has allowed us to provide an empirical evaluation of the feasibility of a cost minimization approach to debt management. We have attempted to avoid the data snooping problems that are highlighted in the finance literature, and inevitably afflict counter factual exercises that are routinely carried out in applied econometrics. Our research, however, does not address adequately the risk management aspects of the debt management. This is partly due to the fact that at present the risks which concern the UK DMA, and their attitude towards them, are not well understood. We have attempted to partially address such concerns by considering explicit constraints on the extent to which public debt portfolio can be altered. This is clearly *ad hoc* and a more satisfactory approach to risk-return analysis of public debt is required.

**Data Appendix**

We used data from four sources. The Bank of England's Monetary Analysis Division 2 (BOE), the Office for National Statistics databank (ONS), Citibase and the Federal Reserve Economic Database (FRED). Researchers interested in acquiring bond data should contact the Monetary Analysis Division 2, Bank of England, Threadneedle Street, London, EC2R 8AH. The ONS and FRED databanks are available online at http://www.data-archive.ac.uk/online/ons/ and http://www.stls.frb.org/fred/index.html.

| | | |
|---|---|---|
| $R_s$ | : | Monthly Holding Period Return, Short Bonds. Source BOE. |
| $R_m$ | : | Monthly Holding Period Return, Medium Bonds. Source BOE. |
| $R_l$ | : | Monthly Holding Period Return, Long Bonds. Source BOE. |
| $X_m$ | : | Medium to Short Return Spread, $R_m - R_s$. |
| $X_l$ | : | Long to Short Return Spread, $R_l - R_s$. |
| $p_s$ | : | ln (Price), Short Bonds, End Month, Clean. Source BOE. |
| $p_m$ | : | ln (Price), Medium Bond, End Month, Clean. Source BOE. |
| $p_l$ | : | ln (Price), Long Bond, End Month, Clean. Source BOE. |
| $B_s$ | : | Quantity, Short Bonds, End Month. Source BOE. |
| $B_m$ | : | Quantity, Medium Bonds, End Month. Source BOE. |
| $B_l$ | : | Quantity, Long Bonds, End Month. Source BOE. |
| $\Delta TB$ | : | Change in 3 Month Treasury Bill rate. Source: ONS, AJNC. |
| $USSP$ | : | Yield Spread between U.S. Long-term Government Bonds and 3 Month Treasury Bills. Calculated as $(1 + LTGOVTBD/100)^{1/12} - (1 + TB3MA/100)^{1/12}$. Source: FRED. |
| $\Delta FT$ | : | Percentage Change in the Financial Times All Share Index. Calculated as $\ln(FT_t/FT_{t-1})$. Source: ONS, AJMA. |
| $\Delta ED$ | : | Percentage Change in the Sterling-Dollar Exchange Rate. Calculated as $\ln(ED_t/ED_{t-1})$. Source: ONS, AJFA |
| $\Delta EM$ | : | Percentage Change in the Sterling-Mark Exchange Rate. Calculated as $\ln(EM_t/EM_{t-1})$. Source: ONS, AJFH. |
| $\Delta POIL$ | : | Percentage Change in the Spot Price of Oil. Calculated as $\ln(POIL_t/POIL_{t-1})$. Source: Citibase, MEEFPP. |
| $\Delta M0$ | : | Year on Year Percentage Change in the Monetary Base. Calculated as $\ln(M0_t/M0_{t-12})$. Source: ONS, AVAD. |
| $\Delta RPIH$ | : | Year on Year Percentage Change in the Retail Price Index (excluding housing). Calculated as $CZBI/100$. Source: ONS. |
| $\Delta IP$ | : | Year on Year Percentage Change in the Industrial Production Index. Calculated as $\ln(IP_t/IP_{t-12})$. Source: ONS, DVZI. |

Notes:

1) There are three other variables in the base sets. These are $(p_m - p_s)$, $(p_l - p_s)$ and $\Delta GAP_s$. The first is the difference between the natural logs of the prices of medium maturity and short maturity bonds. The second is the difference between the natural logs of the prices of long maturity and short maturity bonds.

The third is constructed as follows:

$$\Delta GAP_{st} = GAP_{st} - GAP_{s,t-1}$$

where

$$GAP_{st} = \max(p_{s1}, p_{s2}, \ldots, p_{st}) - p_{st}$$

and $p_s$ is the natural log of the price of short maturity bonds.

2) Bond data

The holding period return data are available from the Bank of England's Monetary Analysis Division 2 on a daily basis. The individual bond data are aggregated into maturity bands: short ($<7$ years), medium (between 7 and $<15$ years) and long ( $\geq 15$ years). Each individual bond is equally weighted. The monthly data used in this study are the sum of underlying daily data.

The daily holding period return on an individual bond is defined as the first difference of (the log of) the closing price, adjusted to reflect "ex-dividend" effects using the 3-month London Interbank Offered Rate ($LIB3$). Namely

$$R_{dt} = \ln\left(\frac{P_{dt} + A_{dt}}{P_{d,t-1}}\right),$$

where $P_{dt}$ denotes the daily close of business dirty price at time $t$ and $A_{dt}$ denotes the adjustment for ex-dividend periods.

3) Macroeconomic indicators

Our macroeconomic indicators were selected based on a review of The Bank of England Quarterly Bulletin from 1980 to 1994. In the listed issues, the following indicators were mentioned in the *Financial Review* section of the Bulletins:

| Measure | Indicator mentioned | Date of the Bulletin |
|---|---|---|
| $TB$ | Treasury Bill sales* | March 1980 |
| $USSP$ | US rates | June 1980 |
| $FT$ | Equity prices* | March 1980 |
| $ED$ | Sterling-US dollar exchange rate | September 1980 |
| $EM$ | Sterling- Mark exchange rates | September 1980 |
| $POIL$ | Oil prices | March 1985 |
| $M0$ | Monetary base | March 1980 |
| $RPIH$ | Inflation | June 1980 |
| $IP$ | Industrial production* | September 1980. |

Indicators denoted by $*$ were mentioned in the Bulletins in connection with the discussions of the money or other markets, rather than the bond markets. References to variables after September 1981 are from the "Gilt-edged" subsection of the "Operation of monetary policy".

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**Table 1: Means and Standard Deviations by Maturity Band, 1980M3–1995M3**

| Period | Holding Period Returns (%) | | | Bond Portfolio Shares (%) | | |
|---|---|---|---|---|---|---|
| | Short | Medium | Long | Short | Medium | Long |
| 1980M3 – 1995M3 | 11.7 | 13.5 | 13.2 | 43.6 | 37.5 | 19.0 |
| | (1.3) | (2.6) | (3.1) | (3.9) | (5.2) | (7.4) |
| 1980M3 – 1984M12 | 14.9 | 16.9 | 17.7 | 40.2 | 32.3 | 27.5 |
| | (1.5) | (3.0) | (3.6) | (1.9) | (5.2) | (6.6) |
| 1985M1 – 1995M3 | 10.3 | 11.9 | 11.1 | 45.2 | 39.9 | 15.0 |
| | (1.2) | (2.4) | (2.9) | (3.6) | (3.0) | (3.1) |
| Selected Years | | | | | | |
| 1980M3 – 1980M12 | 18.4 | 20.1 | 21.8 | 39.0 | 24.7 | 36.3 |
| | (1.0) | (2.6) | (3.4) | (1.7) | (2.0) | (0.9) |
| 1981M1 – 1981M12 | 11.4 | 2.9 | 0.7 | 39.4 | 29.0 | 31.6 |
| | (1.8) | (2.8) | (3.4) | (0.7) | (1.6) | (1.8) |
| 1982M1 – 1982M12 | 30.0 | 44.7 | 49.6 | 38.8 | 32.9 | 28.3 |
| | (1.6) | (3.6) | (4.3) | (0.9) | (1.2) | (0.8) |
| 1983M1 – 1983M12 | 10.2 | 13.1 | 15.5 | 40.4 | 33.6 | 26.0 |
| | (1.0) | (2.5) | (3.3) | (1.1) | (1.3) | (2.0) |
| 1984M1 – 1984M12 | 9.1 | 8.2 | 7.0 | 43.0 | 40.0 | 17.0 |
| | (1.5) | (2.9) | (3.0) | (0.6) | (1.6) | (1.6) |
| 1985M1 – 1985M12 | 11.1 | 13.1 | 11.2 | 43.9 | 40.1 | 16.0 |
| | (0.6) | (1.1) | (1.4) | (0.6) | (1.1) | (0.9) |
| 1986M1 – 1986M12 | 10.7 | 17.1 | 11.4 | 41.7 | 41.5 | 16.8 |
| | (1.8) | (4.2) | (4.8) | (1.4) | (1.4) | (0.4) |
| 1987M1 – 1987M12 | 13.4 | 15.7 | 16.3 | 40.1 | 41.6 | 18.3 |
| | (1.3) | (2.3) | (2.8) | (1.0) | (0.7) | (0.7) |
| 1988M1 – 1988M12 | 5.0 | 7.0 | 9.5 | 42.8 | 42.2 | 15.0 |
| | (1.1) | (1.6) | (1.8) | (0.5) | (0.8) | (0.7) |
| 1989M1 – 1989M12 | 8.7 | 7.9 | 5.5 | 43.2 | 44.4 | 12.4 |
| | (1.0) | (1.8) | (2.0) | (0.9) | (1.1) | (0.8) |
| 1990M1 – 1990M12 | 12.7 | 7.8 | 4.4 | 48.3 | 41.8 | 9.9 |
| | (1.5) | (3.4) | (3.9) | (1.7) | (1.2) | (1.0) |
| 1991M1 – 1991M12 | 13.3 | 18.0 | 18.8 | 50.9 | 39.0 | 10.1 |
| | (0.6) | (1.6) | (1.9) | (1.2) | (1.3) | (0.7) |
| 1992M1 – 1992M12 | 16.7 | 19.7 | 17.7 | 49.3 | 35.8 | 14.9 |
| | (1.7) | (2.8) | (3.0) | (1.8) | (1.2) | (1.9) |
| 1993M1 – 1993M12 | 11.7 | 23.6 | 34.4 | 45.0 | 36.8 | 18.2 |
| | (0.7) | (1.8) | (2.5) | (2.1) | (2.3) | (1.4) |
| 1994M1 – 1994M12 | 0.1 | -8.1 | -12.5 | 46.1 | 36.7 | 17.2 |
| | (0.8) | (2.2) | (2.9) | (2.2) | (1.6) | (1.0) |

Debt shares are expressed as averages of monthly percentages. Holding period returns are calculated using annualised averages of monthly percentages. Standard deviations refer to monthly figures and are in parentheses.

**Table 2: Market Timing Statistics**

| Selection Criterion | Lowest Cost Bond Selected Correctly (%) | Highest Cost Bond Selected Correctly (%) | Generalised H-M Statistic |
|---|---|---|---|
| $\overline{R}^2$ | 50.4 | 43.1 | 2.27 |
| AIC | 50.4 | 41.5 | 1.82 |
| SBC | 45.5 | 41.5 | 2.03 |
| HQC | 54.4 | 41.5 | 1.74 |

The lowest (highest) cost bond selected correctly refers to the proportion of the evaluation period in which the recursively selected models correctly identify the lowest (highest) cost maturity band. The Generalised Henriksson-Merton (H-M) statistic tests the null hypothesis of "no market timing skills" in the 6 by 6 contingency tables in Tables 3a and 3b for different model selection criteria. The test statistic is distributed as a standard normal; the 95% critical value (one-sided) is 1.64.

**Table 3a: Contingency Tables for the Realised and Predicted Orderings of the Bond Returns According to Different Model Selection Criteria**

| Models Selected Recursively by $\bar{R}^2$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| Realised Ordering | Predicted Ordering | | | | | | |
| | S<M<L | S<L<M | L<S<M | M<S<L | M<L<S | L<M<S | Total |
| S<M<L | 27 (0.22) | 6 (0.05) | 1 (0.01) | 1 (0.01) | 2 (0.02) | 7 (0.06) | 44 (0.36) |
| S<L<M | 5 (0.04) | 1 (0.01) | 2 (0.02) | 1 (0.01) | 0 (0.00) | 2 (0.02) | 11 (0.09) |
| L<S<M | 1 (0.01) | 1 (0.01) | 0 (0.00) | 0 (0.00) | 0 (0.00) | 3 (0.02) | 5 (0.04) |
| M<S<L | 1 (0.01) | 1 (0.01) | 3 (0.02) | 0 (0.00) | 0 (0.00) | 0 (0.00) | 5 (0.04) |
| M<L<S | 2 (0.02) | 1 (0.01) | 3 (0.02) | 1 (0.01) | 0 (0.00) | 1 (0.01) | 8 (0.07) |
| L<M<S | 20 (0.16) | 5 (0.04) | 3 (0.02) | 3 (0.02) | 3 (0.02) | 16 (0.13) | 50 (0.41) |
| Total | 56 (0.46) | 15 (0.12) | 12 (0.10) | 6 (0.05) | 5 (0.04) | 29 (0.24) | 123 (1.00) |

| Models Selected Recursively by AIC | | | | | | | |
|---|---|---|---|---|---|---|---|
| Realised Ordering | Predicted Ordering | | | | | | |
| | S<M<L | S<L<M | L<S<M | M<S<L | M<L<S | L<M<S | Total |
| S<M<L | 20 (0.16) | 12 (0.10) | 4 (0.03) | 3 (0.02) | 2 (0.02) | 3 (0.02) | 44 (0.36) |
| S<L<M | 5 (0.04) | 1 (0.01) | 3 (0.02) | 1 (0.01) | 0 (0.00) | 1 (0.01) | 11 (0.09) |
| L<S<M | 0 (0.00) | 2 (0.02) | 0 (0.00) | 0 (0.00) | 1 (0.01) | 2 (0.02) | 5 (0.04) |
| M<S<L | 2 (0.02) | 2 (0.02) | 1 (0.01) | 0 (0.00) | 0 (0.00) | 0 (0.00) | 5 (0.04) |
| M<L<S | 3 (0.02) | 2 (0.02) | 1 (0.01) | 0 (0.00) | 0 (0.00) | 2 (0.02) | 8 (0.07) |
| L<M<S | 17 (0.14) | 8 (0.07) | 5 (0.04) | 2 (0.02) | 1 (0.01) | 17 (0.14) | 50 (0.41) |
| Total | 47 (0.38) | 27 (0.22) | 14 (0.11) | 6 (0.05) | 4 (0.03) | 25 (0.20) | 123 (1.00) |

The returns on long, medium and short dated maturity bonds are denoted by S, M, and L respectively. Proportions shown in parentheses.

**Table 3b: Contingency Tables for the Realised and Predicted Orderings of the Bond Returns According to Different Model Selection Criteria**

| Models Selected Recursively by SBC | | | | | | | |
|---|---|---|---|---|---|---|---|
| Realised Ordering | Predicted Ordering | | | | | | |
| | S<M<L | S<L<M | L<S<M | M<S<L | M<L<S | L<M<S | Total |
| S<M<L | 26 | 8 | 2 | 4 | 1 | 3 | 44 |
| | (0.21) | (0.07) | (0.02) | (0.03) | (0.01) | (0.02) | (0.36) |
| S<L<M | 3 | 2 | 2 | 3 | 0 | 1 | 11 |
| | (0.02) | (0.02) | (0.02) | (0.02) | (0.00) | (0.01) | (0.09) |
| L<S<M | 3 | 0 | 1 | 1 | 0 | 0 | 5 |
| | (0.02) | (0.00) | (0.01) | (0.01) | (0.00) | (0.00) | (0.04) |
| M<S<L | 3 | 0 | 1 | 1 | 0 | 0 | 5 |
| | (0.02) | (0.00) | (0.01) | (0.01) | (0.00) | (0.00) | (0.04) |
| M<L<S | 2 | 3 | 1 | 0 | 0 | 2 | 8 |
| | (0.02) | (0.02) | (0.01) | (0.00) | (0.00) | (0.02) | (0.07) |
| L<M<S | 21 | 9 | 7 | 3 | 2 | 8 | 50 |
| | (0.17) | (0.07) | (0.06) | (0.02) | (0.02) | (0.07) | (0.41) |
| Total | 58 | 22 | 14 | 12 | 3 | 14 | 123 |
| | (0.47) | (0.18) | (0.11) | (0.10) | (0.02) | (0.11) | (1.00) |

| Models Selected Recursively by HQC | | | | | | | |
|---|---|---|---|---|---|---|---|
| Realised Ordering | Predicted Ordering | | | | | | |
| | S<M<L | S<L<M | L<S<M | M<S<L | M<L<S | L<M<S | Total |
| S<M<L | 25 | 10 | 3 | 1 | 0 | 5 | 44 |
| | (0.20) | (0.08) | (0.02) | (0.01) | (0.00) | (0.04) | (0.36) |
| S<L<M | 8 | 0 | 3 | 0 | 0 | 0 | 11 |
| | (0.07) | (0.00) | (0.02) | (0.00) | (0.00) | (0.00) | (0.09) |
| L<S<M | 1 | 1 | 2 | 0 | 0 | 1 | 5 |
| | (0.01) | (0.01) | (0.02) | (0.00) | (0.00) | (0.01) | (0.04) |
| M<S<L | 2 | 3 | 0 | 0 | 0 | 0 | 5 |
| | (0.02) | (0.02) | (0.00) | (0.00) | (0.00) | (0.00) | (0.04) |
| M<L<S | 2 | 3 | 1 | 0 | 0 | 2 | 8 |
| | (0.02) | (0.02) | (0.01) | (0.00) | (0.00) | (0.02) | (0.07) |
| L<M<S | 21 | 6 | 8 | 0 | 2 | 13 | 50 |
| | (0.17) | (0.05) | (0.07) | (0.00) | (0.02) | (0.11) | (0.41) |
| Total | 59 | 23 | 17 | 1 | 2 | 21 | 123 |
| | (0.48) | (0.19) | (0.14) | (0.01) | (0.02) | (0.17) | (1.00) |

See the notes to Table 3a.

**Table 4: Mean Interest Rates from Bond Portfolios (%)**

| Selection Criterion | Maximum Debt Switch | | | | | |
|---|---|---|---|---|---|---|
| | *α=1* | *α=0.1* | *α=0.05* | *α=0.01* | *α=0.005* | *α=0* |
| $\bar{R}^2$ | 8.6 | 9.3 | 9.9 | 10.7 | 10.7 | 10.9 |
| *AIC* | 8.7 | 9.3 | 10.0 | 10.7 | 10.7 | 10.9 |
| *SBC* | 9.2 | 9.3 | 9.8 | 10.5 | 10.5 | 10.8 |
| *HQC* | 8.3 | 9.1 | 9.8 | 10.5 | 10.5 | 10.9 |
| DMA | 11.0 | | | | | |

The mean interest rates are expressed as annualised averages of monthly interest rates for the evaluation period, 1985M1-1995M3. The maximum debt switch is defined as $\alpha$ times the market value of the bond portfolio. It is the maximum value of bonds that can be switched each month.

**Figure 1: Bond Portfolio Shares by Maturity Band**

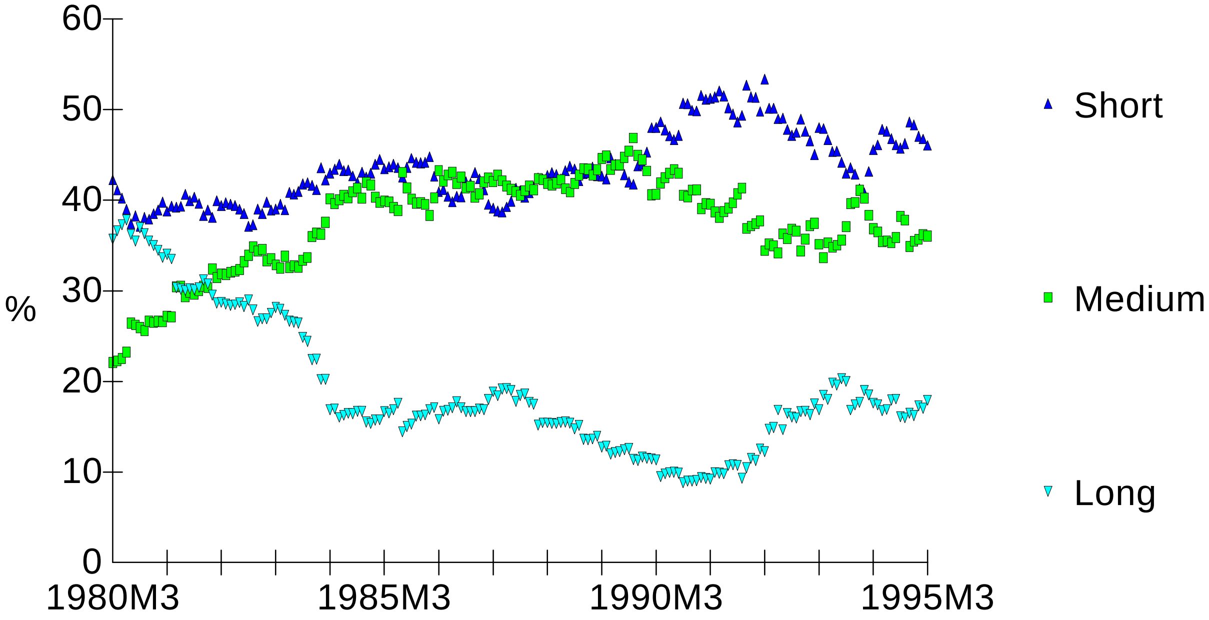



**Figure 2a: Holding Period Return, Short**

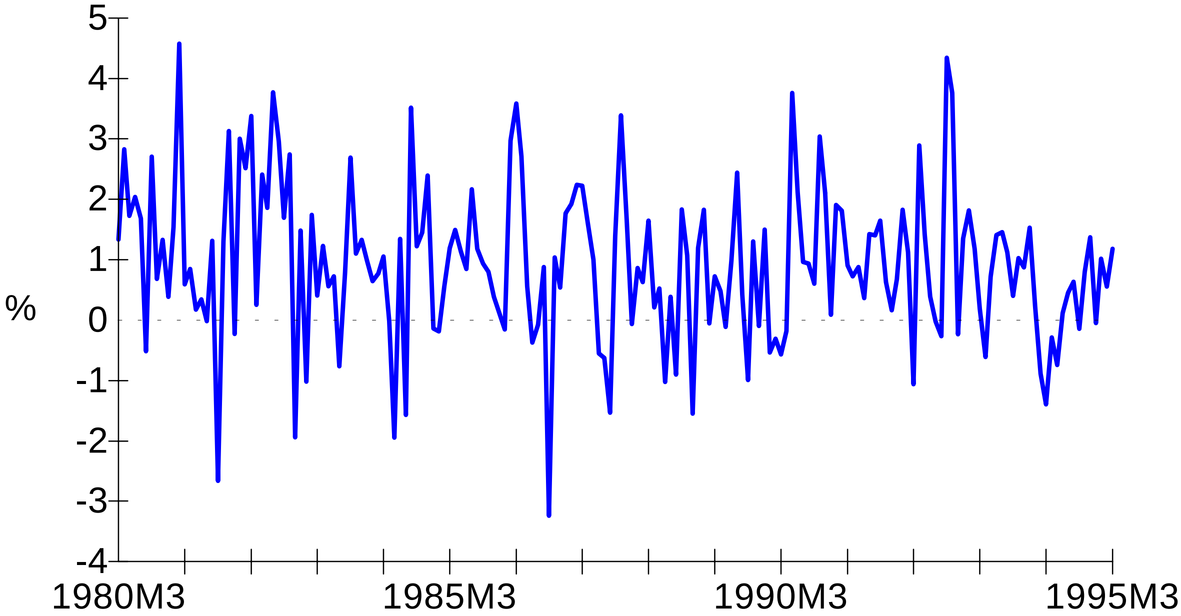


**Figure 2b: Holding Period Return, Medium**

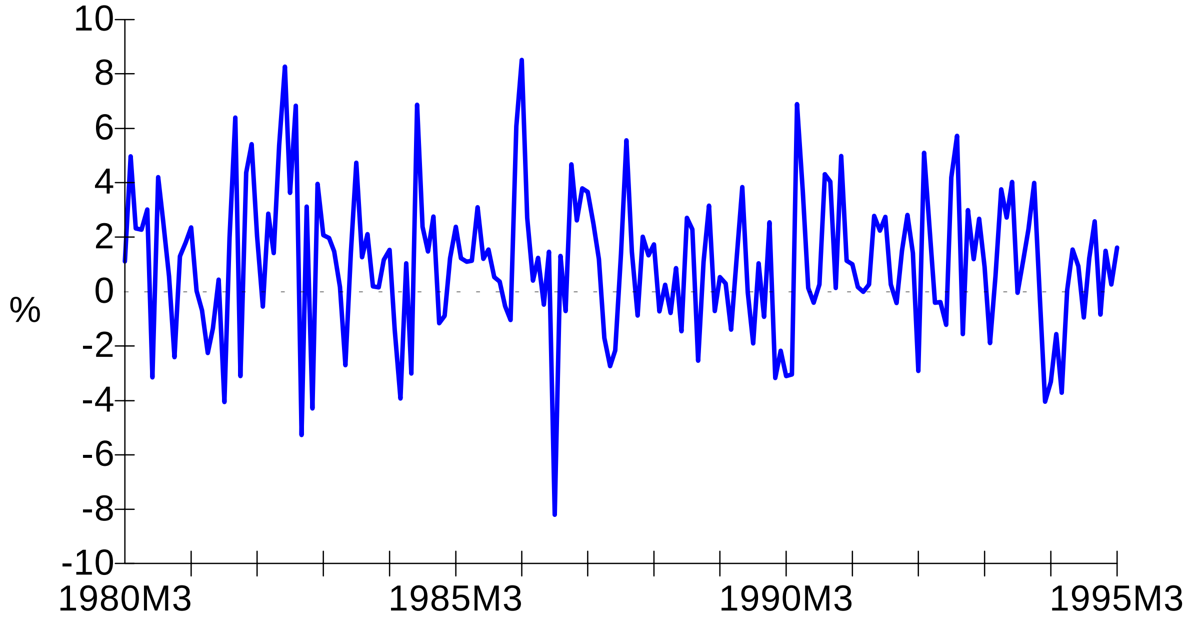



**Figure 2c: Holding Period Return, Long**

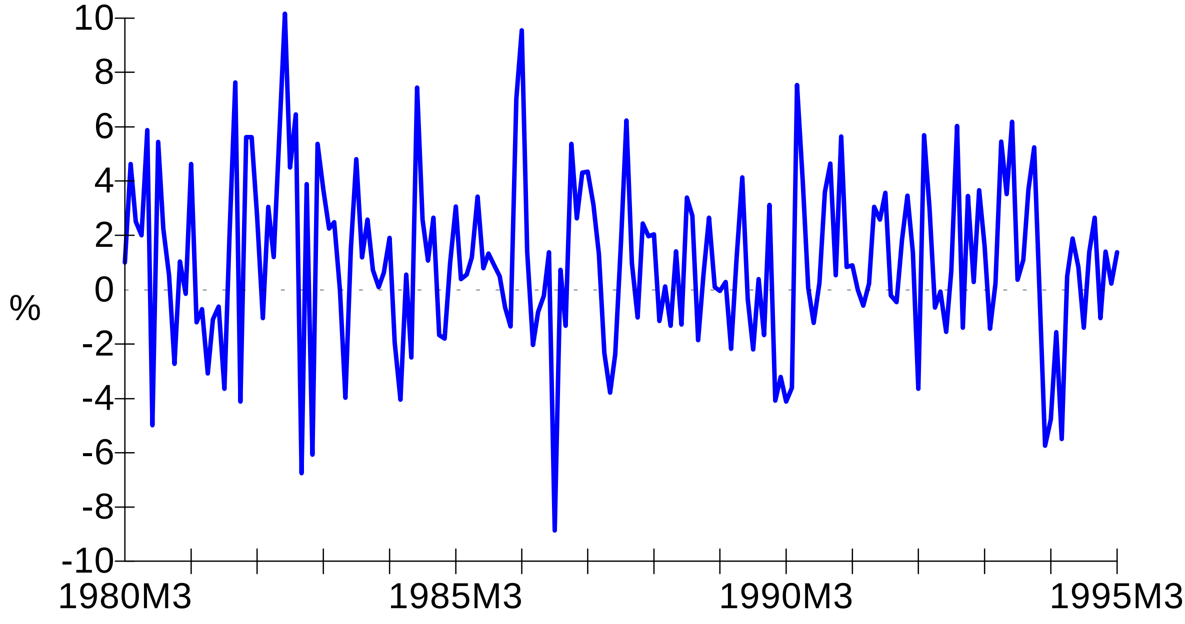
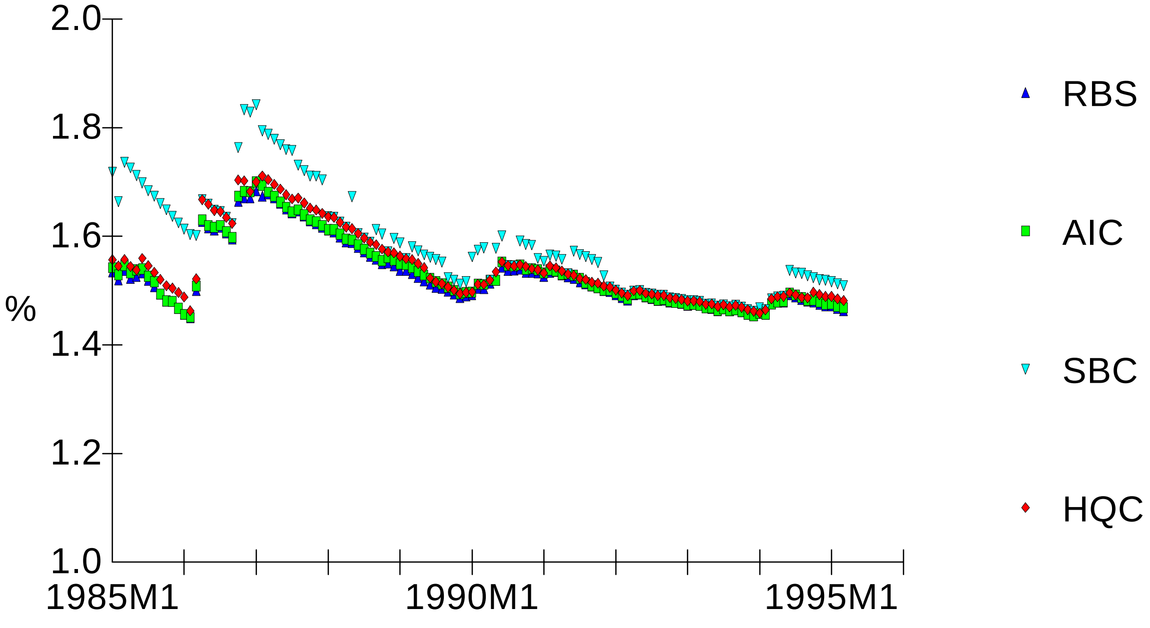
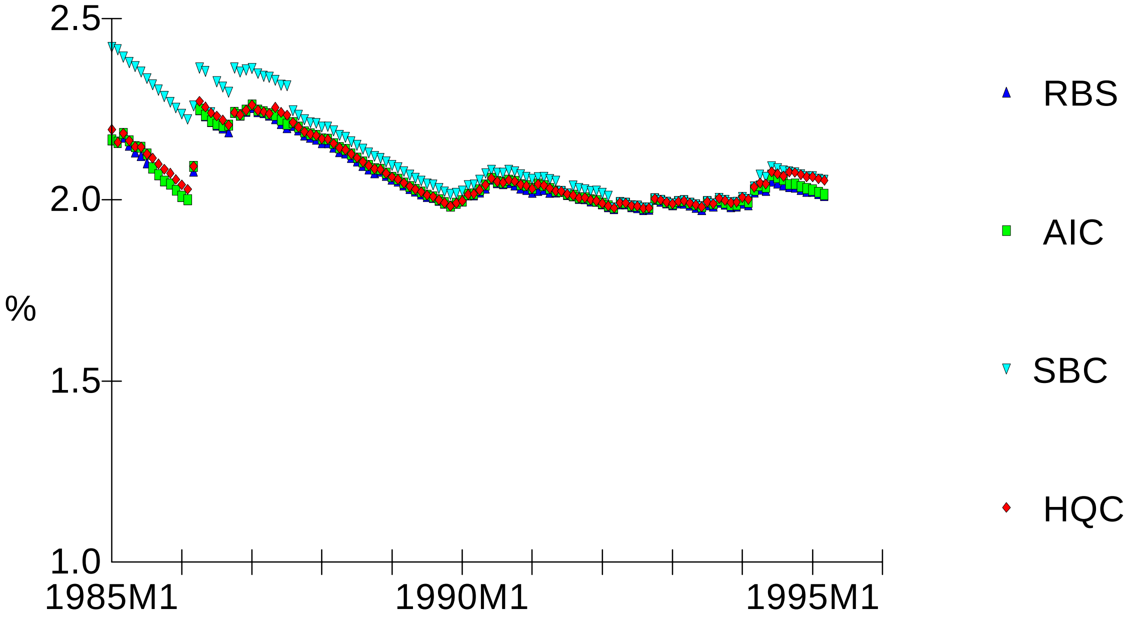


**Figure 3a**: **Recursive Standard Errors, Equation $X_{mt}$: Model Selection Using Various Criteria**




RBS refers to $\bar{R}^2$.

**Figure 3b**: **Recursive Standard Errors, Equation $X_{lt}$; Model Selection Using Various Criteria**




RBS refers to $\bar{R}^2$.

**Figure 4a**: **Forecast and Actual Return Spreads**, $X_{m,t+1}$; **Model Selection Using** $\bar{R}^2$

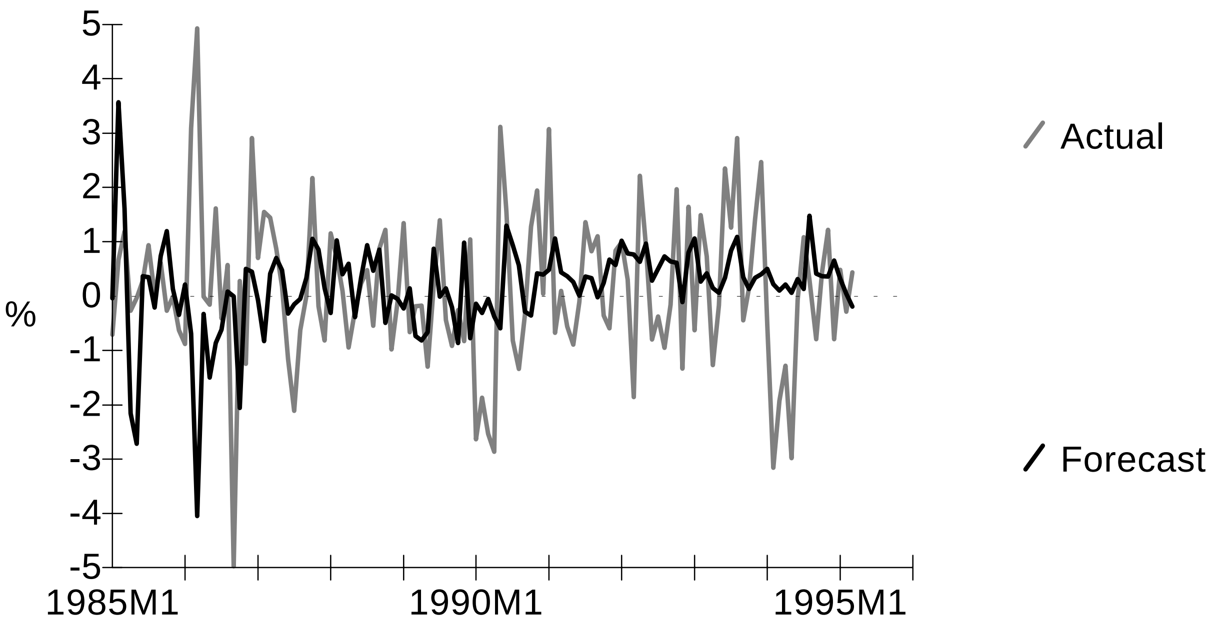



**Figure 4b**: **Forecast and Actual Return Spreads**, $X_{m,t+1}$; **Model Selection Using AIC**

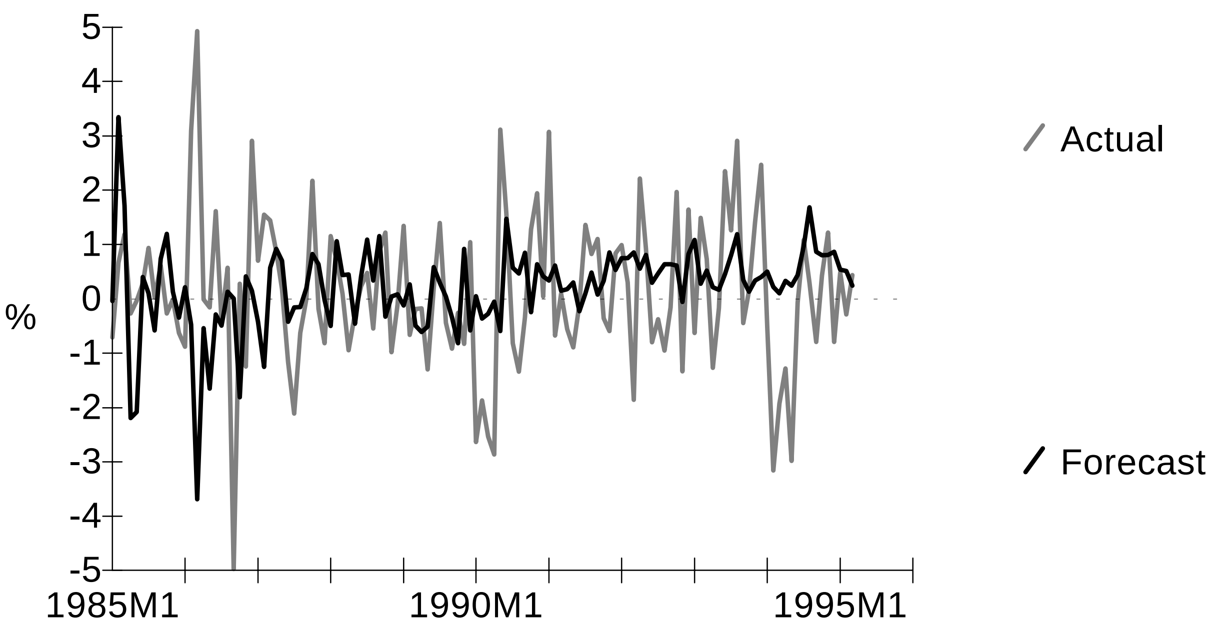


**Figure 4c**: **Forecast and Actual Return Spreads**, $X_{m,t+1}$; **Model Selection Using SBC**

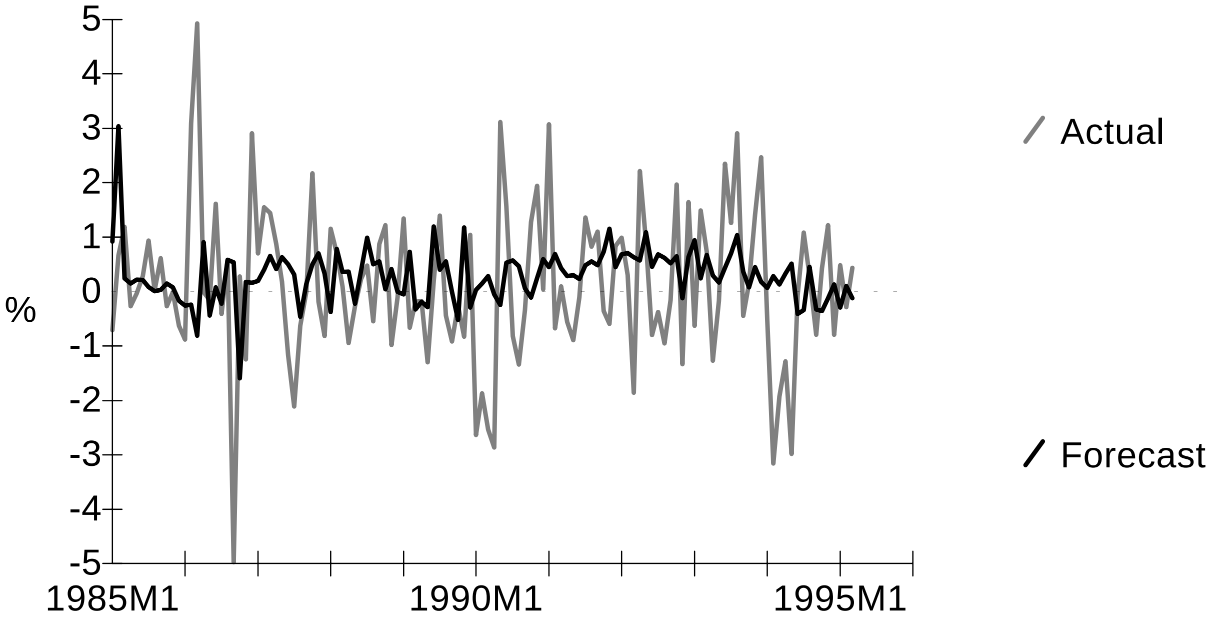



**Figure 4d**: **Forecast and Actual Return Spreads**, $X_{m,t+1}$; **Model Selection Using HQC**

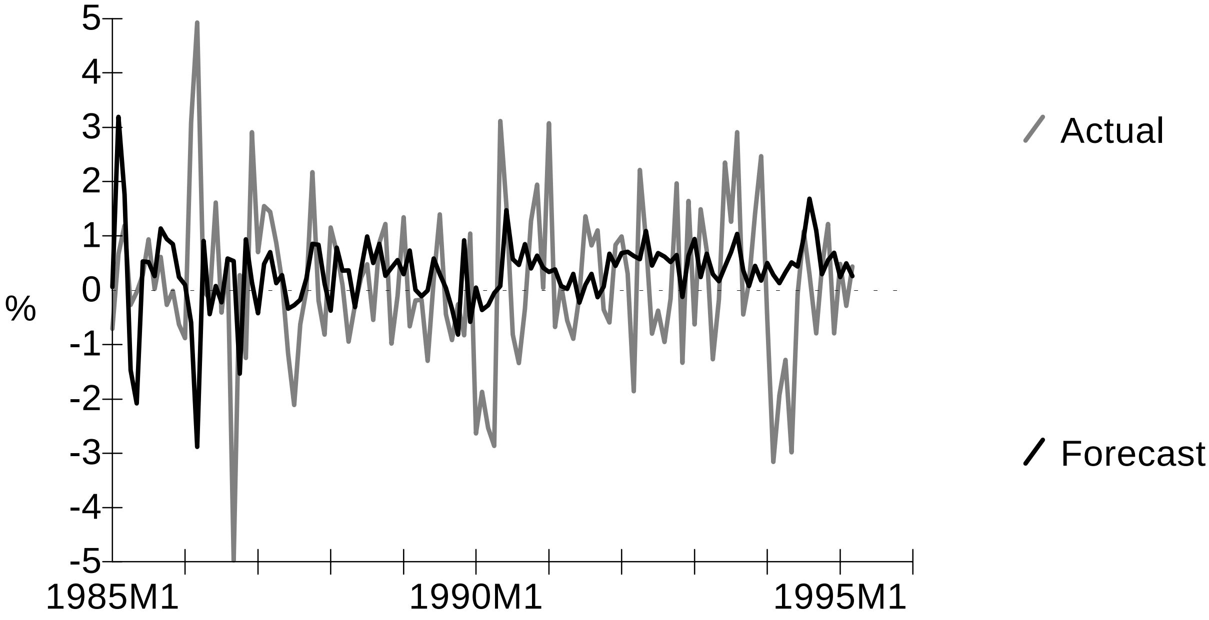


**Figure 4e**: **Forecast and Actual Return Spreads**, $X_{l,t+1}$; **Model Selection Using** $\bar{R}^2$

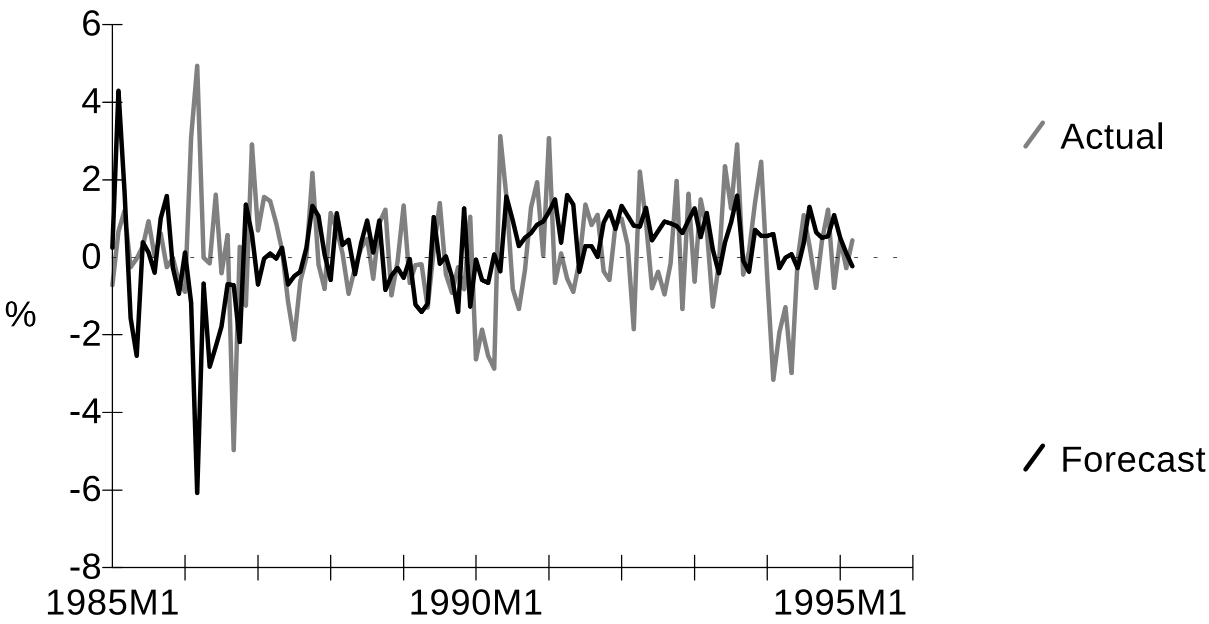



**Figure 4f**: **Forecast and Actual Return Spreads**, $X_{l,t+1}$; **Model Selection Using AIC**

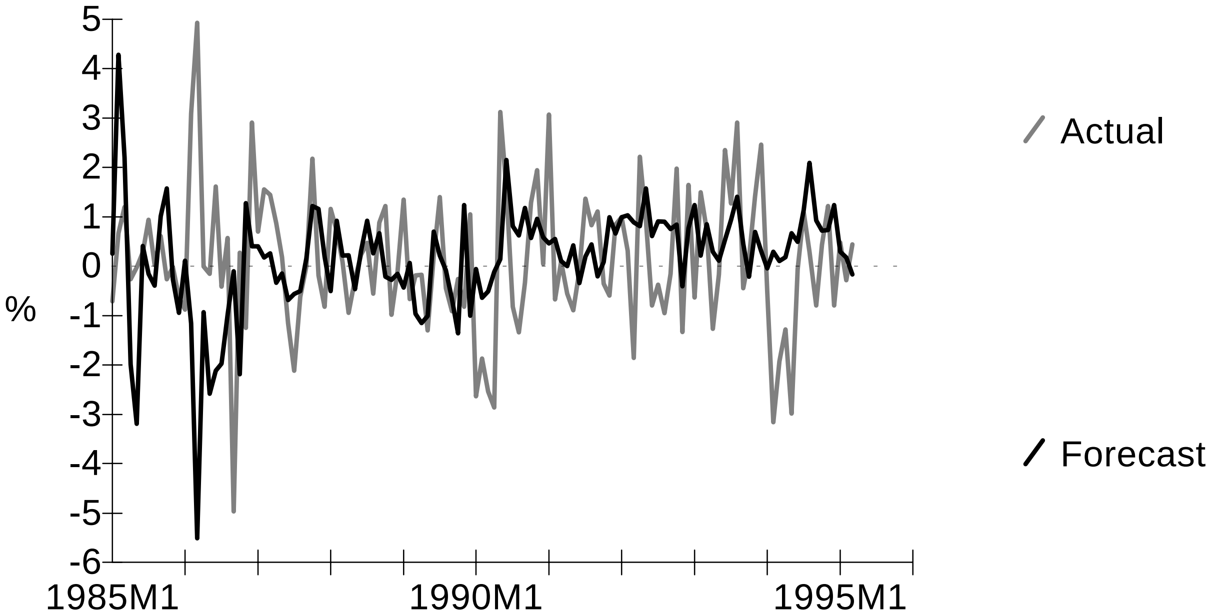


**Figure 4g**: **Forecast and Actual Return Spreads**, $X_{l,t+1}$; **Model Selection Using SBC**

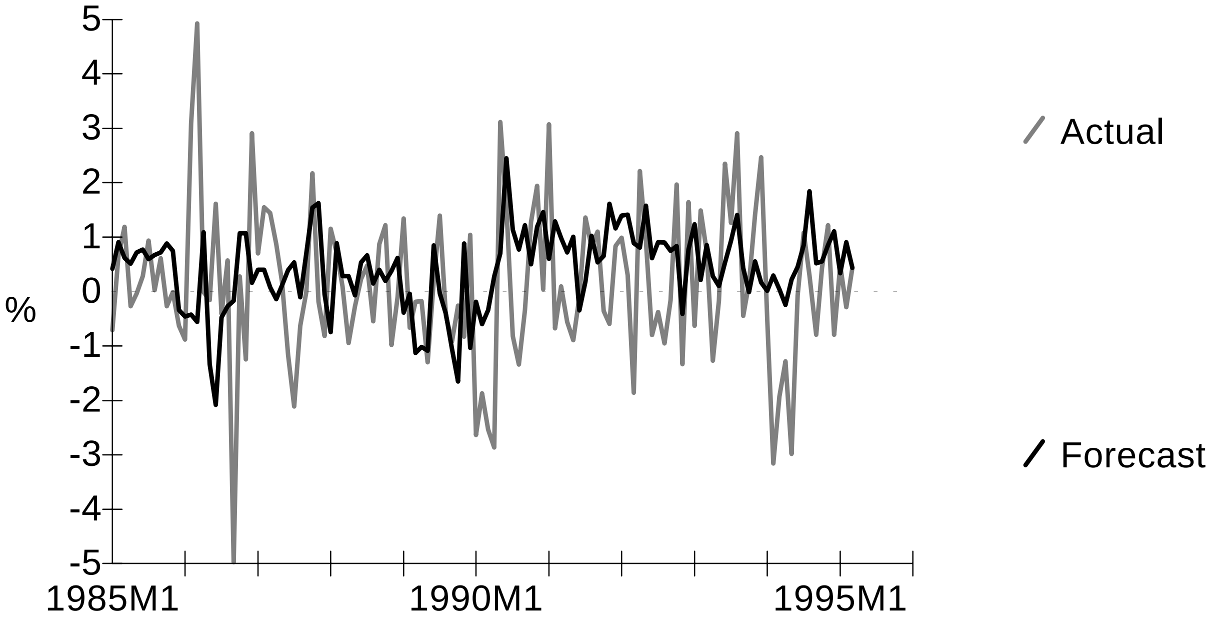



**Figure 4h**: **Forecast and Actual Return Spreads**, $X_{l,t+1}$; **Model Selection Using HQC**

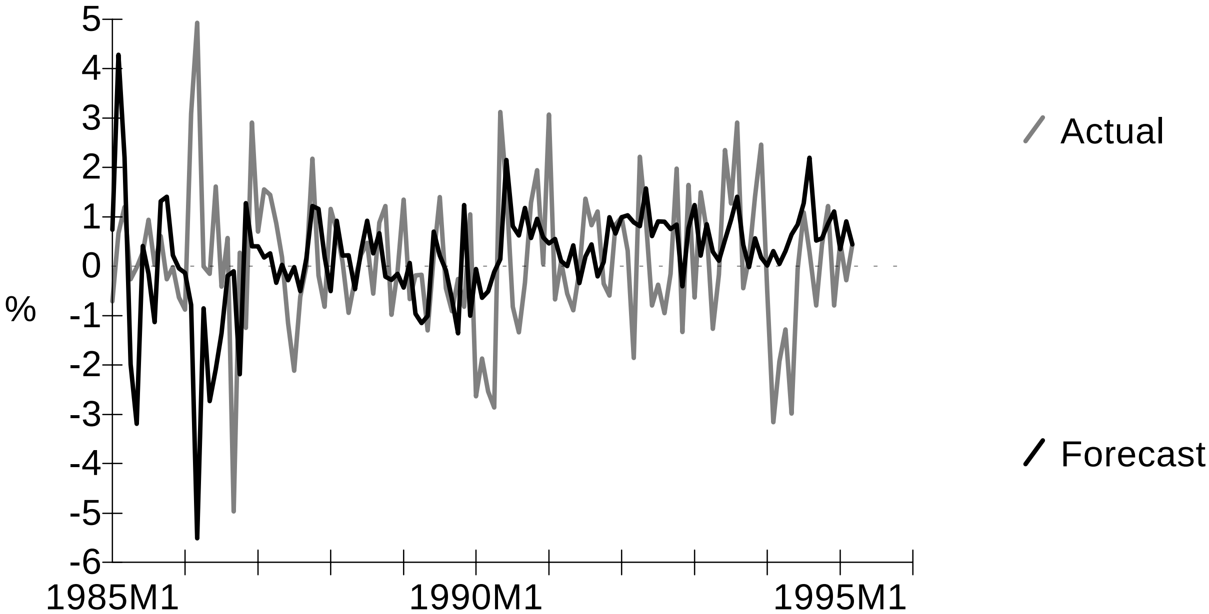


**Figure 5a**: **Recursive Coefficients $(p_{m,t-1} - p_{s,t-1})$, Equation $X_{mt}$, Using AIC and SBC**

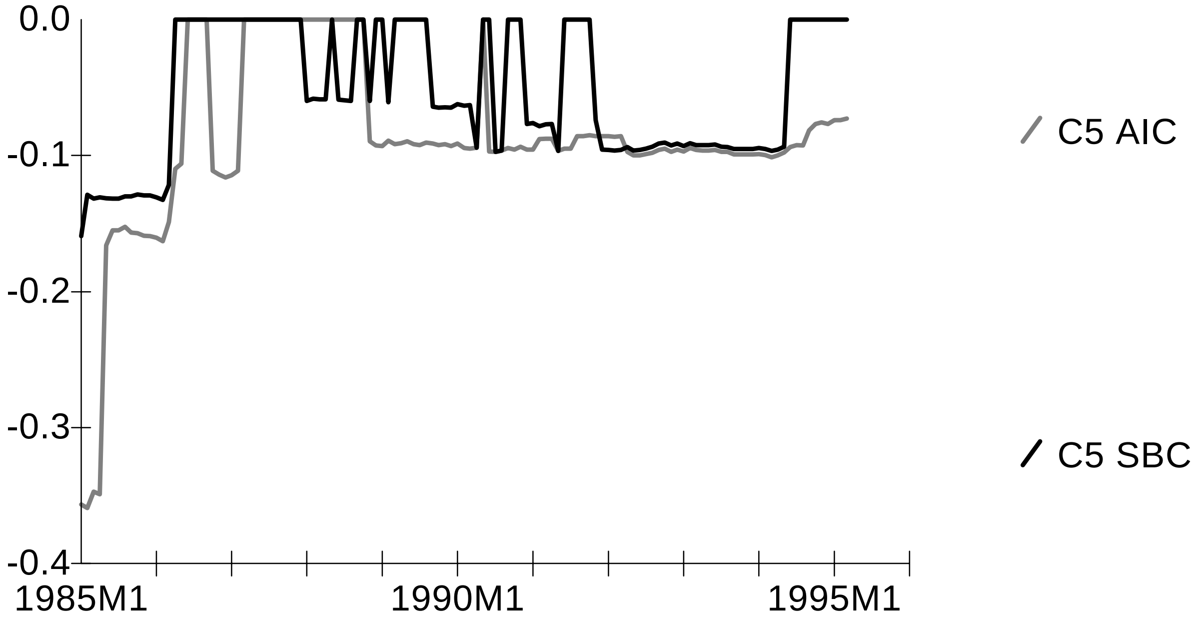



**Figure 5b**: **Recursive Coefficients $(p_{m,t-1} - p_{s,t-1})$, Equation $X_{lt}$, Using AIC and SBC**

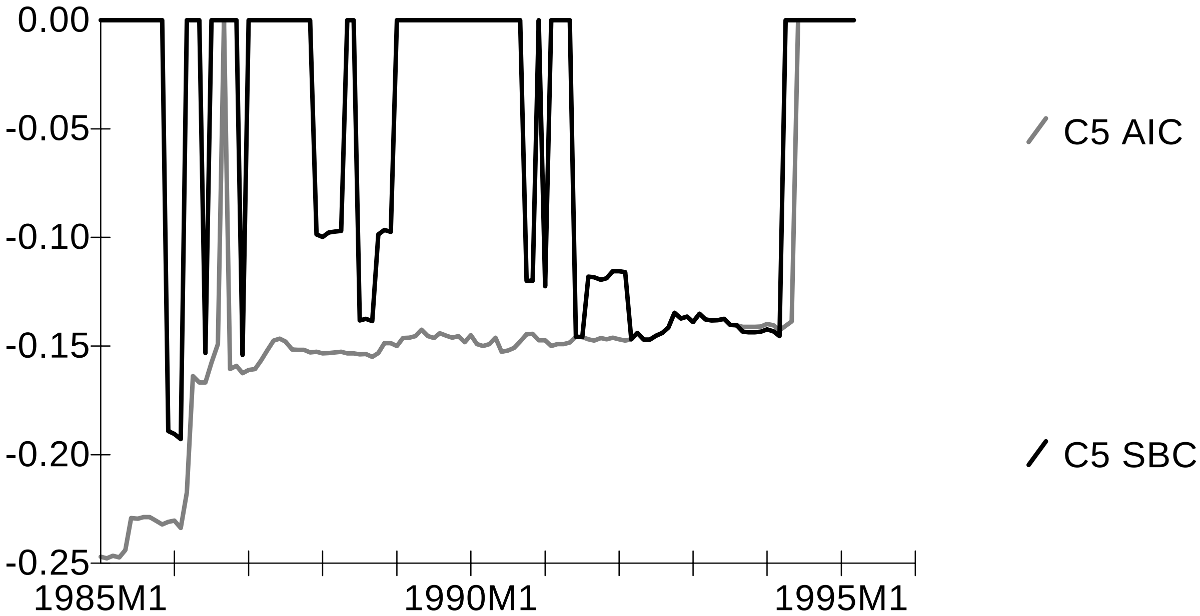


**Figure 5c**: **Recursive Coefficients $\Delta GAP_{s,t-1}$, Equation $X_{mt}$, Using AIC and SBC**

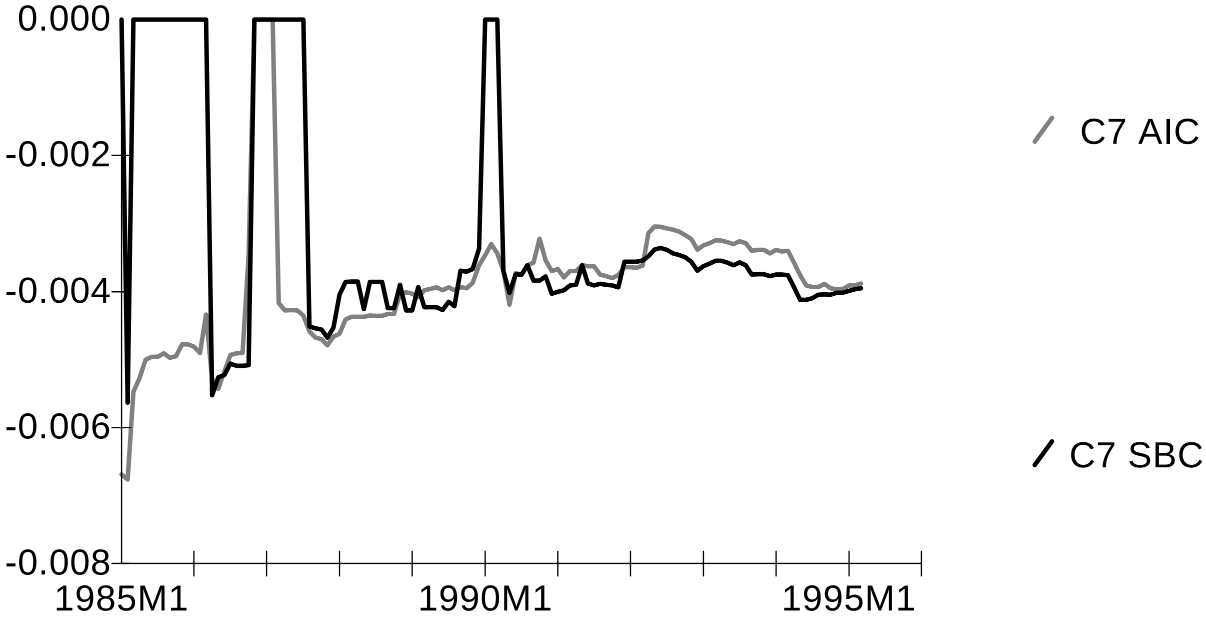



**Figure 5d**: **Recursive Coefficients $\Delta GAP_{s,t-1}$, Equation $X_{lt}$, Using AIC and SBC**

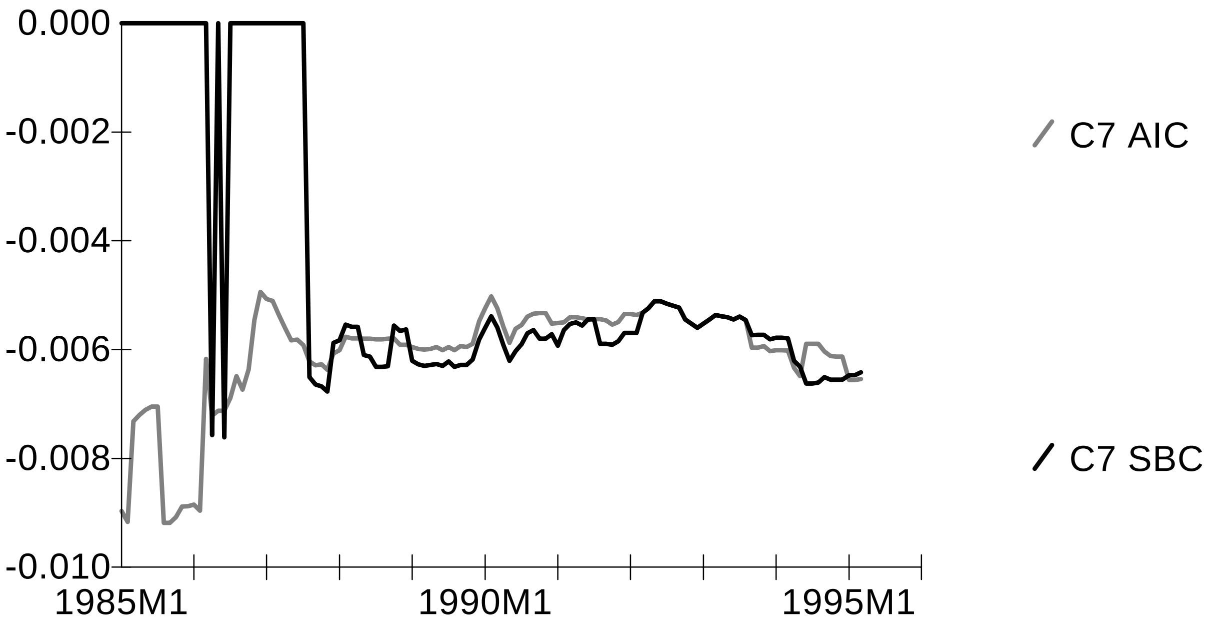


**Figure 5e**: **Recursive Coefficients $\Delta TB_{t-1}$, Equation $X_{mt}$, Using AIC and SBC**

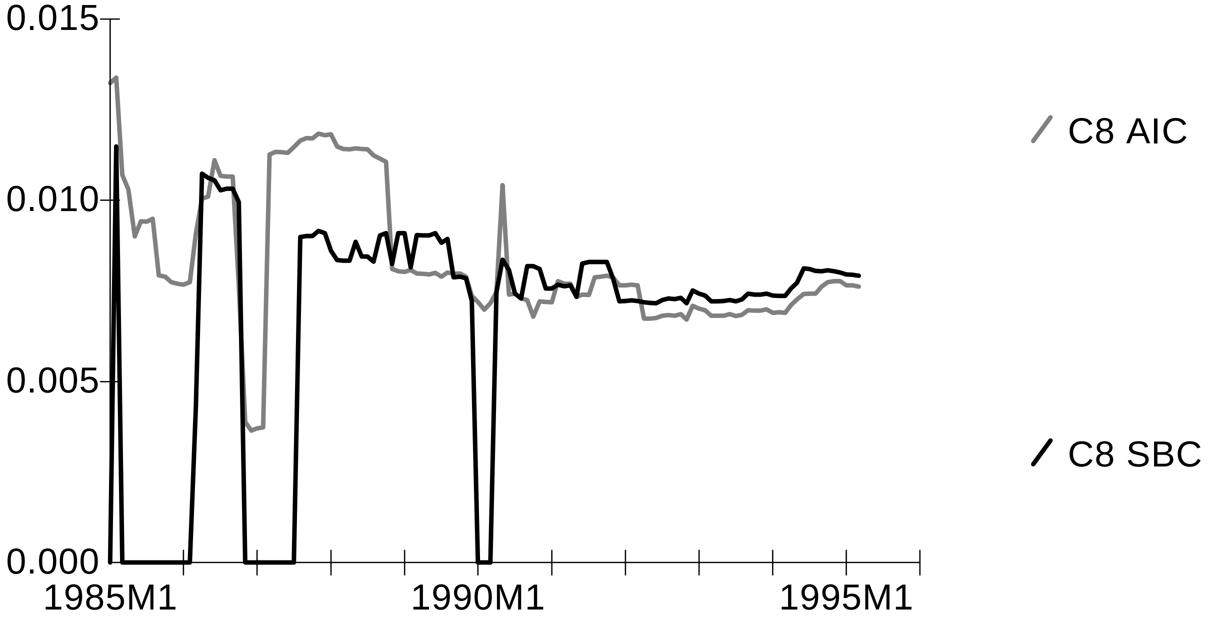



**Figure 5f**: **Recursive Coefficients $\Delta TB_{t-1}$, Equation $X_{lt}$, Using AIC and SBC**

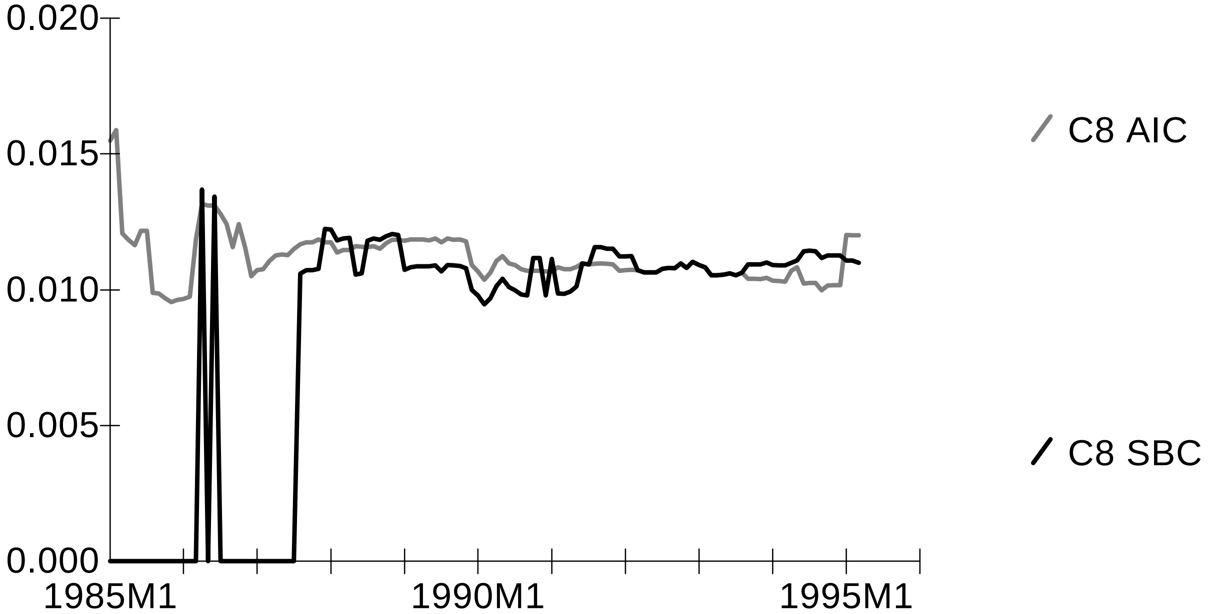


**Figure 5g: Recursive Coefficients $USSP_{t-1}$, Equation $X_{mt}$, Using AIC and SBC**

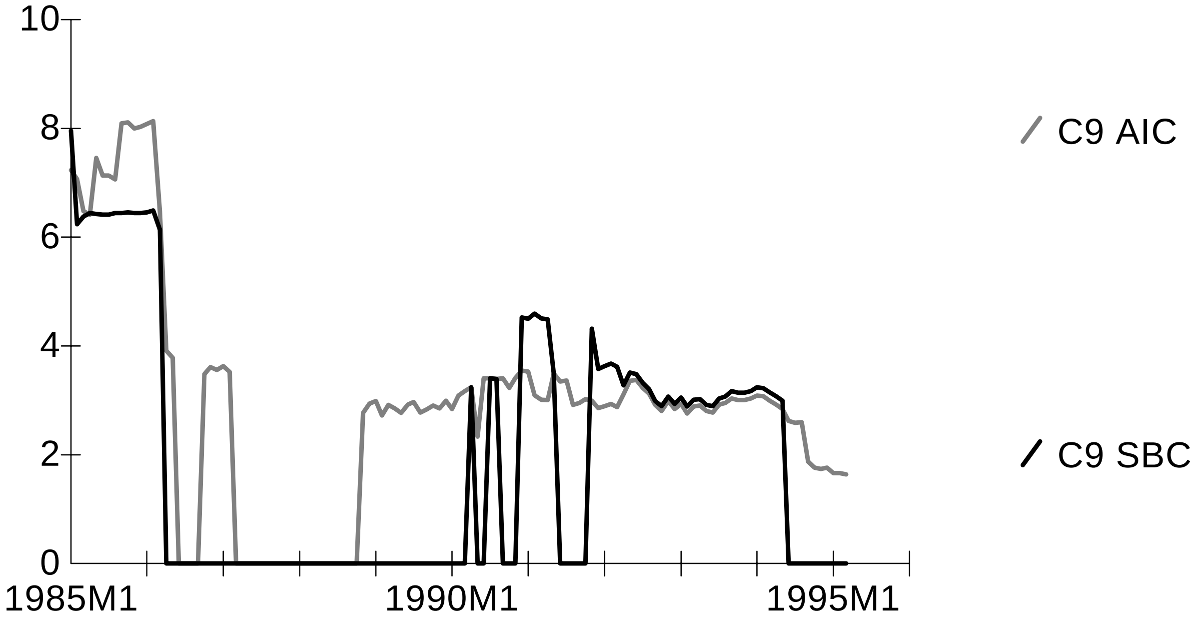



**Figure 5h: Recursive Coefficients $USSP_{t-1}$, Equation $X_{lt}$, Using AIC and SBC**

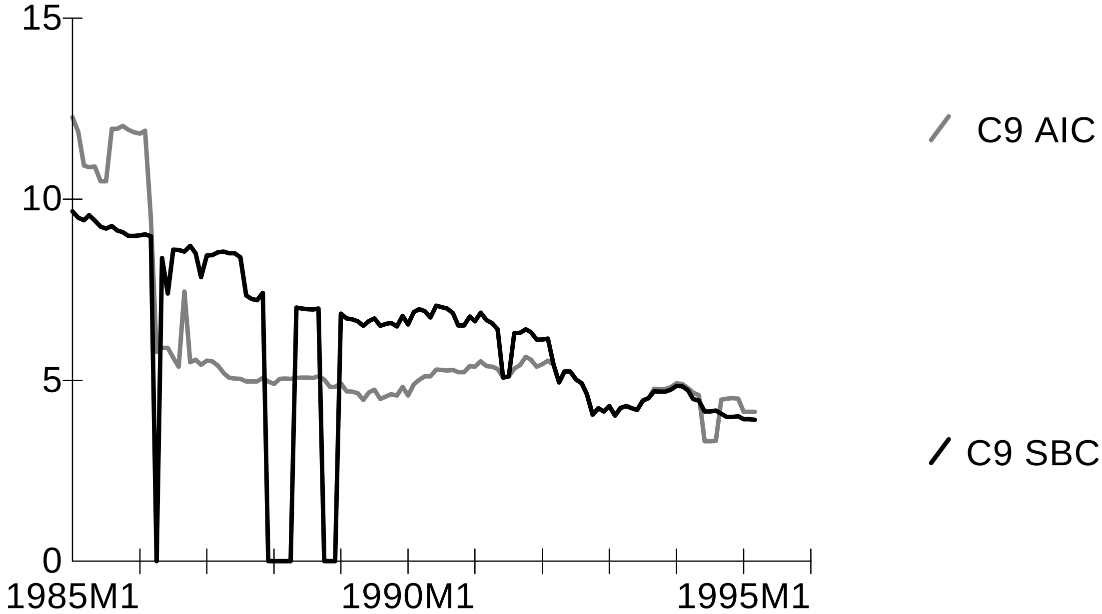


**Figure 5i**: **Recursive Coefficients Δ$IP_{t-3}$, Equation $X_{mt}$, Using AIC and SBC**

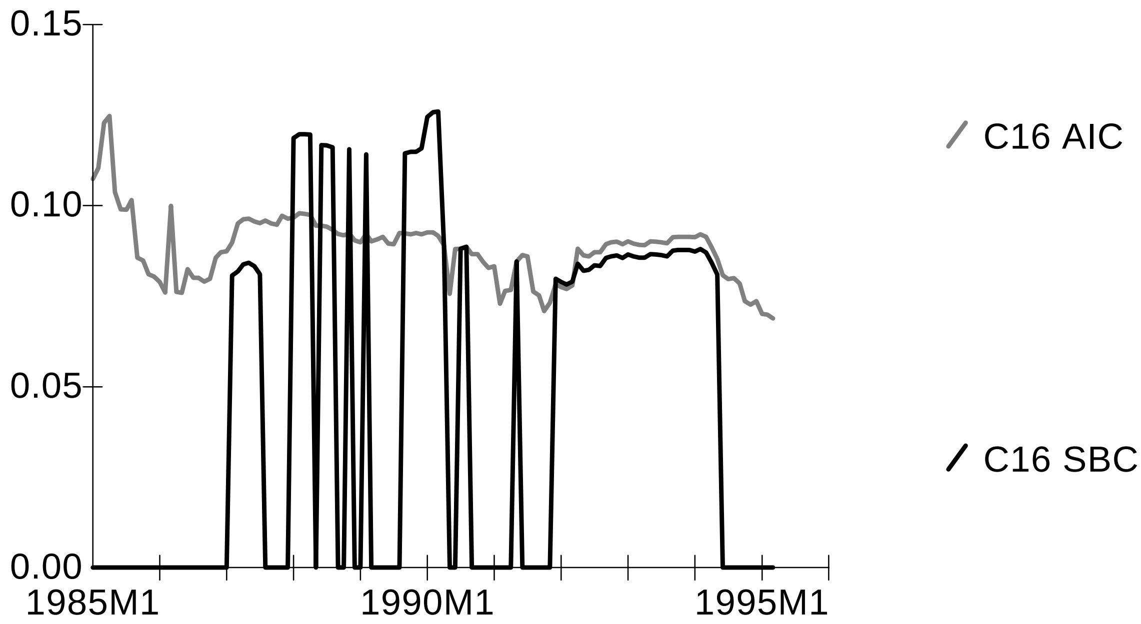



**Figure 5j**: **Recursive Coefficients Δ$IP_{t-3}$, Equation $X_{lt}$, Using AIC and SBC**

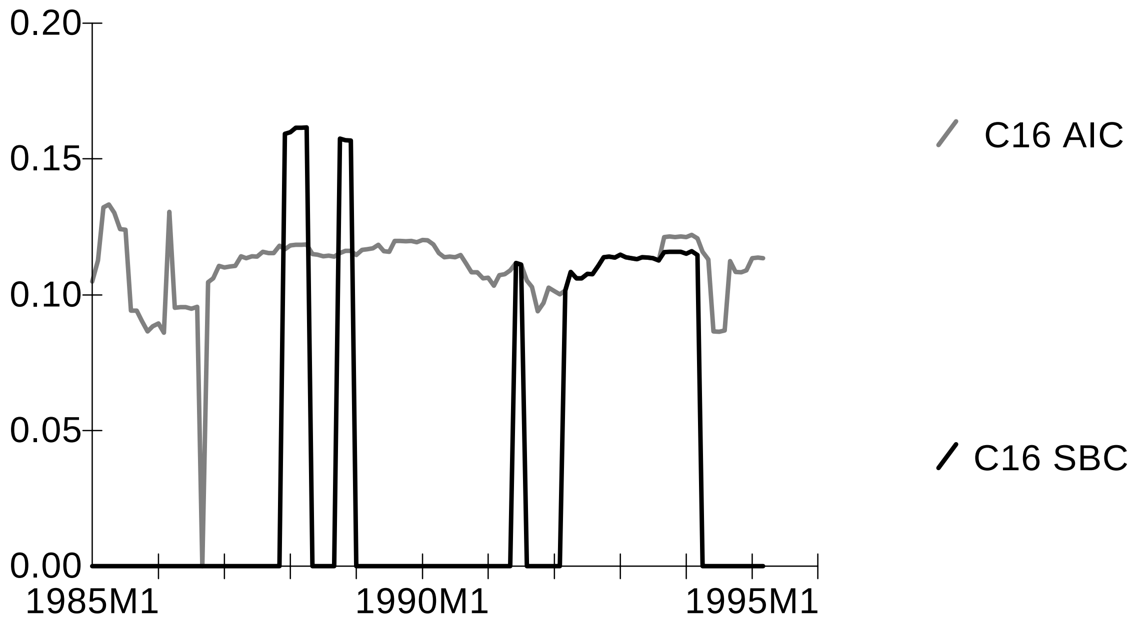
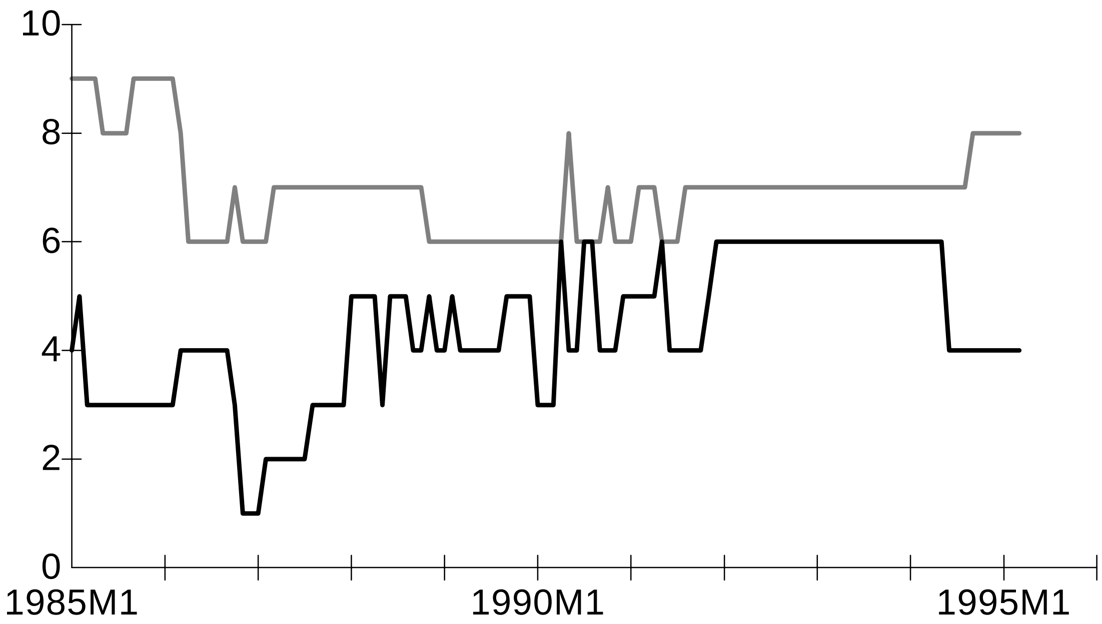
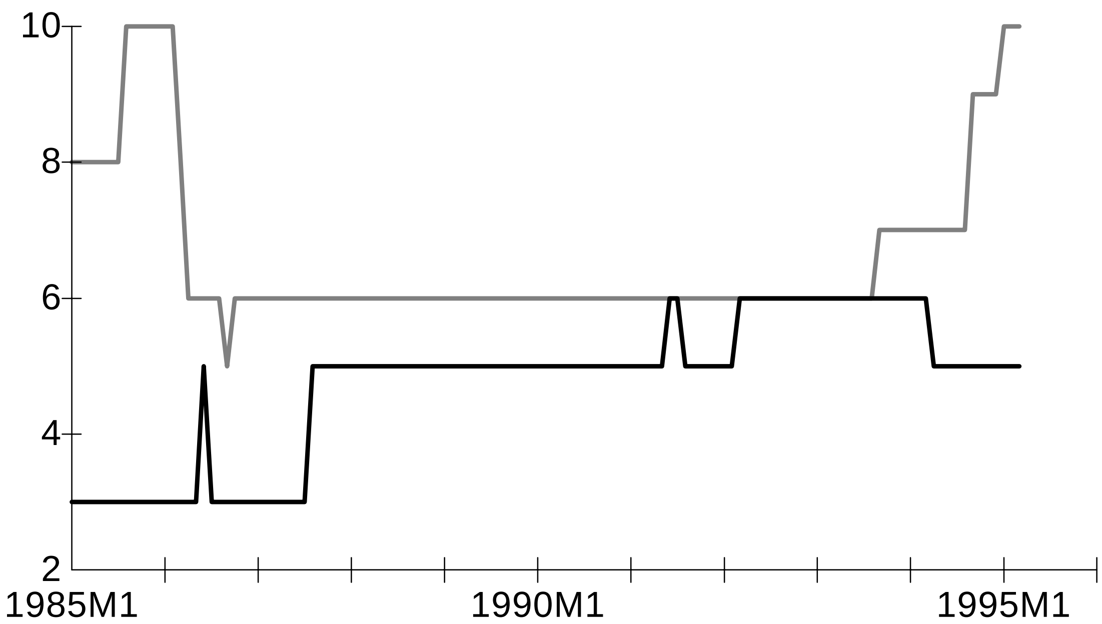


**Figure 6a**: **Number of Recursive Coefficients**, **Equation $X_{mt}$, Using AIC and SBC**



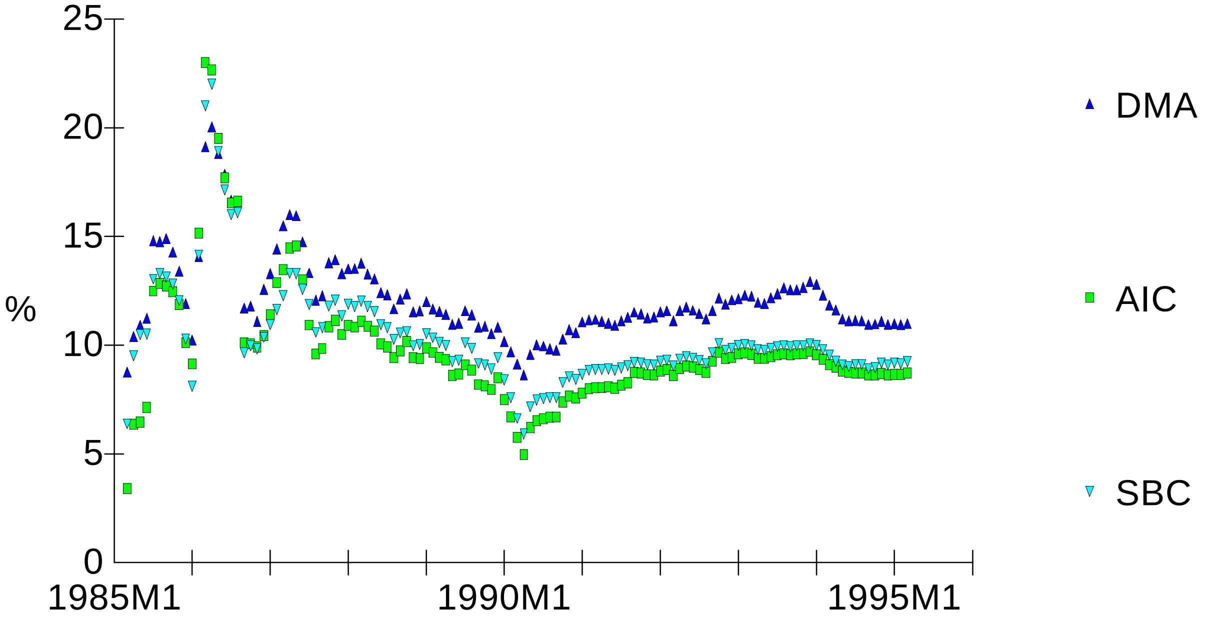

Dark line refers to SBC; light line refers to AIC.

**Figure 6b**: **Number of Recursive Coefficients in Equation $X_{lt}$, Using AIC and SBC**



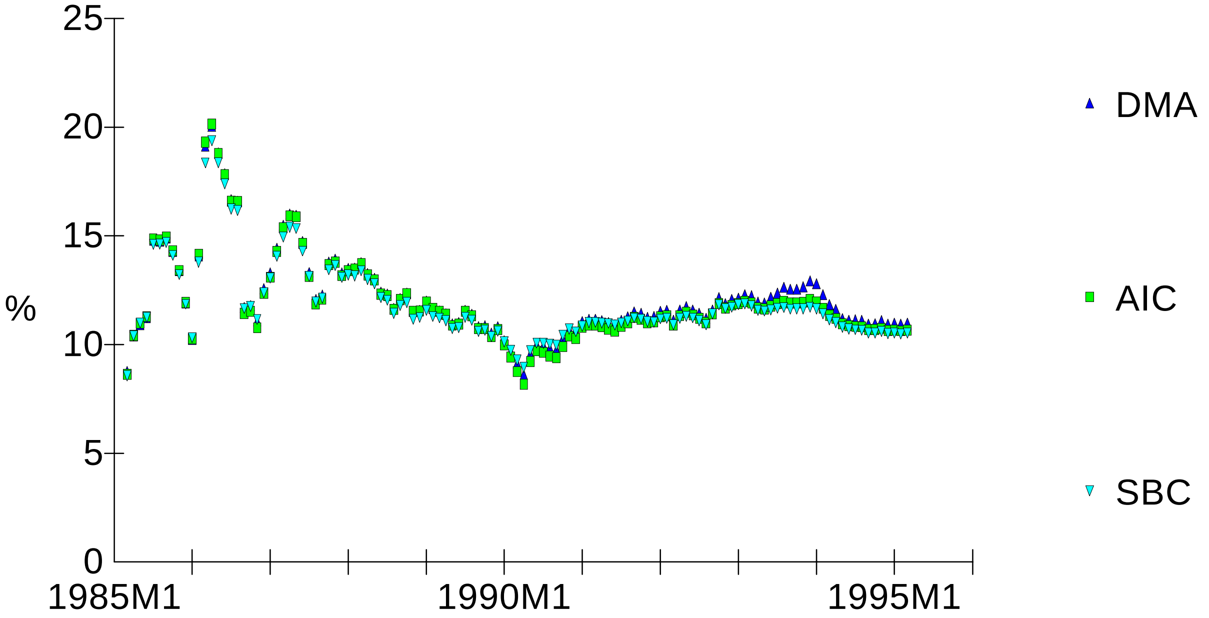

Dark line refers to SBC; light line refers to AIC.

**Figure 7a: Recursive Average Interest Costs, Simulated Portfolios, $\alpha = 1$**



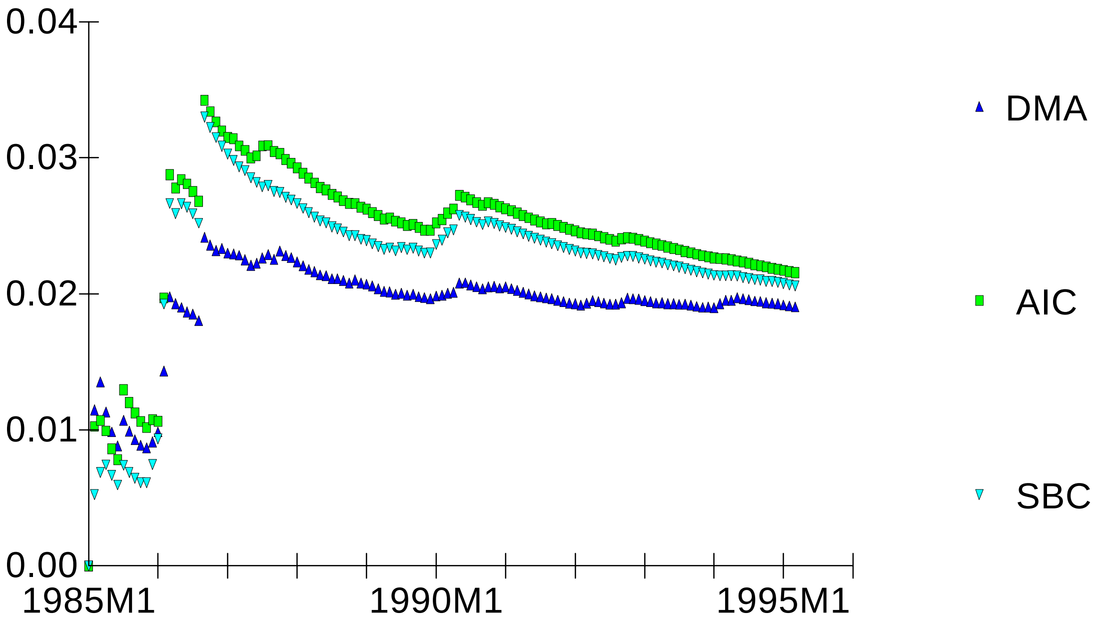

The maximum debt switch is defined as $\alpha$ times the market value of the bond portfolio. It is the maximum value of bonds that can be switched each month.

**Figure 7b: Recursive Average Interest Costs, Simulated Bond Portfolios, $\alpha = 0.01$**



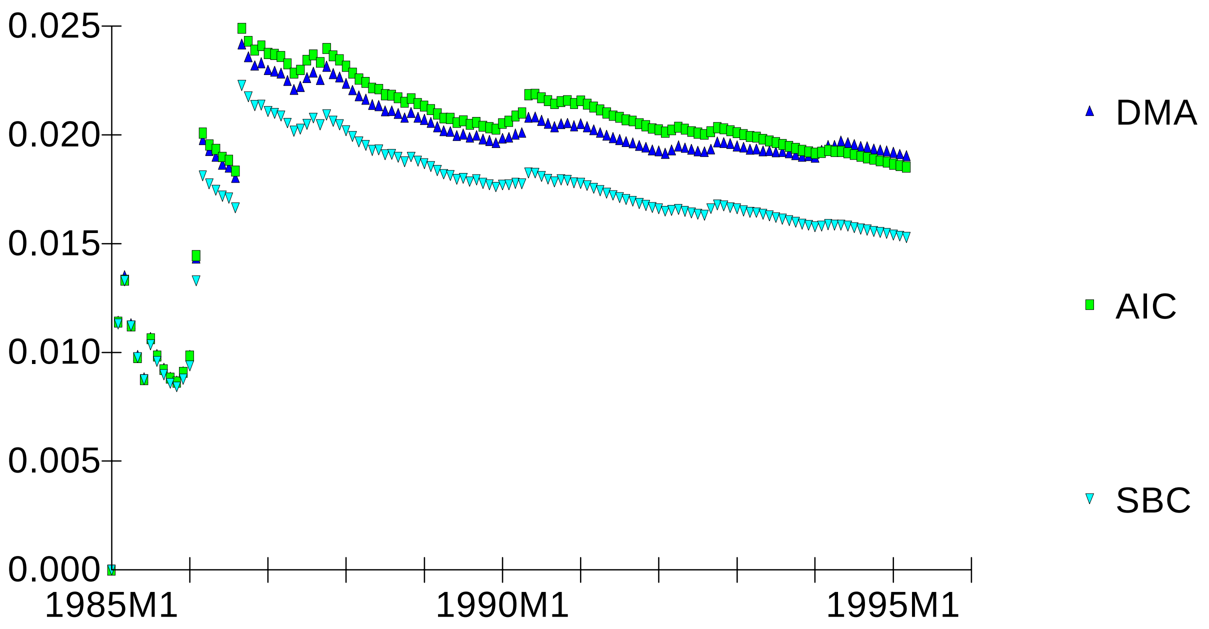

The maximum debt switch is defined as $\alpha$ times the market value of the bond portfolio. It is the maximum value of bonds that can be switched each month.

**Figure 8a**: **Standard Deviations of Interest Costs**, **Simulated Portfolios**, $\alpha = 1$



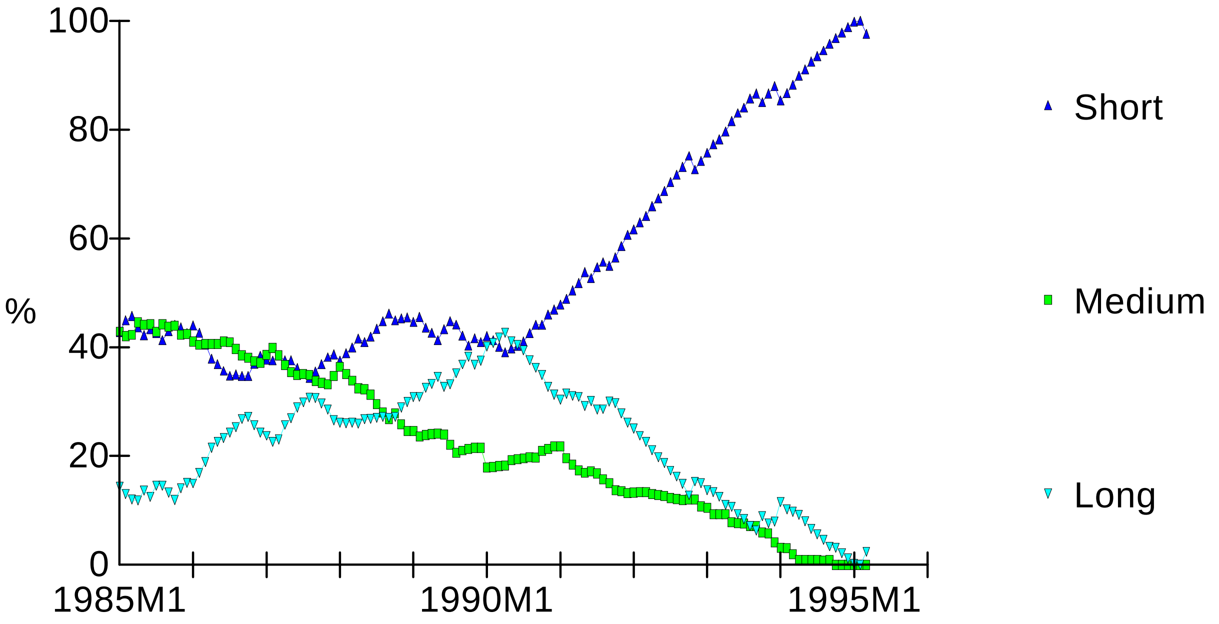

The maximum debt switch is defined as $\alpha$ times the market value of the bond portfolio. It is the maximum value of bonds that can be switched each month.

**Figure 8b**: **Standard Deviations of Interest Costs**, **Simulated Portfolios**, $\alpha = 0.01$



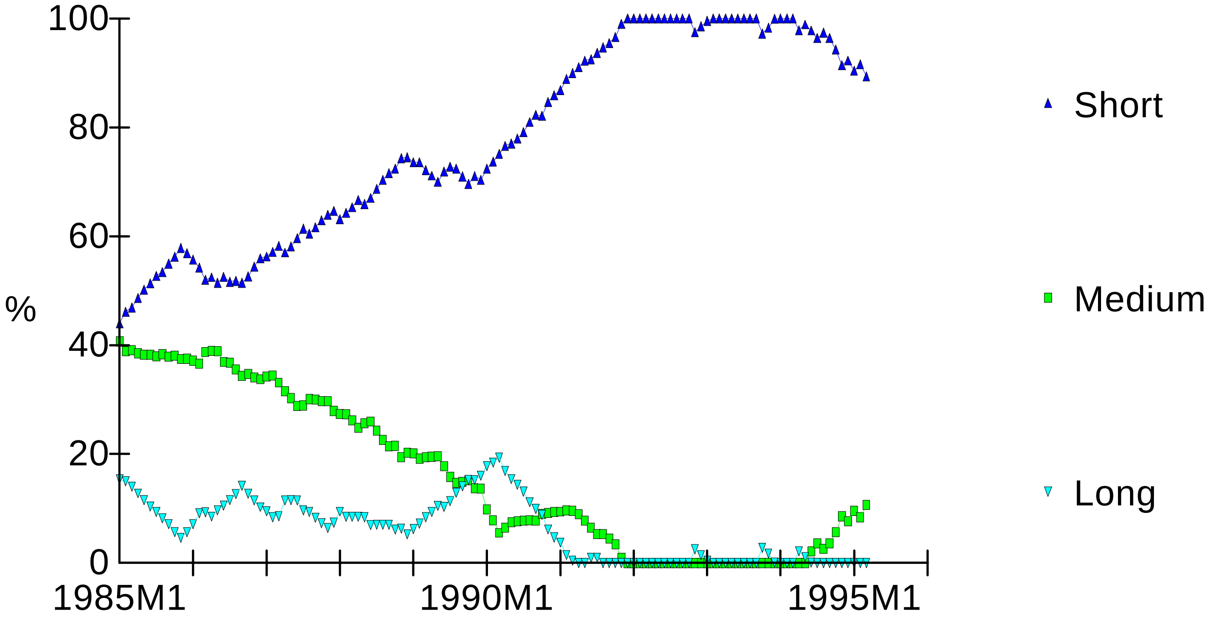

The maximum debt switch is defined as $\alpha$ times the market value of the bond portfolio. It is the maximum value of bonds that can be switched each month.

**Figure 9a: Simulated Bond Portfolio Shares Using AIC, $\alpha = 0.01$**




The maximum debt switch is defined as $\alpha$ times the market value of the bond portfolio. It is the maximum value of bonds that can be switched each month.

**Figure 9b: Simulated Bond Portfolio Shares Using SBC, $\alpha = 0.01$**




The maximum debt switch is defined as $\alpha$ times the market vlaue of the bond portfolio. It is the maximum value of bonds that can be switched each month.