# CHINA'S OFFICIAL RATES AND BOND YIELDS 

Longzhen Fan<br>School of Management, Fudan University

Anders C. Johansson<br>Stockholm School of Economics

## Working Paper 3

March 2009

Postal address: P.O. Box 6501, S-113 83 Stockholm, Sweden.
Office address: Holländargatan 30 Telephone: +46 87369360 Telefax: +46 8313017
E-mail: japan@hhs.se Internet: http://www.hhs.se/cerc

# China's Official Rates and Bond Yields 

Longzhen Fan<br>School of Management, Fudan University<br>Anders C. Johansson*<br>Stockholm School of Economics


#### Abstract

Recent research shows that bond yields are influenced by monetary policy decisions. To learn how this works in an interest rate market that differs significantly from that of the U.S. and Europe, we model Chinese bond yields using the one-year deposit rate as a state variable. We also add the difference between the one-year interest rate and the one-year deposit rate as a factor. The model is developed in an affine framework and closed-form solutions are obtained. It is tested empirically and the results show that the new model characterizes the changing shape of the yield curve well. Incorporating the benchmark rate into the model thus helps us to match Chinese bond yields.


Keywords: China; deposit rate; bond yields; jump process; affine model.
JEL Classification: E43; E44; E52; E58.

[^0]
## 1 Introduction

Early term structure models focus on modeling the short-term interest rate as a continuous stochastic variable. Having specified the short-term interest rate, the medium- and long-term interest rates can then be seen as functions of the short rate. The so-called one-factor term structure models such as the Vasicek (1977) model, the Ho-Lee (1986) model, the Hull-White (1990) model, and the Cox-Ingersoll-Ross (1985) model all define interest rate movements in terms of the dynamics of the short rate. Later models include other factors that may influence the term structure. For instance, the Brennan-Schwartz (1979) model uses both the short- and long-term interest rates to define the term structure. Other possible factors that have been used to model the term structure include current inflation, long-term average expected inflation, credit spreads and so on. Heath, Jarrow, and Morton (1992) also showed that it is possible to use the whole term structure as state variable.

Recently, a number of studies have focused on the relationship between monetary policy and the term structure. In the U.S. and Europe, monetary policy is carried out either through a specific interest rate, such as the Federal Fund Rate, or by adjusting money supply. Monetary policy in this framework focuses on the short-term interest rate, while the medium- and long-term interest rates are set by the market. Based on this standard monetary policy framework, approaches to estimating the term structure and the pricing of fixed income securities have been developed. It is natural to assume that monetary policy directly influences the short-term interest rate and thereby affects the whole term structure. Several recent studies have used this approach to model interest rates for instance by including inflation and real activity (Ang and Piazzesi, 2003) or the Federal Reserve's interest rate target (Piazzesi, 2005).

The new literature that brings macroeconomic and finance perspective together is of significant interest to researchers as well as policy makers and market participants. However, so far, the literature that takes monetary policy into consideration when
modeling the term structure has focused almost exclusively on the U.S. and to some extent Europe. Even though these studies help us understand the underlying processes in the U.S. and European bond markets, they are difficult to apply to countries with monetary systems that differ significantly from that of the U.S. and Europe. One example in which the bond market is markedly different from the U.S. is that of China. For instance, while the U.S. central bank uses the target interest rate to set monetary policy, the Chinese central bank uses a number of official rates of which the one-year deposit rate is arguably the most commonly used benchmark. The deposit rate is thus specified directly by the central bank, and has a direct impact on Chinese market rates. This paper takes the domestic institutional factors in China into account when modeling domestic bond yields. We apply a multifactor model that incorporates the one-year deposit rate to bond yields of up to five-year maturity. The short-term interest rate is assumed to follow a continuous stochastic process in which the one-year deposit rate and the difference between the shortterm interest rate and the one-year deposit rate are used as factors. To model the bond yields, we use an affine approach (see Duffie and Kan, 1996). The difference between the one-year market interest rate and the one-year deposit rate reflects the influence of inflation and other macroeconomic variables. The deposit rate is specified as a jump process and its jump size is modeled as a stochastic process. This paper thus differs from other related studies in that it focuses on the one-year deposit rate and the difference between the deposit rate and the short-term interest rate rather than a target rate as used for the U.S. market. The jump process also differs from similar studies such as Piazzesi (2005) in that we allow for the jump size to follow a stochastic process due to the special features in China's official rates policy. A closed-form solution is derived for the model and a Markov chain Monte Carlo (MCMC) procedure is used to estimate its parameters. It is shown that the model captures the movements in yields for different maturities well during periods of increasing, decreasing, and stable official rates.

The rest of the paper is organized as follows: Section 2 discusses related literature on the relationship between monetary policy and bond yields. Section 3 gives a short introduction to how monetary policy is conducted in China and why the deposit rates are so important in the domestic financial system. Section 4 then introduces the new model. Section 5 briefly explains the estimation procedure and then goes over the data and the empirical results from the estimation. Finally, Section 6 concludes the paper.

## 2 Related Literature

Most studies on yield curves use latent state variables that are estimated using market data (e.g. Dai and Singleton, 2000). While valuable in the sense that they help us to understand and model yield curves, this type of models does not explicitly take macroeconomic factors into account and thus limits our understanding of underlying economic factors that influence the yield curve. There are some contributions that connect term structures with macroeconomic variables in general and monetary policy in particular that relate to this study. In this section, we focus on related research that looks at the relationship between monetary policy and the term structure of interest rates. For a detailed overview of this growing literature, see Diebold, Piazzesi, and Rudebusch (2005).

Balduzzi, Bertola, and Foresi (1997) analyze the Federal Reserve's short-term target rates and their effect on the term structure of interest rates. They find that the spread between the short-term interest rate and the target rate is mainly influenced by expected changes in the target rate, thus showing the importance of central banks' official rates. In Balduzzi, Bertola, Foresi, and Klapper (1998), the authors identify volatile and persistent spreads in federal fund rates and present a model of the term structure that is consistent with this pattern. Farnsworth and Bass (2003) also develop a model in which the short-term interest rate is forced
to keep close to the target rate. The model explains shifts in the yield curve as a result of changes in the target rate. However, the model is not estimated using market data. Ellingsen and Söderström (2001) propose a model that incorporates different theoretical approaches to the relationship between monetary policy and market interest rates and that allows for a variety of market reactions to monetary policy. They classify policy events as exogenous or endogenous and look at the response in the term structure to such events. There are also studies that focus on the relationship between monetary policy and the short rate only. Some of these studies also relates to ours in the sense that they allow for jumps to influence the interest rate. A significant number of empirical studies show that jumps play an important role in explaining changes in interest rates. It is well known that the short-term interest rate exhibits leptokurtosis, or fat tails, a feature that is consistent with the presence of jumps. For instance, Das (2002) shows that a conditional volatility model that includes jumps explains the movements in the U.S. short rate well. His results also indicate that the jumps are more pronounced when the Federal Open Market Committee is having its meetings. There are a number of reasons for why jumps may occur in interest rates, including central bank interventions, macroeconomic surprises, shocks in the exchange rate, extreme market events and other events. Johannes (2004) shows that monetary policy is one of the major factors behind jumps in the short rate (the others being official announcements of the current state of the economy and exogenous political-economic events, domestic and foreign). Piazzesi (2005) focuses on how policy decisions are linked to bond yields. Her results show the value of incorporating monetary policy, in this case in the form of the target rate, when modeling the yield curve. The target rate data clearly improves the fit of the yield curve compared to alternative three-factor model such as Dai and Singleton (2000) and she concludes the paper by advocating additional studies on other markets. Piazzesi (2005) is closely related to our study in the sense that we model the whole term structure and include official rates as state variables. Also,

Andersson, Dillen, and Sellin (2006) analyze how different monetary policy signals influence interest rates. Using signals such as repo rate changes, inflation reports, public speeches and reports from meetings, they show that monetary policy signals has a direct and significant effect on the Swedish term structure of interest rates.

While many of the studies above show the importance relationship between monetary policy and the term structure of interest rates, few of them actually develop models for the term structure and take these models to the data using not only short-rate data but also long yields. One important exception is that of Piazzesi (2005), which is fairly close to our study. However, we focus on monetary policy and its impact on the term structure of interest rates in China, a country with significantly different institutional features compared to that of the U.S. Only a small number of studies have tried to model China's bond yields, and, to our knowledge, no previous study has tried to show how domestic monetary policy directly affects bond yields. ${ }^{1}$

## 3 Monetary Policy and Official Rates in China

As a step in the economic reform program initiated in the end of the 1970s, the People's Bank of China became the country's central bank in 1984. During the first ten years, the primary goal of the central bank was to balance economic growth and inflation. However, even though the economy grew at an impressive rate, inflation was more difficult to control. As a natural result of extreme growth numbers in some years, the inflation spiked several times, including in 1985, 1989 and 1994. Caused primarily by excessive lending to state owned enterprizes, inflation came close to $25 \%$ in 1994. To deal with these outbursts in inflation, the central bank increased the official rates and tightened money supply. In 1995, new central bank guidelines explicitly addressed the inflation problem and made it the top priority for the monetary authorities. This change coincided with the Asian financial crisis

[^1]two years later, which resulted in a fast drop in demand and a cooling down of the domestic economy. As GDP growth dropped below 8\%, the inflation and the interest rates followed suit. During the midst of the regional financial crisis, there was a short period of deflation in China, resulting in very low official and market interest rates. The interest rates remained low until 2005, when inflation started to pick up and the central bank began to adjust the official rates in a number of raises.

The People's Bank of China uses a number of different tools to conduct its monetary policy. The money supply was initially controlled through a system of national bank credit quota, in which the central bank directly controlled the amount of money that each of the banks could use. In October 1998, a major reform resulted in the move to open market operations when the central bank introduced cash bond trading. In 2003, the central bank finally began issuing bills through regular auctions. China's central bank also conducts monetary policy through the fine-tuning of reserve requirements and a general guidance of credit orientation. Even though there is a functioning short-term money market in which the central bank theoretically can control money supply by using repos, this is not an effective channel for monetary policy. Due to excess liquidity, banks are not responding to changes in money market rates. At the same time, outstanding bonds are still quite limited in terms when measured against the size of the economy. With a constant excess demand for bonds, Chinese banks usually have to simply deposit their excess funds with the central bank. The People's Bank of China pays an interest on such excess cash. The fact that the money market in China is not functioning as it does in developed countries means that money market rates are typically not used as benchmark rates. The so-called CHIBOR (China Interbank Offered Rates) came into operation in 1996 to allow for banks to fix interest rates for interbank lending and borrowing. However, the CHIBOR rates have not turned to be a good benchmark tool, mainly because the trading volume of interbank funding activities have been so small. In an attempt to improve the situation, the so-called SHIBOR (Shanghai Interbank

Offered Rate) was launched in January 2007. SHIBOR is set daily and is based on offered rates from 16 banks. The SHIBOR is generally seen as a more marketsensitive benchmark since it is based on quoted rates (CHIBOR is based on actual traded rates). Even though the Chinese government has worked to develop the interbank rates, in practice the traditional traditional official rates are still commonly used as benchmarks by market participants and analysts. The deposit rates, and especially the one-year deposit interest rate, are arguably the most commonly used benchmarks in the bond market. The three-month, one-year, two-year, three-year, and five-year deposit interest rates are shown in Figure 1. The figure shows that the deposit rate is changed quite infrequently. Furthermore, it does not follow the same pattern as a typical official rate in developed countries. For instance, the Federal Fund rate changes in increments of 25 basis points (bps). In China, the official rate is denoted as a nominal annual rate, and changes in multiples of 9 bps . After tax and transferring it into continuous compounding, the changes have no distinct pattern.
[FIGURE 1 HERE]

Figure 1 shows that the central bank has used the deposit rates to deal with the decrease in economic activity after the Asian financial crisis by lowering the official rates. Similarly, the deposit rates have been increased a number of times since 2005 as a response to the inflationary tendencies discussed above. Overall, the official interest rates, and especially the one-year deposit rate, are important policy and benchmark instruments. We therefore believe that the strong focus on the central bank's one-year deposit rate makes it ideal to use as an instrument for monetary policy when modeling bond yields in China

## 4 The Official Rate and Bond Yields

As mentioned earlier, the interest rate that has the most significance in China is that of the one-year deposit rate. We will therefore use it as a state variable that
affects bond yields at different maturities. From the beginning of 1998 to the end of 2007, the one-year deposit rate has been changed 14 times, and has been affected two more times as a result of tax adjustments, which gives us a total number of 15 changes (one of the tax adjustments occurred at the same time as the official rate was changed). For simplicity, we assume that the way the central bank impose changes on the one-year deposit rate follows a constant Poisson process. As mentioned, the policy set by the People's Bank differs from that of the Federal Reserve in that the changes are not made in increments of 25 bps . This means that we need to use a different specification for the official rate changes from that of earlier studies that focus on the U.S. target rate. We assume that the size of the changes in the one-year deposit rate follows a stochastic process:

$$
\begin{equation*}
d r_{t}^{d}=x_{t} d N(h), \tag{1}
\end{equation*}
$$

where $d N(h)$ is the increment in a Poisson process with intensity rate $h$. The jump process reflects the number of times the central bank adjusts the deposit rate and when the change takes place. $x_{t}$ represents the jump size. The nature of the jumps during the sample periods leads us to suggest that the jump size can be modeled as the following stochastic process:

$$
\begin{equation*}
d x_{t}=-\kappa_{x} x_{t} d t+\sigma_{x} d \omega_{t}^{x}, \tag{2}
\end{equation*}
$$

The government bond prices and thus the market interest rates in China behave differently from the official deposit rates. Government bonds are traded frequently and their prices thus react more quickly to changes in the economy. They also reflect money supply, i.e. whether monetary policy is tight or loose as well as the relationship between government bonds and the deposit rates. One way of looking at deposit rates is that they represent the commercial banks' cost for raising funds. One could therefore argue that investments such as those in government bonds should
not yield lower returns than what the commercial banks pay for the capital. This means that the deposit rate should be significantly lower than the government bond rates. Before 2005, this was generally the case, with the spread between the one-year market rate and the one-year deposit rate somewhere around 50 bps . Beginning in 2005, there has been a period of excess liquidity, thus forcing the market interest rates down below the deposit rates several times, even though they have never been below the total cost that the banks faces when raising funds. Because such a large part of the banks' funding comes as very short-term saving at a cost significantly lower than the one-year deposit rate, their cost is most often very low. Due to this, we use the difference between the one-year market rate and the one-year deposit rate as a factor that takes into account how the differences in bonds and deposit rates reflect changes in the economy. The difference between the two rates are thus used as a second factor that decides the bond yields at different maturities. The difference in the two rates can be written as:

$$
\begin{equation*}
s_{t}=r_{t}^{(1)}-r_{t}^{d} . \tag{3}
\end{equation*}
$$

Here, $r_{t}^{(1)}$ represents the one-year market rate. $s_{t}$ is assumed to follow an OrnsteinUhlenbeck process which includes a mean-reverting feature typically seen in the standard Vasicek model. The process can be written as:

$$
\begin{equation*}
d s_{t}=\kappa_{s}\left(\mu_{s}-s_{t}\right) d t+\sigma_{s} d \omega_{t}^{s} \tag{4}
\end{equation*}
$$

As mentioned, the difference between the two interest rates can be seen as incorporating other changes in the economy. It may, for instance, embody the direct effects of money supply and inflation. For example, if money supply increases, liquidity in the financial system goes up. This can in turn result in a decrease in the market interest rates and thus a decrease in the difference between the one-year market interest rate and the one-year deposit rate.

To keep the model as simple as possible, it is assumed that random shocks in the two processes $x_{t}$ and $s_{t}$ are independent from each other:

$$
\begin{equation*}
\operatorname{cov}\left(d \omega_{t}^{x}, d \omega_{t}^{s}\right)=0 \tag{5}
\end{equation*}
$$

From the discussion above, we know that bond prices and market interest rates are decided by the one-year deposit rate, the difference between the one-year market interest rate and the one-year deposit rate and the size of the jumps in the oneyear deposit rate. Setting the par value equal to 1 , the price for a zero-coupon bond at time $\tau$ can be written as $P\left(r_{t}^{d}, s_{t}, x_{t} ; \tau\right)$. The one-year deposit interest rate movements and changes in the difference between the one-year market interest rate and the one-year deposit interest rate are the two important sources of uncertainty in bond prices. The price is thus decided based on their respective level of risk and is determined via the stochastic discount factor. We assume the following process for the stochastic discount factor:

$$
\begin{equation*}
\frac{d \xi_{t}}{\xi_{t}}=-r_{t} d t-\left(\lambda_{0 s}+\lambda_{1 s} s_{t}\right) d \omega_{t}^{s}-\lambda_{J}\left(d N_{t}-h d t\right) \tag{6}
\end{equation*}
$$

where $\lambda_{0 s}$ and $\lambda_{1 s}$ are coefficients for the market price of diffusion risk and $\lambda_{J}$ is the market price of the jump risk. Since $E_{t}\left[\lambda_{J} d N_{t}\right]=\lambda_{J} h d t$, the final part in the last parenthesis, $h d t$, is needed in order to ensure that the expected value of $d \xi_{t} / \xi_{t}$ is equal to $-r_{t} d t$. The expression for the discount factor resembles that of Das and Foresi (1996). In Equation (6), it is assumed that the size of the risk premium of the bond and the size of the difference between the market interest rate and the deposit rate are related. Following standard affine models, it is assumed that the short-term market interest rate is a linear function of a constant and the three state variables:

$$
\begin{equation*}
r_{t}=c_{0}+c_{1} r_{t}^{d}+c_{2} s_{t}+c_{3} x_{t} \tag{7}
\end{equation*}
$$

Using the expression for the state variables in Equations (1), (2), and (4) together with the expressions for the stochastic discount factor in Equation (6) and the shortterm interest rate in Equation (7), the bond price $P\left(r_{t}^{d}, s_{t}, x_{t} ; \tau\right)$ is assumed to follow a linear form:

$$
\begin{equation*}
P\left(r_{t}^{d}, s_{t}, x_{t} ; \tau\right)=\exp \left[-A(\tau)-B_{1}(\tau) r_{t}^{d}-B_{2}(\tau) s_{t}-B_{3}(\tau) x_{t}\right] \tag{8}
\end{equation*}
$$

where $A, B_{1}, B_{2}$, and $B_{3}$ are functions of the maturity $\tau . A(\tau)$ can be seen as a constant, while $B_{1}(\tau), B_{2}(\tau)$ and $B_{3}(\tau)$ embody the sensitivity of the bond price to the deposit rate, the difference between market rate and deposit rate and the jump size, respectively. Using Ito's lemma, the bond price satisfies the following relationship:

$$
\begin{align*}
& -P_{\tau}\left(r_{t}^{d}, s_{t}, x_{t} ; \tau\right)+P_{s}\left(r_{t}^{d}, s_{t}, x_{t} ; \tau\right)\left[\kappa_{s}\left(\mu_{s}-s_{t}\right)-\sigma_{s}\left(\lambda_{0 s}+\lambda_{1 s} s_{t}\right)\right] \\
& -P_{x}\left(r_{t}^{d}, s_{t}, x_{t} ; \tau\right) \kappa_{x} x_{t}+\frac{1}{2} \sigma_{s}^{2} P_{s s}\left(r_{t}^{d}, s_{t}, x_{t} ; \tau\right)+\frac{1}{2} \sigma_{x}^{2} P_{x x}\left(r_{t}^{d}, s_{t}, x_{t} ; \tau\right)  \tag{9}\\
& -r P+h\left(1-\lambda_{J}\right)\left[P\left(r_{t}^{d}+x_{t}, s_{t}, x_{t} ; \tau\right)-P\left(r_{t}^{d}, s_{t}, x_{t} ; \tau\right)\right] \\
& =0 .
\end{align*}
$$

By substituting (7) and (8) into (9), we get

$$
\begin{align*}
& -P\left[-A^{\prime}(\tau)-B_{1}^{\prime}(\tau) r_{t}^{d}-B_{2}^{\prime}(\tau) s_{t}-B_{3}^{\prime}(\tau) x_{t}\right]-P B_{2}(\tau)\left[\kappa_{s}\left(\mu_{s}-s_{t}\right)\right. \\
& \left.-\sigma_{s}\left(\lambda_{0 s}+\lambda_{1 s} s_{t}\right)\right]-P B_{3}(\tau) \kappa_{x} x_{t}+\frac{1}{2} P \sigma_{s}^{2} B_{2}^{2}(\tau)+\frac{1}{2} P \sigma_{x}^{2} B_{3}^{2}(\tau)  \tag{10}\\
& -\left(c_{0}+c_{1} r_{t}^{d}+c_{2} s_{t}+c_{3} x_{t}\right) P+h\left(1-\lambda_{J}\right) P\left[\exp \left(-B_{1}(\tau) x_{t}\right)-1\right] \\
& =0 .
\end{align*}
$$

We can then use a Taylor expansion to obtain an approximation for the last part in
(10):

$$
\begin{equation*}
\exp \left(-B_{1}(\tau) x_{t}\right)-1 \approx-B_{1}(\tau) x_{t} \tag{11}
\end{equation*}
$$

By combining (10) and (11), we get

$$
\begin{align*}
& {\left[-A^{\prime}(\tau)-B_{1}^{\prime}(\tau) r_{t}^{d}-B_{2}^{\prime}(\tau) s_{t}-B_{3}^{\prime}(\tau) x_{t}\right]+B_{2}(\tau)\left[\kappa_{s}\left(\mu_{s}-s_{t}\right)\right.} \\
& \left.-\sigma_{s}\left(\lambda_{0 s}+\lambda_{1 s} s_{t}\right)\right]-B_{3}(\tau) \kappa_{x} x_{t}-\frac{1}{2} \sigma_{s}^{2} B_{2}^{2}(\tau)-\frac{1}{2} \sigma_{x}^{2} B_{3}^{2}(\tau)  \tag{12}\\
& +\left(c_{0}+c_{1} r_{t}^{d}+c_{2} s_{t}+c_{3} x_{t}\right)+h\left(1-\lambda_{J}\right) B_{1}(\tau) x_{t} \\
& =0 .
\end{align*}
$$

For the expression in (12) to be true, the following equations for $B_{1}(\tau), B_{2}(\tau), B_{3}(\tau)$, and $A(\tau)$ must hold:

$$
\begin{align*}
& B_{1}^{\prime}(\tau)=c_{1}  \tag{13}\\
& B_{2}^{\prime}(\tau)=-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right) B_{2}(\tau)+c_{2}  \tag{14}\\
& B_{3}^{\prime}(\tau)=-\kappa_{x} B_{3}(\tau)+h\left(1-\lambda_{J}\right) B_{1}(\tau)+c_{3}  \tag{15}\\
& A^{\prime}(\tau)=\left(\kappa_{s} \mu_{s}-\sigma_{s} \lambda_{0 s}\right) B_{2}(\tau)-\frac{1}{2} \sigma_{s}^{2} B_{2}^{2}(\tau)-\frac{1}{2} \sigma_{x}^{2} B_{3}^{2}(\tau)+c_{0} \tag{16}
\end{align*}
$$

We know that the price of a zero-coupon bond with a par value of 1 is equal to 1 when time to maturity is 0 , which means that we have the boundary conditions for the expression above. We can write this as:

$$
\begin{equation*}
P\left(r_{t}^{d}, s_{t}, x_{t} ; \tau\right)=\exp \left[-A(0)-B_{1}(0) r_{t}^{d}-B_{2}(0) s_{t}-B_{3}(0) x_{t}\right]=1, \tag{17}
\end{equation*}
$$

which means that

$$
\begin{equation*}
A(0)=B_{1}(0)=B_{2}(0)=B_{3}(0)=0 . \tag{18}
\end{equation*}
$$

The one-year market interest rate is equal to the one-year deposit interest rate plus the interest rate difference $s_{t}$. Furthermore, we know that the one-year market rate is determined by the price of a zero-coupon bond with time-to-maturity of one-year as $r_{t}^{(1)}=-\left(\ln \left[P_{t}(1)\right]\right)$. We thus have the following relationship:

$$
\begin{equation*}
-\ln \left[P\left(r_{t}^{d}, s_{t} ; 1\right)\right]=A(1)+B_{1}(1) r_{t}^{d}+B_{2}(1) s_{t}+B_{3}(1) x_{t}=r_{1}^{d}+s_{t}, \tag{19}
\end{equation*}
$$

which means that the following must hold:

$$
\begin{equation*}
A(1)=0, \quad B_{1}(1)=1, \quad B_{2}(1)=1, \quad B_{3}(1)=0 . \tag{20}
\end{equation*}
$$

Using Equation (13) together with the boundary conditions in (18) and (20), we obtain the following expression for $B_{1}(\tau)$ :

$$
\begin{equation*}
B_{1}(\tau)=\tau \tag{21}
\end{equation*}
$$

Similarly, using Equation (14) together with the boundary conditions in (18) and (20), we get the following expression for $B_{2}(\tau)$ :

$$
\begin{equation*}
B_{2}(\tau)=\frac{1}{1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)\right]}\left\{1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right) \tau\right]\right\} . \tag{22}
\end{equation*}
$$

Finally, using Equation (15) together with the boundary conditions in (18) and (20), we obtain an expression for $B_{3}(\tau)$ :

$$
\begin{equation*}
B_{3}(\tau)=\frac{h\left(1-\lambda_{J}\right)}{\kappa_{x}}\left[-\tau+\frac{1-\exp \left(-\kappa_{x} \tau\right)}{1-\exp \left(-\kappa_{x}\right)}\right] . \tag{23}
\end{equation*}
$$

Substituting (21), (22), and (23) into (16) yields:

$$
\begin{align*}
A^{\prime}(\tau)= & \left(\kappa_{s} \mu_{s}-\sigma_{s} \lambda_{0 s}\right) \frac{1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right) \tau\right]}{1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)\right]} \\
& -\frac{1}{2} \frac{\sigma_{s}^{2}}{\left(1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)\right]\right)^{2}}\left[1-2 \exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right) \tau\right]\right.  \tag{24}\\
& \left.+\exp \left[-2\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right) \tau\right]\right]-\frac{1}{2} \frac{\sigma_{x}^{2} h^{2}\left(1-\lambda_{J}\right)^{2}}{\kappa_{x}^{2}} \\
& {\left[\tau^{2}-\frac{2 \tau-2 \tau \exp \left(-\kappa_{x} \tau\right)}{1-\exp \left(-\kappa_{x}\right)}+\frac{1-2 \exp \left(-\kappa_{x} \tau\right)+\exp \left(-2 \kappa_{x} \tau\right)}{\left(1-\exp \left(-\kappa_{x}\right)\right)^{2}}\right]+c_{0} }
\end{align*}
$$

Integrating on both sides and using the fact that $A(0)=0$ enable us to derive the following expression for $A(\tau)$ :

$$
\begin{align*}
A(\tau)= & \left(\kappa_{s} \mu_{s}-\sigma_{s} \lambda_{0 s}\right) \frac{\tau-\frac{1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right) \tau\right]}{\kappa_{s}+\lambda_{1 s} \sigma_{s}}}{1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)\right]}-\frac{1}{2} \frac{\sigma_{s}^{2}}{\left(1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)\right]\right)^{2}} \\
& {\left[\tau-2 \frac{1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right) \tau\right]}{\kappa_{s}+\lambda_{1 s} \sigma_{s}}+\frac{1-\exp \left[-2\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right) \tau\right.}{2\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)}\right] } \\
- & \frac{1}{2} \frac{\sigma_{x}^{2} h^{2}\left(1-\lambda_{J}\right)^{2}}{\kappa_{x}^{2}}\left[\frac{1}{3} \tau^{3}-\frac{\tau^{2}-2\left[\frac{1-\exp \left(-\kappa_{x} \tau\right)}{\kappa_{x}^{2}}-\frac{\tau \exp \left(-\kappa_{x} \tau\right)}{\kappa_{x}}\right]}{1-\exp \left(-\kappa_{x}\right)}\right.  \tag{25}\\
& \left.+\frac{\tau-2 \frac{1-\exp \left(-\kappa_{x} \tau\right)}{\kappa_{x}}+\frac{1-\exp \left(-2 \kappa_{x} \tau\right)}{2 \kappa_{x}}}{\left[1-\exp \left(-\kappa_{x}\right)\right]^{2}}\right]+c_{0} \tau .
\end{align*}
$$

Substituting $A(1)=0$ into (25) makes it possible to extract the following expression for $c_{0}$ :

$$
\begin{align*}
c_{0}= & -\left(\kappa_{s} \mu_{s}-\sigma_{s} \lambda_{0 s}\right) \frac{1-\frac{1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)\right]}{\kappa_{s}+\lambda_{1 s} \sigma_{s}}}{1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)\right]}+\frac{1}{2} \frac{\sigma_{s}^{2}}{\left(1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)\right]\right)^{2}} \\
& {\left[1-2 \frac{1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)\right]}{\kappa_{s}+\lambda_{1 s} \sigma_{s}}+\frac{1-\exp \left[-2\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)\right]}{2\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)}\right] }  \tag{26}\\
& +\frac{1}{2} \frac{\sigma_{x}^{2} h^{2}\left(1-\lambda_{J}\right)^{2}}{\kappa_{x}^{2}}\left[\frac{1}{3}-\frac{1-2\left[\frac{1-\exp \left(-\kappa_{x}\right)}{\kappa_{x}^{2}}-\frac{\exp \left(-\kappa_{x}\right)}{\kappa_{x}}\right]}{1-\exp \left(-\kappa_{x}\right)}\right. \\
& \left.+\frac{1-2 \frac{1-\exp \left(-\kappa_{x}\right)}{\kappa_{x}}+\frac{1-\exp \left(-2 \kappa_{x}\right)}{2 \kappa_{x}}}{\left[1-\exp \left(-\kappa_{x}\right)\right]^{2}}\right] .
\end{align*}
$$

The $\tau$-period yield, written as $r_{t}^{(\tau)}$, and the price of a zero-coupon with time-tomaturity of $\tau$ and a par value of 1 has the following relationship:

$$
\begin{equation*}
r_{t}^{(\tau)}=-\frac{1}{\tau} \ln P\left(r_{t}^{d}, s_{t} ; \tau\right) . \tag{27}
\end{equation*}
$$

Using the expression for the price in Equation (25), we derive an expression for the yield for bonds with maturity at time $\tau$ as:

$$
\begin{align*}
r_{t}^{(\tau)}= & \frac{A(\tau)}{\tau}+\frac{B_{1}(\tau)}{\tau} r_{t}^{d}+\frac{B_{2}(\tau)}{\tau} s_{t}+\frac{B_{3}(\tau)}{\tau} x_{t} \\
= & \frac{A(\tau)}{\tau}+r_{t}^{d}+\frac{1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right) \tau\right]}{1-\exp \left[-\left(\kappa_{s}+\lambda_{1 s} \sigma_{s}\right)\right]} \frac{1}{\tau} s_{t}  \tag{28}\\
& +\frac{h\left(1-\lambda_{J}\right)}{\kappa_{x}}\left[\frac{1-\exp \left(-\kappa_{x} \tau\right)}{\left[1-\exp \left(-\kappa_{x}\right)\right]} \frac{1}{\tau}+1\right] x_{t}
\end{align*}
$$

Finally, using Equation (10), the instantaneous risk premium on a zero-coupon bond with par value of 1 and maturity at time $\tau$ is:

$$
\begin{equation*}
E x R_{t}^{(\tau)}=-B_{2}(\tau) \sigma_{s}\left(\lambda_{0 s}+\lambda_{1 s} s_{t}\right)-B_{1}(\tau) h \lambda_{J} x_{t} \tag{29}
\end{equation*}
$$

The excess return of the bond thus depends on the risk parameters $\lambda_{0 s}$ and $\lambda_{1 s}$, the parameter that embodies the variation in the difference between the market interest rate and the deposit rate, $\sigma_{s}$, the jump intensity, $h$, and the state variables $s_{t}$ and $x_{t}$. Finally, it also depends on $B_{1}(\tau)$ and $B_{2}(\tau)$, the variables that reflect a bond's sensitivity to the state variables.

## 5 Methodology and Empirical Results

### 5.1 Estimation Procedure

In order to estimate the parameters in the model, we first need to discretize the original continuous specification. As seen above, the one-year deposit rate is assumed to follow a stochastic jump process. For simplicity, we assume that the deposit rate
is changed at the most once every month. The deposit rate can then be expressed in the following way:

$$
\begin{equation*}
r_{t+\Delta t}^{d}=r_{t}^{d}+x_{t+\Delta t} \eta_{t+\Delta t}, \tag{30}
\end{equation*}
$$

where

$$
\eta_{t+\Delta t}= \begin{cases}1 & h \Delta t  \tag{31}\\ 0 & 1-h \Delta t\end{cases}
$$

Based on Equation (4), the process of the difference between the one-year market rate and the one-year deposit rate, $s_{t}$, can be written in discrete form as:

$$
\begin{equation*}
s_{t+\Delta t}=s_{t} e^{-\kappa_{s} \Delta t}+\mu_{s}\left(1-e^{-\kappa_{s} \Delta t}\right)+\sigma_{s} e^{-\kappa_{s}(t+\Delta t)} \int_{t}^{t+\Delta t} e^{\kappa_{s} u} d \omega_{u}^{s} \tag{32}
\end{equation*}
$$

which means that the difference between the one-year market interest rate and the one-year deposit rate follows a normal distribution:

$$
\begin{equation*}
s_{t+\Delta t}=N\left(E_{t}\left(s_{t+\Delta t}\right), \operatorname{var}_{t}\left(s_{t+\Delta t}\right)\right) \tag{33}
\end{equation*}
$$

The mean and variance of $s_{t+\Delta t}$ can then be written as:

$$
\begin{align*}
E_{t}\left(s_{t+\Delta t}\right) & =s_{t} e^{-\kappa_{s} \Delta t}+\mu_{s}\left(1-e^{-\kappa_{s} \Delta t}\right)  \tag{34}\\
\operatorname{var}_{t}\left(s_{t+\Delta t}\right) & =\sigma_{s}^{2} e^{-2 \kappa_{s}(t+\Delta t)} \int_{t}^{t+\Delta t} e^{2 \kappa_{s} u} d u \\
& =\sigma_{s}^{2} \frac{1-e^{-2 \kappa_{s} \Delta t}}{2 \kappa_{s}} \tag{35}
\end{align*}
$$

The state variable $x_{t}$ can be expressed the following way in discrete form:

$$
\begin{equation*}
x_{t+\Delta t}=x_{t} e^{-\kappa_{x} \Delta t}+\sigma_{x} e^{-\kappa_{x}(t+\Delta t)} \int_{t}^{t+\Delta t} e^{\kappa_{x} u} d \omega_{u}^{x} \tag{36}
\end{equation*}
$$

Again, the state variable follows a normal distribution:

$$
\begin{equation*}
x_{t+\Delta t}=N\left(E_{t}\left(x_{t+\Delta t}\right), \operatorname{var}_{t}\left(x_{t+\Delta t}\right)\right), \tag{37}
\end{equation*}
$$

where the mean and variance can be expressed as:

$$
\begin{align*}
& E_{t}\left(x_{t+\Delta t}\right)=x_{t} e^{-\kappa_{x} \Delta t}  \tag{38}\\
& \operatorname{var}_{t}\left(x_{t+\Delta t}\right)=\sigma_{x}^{2} \frac{1-e^{-2 \kappa_{x} \Delta t}}{2 \kappa_{x}} \tag{39}
\end{align*}
$$

Finally, we assume that the observed market interest rates $y_{t}^{(\tau)}$, include a random error, $\epsilon_{t}^{(\tau)}$. The error term can be an error caused when fitting the bond yields or due to some other factor such as trading noise. The market interest rates can then be written as:

$$
\begin{aligned}
y^{(\tau)} & =r_{t}^{(\tau)}+\epsilon_{t}^{(\tau)} \\
& =\frac{A(\tau)}{\tau}+\frac{B_{1}(\tau)}{\tau} r_{t}^{d}+\frac{B_{2}(\tau)}{\tau} s_{t}+\frac{B_{3}(\tau)}{\tau} x_{t}+\epsilon_{t}^{(\tau)} \quad(\tau=1,2,3,4,5)(40)
\end{aligned}
$$

It is assumed that the error terms $\epsilon_{t}^{(\tau)}$ are independent of each other and that they follow a normal distribution:

$$
\begin{equation*}
\epsilon_{t}^{(\tau)} \sim N\left(0, \sigma_{\epsilon}^{2}\right) \quad \text { for } \tau=1,2,3,4,5 \tag{41}
\end{equation*}
$$

Since the likelihood function of a model with latent stochastic factors cannot be computed analytically in closed form, we estimate the model using Markov chain Monte Carlo (MCMC) simulation. Here, we introduce the concept of MCMC estimation (for a more complete overview of how MCMC works and how it is used in
finance, see Tsay, 2005).
Suppose we have $j$ number of parameters in the model that we want to estimate Furthermore, $Y$ is the observable data and $M$ denotes the model that we have specified. We want to estimate the parameters in the model in order to be able to make inference. Even though the likelihood function is difficult to obtain, we can still identify the conditional distributions of the parameters given the data, i.e. $p_{i}\left(\theta_{i} \mid \theta_{j \neq i}, Y, M\right)$ is the distribution of the parameter $\theta_{i}$ conditional on all the other parameters, the data, and the specified model. When applying MCMC, we do not need to know the explicit forms of the different conditional distributions. Instead, we draw random numbers from each of them. Using starting values for all parameters, $\theta_{i}^{(0)}$, the parameters can be sampled using the following Markov chain:

1. draw $\theta_{1}^{(1)}$ from $p_{1}\left(\theta_{1} \mid \theta_{2}^{(0)}, \theta_{3}^{(0)}, \ldots, \theta_{m}^{(0)}, Y, M\right)$
2. draw $\theta_{2}^{(1)}$ from $p_{2}\left(\theta_{2} \mid \theta_{1}^{(1)}, \theta_{3}^{(0)}, \ldots, \theta_{m}^{(0)}, Y, M\right)$
m. draw $\theta_{m}^{(1)}$ from $p_{m}\left(\theta_{m} \mid \theta_{1}^{(1)}, \theta_{2}^{(1)}, \ldots, \theta_{m-1}^{(1)}, Y, M\right)$

This completes one iteration in so-called Metropolis-Gibbs sampling. Using the new information from the last draw, the process is repeated $n$ times so that we end up with a sequence of draws:

$$
\left(\theta_{1}^{(1)}, \theta_{2}^{(1)}, \ldots, \theta_{m}^{(1)}\right),\left(\theta_{1}^{(2)}, \theta_{2}^{(2)}, \ldots, \theta_{m}^{(2)}\right), \ldots,\left(\theta_{1}^{(n)}, \theta_{2}^{(n)}, \ldots, \theta_{m}^{(n)}\right)
$$

Under weak regularity conditions, as $n$ becomes large, the drawn values for the parameters are approximately the same as the random draws from the joint distribution $p_{i}\left(\theta_{1}, \theta_{2}, \ldots, \theta_{m} \mid Y, M\right)$. In practice, a number of initial generated samples, commonly know as "burn-ins," are used to achieve convergence. These burn-in samples are then
discarded. Finally, the remaining sample is used to make inference of parameters by way of different location and dispersion measures, e.g. mean and variance. We use WinBugs to estimate the model. We first perform 4,000 iterations, discard them as burn-ins, and then make inference based on the following 11,000 iterations. The vector of observables contains the $\tau$-year yields, where $\tau=1,2,3,4,5$.

We also need priors for $h, \kappa_{s}, \mu_{s}, \sigma_{s}, \kappa_{x}, \mu_{x}, \sigma_{\epsilon}^{2}, \lambda_{0 s}, \lambda_{1 s}$, and $\lambda_{J}$. Looking at Figure 1, we see that People's Bank of China changes the deposit rates quite infrequently, which means that we can assume that the jump intensity, $h$, should be small. We thus use the following prior distribution for the jump intensity at a monthly basis:

$$
\operatorname{Logit}(h / 12) \sim N(-10,100)
$$

The expression for the difference between the one-year deposit rate and the oneyear market interest rate has three parameters. $\kappa_{s}$ reflects the speed of the mean reversion in $s_{t}$ and we can assume that $\kappa_{s}>0$. We also assume that the parameter is normally distributed. Also, it is easy to assume that $\mu_{s}$, the long-term average of $s_{t}$ is normally distributed with a small mean. Finally, we follow a standard assumption of the volatility in the interest rate difference and use a gamma distribution for the inverted value. We thus use the following prior distributions:

$$
\kappa_{s} \sim N(0.2,1) I(0,) \quad \mu_{s} \sim N(0.2,4) \quad 1 / \sigma_{s}^{2} \sim \operatorname{Gamma}(0.1,0.1)
$$

Here, $N(0.2,1) I(0$,$) signifies a normal distribution with mean 0.2$ and variance 1 trimmed for negative values. Similar to $s_{t}$, the state variable $x_{t}$ has two unknown parameters. Again, $\kappa_{x}$ is the speed of return to the long-term value. It is assumed to be positive and follow a normal distribution. Again, the inverse of the variance is assumed to follow a gamma distribution. This means that the priors for the second
state variable can be written as:

$$
\kappa_{x} \sim N(0.2,1) I(0,) \quad 1 / \sigma_{s}^{2} \sim \operatorname{Gamma}(0.1,0.1)
$$

The standard deviation of the observed values for the market interest rate, seen in Equation (41), is assumed to follow the following distribution:

$$
1 / \sigma_{\epsilon}^{2} \sim \operatorname{Gamma}(0.1,0.1)
$$

Finally, we have the three risk parameters, $\lambda_{0 s}, \lambda_{1 s}$, and $\lambda_{J}$. We assume that they are small and normally distributed:

$$
\lambda_{0 s} \sim N(0,1) \quad \lambda_{1 s} \sim N(0,1) \quad \lambda_{J} \sim N(0,1)
$$

It should be noted that the prior distributions allow for the parameters to lie within a wide range of values. Furthermore, as the sampling grows, the influence of the initial values decreases. Having gone over the estimation procedure as well as the prior assumptions in detail, we now present the results of the MCMC estimations.

### 5.2 Data and Empirical Results

The data used to estimate the model include the one-year deposit rate and the one-, two-, three-, four-, and five-year market yields. We use data from January 1998 to December 2007. The time between each observation is $\Delta t=1 / 12$, or one month. This means that we end up with $T=120$ observations. The data is from the Shanghai Stock Exchange and is collected through the database Wind. Following Fisher, Nychka and Zervos (1995), we fit the term structure using smoothing splines that incorporate a roughness penalty. By applying the procedure to the original time series for bonds with different maturities, we obtain a yield curve for up to five years for the Chinese bond market.

The time series for the yields of different maturities are shown in Figure 2. During the period 1998 to 2000, the official rate was cut a number of times. Partly a result of the Asian financial crisis, the real GDP growth rate reached its lowest level since the beginning of the economic reforms in the end of the 1970s. The inflation rate decreased to such an extent that deflation became a problem during this period. In response to this macroeconomic development, the Chinese authorities began to decrease the official rates, including the one-year deposit rate (see Figure 1). As can be seen in Figure 2, the market interest rates followed suit. Due to very high levels of economic growth and signs of overheating, the authorities began to increase the official rates during the period 2005 to 2007. In between these two periods, the deposit rate was stable with only a few smaller changes. The difference between the one-year market interest rate and the one-year deposit rate tended to fluctuate more during this time.

## [FIGURE 2 HERE]

Table 1 presents summary statistics for the yields for bonds with different maturities as well as the one-year deposit rate. The mean of the short-term market market yield is small compared to long-term market yields, with the one-year and fiveyear yields having means of 2.63 and 3.53 respectively. However, the relationship is the opposite when it comes to standard deviation, with short-term market yields exhibiting comparatively higher levels of fluctuation. Also, the mean of the market rate is considerably higher than the mean of the deposit rate, while the standard deviation of the deposit rate is somewhat higher than the standard deviation of the market rate. Both the market interest rates and the deposit interest rate show signs of high kurtosis. Finally, the autocorrelation tests show that all market yields as well as the deposit interest rate exhibit high levels of serial correlation.

## [TABLE 1 HERE]

Having specified the model and the data used in the simulation procedure, we now
turn to the estimation and the empirical results. We use an initial burn-in sample of 4,000 iterations. We then store the following 11,000 iterations for each of the parameters and the latent variables. The saved iterations are finally used for inference and we compute the mean, media, standard deviation, and lower and upper confidence levels for each of the parameters. The results are presented in Table 2. We first look at the parameters influencing the difference between the official rate and the market interest rate, $s_{t}$. The speed of mean reversion in the difference between the official rate and the market interest rate, $\kappa_{s}$ is estimated to 0.24 , which means that the spread between the two interest rates reverts to the long-term average rather slowly and that the spread is lingering once it moves away from the long-term average value. The mean of the difference between the two rates is quite small at 0.11 with a relatively high standard deviation of 0.11 . The short-term market rate tends to follow the deposit rate, but the difference between the market interest rate and the deposit rate fluctuates significantly due to factors other than that of the deposit rates.

The two risk premia included in the expression for the difference between the market interest rate and the deposit rate are both negative. Looking at expression (29), we see that the risk in the interest rate difference variable results in investors demanding a higher return. The reward parameter for the jump risk is significantly positive. Again looking at expression (29), we see that if $x_{t}>0$, the central bank is more likely to increase the official rate. This will then force the bond prices down. Similarly, if $x_{t}<0$, the central bank is more likely to decrease the official rate. Focusing on the process of $x_{t}$, the parameter $\kappa_{x}$ is estimated to 0.41 . Besides the process for $x_{t}$, the expression for the one-year deposit rate in expression (1) shows that we need to take the jump process and the jump intensity rate $h$ into consideration when analyzing changes in the one-year deposit rate. The estimated value for $h$ is 1.07 . This means that the probability that the one-year deposit interest rate jumps in a one-month period is equal to $h / 12=1.07 / 12=0.09$. Finally,
the standard deviation of the error term in the expression for the bond yields, $\sigma_{\epsilon}$ is small but different from zero.
[TABLE 2 HERE]

Having estimated the model, we want to see how well the model is able to capture the movements in the bond yields at different maturities. When we use MCMC to estimate the yield curve model, we also obtain the sample of $r_{t}^{(\tau)}$ from its posterior distributions. The mean of the 11,000 iterations after the burns-in are used as estimates for $r_{t}^{(\tau)}$, which means that we use the smoothed estimate of $r_{t}^{(\tau)}$ from the sample. Unfortunately, we cannot produce filtered estimates for $r_{t}^{(\tau)}$. The summary statistics of the estimated yield curve are presented in Table 3. Comparing the statistics of the estimated yield curve with those of the actual yield curve in Table 1, we see that the model is able to produce good estimates for the different bond yields.
[TABLE 3 HERE]
Figure 3 gives a clear picture of how the estimated yield curves based on the new model compares to the actual yield curves. The first graph presents the one- and five-year actual and estimated yields, respectively, while the other graphs show the corresponding time series for two-, three-, and four-year maturities. All five of the estimated yields are very similar to those of the actual yields in the sample. The one- and five-year estimated yields show slightly larger deviations from the actual yields. The difference between the estimated and actual one- and five-year yields is somewhat more apparent when the change in the yield curve is large. During periods with only smaller changes, the difference is very close to zero. Overall, the model capture the movements in the different yields over time well.
[FIGURE 3 HERE]
The estimation of the affine model that incorporates the impact of changes in the official rate show that it is useful to take monetary policy into consideration when
analyzing the yield curve in China. The significant values for the latent factors indicate that the shifts in the official interest rate has an impact on China's bond yields. To better understand how the model performs, we compare the pricing performance of the new model with that of a standard Vasicek (1977) model in which the dynamics of the short rate can be written as:

$$
\begin{equation*}
r(t)=c+x(t), \tag{42}
\end{equation*}
$$

where $x(t)$ is a state vector that under risk-neutral probability satisfies:

$$
\begin{equation*}
d x(t)=-k x d t+\sigma d \omega(t) \tag{43}
\end{equation*}
$$

Here, $\omega(t)$ is again a Brownian motion. Having derived a closed-form solution for the model, we use a MCMC simulation procedure to estimate the model.

Having estimated the basic short-rate model discussed above, we then compare the performance with that of the new model that incorporates the one-year official interest rate. Following standard conventions in the literature, we report the mean absolute error (MAE) of the two models in Table 4. The new model performs well across all maturities.

## [TABLE 4 HERE]

The positive results in fitting the estimated yields to the actual ones indicate that the model adequately captures the main features of movements in the market interest rates. The model discussed here can be seen as a generalized Vasicek model that allows for the explicit influence of monetary policy in the form of changes in the official rate. Conventional models such as the traditional Vasicek (1977) assume that interest rates follow a normally distributed process and thus cannot fit the term structure of market interest rates with its significant skewness and kurtosis. The new model in this paper can therefore be seen as a significant improvement
compared to standard factor models that do not incorporate monetary policy in China.

## 6 Conclusion

Several recent research articles have focused on how monetary policy influences bond yields in the U.S. and Europe. We take a similar macro-finance approach when we model bond yields in China. While the so-called CHIBOR and SHIBOR were created to enhance the pricing mechanisms in the domestic bond market and improve the monetary transmission system, a number of alternative official rates are still used as benchmarks by market participants and analysts. Of the many different official rates in China, arguably the most often used benchmark interest rate is that of the one-year deposit interest rate. Since the one-year deposit rate has such a significant influence, we argue that it is suitable to include it as a state variable in a model for bond yields in China. We also include a second state variable, the difference between the deposit rate and the short term rate, which reflects other economic variables that influence market interest rates. We use an affine framework in which we model the two state variables explicitly and allow for the changes in the key official rate to occur in the form of jumps at varying magnitudes. Using monthly yield data for different maturities and the one-year deposit rate from the period 1998-2007, we then estimate the model with MCMC simulation. We show that the estimated bond yields fit the bond yields observed in the market well and that the model captures the main features of the bond yields at different maturities in terms of mean, standard deviation, skewness, and kurtosis. The model captures the movements in the yield curve during periods of rising, falling and stable interest rates. The estimates of the model parameters also show how the jump risk in the official interest rate and the difference between the short-term market interest rate and the short-term deposit rate influence the excess returns of bonds. Finally, a
simple comparison with a standard Vasicek (1977) model with one factor shows that the new model performs well with lower pricing errors across all maturities.

This study is an attempt to improve the understanding of how monetary policy influences bond yields in China. As seen in recent research on the U.S. and European term structures of interest rates, a joint modeling strategy that incorporates the short-term interest rate as a policy making instrument and the argument that long rates can be seen as risk-adjusted averages of expected future short rates when modeling the bond yields can have positive results. The results in this study imply that the combination of a macroeconomic approach and a finance approach to bond yields seem to work also in a setting that differs significantly from that of the U.S. and Europe. Given these initial encouraging results, a number of venues for future research opens up, including using alternative economic variables such as inflation and other measures of monetary policy as state variables when analyzing China's bond market.

## References

[1] Andersson, M., Dillen, H. \& Sellin, P., 2006. Monetary Policy Signaling and Movements in the Term Structure of Interest Rates, Journal of Monetary Economics, 53, 1815-55.
[2] Ang, A. \& Piazzesi, M., 2003. A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables, Journal of Monetary Economics, 50, 745-87.
[3] Balduzzi, P., Bertola, G. \& Foresi, S., 1997. A Model of Target Changes and the Term Structure of Interest Rates, Journal of Monetary Economics, 39, 223-49.
[4] Balduzzi, P., Bertola, G., Foresi, S. \& Klapper, L., 1998. Interest Rate Targeting and the Dynamics of Short-Term Rates, Journal of Money, Credit, and Banking, 30, 26-50.
[5] Brennan, M.J. \& Schwartz, E.S., 1979. A Continuous Approach to the Pricing of Bonds, Journal of Banking and Finance, 3, 133-55.
[6] Cox, J.C., Ingersoll, J.E. \& Ross, S.A., 1985. A Theory of the Term Structure of Interest Rates, Econometrica, 53, 385-402.
[7] Dai, Q. \& Singleton, K.J., 2000. Specification Analysis of Affine Term Structure Models, Journal of Finance, 55, 1943-78.
[8] Das, S., 2002. The Surprise Element: Jumps in Interest Rates, Journal of Econometrics, 106, 27-65.
[9] Das, S. \& Foresi, S., 1996. Exact Solutions for Bond and Option Prices with Systematic Jump Risk, Review of Derivatives Research, 1, 7-24.
[10] Diebold, F.X., Piazzesi, M. \& Rudebusch, G.D., 2005. Modeling Bond Yields in Finance and Macroeconomics, Federal Reserve Bank of San Francisco Working Paper Series 2005-4.
[11] Duffie, D. \& Kan, R., 1996. A Yield-Factor Model of Interest Rates, Mathematical Finance, 6, 379-406.
[12] Ellingsen, T. \& Söderström, U., 2001. Monetary Policy and Market Interest Rates, American Economic Review, 91, 1594-1607.
[13] Farnsworth, H. \& Bass, R., 2003. The Term Structure with Semicredible Targeting, Journal of Finance, 58, 839-65.
[14] Fisher, M., Nychka, D. \& Zervos, D., 1995. Fitting the Term Structure of Interest Rates with Smoothing Splines, Working Paper No. 95-1, Finance and Economics Discussion Series, Federal Reserve Board.
[15] Heath, D., Jarrow, R. \& Morton, A., 1992. Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation, Econometrica, 60, 77-105.
[16] Ho, T.S.Y. \& Lee, S.B., 1986. Term Structure Movements and Pricing Interest Rate Contingent Claims, Journal of Finance, 41, 1011-29.
[17] Hull, J. \& White, A., 1990. Pricing Interest Rate Derivative Securities, Review of Financial Studies, 3, 573-92.
[18] Johannes, M., 2004. The Statistical and Economic Role of Jumps in ContinuousTime Interest Rate Models, Journal of Finance, 59, 227-60.
[19] Piazzesi, M., 2005. Bond Yields and the Federal Reserve, Journal of Political Economy, 113, 311-44.
[20] Tsay, R.S., 2005. Analysis of Financial Time Series, New Jersey: John Wiley \& Sons.
[21] Vasicek, O.A., 1977. An Equilibrium Characterization of the Term Structure, Journal of Financial Economics, 5, 177-88.

Table 1: Descriptive Statistics for Monthly Interest Rates

|  |  |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Maturity | Mean | Std.Dev. | Skewness | Kurtosis | $\rho_{1}$ | $\rho_{2}$ | $\rho_{3}$ | $\rho_{6}$ | $\rho_{12}$ |
|  |  |  |  |  |  |  |  |  |  |
| Market Interest Rates |  |  |  |  |  |  |  |  |  |
| 1 | 2.63 | 0.94 | 1.64 | 6.06 | 0.89 | 0.78 | 0.68 | 0.50 | 0.19 |
| 2 | 2.91 | 0.92 | 1.44 | 5.40 | 0.90 | 0.80 | 0.70 | 0.50 | 0.15 |
| 3 | 3.16 | 0.91 | 1.20 | 4.65 | 0.92 | 0.82 | 0.72 | 0.50 | 0.11 |
| 4 | 3.37 | 0.91 | 1.02 | 4.07 | 0.93 | 0.83 | 0.73 | 0.51 | 0.07 |
| 5 | 3.53 | 0.90 | 0.93 | 3.73 | 0.93 | 0.84 | 0.75 | 0.51 | 0.05 |
|  |  |  |  |  |  |  |  |  |  |
| Deposit Interest Rate |  |  |  |  |  |  |  |  |  |
| 1 | 2.25 | 1.05 | 1.82 | 5.00 | 0.93 | 0.86 | 0.81 | 0.62 | 0.29 |

Note: $\rho_{i}$ shows the autocorrelation where $i$ indicates the number of lags.

Table 2: Parameter Estimates

|  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Parameter | Mean | Median | Std.Dev. | $95 \%$ CI Lower | $95 \%$ CI Upper |
|  |  |  |  |  |  |
| $h$ | 1.07 | 1.06 | 0.11 | 0.88 | 1.30 |
| $\kappa_{s}$ | 0.24 | 0.27 | 0.08 | 0.02 | 0.27 |
| $\mu_{s}$ | 0.11 | 0.19 | 0.11 | -0.27 | 0.47 |
| $\sigma_{s}$ | 0.48 | 0.48 | 0.01 | 0.44 | -2.11 |
| $\lambda_{0 s}$ | -2.34 | -2.62 | 0.14 | -2.62 | -0.04 |
| $\lambda_{1 s}$ | -0.49 | -0.57 | 0.17 | 0.87 |  |
| $\kappa_{x}$ | 0.41 | 0.37 | 0.14 | 0.25 | 1.07 |
| $\sigma_{x}$ | 0.87 | 0.89 | 0.11 | 0.62 | 0.72 |
| $\lambda_{J}$ | 0.66 | 0.66 | 0.04 | 0.54 | 0.13 |
| $\sigma_{\epsilon}$ | 0.09 | 0.08 | 0.02 | 0.07 |  |

Table 3: Descriptive Statistics for the Estimated Term Structure

|  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Maturity | Mean | Std.Dev. | Skewness | Kurtosis | $\rho_{1}$ | $\rho_{2}$ | $\rho_{3}$ | $\rho_{6}$ | $\rho_{12}$ |
|  |  |  |  |  |  |  |  |  |  |
| 1 | 2.62 | 0.92 | 1.51 | 5.48 | 0.92 | 0.82 | 0.72 | 0.52 | 0.20 |
| 2 | 2.89 | 0.90 | 1.42 | 5.26 | 0.92 | 0.82 | 0.72 | 0.51 | 0.15 |
| 3 | 3.18 | 0.90 | 1.29 | 4.90 | 0.92 | 0.82 | 0.72 | 0.51 | 0.11 |
| 4 | 3.38 | 0.90 | 1.16 | 4.52 | 0.92 | 0.82 | 0.73 | 0.50 | 0.08 |
| 5 | 3.50 | 0.91 | 1.04 | 4.19 | 0.92 | 0.83 | 0.73 | 0.50 | 0.06 |

Table 4: Model Comparison

|  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 1 Year | 2 Years | 3 Years | 4 Years | 5 Years |
|  |  |  |  |  |  |
| Vasicek Model | 0.475 | 0.240 | 0.108 | 0.262 | 0.425 |
| Official Rates Model | 0.173 | 0.130 | 0.094 | 0.067 | 0.067 |

Note: This table shows the MAE errors over the monthly sample from January 1998 to December 2007 for the standard Vasicek (1977) model and the new model that incorporates the one-year deposit interest rate as the benchmark rate.

Figure 1: Official Deposit Rates


Figure 2: Market Interest Rates $\left(y_{t}^{(1)}, y_{t}^{(2)}, y_{t}^{(3)}, y_{t}^{(4)}, y_{t}^{(5)}\right)$


Figure 3: Estimated $\left(r_{t}^{(\tau)}\right)$ and Actual $\left(y_{t}^{(\tau)}\right)$ Yields



[^0]:    *Corresponding author. E-mail: anders.johansson@hhs.se. Stockholm School of Economics, P.O. Box 6501, 11383 Stockholm, Sweden. Johansson gratefully acknowledges financial support from the Bank of Sweden Tercentenary Foundation and the Swedish Foundation for International Cooperation in Research and Higher Education.

[^1]:    ${ }^{1}$ The very few existing studies on China's bond yields are mostly published in Chinese journals.

