Stochastic Volatility and Pricing Bias in the Swedish OMX-Index Call Option Market

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Abstract

This paper investigates the pricing bias in the Swedish OMX-Index Option market and how a stochastic volatility affects European call option prices. The market is purely European and without dividends for the period studied. A CIR square-root process for the volatility is estimated with non-linear least square minimization, and stochastic volatility option prices are calculated through Fourier-Inversion. These call option prices are compared to Black-Scholes prices as well as observed market prices, and a well-defined bias structure between Stochastic Volatility prices and Black-Scholes prices is observed. With a dynamic hedging scheme, I demonstrate larger *ex ante* profits, excluding transaction costs, for traders using the stochastic volatility model rather than the Black-Scholes model.

JEL Classification Codes: C52, G13.
Keywords: derivatives pricing, stochastic volatility, Fourier inversion.

1 Introduction

An option is a derivative security and its value can, in principle, be determined if all underlying variables are specified. The Black-Scholes (1973) (henceforth "B-S") model is, of course, the outstanding model for this purpose. It is simple and elegant but builds on fairly restrictive assumptions, two of which, the constant stock return volatility and the constant interest rate, have been relaxed in a number of papers in the last decade.

Early studies of the Black-Scholes model and its pricing behavior include Macbeth and Merville (1979), Rubinstein (1985), and Evnine and Rudd (1985). In the case of option pricing with volatility modelled as a stochastic process, both stock and stock index options, (Hull and White (1987), Wiggins (1987), Scott (1987), Stein and Stein (1991), and Ball and Roma (1994)), and currency options (Chesney and Scott (1989), Melino and Turnball (1990), Heston (1993), and Bates (1996)) have been studied. There are also articles where the interest rate is assumed to be stochastic (Heston (1993), Amin and Ng (1993), and Saez (1995)). A common result is that an improvement in pricing (more efficient markets) follows with the inclusion of a stochastic volatility, while the impact of a stochastic interest rate seems less clear.

Several option-pricing models, with different assumptions regarding the return distribution of the underlying asset, have been developed; the vast majority of the models being based on continuous time stochastic processes and Ito calculus. When the model is specified, the option price must be solved for. Normally, this means solving a partial differential equation (PDE) and a number of methods are available. Whether direct numerical solving of the partial differential equation, Monte Carlo simulations, approximation methods, or a combination of numerical and analytical solution methods is used, depends on the kind of option to be priced as well as the processes chosen for the underlying assets. In addition, when introducing a non-traded underlying parameter like stochastic volatility, it is known from financial theory that a non-zero volatility risk premium must be introduced, which complicates the search for the option price, although not in a critical way.

With a randomly changing volatility, the option price is no longer determined by a single stochastic variable, the stock index price, but a second stochastic variable, the volatility of the stock index return, is equally important. We end up having two underlying stochastic processes, two state variables, that may be specified in different ways and may or may not be correlated. In this paper, a Geometric Brownian Motion is assumed for the stock index price and a mean-reverting Cox-Ingersoll-Ross square-root process for the volatility (variance)

$$dS = mSdt + \sigma S dZ_1 \tag{1}$$

$$d\sigma^2 = \alpha(\theta - \sigma^2)dt + \xi\sqrt{\sigma^2}dZ_2, \tag{2}$$

where S is the stock index price, σ^2 is the volatility (variance) of the stock index return, α, θ, ξ, and m are constants, and $dZ_1$and dZ_2 are independent Wiener processes. The parameter α is the degree of mean reversion, θ is the long-run mean volatility, and ξ measures the volatility of the variance process. The choice of model for the volatility behavior is partly due to mathematical tractability where we can draw on interest rate theory and the bond pricing formula in Cox, Ingersoll and Ross (1985), and partly due to feasibility; empirically, volatility is never negative and it has a tendency to revert to a long-run average. Both these phenomena are covered by the mean-reverting square-root process.

To solve for the option price, I use the Feynman-Kac functional and the concept of risk neutrality, i.e. solving the PDE with a stochastic representation formula where the discounting is done with the risk-free rate of interest. In order to find the final stock index price distribution, I use the Fourier-Inversion method introduced by Stein and Stein (1991). They used this technique for the arithmetic Ornstein-Uhlenbeck process, and Ball and Roma (1994) modified the model for the CIR-process. In these studies, the prices given by the Fourier-Inversion model and the B-S model were compared but the pricing methods were not used to back out real-world parameters and biases. In this paper, the aim is to study the pricing bias in the Swedish OMX-Index call option market and the Fourier-Inversion model is used both to estimate volatility process parameters and to price options. While several empirical studies on stochastic volatility option pricing exist, most of these rely on Monte-Carlo methods to find the option price. By instead using the Fourier-Inversion method, I get a quicker and more flexible method. For comparison, I also calculate B-S prices in addition to stochastic volatility prices.

My choice of market is the Swedish OMX-Index option market and, to my knowledge, this is the first paper applying stochastic volatility option pricing methods to this particular market. The OMX-Index option market is smaller than the bigger stock option markets in the US and the UK, but it has many interesting features. All trade is done with a computer system, the contracts are purely European style, and most important, there are no dividends in the market for the time periods studied.

To assess the stability of the results, I have looked at two separate time periods, October 1993 to February 1994 and July 1994 to December 1994. Both these periods are relatively tranquil, compared to, for instance, late 1992 when the Swedish krona was under pressure. Using data from these periods, I back out parameters for the stochastic processes and with these estimates as input, I try to judge how efficient option prices are quoted. The fast Fourier-Inversion method proves to be useful in making it possible to back out the risk neutral (Q-measure) parameters from quoted option prices (compare yield-curve inversion). This is a fairly new approach in the area of option pricing, where most authors use historical stock-return data and moment methods to estimate volatility parameters (Bakshi et al. (1996)). The method has the advantage of directly giving the risk neutral volatility parameters and giving possibilities to infer the sign and size of the volatility risk premium.

The bias study in this paper is divided into a static and a dynamic part. The static study is done on a daily basis and compares, out-of-sample, the market-, the B-S-, and the stochastic volatility-prices by daily updating model inputs. The dynamic efficiency test consists of a hedging scheme, where a hedged position in two call options and the underlying index is formed and daily updated with suitable Δ:s, and where risk-free arbitrage profits are calculated, *ex ante*.

This paper is organized as follows. Chapter 2 looks at the pricing model and the Fourier-Inversion technique. Chapter 3 describes the OMX-Index Option Market and estimates the volatility parameters. Chapter 4 contains the static bias study and the dynamic efficiency test. Finally, chapter 5 concludes the paper.

2 The Model

2.1 Equilibrium Pricing and the Stochastic Volatility Model

In the Black-Scholes model, the call option has a unique price. This is related to the fact that in the B-S model, every contingent claim can be replicated by a self-financed portfolio. In other words, the B-S model is complete.

In the stochastic volatility model, the situation is different. Since volatility is not spanned by assets in the economy, the volatility-risk cannot be eliminated by arbitrage methods. Instead, we must rely on equilibrium methods. It then follows that the market price of volatility risk explicitly enters the general partial differential equation for the option price:

$$\frac{\partial F}{\partial t}(t,s) + rs\frac{\partial F}{\partial s}(t,s) + \frac{1}{2}s^2\sigma^2(t,s)\frac{\partial^2 F}{\partial s^2}(t,s) + \left[\alpha(\theta-\sigma^2)-\lambda\right]\frac{\partial F}{\partial(\sigma^2)}(t,s) + \quad (3)$$
$$+\frac{1}{2}\xi^2\sigma^2\frac{\partial^2 F}{\partial(\sigma^2)^2}(t,s) - rF(t,s) = 0$$

$$F(T,s) = \Phi(s),$$

where F is the option price, S is the price of the underlying asset (the OMX-index), σ^2 is the index return volatility, and finally, λ is the volatility risk premium.

The main difference between the present situation and the B-S setting is that in the B-S model, arbitrage methods are used to find the price, while here, equilibrium arguments are used. The option price will only be unique when supply and demand in the market are equalized, and the forces of supply and demand are, in turn, determined by such phenomena as risk aversion.

One way out is to find situations where the solution to the PDE is independent of risk preferences. This is the case if (a) the volatility is a traded asset or (b) the volatility is uncorrelated with aggregate consumption (Hull and White (1987)). An alternative way is to treat the volatility as the non-traded parameter it actually is but putting the risk premium equal to zero, which is done by Scott (1987) and Hull and White (1987).

The exact form of the risk premium might not be found and one might not be comfortable with assuming a zero risk premium. Then there is the special case of a non-zero constant risk premium for the volatility that does not actually change our solution method *or* the results in any profound way. In this paper, I will assume a non-zero constant risk premium, so that a risk adjusted drift rate for the volatility can be defined in (2). The drift rate $\alpha(\theta - \sigma^2)$ changes to $\alpha(\theta' - \sigma^2)$, where the only change is a shift in the constant long-run mean[1]. λ has now disappeared from (3) and the new parameters, α, θ', and ξ are called risk-adjusted parameters, or Q-parameters[2].

2.2 The Fourier-Inversion Technique and Stochastic Volatility Option Pricing

The Fourier Inversion method as a technique to find the stock price distribution was introduced by Stein and Stein (1991), who focused on the similarity between Moment Generating Functions (MGFs) and Fourier Transforms and combined this with the averaging over time of the stock price variance. Ball and Roma (1994) continued this work by even further emphasizing the important role of the average variance and, in particular, the MGF of the average variance, also showing its importance in other solution methods. Heston (1993) developed a slightly different model and suggested the use of a square-root process, which has the advantage of always giving positive volatilities and being familiar from earlier work by Cox, Ingersoll and Ross (1985) in the different, but related, context of bond pricing.

The closed-form solution of the option price in (3), $F(t)$, is derived by following Stein and Stein (1991) and Ball and Roma (1994) by applying the Feynman-Kac functional (risk adjusted

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[1] In the PDE, all parameters are assumed to be constant. If further assuming a constant risk premium λ, then the variable sustitution $\alpha(\theta - \sigma^2) - \lambda = \alpha(\theta - \frac{\lambda}{\alpha} - \sigma^2) = \alpha(\theta' - \sigma^2)$ can be made.

[2] Under the assumption of a constant risk premium and if the ordinary parameter θ can somehow be found, an estimate of the risk premium, λ, can also be obtained.

expectation)

$$F(t,S) = \frac{1}{e^{r(T-t)}} \int_X^\infty (S_T - X) f(S_T) dS_T \tag{4}$$

to the non-lognormal stock return distribution

$$f(S_T) = \frac{e^{\frac{m(T-t)}{2}}}{2\pi S_T^{\frac{3}{2}}} \int_{-\infty}^{\infty} I[\frac{(\eta^2 + \frac{1}{4})(T-t)}{2}] cos[(ln(S_T) - m(T-t))\eta] d\eta, \tag{5}$$

where (5) is derived by using the similarity between Moment Generating Functions and Fourier-Transforms. We end up with the following expression for the call option price, where S_T is the underlying stock index value at the exercise date, X is the strike price, and η merely an integration variable:

$$F(t) = \frac{1}{2\pi S_T^{\frac{3}{2}} e^{\frac{r(T-t)}{2}}} \int_X^\infty \int_{-\infty}^{\infty} (S_T - X)\, I[\frac{(\eta^2 + \frac{1}{4})(T-t)}{2}] cos[(ln(S_T) - r(T-t))\eta] d\eta dS_T \tag{6}$$

where

$$I[\lambda] = \exp(N + M\sigma_0^2),$$

σ_0^2 is the initial variance, and N and M are the following functions:

$$N = \frac{2\alpha\theta'}{\xi^2} \ln \left[\frac{2\gamma e^{\frac{(\alpha-\gamma)(T-t)}{2}}}{g(T-t)} \right]$$

$$M = \frac{-2(1 - e^{-\gamma(T-t)})}{g(T-t)}$$

where

$$\gamma = \sqrt{\alpha^2 + 2\left(\frac{\lambda}{(T-t)}\right)\xi^2}$$

and

$$g(T-t) = 2\gamma + (\alpha - \gamma)\left(1 - e^{\gamma(T-t)}\right).$$

What remains is an integration giving the explicit solution. Unfortunately, any attempt to find a primitive function to this integrand seems bound to fail, and consequently, we must rely on the best possible approximations found by some kind of numerical integration. To solve this problem, a combination of Simpson's method and the simple Trapetzoid method is chosen, and all programs are written in the *GAUSS* programming language. The stochastic volatility option prices in (6) are calculated with these programs.

3 The Swedish OMX-Index Option Market and Parameter Estimates

3.1 Data

In September 1986, the Swedish exchange for options and other derivative securities (OM) introduced the OMX-index. It consists of a value-weighted combination of the 30 most actively traded stocks on the Stockholm Stock Exchange. The purpose of the introduction was for the OMX-index to serve as an underlying "security" for trading in standardized European style options and forward contracts. A unique feature of the Swedish stock-index options, at least compared to US markets, is that during a large part of the year, there are no dividends at all. The OMX-index must be adjusted for dividends only when the April-, May-, June- and July-contracts are analyzed. This paper looks at dividend-free August to March contracts.

The OMX-index Option Market consists of European style Call- as well as Put-Options with different times to expiration. At any time throughout the year, trading is possible in at least three classes of option contracts with up to one, two and three months left to expiration, respectively. On the fourth Friday each month, when the exchange is open for trading, one class of contracts expires and a new class, with time to expiration equal to three months, is initiated. Furthermore, for options with a given time to expiration, a wide range of exercise prices is available. When options with a new expiration date are introduced, the exercise prices are chosen so that they are centered around the current value of the OMX-index.

The set of data used consists of daily closing bid and ask quotes for the two time periods, October 1993 to February 1994, and July 1994 to December 1994. The option data and index data are obtained from OM and contain both prices and volumes. Options with a time to maturity shorter than 15 days, as well as options with very low liquidity, are removed from the sample. Certain days (very few) have been removed from the data set due to errors in the data (non-feasible prices, missing bid or ask quotations, erroneous strike prices, etc.) and the total number of observations is 1694. Both the options exchange (OM) and the stock exchange (StSE) close at 4.00 P.M., minimizing the possibility of synchronization problems. Interest rates for 30, 60, and 90 days are obtained from Sveriges Riksbank, and the relevant interest rates are computed by interpolation between the two closest interest rates.

3.2 Parameter Estimation

The next step is to estimate the parameters in the volatility process and in the PDE. Most empirical research shows that volatility follows a mean-reverting process, like the one in (2), which gives three parameters to be determined: the reversion rate, α, the long-run mean, θ', and the volatility of the volatility, ξ. Volatility is assumed to start at its long-run level.

There are many alternative ways of estimating these parameters. An approach using the discrete time approximation of continuous time stochastic processes is the method of moments by Chesney and Scott (1989) and Hansen (1982). By looking at the moments of the stock return distribution, estimates of the discrete time process parameters as well as the continuous time parameters can be found.

If choosing to work directly with the continuous time processes, determining the distribution of stock returns as a function of the parameters in question and then applying maximum likelihood methods would be a natural approach. The problem with this approach is that stock returns are dependent over time, and the joint distribution for a sample of observations would be very difficult to derive, Scott (1987).

The estimation approach chosen here is that of calibrating the model to data. This is comparable to the way Brown and Schaefer (1994) fit the "yield curve" of bonds, and this method has the advantage of directly giving the Q- parameters (Martingale), not the objectively observed parameters.

The calibrating technique works as follows. I choose to model the stock return as a Geometric Brownian Motion and the volatility process as a mean reverting square-root process. By specifying these processes under the Q-measure, option prices with the Fourier-Inversion method can be calculated as functions of the volatility parameter set Ω, where Ω is defined as $\Omega = \{\alpha, \theta', \xi\}$. Using empirical data, I calculate the midpoint between daily quoted bid and ask OMX-Index option prices and compare these midpoint values to the modelled option prices. On any given day, I minimize the sum of the squared differences between the model prices and the empirical midpoint prices to get estimates of the parameters, Ω:

$$\min_{\Omega} SSE = \min_{\Omega} \sum_{i=1}^{N} \left(F_{\text{model,i}}(\Omega) - F_{\text{market,i}} \right)^2, \tag{7}$$

where N is the number of options on a particular day. Since the stochastic volatility option price, $F_{\text{model}}(\Omega)$, is highly non-linear in its parameters, the problem (7) is a non-linear least square minimization problem. Thanks to the Fourier-Inversion method, the least square minimization method of estimating the parameters is not too costly in terms of computer resources and fully compiled computer languages are not necessary to implement the routine.

The problem has been implemented in *GAUSS* and the calculations were made on a Pentium 100MHz PC. At the heart of the computation lies an integration routine and for the nonlinear least square minimization, the Gauss-Newton algorithm with numerically calculated derivatives is used. The generalized double integral is truncated to a finite region without too much loss of information. In addition, the high non-linearity creates a number of local minima, and a range

Table 1: Average volatility parameters.

	Oct. 93 to Feb. 94		Jul. 94 to Dec. 94		Both Periods	
	Parameter	Std.	Parameter	Std.	Parameter	Std.
θ'	0.058	0.002	0.037	0.001	0.048	0.001
α	3.96	24.71	4.13	18.39	4.04	21.55
ξ	0.51	0.46	0.39	0.26	0.45	0.36
B-S Implicit Volatility	0.076	0.001	0.036	0.001	0.056	0.001

Parameter = average parameter value over the time period. Std. = average value of the standard deviation coming from the asymptotic covariance matrix for the parameter estimates.

of initial parameter-values has to be tried as inputs to find the global minimum.

Since I look at two separate time periods I can, to some extent, assess the stability of the estimates. Running the program each day in the sample periods gives around 250 estimates each of α, θ', and ξ. Studying how the parameters change over time reveals some time dependency but the model assumption of constant parameters does not seem very strong. Average parameter values and average asymptotic standard deviations (from the covariance matrix) are given in Table 1.[3]

In Table 1, I also include a calculation of the B-S implicit volatility. This volatility is calculated by minimizing the squared difference between the market price and the Black-Scholes price for all options traded on a particular day, which is different from the usual approach of using an at-the-money option only. Our procedure gives a worse correspondence with empirical prices at-the-money but gives a lower pricing bias in-the-money and out-of-the-money[4].

[3]The high asymptotic standard deviation for α might seem to be a subject of concern. It is due to the very low second derivative of the price function F with respect to α; a large variation in α gives only a slight variation in F. However, for different reasons, this should not be subject of too much concern. First, as mentioned, the sensitivity of F with respect to α is very small. This means that predictions of future option prices do not critically depend on the estimated value of α. Second, the aim of this paper is not to find as good estimates of the volatility process as possible.

[4]I have also tried the usual approach of only inverting options at-the-money. The results are not reported here but the two average estimates are fairly similar, even though substantial differences are present on particular days.

4 Pricing Bias

4.1 Static Bias

This chapter looks at the out of sample pricing bias in the Swedish OMX-Index call option market. I have chosen the previous day's (yesterday's) estimates of process parameters as inputs to compute the current day's (today's) model price. These model prices will then be compared to actual market prices and a possible bias is studied. This approach is supposed to replicate the behavior of practitioners and one should be aware of its tendency to favor the shortsighted B-S model; the stochastic volatility model works much better than the B-S model in the unrealistic setting of no (or not very frequent) updating of parameters. Next, the observed market price is subtracted from the model price to compute both the percentage pricing error and the absolute percentage pricing error. This is repeated for all different call options each day in the sample, both for the stochastic volatility model and the B-S model.

The percentage error is defined as

$$e = \frac{100\,(P_{\text{model}}(\Omega) - P_{\text{market}})}{P_{\text{model}}(\Omega)}, \tag{8}$$

and moneyness is defined as

$$m = \frac{100 \cdot \left(\mathit{OMX\text{-}index\ value} - \mathit{Strikeprice} \cdot e^{-r(T-t)}\right)}{\mathit{Strikeprice} \cdot e^{-r(T-t)}}$$

Far-out-of-the-money is defined as $m < -5$, out-of-the-money as $-5 \leq m < -1$, at-the-money as $-1 \leq m < 1$, in-the-money as $1 \leq m < 5$, and finally deep-in-the-money as $m \geq 5$.

If the theoretical prices are compared in a scatter plot (with empirical parameters estimated one day earlier) a smile-shaped bias structure is found between B-S prices and stochastic volatility prices, Fig. 1.

This smile is predicted by theory and is due to the convexity properties of the B-S model (as a function of the mean variance over the life of the derivative security). Jensen's inequality says that, if F is concave, $E[F(\cdot)] < F(E[\cdot])$ where E is the expectation operator (when F is convex the opposite holds). This, together with the fact that (6) can be seen as an expectation of the B-S price over different mean variances and that the B-S price, as a function of the mean variance, is convex for large (and small) values of S/X, and concave for values of S/X close to one, where, as before, S is the stock index price and X is the strike price, gives the observed bias structure.

Fig. 2 is a scatter plot of the bias between stochastic volatility prices and market prices as a function of moneyness. It can be seen how the options are overpriced out-of-the-money and

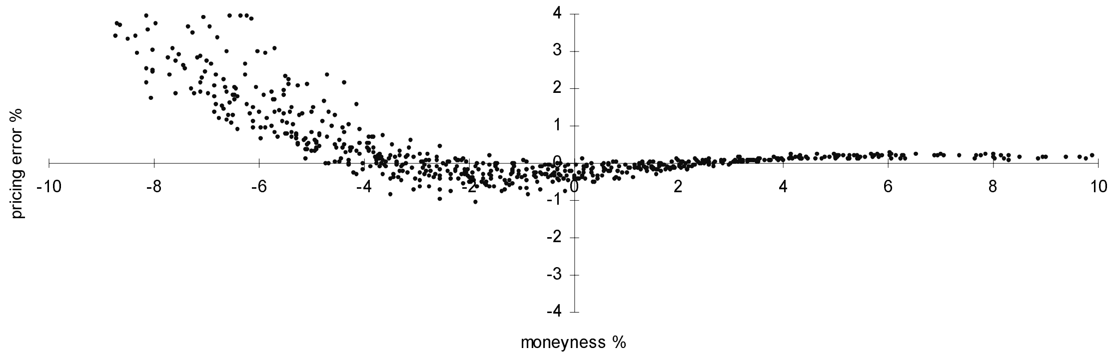

Figure 1: % Bias——Stochastic Volatility Prices minus B-S Prices. Both time periods (1649 obs.).

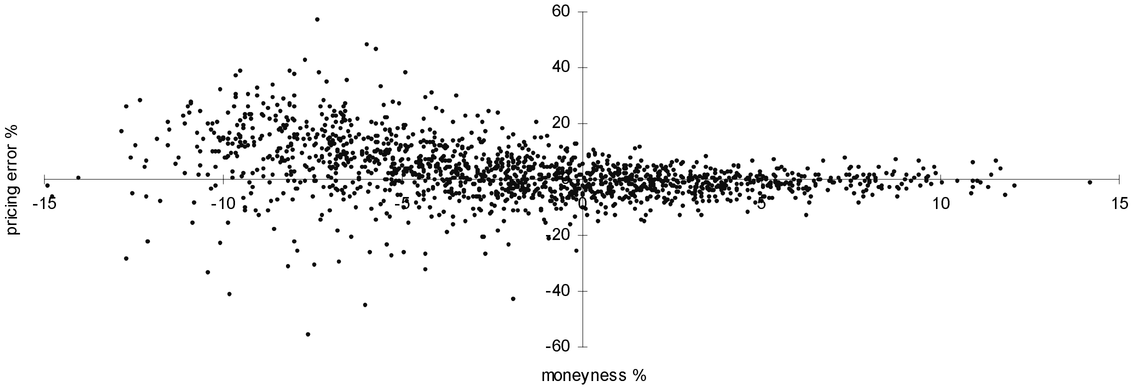

Figure 2: % Bias——Stochastic Volatility Prices minus Market Prices. Both time periods (1649 obs.).

slightly underpriced in-the-money compared to the market valuation. The plot for the B-S model shows a similar bias structure. For a more quantitative analysis of the pricing bias, we can look at the results in Tables 2 and 3. In these tables, the results for the different pricing models are reported, divided into different times to maturity as well as different levels of moneyness. The tables show how the two models demonstrate percentage errors of a similar magnitude, as well as a similar bias structure. This behavior is stable over different time periods as well as different pricing models[5]; overpricing out-of-the-money, underpricing in-the-money, and somewhat better correspondence at-the-money. For both models, the prices deviate significantly from the market price out-of-the-money and in-the-money and the percentage errors (with sign) are significantly different from zero. The absolute percentage errors confirm this evidence of mispricing and both models give significant pricing errors. Overall, the absolute percentage error decreases with moneyness, ranging from around 15% out-of-the-money to 3% in-the-money.

In addition to the strike price bias above, a quick, qualitative look at the time to maturity bias is also interesting. As a whole, the absolute percentage bias decreases with time to maturity. In particular, the options with the longest times to maturity have a smaller bias than the options with shorter maturity. No explanation to this behavior has been found.

For all maturities, both models systematically overprice out-of-the-money and underprice in-the-money. An explanation of this skewness might be a negative correlation between the stock return and the volatility taken into account by the market but not by the models. This would lead to an overvaluation by the models of out-of-the-money options, since the stock return distribution becomes negatively skewed and really high option prices are less likely to be achieved; when the stock price increases, volatility tends to decrease, thereby making large movements in price less likely. The opposite holds when stock prices decrease. On the other hand, several studies have also shown that the skewness is *not* significant in the equity markets.

The results are confirmed by many studies, both for the constant volatility B-S model and the stochastic volatility models. One notable finding, however, is the difference between the B-S prices in my study and the B-S prices in the study by Hansson et al. (1995) in the same market. Hansson et al. find out-of-the-money options to be better priced than at-the-money options and only slightly underpriced compared to the market price. This is somewhat surprising, since they use at-the-money options to back out the implied volatility. The only explanation for the difference in results is the different specification of the implicit volatility and the different data sets[6].

[5]The results for the individual time periods are not presented, but looking at these two time periods, October 1993 to February 1994 and July 1994 to December 1994, separately, there is no significant change in patterns.

[6]Even if not presented in my paper, it is interesting to notice that the B-S model with at-the-money-estimated

Table 2: Stochastic volatility average pricing errors. Both time periods (the total number of options is 1649).

	T-t	15-30 days	31-45 days	46-60 days	all days
	Average % Error	7.17	9.25	8.84	8.81
		2.23	0.95	0.93	0.68
Far Out	Average Abs. % Error	14.73	14.02	12.39	13.61
		1.68	0.67	0.66	0.48
	No. of Options	89	256	156	501
	Average % Error	3.51	2.67	1.10	2.08
		2.15	0.61	0.61	0.47
Out	Average Abs. % Error	11.73	7.30	5.93	7.29
		1.54	0.42	0.37	0.33
	No. of Options	75	258	128	461
	Average % Error	1.02	-0.42	-1.90	-0.75
		1.51	0.60	0.61	0.44
At	Average Abs. % Error	6.38	5.18	4.22	5.01
		1.01	0.36	0.38	0.27
	No. of Options	36	123	62	221
	Average % Error	-0.33	-1.58	-2.30	-1.60
		0.75	0.34	0.43	0.26
In	Average Abs. % Error	3.96	3.64	3.48	3.61
		0.50	0.23	0.30	0.17
	No. of Options	454	176	73	303
	Average % Error	-1.39	-1.12	-0.68	-1.12
		0.53	0.31	0.39	0.24
Deep In	Average Abs. % Error	2.91	2.93	1.59	2.73
		0.35	0.23	0.23	0.17
	No. of Options	44	99	20	163

Small numbers are standard deviations.

Table 3: Black-Scholes average pricing errors. Both time periods (the total number of options is 1649).

	T-t	15-30 days	31-45 days	46-60 days	all days
	Average % Error	-8.86	5.14	5.83	1.16
		3.56	1.09	1.02	0.99
Far Out	Average Abs. % Error	23.57	13.73	11.26	15.55
		2.72	0.74	0.90	0.73
	No. of Options	89	256	156	501
	Average % Error	3.86	2.82	0.58	2.29
		1.73	0.58	0.61	0.47
Out	Average Abs. % Error	11.27	7.33	5.60	7.42
		1.21	0.40	0.38	0.33
	No. of Options	75	258	128	461
	Average % Error	2.15	-0.12	-1.68	-0.38
		1.18	0.59	0.61	0.42
At	Average Abs. % Error	6.08	5.20	8.27	4.97
		0.79	0.35	0.37	0.26
	No. of Options	36	123	62	221
	Average % Error	0.10	-1.54	2.31	-1.51
		0.64	0.34	0.43	0.25
In	Average Abs. % Error	3.77	3.65	3.47	3.59
		0.43	0.23	0.30	0.17
	No. of Options	454	176	73	303
	Average % Error	-1.47	-1.29	-1.41	-1.31
		0.46	0.31	0.66	2.99
Deep In	Average Abs. % Error	2.73	3.04	2.05	2.79
		0.33	0.23	0.31	2.19
	No. of Options	44	99	20	163

Small numbers are standard deviations.

To summarize, the difference between the theoretical B-S price and the stochastic volatility price is in accordance with theory and shows the expected smile-shaped bias structure. The difference between market prices and model prices is larger, though, and both models overprice out-of-the-money and underprice in-the-money. To fully evaluate the pricing performance of the models, we turn to a dynamic test measuring riskfree profits over time by holding, and each day rebalancing, risk-free portfolios designed with each of the models.

4.2 Dynamic Efficiency Test

If the B-S price is closer to the "correct price" than is the market price, then it should be possible to make riskfree arbitrage profits by trading with the B-S model. Further, since the variance is observed to vary randomly, a trader using a random variance model may be even more efficient in identifying mispriced options. To test this hypothesis, I compute *ex ante* net gains from a hedged position of options and the underlying variables[7]. In the B-S case, I use a standard Δ-hedge with a call option and a hedged position in the OMX-Index (or the stocks making up the index). In the random variance case, both the OMX-Index and the volatility must be hedged, which is accomplished by taking positions in two call options as well as the underlying index. In this way, both the random sources in the option pricing model are hedged (Chesney and Scott (1989)).

The dynamic Δ-neutral hedge for the stochastic volatility model is created in the following way. Each day, I take a position in the option that is most mispriced. If the model price is higher than the midpoint of the bid-ask spread, then I buy the option, if it is lower, I sell it. The position in the second option must be the opposite in order to hedge the volatility risk, and if I need to sell the second option, I choose one with a model price below the midpoint price. Finally, a position is taken in the OMX-Index.

The hedged position is

$$F(S,\sigma,t,K_1) + w_1 S_t + w_2 F\left(S,\sigma,t,K_2\right), \tag{9}$$

where w_1 and w_2 are

$$\begin{aligned} w_1 &= -\frac{\partial F\left(S,\sigma,t,K_1\right)}{\partial S} - w_2 \frac{\partial F\left(S,\sigma,t,K_2\right)}{\partial S} \\ w_2 &= -\frac{\frac{\partial F(S,\sigma,t,K_1)}{\partial \sigma^2}}{\frac{\partial F(S,\sigma,t,K_2)}{\partial \sigma^2}}. \end{aligned} \tag{10}$$

implicit volatility shows substantially worse pricing behavior than the other two models out-of-money and in-the-money. This is not very surprising, considering the well-known volatility smile existing in implicit B-S volatilities.

[7] *Ex ante* means that model prices at time t are calculated by using parameters estimated at time $t-1$.

Table 4: Risk-free daily profits (SEK per option and day) from using a dynamic trading rule.

			Oct. 93 to Feb. 94	July 94 to Dec. 94	Both Periods
Profit	Stochastic Volatility	Mean	0.43	0.39	0.41
		Median	0.18	0.37	0.29
		Mean Std.	0.21	0.33	0.20
	Black-Scholes	Mean	0.054	0.14	0.090
		Median	0.023	0.19	0.16
		Mean Std.	0.37	0.10	0.16

Each day the net gain on this hedge is calculated:

$$\begin{aligned}&[F(t_2, K_1) - F(t_1 - K_1)] + w_1\left[S_{t_2} - S_{t_1}\right] + w_2\left[F(t_2, K_2) - F(t_1, K_2)\right] - \\ &r_{t_2 - t_1}\left[F(t_1, K_1) + w_1 S_{t_1} + w_2 F(t_1, K_2)\right].\end{aligned} \tag{11}$$

Every transaction is made at the midpoint price in an attempt to exclude transaction costs, due to the bid/ask spread. For both the B-S model and the stochastic volatility model, positive *ex ante* average profits from using the trading rules are found.

Table 3 shows means, medians, and standard deviations for the B-S model and the stochastic volatility model for the two different time periods. For both time periods, the stochastic volatility model gives higher profits than the B-S model, whose profits are not significant. The existence of these profits indicates mispricing in the OMX-Index market, even though it is important to notice that my hedging scheme assumes that all trade can be done within the bid-ask spread and without transaction costs. In practice, this is the case for large traders and market-makers only.

5 Conclusions

The standard Black-Scholes model for pricing European call options assumes a lognormal probability distribution for the underlying stock-index price and a constant stock-index return volatility. Considering empirical evidence, a more plausible hypothesis is that volatility changes randomly.

In this paper, I specify the volatility process as a mean-reverting square-root process and calculate theoretical option prices with the Fourier-Inversion Technique. The option pricing equation contains a preference term, the volatility risk premium, as volatility is a non-traded asset. It is not obvious, a priori, what constitutes a reasonable value for the price of volatility risk and I have chosen to treat the risk-premium as a constant. In this way, the Fourier-Inversion method can be used.

I quote actual prices on dividend-free European call options from the Swedish OMX-Index Option Market, and from these market prices, I estimate daily volatility process parameters by a non-linear least square minimization of the difference between market and model prices. This procedure has the advantage of directly giving the risk-neutral parameters.

From a static point of view, I find a smile-shaped bias between the Black-Scholes prices and the stochastic volatility prices. Both models give prices showing a similar bias compared to actual prices quoted in the market and both models price options in-the-money and at-the-money more accurately than out-of-the-money. The absolute percentage bias decreases with time to maturity for both models.

The dynamic hedging test reveals riskfree arbitrage profit possibilities, at least for the stochastic volatility model, which supports the existence of mispricing in the OMX-Index Option Market, when transaction costs are not considered.

My conclusion is that the stochastic volatility model dominates the standard Black-Scholes model and produces a more efficient market. Considering the easy implementation of the stochastic volatility pricing model, this model is seen as an alternative to the established Black-Scholes model in actual pricing.

For further research, I suggest a comparative study on stock- and currency options and exchanges versus OTC-trading, and a more thorough study of the risk premium, for instance in the context of different macroeconomics situations (the 1992 currency crisis etc.). The application of the stochastic volatility model in related areas might also prove to be useful.

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