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# Tournaments with prize-setting agents\*

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## Abstract

In many tournaments it is the contestants themselves who determine reward allocation. Labor-union members bargain over wage distribution, and many firms allow self-managed teams to freely determine internal resource allocation, incentive structure, and division of labour. We analyze, and test experimentally, a rank-order tournament where heterogenous agents determine the spread between winner prize and loser prize. We investigate the relationship between prize spread, uncertainty (i.e. noise between effort and performance), heterogeneity and effort. The paper challenges well-known results from tournament theory. We find that a large prize spread is associated with low degree of uncertainty and high degree of heterogeneity, and that heterogeneity triggers effort. By and large, our real-effort experiment supports the theoretical predictions.

## 1 Introduction

In many areas of economic, political and social life, "the rules of the game" are determined by its players: Politicians determine rules of election, sports federations determine rules for leagues and tournaments, and the allocation

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of resources within firms and organizations is often decided by its members/employees.

Tournament theory provides us with a potential tool for analyzing these phenomena. The theory was first introduced by Lazear and Rosen (1981) as an effort to understand situations where wage differences are based on relative differences between the individuals rather than on marginal productivity. The theory has had enormous impact. In many settings, tournaments are found to be at least as good as any other incentive mechanism in terms of inducing effort,<sup>1</sup> and comparative static results on the optimal tournament solution have provided insights into internal wage policies of firms (see Lazear, 1995, for an overview).

So far tournament theory has not been used to analyze games where the players set the rules. In particular, it's always assumed that the spread between winner prize and loser prize (we use the term prize spread throughout this paper) is determined by a non-participating principal. But in many tournaments this is not the case. Prize spread is often set by the contestants themselves. Labor-unions determine prize spread in bargaining over the distribution of fixed wage pools, and many firms allow self-managed teams to freely determine internal resource allocation, incentive structure, and division of labour (Osterman, 1995; and Jehn et al.,1999). One should perhaps expect that the large literature on unions and wage bargaining has addressed tournaments with prize-setting agents, but to our knowledge the tournament feature of decentralized bargaining has not yet been analyzed. In this paper we thus analyze a rank-order tournament where risk neutral heterogeneous agents determine prize spreads.

*Theoretical results:* In a tournament between two risk-neutral agents that differ in ability-levels, the low-ability agent (he) will always prefer zero prize spread. For the high-ability agent (she), however, determining optimal prize spread is not straightforward. A high prize spread is good since she expects

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<sup>1</sup>By tying compensation to the agent's relative performance, the principal can filter out common noise so that compensation to the largest possible extent is based on real effort, not random shocks that are outside the agent's control (see Holmström,1982; and Mookherjee, 1984). With RPE's special form, rank-order tournaments, the agents are also completely insulated from the risk of common negative shocks (see Lazear and Rosen, 1981; Nalebuff and Stiglitz, 1983; Green and Stokey, 1983). Moreover, tournaments need only rely on ordinal performance measures. It may thus be easier and less costly to measure relative than absolute performance (Lazear and Rosen, 1981). In addition, it may be easier for the principal to *commit* to tournament schemes if output is not verifiable (Carmichael, 1983; Malcomson, 1984; Levin, 2002).

to win. But it is bad since it triggers effort, and effort is costly. Since optimal prize spread for the low-ability-agent is always zero, it is sufficient to characterize the high-ability agent's optimal choice in order to understand equilibrium prize spread, i.e. if the high-ability agent has some bargaining-power, then comparative static results on her optimal prize spread hold for the bargaining solution between the agents. We characterize the optimal prize spread for the high-ability agent, and investigate the relationship between prize spread, uncertainty (i.e. noise between effort and performance)<sup>2</sup>, heterogeneity and effort. Our results can be summarized as follows:

First, we find that the high-ability agent's optimization problem entails corner solutions. Either she wants zero prize spread, or she wants maximal prize spread. A marginal parameter change may thus dramatically change prize spread and effort. This is interesting since it can explain why seemingly similar firms may differ substantially in wage structure and performance (see Gibbons et al., 2007, for a discussion on persistent performance differences among seemingly similar enterprises).

Second, we find that more heterogeneity (i.e. larger ability-difference) leads to higher equilibrium effort. This is an interesting result since it challenges theory stating that heterogeneity reduces effort. In Lazear and Rosen (1981), effort suffers from more heterogeneity, or at best is unaffected by ability-difference if the principal can observe the agents' type so that she perfectly can compensate heterogeneity with higher prize spread. We show that higher ability-difference increases prize spread *more* than just to compensate for heterogeneity, leading to higher equilibrium effort.

Third, we find that large prize spread is associated with low degree of uncertainty. This contrasts with the standard tournament result where the optimal prize spread increases with uncertainty. Our result is not trivial, since there are two countervailing effects: As uncertainty increases, the probability of winning decreases *cet. par.* so the high-ability agent might want to decrease prize spread in order to reduce effort costs. However, the high-ability agent can 'remove' the reduced winner probability by increasing the prize spread, since higher prize spread increases the effort-difference between the agents. We show that the former effect dominates under standard assumptions.

From an incentive perspective, the result offers an alternative explana-

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<sup>2</sup>We use "uncertainty" and "noise" synonymously throughout the paper.

tion to a negative relationship between uncertainty and incentives. The standard explanation is risk aversion; the optimal intensity of incentives is negatively related to uncertainty when agents are risk averse. Our model shows that tournaments with prize-setting agents can create such a relationship even if agents are risk neutral.

The result also points to the issue of "desert", or whether or not performance pay is "fair". According to Konow (2003), a common view is that differences owing to luck are unfair, and that only differences attributable to effort are fair. Our high-ability agent seemingly has fairness concerns since her preferences are aligned with the firm's preferences for high prize spread if effort is important. But if luck is important, then her preferences are aligned with the low-ability employee. However, this is not because of fairness concerns; she simply makes a trade-off between effort costs and expected monetary payoff. One should thus be cautious with drawing the conclusion that employees have fairness concerns if they argue that uncertainty makes performance pay unfair.

*Experimental results:* We do not explicitly deduce bargaining solutions between low and high-ability agents, but as noted above, comparative static results on the high-ability agent's optimal prize spread should apply for the bargaining solution between the agents. We conducted a real effort experiment to test this conjecture for some of our theoretical results. We elicited subjects' risk preferences and their ability to do head calculation, and we then got them to bargain over winner prize and loser prize prior to two-player tournaments in head calculation. This enabled us to test the relationship between prize spread, ability-difference and effort. We also imposed two uncertainty levels, high and low, enabling us to study the relationship between prize spread and noise. By large, the experiment supports the theoretical predictions. Here are the results:

First, controlling for risk preferences, we find that prize spread significantly decreases with uncertainty, which supports the theoretical prediction. We find no impact from personality and gender, indicating that fairness concerns do not drive our experimental results.

Second and third, we find that prize spread significantly increases in the ability-difference between the agents, and that effort increases in prize spread. This supports the theoretical prediction that more heterogeneity increases prize spread, which thereby increases effort. Controlling for prize

spread, however, we find a significant negative relationship between ability-difference and effort, supporting previous experimental results.

*Related literature:* As noted above, neither the tournament literature nor the union literature have analyzed rank-order tournaments where heterogeneous agents set prize spread.<sup>3</sup> Brunello (1994) analyzes a case where homogenous agents decide prize spread in a principal-agents game with a flexible wage pool; and Sutter (2006) analyzes an endogenous prize selection tournament where the best member of a team is given a right *ex post* to propose prize spread within the team. Neither of these papers analyze a situation where heterogeneous contestants determine prize spread prior to the tournament. Riis (2007) analyzes a tournament where heterogeneous contestants can choose from a menu of prizes, but the prize menu is defined by the principal *ex ante*. And while Riis focuses on how the principal can structure the prize menu so as to implement first-best effort, we focus on the agents' optimal prize spread and the comparative statics that can be derived from the agents' solution.

Several papers have experimentally tested hypotheses deduced from tournament theory, starting with Bull et. al. (1987).<sup>4</sup> Typically, these papers test the relationship between prize spread, effort and heterogeneity. But there are only a few real effort experiments testing the theory (van Dijk, Sonneman and van Winden, 2001; Gneezy, Niederle and Rustichini, 2003; and Dohmen and Falk, 2006), and no one has examined a case where the agents set prize spread - although Sutter (2006) runs an experiment (not real effort) where he tests his model of endogenous prize selection. Moreover, no one has (to our knowledge) experimentally tested the relationship between uncertainty and prize spread, not even in tournaments where the principal sets prize spread.

The rest of the paper is organized as follows: Section 2 presents the model, Section 3 outlines the experimental design, Section 4 formulates the hypothesis to be tested, Section 5 presents results and analysis from the experiment, while Section 6 concludes. Proofs and tables are in the appendix.

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<sup>3</sup>Unions composed of identical members has been the basis for representations of union preferences (see Cahuc and Zylberberg, 2004), although Ross (1948) already 60 years ago argued that the heterogeneity of union members affects its aims.

<sup>4</sup>See Harbring and Irlenbusch (2004) and Falk and Fehr (2003) for an overview of these experiments.

## 2 The model

Consider a tournament between two risk-neutral agents. The winner of the tournament receives  $w_1$  while the loser receives  $w_2 \leq w_1$ . Output  $y_i$  from agent  $i$  is given by

$$y_i = e_i + z_i$$

where  $e_i$  is effort and  $z_i$  is a random luck component. Expected payoff for agent  $i$  is

$$Pw_1 + (1 - P)w_2 - C_i(e_i) \tag{1}$$

where  $C_i(e_i)$  is the cost of effort ( $C'_i > 0$  and  $C''_i > 0$ ) and  $P$  is the probability of winning. Let

$$P = \text{prob}(e_i + z_i > e_j + z_j) = \text{prob}(e_i - e_j > z_j - z_i) = G(e_i - e_j)$$

denote the probability that agent  $i$  achieves a higher output than agent  $j$ .  $G(\cdot)$  is the cumulative distribution function of the random variable  $z_j - z_i$ , where  $G(e_i - e_j) = 1 - G(e_j - e_i)$ . Each player chooses effort to maximize expected payoff (1). This gives the IC constraint (for interior solution)

$$(w_1 - w_2) \frac{\partial P}{\partial e_i} = C'_i(e_i) \tag{2}$$

From Nash-assumptions it follows that each player optimizes effort against the optimal effort of his opponent. Agent  $i$  thus takes agent  $j$ 's effort as given when choosing his effort level, and it follows that

$$\frac{\partial P}{\partial e_i} = \frac{\partial G(e_i - e_j)}{\partial e_i} = g(e_i - e_j).$$

where  $g(e_i - e_j)$  is the density function of  $G(e_i - e_j)$ . The IC constraint is thus

$$(w_1 - w_2)g(e_i - e_j) = C'_i(e_i) \tag{3}$$

We make the common assumption that the total prize pool is fixed, i.e. that  $w_1 + w_2 = R$ , where  $R$  is exogenous and unaffected by effort levels, but we discuss later how a change in  $R$  affects prize spread. A fixed  $R$  may sound like a strict assumption, but in many tournaments a fixed prize pool is indeed the case. In pure promotion tournaments for example, the sum of

prizes is unaffected by effort-levels. And in larger bureaucratic organizations, total resource provision to organizational divisions may often be exogenously given, or at least perceived as exogenous by the employees. Decentralized wage bargaining is also a good example. In many countries, the *size* of the wage pool that is to be allocated locally in each firm is determined by central bargaining between labor unions and employer federations. The size of the local wage pool is then unaffected by total effort levels, and the only thing bargained over locally is distribution of the fixed wage pool. Finally, note that if the absolute value of output is unverifiable to a third party, then a fixed prize pool may turn out optimal: With a flexible prize pool, total prize payments increase in effort, making the principal's incentive to renege on payments increase in effort. A fixed prize pool removes this problem, and makes it easier for the principal to commit to prize promises (see e.g. Carmichael, 1983).

If  $w_1 + w_2 = R$ , then prize spread is  $w_1 - w_2 = R - 2w_2$ . Agent  $i$ 's optimal prize spread is then the solution to

$$\max_{w_2} [w_2 + P(R - 2w_2) - C_i(e_i)] \quad \text{s.t. (3)}$$

where (3) applies to both agents and is assumed to define the tournament equilibrium. With identical (homogenous) agents,  $P = \frac{1}{2}$  in equilibrium, hence expected prize for each agent is  $\frac{1}{2}R$  and does not vary with prize spread. The agents will simply minimize costs, which is to set zero prize spread,  $R - 2w_2 = 0$ , such that optimal effort level is zero. This is the collusion logic, first thoroughly analyzed by Mookherjee (1984). When prizes are fixed, agents have incentives to collude on low effort equilibria. It follows here that if they are to decide prizes, they set them so that effort is minimized.

*Heterogeneous agents:* A tournament model with prize-setting agents first becomes interesting when we introduce heterogeneity in ability-levels. Of course, the agents still have incentives to collude on zero prize spread by using side payments. It is, however, a quite standard assumption in the tournament literature to assume that collusive contracts are not enforceable. We thus make the assumption that side payments are impossible.

We model differences in ability-level by assuming that the net marginal return from effort is higher for the high-ability agent, and the standard



assumption is then  $C'_i(e_i) < C'_j(e_j)$  for all  $e_i = e_j$ , meaning that the marginal cost from effort is lower for the high-ability agent  $i$ . Symmetric density function  $g(e_j - e_i) = g(e_i - e_j)$  implies that  $C'_i(e_i) = C'_j(e_j)$  in equilibrium, which for interior solutions implies  $e_j < e_i$  and thus  $\frac{1}{2} < G(e_i - e_j) < 1$ .

With no restrictions on prize spread, a prize-setting principal can easily achieve first-best if ability-level is common knowledge. The problem with heterogenous agents arises if their types cannot be identified. For prize-setting agents, however, first-best implementation is not the objective if the wage pool is fixed in advance. For simplicity, we thus assume that the agents know each others' ability-levels. It can easily be shown that the comparative static results we achieve apply also when ability-level is uncertain.

From the restriction  $w_1 \geq w_2$ , it is straight-forward to see that the optimal prize spread for the low-ability agent  $j$  is zero. He has nothing to gain from increasing prize spread, since this implies costly effort and a reduced chance of winning the tournament. Hence,

**Proposition 1** *If  $w_1 \geq w_2$ , and  $C'_i(e_i) < C'_j(e_j)$  for all  $e_i = e_j$ , then the low ability agent  $j$ 's optimal prize spread is zero, yielding zero effort in equilibrium.*

It is less trivial to find the optimal prize spread for the high-ability agent  $i$ . As noted in the introduction, we assume that the parties cannot use side payments in order to collude on low effort / zero prize-spread. Taking this into account, agent  $i$  solves

$$\max_{w_2} W(w_2) = [w_2 + G(e_i - e_j)(R - 2w_2) - C_i(e_i)] \quad \text{s.t.} \quad (3) \quad (4)$$

where effort levels  $e_i, e_j$  are determined as functions of  $w_2$  in the tournament. Agent  $i$  will choose  $(R - 2w_2) > 0$  if there exist equilibrium effort

levels  $e_i, e_j$  where<sup>5</sup>

$$G(e_i - e_j)(R - 2w_2) > C_i(e_i) \quad (5)$$

For the rest of the paper, we assume that the ability-difference is sufficiently large such that (5) holds in equilibrium. This is not a strict assumption. With Inada conditions, i.e.  $C_l(e_l)$ ,  $l = i, j$  continuously differentiable and strictly increasing in  $e_l$  and  $C_l(0) = C'_l(0) = 0$ , then (5) holds in equilibrium for arbitrarily small ability differences.

From (4) we have

$$W'(w_2) = 1 - 2G(e_i - e_j) + (R - 2w_2)g(e_i - e_j)\left(\frac{\partial e_i}{\partial w_2} - \frac{\partial e_j}{\partial w_2}\right) - C'_i(e_i)\frac{\partial e_i}{\partial w_2}$$

From the IC constraint (3) we then get

$$W'(w_2) = 1 - 2G(e_i - e_j) - (R - 2w_2)g(e_i - e_j)\frac{\partial e_j}{\partial w_2} \quad (6)$$

Equation (6) shows that the marginal value for agent  $i$  of increased  $w_2$  (reduced prize spread) has two components. First, for given efforts her expected payment is affected. This marginal payment effect is  $1 - 2G(e_i - e_j) < 0$ , where the inequality follows from the fact that she will exert higher equilibrium effort ( $e_i > e_j$ ) due to her ability advantage, and hence win with a probability exceeding 1/2. Second, there is an indirect effect induced by reduced effort on the part of the other agent ( $\frac{\partial e_j}{\partial w_2} < 0$ ), and this effect will increase agent  $i$ 's probability of winning the tournament.<sup>6</sup> Thus, a reduced prize spread (increased  $w_2$ ) yields one negative and one positive effect for agent  $i$ . We will show below that under reasonable assumptions either the first or the second of these effects will dominate, so that the agent will then choose either maximal spread ( $w_2 = 0$ ) or minimal spread ( $w_2 = R/2$ ).

Consider now the marginal value  $W'(w_2)$ . The IC constraints define

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<sup>5</sup>Note that the tournament equilibrium underlying this analysis will exist only if the IC conditions for the agents' efforts reflect truly optimal choices. In particular, the second-order conditions must hold, hence we must have  $(R - 2w_2)g'(e) - C''_i(e_i) \leq 0$  and  $(R - 2w_2)g'(-e) - C''_j(e_j) \leq 0$ , where  $e = e_i - e_j$ . Since  $g'(e) < 0$  for  $e > 0$ , the first will hold for convex costs, but the second may not, since  $g'(-e) = -g'(e) > 0$ . It follows that the level of uncertainty has to be sufficiently large for a tournament equilibrium to exist. This also applies for standard tournament models where the principal sets prize spread (see Lazear and Rosen, 1981).

<sup>6</sup>The indirect effect induced by agent  $i$ 's own effort response is zero due to the IC constraint.

simultaneously the two effort levels as functions of  $w_2$ . It is convenient here to think of these as being defined recursively; first  $e_i = e_i(e_j)$  being defined by equality of marginal costs ( $C'_i(e_i) = C'_j(e_j)$ ), and then  $e_j = e_j(w_2)$  defined by the IC condition (3) for agent  $j$ , substituting for  $e_i = e_i(e_j)$  in this condition. Let  $e = e_i(e_j) - e_j$  denote the effort difference as a function of agent  $j$ 's effort  $e_j$ , and let  $E_e(e_j)$  denote the elasticity of this function ( $E_e(e_j) = \frac{e_j}{e} \frac{de}{de_j}$ ). The marginal value  $W'(w_2)$  in (6) can then be written in the following form (see the appendix):

$$W'(w_2) = 1 - 2G(e) - \frac{2g(e)e_j}{E_g(e)E_e(e_j) - E_{C'_j}(e_j)} \quad (7)$$

where  $E_g(e) = \frac{g'(e)}{g(e)}e$  is the elasticity of the probability density, and  $E_{C'_j}(e_j) = \frac{C''_j(e_j)}{C'_j(e_j)}e_j$  is the elasticity of the marginal cost function for agent  $j$ .

Consider now the case of minimal wage spread, i.e.  $w_2 = R/2$ . In this case both agents will exert minimal effort ( $e_i = e_j = 0$ ), so the marginal value  $W'(w_2)$  for  $w_2 = R/2$  is given by the expression on the RHS of (7) calculated at  $e = e_j = 0$ . If now the elasticity of agent  $j$ 's marginal cost function is bounded away from zero ( $E_{C'_j}(0) > 0$ ), we see that the value on the RHS is zero, and hence that  $W'(w_2) = 0$  for  $w_2 = R/2$ . Under this mild assumption (the elasticity is bounded away from zero for all strictly convex power functions; see below), it is thus the case that a minimal prize spread and hence minimal effort is a candidate for an optimum.

To examine this issue we introduce further assumptions. In the following we assume  $C_i(e_i) = k_i e_i^n$  and  $C_j(e_j) = k_j e_j^n$  where  $n > 1$  and  $k_j > k_i$ . Note that the elasticity of marginal cost is then constant;  $E_{C'_j}(e_j) = n - 1 > 0$ . The IC constraints imply equality of marginal costs;

$$nk_i e_i^{n-1} = nk_j e_j^{n-1} \quad (8)$$

and the effort difference  $e$  is then given by:

$$e \equiv e_i - e_j = \left( \left( \frac{k_j}{k_i} \right)^{\frac{1}{n-1}} - 1 \right) e_j \equiv \frac{1}{K} e_j \quad (9)$$

where  $K = \left( \left( \frac{k_j}{k_i} \right)^{\frac{1}{n-1}} - 1 \right)^{-1}$ . This yields elasticity  $E_e(e_j) = 1$ , and substituting for the other elasticities and for  $e_j = Ke$  we then see that (7) here

can be written as

$$W'(w_2) = 1 - 2G(e) - K \frac{2g(e)e}{\frac{g'(e)}{g(e)}e - (n-1)} \equiv \hat{F}(e), \quad e = e(w_2) \quad (10)$$

where the effort difference as a function of  $w_2$  ( $e(w_2)$ ) is defined by (9) and the IC constraints.

At an interior optimum we will have  $W'(w_2) = 0$ , and the optimal effort difference  $e$  given by  $\hat{F}(e) = 0$ . The second-order condition for an optimum requires  $W''(w_2) = \hat{F}'(e)e'(w_2) \leq 0$ . From the IC constraints and (9) we can see that  $e'(w_2) < 0$  when  $g'(e) \leq 0$  for  $e \geq 0$ , which we will assume to be the case. The SOC for an optimum thus requires  $\hat{F}'(e) \geq 0$ .

Note that  $e = 0$  is always a solution to  $\hat{F}(e) = 0$ . We will show below that this solution, which corresponds to minimal prize spread ( $w_2 = R/2$ ), is optimal for a range of parameters, and that the other corner solution ( $w_2 = 0$ ) is optimal for other parameters. Moreover, for a class of distributions including the normal and uniform ones, we will show that the optimal solution is always a corner solution.

Assume now that the noise is of the form

$$z_l = \sigma \varepsilon_l + a, \quad \sigma > 0 \quad (11)$$

where  $\varepsilon_l$  has some fixed distribution and  $\sigma, a$  are constants. (This holds e.g. for normal and uniform distributions.) Denote the CDF of  $\varepsilon_j - \varepsilon_i$  by  $\Gamma(d) = \Pr(\varepsilon_j - \varepsilon_i < d)$ , with density  $\gamma(d) = \Gamma'(d)$ . Then we have

$$G(e) = \Pr(z_j - z_i < e) = \Pr(\varepsilon_j - \varepsilon_i < \frac{e}{\sigma}) = \Gamma\left(\frac{e}{\sigma}\right)$$

and  $g(e) = G'(e) = \gamma\left(\frac{e}{\sigma}\right)\frac{1}{\sigma}$ . By defining  $d = \frac{e}{\sigma}$  we have  $g(e)e = \gamma(d)d$  and  $\frac{g'(e)}{g(e)}e = \frac{\gamma'(d)}{\gamma(d)}d$ , and hence (10) can be written as

$$W'(w_2) = F(d) \equiv 1 - 2\Gamma(d) - K \frac{2\gamma(d)d}{\frac{\gamma'(d)}{\gamma(d)}d - (n-1)}, \quad d = \frac{e(w_2)}{\sigma} \quad (12)$$

We see that at an interior optimum the optimal effort difference  $e$  would be given by  $e = \sigma d$ , where  $d$  is a solution to  $F(d) = 0$ . The SOC then requires  $F'(d) \geq 0$ . The other possibilities are corner solutions; either  $w_2 = 0$  or  $w_2 = R/2$ .

For  $w_2 = R/2$  and thus  $e = e(w_2) = 0$ , we see that  $W'(w_2) = F(0) = 0$ . For this to be a maximum, the SOC requires  $F'(0) \geq 0$ . It turns out that his condition is satisfied iff  $K \geq n-1$ , i.e. iff the degree of heterogeneity is 'small' ( $\frac{k_j}{k_i} \leq \left(\frac{n}{n-1}\right)^{n-1}$ ). In such a case, minimal effort and spread ( $e = d = 0$  and  $w_2 = R/2$ ) are then a local maximum.<sup>7</sup> Moreover, this maximum is also a global one if  $F(d) > 0$  for  $0 < d < d_m = e_{\max}/\sigma$ , where  $e_{\max}$  is the largest feasible effort spread for the given  $R$ , i.e. the spread corresponding to  $w_2 = 0$ . We show (see appendix) that this is indeed the case if  $R$  is sufficiently small and/or  $\sigma$  is sufficiently large. For such parameters ( $R$  and  $\sigma$ ) it is thus overall optimal to induce minimal effort and spread when the degree of heterogeneity is small. By a similar reasoning we can also show that for small  $R$  and/or large  $\sigma$  it is optimal to induce maximal effort spread when the degree of heterogeneity is 'large'. We have:

**Proposition 2** (i) For low heterogeneity ( $\frac{k_j}{k_i} < \left(\frac{n}{n-1}\right)^{n-1}$ ) we have: there is  $r_1 > 0$  such that for  $R/\sigma^n < r_1$ , i.e. for  $R$  sufficiently small and /or  $\sigma$  sufficiently large, the optimal solution entails minimal effort and minimal prize spread;  $e = e_i = e_j = 0$  and  $w_2 = R/2$ . (ii) For large heterogeneity ( $\frac{k_j}{k_i} > \left(\frac{n}{n-1}\right)^{n-1}$ ) we have: there is  $r_2 > 0$  such that for  $R/\sigma^n < r_2$  the optimal solution entails maximal effort and prize spread;  $e = e_{\max}$  and  $w_2 = 0$ .

By invoking more assumptions we can be more precise:

**Proposition 3** For a class of distributions including the normal and uniform ones the following holds. (i) For low heterogeneity ( $\frac{k_j}{k_i} < \left(\frac{n}{n-1}\right)^{n-1}$ ) we have: the optimal solution entails either (a) minimal effort and prize spread ( $e = 0$  and  $w_2 = R/2$ ) or (b) maximal effort and prize spread ( $e = e_{\max}$  and  $w_2 = 0$ ). There is  $r_1 > 0$  such that the former is optimal for  $R/\sigma^n < r_1$ , and the latter is optimal for  $R/\sigma^n > r_1$  (provided the tournament equilibrium exists for this case). (ii) For large heterogeneity ( $\frac{k_j}{k_i} > \left(\frac{n}{n-1}\right)^{n-1}$ ) we have: for all parameters  $R, \sigma$  for which the tournament equilibrium exists, the optimal solution entails maximal effort and prize spread;  $e = e_{\max}$  and  $w_2 = 0$ . (iii) When the solution entails maximal spread we have  $e_{\max} = \sigma d_m(R/\sigma^n)$ ,

<sup>7</sup>More precisely, it is a local maximum if strict inequality  $K > n-1$  and thus  $F'(0) > 0$  hold.

where  $d'_m() > 0$ . Effort is then increasing in  $R$  and non-monotone in  $\sigma$  (increasing for  $\sigma$  small and decreasing for  $\sigma$  large).

**Proof.** See appendix. ■

The proposition shows that the high-ability agent's optimal prize spread is high for low uncertainty (low  $\sigma$ ) and low for high uncertainty. Hence, the standard result that prize spread increases in noise when agents are risk neutral is *not* robust to a setting where heterogeneous agents determine prize spread. We also find that prize spread and effort are low for low heterogeneity and high for high heterogeneity. This is not trivial Higher ability-difference increases the chance of winning, *cet. par.* This calls for higher prize spread. But higher ability-difference makes it possible for the high-ability agent to reduce effort, and thereby reduce effort costs without affecting the probability of winning. We show that the former effect dominates under quite general assumptions. Our results thus contrast with the well known result from tournament theory saying that performance suffers from heterogeneity. In existing theory, more heterogeneity never increases effort. If ability-level is common knowledge, then the principal can perfectly compensate more heterogeneity with higher prize spread, making equilibrium effort unaffected by ability difference, while if not, heterogeneity weakens the agents' marginal return from effort. In our setting, heterogeneity is in fact good for effort since it increases prize spread *more* than just to compensate for higher heterogeneity. This result is robust to a setting where the agents do not know their ability-level. If there is a probability  $\alpha < 1$  that an agent has high ability, it would only imply that the high ability agent calculates a probability  $(1 - \alpha)$  that she runs against a weaker contestant. The higher  $\alpha$ , the higher is the threshold heterogeneity and the lower is the threshold uncertainty for when the high ability agent will choose max prize spread.

We also see that  $e_{\max}$  (and hence both efforts) is first increasing in  $\sigma$ , and then decreasing. Hence, the optimal uncertainty-level is strictly positive. This is in contrast to standard tournaments where effort suffers from more uncertainty, or at best is unaffected by the uncertainty-level if the principal can perfectly compensate noise with higher prize spread. The result complements Krakel and Sliwka (2004), who find that more noise may increase effort in a setting where agents can choose risk levels. Finally, observe that effort is increasing in  $R$ , so if the principal can control prize pool but not

prize spread, then one should expect a higher pool,  $R$ , the less heterogeneous the agents are.

We have focused on the agents' optimal choices, and have not deduced a specific bargaining solution between the agents. But if we stick to the restriction that  $w_1 \geq w_2$ ,<sup>8</sup> then in any bargaining game over the prize spread between agent  $i$  and agent  $j$  (prior to effort decisions), comparative statics on the optimal choice for agent  $i$  weakly holds for the bargaining solution, since optimal spread for agent  $j$  is zero. Hence, we can make the following conjecture:

**Conjecture 1** *In any bargaining game over the prize spread between agent  $i$  and agent  $j$ , we have (i) equilibrium prize spread weakly decreases in noise ( $\sigma$ ), (ii) equilibrium prize spread weakly increases in ability difference  $k_j - k_i$  (iii) equilibrium effort weakly increases in ability difference  $k_j - k_i$ .*

In the following we will report on an experiment testing this conjecture.

### 3 Experimental design

The experimental design reflects our aim to investigate conjecture 1. We conducted a real effort experiment in order to make the ability-difference between the subjects natural rather than imposed. We also believe that real effort makes the meaning of noise, or luck, clearer to the subjects.

The work task for the subjects participating in the experiment consisted of doing head calculations; multiplying one- and two-digit numbers (e.g. 7 x 83).<sup>9</sup> The task nicely mimics real world work tasks and also ensures heterogeneity in productivity. Doing head calculations is shown to be rather insensitive to learning and is therefore well-suited for experimentation. A problem with real effort tasks in experiments is the potential for excessive intrinsic motivation, blurring the effect of monetary incentives. We therefore wanted to make the work task boring enough to be affected by monetary incentives. As we shall see, monetary incentives indeed affected performance, and the lack of intrinsic motivation was to some extent confirmed by the

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<sup>8</sup> $w_1 \geq w_2$  is a weak restriction. We did not make this restriction in our experiment, but no one ever proposed  $w_2 > w_1$ .

<sup>9</sup>The actual assignments were borrowed from Thomas Dohmen and Armin Falk who used them in Dohmen and Falk (2006).

subjects' "moaning and groaning" when they learned that the experiment consisted of doing head calculations.<sup>10</sup>

Altogether 108 undergraduate students from the University of Stavanger were recruited by E-mail to participate in the experiment. They were told that they had the opportunity to participate in an economic experiment and if they did well they could earn a nice sum of money. The experiment was programmed in z-Tree (Fischbacher, 1999). The instructions were given both verbally and on the computer screen. The subjects were told that no form of communication was allowed throughout the experiment and that all results were to be held anonymous. We had 18 subjects in each out of six sessions. Each session lasted for about 50 minutes. Total average earnings in the experiment were NOK 302 (38 Euro).

The subjects went through five steps. Subjects were informed that they would go through several steps, but they did not know what these steps would involve, i.e. when they were informed about step 1, they did not know what would happen in step 2 and so on.

*Step 1, risk preferences:* In step 1 we applied a method for eliciting risk preferences similar to Dohmen and Falk's (2006), which is a simple version of Holt and Laury (2002). Upon arrival the subjects were seated at a computer lot and given a table with 12 rows. For each row the subject where asked to decide whether they preferred a lottery or a safe alternative. The lottery was a fifty-fifty probability of NOK 200 or zero, and was the same for all rows. The safe alternative was NOK 15 in row one, increasing with NOK 15 for each row. By examining the shifting point from the lottery to the safe option, we get information on the subjects' risk attitudes. With the chosen value of the safe option, a risk neutral participant with monotonous risk preferences would choose the lottery for the six first situations and then switch to the safe option for the remaining situations.<sup>11</sup>

*Step 2, ability revelation:* In step 2 of the experiment, subjects revealed their ability levels by multiplying one- and two-digit numbers for a period of five minutes. They were paid by a piece rate scheme giving NOK 5 per

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<sup>10</sup>Also Dohmen and Falk (2006), who used exactly the same work task, found that monetary incentives significantly affected performance.

<sup>11</sup>For two reasons, we do not test for reference-dependent risk preferences (see Koszegi and Rabin, 2007, for a general treatment). First, the tournament literature, which our model is deduced from, does assume expected utility maximizing agents with reference independent preferences. Second, in our experiment the reference is approximately the same in the lottery choice and in the tournament.



correct answer. The problems were given on the computer screen and the subject typed the answer to the problem using the keyboard. A message appeared on the screen telling the subject whether the answer was correct or not. After the five minute work period they received either the grade A, B or C, depending on how well they did compared to the others, and they were told that 1/3 received grade A, 1/3 grade B, and 1/3 grade C.

*Step 3, bargaining:* In step 3, subjects were told that they in the next step were going to compete against another subject in doing similar kinds of head calculations for five minutes. The subjects were then asked to split NOK 200 by deciding a winner prize and a loser prize  $(w_1, w_2)$  prior to this competition. Subjects were randomly picked to either propose prizes (*proposer*) or choose to accept or reject prize proposals (*responder*). Accept yielded the proposed solution, but if an offer was rejected, then prizes were set to  $(150, 0)$ .<sup>12</sup> We imposed uncertainty by telling the subjects that a random variable, called a bonus, would be drawn after the competition (tournament) and added to the subjects' number of correct answers. We imposed two levels of uncertainty: the random bonus either had uniform distribution between  $-3$  and  $3$  ("low uncertainty"), or uniform distribution between  $-10$  and  $10$  ("high uncertainty"). Ability-levels (for proposer and responder) and the uncertainty-level were common knowledge when they bargained.

Each subject participated in four rounds of bargaining, where they met new opponents each round. They were told that one out of the four rounds would be picked at random to determine the prizes for the oncoming tournament. There were two bargaining rounds where subjects were told that the random bonus was distributed between  $-3$  and  $3$ , and two rounds where they were told that the bonus was distributed between  $-10$  and  $10$ . After each round of bargaining, subjects were informed about the outcome of the bargaining.

*Step 4, tournament.* Subjects went through a new five-minute work period multiplying one- and two-digit numbers. They knew the grade of their opponent (as well as their own), the size of the prizes and level of uncertainty. The sequence of problems were the same for all subjects and in case of a tie, randomization determined the winner. After the work period,

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<sup>12</sup>The rejection prizes reflect the cost of bargaining break down (lower total surplus), and the idea that a principal in general would like a higher prize spread than the agents.

winners and losers were revealed together with the number of correct answers and the random individual bonus (luck component).

*Step 5, questionnaire.* We gathered questionnaire data on gender, age and personality. Personality was measured by the Big-Five scale used by psychologists, which measures the degree of Openness, Conscientiousness, Extraversion, Agreeableness and Neuroticism.<sup>13</sup>

## 4 Hypothesis

Assume that the subjects believe their ability assignment. In equilibrium, the responder then accepts the proposer's offer in the bargaining game outlined above. In games where the best subject proposes (A to B, A to C or B to C), she has all the bargaining power since the low ability subject has nothing to earn from rejecting the offer. In games where the low-ability subject proposes (B to A, C to A or C to B), the high-ability subject has some bargaining power, since she can gain from refusing an offer with sufficiently low prize spread. The low-ability subject will offer the lowest prize spread that the high-ability subject is expected to accept. Hence, the qualitative comparative statics results on the optimal spread for the high-ability agent applies for the bargaining solution also when the low ability subject proposes. The model thus predicts the following outcomes from our experiment:

**H1:** *Among heterogenous pairs, prize spread is higher when the random bonus has distribution  $U(-3, 3)$ , than when it has distribution  $U(-10, 10)$ .*

**H2:** *Prize spread increases with ability-difference.*

**H3:** *Effort increases with ability-difference.*

Effort in our model equals number of correct answers, while output is number of correct answers plus the randomly chosen bonus.

## 5 Results and analysis

In this section we present the main results. Table 1 displays summary statistics on prize spread by pair composition and level of uncertainty.

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<sup>13</sup>The Big-Five questionnaire measures personality traits by asking subjects how they assess themselves. We used a 20 item version of the questionnaire. The subjects indicate their assessments on a seven-point scale for each item.

	AC	AB	BC	Homogeneous	Heterogeneous pairs	All pairs
Average prize spread	121.9	98.3	105.7	55.0	108.6	99.7
low uncertainty	(33.9)	(48.1)	(34.5)	(39.5)	(40.2)	(44.6)
Average prize spread	102.3	93.7	97.7	62.2	97.9	91.9
high uncertainty	(39.9)	(53.5)	(48.4)	(64.2)	(47.2)	(51.8)
Average prize spread	112.1	96	101.7	58.6	103.3	95.8
total	(38.0)	(50.5)	(41.9)	(52.5)	(44.0)	(48.4)
Observations						
(low/high/total)	30/30/60	30/30/60	30/30/60	18/18/36	180	216

An "AB" pair consists of a subject graded A who is bargaining against a subject graded B. A bargaining solution from an AB pair is either the outcome from an A's offer to a B, or a B's offer to an A. The same goes for "AC" pairs and "BC" pairs. "Homogeneous" pairs consist of bargaining solutions from A vs. A, B vs. B or C vs. C. "Low" refers to random bonus distribution  $U(-3, 3)$ , while "High" refers to random bonus distribution  $U(-10, 10)$ .

Two tendencies are shown in Table 1: First, we observe that prize spread decreases with uncertainty-level. Except for the homogenous pairs, prize spread is lower under high uncertainty than under low uncertainty. This supports H1. Second, we see that prize spread increases quite strongly with ability-difference. It is lowest for the homogeneous pairs and highest for the AC pairs. This also corresponds with the prediction of the model and seems to support H2 above.

Let us examine H1 more closely. First, we report on a t-test of H1, dropping homogenous pairs from the sample (since H1 does not apply for homogenous pairs). We test the hypothesis that prize spread is the same under both low- and high uncertainty against the one-tailed alternative that prize spread is *higher* under low uncertainty than under high uncertainty. A two sample t-test<sup>14</sup> makes us reject the null-hypothesis of equal prize spread; prize spread is significantly higher under low uncertainty ( $t(178) = 1.65$ ,  $p = 0.05$ , one-tailed). When we run a regression, controlling for risk aversion, pair composition (heterogeneity) and gender, we get the same picture<sup>15</sup>, see Table 2. The coefficient on uncertainty-level ("high") is statistically significant within a 90 % confidence interval ( $p = 0.09$ ). Controlling for risk

<sup>14</sup>Since the two samples have unequal variances we use Welch's t-test.

<sup>15</sup>The Breusch-Pagan test cannot reject heteroskedasticity, requiring us to make a robust regression.

preferences, pair composition and gender, the regression shows a decrease in prize spread of NOK 11.0 when going from low uncertainty to high uncertainty. Importantly, we see that risk preferences cannot explain prize spread. It can also be shown that interaction variables on risk preferences and uncertainty-level are highly insignificant. This may seem surprising, but the majority of subjects are risk neutral or close to risk neutral over the relatively low stakes offered here. We thus establish our first main result from the experiment:

**Result 1** *Controlling for risk preferences, prize spread is higher under low uncertainty than under high uncertainty.*

Result 1 supports H1. Note also that the result indicates that effort is costly in our experiment. Recall that there are two effects of more noise on agent  $i$ 's optimal choice: It decreases the probability of winning *cet. par.* so agent  $i$  might want to decrease prize spread in order to save on effort costs, or she can eliminate the reduced winner probability by increasing the prize spread, since higher prize spread increases the effort-difference between the agents. The experiment indicates that the former effect dominates, suggesting substantial effort costs.

An alternative explanation for result 1 is that subjects have some kind of fairness concerns: High prize spread is OK if effort is important, but not if luck is important (see Cappelen et al., 2007, for an interesting experiment on the relationship between effort and distributive justice). In our experimental setting, we cannot make certain conclusions whether or not these effects exist. One would, however, expect that if fairness concerns play a role, then gender and/or personality have an impact on prize spread *per se*, and on the relationship between prize spread and uncertainty. Several studies show that social preferences are stronger among women (see Croson and Gneezy, 2004, for a survey), and concerns for distributive justice<sup>16</sup> have been shown to be correlated with personality traits derived from the Big-Five personality test (see Skarlicki et al. 1999). However, we find no significant effects from personality and gender, indicating that neither gender nor fairness concerns drive result 1. We tested for a number of interaction variables. Only one of these turned out significant: A decrease in prize spread from

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<sup>16</sup>Concerns for distributive justice are in the psychology literature measured by individual differences in reward allocation decisions, and individual differences in reaction to inequity, see Major and Deaux (1982) for an early review.

higher uncertainty is significant among pairs where women are present, while there is no effect in pairs with only men. However, the effect was larger in pairs with one man and one woman than in pairs with only women, making it tendentious to conclude that gender affects the relationship between prize spread and uncertainty.

Let us now consider H2: Prize spread increases with ability-difference. Table 2 indicates that there is a positive relationship between prize spread and ability-difference among heterogenous pairs, but since the corollary also applies when  $k_j$  is reduced from  $k_j = k_i$  to  $k_j < k_i$ , we must also include the homogenous pairs in the sample. We first report on t-tests on the relationship between prize spread and each pair composition. Let  $s(h)$ ,  $h = AC, AB, CB, HOMO$  denote prize spread as a function of ability-difference. The tests support that  $s(AC) > s(AB) = s(BC) > s(HOMO)$ . From Table 3, we see that all tests are significant within a 90 % confidence interval except for  $s(BC)$  vs.  $s(AB)$ , as predicted. We can thus state

**Result 2** *Prize spread increases with ability-difference.*

Result 2 supports H2.<sup>17</sup> As predicted by the model, Result 2 should also imply that effort increases with ability-difference. This leads us to H3. Table 4 displays a robust regression where effort, i.e. number of right answers (random bonus excluded) is the dependent variable. We see that prize spread has a significantly positive effect on effort ( $p = 0.017$ ).<sup>18</sup> For a NOK 1 increase in prize spread, the number of correct answers increases with 0.034. This may seem like a small effect, but it means that an increase from zero prize spread to max prize spread of NOK 200 increases the number of correct answers with 6.8. We thus have

**Result 3** *Effort increases with prize spread.*

Results 2 and 3 support H3: Higher ability-difference increases prize spread, which in turn increases effort. But note from Table 4 that when we *control for prize spread*, ability-difference has a negative effect on effort. This fits with other findings in the literature (starting with Bull et al.,1987) and

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<sup>17</sup>It can be shown that Results 1 and 2 hold when we control for who is proposer and who is responder. In particular, we find the same results when we examine the high-ability subjects' proposals.

<sup>18</sup>Although there is a positive relationship between dependent variables in this regression (ability-difference and prize spread), we do not have an endogeneity problem since prize spread and ability-difference are not determined simultaneously. Moreover, tests shows that the level of multicollinearity is sufficiently limited, allowing us to use the robust OLS-regression presented here.

supports our model. From the IC constraints, we see that for a *given prize spread*, effort decreases with ability-difference. This result is well known and traces back to Lazear and Rosen (1981).

## 6 Conclusion

In this paper we analyze and experimentally test a tournament model where heterogenous agents determine prize spread. We find some results particularly interesting. First, our corner solutions elucidate empirical puzzles on firm characteristics and wage structure, since marginal differences in heterogeneity, uncertainty and size of the prize pool can significantly impact on prize spread.

Second, our theoretical result on the positive relationship between heterogeneity and prize spread, supported by the experiment, challenges the idea that heterogenous agents should not participate in the same contest. Higher ability-difference triggers higher prize spread, resulting in higher effort.

Third, our model shows that if agents set prize spread in an asymmetric tournament, then we can expect a negative relationship between noise/uncertainty and prize spread. This result is supported experimentally and has important empirical implications. It suggests that the relationship between wage structure and uncertainty in an industry is affected by employee power, such as the degree of unionization. Moreover, it implies that an observed negative relationship between prize spread and uncertainty does not have to be explained by risk aversion or fairness concerns.

As noted in the introduction, several wage-setting regimes have the feature of being tournaments where the contestants themselves set prizes, and it is thus important to understand these tournaments. Our paper is just a small contribution as compared to what should be investigated. Future theoretical research should explore sorting, risk aversion, and social preferences within the setting presented here. The constraint on the total prize should also be relaxed, and richer bargaining environments that include bargaining between agents and principal should be investigated. Future experimental work should not only include real-effort experiments, since some control is lost when we do not know the agents' cost functions.

## Appendix

### Proof of (7)

From the IC constraint  $(R - 2w_2)g(e) = C'_j(e_j)$  we have

$$(R - 2w_2)g'(e) \frac{\partial e}{\partial e_j} \frac{\partial e_j}{\partial w_2} - C''_j(e_j) \frac{\partial e_j}{\partial w_2} = 2g(e)$$

Substituting from this and the IC constraint in (6) we then get

$$\begin{aligned} W'(w_2) &= 1 - 2G(e) - (R - 2w_2)g(e) \frac{2g(e)}{(R - 2w_2)g'(e) \frac{\partial e}{\partial e_j} - C''_j(e_j)} \\ &= 1 - 2G(e) - \frac{2g(e)}{\frac{g'(e)}{g(e)} \frac{\partial e}{\partial e_j} - \frac{C''_j(e_j)}{C'_j(e_j)}} \end{aligned}$$

The last expression coincides with the one in (7), and hence proves the formula.

### Proof of Proposition 2

Consider

$$F(d) \equiv 1 - 2\Gamma(d) - K \frac{2\gamma(d)d}{\frac{\gamma'(d)}{\gamma(d)}d - (n-1)} = 1 - 2\Gamma(d) + 2K \frac{\gamma(d)^2 d}{-\gamma'(d)d + (n-1)\gamma(d)}$$

We have

$$F'(d) = -2\gamma'(d) + 2K \frac{(2\gamma(d)d + \gamma(d)^2)(-\gamma'(d)d + (n-1)\gamma(d)) - \gamma(d)^2 d(-\gamma''(d)d - \gamma'(d) + (n-1)\gamma'(d))}{(-\gamma'(d)d + (n-1)\gamma(d))^2}$$

and so

$$F'(0) = 2\gamma(0) \left[ -1 + K \frac{(n-1)\gamma(0)^2}{((n-1)\gamma(0))^2} \right] = 2\gamma(0) \left[ -1 + K \frac{1}{(n-1)} \right]$$

Hence  $F'(0) > 0$  iff  $K > n - 1$ . For  $K > n - 1$ , i.e. low heterogeneity ( $\frac{k_j}{k_i} < \left(\frac{n}{n-1}\right)^{n-1}$ ), we then have by continuity  $F(d) > 0$  for  $0 < d < d_1$ , some  $d_1 > 0$ .

Let  $e_{\max}$  be the effort spread corresponding to  $w_2 = 0$ ; it is from the IC constraint and (8)-(9) given by

$$Rg(e_{\max}) = nk_j(Ke_{\max})^{n-1} \equiv k(e_{\max})^{n-1} \quad (13)$$

where  $k = nk_j(K)^{n-1}$  is defined by the identity. Since  $g(e) = \gamma\left(\frac{e}{\sigma}\right)\frac{1}{\sigma}$  we

then have  $e_{\max} = \sigma d_m$ , where  $d_m$  is given by

$$(R/\sigma^n)\gamma(d_m) = k(d_m)^{n-1}, \quad (d_m = e_{\max}/\sigma) \quad (14)$$

We see that  $d_m \rightarrow 0$  as  $R/\sigma^n \rightarrow 0$ , hence there is  $r_1 > 0$  such that  $d_m < d_1$  for  $R/\sigma^n < r_1$ . It thus follows that for  $R/\sigma^n < r_1$  we have  $F(d) > 0$  for  $0 < d < d_m$ , and hence  $W'(w_2) > 0$  for  $0 < w_2 < R/2$ . The minimal spread  $w_2 = R/2$  (and  $e = 0$ ) is thus optimal here.

By a similar reasoning we can also show that for small  $R$  and/or large  $\sigma$  it is optimal to induce maximal effort spread when the degree of heterogeneity is 'large'. Consider high heterogeneity:  $K < n - 1$ . We then have  $F'(0) < 0$  and hence  $F(d) < 0$  for  $0 < d < d_2$ , some  $d_2 > 0$ . From (14) we now see that there is  $r_2 > 0$  such that  $d_m < d_2$  for  $R/\sigma^n < r_2$ . For  $R/\sigma^n < r_2$  we thus have  $F(d) < 0$  for  $0 < d < d_m$ , and hence  $W'(w_2) < 0$  for  $0 < w_2 < R/2$ . The maximal spread  $w_2 = 0$  (and  $e = e_{\max}$ ) is thus optimal here. This completes the proof.

### Proof of Proposition 3

Consider

$$F(d, K) \equiv 1 - 2\Gamma(d) - K \frac{2\gamma(d)d}{\frac{\gamma'(d)}{\gamma(d)}d - (n-1)}, \quad 0 \leq d \leq \bar{d}$$

where  $\Gamma(\bar{d}) = 1$ . We show below that for normally or uniformly distributed noise terms the following conditions are satisfied:

- (c1)  $F(d, K) < 0$  for all  $d \in (0, \bar{d})$  when  $K \leq n - 1$
- (c2)  $F(d, K) \geq 0$  as  $d \leq d_0$ ,  $d \in (0, \bar{d})$ , when  $K > n - 1$
- (c3)  $\gamma'(d) < 0$  for  $d \in (0, \bar{d})$ , and  $\gamma(d)d \rightarrow 0$  as  $d \rightarrow \bar{d}$ .

Let  $D$  be the class of distributions that satisfy (c1-c3).

(i) Consider first low heterogeneity;  $K > n - 1$ .

Let  $d_m = d_m(R/\sigma^n)$  be defined as in the proof of Proposition 1, see (14). We see that  $d'_m > 0$  and  $d_m \rightarrow \bar{d}$  as  $R/\sigma^n \rightarrow \infty$ , and hence that there is  $r_0 > 0$  such that  $d_m \leq d_0$  as  $R/\sigma^n \leq r_0$ , where  $d_0$  is the root defined in (c2). For  $R/\sigma^n < r_0$  we thus have  $F(d, K) > 0$  all  $d \in (0, d_m)$  and therefore  $W'(w_2) > 0$  all  $w_2 \in (0, R/2)$ . This implies that  $w_2 = R/2$  (and thus  $e = 0$ ) is optimal for  $R/\sigma^n < r_0$ . The optimal value is then  $W(R/2) = R/2$ .

For  $R/\sigma^n > r_0$  we have  $d_m > d_0$  and hence  $F(d, K) \geq 0$  as  $d \leq d_0$ ,  $d \in (0, d_m)$ . There is thus  $w_{20}$  such that  $W'(w_2) \geq 0$  as  $w_2 \geq w_{20}$ ,  $w_2 \in (0, R/2)$ .



Hence either  $w_2 = R/2$  ( $e = 0$ ) or  $w_2 = 0$  ( $e = e_{\max}$ ) is then optimal.

For  $w_2 = 0$  the value is

$$W_0 = G(e_{im} - e_{jm})R - C_i(e_{im})$$

where  $e_{im}, e_{jm}$  are the agents' respective efforts when  $w_2 = 0$ , and thus  $e_{im} - e_{jm} = e_{\max}$ . Consider now how the value  $W_0$  varies with  $R$ . Since efforts depend on prizes via  $R - 2w_2$ , we will have  $\frac{\partial e_{im}}{\partial R} = -\frac{1}{2} \frac{\partial e_l}{\partial w_2}$ . By the same reasoning that led from (6) to (10) and (12) we then obtain

$$\begin{aligned} \frac{\partial}{\partial R} W_0 &= G(e_{im} - e_{jm}) + g(e_{im} - e_{jm}) \left( \frac{\partial e_{im}}{\partial R} - \frac{\partial e_{jm}}{\partial R} \right) R - C'_i(e_{im}) \frac{\partial e_{im}}{\partial R} \\ &= -\frac{1}{2} \left( \hat{F}(e_m) - 1 \right), \quad e_m = e(0) = e_{\max} \\ &= -\frac{1}{2} (F(d_m) - 1), \quad d_m = \frac{e_m}{\sigma} \end{aligned}$$

Comparing the values corresponding to  $w_2 = R/2$  and  $w_2 = 0$  we thus have

$$\frac{\partial}{\partial R} (W(R/2) - W_0) = \frac{1}{2} - \frac{\partial}{\partial R} W_0 = \frac{1}{2} + \frac{1}{2} (F(d_m) - 1) = \frac{1}{2} F(d_m)$$

From the properties of  $F(d_m)$  it then follows that the value difference is increasing for  $R < r_0 \sigma^n$  (where  $d_m < d_0$ ) and decreasing for  $R > r_0 \sigma^n$ . If the difference is negative for  $R$  sufficiently large, it then follows that there is  $R_1 > 0$  such that  $W(R/2) \geq W_0$  as  $R \leq R_1$ .

Consider

$$\begin{aligned} W_0 &= G(e_{im} - e_{jm})R - C_i(e_{im}) = G(e_m)R - K_i(e_m)^n \\ &= \Gamma(d_m)R - K_i(d_m)^n \sigma^n, \end{aligned}$$

where  $K_i = k_i \left( 1 - \left( \frac{k_i}{k_j} \right)^{\frac{1}{n-1}} \right)^{-n}$ , and  $d_m = \frac{e_m}{\sigma}$  is determined by  $\gamma(d_m)(R/\sigma^n) = k d_m^{n-1}$ . So we have

$$\begin{aligned} \frac{W_0}{W(R/2)} &= 2 \left( \Gamma(d_m) - \frac{K_i(d_m)^n}{(R/\sigma^n)} \right) = 2 \left( \Gamma(d_m) - \frac{K_i(d_m)^n}{k d_m^{n-1} / \gamma(d_m)} \right) \\ &= 2 \left( \Gamma(d_m) - \frac{K_i}{k} \gamma(d_m) d_m \right) \end{aligned}$$

where  $\frac{K_i}{k} = \frac{1}{n} \left( 1 - \left( \frac{k_i}{k_j} \right)^{\frac{1}{n-1}} \right)^{-1}$ . Since  $d_m \rightarrow \bar{d}$  and thus  $\gamma(d_m) d_m \rightarrow 0$  as  $R \rightarrow \infty$ , we see that  $W_0 > W(R/2)$  for  $R$  sufficiently large. Since moreover  $d_m$  and hence the ratio  $\frac{W_0}{W(R/2)}$  depends on  $R$  and  $\sigma$  via  $R/\sigma^n$ , we see that there is indeed  $r_1 > 0$  such that  $W_0 > W(R/2)$  iff  $R/\sigma^n > r_1$ . This proves statement (i) in the proposition.

(ii) Consider next high heterogeneity;  $K < n - 1$ .

It follows from property (c1) that we then have  $F(d, K) < 0$  for all  $d \in (0, \bar{d})$ , and therefore  $W'(w_2) < 0$  for all  $w_2 \in (0, R/2)$ , for any (feasible)  $R > 0$ . Hence  $w_2 = 0$  is always optimal in this case. This proves statement (ii).

Finally, consider statement (iii).

We have  $e_{\max} = \sigma d_m(R/\sigma^n)$ , where  $d'_m(\cdot) > 0$ , and hence

$$\frac{\partial}{\partial \sigma} e_{\max} = d_m(R/\sigma^n) + \sigma d'_m(R/\sigma^n)(-nR\sigma^{-n-1}) = d_m(R/\sigma^n) - n(R/\sigma^n)d'_m(R/\sigma^n)$$

Defining  $r = R/\sigma^n$  we have  $d_m$  given by  $\gamma(d_m)r = kd_m^{n-1}$  and hence

$$\gamma(d_m) + r\gamma'(d_m)d'_m = k(n-1)d_m^{n-2}d'_m, \quad \text{i.e.}$$

$$d'_m = \frac{\gamma(d_m)}{k(n-1)d_m^{n-2} - r\gamma'(d_m)} = \frac{\gamma(d_m)}{r\gamma'(d_m)(n-1)d_m^{-1} - r\gamma'(d_m)} > 0$$

(The inequality follows from  $\gamma' < 0$ .) Hence

$$\begin{aligned} \frac{\partial}{\partial \sigma} e_{\max} &= d_m - nr d'_m = d_m - nr \frac{\gamma(d_m)}{r\gamma'(d_m)(n-1)d_m^{-1} - r\gamma'(d_m)} \\ &= d_m \left[ 1 - \frac{n\gamma(d_m)}{\gamma'(d_m)(n-1) - \gamma'(d_m)d_m} \right] \end{aligned}$$

For  $\sigma \rightarrow 0$  we have  $r \rightarrow \infty$  and  $d_m \rightarrow \bar{d}$ , hence  $(\frac{\partial}{\partial \sigma} e_{\max})/d_m \rightarrow [1 - 0] > 0$

For  $\sigma \rightarrow \infty$  we have  $r \rightarrow 0$  and  $d_m \rightarrow 0$ , hence  $(\frac{\partial}{\partial \sigma} e_{\max})/d_m \rightarrow \left[ 1 - \frac{n\gamma(0)}{\gamma(0)(n-1)+0} \right] < 0$ .

This proves statement (iii).

It remains to show that uniformly or normally distributed noise yields distributions that satisfy (c1 - c3).

Consider first the uniform case:  $\varepsilon_l \sim U[0, 1]$  (so  $z_l \sim U[a, a + \sigma]$ ). Here we have  $\Gamma(d) = \Pr(\varepsilon_j - \varepsilon_i < d) = 1 - \frac{1}{2}(1-d)^2$  for  $0 \leq d \leq 1$ , and thus  $\gamma(d) = 1 - d$ . From (12) we then have

$$\begin{aligned} F(d) &= 1 - 2 \left( 1 - \frac{1}{2}(1-d)^2 \right) - K \frac{2(1-d)d}{\frac{-1}{(1-d)}d^{-(n-1)}} \\ &= -1 + (1-d)^2 + K \frac{2(1-d)^2 d}{d+(n-1)(1-d)} \end{aligned}$$

This yields

$$\begin{aligned} F'(d) &= 2(1-d) \frac{K[-2d(d+(n-1)(1-d))+(1-d)(n-1)]-(d+(n-1)(1-d))^2}{[d+(n-1)(1-d)]^2} \\ &\equiv 2(1-d) \frac{f(d, K)}{[d+(n-1)(1-d)]^2} \end{aligned}$$

Note that the sign of  $F'(d)$  is determined by the 2nd-order polynomial  $f(d, K)$ . We have

$$\begin{aligned} f(0, K) &= (n-1)(-n+1+K), & f(1, K) &= -2K-1 < 0, \\ f''(d; K) &= 2(n-2)(2-n+2K) \end{aligned}$$

For  $K \leq n-1$  we have  $f(0, K) < 0$  and  $f(1, K) < 0$ . Suppose there is  $d \in (0, 1)$  such that  $f(d, K) \geq 0$ . Then the term multiplying  $K$  in the

definition of  $f()$  must be positive, and hence

$$f(d, K) \leq f(d, n-1) = d((n-2)nd + 1 - n^2)$$

But the last parenthesis is smaller than  $((n-2)n + 1 - n^2) = 1 - 2n < 0$ , hence we have a contradiction. Thus  $f(d) < 0$  and hence  $F'(d) < 0$  for all  $d \in (0, 1)$ , thus condition (c1) holds.

For  $K > n - 1$  we have  $f(0, K) > 0$ ,  $f(1, K) < 0$  and  $f''(d, K) > 0$ , hence  $f(d, K) \geq 0$  as  $d \leq d_1$  for some  $d_1 \in (0, 1)$ , and the same is then true for  $F'(d)$ . Since  $F(1) < 0$ , it is thus the case that  $F(d) = 0$  has a unique root for  $d \in (0, 1)$ , and hence that condition (c2) holds.

Consider finally normally distributed noise. As a normalization suppose  $\varepsilon_i - \varepsilon_j \sim N(0, 1)$ , and let  $\phi()$  and  $\Phi()$  be the corresponding density and CDF, respectively. Note that we have  $\phi'(d) = -d\phi(d)$ , and thus from (12)

$$F(d, K) \equiv 1 - 2\Phi(d) - K \frac{2\phi(d)d}{\frac{\phi'(d)}{\phi(d)}d - (n-1)} = 1 - 2\Phi(d) + 2K \frac{\phi(d)d}{d^2 + (n-1)}$$

This yields

$$\begin{aligned} F'(d, K) &= -2\phi(d) + 2K \frac{(\phi(d) + \phi'(d)d)[d^2 + (n-1)] - \phi(d)d2d}{[d^2 + (n-1)]^2} \\ &= 2\phi(d) \left[ -1 + K \frac{(1-d^2)[d^2 + (n-1)] - 2d^2}{[d^2 + (n-1)]^2} \right] \end{aligned}$$

Consider first  $K \leq n - 1$ . Suppose  $F'(d, K) \geq 0$  for some  $d > 0$ . Then the term multiplying  $K$  must be positive and hence

$$F'(d, K) \leq F'(d, n-1) = 2\phi(d) \left[ -d^2 \frac{n-2+d^2n+n^2}{(d^2+n-1)^2} \right]$$

This yields a contradiction, hence  $F'(d, K) < 0$  for all  $d > 0$  when  $K \leq n - 1$ . This shows that (c1) holds.

Consider next  $K > n - 1$ . Let  $y = d^2$ , and note that  $F'(d, K)$  has the same sign as the 2nd-order polynomial

$$f(y, K) = -(y + (n-1))^2 + K((1-y)(y + (n-1)) - 2y)$$

We have here

$$f(0, K) > 0, \quad f(1, K) < 0, \quad f''(y; K) = -2K - 2 < 0$$

Hence there is  $y_1 \in (0, 1)$  such that  $f(y, K) \geq 0$  as  $y \leq y_1$ , and consequently  $F'(d, K) \geq 0$  as  $d \leq d_1$  for some  $d_1 > 0$ . Since  $F(d, K) \rightarrow -1$  as  $d \rightarrow \infty$ , we then see that (c2) holds.

This completes the proof.

## Tables and figures

**Table 2: Prize spread and uncertainty**

Dependent variable: Prize spread	
High	-10.976* (p=0.093)
AB	-15.780* (p=0.053)
BC	-10.935 (p=0.139)
Male	11.113 (p=0.326)
Averse	-3.364 (p=0.682)
Love	-5.552 (p=0.501)
Non-monotonic	-8.891 (p=0.200)
Constant	119.826*** (p<0.000)
R-squared	0.052
Sample size	180

*Notes:* Robust OLS estimates. Level of significance: \* = 0.10, \*\* = 0.05, \*\*\* = 0.01. Homogenous pairs not included. "High" is the uncertainty dummy, which is equal to one if uncertainty is high and zero if uncertainty is low. "Male" is the gender dummy, which equals one if the pair consists solely of men and zero if a woman is a part of a pair. The reference group for pair composition is AC pairs. The dummy "AB" equals one if the pair is an AB pair and zero otherwise. The same goes for the "BC" dummy. For risk preferences we use the lottery choices elicited in step 1 of the experiment, and categorize pairs in four different risk categories. As in Dohmen and Falk (2006) there were some subjects that did not have a unique switching point, making us have a "non-monotonic" dummy. The "averse" dummy is equal to one if the pair is risk averse and zero if not, and "love" equals one if a pair is risk loving, and zero otherwise. Risk neutral pairs are the reference group. In risk averse pairs at least one is risk averse and no-one is risk loving. In risk loving pairs at least one is risk loving and no-one is risk averse, while in risk neutral pairs both are risk neutral, or one is risk loving while the other is risk averse.

**Table 3: Prize spread and ability-difference**

Prize spread	Welch's t-test
AC versus AB	p = .03
AC versus BC	p = .08
BC versus AB	p = .51
AC versus HOMO	p < 0.001
AB versus HOMO	p < 0.001
BC versus HOMO	p < 0.001

*Notes:* The t-tests are one-sided except for BC versus AB which is a two-sided test.

**Table 4: Effort and ability-difference**

Dependent variable:	
Effort	Coef.
Prize spread	0.034** (p=0.017)
AC	-5.267** (p=0.022)
AB	-1.458 (p=0.528)
BC	-3.206 (p=0.104)
A	9.186*** (p<0.001)
C	-6.141*** (p<0.001)
Constant	10.981***
R-squared	0.510
Sample size	108

Notes: Robust OLS estimates. Level of significance: \*=0.10, \*\*=0.05, \*\*\*=0.01.  
Here the data are on individuals, not pairs, making the dummies AB, AC and BC equal one if the subject was part of the relevant pair. The reference group consists of subjects that were part of homogeneous pairs. The dummies A and C are ability-levels, with B as reference group.

**Table 5: Effort data****Effort piece rate**

Ability level:	A	B	C
Mean effort:	21,4	9,5	3,2
Std. Dev	4,76	3,38	1,62
Median	20,5	9,5	3
Min	16	4	1
Max	33	17	6
# Obs	36	36	36

**Effort tournament**

Ability level:	A	B	C
Mean effort:	20,4	12,6	4,6
Std.Dev	9,71	6,34	3,1
Median	19,5	13	4
Min	5	1	1
Max	38	25	12
# obs	36	36	36

Effort means number of right answers in a 5 minute work period

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