View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by Research Papers in Economics

NHH

INSTITUTT FOR FORETAKSØKONOMI

DEPARTMENT OF FINANCE AND MANAGEMENT SCIENCE

FOR 9 2008

ISSN: 1500-4066

MARCH 2008

Discussion paper

Credit Spreads and Incomplete Information

BY

SNORRE LINDSET, ARNE-CHRISTIAN LUND, AND **SVEIN-ARNE PERSSON**

Norges Handelshøyskole

NORWEGIAN SCHOOL OF ECONOMICS AND BUSINESS ADMINISTRATION

Credit Risk and Asymmetric Information: A Simplified Approach

Snorre Lindset, Arne-Christian Lund, and Svein-Arne Persson*

August 31, 2011

Abstract

If only debtholders receive delayed information about the state of a company, the credit spread on risky corporate debt is wider than if both debt- and equityholders receive information with the same delay. Information asymmetry leads to wider credit spreads than only delayed, but symmetrically distributed, information. Incomplete, in particular asymmetric and delayed, information's impact on the pricing of risky corporate debt is analyzed in a simplified version of the seminal Duffie and Lando (2001) model. Incomplete information is costly in the sense that companies default earlier and credit spreads, in most, but not all, cases widen, both compared to the case of full information. Delayed, but symmetrically distributed information has, for realistic parameter values, only a minor effect on credit spreads compared to the full information case.

Keywords and phrases: Optimal default, credit spreads, asymmetric information.

JEL-classification: G12, G33

1 Introduction

We show that information asymmetry leads to wider credit spreads than only delayed, but symmetrically distributed, information in most realistic cases.

*Lindset is a Professor at the Norwegian University of Science and Technology, Department of Economics, Lund is a post. doc fellow at the Norwegian School of Economics, and Persson is a Professor at the Norwegian School of Economics. The authors would like to thank Darrel Duffie, Hans Marius Eikseth, Steinar Ekern, Chris Florackis, Nadine Gatzert, Kay Giesecke, Jørgen Haug, Kristian Miltersen, Aksel Mjøs, Jøril Mæland, Øyvind Norli, Per Östberg, and participants at faculty seminars at Trondheim Business School, the Norwegian School of Economics, Princeton University, the University of Stavanger, Norwegian University of Science and Technology, Department of Economics, participants at European Financial Management Association Annual Meeting in Athens 2008, and European Group of Risk and Insurance Economists Meeting in Toulouse 2008 for useful comments and discussions. *First version: March 4, 2008.*

Incomplete, in particular asymmetric and delayed, information's impact on the pricing of risky corporate debt is analyzed in a version of the seminal Duffie and Lando (2001) model. Duffie and Lando (2001) use a more complicated (and realistic) information structure than we do, including noisy accounting information. As their model, our model is a structural model of credit risk for which a default intensity exists, but we simplify their model by not including noisy accounting information. We get qualitatively similar results, suggesting that the presence of asymmetric information between two groups of agents is the important property of a credit risk model if the objective is to obtain wider credit spreads on risky corporate debt than those obtained from standard structural models of credit risk. In order to analyze the effect of delayed, either symmetrically or asymmetrically distributed information, we also analyze the case where even the better informed group of agents receives delayed information. This situation leads to the presence of a bankruptcy wild card, a non-negative cashflow (strictly positive if the liquidation value of the company exceeds the sum of debt and bankruptcy costs). This novel property of our model captures the fact that even the better informed group of agents, the equityholders, must make the default decision based on incomplete information. The presence of the bankruptcy wild card tends to accelerate the default decision. However, only in situations with extraordinary long information delays and/or high volatilities are credit spreads on risky corporate debt wider, compared to the situation when equityholders have complete information.

Incomplete information is costly in the sense that companies default earlier and credit spreads in most, but not all, cases widen. From our model we learn that the major cause of this widening is asymmetric information between two groups of agents, and not the delay in the information per se.

Our model contains two distinct types of agents; bond- and equityholders.[1] The two types of agents are grouped into one or two groups. A group is characterized by the information available to the agents in the group. Equityholders are always included in the better informed group. Incompleteness of information is caused by delayed observations of the state variable. The better informed group observes the state variable with shorter delay than the less infomed group. We calculate the optimal bankruptcy barrier in the presence of the bankruptcy wild card. To highlight the insights of the model, we analyze the following four cases:

- Case 1: The complete information case, cf. Leland (1994), our benchmark case, where all agents are perfectly informed.
- Case 2: The general case where both bond- and equityholders have incomplete information, but equityholders are better informed than bondholders.

[1] For future research we propose in section 5 to include a third group (management).

- Case 3: Equityholders have perfect information, bondholders have acces to delayed information only, cf. Duffie and Lando (2001).
- Case 4: All agents have access to the same delayed information.

Our paper is also related to Giesecke (2006). He analyzes two classes of models of imperfect information: 1) Models where the bankruptcy barrier is not observable to all agents (see also Giesecke and Goldberg (2004) for more on this). 2) Models with incomplete information about the value of the company's assets. Our model belongs to his category 2). From his results (Proposition 6.4) we know that a default intensity exists for our model. His results in the case where agents do not have any information about the state variable, may be seen as a special case of our model where agents have access to delayed information. Collin-Dufresne, Goldstein, and Helwege (2003) calculate the default intensity explicitly in a similar model, analyzing somewhat different issues.

Credit risk is a topic that has received attention in both the academic literature and among practitioners. There are two dominating approaches to credit risk modelling in the finance literature; *structural models* and *reduced form models*. The first was pioneered by Merton (1974). Merton models the value of a company's assets by a stochastic process and debt and equity are considered as contingent claims on total asset value. Some of the papers in this tradition include Black and Cox (1976), Geske (1977), Longstaff and Schwartz (1995), Leland (1994), and Duffie and Lando (2001). The second approach assumes the existence of a default arrival intensity. This approach was pioneered by Jarrow and Turnbull (1992), for extensions see e.g., Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997), and Schönbucher (1998).[2] Coculescu, Geman, and Jeanblanc (2008) and Guo, Jarrow, and Zeng (2008) analyze technical aspects of credit risk and incomplete information. Giesecke (2006) and Giesecke and Goldberg (2004) analyze how different information sets in structural models affect default arrival intensities.

Jarrow and Protter (2004) argue that the essential difference between structural and reduced form models is the assumption regarding which information that is available to the modeler. In their terminology, a model is structural if the modeler can observe the state of the company and reduced form if not. They write (page 2): "...there appears to be *no* disagreement that the asset value process is unobservable by the market... Although not well understood in terms of its implications, this consensus supports the usage of reduced form models." In our set-up we find that it is *not* the incompleteness of information about the asset value process that is important

[2] Comprehensive treatments of these two approaches can be found in the enclopedic monograph by Bielecki and Rutkowski (2002) or in the more accessible monograph by Duffie and Singleton (2003).

for pricing of credit risk. If different groups of agents have access to the same incomplete information about the process, the error made by using a structural model, compared to a reduced form model, interpreted as in Jarrow and Protter (2004), is negligible. The important factor is any information *asymmetry* between the agents in the credit market (bondholders) and the agents operating the company (equityholders).

The paper is organized as follows: In section 2 we present our economic model. In section 3 we analyze the optimal default barrier and credit risk valuation. Special cases with numerical examples are presented and analyzed in section 4. Section 5 concludes the paper and gives suggestions for future reserach.

2 Economic Model

This section presents our model of a company with incomplete secondary market information about the credit quality of its debt. In particular, we show how incomplete information affects the valuation of debt in secondary markets. Therefore, we do not address whether debt is issued in an optimal way, i.e., whether the capital structure of the issuer is optimal or not. Our model is standard, and we follow closely the set-up by Duffie and Lando (2001).

We assume that the only state variable is the *stock of assets*. It is given as the solution to the stochastic differential equation

$$dV_t = \mu V_t dt + \sigma V_t dB_t, \tag{1}$$

where μ and σ are constants. We assume that $\mu < r$, where r is the constant risk free interest rate, and that its time 0 value, V_0, is a given constant. Here, the process $B = \{B_t\}_{t\geq 0}$ is a standard Brownian motion defined on a fixed, filtered probability space $(\Omega, \mathcal{F}, P)$. The process $V = \{V_t\}_{t\geq 0}$ is known as a geometric Brownian motion and V_t is log-normally distributed. Also, P represents the original probability measure. All agents in the economy are assumed risk neutral. The information filtration $\mathcal{F}_t$ is generated by the process $\{V_s,\ 0 \leq s \leq t\}$ (augmented with all sets of measure zero).

To incorporate asymmetric information, we assume that two distinct groups of agents are completely characterized by their information filtrations $\mathcal{F}_t^m$ and $\mathcal{F}_t^l$, respectively. Superscripts m and l signify *more* and *less* information. We define

$$\mathcal{F}_t^m = \mathcal{F}_{t-m}, \text{ for all } t \geq m,$$

$$\mathcal{F}_t^l = \mathcal{F}_{t-l}, \text{ for all } t \geq l,$$

and $0 \leq m \leq l$. Clearly, from this specification the filtrations can be nested as

$$\mathcal{F}_t^l \subseteq \mathcal{F}_t^m \subseteq \mathcal{F}_t.$$

Thus, in the case $m > 0$, also the better informed agents have incomplete information about the state variable, whereas in the case $m = 0$, the better informed agents have complete information. We interpret m as a measure of *information delay* and $l - m$ as a measure of *information asymmetry* in the remainder of the paper. Thus, information delay refers to information available to the better informed agent, whereas information asymmetry refers to the difference in the information available to the two groups of agents.

At time t the asset process generates a dividend δV_{t-m}, for some constant $\delta > 0$, to the equityholders. Observe that the dividend payment at time t is determined by the delayed value V_{t-m} of the asset process.

The time t present value of all future dividends, $\hat{V}_t^m$, is therefore

$$\hat{V}_t^m = E\Big[\int_t^\infty e^{-r(s-t)}\delta V_{s-m}ds\Big|\mathcal{F}_t^m\Big] = \frac{\delta V_{t-m}}{r-\mu}. \tag{2}$$

Observe that the present value $\hat{V}_t^m$ is just a multiple of V_{t-m}, either one of these quantities could therefore be used as the state variable. We assume that the dividends are not observable for the less informed group of agents, otherwise they could calculate the better informed group's assessed value of the company, and, thus, eliminate the information asymmetry. The quantity $\hat{V}_t^m$ is sometimes called *the unlevered value of the company.*

As in Leland (1994), we assume that the company has issued perpetual debt with face value D. The debt is serviced by a constant rate of coupon payments C. These payments are tax deductible (only interest is paid on perpetual debt). The tax benefit rate is θC, where θ is the tax rate.

We define the stopping time τ with respect to the filtration $\mathcal{F}_t^m$ for fixed $t \geq m$ as

$$\tau = \inf\{u \geq t : V_{u-m} \leq W^m\}, \tag{3}$$

where W^m is $\mathcal{F}_t^m$-measurable (i.e., a constant for agents with information given by $\mathcal{F}_t^m$). In this model, the company is bankrupt and liquidated the first time $V_{t-m} = W^m$, i.e., τ represents the time of bankruptcy. In the special case where $m = 0$, we denote W^0 by W. At the time of bankruptcy, the value of the company is immediately revealed and publicly available, and we define the liquidation value of the company as

$$\frac{\delta V_\tau}{r-\mu},$$

i.e., as the unlevered value of the company given full information at the time of bankruptcy. This value does not take any effects of possibly optimally restructured debt at bankruptcy into account. Furthermore, at the time of bankruptcy, a bankruptcy cost $\alpha\frac{\delta V_\tau}{r-\mu}$, $\alpha \in [0, 1]$, proportional to the liquidation value of the company occurs.

In addition to the information contained in $\mathcal{F}_t^l$, the less informed agents also observe whether the company is bankrupt or not. Formally, we define the information available to the less informed agents as

$$\mathcal{G}_t^l = \mathcal{F}_t^l \vee \sigma(1_{\{\tau > s\}}, s \leq t),$$

where $1_{\{.\}}$ denotes the usual indicator function.

The better informed agents are restricted from trading in the financial market, and the market therefore only consists of the less informed agents. Thus, market prices of credit risky instruments are set by the less informed agents. In this paper we assume that equityholders belong to the better informed group. The bondholders belong to the better informed group in the case of symmetric information and to the less informed group in the case of asymmetrric information.

3 Optimal Default Barrier and Credit Risk Valuation

3.1 Bankruptcy Wild Card

The decision to file for bankruptcy is taken by the better informed agents, i.e., by equityholders. Due to the delay m, there is a positive probability that the full information value of the stock of assets, V_τ, is larger than the delayed value of the stock of assets, $V_{\tau-m}$, on which the bankruptcy decision is made. From standard properties of geometric Brownian motions, the complete-information value of the stock of assets at the bankruptcy time τ is given by

$$V_\tau = V_{\tau-m} e^{(\mu - \frac{1}{2}\sigma^2)m + \sigma(B_\tau - B_{\tau-m})} \tag{4}$$

and is also log-normally distributed. Using the definition of the barrier W^m in expression (3), the time τ value of the stock of assets can also be written as

$$V_\tau = W^m e^{(\mu - \frac{1}{2}\sigma^2)m + \sigma(B_\tau - B_{\tau-m})}.$$

In case of bankruptcy, the debtholders, according to *absolute priority*, require the face value of the debt D. As explained above, a positive delay m has the consequence that $\frac{\delta V_\tau}{r-\mu} - \alpha \frac{\delta V_\tau}{r-\mu} - D > 0$ with positive probability. In this case, the time τ liquidation value of the company is more than sufficient to cover debt and bankruptcy costs, and any proceeds are paid to the equityholders. By deciding to file for bankruptcy at time τ, the equityholders get a bankruptcy wild card[3] with payoff $(\frac{\delta(1-\alpha)}{r-\mu} V_\tau - D)^+$.

[3]This wild card has some resemblance to the *wild card play* that is present when trading the CBOT Treasury bond futures, see e.g., Hull (2006, pp. 135-136).

The time τ value of this wild card is

$$\pi(W^m) = E[(\frac{\delta(1-\alpha)}{r-\mu}V_\tau - D)^+ \big| \mathcal{F}_\tau^m] \\ = \frac{\delta(1-\alpha)}{r-\mu}e^{\mu m}W^m N(z) - DN(z - \sigma\sqrt{m}), \tag{5}$$

where

$$z = \frac{\ln\left(\frac{\delta(1-\alpha)W^m}{(r-\mu)D}\right) + (\mu + \frac{1}{2}\sigma^2)m}{\sigma\sqrt{m}}$$

and $N(\cdot)$ is the cumulative standard normal probability distribution function.

Proof. The result follows from the standard Black-Scholes-Merton formula for a European call option, but without discounting because the payoff is received instantaneously at the time of bankruptcy, i.e., when $V_{\tau-m} = W^m$. □

3.2 Optimal Default Barrier

Equityholders maximize the value of their investment by determining when to default on the loan payments. Default is declared the first time $V_{t-m} = W^m$. At time t they face the optimal stopping problem

$$\phi(v) = \sup_{\tau \in \mathcal{T}_m} E\Big[\int_t^\tau e^{-r(s-t)}(\delta V_{s-m} - (1-\theta)C)ds + e^{-r(\tau-t)}\pi(W^m)\big|\mathcal{F}_t^m\Big]. \tag{6}$$

The first term inside the integral at the right hand side in expression (6) is the discounted value of the dividends net of after-tax coupon payments. The second term is the present value of the bankruptcy wild card. There are three differences between the optimization problem in expression (6) and the standard complete information optimization problem, see e.g., Duffie (2001), chapter 11.C. The first is the inclusion of the bankruptcy wild card in the optimization problem. Second, the lagged state variable V_{t-m} enters, and third, the information set at time t is lagged.

We now use the results of Øksendal (2004) to transform the optimal stopping problem with delayed information into an optimal stopping problem with non-delayed information. To this end observe that expression (6) can be written as

$$\phi(v) = \sup_{\tau^* \in \mathcal{T}^*} E\Big[\int_{t-m}^{\tau^*} e^{-r(s-(t-m))}(\delta V_s - (1-\theta)C)ds \\ + e^{-r(\tau^*-(t-m))}\pi(W^m)\big|\mathcal{F}_{t-m}\Big], \tag{7}$$

where the stopping time $\tau^* = \tau - m$ and $\mathcal{T}^*$ is the set of all $\mathcal{F}_{t-m}$-adapted stopping times. We recognize expression (7) as a standard optimal stopping problem and its connection to ordinary differential equations (ODEs) is known.

The value function in expression (7) satisfies the ODE

$$\mu v \phi_v + \frac{1}{2}\sigma^2 v^2 \phi_{vv} - r\phi + (r - \mu)v - (1 - \theta) = 0, \tag{8}$$

where subscripts denote partial derivatives. The general solution to this equation is

$$\phi(v) = A_1 v^{\gamma_1} + A_2 v^{\gamma_2} + v - (1 - \theta)\frac{C}{r},$$

where A_i, $i = 1, 2$, are constants to be determined from boundary conditions and

$$\gamma_i = \frac{\frac{1}{2}\sigma^2 - \mu \pm \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2},$$

with $\gamma_1 < 0$ and $\gamma_2 \geq 1$. Differentiating ϕ with respect to v yields

$$\phi_v = \gamma_1 A_1 v^{\gamma_1 - 1} + \gamma_2 A_2 v^{\gamma_2 - 1} + 1.$$

When the value of the company approaches infinity, only equityholders benefit from a marginal increase in asset value, thus

$$\lim_{v \to \infty} \phi_v = 1. \tag{9}$$

As $\gamma_2 \geq 1$, condition (9) implies that $A_2 = 0$, i.e.,

$$\phi(v) = A_1 v^{\gamma_1} + v - (1 - \theta)\frac{C}{r}. \tag{10}$$

We impose the usual *value matching* and *high contact* conditions

$$\phi(W^m) = \pi(W^m) \tag{11}$$

and

$$\phi_v(W^m) = \frac{\delta(1 - \alpha)}{r - \mu} e^{\mu m} N(z). \tag{12}$$

Equations (11) and (12) can be solved for A_1 and W^m. Note in particular that W^m also can be determined by bondholders, i.e., W^m is $\mathcal{F}_t^l$-measurable, i.e., a constant for *all* agents in our model.

3.2.1 A closed form solution for equations (11) and (12)

In this subsection we express the company's debt D as a multiple λ of the default barrier W^m, i.e., $D = \lambda W^m$, for a constant $\lambda > 0$. Any relationship between W^m and D can be expressed in this manner, so our approach is without any loss of generality (although it may be somewhat unfamiliar to think of the face value of debt as a multiple of the optimal default barrier). We can write the time τ value of the bankruptcy wild card from expression (5) as

$$\pi(W^m) = W^m \left[\frac{\delta(1-\alpha)}{r-\mu} e^{\mu m} N(z) - \lambda N(z - \sigma\sqrt{m}) \right], \tag{13}$$

where

$$z = \frac{\ln\left(\frac{\delta(1-\alpha)}{(r-\mu)\lambda}\right) + (\mu + \frac{1}{2}\sigma^2)m}{\sigma\sqrt{m}}.$$

By the assumed parametrization of D, z does not depend on W^m and $\pi(W^m)$ is, thus, linear in W^m.

We further simplify notation by defining

$$K \equiv \frac{\delta(1-\alpha)}{r-\mu} e^{\mu m} N(z)$$

and

$$Q \equiv \lambda N(z - \sigma\sqrt{m}).$$

We can then write

$$\pi(W^m) = W^m(K - Q).$$

By using this notation, equation (12) can be written as

$$\phi_v(W^m) = K. \tag{14}$$

Next, we solve the equations (11) and (14). The solution for W^m is

$$W^m = \frac{\gamma_1}{\gamma_1 - 1} \frac{(1-\theta)C}{r} \frac{1}{1 - \left(K - \frac{\gamma_1}{\gamma_1 - 1} Q\right)}. \tag{15}$$

Observe that in cases where the bankruptcy wild card has no value, i.e., $\pi(W^m) = 0$, $K = Q = 0$. Then the above solution for W^m collapses to the classical solution

$$W = \frac{\gamma_1}{\gamma_1 - 1} \frac{(1-\theta)C}{r}. \tag{16}$$

Combining expressions (15) and (16) we write

$$W^m = W \frac{1}{1 - f(m)},$$

where $f(m) = K - \frac{\gamma_1}{\gamma_1 - 1}Q$. The right hand side of the above equation is a hyperbolic function of the delay m. There exists an asymptotic value of m where W^m is not well defined. For values of m below this asymptote, W^m is positive and increasing. Realistic values of m is below the asymptote for reasonable parameter values. In general, for $\pi(W^m) > 0$, implying that $K > Q$, thus, $K > \frac{\gamma_1}{\gamma_1 - 1}Q$, the bankruptcy barrier W^m is higher than in the case without a bankruptcy wild card. Thus, the existence of a bankruptcy wild card leads to earlier default compared to the case with no bankruptcy wild card.

In expression (15) we establish that imperfect information leads to earlier defaults than under perfect information. The value of equity can be thought of as the value of a real option the equityholders hold to continue servicing the loan payments and thereby receiving dividend payments. Delayed information reduces the value of this real option and equityholders will therefore pay less to keep the option alive, i.e., they increase the default barrier and therefore default earlier. Exercising the bankruptcy wild card is an alternative for the equityholders to continue operating the company. As the alternative becomes more lucrative, i.e., its value increases, running the company becomes relatively less rewarding and the equityholders will file for bankruptcy earlier.

3.3 Credit Risk Valuation

The only credit sensitive asset of the company is a bank loan. The bank belongs to the less informed group of agents and assesses the fair terms of the loan. The bank securitizes the loan into a continuum of zero coupon bonds, where the continuum is with respect to the time to maturity. Duffie and Lando (2001) explain the connection between perpetual debt and unit discount bonds. Throughout the paper we analyze one of these zero coupon bonds. The bond matures at time s, with recovery function $R(\tau, s)$ in the case of default at time $\tau < s$. The price of the bond consists of two parts: 1) The discounted value of the principal paid at maturity and 2) the discounted value of the recovery payment in case of default. More formally, the time t price of a bond maturing at time s is the conditional expected discounted payoff, i.e.,

$$\begin{aligned} \varphi(t,s) &= E\Big[e^{-r(s-t)}1_{\{\tau>s\}} + e^{-r(\tau-t)}R(\tau,s)1_{\{\tau\le s\}}\big|\mathcal{G}_t^l\Big] \\ &= e^{-r(s-t)}\Big(\alpha P(\tau > s\big|\mathcal{G}_t^l) + (1-\alpha)]\Big). \end{aligned} \tag{17}$$

In the second equality, we have used the recovery function $R(u, s) = (1 - \alpha)e^{-r(s-u)}$, $u \in (t, s]$, i.e., the same recovery function as in Duffie and Lando (2001). This recovery function is used throughout the paper.

For a credit risky zero-coupon bond, the credit spread $\eta^{l,m}$ is defined as the excess yield compared to the yield on a riskfree zero-coupon bond. From expression (17) we have that

$$e^{-(r+\eta^{l,m})(s-t)} = e^{-r(s-t)}\Big(\alpha P(\tau > s\big|\mathcal{G}_t^l) + (1-\alpha)]\Big),$$

so

$$\eta^{l,m} = \frac{-\ln\Big(\alpha P(\tau > s\big|\mathcal{G}_t^l) + (1-\alpha)\Big)}{s-t}. \tag{18}$$

Notice that the credit spread vanishes as $\alpha \to 0$. Furthermore, the credit spread tightens as the survival probability $P(\tau > s\big|\mathcal{G}_t^l)$ increases.

Collin-Dufresne et al. (2003) find that since 1937 only four companies have defaulted on bonds with an investment grade rating from Moody's. This empirical observation suggests that the information asymmetry $l-m$ cannot be too large. With a sufficiently large information asymmetry, even a company issuing bonds rated investment grade may have time to move into default.[4]

4 Special Cases and Numerical Examples

In this section we look at the four special cases from the introduction and provide some numerical examples. We start with the simplest case of all, i.e., the base-case with complete information, i.e., no information asymmetry, $l-m=0$, and no information delay, $m=0$. This case serves as a benchmark case. We then look at the most general case with information asymmetry, $l-m>0$, and information delay, $m>0$. We end this section by looking at the cases with 1) information asymmetry, but no information delay, $l-m>0$, $m=0$, and 2) no information asymmetry, but information delay, $l-m=0$, $m>0$. The numerical examples in this section are based on the parameter values in table 1.

In the 12 year period from 1995-2007 the average recovery rate for senior secured loans in the US was 72.3%, while the corresponding recovery rate for high yield bonds was 42.2%, cf. Altman, Resti, and Sironi (2004). Our model does not include an exogenously specified recovery rate parameter. Instead, the use of a bankruptcy cost parameter α induces different (expected) recovery rates, depending on the numerical values of the other parameters that are used. A tax rate of 30% seems reasonable for many companies. A volatility of 30% means that the instantaneous standard deviation of the return on the stock of assets is 30%. Some of the parameters are altered in the numerical examples.

[4]This observation also gives some justification for using a diffusion model instead of a jump-diffusion model. Few companies "jump" into default when their bonds are rated investment grade. However, it should be pointed out that many bonds jump several categories down to junk rating from investment grade and default shortly thereafter. This happened to Enron who defaulted from a junk rating, but who was rated investment grade two days before it defaulted.

Table 1: Base-case parameters.

V_0	100	initial value of stock of assets
δ	0.035	fraction of stock of assets paid as dividend
r	0.08	riskfree interest rate
μ	0.045	growth rate of stock of assets
σ	0.3	volatility of stock of assets
θ	0.3	tax rate
α	0.5	bakruptcy cost parameter
C	13	coupon payment
λ	$\frac{90}{65}$	ratio between debt and bankruptcy barrier

4.1 The Base-case ($l - m = 0, m = 0$)

In this case there is neither information asymmetry, nor information delay, both bond- and equityholders have perfect information. Thus, $\mathcal{G}_t^l = \mathcal{F}_t^m = \mathcal{F}_t$. If $\mu = r$ and $\delta = 0$, this case corresponds to the model by Leland (1994). Also, with $r > \mu$ and $\delta > 0$, this is a well-known case and serves well as a benchmark case when delayed information is included in the subsequent subsections.

Equityholders are faced with the same optimal stopping problem as in equation (6), but the bankruptcy wild card is not present (it has zero value). The value matching and the high contact conditions therefore become $\phi(W) = 0$ and $\phi_v(W) = 0$. The solution for W is given in expression (16)[5] . For the base-case parameter values, $W = 65$.

Bond prices are calculated by expression (17) with $l = 0$. Thus,

$$\varphi(t, s) = P(\tau > s \big| \mathcal{F}_t) = \Psi(s - t, \ln(W/V_t)),$$

where an analytical expression for $\Psi(\cdot, \cdot)$ is given in expression (22) in appendix A.

In figure 1 we plot the credit spreads bondholders require for bonds with maturities between 0 and 3 years for three different levels of the tax rate θ: 20%, 30%, and 40%. The widest credit spreads are required for the lowest tax rate and the tightest spreads for the highest tax rate. The explanation for this observation is that, *ceteris paribus*, lending money is less risky when the tax rate is high because the value of the tax-shield from interest payments is worth more to equityholders and they will therefore wait longer, i.e., accept lower dividend payments before they default on their loan payments. The survival probability is increasing in the tax rate.

Observe how the credit spreads vanish as the time to maturity approaches zero, a typical feature of structural models of credit risk, but it

[5] In the special case considered by Leland (1994) where $\mu = r$, $W = (1-\theta)C/(r+0.5\sigma^2)$.

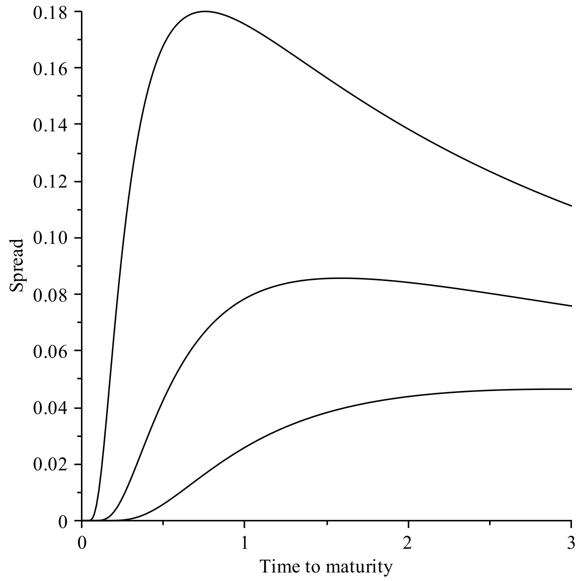


Figure 1: **Credit spreads base case** The figure shows credit spreads for zero-coupon bonds with up to three years to maturity. The tax rates θ are 20% (widest spreads), 30%, and 40% (tightest spreads).

contradicts empirical observations in credit markets.

4.2 The General Case ($l - m > 0, m > 0$)

We now assume that both information asymmetry and information delay are present. I.e., both groups of agents have incomplete information about the value of the state variable, and the information is asymmetrically distributed between the two groups. More formally, $\mathcal{G}_t^l \subset \mathcal{F}_t^m \subset \mathcal{F}_t$.

The bankruptcy barrier is not a function of the current value of the state variable. Thus, even though equityholders are better informed than bondholders, also bondholders can determine W^m. Denote the minimum value of the process V_t over a period $[u, v]$ by $M_{u,v}$, i.e.,

$$M_{u,v} = \min\{V_t; u \leq t \leq v\}.$$

The survival probability, as seen from the bondholders' point of view, is given by

$$P(\tau > s \big| \mathcal{G}_t^l) = P(\tau > s \big| M_{t-l,t-m} > W^m; \mathcal{F}_t^l). \tag{19}$$

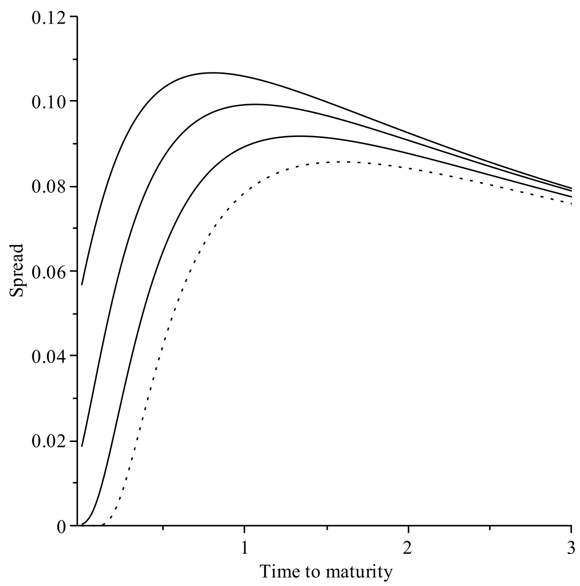


Figure 2: **Credit spreads general case** The figure shows credit spreads for zero-coupon bonds with up to three years to maturity. The information delays m are 0.1 (widest spreads), 0.2, and 0.3 (tightest spreads). The lower, dotted line represents the complete information case.

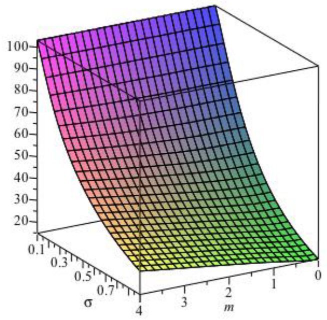
Using Baye's rule, expression (19) can be written as

$$\begin{aligned} P(\tau > s \big| \mathcal{G}_t^l) &= \frac{P(\tau > s \cap \ M_{t-l,t-m} > W^m \big| \mathcal{F}_t^l)}{P(M_{t-l,t-m} > W^m \big| \mathcal{F}_t^l)} \\ &= \frac{P(M_{t-l,s-m} > W^m \big| \mathcal{F}_t^l)}{P(M_{t-l,t-m} > W^m \big| \mathcal{F}_t^l)} \\ &= \frac{\Psi(s-m-(t-l), \ln(W^m/V_{t-l}))}{\Psi(l-m, \ln(W^m/V_{t-l}))}, \end{aligned} \tag{20}$$

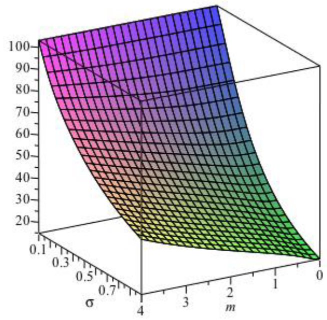
where the expression for $\Psi(\cdot,\cdot)$ is given in expression (22) in appendix A.

Assume the same parameter values as in table 1. In addition, $l = 0.4$. Figure 2 shows the credit spreads for $m = 0.1$ (widest spreads), $m = 0.2$, and $m = 0.3$ (tightest spreads with solid line, the dotted line represents the complete information case (base-case)). Note in particular how asymmetric information leads to wider credit spreads for short-term bonds.

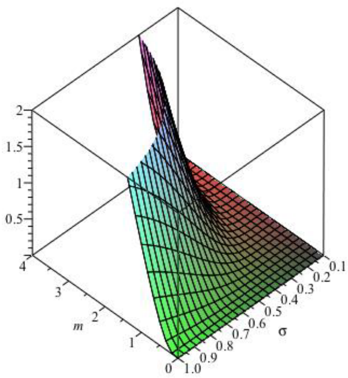
It may at first seem counter intuitive that the spreads *decrease* as equityholders become *less* informed (m increases). However, recall that the degree of asymmetric information between bond- and equityholders, $l - m$, *decreases* as m increases. Thus, the *decrease* in credit spreads is a result of a *decreased* degree of asymmetric information.



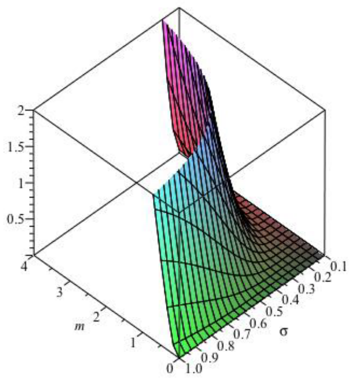
(a) The optimal default barrier W^m as a function of m and σ for $\alpha = 0.5$.



(b) The optimal default barrier W^m as a function of m and σ for $\alpha = 0.3$.



(c) The value of the bankruptcy wild card $\pi(m)$ as a function of m and σ for $\alpha = 0.5$.



(d) The value of the bankruptcy wild card $\pi(m)$ as a function of m and σ for $\alpha = 0.3$.

Figure 3: Effects on the optimal default barrier W^m and the value of the bankruptcy wild card $\pi(m)$ for different levels of information delay m and volatiliy σ, for two different levels of the bankruptcy parameter α.

The optimal default barrier is relatively insensitive to which degree the information is delayed. The obvious explanation for this observation is that even for long delays, the value of the bankruptcy wild card is modest.

From figure 3, part (a) and (b), we see that even a delay of one year has only a negligible effect on the optimal default barrier W^m. The graph is increasing in volatility, but rather flat in the delay (in particular for $\alpha = 0.3$), suggesting that the delay has only low influence on the optimal default barrier.

The value of the bankruptcy wild card is decreasing in the bankruptcy cost parameter α, cf. figure 3 part (c) and (d). The delay is therefore more important for the optimal default barrier when bankruptcy costs are low than when they are high. The probability that the bankruptcy wild card matures in-the-money decreases in α.

The bankruptcy wild card is a type of call option and its value is therefore non-decreasing in volatility. Figure 3 illustrates the effect of volatility on the default barrier and on the value of the bankruptcy wild card. A higher α means that the wild card is more out-of-the-money and therefore has a lower value.

4.3 Duffie-Lando ($l - m > 0, m = 0$)

In this case there is information asymmetry, but no information delay. Similar to the model by Duffie and Lando (2001), the information structure is as follows: $\mathcal{G}_t^l \subset \mathcal{F}_t^m = \mathcal{F}_t$. As already mentioned, they use a richer and more complicated information structure than we do here, but our simplified information structure produces qualitatively similar results.

When no information delay is present, i.e, $m = 0$, the bankruptcy wild card is not present and the default barrier is W, the same as in the case of complete information, see subsection 4.1. The survival probability is, thus, given in expression (20) with $m = 0$.

If we plot credit spreads for $l = 0.1$, $l = 0.2$, and $l = 0.3$ in this case, using the base case parameters, we get a figure identical to figure 2. The explanation is that the information asymmetry $l - m$ is the same as in the previous subsection, i.e., these values of l are the same as the the values of $l - m$ in previous subsection. In addition, the value of the bankruptcy wild card is not significant for the base case parameters. We show later that in (extreme) cases where the delay m is sufficiently long and/or the voltility is extremely high, the optimal default barrier and credit spreads in the present case and in the general case in subsection 4.2 differ.

4.4 Case of Symmetrically Delayed Information ($l - m = 0, m > 0$)

The final case we consider is with delayed information, but with no information asymmetry. The information is *symmetrically* distributed between bond- and equityholders, i.e., $l = m, m > 0$.

To price corporate bonds, bondholders also in this case make use of the survival probability in expression (20). Because $l = m$, the expression simplifies to

$$P(\tau > s \big| \mathcal{G}_t^l) = P(\tau > s \big| \mathcal{F}_t^m) = \Psi(s - t, \ln(W^m / V_t^m)). \tag{21}$$

By comparing this expression with the corresponding expression for the case of full information, the only way symmetric, but delayed information, can affect credit spreads is if the default barrier W^m differs from W.

In the figures 4 and 5, the credit spreads for the four cases are plotted. For the base-case volatility and no information delay ($m = 0$) and asymmetric information $l - m = 0.2$, part (a) of both figures show that the general case coincides with the Duffie-Lando case. The assumption of no information delay $m = 0$ reduces the general model to the Duffie-Lando model. Similarily, considering symmetrically delayed information, i.e., $m = 0.2$ and $l - m = 0$, the credit spreads coincide with the spreads in the complete information case. Thus, *asymmetry* $l - m$ is far more important for credit spreads than a symmetric delay of information. In the Duffie-Lando case, the less informed agents observe the information with a delay of 0.2. In the case with symmetrically delayed information, all agents observe the information with a delay of 0.2. The case where only some of the agents observe the information with a delay of 0.2 produces the wider credit spreads.

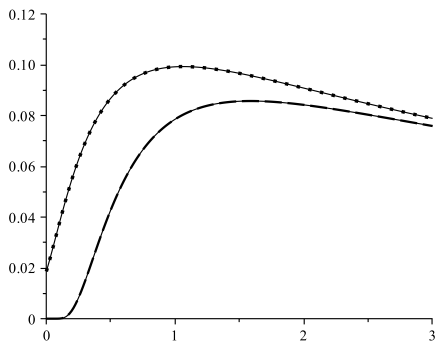
In panel (b) of the figures, the information asymmetry is doubled to $l-m = 0.4$. The increased information asymmetry widens the credit spreads, in particular for bonds with short time to maturity.

We investigate two strategies to enhance the differences between the various cases; we increase the information delay and the volatility. In part (c) of the figures, the information delay is increased to $m = 2$, maintaining the same information asymmetry of $l - m = 0.2$ as in part (a). This increase creates wider credit spreads in the general case than in the Duffie-Lando case. Wider credit spreads are obtained from the case with symmetrically delayed information compared to the complete information case. This effect is solely due to the changed optimal default barrier. Note that we have to increase m to around 2 to visualize this effect for $\alpha = 0.5$. For more realistic values of m, say, below 1 year, this difference is not visible.

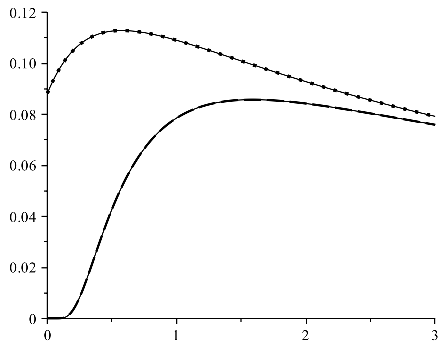
In part (d) we apply moderate information delay and information asymmetry ($l - m = 0.5$, $m = 0.2$), and more drastically increase the volatility to $\sigma = 1.2$. We see similar effects as in part (c) of the figures. Also, note that in this case we increase the volatility together with the delays to about these levels ($\sigma = 1.2$) to visually detect differences between the two cases of asymmetric and symmetric information (for $\alpha = 0.5$), respectively.

In the case of high volatility and low bankruptcy costs, see figure (5), panel (c), we observe lower spreads in the asymmetric Duffie-Lando case than in the symmetrical delayed case even for relatively short times to maturity. In this case the effect on the spreads from the increased value of the bankruptcy wild card (which is increasing in m and σ) dominates the standard increasing effect on the spread from asymmetric information.

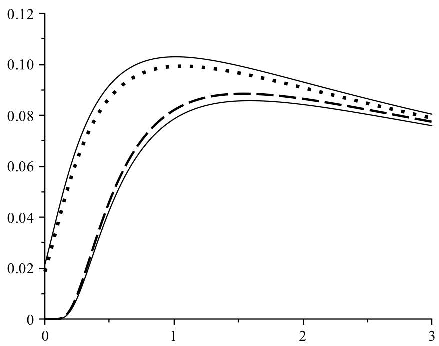
Finally, comparing figures 4 and 5, we see that one effect of a lower bankruptcy cost is to tighten credit spreads, cf. the definition of credit spreads in expression (18).



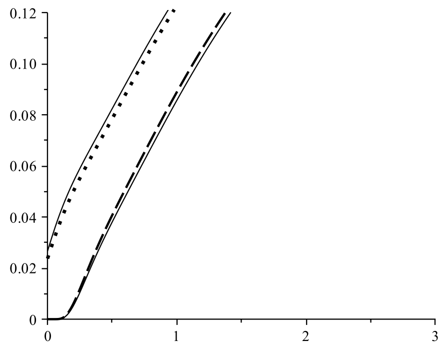
(a) No information delay, moderate information asymmetry, $\sigma = 0.3$, $l - m = 0.2$, $m = 0$.



(b) Moderate information delay, high information asymmetry, $\sigma = 0.3$, $l - m = 0.4$, $m = 0.2$.



(c) Long information delay, moderate information asymmetry, $\sigma = 0.3$, $l - m = 0.2$, $m = 2$.



(d) High volatility, moderate information delay, moderate information asymmetry, $\sigma = 1.2$, $l - m = 0.2$, $m = 0.2$.

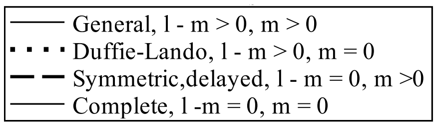
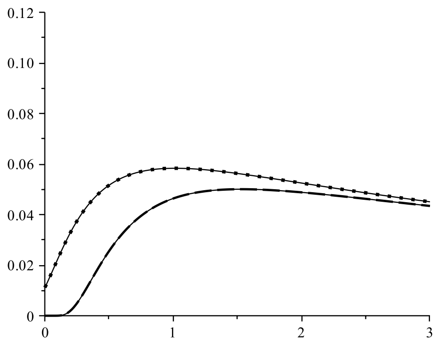
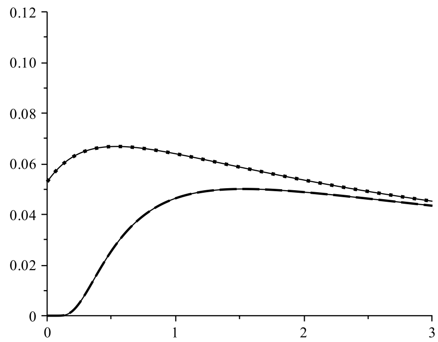


Figure 4: Examples of credit spreads for four the four cases for $\alpha = 0.5$

5 Conclusions and Suggestions for Future Research

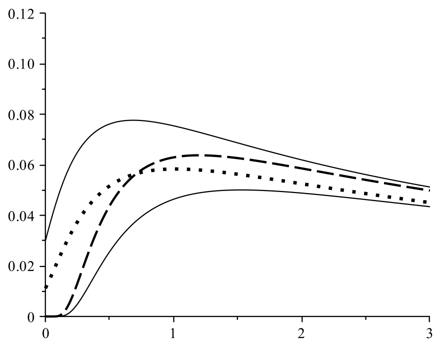
In this paper we propose a simple approach to include delayed and asymmetric information between groups of agents in a secondary credit market. We articulate the effect of delayed information on shareholders' endogenous decision to default. In particular, we show that incomplete information to equityholders about the true company value only has a small effect on their decision to default on the loan payments for realistic parameter values. Any effect is to accelerate a default. The decision to default is accelerated because by defaulting, equityholders receive a bankruptcy wild card with non-negative value. If both bond- and equityholders have access to the same delayed information, the only reason for changed credit spreads is a

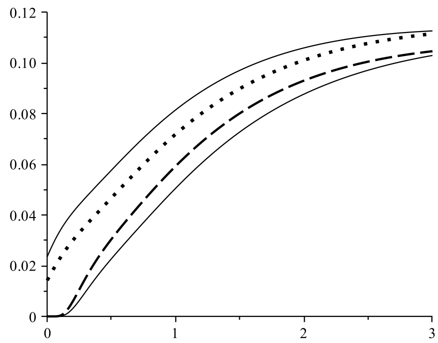




(a) No information delay, moderate information asymmetry, $\sigma = 0.3$, $l - m = 0.2$, $m = 0$.

(b) Moderate information delay, high information asymmetry, $\sigma = 0.3$, $l - m = 0.4$, $m = 0.2$.





(c) Long information delay, moderate information asymmetry, $\sigma = 0.3$, $l - m = 0.2$, $m = 2$.

(d) High volatility, moderate information delay, moderate information asymmetry, $\sigma = 1.2$, $l - m = 0.2$, $m = 0.2$.

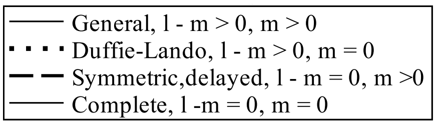


Figure 5: Examples of credit spreads for four the four cases for $\alpha = 0.3$

potential change in equityholders' optimal default barrier, compared to the complete information case. For realistic parameter values, these changes are small. We further find that asymmetric information between bond- and equityholders is important for credit spreads, more important than delayed, symmetrically distributed information. In most interesting cases, increased information asymmetry leads to wider credit spreads. Our model produces short-term credit spreads more in line with empirical observations than most standard structural models of credit risk.

The results in this paper have empirical testable implications: Do companies where there is likely to be more asymmetric information between bond- and equityholders pay higher interest rates on their loans? Do companies where there is more uncertainty about asset values, i.e., a higher degree of incomplete information, default earlier than other companies? One indication that may lead to a confirmative answer to the last question is if equityholders tend to receive payments from the bankruptcy wild card more often than equityholders of companies with a lower degree of incomplete information. Our model also predicts that companies with a high α, for instance because of relatively illiquid assets, wait longer before they default. A typical reason for illiquid assets is a high degree of asset specificity.

For future extensions of the results in this paper, we think it would be interesting to include a third group of agents: management. Management would always belong to the better informed group. With three groups of agents, we could for instance assume that bond- and equityholders both belong to the less informed group of agents. By this assumption one could extend the analysis to also include companies whose equity is traded in the financial market. Unfortunately, this assumption makes the model hard to solve. For the model to be consistent, the dividends paid at time t would have to be δV_{t-u}, where $u \leq l$. If the default barrier is based on the information set of the management ($\mathcal{F}_t^m$), the connection between the optimal stopping problem and the ODEs is, to the best of our knowledge, not known. Most likely such a set-up would result in W^m no longer being $\mathcal{G}_t^l$-measurable.

A Survival Probability

Consider a geometric Brownian motion with dynamics as in expression (1) with initial value v, and a barrier $v_b < v$. Consider the arithmetic process $dX_t = (\mu - \sigma^2)t + \sigma W_t$, starting at $X_0 = 0$. The first time the process V_t hits v_B is equivalent to the first time the process X_t hits $x = \ln(v_B/v)$. The probability for process V_t of not crossing the barrier v_b in a time period of length s is identical to the probability for process X_t of not crossing the barrier $x = \ln(v_B/v)$ in a time period of length s is

$$\Psi(s,x) = N\Big(\frac{-x+\nu s}{\sigma\sqrt{s}}\Big) - e^{\frac{2x\nu}{\sigma^2}} N\Big(\frac{x+\nu s}{\sigma\sqrt{s}}\Big), \tag{22}$$

where $\nu = \mu - \frac{1}{2}\sigma^2$, see e.g., Musiela and Rutkowski (1997) Corollary B.3.4.

References

Altman, E., Resti, A., and Sironi, A. (2004). "Default recovery rates in credit risk modelling: a review of the literature and empirical evidence", *Economic Notes*, *33*(2), 183–208.

Bielecki, T. R. and Rutkowski, M. (2002). *Credit Risk: Modeling, Valuation and Hedging.* Springer-Verlag, Berlin-Heidelberg.

Black, F. and Cox, J. C. (1976). "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions", *Journal of Finance*, *31*(2), 351–367.

Coculescu, D., Geman, H., and Jeanblanc, M. (2008). "Valuation of default-sensitive claims under imperfect information", *Finance and Stochastics*, *12*, 195–218.

Collin-Dufresne, P., Goldstein, R. S., and Helwege, J. (2003). "Is Credit Event Risk Priced? Modeling Contagion via the Updating of Beliefs", Working Paper, Carnegie-Mellon University.

Duffie, D. (2001). *Dynamic Asset Pricing Theory.* Princeton University Press, Princeton, New Jersey.

Duffie, D. and Lando, D. (2001). "Term Structures of Credit Spreads with Incomplete Accounting Information", *Econometrica*, *69*(3), 633–664.

Duffie, D. and Singleton, K. J. (2003). *Credit Risk* (Third edition). Princeton series in finance, Princeton, New Jersey.

Geske, R. (1977). "The Valuation of Corporate Liabilities as Compound Options", *Journal of Financial and Quantitative Analysis*, *12*, 541–552.

Giesecke, K. (2006). "Default and Information", *Journal of Economic Dynamics and Control*, *30*(11), 2281–2303.

Giesecke, K. and Goldberg, L. R. (2004). "Forcasting Default in the Face of Uncertainty", *Journal of Derivatives*, *12*(Fall), 11–25.

Guo, X., Jarrow, R. A., and Zeng, Y. (2008). "Credit Risk Models with Incomplete Information", Working Paper Cornell University.

Hull, J. C. (2006). *Options, Futures and other Derivatives* (Sixth edition). Pearson-Prentice Hall, Upper Saddle River, New Jersey.

Jarrow, R., Lando, D., and Turnbull, S. (1997). "A Markov Model for the Term Structure of Credit Risk Spreads", *Review of Financial Studies*, *10*(2), 481–523.

Jarrow, R. and Protter, P. (2004). "Structural versus Reduced Form Models: A New Information Based Perspective", *Journal of Investment Management*, *2*(2), 1–10.

Jarrow, R. and Turnbull, S. (1992). "Credit Risk: Drawing the Analogy", *Risk Magazine*, *5*(9), 53–85.

Jarrow, R. and Turnbull, S. (1995). "Pricing Derivatives on Financial Securities Subject to Credit Risk", *Journal of Finance*, *50*(1), 53–85.

Leland, H. E. (1994). "Corporate Debt Value, Bond Coventnats, and Optimal Capital Structure", *Journal of Finance*, *49*(4), 1213–1252.

Longstaff, F. A. and Schwartz, E. (1995). "A Simple Approach to Valuing Risky Debt", *Journal of Finance*, *50*(3), 789–821.

Merton, R. C. (1974). "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates", *Journal of Finance*, *29*(2), 449–470.

Musiela, M. and Rutkowski, M. (1997). *Martingale Methods in Financial Modeling*. Springer Verlag, Berlin Heidelberg.

Øksendal, B. (2004). "Optimal Stopping with Delayed Information", Working paper Pure Mathematics no. 23/04, Department of Mathematics, University of Oslo.

Schönbucher, P. (1998). "Term Structure Modelling of Defaultable Bonds", *Review of Derivatives Research*, *2*(2-3), 161–192.