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Discussion paper

Condorcet Methods
– When, Why and How?

BY
EIVIND STENSHOLT

Norges Handelshøyskole

CONDORCET METHODS - WHEN WHY AND HOW ?

BY EIVIND STENSHOLT

Norwegian School of Economics and Business Administration

Helleveien 30, 5045 Bergen, Norway

email: eivind.stensholt@nhh.no

Abstract

Geometric representations of 3-candidate profiles are used to investigate properties of preferential election methods. The representation visualizes both the possibility to win by agenda manipulation, i.e. introducing a third and chanceless candidate in a 2-candidate race, and the possibility to win a 3-candidate election through different kinds of strategic voting. Here the focus is on the "burying" strategy in single-winner elections, where the win is obtained by ranking a main competitor artificially low. Condorcet methods are compared with the major alternatives (Borda Count, Approval Voting, Instant Runoff Voting). Various Condorcet methods are studied, and one method is proposed that minimizes the number of noncyclic profiles where burying is possible.

1 Introduction

1.1 The Condorcet methods and some main alternatives

For $n=2$, the best known methods for n-candidate preferential elections all agree on the principle that the majority decides, but this principle may be formulated in many different ways and generalized accordingly. Thus the Borda Count lets each voter give (n-1, n-2, ... , 1, 0) points to number (1, 2, ..., n) in the voter's ranking of the candidates. One may allow the ordering R_i from voter i to include equalities and handle them by means of symmetrization[1]. Other methods preserve the feature from $n=2$ that R_i has just 2 indifference classes: In AV (Approval Voting) voter i gives (1, 1, ..., 1, 0, ..., 0) points, choosing the class sizes r_i and n-r_i for approved and disapproved candidates. A mandatory r_i=1 or r_i=n-1 for all i defines Plurality or Antiplurality Voting. After symmetrization they may all be tallied as a Borda Count[2]. Also IRV (Instant Runoff Voting) generalizes the majority principle from the $n=2$ case, as it iteratively eliminates the Plurality loser.

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1. If e.g. voter i ranks 4 candidates $x(yzw)$, with y, z and w sharing ranks 2, 3 and 4, symmetrization replaces the ballot by 6 "miniballots" $xyzw, xywz, xwyz, xwzy, xzwy, xzyw,$ each counting as 1/6 of a full vote. That means 3 Borda-points to x and (2+2+1+1+0+0)/6=1 Bordapoint to each of the others.
2. Voter i gives (n-1+n-r_i)/2 Borda-points to each of the r_i approved candidates and (n-r_i-1)/2 to each of the n-r_i non-approved candidates; the difference is n/2 and thus independent of r_i.

Each way to generalize has merits and harmful side effects. Condorcet's idea, however, sticks more consistently to the basic $n=2$: just tally each of the $n(n-1)/2$ candidate pairs as a 2-candidate election! We consider elections with linear R_i, which of course may be obtained from complete R_i's through symmetrization. The Condorcet relation R is determined by the election:

$$xRy \text{ if and only if } \left|\{i|xP_iy\}\right| - \left|\{i|yP_ix\}\right| \geq 0$$

P, P_i and I, I_i are the relations for strict preference and indifference associated with R, R_i, and xyz means xP_iyP_iz etc. when the relation P_i is understood. Candidate w is called a Condorcet winner if in Condorcet's relation, wPx for every other candidate x.

Arrow's IIA-axiom ("Independence of Irrelevant Alternatives") [Arrow 1963] generalizes Condorcet's idea to a wider class of social relations: Whether xPy, yPx, or xIy ($x \neq y$) is -somehow - determined by a partitioning of the voter set V, i.e.

$$V = \{i|xP_iy\} \cup \{i|yP_ix\}$$

Even before Arrow it was reason to doubt that any practical fair election method could satisfy IIA and always produces a complete ordering R. A reason is that if the method should be fair to voters and fair to candidates (i.e. satisfy the symmetry conditions of anonymity[3] and neutrality[4] and be monotonic[5]), then it is easily seen that the possibility is reduced to a rule like this: There is a $\Lambda \geq 0$ so that

$$xRy \text{ if and only if } \left|\{i|xP_iy\}\right| - \left|\{i|yP_ix\}\right| \geq -\Lambda \text{ [6]}$$

Here $\Lambda > 0$ means a rule of qualified majority. Even so, cycles may occur[7]. Being aware of Condorcet cycles, one should not be surprised by the message in Arrow's Impossibility Theorem, i.e. that IIA is essentially incompatible with the requirement that R should always be a complete ordering. IIA offers no useful substitute for Condorcet's relation. More remarkable is how Arrow's axiomatic method paved

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3. Under anonymity, the election result R is the same if two voters switch ballots.
4. Under neutrality, two candidates switch positions in the result R if they switch positions in every ballot.
5. Under monotonicity a candidate is never harmed by being moved upwards in any ballot.
6. Thus strict preference xPy means $\left|\{i|xP_iy\}\right| - \left|\{i|yP_ix\}\right| > \Lambda$.
7. If $|V|=4$ voters rank $n=4$ candidates $xyzw$, $yzwx$, $zwxy$, $wxyz$, and $2 \leq |\Lambda| < 3$, there is a Condorcet cycle of length 4: $xPyPzPwPx$ with 3/4 majorities.

the way for further progress, e.g. for the Gibbard-Satterthwaite theorem: Except in certain trivial methods, there will exist profiles that allow strategic voting - although in a very wide meaning [Gibbard 1973, Satterthwaite 1975].

IIA must be relaxed, and the most important types of strategic voting exploit a violation of IIA. A single winner Condorcet election method picks the Condorcet winner when one exists, and is defined by how the winner is determined when there is no Condorcet winner. The possibility for strategic voting then depends on how cycles are handled. Other methods, that are not explicit about problems caused by cycles, may be said to sweep the theme under the carpet - out of sight, but not without consequences. Any election method will be criticized if candidate y is elected while a majority prefers another candidate x. As such cases cannot be completely avoided, it is a point in favor of any Condorcet method both that it avoids them as well as possible, and that it is explicit about how cycles are handled.

Cycles may occur in real elections by accident. How serious will an occurrence be? Fortunately, the probability of a cycle, in elections with many independent voters with non-strategic behaviour, is very small [Gehrlein 2002], and does not warrant much worry. But how different politically is a Condorcet winner from a Plurality winner or an IRV-winner? Even with $n=3$, a Condorcet winner may be top ranked by very few voters and thus be chanceless in Plurality or IRV. And, of course, what about strategic voting?

1.2 Strategic voting

In general, a strategic voting possibility in a single winner election method is defined as the following condition:

A set U of voters may change their votes from R_i to R_i^*, $i \in U$, and thereby change the

winner from w to w^*, so that $w^* P_i w$ for all $i \in U$.

Thus, by voting "strategically" R_i^* instead of "honestly" R_i, the result is improved according to the "honest" opinion R_i of all $i \in U$. In most common types of strategic voting, $w^* P_i^* w$, i.e. they do not require voters also to misrepresent their internal ranking of w and w^*.

How strong are the incentives for a voter subset to make an attempt to vote strategically? An attempt at strategic voting may be difficult and even hazardous; the prospects depend a lot on the election method. It is more reason for concern that a missed opportunity for strategic voting will be discovered in a post-election analysis. Just consider the opposite change, from R_i^* to R_i! All sorts of opinion change take place during an election campaign; it is a purpose of campaigns to influence voters. Those who honestly did change from R_i^* and voted R_i see the result worsened from w^* to w. They may well feel unfairly punished for their honesty, and lose trust in the election method.

However, the definition of strategic voting is very broad, and so allows the very general impossibility theorem of Gibbard and Satterthwaite. The definition is purely technical. Quite likely, it is a perfectly honest choice between an "expressive" R_i and an "instrumental" R_i^*. Democracy depends on letting people make that choice. There is no reason to consider all possibilities for strategic voting as potential democratic problems.

There are many types of strategic voting. Some "pure" strategies let all voters in U make the same change, and 3 types have received particular attention. For $n=3$, they are:

Strategy 1: Voter i switches from xyz to yxz, $i \in U$, the win passes from z to y.

Strategy 2: Voter i switches from xyz to xzy, $i \in U$, the win passes from y to x.

Strategy 3: Voter i switches from xyz to yxz, $i \in U$, the win passes from z to x.

All these strategies show a violation of IIA, since all voters keep their internal ranking between the old and the new winner.

Strategy 1 is common in Plurality elections when x and y are politically close, but y is considered more qualified to beat the common adversary z. An incentive to vote instrumentally for y rather than expressively for x is the driving force behind "Duverger's law", i.e. that the Plurality method leads to a 2-party system. Those who favor a 2-party system may regard strategy 1 as a useful tool rather than a weakness. Since voter i wants to support y more than x in the competition with z, there is no misrepresentation of i's real intention. Strategy 1 in British elections is studied by [Alvarez et al 2006].

Among the two popular names for strategy 1, "Compromise" is certainly closer to common practise and more fair than "Favorite betrayal".

Strategy 2 is, unfortunately, a disturbing possibility in many preferential voting methods. One major benefit of preferential voting should be to give different incentives to both candidates and voters than is commonly seen in Plurality voting. Instead of attacking political neighbours for splitting the votes and "spoiling" the election, a candidate should rather try to obtain their subsidiary support. But then it is important that voters do not have incentives to misrepresent their subsidiary rankings. An honest second choice should not harm a voter's first choice. Voters who practise strategy 2, commonly called "burying", do so from fear of harming their favorite and not from unethical shrewdness.

With Condorcet methods, strategy 2 is our particular concern. In the typical case, where there is a Condorcet ranking, strategy 2 involves creating a voting cycle. For $n=3$, it is only the supporters of nr 2 in the Condorcet ranking who may create a cycle. By voting "honestly" xyz the voters in U only obtain the Condorcet ranking $yPxPz$. By voting "strategically" xzy instead, they obtain zPy, so that a Condorcet cycle $yPxPzPy$ emerges.

Different Condorcet methods give very different opportunities for strategic voting. We address all these problems by means of geometric models. The same models illustrate that the Borda Count is extremely vulnerable to strategy 2.

IRV reduces the urge of Plurality to use strategy 1, but close enough to avoid strategy 2: The tally officers are, in fact, instructed to respect each voter's ranking. Only the current first preference can influence the tally process. However, there is a snag:

Strategy 3 is occasionally possible in IRV and other elimination methods: the idea is to eliminate z instead of y in the first part of the tally, so that x in the second part will win over y instead of lose to z - even though y is moved up to pass x. It then rewards a very clear misrepresentation of the voter's real intention, but an attempt is likely to be very risky, and other voter groups may well use counter strategies [Stensholt 2002, 2004]. However, there is good reason to be concerned about the opposite change, where

voters who honestly switched from *yxz* to *xyz* learn afterwards that they robbed their new favorite *x* of the victory and handed it to *z*.

1.3 Agenda manipulation

In the "Arrovian framework" there is a fixed number, *n*, of candidates[8]. The major election methods are defined for any *n*, but without different *n*-values being linked axiomatically[9]. Therefore the question of agenda manipulation, i.e. improving the result by entering or withdrawing candidates, must be dealt with ad hoc.

The obvious possibility of strategy 1 in Plurality elections is linked to the spoiling effect of two similar candidates, and so there is an associated incentive not to enter similar candidates. Small parties are urged to come together and present one alternative of broad appeal.

The equally obvious possibility of strategy 2 in the Borda Count is linked to a similarity effect in the opposite direction. By voting *xzy* the *xyz*-preferrer *i* acts as though *z* were politically closer than *y* to *i*'s position. Even better for *x* in a 2-candidate race vs *y* might be a third candidate *z* who really was positioned to cause many sincerely meant *xzy*-ballots. If *y* (moderate left) wins clearly over *x* (central right) in a 2-candidate election, *x* may win if *z* (ultra-right) enters the race. It can hardly serve any democratic purpose that an election method invites to such manoeuvres.

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8. Strictly speaking, there is also a fixed number of voters in Arrow's setting, but the election methods we consider here, are defined in terms of the relative profiles.
9. One may of course introduce axioms that rule out methods like "Borda Count for odd *n* and IRV for even *n*". To do so may be worth while provided the axioms lead to useful results e.g. about agenda manipulation.

2 Models of 3-candidate profiles

2.1 A model of perfect pie-sharing

In a preferential election a ballot reflects the voter's perception of the political landscape and the position in that landscape of the voter's "ideal point". The ballot ranking may, hopefully, be described as corresponding to some kind of "distance" between the ideal points of the candidates and the ideal point of the voter.

With 3 candidates, A, B and C, the voters usually have perceptions of the landscape, i.e. of the candidates' ideal points, that are sufficiently similar to be replaced by an average picture. Imagine the voters distributed with uniform density in the unit circle. In Figure 1 the candidates have been assigned ideal points as follows:

$$A = (-0.2, 0.3), \quad B = (0.2, 0.75), \quad C = (-0.2, -0.5)$$

The ideal points of A, B, and C are corners in the "candidate triangle". The candidates divide the "electoral pie" in 6 pieces along the mid-normals. Rounded off to 10000 voters, we get the profile with components in the following (anti-clockwiswe) order:

$$(|ABC|, |ACB|, |CAB|, |CBA|, |BCA|, |BAC|) = (1739, 1433, 4260, 63, 42, 2463).$$

The circle center is in the area where voters say ACB. The Condorcet relation is also the ranking A*PCP*B because *xPy* simply means that *x* is closer to the center than *y* is. Cycles do not occur in this pie-sharing model. Obviously, the profile only determines the shape of the candidate triangle. In a real case the voters may answer additional questions, so that also the size of the candidate triangle and the location of the ideal points of A, B, and C may be determined.

2.2 Pictogram models

How accurate can we expect the model of perfect pie-sharing to be? Obviously it cannot fit exactly for profiles with a Condorcet cycle. However, to every 3-candidate profile corresponds an exact "pictogram" model which is unique up to rotations and reflections. It consists of a circle and 3 chords that intersect pairwise in the circle, forming an "exceptional triangle", T. The 6 other areas are proportional to the profile components. Consider e.g. the profile obtained by moving 3200 voters from CAB to CBA in

Figure 1; then the "pictogram" of Figure 2 fits exactly, in the sense that the 6 areas defined by 2 chords and the circle periphery are, in corresponding clockwise order, proportional to the new profile components.

Empirically, real profiles have pictograms with a small T, so it is visually natural to fit a pie-sharing model with a candidate triangle. Without massive strategic voting, a profile with T as large as in Figure 2 would hardly ever occur with 10000 independent voters. Actually, because of round-offs, the pie-sharing model in Figure 1 does not fit the integer profile exactly either. In a pictogram T covers about 10^{-7} of the circle area. But real election profiles generally have pictograms with a small T.

One reason why the pie-sharing model generally fits well is that the ballot rankings reflect not just the voters' different opinions, but also reflect a certain uniformity in their perceptions of the political landscape, i.e. of the location of the candidate ideal points. Even though these perceptions also vary, the robustness of the pie-sharing model allows a candidate triangle to represent a reasonable average of the different views of the political reality. A discussion of the pictogram's properties and calculation is in [Stensholt 1996].

2.3 Condorcet cycles

It is easily seen that in the pictogram of a cyclic profile, T must cover the circle center, as it does in Figure 2. But in real election profiles T is very small. If we fit a pie-sharing model as well as possible, we must then place the corners of the candidate triangle at about the same distance from the circle center. Actually, the location of T in the pictogram will depend on a stochastic component of the election profile (like last day changes in voter preferences or the weather's influence on participation).

Thus there can only be a significant probability of a cycle when all 3 pairwise encounters are pretty close to 50-50. As the exact positions of the chords are stochastic, the circle center may then be covered by T or by any of the 6 other areas. Assume the profiles are distributed according to IC[10] (Impartial Culture)

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10. In IC each voter independently picks one of the $n!$ linear rankings of the n candidates, each with probability $1/n!$.

or IAC[11] (Impartial Anonymous Culture). As the number of voters increases, the well known limit probabilities of a cycle are, respectively, Guilbaud's number $\arccos(23/27)/(2\pi) \approx 0.09$ and $1/16$. See e.g. [Stensholt 1996]. This indicates that even in the most favourable setting (for a cycle) the probability will be less than 10%.

Moreover, in most 3-candidate cases, the conditions are not favourable for cycles at all, because there is one dominating "dimension" in the voters' perceptions. Then one candidate appears as intermediate between the others, and T is located far from the circle center. The profile may even become roughly single-peaked, as in Figure 1.[12]

However, cycles are occasionally found in assemblies. Two particular circumstances raise the probability of a cycle. One is that both primary and subsidiary voting are influenced by party discipline. Another is that the theme makes it difficult to formulate any compromise proposal.[13] These circumstances combined in an decision in the Norwegian national assembly (Stortinget) in 1992 on the location for a new airport [Stensholt 1999]. According to the party leaders' statements, and assuming that party discipline would prevail if necessary, the 165-politician profile, restricted to the 3 main alternatives[14] F, G, and H, was

$$(|FGH|, |FHG|, |HFG|, |HGF|, |GHF|, |GFH|) = (0, 42, 22, 37, 1, 63).$$

Here is a Condorcet cycle F*P*H*P*G*P*F with a vast "rotating majority". The triangle T covers 19% of the circle area. What actually happened in 1992, was in fact a use of strategy 1. G won by strategic voting in the serial voting procedure. The planned voting sequence started with G (yes or no)[15]. A "no" to G would lead to a win for F. Most members of the HGF group voted as though they belonged to GHF in

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11. In IAC the profiles are seen as lattice points in a simplex of dimension $n!-1$, each lattice point with the same probability.
12. If the midnormals intersect outside the circle in a perfect pie-sharing model, the profile is single-peaked and does not determine the model uniquely. But they will intersect exactly at the periphery in a unique model, which therefore also is a pictogram with T shrinked to a point.
13. Some cyclic profiles may disappear through deals between parties on how to vote in several cases.
14. The alternatives were F (traffic shared between G and then existing airport Fornebu); G (winner proposal: all traffic at location Gardermoen); H (all traffic at alternative site Hobøl). A compromise site within or near the FGH triangle was hardly conceivable.

order to prevent a win for F. So the strategy may be classified as type 1, which should be seen as democratically approvable, or at least acceptable.

All this took place in front of the public eye, i.e. national TV, and caused lots of comments, including unfair criticism against the politicians - who, despite some displays of hot temper, actually did their best on that occasion and should not be blamed.

3 Opportunities for strategy in Borda and Condorcet

3.1 The similarity effect and agenda manipulation

Consider a 3-candidate pie-sharing profile. In Figure 3, A and C are located as in Figure 1, but B is treated as a variable.

In any Condorcet method x beats y if and only if x is closer to the center than y is. Thus A wins against C in their pairwise encounter. There are 3 Condorcet rankings A$PC PB$, A$PB PC$, B$PA PC$ as the ideal point for B moves from the circle periphery towards the center, apart from possible exact ties when B crosses one of the concentric circles through A or C. The location of B never changes the fact that A is clearly ahead of C in Condorcet's sense with 5635 against 4365 votes.

The Borda Count may give another result if B is more similar to C than to A. B helps C to win if B is roughly "south"of C, despite A's clear direct win over C if B does not run. Similarly, if B is north of C, B may win with help from C. Nomination of a third candidate B who "outflanks" C, lets the C-party snatch the victory from A by "agenda manipulation".

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15. The assembly's president, an HGF-member, had proposed to start with H. He was over-ruled by the combined GFH- and FHG-groups, who put G first on the agenda. In fact, together they eliminated H. This must be seen as rational behaviour also from the FHG-members, because H seemed much closer to G than to F in their cardinal preference. The HGF-members were under pressure. They asked for time-out, and afterwards most of them voted "yes" to G.

If C is moved northwards to (-0.2, -0.4), then A's direct win over C drops to 5318-4682. The CAB area is approximately doubled, and A becomes much more vulnerable to agenda manipulation.

Figure 3 indicates that apart from the similarity effect, the Borda Count and the Condorcet methods behave much the same way for $n=3$. The similarity effect is related to the "clone effect" studied in some theoretical work; two candidates are clones if they occupy consecutive places in all ballot rankings. The clone effect is maximal if one clone is always ranked immediately after the other. The profile obtained by inserting a clone in the ballots may be unrealistic, but the very realistic similarity effect is disturbing enough[16]. Of course the similarity effect gets even stronger for $n>3$. Dummett [1998] was, with good reason, concerned about the similarity effect. He also proposed a modified Borda Count[17] intended to counteract the similarity effect. However, any proponent of this or other Borda modifications still has a burden of proof. Is there no significant similarity effect left? And what advantage will a Borda modification have over the much simpler Condorcet methods?

3.2 Creating a majority cycle

For what profiles is strategy 2 available when a Condorcet method is used? We concentrate on noncyclic profiles, because the natural occurrence of a cycle in real elections with many independent voters is an event too rare to worry about: For a cycle to be a realistic possibility, the pairwise contests must then be so close to 50-50 that the outcome is a random event anyway. Thus strategies that require a cyclic profile may appear as useful only *ex post*, not *ex ante*.

A switch from *xyz* to *xzy* will never turn *x* into a Condorcet winner. The only way to win is that the switch creates a cycle, and the strategy must change *yPz* into *zPy*. Thus, without strategic voting, *z* must be Condorcet loser and *y* Condorcet winner. The strategic voting starts in a non-cyclic profile where *yPxPz*, it is performed by the supporters of *x*, and it creates a cycle *yPxPzPy*. The switch works as intended if the cycle-break rule, which defines the particular Condorcet method, actually lets *x* win.

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16. For a party that enters 2 or more similar candidates, the similarity effect is positive in the Borda Count (but bad for democracy) and negative in the Plurality method (but, at least arguably, good for democracy).
17. This is not the modified Borda Count of [Emerson et al, 2007]

Clearly strategy 2 never works if *y* has more than 50% of the topranks. In Figure 4, A and C are located as in Figure 1, and B is considered as a variable[18]. Realistic profiles are generated by means of the pie-sharing model. When B is south of the curve from g to h, A has > 50 % of the topranks, and strategy 2 is not available. When B is north of the curve, *x* (= C, B, A as B moves from periphery to center) may always create a cycle. Whether *x* then wins the election depends on the particular cycle-break rule. The Table has 14 profiles obtained by placing B in 14 positions with the same first coordinate, as indicated in Figure 4, and some possibilities are shown in the Table and in the next section.

The contrast between all Condorcet methods and the Borda Count is most striking when B is located in the CAB-area in Figure 3. In the Borda Count, candidate C then becomes winner instead of A even without strategic voting. Moreover, Borda generally urges the use of strategy 2, while in any Condorcet method, strategy 2 cannot work at all with B south of the gh-curve in Figure 4.

3.3 Overlapping majority cycles

Consider the $n(n$-$1)/2$ candidate pairs in an *n*-candidate round robin tournament without draws. The results may be recorded in an *n*x*n* tournament matrix (m_{xy}) where $m_{xy}=1$, $m_{yx}=0$ if *x* defeats *y* in their pairwise encounter. In a chess tournament with no drawn games all tournament matrices are obviously possible. McGarvey [1953] showed that every tournament matrix also describes the Condorcet relation *P* for a suitably constructed profile of linear ballot rankings[19].

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18. If B is too close to the line with points (-0.2, t) through A and C (actually between the two circular arcs in Figure 4), the profile becomes "single-peaked", i.e. one of the 3 candidates is not ranked last by any voter. Realistic profiles are at most approximately single-peaked, as in Figure 1. B is rarely close to A or C, since too few voters would then rank C or A, respectively, in second place. Moreover, the profile may change fast with a small movement of B near A or C. As "distance" between two locations for B one may consider using the distance between the corresponding profiles.

19. Organize the pairwise encounters in rounds as is done in chess tournaments. Assign 2 voters to each round. Consider one of the 7 rounds in a 7 player tournament, with "games" (*a*, *b*), (*c*, *d*), (*e*, *f*) and a bye for *g*. In order to obtain e.g. *aPb*, *dPc*, *fPe* the 2 voters vote *gabdcfe* and *fedcabg*. They agree exactly on the 3 "games" of the round, and cancel each other in the 18 other "games". Thus 14 voters suffice to construct a Condorcet relation for any 7x7 tournament matrix, but much more efficient ways are known. As more qualified majorities are required, the possibilities for constructing an election that leads to a given tournament matrix are reduced [Mala 1999]. E.g. are 3-cycles *aPbPcPa* obviously impossible when higher scores than 2/3 vs 1/3 is required.

A Condorcet method must of course cover the theoretical possibility of more complicated profiles, i.e. with several overlapping 3-cycles. Many methods have been worked out in order to satisfy some normative properties, like monotonicity or independence of clones.

However, there seems to be no evidence that complicated Condorcet relations ever have occured naturally with many independent voters. Even a "Smith set"[20] $\{x, y, z\}$ in the shape of a single 3-cycle $xPyPzPx$, i.e. where $x, y,$ and z defeat all other candidates, must be quite rare with many independent voters.

It then seems over-cautious of an election designer to be motivated by a wish to preserve various normative properties if large Smith sets occur. A more likely source for dissatisfaction with Condorcet methods may be criticism based on post-election analysis: the x-party members who decided to vote xyz instead of xzy caused y to be elected at the expense of x. What can be done in order to minimize the incentives to attempt strategy 2 (which involves the creation of a cycle) or in order to minimize the number of missed opportunities to apply strategy 2 (since a miss may lead to voters feeling cheated)?

We show how various Condorcet methods for 3 candidates may be compared by means of geometric considerations, and study 3 particular methods: Baldwin's method, Nanson's method, and the suggested BPW-method, designed to minimize the number of profiles where strategy 2 is possible.

4 How to compare different Condorcet methods?

4.1 Baldwin's method

Baldwin's method [Baldwin 1926] is an elimination method similar to Instant Runoff, but with a different elimination criterion: The count is done in several rounds, each time the Borda sums are recalculated, and the candidate with the lowest sum is eliminated. A Condorcet winner never has less than average Borda sum, and therefore wins in the end.

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20. The Smith set is the smallest nonempty subset S of candidates so that xPy whenever $x \in S, \; y \notin S$.

For $n=3$ candidates, Figure 5 shows how the possibility to win by means of strategy 2 depends on the location of B with A and C fixed at (-0.2, 0.3) and (-0.2, -0.5). Comparison with Figure 4 indicates that Baldwin's method quite often will work if nr 2 in Condorcet's ranking is able to create a cycle.

Since A's direct win over C is so clear, the number of voters required for strategy 2 is pretty high, as shown for selected profiles in the Table. Strategy 3 is sometimes possible in most elimination methods, but will never work when there is a Condorcet winner. However, Baldwin's method occasionally allows strategy 3 when there is a cycle[21].

4.2 Nanson's method

Nanson's method [Nanson 1982] is like Baldwin's except that all candidates with less than average Borda score are eliminated at the same time. For $n=3$ candidates, Figure 6 shows how the possibility to win by means of strategy 2 depends on the location of B when A and C are fixed at (-0.2, 0.3) and (-0.2, -0.5). In Baldwin's method it is often possible for x (= A, B or C) to win with strategy 2 even though x would actually be eliminated with Nanson's method. But to get the Condorcet-winner y eliminated, Baldwin demands a higher number of voters to join the strategy attempt than Nanson does: it takes more to inflict the lowest Borda score on y than just to bring y below the average score (10000 in the Figure profiles). Thus there also are profiles where the x-supporters may win by strategy 2 in Nanson's method, but cannot afford it in Baldwin's.

For $n=3$, Nanson's method ignores the smallest pairwise defeat in a 3-cycle.[22] Many other Condorcet methods therefore coincide with Nanson's for $n=3$.

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21. E.g in the profile of Figure 1, the Table shows that a transfer of 3100 voters from CAB to CBA creates a 3-cycle, but the Borda scores are A: 9968, B: 9913, C: 10119, and B will still be eliminated. Elimination of A and win for C by means of strategy 3 may then be achieved by t voters moving from CBA to BCA, $55 < t < 151$. However, both simpler and safer for the conspirators in C's party would be to apply strategy 2 alone and let 100 more transfer from CAB to CBA. Thus strategy 3 may be possible in Baldwin, but is hardly of real significance.

22. Consider a profile $(|xyz|, |xzy|, |zxy|, |zyx|, |yzx|, |yxz|) = (p, q, r, s, t, u)$ which is cyclic, $xPzPyPx$. x beats z with $u+p+q-r-s-t$ votes, z beats y with $q+r+s-t-u-p$ votes, y beats x with $s+t+u-p-q-r$ votes. The "smallest defeat rule" lets z win if the result in the pair $\{x, z\}$ is ignored, i.e. if
$$u+p+q-r-s-t < q+r+s-t-u-p \text{ and } u+p+q-r-s-t < s+t+u-p-q-r.$$
But these simplify to $u+p < r+s$ and $p+q < s+t$, which again mean that z has more topranks than bottomranks (i.e. better than average Borda score) and x has more bottomranks than topranks (i.e. worse than average Borda score). Thus x is eliminated and z survives to become Nanson-winner. It makes no difference whether y is eliminated in round 1 or is promoted to round 2, since zPy.

4.3 The BPW method - "Beat the Plurality Winner".

An attractive feature of Nanson's method is that, in the case of a 3-cycle, it minimizes the necessary violation of a pairwise result. However, as Figure 6 indicates, there are many noncyclic profiles where a post-election analysis will show that the party of the runner-up candidate missed an opportunity to win by strategy 2. More upsettingly formulated, a party lost the election by giving too high subsidiary support to the winner.

We may instead design a method to minimize the number of noncyclic profiles which allow the Condorcet runner-up to win by strategy 2 or - more realistically - to discover after an election that a possibility to win by strategy 2 was missed. In a profile

$$(|xyz|, |xzy|, |zxy|, |zyx|, |yzx|, |yxz|) = (p, q, r, s, t, u),$$

a 3-cycle $xPzPyPx$ may have arisen from a noncyclic profile by means of strategy 2 in 3 different ways: The x- or y- or z-party has transferred h_x or h_y or h_z votes from xyz to xzy or from yzx to yxz or from zxy to zyx. The original profile is obtained by undoing the transfer, and so it was

$$(p + h_x, q - h_x, r, s, t, u) \text{ or } (p, q, r + h_z, s - h_z, t, u) \text{ or } (p, q, r, s, t + h_y, u - h_y).$$

How many h_x-values make the first of these noncyclic? It is made noncyclic if yPz, i.e. if $t + u + p + h_x > q - h_x + r + s$. Thus $q + r + s - t - u - p \le 2h_x \le 2q$.

With $|V| = p + q + r + s + t + u$ voters, $2h_x$ therefore belongs to an interval of length

$$2q - q - r - s + t + u + p = |V| - 2(r + s) = |V| - 2(|zxy| + |zyx|).$$

Similarly, $2h_y$ and $2h_z$ belong to intervals of lengths $|V| - 2(p + q)$ and $|V| - 2(t + u)$.

The Condorcet method suggested here minimizes the length of the interval. This means to declare as winner the candidate who defeats the Plurality winner. Strategy 2 is possible exactly in those noncyclic profiles where the Plurality winner is also the Condorcet loser.

For $n=3$ candidates, Figure 7 shows how the possibility to win by means of strategy 2 in this method depends on the location of B with A and C fixed at (-0.2, 0.3) and (-0.2, -0.5). Typically, there are 2 candidates, like A and B in Figure 7, who split a majority in two parts which are not too different in size:

C becomes Plurality Winner but loses both to A and to B in pairwise contests. Since strategy 2 only could have been used by one majority candidate against the other, few voters are likely to be very upset if a post-election analysis shows that there was indeed a missed opportunity to win by means of strategy 2.

In Condorcet methods, cases of nonmonotonicity only occur in cyclic profiles. With elimination methods the possibility of strategy 3 must be expected. When there is a 3-cycle, BPW in fact eliminates the Plurality winner. Strategy 3 occurs, but also a more curious way of exploiting nonmonotonicity.[23]

In order to define the BPW-method for any $n>3$ and any Smith set, one may e.g. tally each of the $n(n-1)(n-2)/6$ candidate triples separately and give the BPW-winner 1 point.

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23. When there is a cycle, BPW may allow a peculiar strategy different from types 1, 2 and 3:

Strategy 4: Voter i switches from xyz to yxz, $i \in U$, the win passes from y to x.

Unlike strategies 1, 2, and 3, strategy 4 is compatible with IIA, but strategies 3 and 4 both exploit non-monotonicity. Strategy 4 does not work if y is Condorcet winner. It requires a cyclic profile. For an example, choose B at (0.3, 0.38) in Figure 7. We get the noncyclic profile

(|ABC|, |ACB|, |CAB|, |CBA|, |BCA|, |BAC|) = (1866, 1065, 2724, 1036, 605, 2704).

with Condorcet ranking $APBPC$. B may win by strategy 2: B moves 700 voters from BAC to BCA, there is a Condorcet cycle $APBPCPA$, C is plurality winner, and the BPW-method lets B win. However, in the cyclic profile created,

(|ABC|, |ACB|, |CAB|, |CBA|, |BCA|, |BAC|) = (1866, 1065, 2724, 1036, 1305, 2004),

A may win by strategy 4: A moves 500 voters from ABC to BAC, making B plurality winner instead of C, but the Condorcet cycle persists and A becomes BPW-winner.

An opportunity to perform strategy 4 in a general election will be extremely rare, since it requires a cycle to exist. In some noncyclic profiles, strategy 4 might be a counterstrategy if B could win by strategy 2. Such theoretical use as counterstrategy may perhaps occasionally be a consolation for those who missed out on strategy 2: The cycle they might have created, might have been the stepping stone for strategy 4 or some other counter-strategy.

However, in assemblies with a few parties, strange things do happen. In the profile discussed in section 2.3, concerning an airport location in 1992, there was a Condorcet cycle $FPHPGPF$ and a Plurality ranking G (64), H (59), and F (42). So by beating G, H is the BPW-winner, but BPW would allow F to win by strategy 4: Transfer t voters from FHG to HFG, $5 < t < 23$ and turn H into Plurality winner!

Moreover, even G might win by strategy 3, transferring u voters from GFH to FGH, $17 < u < 19$. With any of the two common sequential voting methods described in section 5.3, strategies 3 and 4 would hardly ever be practical.

5 Discussion

5. 1 Proportionality or not?

Sometimes election methods are discussed with focus on proportional representation. Roughly proportional representation is often achieved by letting voters choose between party lists in multi-seat constituencies, perhaps in combination with extra seats distributed according to some formula. STV (Single Transferable Vote) is a family of preferential election methods, developed over a long time, that also achieve proportional representation [Tideman 1995]. Its main ingredients are a round-by-round tally where a criterion for electing or a criterion for eliminating one candidate is applied. Each voter has a "voting power" that is diminished when the voter helps to get a candidate elected; thus the voter has reduced influence in later rounds. In most STV methods that are actually used, both criteria are based only on the current topranked candidate in each ballot. For that reason, strategy 2 is impossible: no voter can then harm or help the candidate ranked as number k by means of rearranging the candidates in places $k+1, ..., n$.

There is nonmonotonicity in STV due to its use of eliminations: a candidate may be harmed by being raised in some ballots. However, since voters generally contribute to the election of several candidates, a voter who inadvertently harms a favorite candidate, is likely to get some recompensation by helping a political neighbor. Meek's algorithm[24] will help to achieve a fair seat distribution [Meek 1969]. Among the preferential election methods that achieve a rough proportionality, STV methods are well known and in actual use.

Some other preferential methods have been suggested with the purpose of achieving proportionality without the combination of eliminations and vote transfers, e.g. the QBS (Quota Borda System) proposed by Dummett [1984]. It is not clear what the elaborate system of "solid coalitions" in QBS can do to match the flexible vote transfer that the elimination rule achieves in STV [Schulze 2002].

If a national assembly is composed by Condorcet winners in single-seat constituencies, it is likely to become too politically homogeneous to serve as a political forum. But in order to elect the most central

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24. Meek's algorithm in effect "bills" a voter i retroactively when a new candidate advancing to i's ballot top has already been elected.

and most widely accepted candidate, e.g. for a party leader, the Condorcet methods are natural and without the weaknesses of the Borda Count described above - the similarity effect and the strong associated urge to use strategy 2.

5.2. Compromise candidates

Other preferential election methods than STV are mainly considered for single-seat constituencies. IRV, the single-seat version of STV, have more obvious competitors. Strategy 3 is possible in certain realistic profiles, but fortunately these are relatively few.[25]

Single winner preferential election methods generally favor compromise candidates - but not always in the same way. Condorcet methods favor the political center. IRV gradually concentrates the votes of increasing political groups onto a common candidate. When 3 candidates remain, they are probably not politically close. They are, say, a left wing compromise x, a center candidate y, and a right wing compromise z. If x and z are moderate wing candidates, y may be squeezed and eliminated. In that case IRV has an effect similar to Plurality, but IRV relies more on gentle vote transfers and less on urging the voters to apply strategy 1.

To become an IRV-winner it is important to be a balanced candidate with significant supply of both primary and subsidiary support. With small primary support, a candidate may be Condorcet-winner, but get eliminated in IRV. With sufficient subsidiary support it is possible to become Plurality winner through strategy 1, but in IRV an alliance builder has the advantage.

The elimination process in IRV has received some attention. Even for single-peaked preferences, with candidates allocated on a left-right interval, there is an "anti-domino" effect: When one candidate is eliminated, the immediate neighbors are usually safer for a while because of vote transfers. Sometimes

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25. In third profile of the Table, IRV allows the Plurality winner C to win by strategy 3, transferring t votes from CBA to BCA, $108 < t < 216$. The transfer will change the profile to:
   (|ABC|, |ACB|, |CAB|, |CBA|, |BCA|, |BAC|) = (1722, 1213, 4023, 215-t, 127+t, 2700).
   Thus C achieves the elimination of the Condorcet winner A and wins against B in the second tally round. In the second profile in the Table, shown in Figure 1, |CBA| is too small for strategy 3, but a number of CAB-preferrers may join the conspiracy and vote BCA.
   Strategies 2 and 3 both become impossible in "Conditional IRV", where the rule is as follows:
       Let x, y, z, ... have $n_x > n_y > n_z > ...$ topranks, and let the others have less.
       If $2n_y > n_x + n_z$ have an instant runoff with x and y. If not, elect x.
   In the profile considered here, C then wins in the first round.

y survives in the scenario above and wins by being close enough to *x* or *z* to take enough topranks from them. In simulations, the process seems somewhat erratic. Although candidates near an end of the interval may be chanceless, small changes in the allocations may still change the result drastically. It does not follow that IRV in practice will suffer from such "chaos", but chaos in simulation may indicate a factor likely to modify voter behaviour. When there are more than two viable candidates, voters will be motivated for strategic voting similar to the use of strategy 1 in Plurality elections. There is long experience with IRV, e.g. in Irish presidential elections, and good reason to ask if voter behaviour counteracts intolerable unpredictability in the elimination process.

AV [Weber 1995, Regenwetter and Grofman 1998, Brams and Fishburn 2003] is often seen as a way to obtain a solution most voters can accept. It cuts down the number of possible ballots from $n!$ to 2^n-2, which is likely to reduce the possibilities for agenda manipulation or strategic voting. For $n=3$, however, there is no reason to think that the similarity effect is weaker than in the Borda Count. If one half of the voters in the *xyz*-category approve only *x* and one half approves *x* and *y*, etc, then *x* receives $|zxy|/2 + |xzy| + |xyz| + |yxz|/2$ approvals, which is one half of the Borda-points. Then both methods give the same final ranking. Other models for how the *xyz*-preferrers will vote may of course strengthen or weaken the similarity effect. There is reason to ask how strong the similarity effect will be in AV.

Emerson et al [2007] are particularly concerned with voting methods in societies that are split along an ethnical or religious divide, and where a minority in fact is not taking part in the political process. Their criticism against Plurality is, in this author's opinion, too general, but absolutely justified when Emerson's necessary criterion for a democracy is not fulfilled: "A minimal interpretation might describe it as a means by which power is transferred without bloodshed." As long as there is a political middle segment in society that is not firmly committed to one side of a divide, IRV (and probably also Plurality) may suffice at least to meet this minimum requirement. But with a sharp perpetual divide, it is essential to promote real minority participation in the political process.

Minority representation has been counteracted [e.g. Trebbi et al 2008] or promoted [e.g. Pande 2003] by various rules. With STV (multi-seat), proportional representation in national assemblies may be

achieved for minorities of sufficient size, but this will not necessarily mean much minority influence in that assembly. Can the citizens vote across the divide in a way that promotes Emerson's "All-Inclusive Democracy"?

In IRV/STV a minority voter may state in the ballot that x is the best candidate from the majority side, but quite likely the voter will see x eliminated before the tally officers are even allowed to consider that statement. A Condorcet method may function as a radical device to promote meaningful voting across the divide. Although it may be practically certain that a candidate from the religious/ethnic/racial ... majority will be elected, every majority candidate has a strong incentive to campaign - also on the minority side of the divide - for a higher relative ranking than other candidates - from the majority side - get.

5.3 Condorcet methods, strategic voting and agenda manipulation

In elections with many independent voters, cycles are rare stochastic events and realistic only if all 3 pairwise encounters are close to 50-50. Voting will then necessarily have an element of gambling. Strategy 2, which is the type worth considering in Condorcet methods, will harm rather than help. A party that learns about a missed possibility after an election, is similar to a football pool gambler who learns about an upset result when it is already too late to play on it.

But in assemblies with a few dominating parties that impose party discipline on subsidiary rankings, cycles are more likely to occur and even to be predictable once in a while.

Voting over proposals in national assemblies generally pick a Condorcet winner, but not through a tally of submitted ballots. The proposals are numbered, P_1, P_2, ..., P_n. and the voting is done in several rounds. One method lets the assembly vote for or against P_k in round k: the voting stops when there is a majority for a proposal. A second method matches P_2 against P_1 in round 1, and in round k P_{k+1} against the winner of round k-1.

Provided there is a Condorcet winner and no member votes strategically, the second method may be the safest way to pick up the Condorcet winner. The first method will work the same way if P_n or P_{n-1} is

Condorcet winner, and members traditionally vote against P_k if a proposal they like better will come later.

If there is a cycle, it will usually be known through the debate, and it is natural to ask if one method then is better than another to solve the problem.[26] In both methods, the open sequence of votes either with two alternatives or with a "yes-no"-decision, under public scrutiny, is likely to keep members of parliament away from the most obvious and obnoxious violations of their own declared preferences. An intentional elimination of a Condorcet winner in a pairwise contest might, e.g., appear as "justice obstructed" by a group of voters.

In elections with many independent voters, there are few incentives to attempt strategic voting with a Condorcet method. Although a profile from opinion polls may allow strategic voting, in real life the uncertainty will make an attempt too hazardous. But, also with normal, noncyclic profiles, it is worth while considering the disappointment of a voter group on learning that its candidate y failed to win just because of the group's generous subsidiary support to the winner x. BPW minimizes the number of cases where a post-election analysis will reveal that an opportunity for strategy 2 was missed.

The three Condorcet methods discussed above may all be applied to $n>3$ candidates, and a 3-cycle may be analyzed as above by disregarding all candidates not in the cycle. The examples show how different various Condorcet methods can be. With the BPW-method the requirements for winning by applying strategy 2 in a noncyclic profile are quite strict: with Condorcet order $xPyPz$, y's party may win by strategy 2 if and only if z is Plurality winner. Then the most likely scenario is special: x and y are political neighbors, each of them would defeat z, but they are so close politically that they would spoil a Plurality election for each other. The y-supporters who learn too late that an opportunity to win by strategy 2 had been missed, will at least have the consolation that a neighbor candidate won anyway.

Regard the profile as a function $B(i, x, y)$: $B(i, x, x) = 0$, and for $x \neq y$,

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26. A point in favor of a serial "yes-no"-procedure may be that it in the case of a cycle it is better suited to facilitate a reasonable use of strategy 1, as it happened in the airport voting described in section 2.3. It might e.g. have felt tougher for the HGF-group to vote for G in a match between H and G than just to vote "yes" to G. However, if there is a Condorcet winner, and an agenda manipulator lets it come early in the procedure, it runs a risk of getting eliminated.

$$B(i, x, y) = 1 \text{ if } xP_iy \text{ and } B(i, x, y) = 0 \text{ if } yP_ix.$$

The usual Borda tally just needs the Borda points $\Sigma_z B(i, x, z)$ given by voter i to candidate x. However, Borda for n candidates may also be done as a round robin tournament of $n(n-1)/2$ 2-candidate elections with $\Sigma_i B(i, x, y)$ and $\Sigma_i B(i, y, x)$ recorded e.g. off the diagonal in an nxn-matrix M. R_i might as well be intransitive. The same is true for many Condorcet methods[27]:

In the Condorcet relation R, xRy means $\Sigma_i B(i, x, y) \geq \Sigma_i B(i, y, x)$

In the Borda relation R', $xR'y$ means $\Sigma_z \Sigma_i B(i, x, z) \geq \Sigma_z \Sigma_i B(i, y, z)$

IIA is just a generalization of Condorcet's idea. The high aggregation level of Borda is a flagrant violation of IIA. The $\{x, y\}$-contest is strongly influenced[28] by every other alternative z through every voter i. Borda urges the use of strategy 2 and of an agenda manipulation that makes many voters react naturally as though they participated in strategy 2. With increasing number of candidates, Borda increases each voter's power to participate in strategy 2.

Condorcet methods are immune to agenda manipulation. In all Condorcet methods strategy 2 requires participation from many voters to upset a clear pairwise decision, which makes the strategy impractical. Besides minimizing the theoretical opportunity to use strategy 2, BPW also limits the opportunity to cases where the incentive to attempt strategy 2 should be low.

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27. E.g. Baldwin and Nanson. They just need M to calculate Borda scores. BPW is less blunt. It needs more than M because it requires transitive R_i in order to find the Plurality winner in a 3-cycle.

28. As the number n of candidates increases, the "irrelevant alternatives" $z \notin \{x, y\}$ become dominating.
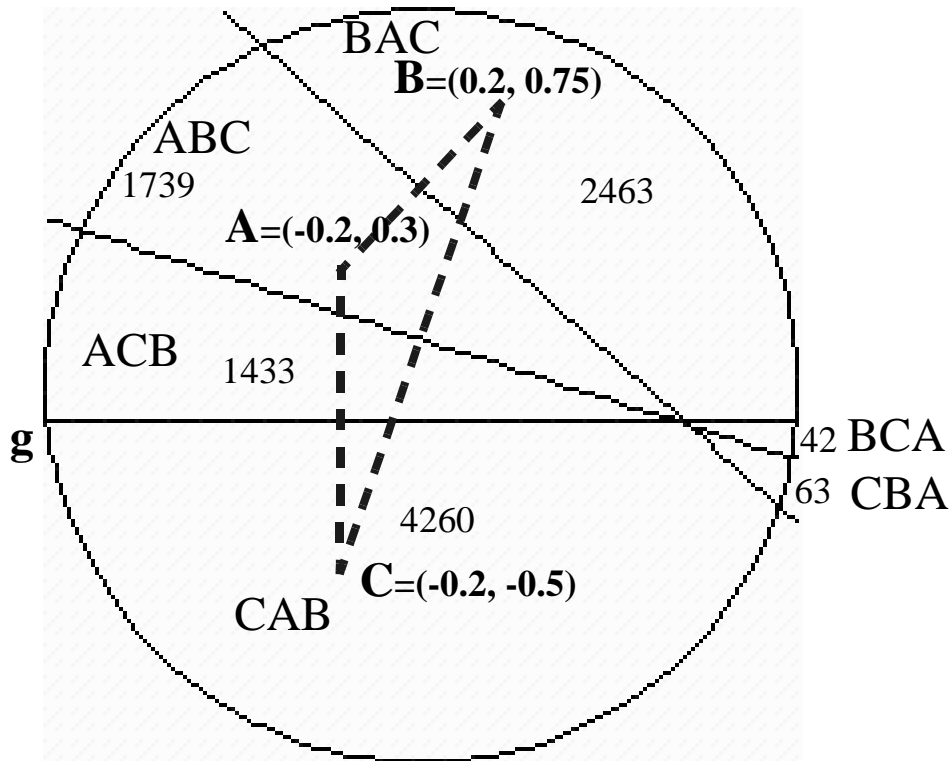
Figure 1 Almost single-peaked profile

The ideal points for A, B, and C are (-0.2,0.3), (0.2,0.75), and (-0.2,-0.5). The numbers sum to 10000.

The voters perceive A as situated roughly between B and C, and 99% ranks A as nr 1 or nr 2. In an

exact pictogram for this integer profile, the chords form a triangle T which is about 10^{-7} of the circle

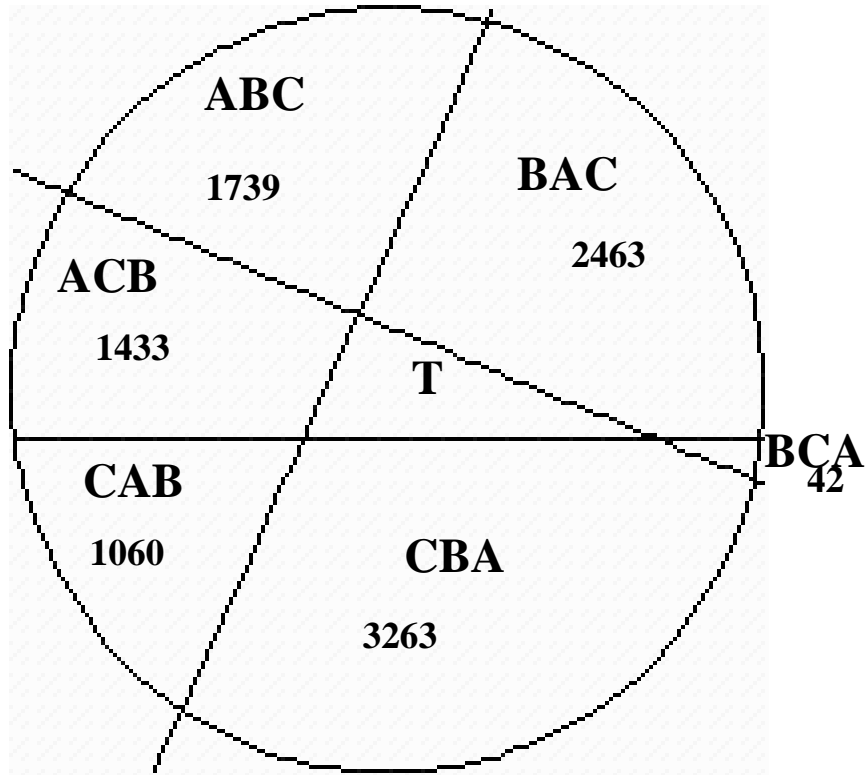area. The profile is the second in the Table; see also Figure 4.

Figure 2 A Condorcet cycle

A beats C beats B beats A. The triangle T is 5% of the circle area and covers the circle center. This profile is obtained from the noncyclic profile of Figure 1 when C's party transfer 3200 voters from CAB to CBA. All Condorcet methods that award victory to C in this cyclic profile, therefore allow C to win by strategy 2 in Figure 1. Among these methods are Baldwin's and all those, including Nanson's, that overrule the smallest pairwise defeat (C vs A 4365 - 5635; A vs B 4232 - 5768; B vs C 4244 - 5756)
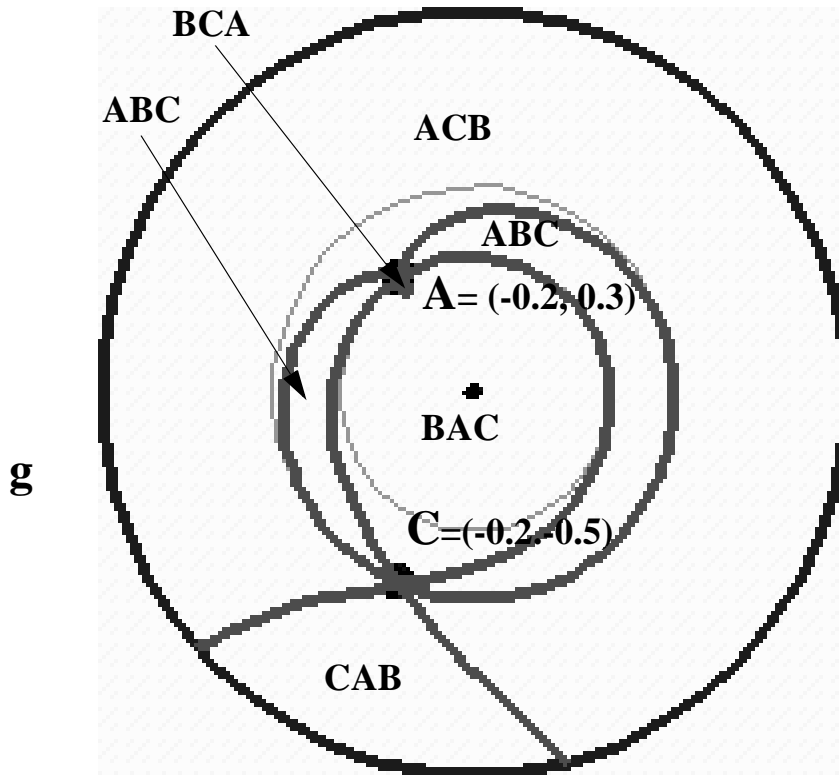
Figure 3 A Borda-Condorcet comparison

A defeats C (5635-4365) in a pairwise encounter, but the Borda rankings are ABC, ACB, BAC, BCA or CAB according to the location of B. In the CAB-area B is third in Condorcet's ranking but helps C to become Borda winner. This is the similarity effect, related to the theoretical cloning effect. Notice also a part of the area between the circles through A and C, where B is second in Condorcet's ranking but becomes Borda winner with help from C. Actually, the term "outflanking effect" might be more appropriate, because the candidate helped and the candidate helping may be very different politically. Thus, with B at (-0.30, 0.202) the profile becomes

$$(|ABC|, |ACB|, |CAB|, |CBA|, |BCA|, |BAC|) = (3860, 186, 972, 3130, 263, 1589)$$

A is Condorcet winner and would defeat B 5018-4982 but the entry of C makes B the Borda winner.
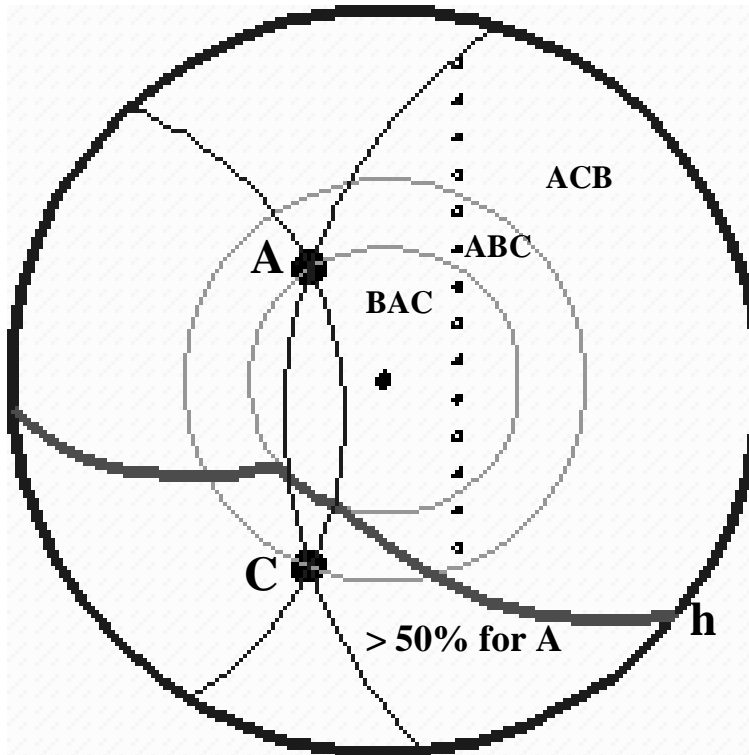
Figure 4 The possibility for Strategy 2 and the Single Peak condition

The Condorcet ranking changes from ACB through ABC to BAC as B moves from the periphery to the center. Generally x beats y when x is closer to the center than y is.

The curve from g to h marks where the location of B causes the Condorcet winner (A) to get 50% of the topranks. With B below the curve, strategy 2 is not possible in any Condorcet method because A gets more than 50% of the topranks. If B is above the curve, then number 2 in the Condorcet ranking may create a cycle by means of strategy 2. The 14 positions of B giving the profiles in the table are shown.

In any Condorcet method, for B to snatch the win from A by strategy 2, at least 636 voters must move from BAC to BCA just to create a cycle, when A defeats C with 2x635. A small number of voters can do it only if A's win over C is also small. That forces A, B, and C to be approximately equally far from the center. Then the result cannot be reliably predicted, and an attempt at strategy 2 is most likely not the best choice ex ante.

The thin arks through A and C show where the location of B causes the secants to meet at the periphery of the circle. If B is in one of the 3 areas which the arcs define around the vertical through A and C, then the profile is single-peaked.
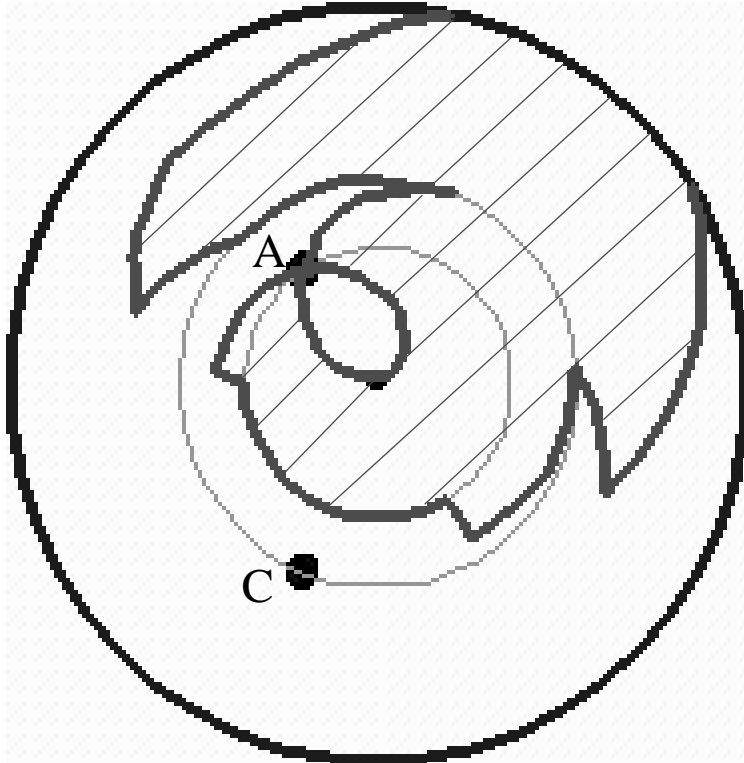
Figure 5 Strategy 2 with Baldwin's method

B in the marked area creates a profile so that strategy 2 becomes possible. With B outside the circle S' through C, C can win instead of A. With B inside the circle S through A, A can win instead of B. With B between S and S', B can win instead of A.
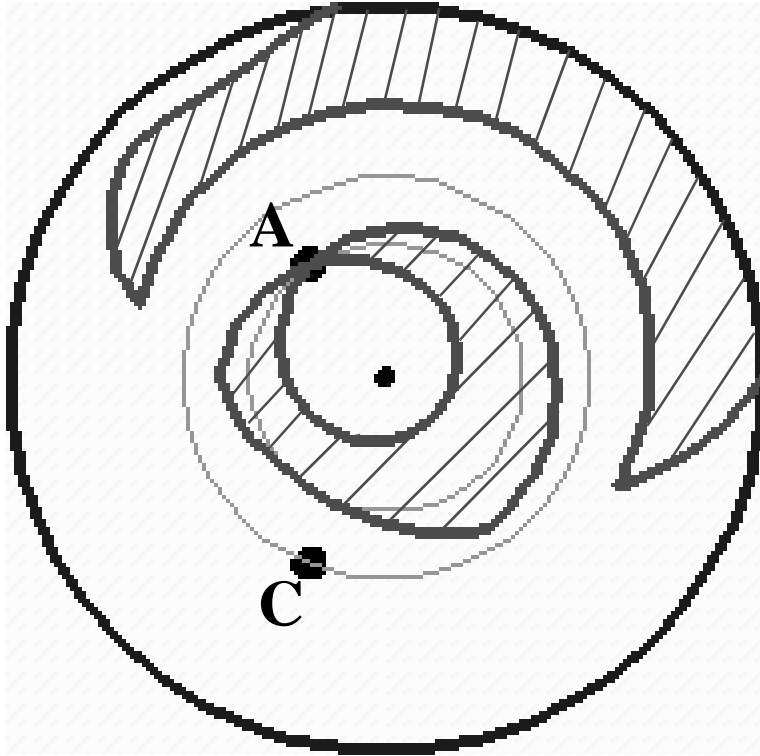
Figure 6   Strategy 2 with Nanson's method

There are two areas where candidate B creates a profile that allows strategy 2. With B in the NE area, C may win instead of A by use of strategy 2. Then assume B is inside the area that contains the inner circle S (through A): then B outside S may use strategy 2 to win instead of A and B inside S may be defeated by A by means of strategy 2.
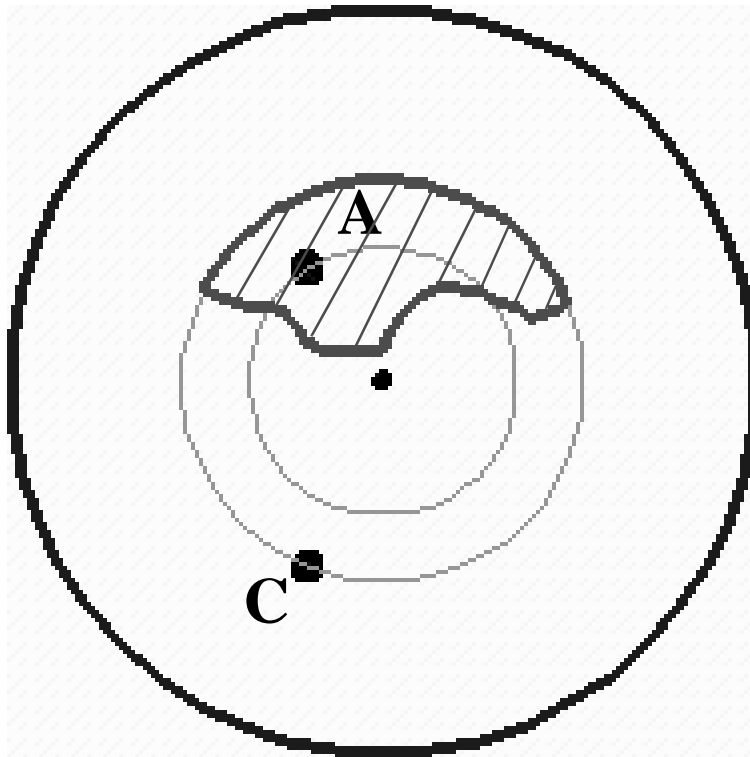
Figure 7   Strategy 2 with the "BPW"-method (Beat the Plurality Winner)

B in the marked area around A creates a profile where strategy 2 becomes possible. With B inside the

marked area and inside the circle S through A, A may win instead of B by strategy 2. With B outside S

but inside the marked area, B may win instead of A by strategy 2.

Table
Strategy 2 in 14 selected profiles

B_y	ABC	ACB	CAB	CBA	BCA	BAC		xyz	Bal	Nan	BPW
0.85	1789	1701	4360	3	2	2145		ACB	3699	3485	
0.75	1739	1433	4260	63	42	2463		ACB	3127	3068	
0.65	1722	1213	4023	215	127	2700		ACB	2501		
0.55	1748	1037	3637	475	252	2851		ACB	1814		
0.45	1826	899	3110	844	411	2910		ABC	1126		635
0.35	1963	801	2493	1274	597	2872		ABC	980	893	636
0.25	2155	739	1889	1672	804	2741		BAC	1126	917	700
0.15	2385	716	1397	1942	1026	2534		BAC	1518	1447	
0.05	2620	738	1052	2058	1255	2277		BAC	1764	1742	
-0.05	2822	820	835	2046	1483	1994		BAC	1878	1822	
-0.15	2948	985	711	1952	1701	1703		BAC	1848	1708	
-0.25	2944	1275	652	1816	1896	1417		BAC	1639	1386	
-0.35	2752	1742	642	1669	2053	1142		ABC		772	
-0.45	2334	2416	671	1538	2156	885		ABC			

The ideal points of A and C are at (-0.2, 0.3) and (-0.2.-0.5). The ideal point of B is at (0.2, B_y).

The Condorcet ranking is *xyz*, and candidate *y* will succeed with strategy 2 by transferring *t* votes from *yxz* to *yzx* if *t* is larger than the number indicated. E.g., with B_y= 0.25 the Condorcet ranking is BAC, but A wins by strategy 2, changing the numbers in columns 1 and 2 (lightly shaded) provided *t*>1126, *t*>917, *t*>700 in the Baldwin method, the Nanson method, and the BPW-method.

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