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# **Continuous Monitoring: Look before You Leap**

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# Continuous Monitoring: Look before You Leap\*

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## Abstract

We present a model for pricing credit risk protection for a limited liability non-life insurance company. The protection is typically provided by a guaranty fund. In the case of continuous monitoring, i.e., where the market values of the company's assets and liabilities are continuously observable, and where the market values of assets and liabilities follow continuous processes, the regulators can liquidate the insurance company at the instant the market value of its assets equals the market value of its liabilities, implying that the credit protection is worthless. When jumps are included in the claims process, the protection provided by the guaranty fund has a strictly positive market value. We argue that the ability to continuously monitor the equity value of a company can be a new explanation for why jump processes may be important in models of credit risk.

*Keywords:* credit risk for non-life insurers, guarantee fund, continuous monitoring, barrier options.

*JEL classifications:* G13, G23, G33.

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# 1 Introduction

In this paper we show how monitoring frequency influences the value of credit risk protection. We demonstrate that under our definition of bankruptcy the seminal Merton (1974) bankruptcy model breaks down if we assume that the market value processes for the assets and liabilities of the company are continuously observable. Continuously observable asset price processes are parts of the standard set-up in the continuous time finance model of Merton (1974), where the arbitrage argument depends on the possibility to continuously in time (i.e., at *any* time!) rebalance portfolios in order to replicate payoffs of contingent claims.

Our approach is applicable to all limited liability corporations, but our focus is a non-life insurance company. There are two reasons for that: First, an external regulator with power to initiate liquidation negotiations is consistent with our definition of bankruptcy. Second, this paper fits into and extends the existing literature on guaranty fund, a common credit protection mechanism in insurance.

Supervision or regulation of the insurance industry is common in most, if not all, countries. It is considered desirable for a society to be able to trust its insurance industry. Regulation is imposed in order to avoid hazardous management which again may lead to unwanted defaults. In most industrialized countries insurance policyholders are protected through a guaranty fund from losses in the case of insurance company insolvencies. The exact implementation of such funds seems to vary from one country to another, e.g., in the USA the insurance industry itself, rather than the government, is the ultimate guarantor.

Cummins (1988) analyzes guaranty funds. His analysis is based on the seminal Merton (1974) model, which includes a fixed time horizon - interpreted by Cummins as the time when the guaranty fund audits the insurer. A guaranty fund typically audits the insurance companies at given points in time, for instance once a year. A possible bankruptcy is both detected and declared after an audit has taken place. Furthermore, Cummins (1988) argues that because of the physical characteristics of insurance risks, it is natural to include jumps in realistic models of the claims against the insurance company. The use of *risk based* premiums for the bankruptcy protection from the guaranty fund is strongly advocated in his paper. Charging the insurers “correct” premiums is important because different insurers represent different risks. Not differentiating among the level of risk can lead to moral hazard and unwanted economic behavior through unreasonable risk taking.

Our main addition to Cummins’ approach is, instead of only letting the guaranty fund audit the insurer at a fixed point in time, to allow the guaranty fund to declare the insurer bankrupt the first time the market value of the assets is less than the market value of the liabilities. This is a natural definition of bankruptcy in our setting. For a general, non regulated company, our definition of bankruptcy is consistent with the use of bond covenants, i.e., the bond holders of a company have the right to declare the company default under certain conditions, the typical example is that the value of the company is below some threshold, see e.g., Black and Cox (1976). Analogously, in our model a guaranty fund declares the company bankrupt on behalf of the liability holders. This bankruptcy mechanism is in contrast to letting the equityowners determine the default of the company, an

approach which may not be appropriate in the case of regulated industries.

As Cummins (1988) we also use jump-diffusion processes to model the value of the insurer's liabilities and diffusion processes to model the assets. However, our starting point is somewhat different from Cummins' in that we use the EBIT approach of Goldstein, Ju, and Leland (2001). By the EBIT approach the market value of the liabilities is calculated as the market value of all future discounted claims, and the market value of the assets is the market value of all future discounted premium income.

We show that if there are no jumps in the price processes for the assets or the liabilities, our proposed monitoring mechanism completely eliminates the credit risk. Cummins' argument for including jumps is that jumps represent natural characteristics of insurance risk. This paper therefore provides an additional argument for why it is important to include jumps in models of credit risk.

In potential applications of our model the main argument against continuous monitoring may be the cost of frequent audits. However, abstracting from jump risk, we show that the value of the equity of the insurer, which normally is a convex function of the value of the company, becomes linear under continuous monitoring. Thus, the value of the equity is simply the difference between the market value of the assets and the liabilities. Auditing a stock-listed insurer is therefore unnecessary. Financial analysts audit the insurer for free, and the guaranty fund only has to monitor the value of the insurer's equity, a quantity that can be observed on any Reuter screen. In the case of jump risk we show that increasing monitoring frequency severely reduces the cost of bankruptcy protection, although this cost is not completely eliminated as in the case without jumps.

The paper is organized as follows: In section 2 we present an EBIT based model of the insurance company. In section 3 we analyze the special case of our economic model without jumps. The analysis is extended in section 4 to also include jumps in the claims process. Section 5 concludes the paper.

## 2 A Model of a Property-Liability Insurer

We consider an insurance company whose only liabilities are the potential insurance claims to its policyholders. We further take as given an equivalent martingale measure  $Q$  where the discounted price processes for the company's assets and liabilities are martingales. The risk free interest is denoted  $r$  and is assumed to be a constant.

Let  $x_s$  be the rate of new claims filed against the insurer at time  $s > t$ ,  $t$  represents a fixed, initial point in time. Under the measure  $Q$   $x_s$  is given by

$$x_s = x_t e^{(\mu_x - \gamma m - \frac{1}{2} \|\sigma_x\|^2)(s-t) + \sigma_x W_s} \prod_{i=1}^{N_s} Y_i. \quad (1)$$

Here  $N_s$  is a Poisson process with constant intensity  $\gamma$ ,  $W_s$  a two-dimensional vector of independent, standard Brownian motions,  $\mu_x$  is a drift parameter,  $\sigma_x = (\sigma_{11}, \sigma_{12})$ , where the  $\sigma_{ij}$ 's are constants, is the volatility vector of the continuous part of the process. Here the  $Y_i$ 's represent a sequence of jump magnitudes and are independent and identically distributed. In addition, the  $Y_i$ 's are independent of  $W_s$  and  $N_s$ , and also  $W_s$  and  $N_s$  are independent. In particular we assume that  $\ln(Y_i) \sim \mathcal{N}(a, b^2)$ . Also,  $m = E[Y_i] - 1 = e^{a + \frac{1}{2}b^2} - 1$ . Every time a jump occurs, the level of  $x_t$  is permanently changed.

Observe that  $E[\prod_{i=1}^{N_s} Y_i | \mathcal{F}_t] = e^{\gamma m(s-t)}$ , so  $E[x_s | \mathcal{F}_t] = x_t e^{\mu_x(s-t)}$ . The

initial values  $W_t = N_t = 0$ , and  $x_t$  is a given constant. Finally,  $\{\mathcal{F}_s, s \geq t\}$  is a filtration, where  $\mathcal{F}_s$  is interpretable as the information available at time  $s$ , in particular  $\mathcal{F}_t$  is *trivial*. The notation  $\|\cdot\|$  indicates the standard Euclidean norm.

The time  $t$  market value of the stream of claims is calculated as the expected discounted value under the equivalent martingale measure  $Q$ , i.e.,

$$L_t = E \left[ \int_t^\infty e^{-r(s-t)} x_s ds | \mathcal{F}_t \right] = \frac{x_t}{r - \mu_x}, \quad (2)$$

where by assumption  $\mu < r$ . Expression (2) is some places known as *Gordon's formula*.

At the future fixed time  $T > t$  the random value of the liabilities is given by

$$L_T = L_t e^{(\mu_x - \gamma m - \frac{1}{2} \|\sigma_x\|^2)(T-t) + \sigma_x W_T} \prod_{i=1}^{N_T} Y_i,$$

where  $L_t$  is given by (2).

In a similar manner, we let the rate of premium income at time  $t$  be given by

$$p_s = p_t e^{(\mu_p - \frac{1}{2} \|\sigma_p\|^2)(s-t) + \sigma_p W_s}, \quad (3)$$

where  $\mu_p$  is a constant and  $\sigma_p = (\sigma_{21}, \sigma_{22})$ , with constant  $\sigma_{ij}$ . Here  $p_t$  is a given constant.

The time  $t$  market value of the future stream of premium income is the insurer's assets and is given by

$$A_t = E \left[ \int_t^\infty e^{-r(s-t)} p_s ds | \mathcal{F}_t \right] = \frac{p_t}{r - \mu_p}. \quad (4)$$

At the future time  $T > t$  the random market value of the assets is

$$A_T = A_t e^{(\mu_p - \frac{1}{2} \|\sigma_p\|^2)(T-t) + \sigma_p W_T}.$$



The use of a two-dimensional Brownian motion allows a possible non-zero covariation between the asset and liability processes.

Note that both future asset and liability market values have similar probability distributions as in the model of Cummins (1988).

The variances of  $A_T$  and  $L_T$  are given by

$$\text{var}(A_T) = A_t^2 e^{2\mu_p(T-t)} (e^{\|\sigma_p\|^2(T-t)} - 1) \quad (5)$$

and

$$\text{var}(L_T) = L_t^2 e^{2\mu_x(T-t)} (e^{(-2\gamma m + \|\sigma_x\|^2 + \gamma((m+1)^2 e^{b^2} - 1))(T-t)} - 1), \quad (6)$$

respectively, whereas the covariance between  $A_T$  and  $L_T$  is given by

$$\text{cov}(A_T, L_T) = L_t A_t e^{(\mu_x - \mu_p)(T-t)} (e^{\sigma_x \sigma_p^\top (T-t)} - 1), \quad (7)$$

where  $k^\top$  denotes the *transpose* of some vector  $k$ .

### 3 The Diffusion Case

In this section we assume that  $Y_i = 1$  for all  $i$  (or equivalently, that  $a = b = 0$ , so  $m = 0$ ), i.e., there is no jump risk in the model and it is therefore equivalent to a pure diffusion model where the processes for the value of both assets and liabilities have continuous sample paths.

#### 3.1 Audit only at time $T$

In the model of Cummins (1988) a guaranty fund evaluates the insurer at some future point in time  $T$ . If the insurer is insolvent, i.e.,  $A_T < L_T$ , the insurance company is dissolved and the policyholders are compensated for their claims against the insurer. The shortage of funds, i.e.,  $L_T - A_T$

is supplied by the guaranty fund. Thus, the payment at time  $T$  by the guaranty fund is

$$\pi_T = \max(L_T - A_T, 0). \quad (8)$$

Since both the market value of the assets and the liabilities follow stochastic processes, the contingent cashflow in (8) can be interpreted as an exchange option and has time  $t$  market value (see e.g., Fischer (1978) and Margrabe (1978))<sup>1</sup>

$$\pi_t = L_t e^{(\mu_x - r)(T-t)} \Phi(d_1) - A_t e^{(\mu_p - r)(T-t)} \Phi(d_2), \quad (9)$$

where  $\Phi(\cdot)$  is the standard normal distribution function,

$$d_1 = \frac{\ln\left(\frac{L_t}{A_t}\right) + (\mu_x - \mu_p + \frac{1}{2} \|\sigma_x - \sigma_p\|^2)(T-t)}{\|\sigma_x - \sigma_p\| \sqrt{T-t}},$$

and

$$d_2 = d_1 - \|\sigma_x - \sigma_p\| \sqrt{T-t}.$$

The guaranty fund covers the policyholders' economic losses in case the insurer is declared bankrupt at time  $T$ , but is not in a position to take any actions against the insurer to limit its losses if the insurer's financial situation becomes difficult prior to time  $T$ . Action can only be taken at time  $T$ . A possible real world explanation for this model may be that the actual market values of the assets and liabilities may not be readily available in real life. Detailed information about these values is only available after a closer revision of the insurer. This explanation may seem somewhat extreme, but is not necessarily unrealistic. However, it contradicts the assumption of

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<sup>1</sup>Note that this is a slight extension of the Fischer (1978) and Margrabe (1978) formula in that both the assets and the liabilities have a drift rate different from the risk free rate.

continuous observable price processes inherent in the Merton model (which is implicitly used here).

**Example 1.** Assume the following parameter values and that  $t = 0$ :

$x_0$	$\sigma_{11}$	$\sigma_{12}$	$\mu_x$	$p_0$	$\sigma_{21}$	$\sigma_{22}$	$\mu_p$	$r$	$T$
10	0.2	0	0.05	12	0.1	0.05	0.05	0.1	1

Using the formula in expression (9) we calculate the market value of the credit protection provided by the guaranty fund to 0.5029. For these parameter values (see expressions (5)-(7))

$$\text{var}(A_T) = 800.72,$$

$$\text{var}(L_T) = 1804.12,$$

and

$$\text{cov}(A_T, L_T) = 969.66.$$

These numbers imply a correlation coefficient of 0.81.

### 3.2 Continuous auditing

Consider now the opposite extreme case, i.e., the case of *continuous* monitoring. If the market values of the assets and liabilities are continuously observable, continuous monitoring can be performed. We suppose this is the case and that the insurer will be liquidated the first time  $A_s \leq L_s$ ,  $s \in (t, T]$ . We also assume that the policyholders' claims have higher priority than the equity at the time of bankruptcy. Define the stopping time  $\tau$  as

$$\tau = \inf_{s \in (t, T]} \left( \frac{L_s}{A_s} \geq 1 \right),$$

i.e.,  $\tau$  is the time the insurer is insolvent and therefore will be declared bankrupt. The cashflow from the guaranty fund is now

$$\pi_\tau = \max(L_\tau - A_\tau, 0)1_{\tau \leq T}.$$

Note that this cashflow is identical to the cashflow from a barrier exchange option. The first time  $L_s$  hits  $A_s$  from below, the option expires immediately. Thus, the guaranty fund has issued a barrier exchange option expiring at whatever comes first of the stopping time  $\tau$  or  $T$ . Because the option matures the first time  $L_s = A_s$ , it must always be the case that  $\pi_\tau = 0$ , and the option issued by the guaranty fund must therefore be worthless.<sup>2</sup> Thus, by invoking continuous monitoring of the insurance company's balance sheet and liquidating the insurer the first time his assets and liabilities have the same market value, the guaranty fund will never have to pay money in case the insurer is declared bankrupt. In the case of only time  $T$  auditing, the guaranty fund is a vehicle providing financial security for the policyholders. Under continuous monitoring the guaranty fund will never have to pay money and has now changed its purpose into a vehicle for surveillance of an insurance business with insolvency risk.

Cummins (1988) explains that insurers are subject to a revision once a year with more detailed revisions every three to five years. Revisions are costly and are therefore not performed more frequently. However, we argue that in the diffusion case this is not necessarily a problem for stock listed insurers. To see this, let us take a closer look at an insurer's equity.

The purpose of the guaranty fund is to protect the policyholders from the credit risk of the insurer. In the case where the insurer has unlimited liability,

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<sup>2</sup>For a discussion of this kind of barrier exchange options, see e.g., Lindset and Persson (2006).

there is in principle no credit risk for the policyholders. The value of the equity of the insurer at time  $s$  is then  $A_s - L_s$ . In practice most insurers have limited liability. In this case the guaranty fund pays  $\pi_T = L_T - A_T$  in case of bankruptcy and zero otherwise. This guaranty is a state contingent claim for the insurer, and the value at time  $t < T$ ,  $\pi_t$ , increases the value of the equity by the same amount, compared to the case with unlimited liability. When the guaranty fund instead uses continuous monitoring, we saw above that  $\pi_t = 0$ , thus, even though the insurer may have limited liability, the value of the equity is the same as in the case of unlimited liability, i.e.,  $A_t - L_t$ . Publicly traded companies are subject to “continuous” scrutiny by investors and financial analysts afraid of losing their money and that are looking for new investment opportunities. The guaranty fund does therefore only have to observe the financial market and watch the stock price of the insurance company. If the stock price at time  $s > t$  gets low, this is evidence that  $L_s$  is approaching  $A_s$ , and closer monitoring could be implemented. Even though the guaranty fund may use continuous monitoring, the financial market can probably perform most of the monitoring, severely reducing the monitoring costs for the guaranty fund and almost eliminating the costs from insolvency.

The difference between Cummins’ monitoring only at time  $T$  and our proposed continuous monitoring has the same effect as including a covenant in a debt contract, a feature frequently discussed in corporate finance. In the model by Cummins (1988), the value of the guarantee provided by the guaranty fund is convex in the difference  $L_T - A_T$ . From the discussion above, it is clear that this also means that the value of the equity is convex in this difference. By increasing the number of monitoring points, the value of the equity becomes less convex, and in the limit (i.e., under continuous

monitoring), the value of the equity is linear in  $L_T - A_T$ . Thus, our proposed monitoring scheme represents a “de-convexification” of the market value of the equity.

## 4 The General Case

### 4.1 Audit only at time $T$

Jump-diffusion models are proposed in the finance literature by Merton (1976) in the pricing of options and have found some empirical support in Jorion (1988). The importance of jumps is quite intuitive for a property-liability insurer; earthquakes, hurricanes, and changes in judicial interpretations can all lead to sudden shifts in the value of an insurer’s liabilities. Cummins (1988) therefore proposes to model the market value of insurance liabilities by a jump-diffusion model.

As mentioned in section 2, we assume, for some constants  $a$  and  $b$  that  $\ln Y_i \sim \mathcal{N}(a, b^2)$ , i.e., the jumps are lognormally distributed. Following the arguments of Merton (1976), it can be shown that  $\Pi_t$ , the initial market value of the guarantee provided by the guaranty fund, is given by (a related formula is derived in Lindset (2007), where a proof can be found)

$$\Pi_t = \sum_{n=0}^{\infty} e^{-\gamma(T-t)} \frac{(\gamma(T-t))^n}{n!} \pi_t^n, \quad (10)$$

where

$$\pi_t^n = L_t e^{(\mu_x - r - \gamma m)(T-t) + n \ln(1+m)} \Phi(d_1^n) - A_t e^{(\mu_p - r)(T-t)} \Phi(d_2^n).$$

Here the functions  $d_1^n$  and  $d_2^n$  are defined as

$$d_1^n = \frac{\ln\left(\frac{L_t}{A_t}\right) + (\mu_x - \mu_p - \gamma m + \frac{1}{2}\sigma_n^2)(T-t) + n \ln(1+m)}{\sigma_n \sqrt{T-t}}$$

and

$$d_2^n = d_1^n - \sigma_n \sqrt{T - t},$$

respectively. The volatility  $\sigma_n$  is defined as

$$\sigma_n = \sqrt{\|\sigma_x - \sigma_p\|^2 + nb^2/(T - t)}.$$

Notice that the formula in expression (10) is close to a weighted sum of market values of exchange options in a diffusion model. The inclusion of the jumps has affected the terminal distributions of the asset and the liability values, but the timing of the jumps does not affect the cost for the guaranty fund. Much of the generality that is gained by including the jumps could therefore have been gained by adjusting the input parameters in the diffusion model, for instance by using an “implied volatility approach”, see Example 3 below.

**Example 2.** *We now assume that  $a = 0$  and that  $x_0, \mu_x, p_0, \mu_p, \sigma_{12}, r, T,$  and  $t$  are as in Example 1. We construct the table below by varying the jump intensity  $\gamma$  and the volatility parameter  $b$  of the jump.*

Case	$\gamma$	$b$	$\sigma_{11}$	$\sigma_{21}$	$\sigma_{22}$	$\Pi_0$
1	0.5	0.04	0.1980	0.1010	0.0479	0.5076
2	1	0.04	0.1959	0.1021	0.0456	0.5122
3	2	0.04	0.1918	0.1043	0.0403	0.5217
4	0.5	0.08	0.1917	0.1043	0.0402	0.5681
5	1	0.08	0.1831	0.1092	0.0239	0.6398
6	2	0.08	<i>n/a</i>	<i>n/a</i>	<i>n/a</i>	<i>n/a</i>

*These choices of parameter values imply that  $\text{var}(A_T), \text{var}(L_T),$  and  $\text{cov}(A_T, L_T)$  are the same as in Example 1 in all cases. Here *n/a* indicates that it is not*

possible in case 6 (i.e.,  $\gamma = 2$  and  $b = 0.08$ ) to adjust the three volatility parameters  $\sigma_{11}$ ,  $\sigma_{12}$ , and  $\sigma_{21}$  to obtain the same variances and covariance as used in Example 1 and the other 5 cases.

Compared to Example 1 jump risk is added. In order to keep the total variance of  $L_T$  constant  $\sigma_{11}$  is reduced in all cases compared to the value of 0.2 used in Example 1. We also require the covariance between  $A_T$  and  $L_T$  to be the same as in Example 1. Therefore  $\sigma_{21}$  must increase (given that  $\sigma_{11}$  decreases) relative to 0.1. Finally, in order to maintain the same variance of  $A_T$   $\sigma_{22}$  must decrease (given that  $\sigma_{21}$  increases) relative to 0.05.

**Example 3.** In this example we show how to adjust the volatility parameter  $\sigma_{11}$  (column 2) to  $\hat{\sigma}_{11}$  (column 5) in the formula (9) (column 4 based on original  $\sigma_{11}$ ), which is based on continuous processes (no jumps), in order to obtain the same market price as in expression (10) (column 3), which is based on processes including jumps.

Case	$\sigma_{11}$	$\Pi_0$	$\pi_0$	$\hat{\sigma}_{11}$
1	0.1980	0.5076	0.4268	0.2023
2	0.1959	0.5122	0.3528	0.2046
3	0.1918	0.5217	0.2260	0.2095
4	0.1917	0.5681	0,2242	0.2117
5	0.1831	0.6398	0.0515	0.2244

## 4.2 Continuous auditing

When we allow for more frequent monitoring of the insurer, the inclusion of jumps becomes important. In the case with continuous monitoring analyzed above, we found the guaranty fund to have issued an exchange option that is worthless and the value of the equity is simply the difference between



the value of the assets and the liabilities, i.e., the option element of the equity is worthless. The insurer is declared bankrupt immediately when the ratio  $\frac{L_t}{A_t}$  hits one from below. Because the sample paths for the stochastic process  $\{A_t\}_{t \in [0, T]}$  are discontinuous in a jump-diffusion model, we have that  $P(\{\frac{L_{t-}}{A_{t-}} < 1\} \cap \{\frac{L_t}{A_t} > 1\}) > 0$ . In words, the guaranty fund can observe a solvent insurer at time  $t$  and an instant of time later, observe that the value of the liabilities jump so that the insurer becomes insolvent. The associated cost for the guaranty fund is  $L_t - A_t > 0$ , a strictly positive payoff of the issued exchange option.

**Example 4.** *We now present examples where the value of the bankruptcy protection provided by the guaranty fund is estimated. In addition to the six cases used in Example 2, we have also included the diffusion case.*

Case	$\gamma$	$b$	Number of monitoring points					
			$10^0$	$10^1$	$10^2$	$10^3$	$10^4$	$10^5$
<i>Diffusion</i>	–	–	0.5029	0.3064	0.1241	0.0441	0.0140	0.0044
1	0.5	0.04	0.5076	0.3112	0.1369	0.0567	0.0309	0.0229
2	1	0.04	0.5122	0.3200	0.1539	0.0672	0.0442	0.0261
3	2	0.04	0.5217	0.3327	0.1736	0.0978	0.0770	0.0840
4	0.5	0.08	0.5681	0.3799	0.2544	0.1867	0.1735	0.1698
5	1	0.08	0.6398	0.4697	0.3689	0.3116	0.3114	0.3048
6	2	0.08	<i>n/a</i>	<i>n/a</i>	<i>n/a</i>	<i>n/a</i>	<i>n/a</i>	<i>n/a</i>

*All numbers are calculated based on 100,000 simulations.*

Example 4 illustrates how the market value of the exchange option issued by the guaranty fund varies when the continuous monitoring of the insurer

is approximated by different numbers of monitoring points.<sup>3</sup> It is clear that a high number of monitoring points is required for the value of the issued guarantee to converge to zero, i.e., the value when the insurer is monitored continuously in the diffusion model.<sup>4</sup> Although the convergence rate is also slow in the jump-diffusion model, we can clearly see that the value does not converge to zero, demonstrating that even when the insurer is monitored continuously the guarantee has positive value. See also Figure 1. No matter how the volatilities and the initial values of the assets and the liabilities are changed in the diffusion model, this result cannot be obtained since the value is always zero.<sup>5</sup> Explicitly modeling the jumps is therefore important in this case.

The market values of the guarantee provided by the guaranty fund we estimated in Example 4 contain more information than the convergence rate to continuous monitoring. They also show that more frequent monitoring reduces the market value of the guarantee. Table 1 illustrates that monitoring the insurer twice a year reduces the market value of the bankruptcy protection by from 8.8% to 13.5%. It also shows that quarterly monitoring further reduces this value by from 4.5% to 11.6% relative to the market value of annual auditing.

If the guaranty fund finds it difficult to rely on information from the

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<sup>3</sup>The calculations are performed using Ox, see Doornik (1999).

<sup>4</sup>Faster convergence can be obtained if the insurer is declared insolvent the first time

$$\frac{L_t}{A_t} = e^{-0.5826|\sigma_x - \sigma_p|\sqrt{dt}},$$

where  $dt$  is the time between each monitoring point (see e.g., Broadie, Glasserman, and Kou (1997)).

<sup>5</sup>The uninteresting case where the insurer is insolvent at the beginning of the period is disregarded here.

Table 1: The table shows the value of the guarantee provided by the guaranty fund for two and four monitoring points a year for the diffusion case and for the six cases considered in Example 2 (the other parameter values are as in the Examples 1 and 2).

<i>Case</i>	$\gamma$	$b$	<i>Number of monitoring points</i>		
			1	2	4
<i>Diffusion</i>	–	–	0.5029	0.4516	0.3935
1	0.5	0.04	0.5076	0.4602	0.4017
2	1	0.04	0.5122	0.4670	0.4125
3	2	0.04	0.5217	0.4513	0.4090
4	0.5	0.08	0.5681	0.5176	0.4741
5	1	0.08	0.6398	0.5653	0.5362
6	2	0.08	<i>n/a</i>	<i>n/a</i>	<i>n/a</i>

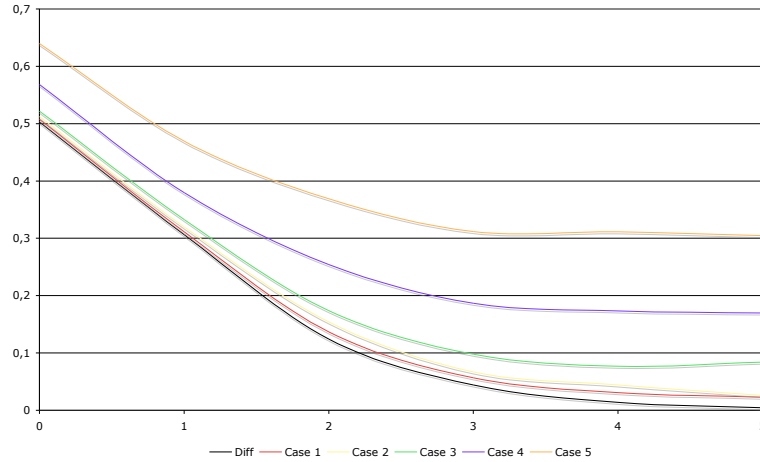


Figure 1: This graph shows the convergence of the market values of the credit protection as the frequency of monitoring increases. All numbers are from Example 4. The numbers on the  $x$  axis are the logarithms of the number of simulations.

financial market and the cost of revising an insurer is known, an optimal frequency of audits can be estimated.

Although the focus in this paper has been on property-liability insurance, the main idea in this paper, i.e., the benefits of more frequent monitoring, has a much wider field of applications and implications. It is also common to have guaranty funds protecting holders of life insurance policies. Much of the same reasoning as we have done for the guaranty fund for property-liability insurance also applies for the guaranty funds used in life insurance. This is also true for deposit insurance used to protect customers from (savings) bank default. Even banks themselves try to get a better understanding of their loan customers. This basically requires two things; more frequent

monitoring of the customers and better ways to evaluate each customer at each monitoring point. For a bank it is relatively easy to locate its least risky and its most risky borrowers. The difficult part is to distinguish between the different customers in the middle. The bank that has the best system to also categorize these customers clearly has an edge when it comes to setting competitive borrowing rates to the above average solid customers. This requires both good and frequent monitoring.

## 5 Conclusions

In this paper we present a framework based on the EBIT approach and Cummins (1988)'s jump-diffusion model for valuing the credit protection provided by a guaranty fund. We show that if jumps are not included in the model and the insurer can be monitored continuously, the credit risk vanishes, clearly demonstrating the importance of including jumps. We explain in which sense more frequent monitoring represents de-convexification of the equity. Monitoring costs may be reduced by exploiting the monitoring that already takes place for stock-listed companies in the financial marketplace. Most of the information needed by the guaranty fund is present in the quoted stock price of the insurer.

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