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## **MARKET SHARING AND PRICE LEADERSHIP**

by

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## MARKET SHARING AND PRICE LEADERSHIP\*

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*Abstract:* This paper proposes an alternative to the traditional model of supply and demand in markets where consumers take prices as given. Within the framework of “no side payments and partial preplay communication” firms are assumed to decide non-cooperatively on production and marketing while the market price is set by a *competitive price leader*, i.e. a firm preferring the lowest market price. Predictions include excess supply and a revenue-maximizing market price in markets where production precedes sales. In markets where sales precede production competitive price leadership predicts monopoly pricing but not necessarily monopoly profits if firms are “sufficiently similar”, while the presence of firms with high costs or low capacities will make it possible for the price leader, in some circumstances, to increase its market share and also its profits by reducing its price. And the threat of costly competition for market shares may reduce the market price even for identical firms.

*Keywords:* Pricing, oligopoly, price leadership, market sharing

*JEL classification:* L13 Oligopoly

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## 1. Introduction

Fundamental to economics is the notion of a market price determined by supply and demand. To give substance to this proposition when price-setting is decentralized to profit-maximizing firms one must show, firstly, why and how firms choose the same price and, secondly, that this price is determined by the market's demand and supply curves. This we shall do in the present paper. But we shall also see that the market price is not necessarily determined by *equality* between demand and supply.

Our point of departure is the traditional text-book model, where not only all buyers but also all sellers are price-takers. The demand curve describes what buyers are willing to buy at various prices, and the supply curve describes what sellers are willing to supply at various prices on the assumption that they can sell what they like. And then excess supply is supposed to trigger a process of price adjustment where firms try to sell their excess supplies by cutting prices until the market clears.

Now, suppose that producers do take a market price as given during the market period, even when it implies excess supply. Then they can no longer stick to the presumption that they can sell everything they produce. Instead they will realize that there is rationing and they will adapt to this. But how are producers rationed, or, in other words, how will the market be shared? This question is answered in Section 3 for markets where production precedes sales (where excess supply may arise in equilibrium after output adjustment) and in Section 4 for markets where sales precede production (where excess supply is excluded by definition). And it turns out that market sharing is a crucial element in a theory of price formation.

Having derived firms' profits as functions of the market price, in equilibrium after adjustment of production or marketing, we can formulate an alternative principle of price adjustment, namely that the market price goes down if and only if a price cut appears profitable to a firm *even if its competitors follow suit*, while the market price goes up if and only if a higher market price is profitable to every firm.

This means that the market price is determined by the lowest market price preferred by a firm, an idea which goes back at least to Boulding (1941 p. 610). What's new here is the emphasis on market sharing and its consequences for price formation.

More precisely, we shall consider a market form where the market price is set by a *price leader* in the beginning of the market period. When all firms prefer the same market price, the choice of price leader is immaterial, and then we have a *barometric price leader*, i.e. a price leader who "commands adherence of rivals to his price only because, and to the extent that, his price reflects market conditions with tolerable promptness" (Stigler 1947). It might be

thought that the market price will always be monopolistic in this case, but competition in other variables than prices will often enforce a lower market price, as we shall see in this paper. And when price preferences differ, and the market price is set by a firm preferring the lowest market price, I will call this firm a *competitive price leader*.

This version of price leadership will be developed successively in the following sections within the framework of “no side payments and partial preplay communication”, which Luce and Raiffa (1957 p. 169) once characterized as the most surprising omission in the literature on games. More precisely, I do assume that firms decide non-cooperatively on production or marketing, but I do not assume that they also decide non-cooperatively on prices. My reason for this approach is simply that firms cannot decide non-cooperatively on both prices and quantities, as shown in Appendix 1. Appendix 1 also reviews some of the literature on non-cooperative pricing – and concludes that even models where firms only choose prices are too complicated.

## 2. Assumptions

We assume that consumers are free to choose among producers (consumer sovereignty), excluding, for instance, the possibility for producers to fix market shares. We also assume that consumers take prices as given, excluding haggling or bargaining. The exclusion of haggling reduces transaction costs and facilitates price comparisons. By excluding bargaining we exclude the possibility for consumers to organize and bargain with producers over prices. And when consumers take prices as given, there is a well-defined demand function, which determines what consumers buy at a given market price. We assume that this demand function is decreasing in the market price (so that its inverse exists) and that its price-elasticity is non-decreasing and greater than 1 for some price.

For simplicity we assume throughout the paper (with one exception) that a firm’s marginal cost is constant up to a certain fixed capacity. In other words, assuming that it can sell everything produced, a firm’s supply is equal to its capacity for every market price higher than its marginal cost (and is otherwise 0 or indeterminate). However, this supply should perhaps be called *potential* supply, since it may differ from what the firm actually supplies. A firm’s potential supply will usually be called its capacity in this paper, where there will be an important difference between excess supply and excess capacity, and an important difference between market clearing and capacity clearing.

When it comes to rules for price formation, let us modify the classical price-taking postulate as little as possible. Thus we assume that prices are taken as given *during* the market

period not only by all consumers but also by all producers. We also assume that all producers *except one* take prices as given at the beginning of the market period. The choice of price leader is immaterial when all firms prefer the same price, while we assume that the price leader is the firm preferring the lowest market price when price preferences differ (excluding the possibility of side payments). This is a well-defined market form which I will call *competitive price leadership* (which includes barometric price leadership as a special case).

Now, what is a market form and how can it be observed? A market form is a set of rules for price formation which determines a market price. Examples of market forms which are easy to observe include Walras markets, where an auctioneer first asks for demand and supply at various market prices and then sets that price which clears the market. And there are plenty of such markets, including markets for gold and securities. Second, there are Cournot markets, where there also is an auctioneer, but an auctioneer who doesn't ask for demand and supply before announcing the price but sets that price which equals demand to that supply which has been brought to the market place. Such markets also exist, namely markets for agricultural products and other raw materials. Third, there are Bertrand markets, where firms independently and simultaneously commit to prices. Such markets also exist, but only in markets with big buyers, like in construction. And Bertrand models should be appropriate for industries with sealed bidding and excess capacity, as emphasized by Shapiro (1989 p. 351).

How can price leadership be observed? A necessary condition which is particularly easy to observe is that there is *not* an auctioneer in the market or a big buyer enforcing sealed bidding. Moreover, when market conditions change there should be a short period of price adjustment. This may be initiated by one of the firms and followed by the other firms in the market, in which case price leadership is particularly obvious. Even if firms simultaneously announce new list prices, price leadership is obvious if some firms adjust their prices after the initial announcement. And if there is no adjustment at all this may be a sign of particularly good coordination (when all firms prefer the same market price and the choice of price leader is immaterial). It cannot be interpreted as a price cartel unless some firms object to the price agreement and would have preferred a lower market price.

Of course, the existence of price leadership can also be observed indirectly by checking its predictions. These include, as we shall soon see, market clearing or capacity clearing in some cases, excess supply in some situations and, above all, in many cases mark-up pricing with a mark-up over variable costs which depends on the price elasticity of demand.

However, checking these mark-up formulas presupposes information on the price elasticity of demand which may be difficult to find for a researcher and even more difficult to

find for a firm. And even if a producer is able to exploit inelastic demand it is not certain that she always is willing to do it.

A crucial point when checking the predictions of competitive price leadership is consequently whether a mark-up depends on the price elasticity of demand or not. And “not” in many cases means full cost pricing, implying that firms set prices to cover all costs, including capital costs, where the contribution of capital costs to the price is obtained by dividing capital costs for the market period by estimated sales.

Now, a mark-up according to full cost pricing is not necessarily profit-maximizing. To see this, consider a monopoly with marginal cost  $c$ , capacity  $K$  and capital costs  $rK$ . Suppose, for simplicity, that the demand curve is linear, with  $D(c + mc) = 0$ , where  $m$  measures the steepness of the demand curve. Then it is easy to verify that

$$(1) \quad D(p) = D(c)(1 - x),$$

$$(2) \quad (p - c)D(p) = mcD(c)x(1 - x),$$

$$(3) \quad \text{where } p = c + xmc \text{ with } 0 \leq x \leq 1,$$

so that  $\max(p - c)D(p) = mcD(c)/4$  for  $x = 1/2$ . It follows that, if the firm sets a mark-up on the assumption that its sales will be equal to its capacity, its mark-up  $rK/cK$  will not be profit-maximizing unless  $r/c = m/2$ . And then sales will not be equal to capacity unless  $K = D(c)/2$ .

Moreover, for a given mark-up  $xm = \mu$  the firm's profits will be positive if and only if

$$(4) \quad mcD(c) \frac{\mu}{m} \left(1 - \frac{\mu}{m}\right) > rK,$$

which, if  $\mu = r/c$ , is equivalent to

$$(5) \quad m > \mu \frac{D(c)}{D(c) - K}.$$

Thus, full cost pricing ( $\mu = r/c$ ) yields positive profits if demand is sufficiently inelastic. And as long as profits are positive it may be rational for the firm to be satisfied with full cost pricing – if further information on demand is costly.

### 3. Price formation in markets where production precedes sales

In this section we focus on markets where production precedes sales. We begin with an atomistic market, where firms are too small to perceive any influence on aggregate output, and then proceed to a market with an arbitrary number of firms.

#### 3.1 Price formation in atomistic markets<sup>1</sup>

Define supply  $S(p)$  at the market price  $p$  in the usual way as aggregate competitive supply, i.e.  $S(p) = \sum s_j(p)$ , where individual supply  $s_i(p)$  is derived on the presumption that everything produced will be sold. This presumption is also true if  $p = p^c$ , where  $p^c$  clears the market,  $D(p^c) = S(p^c)$ , where  $D(p)$  denotes demand at  $p$ . But it is not true if  $p > p^c$ . Producers with a “disequilibrium awareness” (Fisher 1983) should realize this and adjust production accordingly – assuming that they do take the market price  $p$  as given when production is determined.

At this stage market sharing has to be specified. Assuming (for simplicity) homogeneous goods, market shares can only be influenced by making goods easily available to consumers in shops. And assuming that availability in shops is proportional to output distributed among shops in the market, a firm’s market share will be

$$(6) \quad \alpha_i = q_i / \sum q_j,$$

where  $q_i$  denotes a firm’s production. This is *proportional rationing*, where every unit of supply of a homogeneous good has the same probability of being sold in the market.

It follows that a firm’s profit function is

$$(7) \quad \pi_i = pD(p)q_i / \sum q_j - c_i(q_i),$$

where  $c_i(\cdot)$  denotes a firm’s cost function, assuming in addition (for simplicity) that output remaining at the end of the market period is without value.<sup>2</sup> Differentiation yields

$$(8) \quad \frac{\partial \pi_i}{\partial q_i} = p(1 - \alpha_i)d - c'_i(q_i),$$

where  $d = D(p) / \sum q_j$ . It follows that  $(q_i)$  is an equilibrium point if

$$(9) \quad p(1 - \alpha_i)d = c'_i(q_i) \text{ for every } i.$$

<sup>1</sup> This subsection is a revised version of ch. 4 in my thesis (Farm 1986).

<sup>2</sup> Also note that stocks remaining at the end of the market period are often sold at a reduced price (or simply scrapped). In any case, adding an inventory evaluation function will not change the substance of the analysis.

This system of equations cannot in general be solved without information on individual cost functions. But in atomistic industries, where firms are “small”, we can set  $\alpha_i = 0$ , so that  $pd = c'_i(q_i)$  or, equivalently,  $q_i = s_i(pd)$ . Hence  $\sum q_i = S(pd)$  and  $d = D(p)/S(pd)$  so that  $d = d(p)$  solves the equation

$$(10) \quad D(p) = S(pd)d.$$

Assuming, as we always do in this paper, that  $D(p)$  is decreasing in  $p$  and  $S(p)$  increasing or constant, the solution to this equation is unique, and then we have the following result:

**PROPOSITION 1.** In a market where production precedes sales and rationing is proportional, “small” firms taking a market price  $p > p^c$  as given will in equilibrium produce

$$(11) \quad q_i^e(p) = s_i(pd(p)),$$

where  $d(p)$  is defined above.

Note that  $d(p) < 1$  for  $p > p^c$  so that (11) is indeed an interior solution (excluding market clearing) and  $1 - d(p)$  is the equilibrium rate of excess supply.

Also note that  $pD(p) = S(pd(p))pd(p)$ , so that, with our assumptions on demand and supply, it follows from  $(pD)' = S'(pd)(pd)'pd + S(pd)(pd)'$  that

$$(12) \quad \text{sign}(pd(p))' = \text{sign}(pD(p))'.$$

Hence the equilibrium supply curve

$$(13) \quad Q^e = Q^e(p) = S(pd(p))$$

is backward-bending at  $p$  if the usual potential supply curve  $S(p)$  is forward-bending and demand is elastic,  $-pD'(p)/D(p) = \eta(p) > 1$ , since  $(pD)' = D(1 - \eta)$ . If demand is elastic for every  $p > p^c$ , equilibrium supply will be less than  $D(p^c)$  for every  $p > p^c$ . And if demand is inelastic at  $p^c$ , equilibrium supply will be increasing up to  $p^o = \arg \max pD(p)$  and then decreasing.



The traditional supply curve  $S(p)$  reflects potential output from every potential firm. The equilibrium supply curve  $Q^e(p)$  reflects endogenous output restriction, including exit. A firm's output will be positive if and only if  $s_i(pd(p)) > 0$ .

Next we find that every firm prefers the same market price, irrespective of its cost function:

**PROPOSITION 2.** In a market where production precedes sales, rationing is proportional and firms are "small", all firms prefer the same market price, namely

$$(14) \quad \max(p^c, p^o),$$

$$(15) \quad \text{where } D(p^c) = S(p^c) \text{ and } p^o = \arg \max pD(p).$$

*Proof.* Recall that a firm's profits in equilibrium after quantity adjustment are

$$(16) \quad \pi_i^e(p) = pd(p)q_i^e - c_i(q_i^e),$$

where  $d(p)$  is defined above and  $q_i^e = s_i(pd(p))$  maximizes  $\pi_i = pd(p)q_i - c_i(q_i)$ . It follows from the envelope theorem that

$$(17) \quad \frac{d\pi_i^e(p)}{dp} = \frac{\partial(pd(p))}{\partial p} s_i(pd(p)),$$

and hence that  $\pi_i^e(p)$  is maximized by  $\arg \max pd(p)$ , which is equal to  $\arg \max pD(p)$  according to (12).

The intuition of this result is as follows. A firm with a disequilibrium awareness should realize that its profits for  $p > p^c$  will not be  $pq_i - c_i(q_i)$  but  $pd(p)q_i - c_i(q_i)$ , where  $pd(p)$  denotes average revenues per unit of supply (in equilibrium after quantity adjustment). Moreover, assuming that the firm is too small to perceive any influence from  $q_i$  on  $pd(p)$ , it quite naturally wishes to maximize average revenues per unit of supply, irrespective of its output and cost function. And maximization of  $pd(p)$  turns out to be equivalent to maximization of the industry's collective sales revenues  $pD(p)$ .

Finally, to complete the model it is hardly realistic in this case to assume that one of the small firms is a price leader. Instead we assume, in order to model an orderly market, that the industry has a trade association which sets the market price. And realizing that every firm

prefers that market price which maximizes the industry's revenues, the trade association's problem is that of a *statistician*, namely to estimate the demand function and especially its elasticity.

### 3.2 Price formation in oligopolistic markets<sup>3</sup>

The result that every firm prefers the same market price, irrespective of its cost function, is remarkable and can probably not be generalized from an atomistic to an oligopolistic market when production precedes sales (and it is definitely not true when sales precede production, as we shall see in the next section). But let us now see what can be generalized in the simplest possible framework. Assuming identical firms and constant returns it is also possible to provide explicit formulas for excess supply and profits in equilibrium.

**PROPOSITION 3.** Consider a market where production precedes sales, rationing is proportional and there are  $n$  firms producing at constant returns with the same marginal cost  $c$  at a market price  $p \geq c$ . Then each firm in equilibrium produces:

$$(18) \quad q_i^e(p) = \frac{D(p)}{nd(p)} \text{ if } p > p^u,$$

$$(19) \quad q_i^e(p) = \alpha_i D(p) \text{ if } c \leq p \leq p^u,$$

$$(20) \quad \text{where } p^u = c/(1-1/n), \quad d(p) = p^u/p, \text{ and}$$

$$(21) \quad \alpha_i = 1 - c/p + \varepsilon_i, \text{ where } \varepsilon_i \geq 0 \text{ and } \sum \varepsilon_j = 1 - n(1 - c/p).$$

*Proof.* In this case (9) reduces to the system of equations  $p(1 - \alpha_i)d = c$ , which is solved by  $\alpha_i = 1/n$  and  $d = (c/p)/(1-1/n)$  if  $d < 1$  or, equivalently,  $p > p^u$ . And then we also have  $\sum q_j^e = Q^e = D(p)/d(p)$  and  $q_i^e = Q^e/n$ . Next we observe that a point on the demand curve (where  $d = 1$ ) will be an equilibrium point if  $\partial \pi_i / \partial q_i = p(1 - \alpha_i) - c \leq 0$  for every  $i$ . And this condition is satisfied for the market shares specified above if  $c \leq p \leq p^u$ .

The assumption of constant returns should not be taken literally. Instead it models a situation when demand is so low that capacity constraints can be ignored. In general, potential aggregate supply is of course not equal to infinity but some total capacity  $K$  if  $p > c$ .

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<sup>3</sup> This subsection is a summary of Farm (1988).

Moreover, potential aggregate supply is indeterminate – between  $D(c)$  and  $K$  – if  $p = c$ . On the other hand, we now see that firms taking the market price as given will restrict production so that actual aggregate supply is always determinate and limited. In fact the market even clears if  $c \leq p \leq p^u$ . But the market shares are not uniquely determined in this case (unless  $p = p^u$ ). They are completely indeterminate if  $p = c$ , since then  $\alpha_i = \varepsilon_i$  and  $\sum \varepsilon_j = 1$ .

However, at market prices between  $c$  and  $p^u$  all market shares will be at least as great as  $1 - c/p$ , and they “tend towards uniqueness” (in fact towards  $\alpha_i = 1/n$ ) as  $p \rightarrow p^u$ . And if  $p > p^u$  there will be excess supply in equilibrium. In fact firms will produce

$$(22) \quad \sum q_j^e(p) = (1 - 1/n)(p/c)D(p) \text{ if } p > p^u = c/(1 - 1/n).$$

**PROPOSITION 4.** Consider a market where production precedes sales, rationing is proportional and there are  $n$  firms producing at constant returns with the same marginal cost  $c$ . Then all firms prefer the same market price, namely

$$(23) \quad p^o \text{ if } p^o > p^u,$$

$$(24) \quad \min(p^u, p^m) \text{ if } p^o \leq p^u,$$

$$(25) \quad \text{where } p^o = \arg \max pD(p), \quad p^u = c/(1 - 1/n) \text{ and } p^m = \arg \max (p - c)D(p).$$

*Proof.* It follows from  $\pi_i = (pd - c)q_i$  and Proposition 3 that in equilibrium after quantity adjustment at the market price  $p$ ,

$$(26) \quad \pi_i^e(p) = pD(p)/n^2 \text{ if } p \geq p^u,$$

$$(27) \quad \pi_i^e(p) = \alpha_i(p - c)D(p) \text{ if } p \leq p^u.$$

As noted in Proposition 3, the market shares  $\alpha_i$  are not completely determinate in our model if  $p \leq p^u$ . Since they are equal for  $p > p^u$ , we assume, however, that they are equal,  $\alpha_i = 1/n$ , for  $p \leq p^u$  as well. And then the proposition follows immediately from the expressions above for a firm’s profits, since  $(p - c)D(p)$  is increasing in  $p$  up to  $p^m$  and  $p^o < p^m$  with our assumptions on demand.

Substituting  $p^m = c/(1 - 1/\eta(p^m))$  and  $p^u = cn/(n - 1)$  in  $p^m < p^u$  we also find that

$\min(p^u, p^m) = p^m$  if and only if  $n < \eta(p^m)$ . Excluding exceptional cases with elastic

demand and few firms, however, all firms prefer  $\max(p^u, p^o)$  as the market price, where  $p^u$  clears the market, just as  $p^c$  does in Proposition 2. (But note that  $p^u > c$ , even if  $p^u \rightarrow c$  as  $n \rightarrow \infty$ .) Thus, the essence of Proposition 2 also applies to oligopolistic industries with identical firms (when capacity constraints can be ignored). And note that an oligopolist's price preference is independent of the number of firms when demand is sufficiently inelastic.

Moreover, when demand is inelastic it should be possible for producers to exploit this when consumers are price-takers and price-setting is up to the producers. But instead of postulating a statistician, as in an atomistic market, it may be more reasonable here to complete the model by postulating a *price leader*. And when all firms prefer the same market price, the choice of price leader is immaterial.

Note, however, that a revenue-maximizing market price comes at a cost, namely costly competition for market shares through non-cooperatively chosen quantities, taking the market price as given. In fact it follows from  $\pi_i = (pd(p) - c)q_i$  and Proposition 3 that

$$(28) \quad \pi_i^e(p^o) = \frac{p^o D(p^o)}{cn^2} ,$$

so that total profits in the industry will tend towards 0 as  $n \rightarrow \infty$ . A corollary of this result is that new entrants would reduce profits for incumbents not only at the rate of  $1/n$ , because of more firms sharing the same revenues, but at the rate of  $1/n^2$ , because of additional supply in equilibrium.

#### 4. Price formation in markets where production precedes sales

This section deals with markets where sales precede production or, in other words, firms produce to order. Carlton (1989 p. 941) expects “that our economy has increased its reliance on industries that produce to order”, even if he “has not seen much research on this topic”. Since production to orders eliminates costly excess supply, it may also appear profitable for all firms in an industry to introduce this market form – whenever it is possible.

Services are, of course, always produced to order. Otherwise production to orders is possible whenever consumers can accept some waiting time between purchase and delivery. If consumers want to inspect a product before purchase, they will prefer shops where products are demonstrated, but they may accept some waiting time before a replica of the product is delivered from the factory, implying production to orders. And with the advent of internet, not even a visit to a shop with inventories may be necessary.

In markets with production to orders, supply is always equal to demand, so the notion of a market price determined by market-clearing is meaningless. And then, assuming that market shares  $\alpha_i$  are exogenously given, profits of a firm in an industry with identical firms taking a market price  $p \geq c$  as given are  $\pi_i = \alpha_i (p - c) D(p)$ , where  $c$  denotes marginal cost, so that all firms prefer the monopoly price  $p^m = \arg \max (p - c) D(p)$ , when all firms are producing below capacity in a recession.

In general, however, the market price will be  $\max(p^m, p^k)$ , where  $p^k$  denotes the capacity-clearing price,  $p^k = P(K)$ , where  $P(\cdot)$  denotes the inverse of the demand function. In a boom the market price will consequently be capacity-clearing if demand is so strong that  $p^k > p^m$ . In a recession, on the other hand, the market price will be monopolistic with respect to variable cost, but it will not be particularly high unless demand is very inelastic. Note that a firm's profits are not even positive unless  $\alpha_i (p^m - c) D(p^m) > r_i K_i$ , where  $r_i K_i$  denotes the firm's capital costs.

Moreover, in a market with homogeneous goods and identical firms the assumption of exogenous markets shares is not exceptional, since in this case every firm has the same probability of being contacted by a consumer, so the market shares must be equal (according to the law of large numbers). And if firms have different capacities  $k_i$ , it may sometimes be reasonable to assume that investment in outlets has been adjusted to capacities, so that markets shares are predetermined and thus exogenous during the market period even in this case, with  $\alpha_i = k_i / \sum k_j$ . It follows that if we complete the model with a barometric price leader, both the market price and the industry's profits will be the same as with a monopoly.

We shall now see how marketing, cost differentials and capacity differentials can modify this bench-mark model of price formation in markets with production to orders.

#### 4.1 Effects of marketing

In markets with production to orders, a firm's output does not determine but is determined by its market share. However, a firm can influence its market share by other means than output. Following Shubik with Levitan (1980 p. 192-194), we assume that a firm's market share is

$$(29) \quad \beta_i = (1 - \gamma) \alpha_i + \gamma a_i / \sum a_j, \quad 0 < \gamma \leq 1,$$

where  $\alpha_i$  denotes its market share in the absence of marketing,  $a_i$  denotes the firm's expenditures on marketing and  $\gamma$  measures the effect of this marketing.

Shubik with Levitan (1980 p. 194) interprets  $a_i$  as expenditures on advertising and  $1-\gamma$  as the proportion of customers who are not influenced by advertising, but other interpretations are possible, provided they only include expenditures on marketing which are made and have effects during the market period. The market shares  $\alpha_i$  may be equal to  $1/n$  or  $k_i/\sum k_j$  or, in general, determined by previous marketing expenditures, including expenditures on design.

With this marketing technology a firm's profit function is

$$(30) \quad \pi_i = (p-c)D(p)\left[(1-\gamma)\alpha_i + \gamma a_i/\sum a_j\right] - a_i,$$

so that

$$(31) \quad \partial\pi_i/\partial a_i = (p-c)D(p)\gamma\frac{1-a_i/A}{A} - 1,$$

where  $A = \sum a_j$ . It follows that in equilibrium at  $p > c$ ,

$$(32) \quad a_i/A = 1/n,$$

$$(33) \quad A = (p-c)D(p)\gamma(1-1/n),$$

$$(34) \quad \pi_i = (p-c)D(p)\left[(1-\gamma)\alpha_i + \gamma/n^2\right].$$

Marketing will consequently affect profits but not preferred prices in equilibrium.

However, introducing capacity constraints, and assuming for simplicity that all firms have the same size ( $k_i = K/n$ ) and the same  $\alpha_i$ , a firm's profits as a function of the market price  $p$  (in marketing equilibrium) will be

$$(35) \quad \pi_i(p) = (p-c)D(p)/m \text{ if } p > P(K),$$

$$(36) \quad \pi_i(p) = (p-c)K/n \text{ if } p \leq P(K),$$

where  $P(\cdot)$  denotes the inverse of the demand function, and

$$(37) \quad 1/m = (1-\gamma)/n + \gamma/n^2,$$

assuming that marketing when there is excess demand or capacity-clearing ( $p \leq P(K)$ ) can be ignored. Note that  $\pi_i(p)$  is discontinuous at  $p = P(K)$ , since  $1/m < 1/n$  when  $\gamma > 0$  and  $n > 1$ .

We now have the following result:

PROPOSITION 5. Consider  $n$  firms with the same constant marginal cost  $c$  up to capacity and the same capacity ( $K/n$ ) in a market with production to orders and marketing according to (29). Then all firms prefer the same market price, namely

$$(38) \quad p^m \text{ if } K > K_d,$$

$$(39) \quad P(K) \text{ if } K \leq K_d,$$

(40) where  $p^m = \arg \max (p - c)D(p)$  and  $K_d$  is determined by the equation

$$(41) \quad (P(K_d) - c)K_d = ((1 - \gamma) + \gamma/n)(p^m - c)D(p^m).$$

*Proof.* Follows immediately from (35) and (36), since  $(p - c)D(p)$  is increasing in  $p$  up to  $p^m$  with our assumptions on demand.

Note that, in this case, a price leader will set a capacity-clearing price  $P(K)$  not only for  $K \leq D(p^m)$ , as in a price leader model with exogenous market shares. Instead we have capacity clearing and a market price below  $p^m$  all the way up to  $K_d$ , with  $K_d$  even approaching  $D(c)$  as  $n \rightarrow \infty$  if  $\gamma = 1$ . The threat of costly competition for market shares in excess-capacity situations will enforce capacity clearing provided that  $K \leq K_d$ , so that the profit guarantee at capacity clearing is sufficiently high.

#### 4.2 Effects of different costs

Let us now ignore marketing as well as capacity constraints and focus on costs. Suppose there are  $\nu$  low-cost firms in the market with the same marginal cost ( $c_1$ ), and let  $p_1^m$  maximize  $(p - c_1)D(p)$ . Taking the market price as given by  $p_1^m$ , it will be tempting for a high-cost firm ( $c_n$ ) to enter the market, provided that  $c_n < p_1^m$ . But the market share and the profits of the price leader (one of the low-cost producers) will decline as high-cost firms enter the market. Assuming in addition (for simplicity) that each firm captures an equal share of the market, a low-cost firm will prefer  $c_n$  instead of  $p_1^m$  as the market price if

$$(42) \quad (c_n - c_1)D(c_n)/\nu > (p_1^m - c_1)D(p_1^m)/n,$$

i.e., if the number of firms ( $n$ ) is so large that the profits at a low price ( $c_n$ ) and a big market share ( $1/\nu$ ) is higher than the profits at a high price ( $p_1^m$ ) and a small market share ( $1/n$ ).

PROPOSITION 6. Consider  $\nu$  low-cost firms ( $c_1$ ) and  $n - \nu$  high-cost firms ( $c_n$ ) in a market with production to orders and constant returns, and suppose that  $c_n < p_1^m$  where

$p_1^m = \arg \max (p - c_1)D(p)$ . Then all low-cost firms prefer the same market price, namely

$$(43) \quad p_1^m \text{ if } c_n \leq c_s,$$

$$(44) \quad c_n \text{ if } c_n > c_s,$$

where  $c_s$  is defined by

$$(45) \quad (c_s - c_1)D(c_s) = (\nu/n)(p_1^m - c_1)D(p_1^m).$$

*Proof.* Follows immediately from (42), since  $(p - c_1)D(p)$  is increasing in  $p$  up to  $p_1^m$  with our assumptions on demand.

Thus, threat of entry of high-cost firms will force low-cost firms to marginal cost pricing with respect to the high cost if the high cost is not too low,  $c_n > c_s$ . Note that here the price leader cuts its price in order to *eliminate* high-cost competitors (“cut-throat” competition). In the next subsection, with decreasing returns, a price leader may find a price-cut profitable even if it does not eliminate other firms.

#### 4.3 Effects of different capacities

Consider an industry with  $\nu$  small firms (with capacity  $k_1$ ) and  $n - \nu$  big firms (with capacity  $k_n > k_1$ ), where every firm has the same (constant) marginal cost ( $c_i = c$ ) up to its (fixed) capacity. We assume that each firm has the same market share at the market price  $p$  when no capacity constraint is binding, i.e. when  $D(p)/n \leq k_1$  or, equivalently, when  $p \geq P(nk_1)$ , where  $P(\cdot)$  denotes the inverse of the demand function. For lower price levels the small firms will produce at capacity and rationed customers will turn to other firms.

The profits of a small firm as a function of the market price  $p$  will consequently be

$$(46) \quad \pi_1(p) = (p - c)D(p)/n \text{ if } p \geq p^u,$$

$$(47) \quad \pi_1(p) = (p - c)k_1 \text{ if } p \leq p^u,$$

$$(48) \quad \text{where } p^u = P(nk_1),$$

while the profits of a big firm as a function of the market price  $p$  will be



$$(49) \quad \pi_n(p) = (p - c)D(p)/n \text{ if } p \geq p^u,$$

$$(50) \quad \pi_n(p) = (p - c)\alpha_n(p)D(p) \text{ if } p^k \leq p \leq p^u,$$

$$(51) \quad \pi_n(p) = (p - c)k_n \text{ if } p \leq p^k,$$

$$(52) \quad \text{where } \alpha_n(p) = \frac{1 - \nu k_1 / D(p)}{n - \nu} \text{ and } p^k = P(\nu k_1 + (n - \nu)k_n).$$

It follows immediately that a small firm will never prefer a lower market price than a big firm, and that the market price preferred by a small firm is

$$(53) \quad p_1^* = \arg \max \pi_1(p) = \max(p^m, p^u),$$

$$(54) \quad \text{where } p^m = \arg \max (p - c)D(p).$$

The market price preferred by a big firm depends on the size of a small firm, and we shall here focus on the case when  $k_1 \geq D(p^m)/n$  or, equivalently,  $p^u \leq p^m$ , when small firms always prefer the monopoly price  $p^m$  (while results for  $k_1 < D(p^m)/n$  are reported in Appendix 2). In this case the “monopolistic option”,

$$(55) \quad \pi_n(p^m) = (p^m - c)D(p^m)/n,$$

is always available to a big firm. But note that  $\alpha_n(p)$  is decreasing in  $p$ , so that a lower market price will increase the market share of a big firm, and sometimes also, as we shall see below, its profits.

To derive that price which maximizes  $\pi_n(p)$  we begin by noting that

$$(56) \quad \frac{\partial \pi_n(p)}{\partial p} = \frac{D(p)}{n - \nu} ((n - \nu)\alpha_n(p) - \varphi(p)) \text{ if } p^k \leq p \leq p^u,$$

$$(57) \quad \text{where } \varphi(p) = \frac{p - c}{p} \eta(p) \text{ and } \eta(p) = -pD'(p)/D(p).$$

Note that  $\varphi(p)$  is increasing in  $p$ , with  $\varphi(c) = 0$  and  $\varphi(p^m) = 1$ . Our assumptions on demand imply that  $\pi_n'(p)$  is decreasing in  $p$ . Hence, whenever there is an interior maximum on  $p^k \leq p \leq p^u$ , it is defined implicitly by the equation  $\varphi(p^o) = (n - \nu)\alpha_n(p^o)$ , or, equivalently, by the equation

$$(58) \quad D(p^o)(1 - \varphi(p^o)) = \nu k_1.$$

Note that  $p^o$  is less than  $p^m$ , independent of  $k_n$  and decreasing in  $k_1$ . And assuming a linear demand function it is easy to verify (see Appendix 2) that

$$(59) \quad \frac{p^o - c}{p^m - c} = 1 - \frac{\nu k_1}{2D(p^m)}.$$

PROPOSITION 7. Consider  $\nu$  small firms (with capacity  $k_1$ ) and  $n - \nu$  big firms (with capacity  $k_n > k_1$ ) producing at constant marginal cost ( $c$ ) in a market with production to orders. Suppose that  $k_1 \geq D(p^m)/n$  so that all small firms prefer monopoly pricing. Then all big firms prefer the same market price, namely

$$(60) \quad p^m \text{ if } k_1 \geq k_1^*,$$

$$(61) \quad p^m \text{ if } k_1 \leq k_1^* \text{ and } k_1 \leq k_n \leq k_n^r,$$

$$(62) \quad p^k \text{ if } k_1 \leq k_1^* \text{ and } k_n^r \leq k_n \leq k_n^*,$$

$$(63) \quad p^o \text{ if } k_1 \leq k_1^* \text{ and } k_n \geq k_n^*,$$

where  $p^m = \arg \max(p - c)D(p)$ ,  $p^k = P(\nu k_1 + (n - \nu)k_n)$ ,  $p^o$  is defined above and the critical capacities  $k_1^*$ ,  $k_n^*$  and  $k_n^r$  are defined in Appendix 2.

*Proof.* See Appendix 2.

According to Scherer (1980 p. 176), *collusive price leadership* is most likely to emerge when, among other things, “the oligopolists’ cost curves are similar”. But how “similar” must the cost curves be? Proposition 7 suggests an answer, since every firm prefers  $p^m$  as market price if the firms’ capacities are “sufficiently large” ( $k_1 \geq k_1^*$ ) or “sufficiently similar” ( $k_1 \leq k_n \leq k_n^r$ ).

On the other hand, Chamberlin (1929 p. 86) envisages a *disintegration of monopoly pricing* in an oligopolistic market when the number of firms increases, even if he finds it “impossible to say at just what point” this will happen. But Proposition 7 suggests an answer, provided that we can assume that an increasing number of firms also makes “dissimilar” firms more probable. A necessary condition for the breaking up of monopolistic pricing is that some firms are “sufficiently small”, or more precisely that  $k_1 \leq k_1^*$ . Moreover, given the presence of such small firms, monopoly pricing will break up if (and only if) some firms become “sufficiently big”, or more precisely if  $k_n > k_n^r$ . For then all big firms prefer a market price below  $p^m$ . And with one of the big firms as a price leader this preferred price will also be the market price.

It might be argued, however, that price leadership is not a robust market form in this case. In fact it can be shown that small firms will prefer to stick to  $p^m$  in some cases, even if the big firms set  $p^k$  or  $p^o$ . However, this will either not affect the profits of big firms (if  $p_n^* = p^k$ , where  $p_n^*$  is the market price preferred by a big firm) or increase them (if  $p_n^* = p^o$ ), since  $\pi_n(p_n^* | p_1 > p_n^*) = (p_n^* - c)k_n$  whenever it is possible for small firms to raise profits by exploiting a contingent demand curve. This might be an explanation of *price dispersion* in some cases, but I will not pursue this issue any further in this paper.

Now, what kind of pricing will obtain when monopoly pricing has broken up? A *capacity-clearing price* is a particularly interesting candidate, representing, as it does, the classical notion of an equilibrium price determined by equality between demand and (potential) supply. And a competitive price leader does find  $p^k$  profit-maximizing in some circumstances, namely if the small firms are “sufficiently small” ( $k_1 \leq k_1^*$ ) and the big firms are both “sufficiently large” ( $k_n \geq k_n^r$ ) and “sufficiently small” ( $k_n \leq k_n^*$ ).

If, however, the big firms are “sufficiently large” ( $k_n \geq k_n^*$ ), while the small firms still are “sufficiently small” ( $k_1 \leq k_1^*$ ), a big firm will prefer a market price  $p^o$  at which small firms produce at capacity but big firms produce below capacity and consequently maximize profits with respect to the residual demand curve. This suggests *dominant-firm price leadership*, as defined, for instance, in Scherer (1980 p. 176), since this market form is characterized by the following assumptions, assuming, for simplicity, that there is only one big firm.

Firstly, the market price is set by the big firm, while the small firms (the “competitive fringe”) take the price as given. Secondly, the small firms produce “competitively” at the given price, i.e. at full capacity. Thirdly, the big firm sets that price which maximizes its individual profits, given its residual demand curve. And a central prediction of the dominant firm theory is that the price set by the dominant firm is decreasing in the total capacity of the small firms, including marginal cost pricing as a special case.

Proposition 7 suggests a rationale for dominant-firm price leadership – as well as boundaries for its applicability. A big firm will indeed anticipate the supply reactions of other firms, or, more precisely, their market shares at different market prices. In doing this, the big firm will also find it optimal, *in some circumstances*, to set a price at which the small firms produce all they want at the ruling market price. And then the market price is indeed decreasing in the total capacity of the small firms, according to (59). But note that  $p^o$  only

applies as long as  $k_1 \leq k_1^*$ . This means that there is a lower limit (above marginal cost) to the price set by a dominant firm, in contrast to traditional dominant firm analysis.

## 5. Conclusions

The most important predictions of competitive price leadership are, in summary, as follows. First, the basic determinants of the market price are the relation between demand and capacity in the industry and the price elasticity of demand. If demand is sufficiently strong in relation to capacity (so that  $p^k > p^m$ ), a price leader will set the capacity-clearing price, and variations in demand will affect price but not production. With excess capacity, on the other hand, there will be mark-up pricing, with a mark-up over variable costs which depends on the price elasticity of demand.

Second, the market price set by a competitive price leader depends on whether production precedes sales or not. In markets where production precedes sales, the market price will maximize the industry's sales revenues.

Third, in markets where sales precede production, pricing will be monopolistic if the firms' cost curves are "sufficiently similar" or their capacities "sufficiently large" (as specified more precisely in Section 4). But a monopolistic price in this context means monopolistic with respect to variable cost, so that fixed costs are not covered and profits are not positive unless demand is sufficiently inelastic.

Fourth, the market price set by a competitive price leader may be reduced by the presence of firms with high costs or low capacities, since this will make it possible for the price leader, in some circumstances, to increase its market share and also its profits by reducing its price. And the threat of costly competition for market shares may reduce the market price even for identical firms.

Fifth, a fall in demand during a recession need not reduce the market price. Sales are reduced but not necessarily the market price. And if the market price responds at all, it *increases* if it before the recession was lower than the monopoly price, since excess capacity is conducive to monopolistic pricing (as we have seen in Section 4), while it *decreases* only if it before the recession was higher than the monopoly price.

Sixth, the market price depends on the number of firms only in special cases. The breaking up of a monopoly, for instance, does not necessarily lower the market price. But it leads to competition in other variables than prices, which may increase availability and

quality of the products. Deregulation of a taxi market, for example, will not lower the market price but increase the number of cabs.

Seventh, at the market price set by a competitive price leader there will be excess supply in markets where production precedes sales, even in equilibrium, when firms have realized that they cannot sell all they want.

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### **Appendix 1. Notes on the literature**

Suppose that firms choose quantities ( $q_i$ ) as well as prices ( $p_i$ ) non-cooperatively. It is sometimes taken for granted that excess capacity is sufficient to guarantee a competitive equilibrium even in this case (see e.g. Levitan and Shubik 1980 p. 66). But a competitive state which clears the market can *never* be a non-cooperative equilibrium in a price-quantity game in which the strategy of each player consists of two numbers, namely a price ( $p_i$ ) at which he will sell his product and a quantity ( $q_i$ ) which he will bring to the market place.

To prove this, let us take  $(c, q_j)$  as given for  $j \neq i$  and contemplate strategies  $(p_i, x_i) \neq (c, q_i)$  for firm  $i$ . Then consumers will buy  $D(c) - q_i$  (but no more) from the other

firms at price  $c$  and the contingent demand  $x_i$  from firm  $i$  at price  $p_i \geq c$ . And with  $\pi_i = p_i x_i(p_i) - c x_i(p_i)$  we always have  $\partial \pi_i / \partial p_i = x_i(c) = q_i$  at  $p_i = c$ . And  $q_i > 0$  for at least some  $i$ , since  $\sum q_i = D(c)$ .

### *Static Bertrand models*

Assuming production to orders (preventing production from being a firm's decision variable), firms cannot select their own prices ( $p_i$ ) non-cooperatively without being forced to marginal cost pricing. The well-known argument, assuming equal and constant marginal cost ( $c$ ), is that  $p_i = p > c$  cannot define an equilibrium since every firm can increase its sales discontinuously by choosing a price slightly less than  $p$ . More precisely,  $p_i = p$  is not optimal against  $p_j = p$  ( $j \neq i$ ) for any  $p > c$ , while  $p_i = c$  is optimal against  $p_j = c$  ( $j \neq i$ ) for every  $i$ .

One assumption upon which this argument is based is also well-known, viz. the existence of excess capacity. For if capacities ( $k_i$ ) were limited in the sense that a firm's rivals could not satisfy the whole market ( $\sum_{j \neq i} k_j < D(c)$  where  $D(p)$  denotes the market demand function), a firm's sales would not reduce to zero for  $p_i > c$ , given  $p_j = c$  ( $j \neq i$ ). In this case the market price is indeterminate, as emphasized, for instance, by Edgeworth (1925 p. 125), Shubik (1959 ch. 5) and Shapley and Shubik (1969). A Nash equilibrium in pure strategies does not exist.

Equilibria in mixed strategies may exist (Dasgupta and Maskin 1986), and some characterizations are also available in the literature. In situations with binding capacity constraints, the monopoly price is quoted with positive probability not only in a duopoly (Davidson and Deneckere 1986, Osborne and Pitchik 1986), but also in an industry with many firms (Allen and Hellwig 1986). At the same time prices converge in a probabilistic sense (more precisely in distribution) to the market-clearing price, when the number of firms increases (Allen and Hellwig 1986).

Mixed strategy solutions to oligopoly problems are also presented by e.g. Shubik (1959 ch. 5) and Levitan and Shubik (1980 ch. 8). But they do this with many reservations, and the use of mixed strategies is also much harder to justify in non-constant-sum games (where they may lead to unstable equilibria) than in constant-sum games (see Shubik 1982 p. 249-251). And, as noted by Shapiro (1989 p. 346), each firm in a mixed equilibrium "would have an incentive to change its ex ante optimal but ex post suboptimal price".



The paper by Kreps and Scheinkman (1983) shows what happens if the auctioneer in a Cournot model is replaced by Bertrand pricing, but also what happens in a Bertrand model if the assumption of production to orders is replaced by production before sales. An important conclusion is that outcomes are sensitive not only to the way the price-setting stage is specified, but also to the relation in time between production and pricing. Moreover, outcomes are sensitive to how demand is rationed by a firm with a low price, as demonstrated by Davidson and Deneckere (1986).

### *Dynamic Bertrand models*

Repetition of a Bertrand game will sometimes neutralize its competitive implications. The basic idea is that the long-run loss of a price war will outweigh the short-run gain of a price cut if the discount factor is sufficiently high. Assuming  $n$  identical firms and constant returns, pricing will be monopolistic if  $n(1-\delta) \leq 1$  and competitive if  $n(1-\delta) > 1$ , where  $\delta$  is the discount factor; see, for instance, Shapiro (1989 p. 370).

Introducing capacity constraints, the degree of sustainable collusion is studied by Brock and Scheinkman (1985) in a model with identical firms. They find that an increase of the number of firms sometimes will raise the cartel price (if total capacity is larger than monopoly capacity but not “too large”). Benoit and Krishna (1987) study capacity choice with repeated price competition in a duopoly, focusing on the possibility of excess capacity in equilibrium. Some results on pricing by two firms with different capacities are also available in Davidson and Deneckere (1990, Section 4). Assuming that prices are chosen, subject to “no cheating”, so as to maximize a certain “cartel welfare function”  $F(\pi_1, \pi_2)$ , where  $\pi_i$  denotes the profits of firm  $i$  and  $F_1 > 0$  and  $F_2 > 0$ , they find that pricing will be monopolistic if the interest rate is sufficiently low.

Models of “alternating price competition”, originating with Maskin and Tirole (1988), do not represent repeated sealed bidding, but like sealed bidding they start from the concept of “commitment”. Thus price-setters are committed to their prices for at least some time. This means that a price cut will always raise a firm’s market share and its profits, even if it only is for a very short time, until competitors have retaliated. In this set-up a firm will abstain from price-cutting if – and only if – its long-run loss (due to a price-war) will outweigh its short-run gain.

*Quick response models*

A repeated game has not been the only attempt to model the seminal idea in Chamberlin (1929) that threats to match price cuts will prevent price cuts. An alternative is the “quick response” approach, which postulates a period of price adjustment before trade takes place. During this period initial price announcements are observed and reacted upon “instantaneously”, and all transitory profits, which the firms might earn before the responses are complete, are assumed to be negligible.

The intuition of the quick response approach is straightforward, assuming that firms are free to observe and change their prices at any time with negligible costs (perfect price flexibility). Contemplating a price cut a firm must reckon with responses evoked by it. In equilibrium a firm’s pricing strategy must consequently be optimal against other firms’ complete pricing strategies, including not only their expected initial prices but also their expected price response functions. Moreover, in a non-cooperative equilibrium such expectations must be rational, as emphasized not least by Johansen (1982) and Friedman (1983), including, in particular, rational expectations of rivals’ response functions.

Suppose that every firm quotes the monopoly price to begin with. Then a price cut might appear profitable to an individual firm, but only if other firms do not match the cut. Now, if other firms do not match a price cut, “almost every” firm will quote the monopoly price, and the rest will quote a price only slightly less. On the other hand, if other firms do match a price cut, an individual firm cannot rationally go on believing that rivals will not match price cuts. If price cutting occurs, its function is not to enforce a competitive price but rational expectations of rivals’ price response functions. And having succeeded in doing this, price cutting expires. In fact a rather heroic degree of myopia is required to insist on taking rivals’ prices as given when they are constantly falling. We conclude that price cutting is impossible as a part of equilibrium behaviour, as emphasized, for instance, by Friedman (1983 p. 228).

For this argument to hold it is only necessary that price cutting by a firm can be detected by other firms. This assumption may not be valid with large buyers (cf. Stigler 1964), but in markets with small buyers – as in consumer markets – it is certainly applicable, since “no one has yet invented a way to advertise price reductions which brings them to the attention of numerous customers but not to that of any rival” (Stigler 1964).

In consumer markets, where buyers take prices as given and firms are free to observe and revise their prices at any time, it is also reasonable to assume that a firm can – if it so wishes – set the same price as another firm. Also note that price-taking behaviour and price leadership

“is not apt to be found contrary to the antitrust laws unless the leader attempts to coerce other producers into following its lead” (Scherer 1980 p. 520).

Formal models of the quick-response approach include Marschak and Selten (1978), Farm and Weibull (1987) and Bhaskar (1989). Bhaskar (1989) assumes that price decisions  $p_i$  are taken at time  $t$ , and set equal to the price announcement at time  $t$ ,  $p_i = p_i(t)$ , if and only if, for every  $j$ ,  $p_j(t) = p_j(t-1)$ . Price announcements are consequently not perceived as final price decisions (trading prices) until, after having been observed, they are repeated by every firm. (If there is no repetition for a finite  $t$ , Bhaskar assumes that no trade takes place.)

We interpret repetition as acceptance. Every firm can veto or “vote against” the current price vector merely by changing its own price. On the other hand, a firm accepts or “votes for” the current price vector by not changing its own price. Price announcements become price decisions when accepted by every firm in this sense. Moreover, since no firm is committed to its initial price announcement, it is not restrictive to assume that  $p_i(0) = p_i^*$ , where  $p_i^*$  denotes the market price preferred by firm  $i$ .

Consider for simplicity a duopoly and define price-taking behaviour for firm  $i$  by the pricing strategy  $p_i(t+1) = p_j(t)$ ,  $t = 0, 1, \dots$ . Then it is easy to see, when the firms prefer the same price,  $p_1^* = p_2^* = p^*$ , that price-taking strategies (with preferred prices as initial price announcements) constitute a Nash equilibrium with  $p_i = p^*$  as final price decisions.

On the other hand, if firms can agree on playing a non-cooperative game with rules as specified above, they should also be able to agree on price leadership.

### *Price leadership*

Price leadership is not even mentioned in the index to the *Handbook of Industrial Organization* (Schmalensee and Willig 1989, Armstrong and Porter 2007), while it is frequently discussed in traditional literature, including Scherer (1980). In the traditional literature price leadership means that one of the firms sets a price which the other firms match. This is also the definition I use in this paper – but note, for instance, the difference between my analysis of dominant-firm price leadership (in Section 4.3) and the traditional one.

In modern (strictly non-cooperative) literature, however, there is also another interpretation of price leadership, namely that the followers optimize against the price set by the leader (Stackelberg leadership). Of course this means marginal cost pricing when the followers have sufficient capacity. But it also means that a pure strategy equilibrium exists

when capacity constraints prevent marginal cost pricing. The basic idea, as noted for instance by Shubik with Levitan (1980 p. 143) for a duopoly, is that the leader will set the highest price which makes it more profitable for the follower to set a high price (exploiting contingent demand) than to undercut. Shubik with Levitan add, however, that “(w)hile these solutions may be formally correct under static conditions, they are highly unrealistic”.

Which firm will be the leader in a Stackelberg game? Hviid (1990) considers a pricing game where two firms sequentially decide whether or not they want to commit to a price and become a Stackelberg leader. When capacities differ the big firm is indifferent between being a leader and a follower, while the small firm prefers being a follower, suggesting that the big firm becomes the leader (at least when discounting is introduced).

Deneckere and Kovenock (1992) investigate three potential price setting games in a duopoly with one big firm and one small firm, namely a Bertrand game, a Stackelberg game with the big firm as a leader, and a Stackelberg game with the small firm as a leader. Letting  $\pi_i^S$  denote the payoff to firm  $i$  in the simultaneous move game,  $\pi_i^L$  the payoff to firm  $i$  as a leader and  $\pi_i^F$  the payoff to firm  $i$  as a follower in a Stackelberg game, they find that  $\pi_2^S = \pi_2^L = \pi_2^F$  and  $\pi_1^S = \pi_1^L < \pi_1^F$ , provided that the firms' capacities  $k_1$  and  $k_2 > k_1$  are in the range where the simultaneous move game has an equilibrium in mixed strategies. In this case firm 2 (the big firm) is consequently indifferent as to which game it plays, while firm 1 (the small firm) is indifferent between being a leader and moving simultaneously, but strictly prefers to be a follower. It is easy to conclude from this result that the large firm will become a leader in a model where leadership is endogenized. As emphasized by Deneckere and Kovenock (1992, Section 6), however, the firms' ranking of the games depends on the specification of contingent demand.

### *Monopolistic competition*

Another common approach in the price-setting literature is based on firms' *individual demand functions*, usually attributed to heterogeneity (Chamberlin 1962), disequilibrium (Arrow 1959) or uncertainty (Diamond 1971). The concept is often introduced in unorthodox and highly simplified models, as in the seminal paper by Hotelling (1929). But the concept can also be defined in a classical homogeneous market: taking his competitors' prices as given, an oligopolist's contingent demand function (conditional on these prices) is well-defined (Shubik 1959).

On the other hand, the derivation of individual (contingent) demand curves is a complex problem (Shubik 1959 ch. 5). Moreover, as also emphasized by Shubik, imperfections like product differentiation only adds to this complexity. The prevalence of subjectively defined “perceived” or “conjectured” individual demand curves in the literature following Chamberlin (1962), and including Negishi (1960-61, 1979) and Hahn (1978), is therefore not surprising. In general firms can only guess at their individual demand curves.

With a game-theoretic approach even more information is required. Note, for example, that oligopolistic pricing in models with differentiated products, as in Friedman (1983 ch. 3), presupposes not only that every producer has perfect information about his own individual demand curve, where other prices appear only as parameters (equal to their equilibrium values), but also that every producer has perfect information on every firm’s individual demand as an explicit function of every firm’s price.

## Appendix 2. Proofs

Consider an industry with  $\nu$  small firms (with capacity  $k_1$ ) and  $n - \nu$  big firms (with capacity  $k_n$ ), where every firm has the same (constant) marginal cost ( $c_i = c$ ) up to its (fixed) capacity.

Then the profits of a big firm as a function of the market price  $p$  is

$$\begin{aligned}\pi_n(p) &= (p - c)D(p)/n \text{ if } p \geq p^u, \\ \pi_n(p) &= (p - c)\alpha_n(p)D(p) \text{ if } p^k \leq p \leq p^u, \\ \pi_n(p) &= (p - c)k_n \text{ if } p \leq p^k,\end{aligned}$$

$$\text{where } p^u = P(nk_1), \quad p^k = P(\nu k_1 + (n - \nu)k_n) \text{ and } \alpha_n(p) = \frac{1 - \nu k_1 / D(p)}{n - \nu}.$$

We wish to find  $\arg \max \pi_n(p)$  and begin by noting that

$$\frac{\partial \pi_n(p)}{\partial p} = \frac{D(p)}{n - \nu} ((n - \nu)\alpha_n(p) - \varphi(p)) \text{ if } p^k \leq p \leq p^u,$$

$$\text{where } \varphi(p) = \frac{p - c}{p} \eta(p) \text{ and } \eta(p) = -pD'(p)/D(p).$$

Note that  $\varphi(p)$  is increasing in  $p$ , with  $\varphi(c) = 0$  and  $\varphi(p^m) = 1$ . Our assumptions on demand imply that  $\pi_n'(p)$  is decreasing in  $p$ . Hence, whenever there is an interior maximum on

$p^k \leq p \leq p^u$ , it is defined implicitly by the equation  $\varphi(p^o) = (n - \nu)\alpha_n(p^o)$ , or, equivalently, by the equation

$$D(p^o)(1 - \varphi(p^o)) = \nu k_1.$$

Note that  $p^o$  is less than  $p^m$ , independent of  $k_n$  and decreasing in  $k_1$ .

LEMMA 1. Let  $\tilde{p}$  denote  $\arg \max \pi_n(p)$  for  $p^k \leq p \leq p^u$ . Then

$$\begin{aligned} \tilde{p} &= p^u \text{ if } k_1 \geq D(p^b)/n, \\ \tilde{p} &= p^k \text{ if } k_1 \leq D(p^b)/n \text{ and } k_1 \leq k_n \leq k_n^*, \\ \tilde{p} &= p^o \text{ if } k_1 \leq D(p^b)/n \text{ and } k_n \geq k_n^*, \end{aligned}$$

where  $p^b$  and  $k_n^*$  are defined by the equations

$$\begin{aligned} \varphi(p^b) &= 1 - \nu/n, \\ \varphi(p^k(k_n^*)) &= \frac{(n - \nu)k_n^*}{\nu k_1 + (n - \nu)k_n^*}. \end{aligned}$$

Moreover,  $\pi_n(p^k(k_n))$  is increasing in  $k_n$  for  $k_1 \leq k_n \leq k_n^*$  with  $\pi_n(p^k(k_n^*)) = \pi_n(p^o)$ , while  $\pi_n(p^o(k_1))$  is decreasing in  $k_1$ , with  $\pi_n(p^o(D(p^m)/n)) > (p^m - c)D(p^m)/n$  and  $\pi_n(p^o(D(p^b)/n)) < (p^m - c)D(p^m)/n$ .

*Proof.* Since  $\pi'_n(p)$  is decreasing in  $p$ ,  $\tilde{p} = p^u$  if  $\pi'_n(p^u) \geq 0$ , i.e. if  $1 - \nu/n \geq \varphi(p^u)$  or  $p^u \leq p^b$  or  $\nu k_1 \geq D(p^b)$ . Next, suppose that  $k_1 \leq D(p^b)/n$  or, equivalently,  $\pi'_n(p^u) \leq 0$ . Then  $\tilde{p} = p^k$  if  $\pi'_n(p^k) \leq 0$  while  $\tilde{p} = p^o$  if  $\pi'_n(p^k) \geq 0$ . And  $\pi'_n(p^k) \leq 0$  if and only if  $(n - \nu)k_n/K \leq \varphi(p^k)$ , where  $K = \nu k_1 + (n - \nu)k_n$ . Note that

$$(n - \nu)k_n/K = \frac{(n - \nu)k_n}{\nu k_1 + (n - \nu)k_n} = f(k_n)$$

is an increasing function of  $k_n$  with  $f(k_1) = 1 - \nu/n$ . Moreover,

$$\varphi(p^k) = \varphi(P(\nu k_1 + (n - \nu)k_n)) = g(k_n)$$

is a decreasing function of  $k_n$  with  $g(k_1) = \varphi(P(nk_1))$ . It follows that  $f(k_n)$  and  $g(k_n)$  have a unique intersection for  $k_n \geq k_1$  if and only if  $1 - \nu/n \leq \varphi(P(nk_1))$ , which is equivalent to  $P(nk_1) \geq p^b$  or  $nk_1 \leq D(p^b)$ . And then  $f(k_n) \leq g(k_n)$  if and only if  $k_n \leq k_n^*$ , where  $f(k_n^*) = g(k_n^*)$ . It follows that  $\tilde{p} = p^k$  if and only if  $k_n \leq k_n^*$ . Moreover,

$$\pi_n(p^k(k_n)) = (p^k - c)k_n = (P(\nu k_1 + (n - \nu)k_n) - c)k_n, \text{ so that}$$

$$\frac{\partial \pi_n}{\partial k_n} = \frac{n - \nu}{D'(p^k)} k_n + (p^k - c) = \frac{D(p^k)}{D'(p^k)} \left[ \frac{n - \nu}{K} k_n - \varphi(p^k) \right].$$

Since  $D'(p^k) < 0$  it follows that  $\partial \pi_n / \partial k_n \geq 0$  if  $(n - \nu)(k_n / K) - \varphi(p^k) \leq 0$ , or if

$f(k_n) \leq g(k_n)$  or  $k_n \leq k_n^*$ . It follows that  $\pi_n(p^k(k_n))$  is increasing in  $k_n$  for  $k_1 \leq k_n \leq k_n^*$  with  $\pi_n(p^k(k_n^*)) = \pi_n(p^o)$ .

To see that  $\pi_n(p^o(k_1))$  is decreasing in  $k_1$ , note that  $p^o(k_1)$  is decreasing in  $k_1$  and that  $\pi_n(p^o)$  is increasing in  $p^o$  because  $p^o < p^m$  and

$$\pi_n(p^o) = (p^o - c)\alpha_n(p^o)D(p^o) = (p^o - c)D(p^o)\varphi(p^o)/(n - \nu).$$

Moreover, for  $k_1 = D(p^b)/n$  we obtain

$$D(p^o)(1 - \varphi(p^o)) = \nu k_1 = (\nu/n)D(p^b) = (1 - \varphi(p^b))D(p^b)$$

so that  $p^o = p^b$  in this case. And then

$$\pi_n(p^o) = (p^o - c)D(p^o)\varphi(p^o)/(n - \nu) = (p^b - c)D(p^b)/n < (p^m - c)D(p^m)/n.$$

Finally, to see that  $\pi_n(p^o(D(p^m)/n)) > (p^m - c)D(p^m)/n$  we note that

$$\pi_n(p^o(k_1, k_n)) = \pi_n(p^k(k_1, k_n^*)) > \pi_n(p^k(k_1, k_1)),$$

and that if  $k_n = k_1 = D(p^m)/n$  then  $\pi_n(p^k(k_1, k_1)) = (p^m - c)D(p^m)/n$ .

**PROPOSITION 1.** If  $p^u \geq p^m$  or, equivalently,  $k_1 \leq D(p^m)/n$ , then:

$$\arg \max \pi_n(p) = p^k \text{ if } k_1 \leq k_n \leq k_n^*,$$

$$\arg \max \pi_n(p) = p^o \text{ if } k_n \geq k_n^*.$$

*Proof.* If  $p^u \geq p^m$  then obviously  $\arg \max \pi_n(p) = \tilde{p}$  and the rest follows from Lemma 1 since  $nk_1 \leq D(p^m) < D(p^b)$ .

Next we consider the case when  $p^u \leq p^m$  or, equivalently,  $k_1 \geq D(p^m)/n$ . Then the monopolistic option,

$$\pi_n(p^m) = (p^m - c)D(p^m)/n,$$

is always available to a big firm and must be compared to  $\pi_n(\tilde{p})$ . The outcome of this comparison is as follows:

**PROPOSITION 2.** If  $p^u \leq p^m$  or, equivalently,  $k_1 \geq D(p^m)/n$ , then:

$$\arg \max \pi_n(p) = p^m \text{ if } k_1 \geq k_1^*,$$

$$\arg \max \pi_n(p) = p^m \text{ if } k_1 \leq k_1^* \text{ and } k_1 \leq k_n \leq k_n^r,$$

$$\arg \max \pi_n(p) = p^k \text{ if } k_1 \leq k_1^* \text{ and } k_n^r \leq k_n \leq k_n^*,$$

$$\arg \max \pi_n(p) = p^o \text{ if } k_1 \leq k_1^* \text{ and } k_n \geq k_n^*,$$

where  $k_1^*$  and  $k_n^r$  are defined by the equations

$$\pi_n(p^o(k_1^*)) = \pi_n(p^m),$$

$$\pi_n(p^k(k_n^r)) = \pi_n(p^m).$$

*Proof.* If  $nk_1 \geq D(p^b)$  then  $\tilde{p} = p^u$  (according to Lemma 1), and since  $p^u \leq p^m$  (by assumption) it follows that  $\arg \max \pi_n(p) = p^m$  if  $k_1 \geq D(p^b)/n$ . If  $nk_1 \leq D(p^b)$  we have  $\tilde{p} = p^k$  if  $k_1 \leq k_n \leq k_n^*$  and  $\tilde{p} = p^o$  if  $k_n \geq k_n^*$ , according to Lemma 1. Moreover,  $\pi_n(p^o) = \pi_n(p^o(k_1))$  is decreasing in  $k_1$  so that  $\pi_n(p^o) \leq \pi_n(p^m)$  if  $k_1$  is sufficiently large,  $k_1 \geq k_1^*$ , where  $k_1^*$  is determined by the equation  $\pi_n(p^o(k_1^*)) = \pi_n(p^m)$ . Note that  $D(p^m)/n < k_1^* < D(p^b)/n$ , since  $\pi_n(p^o(D(p^m)/n)) > \pi_n(p^m)$  and  $\pi_n(p^o(D(p^b)/n)) < \pi_n(p^m)$  according to Lemma 1. Moreover, if  $k_1^* \leq k_1 \leq D(p^b)/n$  then  $\pi_n(\tilde{p}) = \pi_n(p^k) \leq \pi_n(p^o) \leq \pi_n(p^m)$  if  $k_1 \leq k_n \leq k_n^*$  and  $\pi_n(\tilde{p}) = \pi_n(p^o) \leq \pi_n(p^m)$  if  $k_n \geq k_n^*$ , so that  $\arg \max \pi_n(p) = p^m$  not only for  $k_1 \geq D(p^b)/n$  but for  $k_1 \geq k_1^*$ . On the other hand, if



$k_1 \leq k_1^*$  then  $k_1 \leq D(p^b)/n$  and  $\pi_n(p^o) \geq \pi_n(p^m)$ , so that  $\pi_n(\tilde{p}) = \pi_n(p^o) \geq \pi_n(p^m)$  if  $k_n \geq k_n^*$ , while  $\pi_n(\tilde{p}) = \pi_n(p^k) \geq \pi_n(p^m)$  if  $k_n^r \leq k_n \leq k_n^*$ , where  $k_n^r$  is determined by the equation  $\pi_n(p^k(k_n^r)) = \pi_n(p^m)$ .

We finally derive expressions for  $p^o$  and the critical capacities  $k_n^*$ ,  $k_1^*$  and  $k_n^r$  when the demand function is linear.

LEMMA 2. Suppose that the demand function is linear,  $D(p) = D(c) - b(p - c)$ . Then

$D(p)/D(p^m) = 2 - z$  and  $\varphi(p) = z/(2 - z)$ , where  $z = (p - c)/(p^m - c)$ , and

$$\frac{p^k - c}{p^m - c} = \frac{2D(p^m) - K}{D(p^m)}.$$

*Proof.* It is easy to see that  $p^m - c = D(c)/2b$ ,  $D(p^m) = D(c)/2$  and  $b = D(p^m)/(p^m - c)$ .

Hence 
$$D(p) = 2D(p^m) - \frac{D(p^m)(p - c)}{p^m - c} = D(p^m)(2 - z),$$

and 
$$\varphi(p) = \frac{p - c}{p} \eta(p) = -(p - c)D'(p)/D(p) = (p - c)b/D(p),$$

so that 
$$\varphi(p) = \frac{(p - c)D(p^m)}{(p^m - c)D(p^m)(2 - z)} = \frac{z}{2 - z}.$$

Moreover 
$$K = D(c) - b(p^k - c),$$

so that 
$$p^k - c = \frac{D(c) - K}{b} = \frac{2D(p^m) - K}{D(p^m)/(p^m - c)}.$$

PROPOSITION 3. If the demand function is linear then:

$$\begin{aligned} \frac{p^o - c}{p^m - c} &= 1 - x_1 v/2n, \\ \frac{k_n^*}{D(p^m)/n} &= \frac{1 - x_1 v/2n}{1 - v/n}, \\ \frac{k_1^*}{D(p^m)/n} &= \frac{2(1 - \sqrt{1 - v/n})}{v/n}, \end{aligned}$$

$$\frac{k_n^r}{D(p^m)/n} = \frac{k_n^*}{D(p^m)/n} - \Delta,$$

$$\text{where } x_1 = \frac{k_1}{D(p^m)/n} \text{ and } \Delta = \frac{\sqrt{(1-x_1 v/2n)^2 - (1-v/n)}}{1-v/n}.$$

*Proof.* Using Lemma 2 the equation for  $p^o$  can be written as

$$(2-z) \left( 1 - \frac{z}{2-z} \right) = vk_1/D(p^m)$$

which is solved by  $z = 1 - x_1 v/2n$ .

Secondly, according to Lemma 2,

$$\varphi(p^k) = \frac{2 - \frac{K}{D(p^m)}}{2 - \left( 2 - \frac{K}{D(p^m)} \right)} = \frac{2D(p^m) - K}{K},$$

so that the equation for  $k_n^*$  becomes

$$\frac{2D(p^m) - K}{K} = \frac{(n-v)k_n}{K},$$

or

$$2D(p^m) - vk_1 = 2(n-v)k_n,$$

which is solved by  $k_n^*$ .

Thirdly,

$$\begin{aligned} \pi_n(p^o) &= (p^o - c) \left( (D(p^o) - vk_1) / (n-v) \right) = (\text{definition of } p^o) = \\ &= (p^o - c) D(p^o) \varphi(p^o) / (n-v) = (\text{Lemma 2}) = \\ &= (p^o - c) \frac{p^o - c}{p^m - c} D(p^m) / (n-v) = (\text{expression for } p^o) = \\ &= (p^m - c) D(p^m) (1 - x_1 v/2n)^2 / (n-v), \end{aligned}$$

$$\text{where } x_1 = \frac{k_1}{D(p^m)/n},$$

so that the equation  $\pi_n(p^o(k_1)) = (p^m - c) D(p^m) / n$  becomes

$$(1 - x_1 v/2n)^2 = (n-v)/n,$$

which is solved by  $k_1^*$ .

Finally, the equation  $\pi_n(p^k(k_n)) = \pi_n(p^m)$ , or  $(p^k - c)k_n = (p^m - c)D(p^m)/n$ , becomes

$$\frac{2D(p^m) - K}{D(p^m)} k_n = D(p^m)/n,$$

according to Lemma 2. From this we obtain, with  $x_n = \frac{k_n}{D(p^m)/n}$ ,

$$\frac{2D(p^m) - \nu k_1 - (n - \nu)k_n}{D(p^m)} x_n = 1,$$

$$\left(2 - \frac{\nu}{n} x_1 - \left(1 - \frac{\nu}{n}\right) x_n\right) x_n = 1,$$

$$(1 - \nu/n)x_n^2 - (2 - x_1 \nu/n)x_n + 1 = 0,$$

which is solved by

$$x_n = \frac{2 - x_1 \nu/n}{2(1 - \nu/n)} \pm \sqrt{\frac{(2 - x_1 \nu/n)^2}{4(1 - \nu/n)^2} - \frac{1}{1 - \nu/n}}.$$