Corrective Taxation of a Consumption Externality in the Presence of an Optimal Non-linear Income Tax*

by

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ABSTRACT

This paper is concerned with the problem of combining a non-linear income tax with an indirect externality correcting tax. The analysis is performed in a model economy with two types of individuals and two types of consumption goods. The government wants to redistribute from the more able individuals to the less able, and also to correct for the externality arising from the total consumption of the dirty good. It turns out that the optimal tax structure depends on the complementarity or substitutability between the dirty good and leisure. The second-best externality correcting tax can be interpreted as consisting of a redistributive and an environmental component. Consequently, the dirt tax can be lower or higher than the first-best Pigouvian tax.

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1. Introduction

Economic theory recognizes environmental problems as a misuse of environmental resources due to ill-defined property rights. There is a *Tragedy of the Commons*, where the individual optimal use of a common resource contradicts the social optimal use, and this causes misallocation and externalities. Even though these problems can be taken care of by some kind of social control (social norms), there are several reasons why social norms might not work or why, if they work to begin with, they might be destroyed. Consequently, public or governmental intervention might be necessary.

Public intervention might come through the market mechanism or via a package of regulatory and/ or incentive instruments. However, as Bovenberg & Cnossen remark, each of these intervention measures are imperfect instruments designed by imperfect people to be applied in imperfect markets, the introduction of Pigouvian taxes, in order to internalize the environmental damages, might be a solution to the problem.

In fact, many developed countries of today rely on environmentally motivated taxes, in order to improve the quality of the environment and there has been an ongoing discussion of how these environmental taxes affect the rest of the tax system. According to economic theory the Pigouvian solution results in a *first-best* outcome, if there are no other distortions that affect the allocation of resources in the economy. If the government is not able to set the rest of the tax system in an optimal way, that is, no lump-sum taxes or transfers are available, then the government faces a *second-best* world.

In the optimal tax literature, the second-best problem is seen as a problem of indirect control. The government would like to control the individual consumption decisions, but lacks the relevant information. Consequently, the government tries to control individuals indirectly by using instruments that affect individual behavior. For example, in Stiglitz (1987) and Edwards-Keen-Toumala (1994) the government is unable to observe and tax individual ability, and therefore income is observed and taxed. The available tax instruments in these models are a non-linear income tax and linear commodity taxes. With focus on the self-selection (or incentive compatibility) conditions, Stiglitz determines the marginal income tax rate to be zero on the top of the tax schedule and strictly positive at the bottom. Edwards et al.

show that the introduction of commodity taxes into the model may slacken the self-selection constraint and thereby change the result of zero marginal tax for the highest income in the economy. However, they also show that defining effective or total marginal tax gives back the results from the Stiglitz' analysis. Consequently, the zero on the top result depends on, whether or not, both indirect and direct taxes are included in the definition of the marginal tax.

Since the pioneering work by Sandmo (1975), there have been many studies of optimal environmental taxes in a second-best setting. The findings of Sandmo is that, the Pigouvian taxation principle is valid in a general model of optimal indirect taxation and, that this is true whether equity considerations are accounted for or not. Further, Sandmo concludes that the optimal second-best tax on the externality generating good is a weighted average of a Ramsey and a Pigouvian term.

In many of the later studies, within this field, the focus is on the so called double-dividend hypothesis. According to this hypothesis environmental taxes will not only improve the environment, but also make it possible to reduce other distortionary taxes, and thereby improve the efficiency of the overall tax system. This issue has been surveyed by Goulder (1995).

Two recent contributions on this subject is Bovenberg-de Mooij (1994) and Bovenberg-van der Ploeg (1994). Among other things, they find that in a tax system with a linear labor tax and an additional tax on the dirty good the second-best optimal tax on the dirty good has to be below the first-best Pigouvian tax. Thereby, the dirt tax does not cover the total marginal environmental damage.

The aim of this work is to se if an environmentally harmful consumption externality affects the conventional results of the marginal income tax in the optimal tax literature and, whether or not this is the case, if it makes any difference if the government have access to a commodity tax besides the income tax. In other words, we try to combine the problem of optimal non-linear income taxation with that of optimal corrective taxation. In order to do so, we introduce an external consumption effect into the Stiglitz' model. From the Edwards et al. study we know that the commodity tax has an important role in weakening the self-selection constraint, but the question here is if it also plays any role in correcting for the externality. (I.

e. does it matter if the government is correcting for the externality via income taxes or commodity taxes?)

The paper is organized as follows. Section 2 outlines the model, which combines the model in the first sections of the Edwards et al. study with a consumption externality. In section 3 the marginal income tax is analyzed, within the presence of the externality, and the results are compared to the traditional results regarding the marginal income tax from the Stiglitz' analysis. In section 4 the analysis from section 3 is extended by introducing a commodity tax. The findings from this section are compared to the results from the Edwards et al. paper and the results from the double-dividend literature, where conditions under which the dirt tax will be different from the first-best Pigouvian tax are examined. Section 5 just briefly touches the externality implications for the total or effective marginal tax rate defined in the Edwards et al. study, and section 6 summarizes and concludes.

2. The Model

The consumers

Consider a single period model for a closed economy. There are two types of individuals, N_1 of type 1 and N_2 of type 2. The individuals are identical in all respects, except that type 2 are more productive than type 1. That is, type 2 have a higher pre-tax wage, $w^2 > w^1$. Each individual spends L hours at work at the given wage rate, w^h, and this results in a before-tax income:

(1) $$y^h = w^h L^h \qquad h = 1,2 \qquad (h \text{ indicates individual type})$$

$T(y)$ denotes the income tax schedule and after-tax (or disposable) income becomes:

(2) $$B^h = y^h - T(y^h) = w^h L^h - T(w^h L^h) \qquad h = 1,2$$

There are two types of normal consumption goods, x and z. Good x is clean, i.e. the consumption of good x causes no externality. The consumption of good z, called the dirty good, creates a negative externality, *E*. The externality depends on the total consumption of the dirty good, which is a common assumption in the case of *atmospheric* externalities:

(3) $$E = N_1 z^1 + N_2 z^2$$

An individual of type *h* receives utility from the consumption of goods and disutility from hours of work, *L*, and from the externality caused by, *z*. The preferences are described by a well-behaved utility function:

(4) $$U[F(L^h, x^h, z^h), E]$$

$$\text{where } U_F \frac{\partial U}{\partial x} > 0,\ U_F \frac{\partial U}{\partial z} > 0,\ U_F \frac{\partial U}{\partial L} < 0,\ \frac{\partial U}{\partial E} < 0 \text{ and } \frac{\partial^2 U}{\partial E^2} < 0 .$$

For analytical convenience, it is assumed that *E* enters preferences in either a weakly, or in an additively, separable way. Further, each individual considers himself being very small compared to the economy and therefore ignores his own contribution to the externality, i.e. $\partial E / \partial z^h = 0$. This assumption is in conformity with the usual competitive assumption.

Before proceeding with the individual optimization problem, it is convenient to rewrite the individual utility function (4) in terms of before-tax income, *y*.

(5) $$U[F(L^h w^h/w^h, x^h, z^h), E] = U[F(y^h/w^h, x^h, z^h), E]$$

We follow Christiansen (1983), and divide the consumer optimization problem into two stages. At the first stage the disposable income is optimally allocated over the consumption goods holding the labor supply constant. That is, (5) is maximized with respect to *x* and *z* subject to the individual after-tax budgetconstraint:

(6) $$x^h + p_z z^h = B^h \qquad \text{where } p_z = 1 + t_z$$

p_z is the consumer price, and t_z denotes the tax on good z. The producer prices are normalized to unity, a common arbitrary normalization in this kind of optimum tax models, whereby the tax on good x is set equal to zero.

The individual optimizing problem gives demand functions conditional upon the value of y.

(7) $$x^h(p_z, B^h, y^h/w^h)$$
$$z^h(p_z, B^h, y^h/w^h)$$

According to (7), demand is a function of the consumer prices, the wage rate and after-tax and before-tax income, but, because of the separability assumption, demand is not affected by the level of the externality. Consequently, there is no feedback effect caused by the externality.

$\partial x/\partial L = (\partial x/\partial y)w$ is the marginal effect on the demand for good x of an increase in work effort when after-tax income, B, is held constant. Later on, we will abstract from the pre-tax wage and refer to this marginal effect as the substitution/ complementarity relationship between leisure and the good in question.

The indirect utility function which corresponds to the conditional demand functions becomes:

(8) $$V[G^h(p_z, B^h, y^h/w^h), E] \equiv U[F(y^h/w^h, x^h(p_z, B^h, y^h/w^h), z^h(p_z, B^h, y^h/w^h)), E]$$

In what follows w^h is left out and V^h indicates the individual type, i.e. (8) is written as:
$V^h[G^h(p_z, B^h, y^h), E]$

At the second stage of the consumer's problem, hours at work (or pre-tax income since we have rewritten the problem in terms of y) are chosen given the connection between before-tax and after-tax income:

(9) $$\max_{B^h, y^h} V^h[G^h(p_z, B^h, y^h), E]$$
$$s.t\ B^h = y^h - T(y^h)$$
$$\max V^h[G^h(p_z, y^h - T(y^h), y^h), E]$$

y^h

The first order conditions is:[1]

$$(10) \qquad \left(-\frac{V_y^h}{V_B^h}\right) = 1 - T'(y^h),$$

where $T'(y^h)$ is the marginal income tax rate. There is one further assumption to be made before getting on to the government's problem:

$$(11) \qquad \frac{\partial}{\partial w}\left(-\frac{V_y}{V_B}\right) < 0$$

so that individuals with higher ability (wage) will have flatter indifference curves in $\{B, y\}$ space.[2]

The production sector

The production side of the economy is characterized by a linear production structure, i.e. linear in labor supply, and labor supply is normalized so that the wage-rate becomes equal to each person's productivity. Further, it is assumed that the externality has no influence on production.

The government

The government wants to correct for the externality and to redistribute from the more able individuals to the less able. However, the government cannot observe the individual productivity (or equivalently wage). This informational limitation excludes lump sum taxes conditioned on individual wage. Therefore, the government has to rely on what it can observe, namely before-tax income, y.

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[1] More correctly, the F.O.C becomes $(-G_y/G_B) = 1 - T'(y)$, but in what follows, $V_G G_y$ and $V_G G_B$ will be denoted V_y and V_B.
[2] A sufficient condition for (11) hold is that total consumption, B, is normal.

Accordingly, the government's problem is to choose a tax structure, i.e. a non-linear income tax ($T(y)$) and a commodity tax (t_z), to achieve a Pareto efficient allocation. For this purpose the government will select after-tax and before-tax income (B and y) and the consumer price of good z (p_z). The individuals will have to choose between the B and y packages offered by the government. Since there are only two groups of individuals, it is enough to characterize only two points on the tax schedule.

There is also need for a self-selection constraint, since the individuals with the higher wage (type 2) might want to imitate the low ability individuals. The self-selection constraints imply that each type prefers the $\{B, y\}$ set meant for them, to that meant for the other type. However, the only case considered here, is redistribution from the high ability group to the low ability one, and therefore, only one self-selection constraint will be binding. Assumption (11) makes sure that both self-selection constraints cannot bind at the same time.

Until now we have assumed that the externality enters preferences in a weakly separable way. The individual optimizing problem will not be affected if we instead assume additive separability, since demand is independent of the externality under either separability assumption. This is not true for the government's problem, since there is a difference between the two separability assumptions regarding the distributional implications of the externality. In appendix A we present the governments problem when utility is weakly separable in the externality, while the presentation here assumes additive separability. The reason for this is that the presentation is less space consuming under the additive assumption. However, the analysis of the marginal income tax, section 3, the commodity tax, section 4, and the effective marginal tax, section 5, covers both approaches.

The formal description of the set of Pareto efficient allocations is characterized by the solution to the government's optimizing problem:

$$(12) \qquad \underset{y^1, B^1, y^2, B^2, p_z}{Max} \quad V^1 = G^1(p_z, B^1, y^1) + m(E)$$

subject to

$$(13) \qquad V^2 \geq \bar{V}^2 \qquad \text{where } V^2 = G^2(p_z, B^2, y^2) + m(E)$$

$$(14) \qquad V^2 \geq V^{im} \qquad \text{where } V^{im} = G^2(p_z, B^1, y^1) + m(E)$$

$$(15) \qquad N_1(y^1 - B^1) + N_2(y^2 - B^2) + t_z(N_1 z^1 + N_2 z^2) \geq R$$

The first constraint ensures that the high ability type 2 achieves a utility level of at least $\bar{V}^2$. The second constraint is the self-selection constraint, which requires that type 2 cannot obtain a higher utility level by imitating the type 1 individuals (imitation is denoted by im). The last constraint is the government's budget constraint, where R is the exogenously determined revenue requirement. The Lagrange multipliers that correspond to the restrictions (12) - (14) are: δ, λ and μ. Except for λ, these Lagrange multipliers are always assumed to be strictly positive, since we only study solutions where the constraints are strictly binding.

The first order conditions for the government's problem may be written:

$$(16) \qquad V_B^1 = \lambda V_B^{im} - \frac{\partial E}{\partial z^1}\frac{\partial z^1}{\partial B^1}(V_E + \delta V_E) + \mu N_1\left(1 - t_z \frac{\partial z^1}{\partial B^1}\right)$$

$$(17) \qquad V_y^1 = \lambda V_y^{im} - \frac{\partial E}{\partial z^1}\frac{\partial z^1}{\partial y^1}(V_E + \delta V_E) - \mu N_1\left(1 + t_z \frac{\partial z^1}{\partial y^1}\right)$$

$$(18) \qquad (\delta + \lambda)V_B^2 = -\frac{\partial E}{\partial z^2}\frac{\partial z^2}{\partial B^2}(V_E + \delta V_E) + \mu N_2\left(1 - t_z \frac{\partial z^2}{\partial B^2}\right)$$

$$(19) \qquad (\delta + \lambda)V_y^2 = -\frac{\partial E}{\partial z^2}\frac{\partial z^2}{\partial y^2}(V_E + \delta V_E) - \mu N_2\left(1 + t_z \frac{\partial z^2}{\partial y^2}\right)$$

(20) $$V_{p_z}^{1} + (\delta + \lambda)V_{p_z}^{2} - \lambda V_{p_z}^{im} + \left(\frac{\partial E}{\partial z^{1}} \frac{\partial z^{1}}{\partial p_z} + \frac{\partial E}{\partial z^{1}} \frac{\partial z^{1}}{\partial p_z} \right)(V_E + \delta V_E)$$
$$+ \mu \left(N_1 z^{1} + N_2 z^{2} + t_z (N_1 \frac{\partial z^{1}}{\partial p_z} + N_2 \frac{\partial z^{2}}{\partial p_z}) \right) = 0$$

(See also appendix A.)

3. Properties of the Income Tax

As already mentioned, the conventional results in this kind of analysis, Stiglitz (1987), is that the marginal income tax rate on the high ability individual is zero and that for the low ability type is positive. Edwards et al. show that the introduction of commodity taxes changes the result of zero marginal income tax on the top in a formal sense. Their conclusions is that commodity taxes should be used only if they discourage mimicking and that the total marginal tax paid should be set to zero on the top, and strictly positive at the bottom[3].

One of the purposes of this paper is to see if and how the introduction of a consumption externality changes the conventional results regarding the marginal income tax rate. The analyses are performed under various separability assumptions considering the individual preferences over the externality.

No commodity tax, utility is additively separable in the externality

The first order conditions (16) - (20) above, incorporate the consumption externality, a binding self-selection constraint and a commodity tax on the dirty good. To begin with we assume that the government does not have access to the commodity tax. That is, we make no use of equation (20) and in the remaining equations, t_z is set equal to zero. This makes it possible to see how the consumption externality affects the marginal income tax.

[3] Total marginal tax, or effective marginal tax, is defined in section 5.

Recalling from equation (10) that $T'(y) = 1 + V_y / V_B$, i.e. the marginal tax rate is one plus the MRS between labor and income, or the total consumption expenditure. Consequently, it is sufficient to analyze this MRS, for each individual type, in order to determine the marginal income tax and the effect upon it due to the consumption externality.

Dividing equation (19) by equation (18) with $t_z = 0$ gives the MRS for the high ability type.

$$\frac{V_y^2}{V_B^2} = \frac{-\mu - \frac{\partial z^2}{\partial y^2}\left(V_E + \delta V_E\right)}{\mu - \frac{\partial z^2}{\partial B^2}\left(V_E + \delta V_E\right)} \tag{21}$$

By dividing equation (17) by (16) we get the corresponding MRS for the low ability type:

$$\frac{V_y^1}{V_B^1} = \frac{\lambda V_y^{im} - N_1 \frac{\partial z^1}{\partial y^1}\left(V_E + \delta V_E\right) - \mu N_1}{\lambda V_B^{im} - N_1 \frac{\partial z^1}{\partial B^1}\left(V_E + \delta V_E\right) + \mu N_1} \tag{22}$$

Both equation (21) and (22) indicate that the marginal income tax rate depends on the income effect, the substitution effect between leisure and the dirty good and the externality, represented by the term $(V_E + \delta V_E)$. V_E is negative and denotes the individual marginal disutility from the externality, which is the same for all individuals when utility is additively separable in the externality, whereby we can exclude the superscripts. The value of δ reflects how the optimal value of V^1 must change when $\overline{V}^2$ changes and can be interpreted as the relative social welfare weight given to the second type. Accordingly, the term $(V_E + \delta V_E)$ characterizes the weighted marginal social damage, MSD, due to the externality, which of course is negative.

Because of the assumption of increasing marginal disutility in the externality, the individual marginal disutility and thereby the MSD will be greater if the demand of the dirty good somehow increases in the economy. Since the individuals consumption of *z* are perfect substitutes as far as the externality is concerned, it is important to analyze what will influence the demand of z in order to se the magnitude of a change in the externality. However, this is an empirical question which is not possible to answer in this theoretical framework. Consequently, presumptions about the sign of the income effect and the substitution effect between leisure and the dirty good is necessary in order to perform the analysis here.

The most essential assumption is that z is a non-inferior good, i.e. the income effect on the demand of z is non-negative, $\partial z/\partial B \geq 0$. This is a very convenient assumption to do, since both the clean and the dirty good cannot be inferior at the same time according to our model. Further, we assume that z and leisure are, either some kind of substitutes, $\partial z/\partial y > 0$ (i.e. labor is complement to z), or complements, $\partial z/\partial y < 0$.

With these assumptions in our mind, equation (21) tells us that the marginal income tax rate for the high ability individual is positive, whenever leisure and the dirty good are substitutes. I. e. the numerator in (21) will be strictly greater than minus one, while the denominator will be greater than one. This means that, as long as more work gives rise to a greater consumption of the externality producing good, the marginal income tax rate for the high ability type is not only different from the tax when no externality is present in the model, it is also greater.

If, on the contrary, leisure and the externality creating good are complements, the marginal income tax rate at the top can be negative, zero or positive depending on which of the income and substitution effects who are greatest in absolute magnitude. This result is shown in table 1.a).

Table 1.a): The Marginal income tax rate for the high ability individual when leisure and the dirty good are complements, i. e. $\partial z^2 / \partial y^2 < 0$.

$\frac{\partial z^2}{\partial B^2} = 0$	$T'(y^2) < 0$
$\frac{\partial z^2}{\partial B^2} > 0$	$\left\|\frac{\partial z^2}{\partial y^2}\right\| > \left\|\frac{\partial z^2}{\partial B^2}\right\| \Rightarrow \|numerator\| > \|denominator\| \Rightarrow T'(y^2) < 0$ $\left\|\frac{\partial z^2}{\partial y^2}\right\| < \left\|\frac{\partial z^2}{\partial B^2}\right\| \Rightarrow \|numerator\| < \|denominator\| \Rightarrow T'(y^2) > 0$ $\left\|\frac{\partial z^2}{\partial y^2}\right\| = \left\|\frac{\partial z^2}{\partial B^2}\right\| \Rightarrow \|numerator\| = \|denominator\| \Rightarrow T'(y^2) = 0$

In order to determine the marginal income tax rate for the low ability person we need to transform equation (22). (See Appendix B)

(23) $$\frac{V_y^1}{V_B^1} > \frac{-\mu - \frac{\partial z^1}{\partial y^1}\left(V_E + \delta V_E\right)}{\mu - \frac{\partial z^1}{\partial B^1}\left(V_E + \delta V_E\right)}$$

From inequality (23) we se that the marginal tax rate for the low ability individual is positive, as for the no externality case, as long as leisure is a substitute for the dirty good. The result is in line with that for the high ability individual, since the externality introduces a Pigouvian term into the tax for both ability types. It is therefore to be expected, that the sign of the tax changes in the same direction as for the high ability individual, when we reverse the case, by letting the dirty good become a complement for leisure. This is also shown in table 1.b).

Table 1.b): The Marginal income tax rate for the low ability individual when leisure and the dirty good are complements, i. e. $\partial z^1 / \partial y^1 < 0$.

$\frac{\partial z^1}{\partial B^1} = 0$	$T'(y^1) \gtreqless 0$
$\frac{\partial z^1}{\partial B^1} > 0$	$\left\|\frac{\partial z^1}{\partial y^1}\right\| < \left\|\frac{\partial z^1}{\partial B^1}\right\| \Rightarrow T'(y^1) > 0$ $\left\|\frac{\partial z^1}{\partial y^1}\right\| = \left\|\frac{\partial z^1}{\partial B^1}\right\| \Rightarrow T'(y^1) > 0$ $\left\|\frac{\partial z^1}{\partial y^1}\right\| > \left\|\frac{\partial z^1}{\partial B^1}\right\| \Rightarrow T'(y^1) \gtreqless 0$

So far we have concluded that the presence of a consumption externality clearly influences the marginal income tax rate for both types of individuals. In addition, the externality (or more correctly, the MSD due to the externality) affects the two tax rates in the same direction, via the substitution/ complementarity effect between leisure and the dirty good. Since these effects are off the opposite sign, the effect on the marginal income tax depends on which case we assume.

When the dirty good is a substitute for leisure, both individual types have to pay a positive income tax at the margin. The income tax has some of the properties of a Pigouvian tax, and from an environmental point of view mimicking is good, since, in this case, type 2 consumes more of the dirty good than the mimicker. Consequently, the government is taxing the high ability type at a positive rate, at the margin, in order to depress the consumption of the dirty good and thereby protect the environment. With a complementary relationship between the dirty good and leisure, type 1 is the larger consumer of the good in question and the size of the marginal income tax depends on the size of the substitution effect relative to the income effect. This implies, that there are cases when the tax at the margin, from an environmental point of view, should be negative for the high ability individual. Technically this occurs when the dirty good becomes a very strong complement for leisure, i.e. $|\partial z/\partial y| > |\partial z/\partial B|$ (see table 1.a)).

From table 1.a) and table 1.b), it is also evident that the marginal income tax is positive and greater for the high ability type, compared to the no externality case, and positive for the low ability one, when the consumption effect on the dirty good due to more work is small in absolute value compared to the income effect.

It is evident from this section that the marginal income tax has not only a redistributive purpose in the presence of a consumption externality, it has also a corrective aim.

Utility is weakly separable in the externality

When utility is weakly separable in the externality the analysis of the marginal income tax becomes a bit more complicated. (Appendix A gives a formal description of the government's problem under the weak separability assumption.) The MRS between labor and total consumption expenditure for the low ability individual, equation (21), takes the form:

$$(24) \qquad \frac{V_y^1}{V_B^1} = \frac{\lambda V_y^{im} + N_1 \frac{\partial z^1}{\partial y^1}\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] - N_1\mu}{\lambda V_B^{im} + N_1 \frac{\partial z^1}{\partial B^1}\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] + N_1\mu}$$

The corresponding MRS for the high ability type, equation (22), becomes:

$$(25) \qquad \frac{V_y^2}{V_B^2} = \frac{\frac{\partial z^2}{\partial y^2}\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] - \mu}{\frac{\partial z^2}{\partial B^2}\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] + \mu}$$

As for the "additive case", equation (24) and (25) indicate that the marginal tax rate depends on the assumptions about the income effect of the dirty good and the substitutability/ complementability between the dirty good and leisure. The difference compared to the preceding section is captured in the expression:

(26) $$[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)]$$

Expression (26) consists of two parts, and both originate from the introduction of the externality into the model. The second part is the, already familiar, MSD due to the externality, with the exception that the individuals marginal disutilities no longer are equal. This inequality between the individuals experience of the externality and the first part of (26) arises from the changed separability assumption.

The first part in expression (26), $\lambda(V_E^{im} - V_E^2)$, indicates that there is a difference in how the mimicking type 2 and the actual type 2 experience the externality, and that this difference should be reflected in the marginal tax rate. To be able to determine the sign of the tax, we therefore need to know the sign of expression (26) and of the $\lambda(V_E^{im} - V_E^2)$ -term. That is, we need to know the size of V_E^{im} relative V_E^2. However, it is not possible to determine what would make the mimicker and the actual type 2 to experience different marginal disutilities due to the externality, since this is an empirical question.

Nevertheless, if the high ability individual suffers from less marginal disutility, due to the externality, when imitating the low ability individual compared to when imitation does not occur, then $V_E^{im} > V_E^2$ and the $\lambda(V_E^{im} - V_E^2)$ -term and expression (26) becomes positive. If, on the other side, the high ability type experiences greater disutility when imitating, i.e. $V_E^{im} < V_E^2$, the $\lambda(V_E^{im} - V_E^2)$ -term becomes negative. In this latter case, we need to know if the $\lambda(V_E^{im} - V_E^2)$ -term exceeds or is less than the MSD-term, in order to establish the sign of (26) and thereby the sign of the marginal income tax.

Table 2a) and 2b) present the sign of the marginal income tax for the high ability person under the two assumptions about expression (26). In table 2a) leisure and the dirty good are substitutes, while in table 2b) they are complements.

Table 2a): The Marginal income tax for the high ability individuals when z and leisure are substitutes, i. e. $\partial z^2 / \partial y^2 > 0$.

	$\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] > 0$	$\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] < 0$
$\frac{\partial z^2}{\partial B^2} = 0$	$T'(y^2) > 0$	$T'(y^2) < 0$
$\frac{\partial z^2}{\partial B^2} > 0$	$T'(y^2) > 0$	$T'(y^2) < 0$

Table 2b): The Marginal income tax for the high ability individuals when z and leisure are complements, i. e. $\partial z^2 / \partial y^2 < 0$.

	$\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] > 0$	$\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] < 0$
$\frac{\partial z^2}{\partial B^2} = 0$	$T'(y^2) < 0$	$T'(y^2) > 0$
$\frac{\partial z^2}{\partial B^2} > 0$	$\left\lvert\frac{\partial z^2}{\partial y^2}\right\rvert > \left\lvert\frac{\partial z^2}{\partial B^2}\right\rvert \Rightarrow T'(y^2) < 0$ $\left\lvert\frac{\partial z^2}{\partial y^2}\right\rvert < \left\lvert\frac{\partial z^2}{\partial B^2}\right\rvert \Rightarrow T'(y^2) > 0$ $\left\lvert\frac{\partial z^2}{\partial y^2}\right\rvert = \left\lvert\frac{\partial z^2}{\partial B^2}\right\rvert \Rightarrow T'(y^2) = 0$	$\left\lvert\frac{\partial z^2}{\partial y^2}\right\rvert > \left\lvert\frac{\partial z^2}{\partial B^2}\right\rvert \Rightarrow T'(y^2) > 0$ $\left\lvert\frac{\partial z^2}{\partial y^2}\right\rvert < \left\lvert\frac{\partial z^2}{\partial B^2}\right\rvert \Rightarrow T'(y^2) < 0$ $\left\lvert\frac{\partial z^2}{\partial y^2}\right\rvert = \left\lvert\frac{\partial z^2}{\partial B^2}\right\rvert \Rightarrow T'(y^2) = 0$

By transforming equation (24) into inequality (27), see Appendix B, we can derive the marginal income tax for the low ability person.

$$(27) \qquad \frac{V_y^1}{V_B^1} > \frac{\frac{\partial z^1}{\partial y^1}\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] - \mu}{\frac{\partial z^1}{\partial B^1}\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] + \mu}$$

The results of this exercise are given in table 2c) and 2d). Table 2c) assumes that leisure is a substitute for the dirty good and table 2d) assumes the opposite case.

Table 2.c): The Marginal income tax rate for the low ability individual when z and leisure are substitutes, i. e. $\partial z^1 / \partial y^1 > 0$.

	$\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] > 0$	$\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] < 0$
$\frac{\partial z^1}{\partial B^1} = 0$	$T'(y^1) > 0$	$T'(y^1) \gtreqless 0$
$\frac{\partial z^1}{\partial B^1} > 0$	$T'(y^1) > 0$	$T'(y^1) \gtreqless 0$

Table 2.d): The Marginal income tax rate for the low ability individual when z and leisure are complements, i. e. $\partial z^1 / \partial y^1 < 0$.

	$\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] > 0$	$\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] < 0$
$\frac{\partial z^1}{\partial B^1} = 0$	$T'(y^1) \gtreqless 0$	$T'(y^1) > 0$
$\frac{\partial z^1}{\partial B^1} > 0$	$\left\|\frac{\partial z^1}{\partial y^1}\right\| > \left\|\frac{\partial z^1}{\partial B^1}\right\| \Rightarrow T'(y^1) \gtreqless 0$ $\left\|\frac{\partial z^1}{\partial y^1}\right\| < \left\|\frac{\partial z^1}{\partial B^1}\right\| \Rightarrow T'(y^1) > 0$ $\left\|\frac{\partial z^1}{\partial y^1}\right\| = \left\|\frac{\partial z^1}{\partial B^1}\right\| \Rightarrow T'(y^1) > 0$	$\left\|\frac{\partial z^1}{\partial y^1}\right\| > \left\|\frac{\partial z^1}{\partial B^1}\right\| \Rightarrow T'(y^1) > 0$ $\left\|\frac{\partial z^1}{\partial y^1}\right\| < \left\|\frac{\partial z^1}{\partial B^1}\right\| \Rightarrow T'(y^1) \gtreqless 0$ $\left\|\frac{\partial z^1}{\partial y^1}\right\| = \left\|\frac{\partial z^1}{\partial B^1}\right\| \Rightarrow T'(y^1) > 0$

It is evident from table 2a) and 2c) that the marginal income tax will be positive for both individual types, if leisure and the dirty good are substitutes and if the mimicking type 2 experiences less disutility from the externality than the actual type 2. Since the $\lambda(V_E^{im} - V_E^2)$ -term amplifies the MSD-term in this case, the marginal tax will be greater, for both types, than under the additive separability assumption. However, if the mimicker suffers from more disutility due to the externality than the actual type 2, the tax will be lower than in the "additive case", since the $\lambda(V_E^{im} - V_E^2)$ -term abates the MSD-term under these circumstances. Nevertheless, as long as the $\lambda(V_E^{im} - V_E^2)$ -term does not exceed the MSD-term in absolute value, expression (26) and thereby the marginal income tax will be positive. The reason for this is, as for the case where utility is additively separable in the externality, that the introduction of a good whose consumption is a complement for labor and also produces an externality, forces the government to raise the marginal tax, compared to the no externality case, in order to internalize the environmental damage. If the government only taxed the high ability individual at a higher rate, mimicking would be encouraged. Consequently, the government taxes both types at a higher rate in order to accomplish the wanted redistribution and to correct for the externality.

Moreover, if the $\lambda(V_E^{im} - V_E^2)$ -term exceeds the MSD-term in absolute value, expression (26) becomes negative and so does the marginal income tax for the high ability type, while the tax for the low ability one becomes indeterminable.

When the dirty good and leisure are complements, see table 2b) and 2d), imitation implies a greater consumption of *z*. If the mimicker then suffers less from environmental damage than the actual type 2, the marginal tax becomes negative for the high ability type and indeterminable for the low ability individual, if the income effect is sufficiently small. Otherwise the tax will be positive for both types. However, if expression (26) becomes negative, the mimicker suffers more from environmental damage than the actual type 2, and the marginal tax becomes positive for both types, if the income effect is small enough. An income effect that exceeds the effect due to the complementary relationship between leisure and *z*, makes the tax negative for the high ability type and indeterminable for the low ability one.

Accordingly, when utility is weakly separable in the externality the two purposes of the marginal income tax and the difficulty in separating them, become more evident. This is because the externality, in itself, introduces distributional implications into the marginal tax in addition to the distributional intent the government might have.

No binding self-selection constraint

If none of the self-selection constraints binds, the marginal income tax becomes zero for both individual types in the Stiglitz' model. The reason for this is that there is no longer any need for the government to distort the individuals decisions when constructing the optimal tax schedule.

When there is an externality present in the model this result no longer holds since the tax also needs to reflect the MSD descending from the externality. However, the interesting thing to note here is that the two separability assumptions regarding the individual utility functions lead to almost the same expressions of the marginal tax. This follows from the fact that, when imitation does not occur expression (26) reduces to the MSD-term. Although, there is a difference in the MSD-term under the two separability cases. Weak separability indicates that the individual marginal disutilities can be different, which implies that the distributional implications of the externality remain.

Anyhow, in what follows, no binding self-selection constraint means that we only perform the analysis under the weak separability assumption. The MRS for the two types when the self-selection constraint does not bind reduces to:

$$(28) \qquad \frac{V_y^h}{V_B^h} = \frac{-\mu - \frac{\partial z^h}{\partial y^h}\left(V_E^1 + \delta V_E^2\right)}{\mu - \frac{\partial z^h}{\partial B^h}\left(V_E^1 + \delta V_E^2\right)} \qquad h = 1,2$$

The *h* superscripts indicate that the tax expression, although similar for the two types, is evaluated at different points. Since the $\lambda(V_E^{im} - V_E^2)$ -term has disappeared and the MSD term is clearly negative, it is possible to determine the sign of the tax for both individual types unambiguously.

Both types will face a positive tax if leisure and the dirty good are substitutes, and if leisure and the dirty good are complements and the substitution effect is less than the income effect in absolute value. Further the tax, for both types, becomes negative or zero if the substitution effect is greater than or equal to the income effect in absolute value, when there is a complementary relationship between leisure and the dirty good.

One interesting fact that has not yet been mentioned is that it is possible to interpret the MSD in terms of the externality impact on the low ability person. Then, V_E^1 indicates the direct utility effect, while δV_E^2 is the indirect effect, or the cost of giving person 2 a certain utility level, since δ reflects how V_E^1 must change when $\bar{V}^2$ changes.

Income tax and commodity tax

Even though results regarding the commodity tax will not be presented until section 4, it can be valuable to inquire if and how the commodity tax affects the marginal income tax already in this section.

When the self-selection constraint does not bind, the commodity tax will mirror the MSD due to the externality and the marginal income tax no longer needs to reflect, nor be affected by, the externality. Consequently, the marginal income tax becomes zero for both individual types.

If, on the other side, the self-selection constraint is binding, and utility is weakly separable in the externality, the MRS for the high ability type, in the presence of a commodity tax becomes: (Divide equation (A.7) by equation (A.8), in Appendix A.)[4]

(29) $$\frac{V_y^2}{V_B^2} = \frac{N_2 \frac{\partial z^2}{\partial y^2}\left[\lambda(V_E^m - V_E^2) - (V_E^1 + \delta V_E^2)\right] - \mu N_2 (1 + t_z^* \frac{\partial z^2}{\partial y^2})}{N_2 \frac{\partial z^2}{\partial B^2}\left[\lambda(V_E^m - V_E^2) - (V_E^1 + \delta V_E^2)\right] + \mu N_2 (1 - t_z^* \frac{\partial z^2}{\partial B^2})}$$

and the corresponding MRS for the low ability type:

(30) $$\frac{V_y^1}{V_B^1} = \frac{\lambda V_y^m + N_1 \frac{\partial z^1}{\partial y^1}\left[\lambda(V_E^m - V_E^2) - (V_E^1 + \delta V_E^2)\right] - \mu N_1 (1 + t_z^* \frac{\partial z^1}{\partial y^1})}{\lambda V_B^m + N_1 \frac{\partial z^1}{\partial B^1}\left[\lambda(V_E^m - V_E^2) - (V_E^1 + \delta V_E^2)\right] + \mu N_1 (1 - t_z^* \frac{\partial z^1}{\partial B^1})}$$

By substituting the formula for the commodity tax (equation (40) in chapter 4) into equation (29) we get, after some manipulation:

(31) $$\frac{V_y^2}{V_B^2} = \frac{-1 - t_z^s \frac{\partial z^2}{\partial y^2}}{1 - t_z^s \frac{\partial z^2}{\partial B^2}}$$

[4] The results concerning the marginal income tax, in the presence of a commodity tax, are the same whether weak or additive separability regarding the externality is assumed.

In section 4 we determine the sign of t_z^s to depend upon the sign of the substitutability/ complementarity relationship between the dirty good and leisure. It is shown that if the dirty good is a substitute for leisure, then $t_z^s < 0$, and if the dirty good and leisure are complements, $t_z^s > 0$. Consequently, as long as leisure and the dirty good are substitutes the marginal income tax for the high ability individual will be positive. This will also be the case when leisure and the dirty good are complements, as long as the income effect is sufficiently small.

By performing a Stiglitz' transformation on equation (30), see appendix B, and substituting for the commodity tax we get inequality (32):

$$(32) \qquad \frac{V_y^1}{V_B^1} > \frac{-1 - t_z^s \dfrac{\partial z^1}{\partial y^1}}{1 - t_z^s \dfrac{\partial z^1}{\partial B^1}}$$

From inequality (32) it is evident that the results regarding the marginal income tax for the low ability individual are in accordance with those for the high ability person. I. e. the marginal income tax becomes positive for both the high and low ability types as long as leisure and the dirty good are substitutes. This result also holds when more work reduces the consumption of the dirty good, if the income effect is sufficiently small. However, with a large enough income effect the tax becomes negative for the high ability type and indeterminable for the low ability one.

It is clear from this section that the introduction of a commodity tax on the dirty good removes the Pigouvian properties, as well as the redistributional implications due to the externality, from the income to the commodity tax.

4. The Commodity Tax/ Externality Tax

We have seen above that the externality clearly affects the marginal income tax when the government does not have access to a commodity tax. In Edwards et al. the commodity tax plays an important role in weakening the self-selection constraint. This is by taxing goods that the mimicking type 2 consumes more of and subsidizing goods where type 1 is the major consumer. When there is no binding self-selection constraint in the Edwards et al. paper, there is also no need for a commodity tax. What is the role of the commodity tax in the presence of an externality? We are here going to study three cases, namely when the self-selection constraint does not bind, when the self-selection constraint binds and utility is additively separable in the externality and a binding self-selection constraint with weak separability in the externality.

No binding self-selection constraint

In order to analyze the effects of the commodity tax on the dirty good, we need to make use of equation (A.9), in appendix A. Since we begin the analysis without the self-selection constraint, λ becomes zero in the first order conditions, (A.5) - (A.9), in appendix A.

By combining Roy's identity with equation (A.9), when $\lambda = 0$, we get:

(33)
$$-V_B^1 z^1 - \delta V_B^2 z^2 + (V_E^1 + \delta V_E^2)(N_1 \frac{\partial z^1}{\partial p_z} + N_2 \frac{\partial z^2}{\partial p_z})$$
$$+\mu\left(N_1 z^1 + N_2 z^2 + t_z (N_1 \frac{\partial z^1}{\partial p_z} + N_2 \frac{\partial z^2}{\partial p_z}) \right) = 0$$

Further, if we substitute equation (A.5) and (A.7) into equation (33), then, after some manipulation the result becomes:

(34)
$$(V_E^1 + \delta V_E^2) N_1 (\frac{\partial z^1}{\partial p_z} + \frac{\partial z^1}{\partial B^1} z^1) + (V_E^1 + \delta V_E^2) N_2 (\frac{\partial z^2}{\partial p_z} + \frac{\partial z^2}{\partial B^2} z^2) +$$
$$+ N_1 \mu t_z (\frac{\partial z^1}{\partial p_z} + \frac{\partial z^1}{\partial B^1} z^1) + N_2 \mu t_z (\frac{\partial z^2}{\partial p_z} + \frac{\partial z^2}{\partial B^2} z^2) = 0$$

Using the Slutsky relation, $\partial z/\partial p_z|_{\bar{u}} = \partial z/\partial p_z + \partial z/\partial p_z \, z$,[5] in equation (34) brings us to the below expression of the commodity tax:

$$(35) \qquad t_z = -\frac{1}{\mu}(V_E^1 + \delta V_E^2)$$

What is the interpretation of equation (35)? $1/\mu(V_E^1 + \delta V_E^2)$ is the MSD due to the externality and always negative. Consequently, when the self-selection constraint does not bind, the commodity tax is always positive. In addition, the tax reflects the whole MSD and therefore it is a Pigouvian tax and we rename it to t_z^p.

Binding self-selection constraint, utility is additively separable in externality

As already mentioned, it is not necessary to perform the analysis under the additive separability assumption when the self-selection constraint does not bind, since nothing but the superscripts changes[6]. But with a binding self-selection constraint, it is valuable to separate the two cases of separability.

When utility is additively separable in the externality, equation (35) changes to:

$$(36) \qquad t_z\left(N_1\frac{\partial z^1}{\partial p_z}|_{\bar{u}} + N_2\frac{\partial z^2}{\partial p_z}|_{\bar{u}}\right) = \lambda^*(z^1 - z^{im}) - \frac{1}{\mu}(V_E + \delta V_E)\left(N_1\frac{\partial z^1}{\partial p_z}|_{\bar{u}} + N_2\frac{\partial z^2}{\partial p_z}|_{\bar{u}}\right)$$

where $\lambda^* = \dfrac{\lambda V_B^{im}}{\mu} > 0$.

From (36) it is clear that a binding self-selection constraint has an important influence on the tax. As before $(V_E + \delta V_E)$ is the MSD of the externality and negative. Since the

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[5] $\dfrac{\partial z}{\partial p_z}|_{\bar{u}}$ is the substitution effect of a change in p_z, and hence always negative.

[6] Remember that these superscripts indicate that there is a difference in the two separability assumptions when it comes to the government's optimal policy, but that the empirical implications concerning the individual demands are the same.

compensated demands also are negative, the size of the tax will depend on the sign of the term (z^1- z^{im}). This term reflects the substitutability/ complementability between good z and leisure.

If z and leisure are complements, $(z^1 - z^{im}) < 0$, and the term $(z^1 - z^{im})$ has a positive impact on t_z. On the other side, if z and leisure are some kind of substitutes, $(z^1 - z^{im}) > 0$, then the term $(z^1 - z^{im})$ tends to reduce t_z. This brings about that a binding self-selection constraint will introduce a redistributive term (or some kind of Ramsey term) into the commodity tax on z.

We can rewrite equation (36) as:

(37) $$t_z^*\left(N_1 \frac{\partial z^1}{\partial p_z}\Big|_{\bar{u}} + N_2 \frac{\partial z^2}{\partial p_z}\Big|_{\bar{u}} \right) = \lambda^*(z^1 - z^{im}) + t_z^p\left(N_1 \frac{\partial z^1}{\partial p_z}\Big|_{\bar{u}} + N_2 \frac{\partial z^2}{\partial p_z}\Big|_{\bar{u}} \right)$$

or

(38) $$t_z^* = t_z^s + t_z^p \qquad \text{where } t_z^s = \lambda^*(z^1 - z^{im}) \Big/ \left(N_1 \frac{\partial z^1}{\partial p_z}\Big|_{\bar{u}} + N_2 \frac{\partial z^2}{\partial p_z}\Big|_{\bar{u}} \right)$$

It is evident from equation (38) that the commodity tax can be partitioned into two terms. The first term, t_z^s, reflects the effect upon the self-selection constraint and the second one, t_z^p, is the Pigouvian tax. This additive property of the commodity tax was first derived by Sandmo in his (1975) paper. However, as Sandmo pointed out, the additive property of the tax does not mean that the two terms can be set independently of each other.

When z and leisure are complements, the mimicking type 2 consumes relatively more of z than type one. Consequently, the government would like to prevent the high ability type from mimicking by letting $t_z^s > 0$. In this case the distributional purpose amplifies the corrective purpose of the tax and the resulting total tax on z, t_z^*, becomes strictly positive and greater than the first-best Pigouvian tax.

If leisure is a substitute for the dirty good, the low ability person consumes relatively more of the dirty good than the mimicking type 2. This enables the government to achieve more redistribution towards the low ability type by introducing a subsidy on the dirty good, i.e. $t_z^s < 0$. However, since the government also wants to correct for the externality by setting

$t_z^p > 0$ the total tax on z can be positive, zero or negative. Consequently, the two purposes of the tax compete with each other and the only thing we can conclude from this is that the commodity tax on z must be greater than if no externality existed.

According to the "double dividend literature", the tax rate for polluting goods should lie above the Pigouvian tax in models with only indirect taxes. Although, if it exists linear labor taxes and the tax on the clean consumption good is normalized to zero, the dirt tax should lie below the Pigouvian tax. This later view is presented in Bovenberg-de Mooij (1994) and Bovenberg-van der Ploeg (1994).[7]

In our model, the above analysis concludes that, the commodity tax on the dirty good will be higher than the Pigouvian tax, whenever leisure and the dirty good are complements, and lower than the Pigouvian tax if they are substitutes. Accordingly, our results regarding the dirt tax seem to differ, somewhat, from the Bovenberg et al. results, even though the same definition of the dirt tax is used. However, for reasons explained below, these differences in results should not be exaggerated.

The explanation for the differences lies presumably in the objectives of the tax systems. Bovenberg et al. abstract from distributional considerations by assuming identical individuals. Thereby the goal of their tax systems is, in addition to correcting for the externality, to finance public spending through distortionary taxation (the possibility of using lump-sum taxes is assumed away). The dirt tax falls below the first-best Pigouvian tax because the tax system focuses more on revenue-raising than externality-internalizing. This is reflected by a high marginal cost of public funds, which incorporates the cost of distortionary taxation, in the weighted average formula for the dirt tax. In our model redistribution is a main objective of the tax system. This implies that a commodity tax will both supplement the income tax in achieving the wanted distribution and correcting for the externality. Even though the dirt tax

[7] The double-dividend hypothesis claims that green taxes will both improve the environment and reduce the existing distortions of the tax system. The first, environmental, dividend is not controversial. It is the second dividend that provoke disagreement. There are, at least, two views regarding this second dividend in the double-dividend literature. Schöb (1994) distinguishes these two views as the "environmental" and "public finance" view. The environmental view claims that revenues from green taxes can be used to cut an ordinary, distortionary tax and hence improve the efficiency of the rest of the tax system, while the public finance view argues that green taxes exacerbate rather than alleviate preexisting distortions. The public finance view thereby focuses on the effect of a green tax on the efficiency of the whole tax system. According to Schöb there is no real contradiction

in our model fulfills the additive property, it is not a *weighted average* of a Ramsey and a Pigouvian term. According to this, it is hard to make any direct comparisons between the results of the two types of models.

Binding self-selection constraint, utility is weakly separable in externality

When utility is weakly separable in the externality and the self selection constraint binds, the expression that describes the commodity tax becomes:

(39) $$t_z^* \left(N_1 \frac{\partial z^1}{\partial p_z}|_{\bar{u}} + N_2 \frac{\partial z^2}{\partial p_z}|_{\bar{u}} \right) =$$

$$= \lambda^* (z^1 - z^{im}) + \frac{\lambda}{\mu} (V_E^{im} - V_E^2) \left(N_1 \frac{\partial z^1}{\partial p_z}|_{\bar{u}} + N_2 \frac{\partial z^2}{\partial p_z}|_{\bar{u}} \right) + t_z^p \left(N_1 \frac{\partial z^1}{\partial p_z}|_{\bar{u}} + N_2 \frac{\partial z^2}{\partial p_z}|_{\bar{u}} \right)$$

or

(40) $$t_z^* = t_z^{sp} + t_z^p$$

where $t_z^{sp} = \lambda^* (z^1 - z^{im}) \Big/ \left(N_1 \frac{\partial z^1}{\partial p_z}|_{\bar{u}} + N_2 \frac{\partial z^2}{\partial p_z}|_{\bar{u}} \right) + \frac{\lambda}{\mu} (V_E^{im} - V_E^2) = t_z^s + \frac{\lambda}{\mu} (V_E^{im} - V_E^2)$

It is not possible to determine the sign of t_z^* unambiguously, except under very special circumstances. These occur when there is a complementary relationship between the dirty good and leisure and both the term $\lambda(V_E^{im} - V_E^2)$ and expression (26) are positive. Under these assumptions, t_z^* becomes unambiguously positive. In all other situations we cannot tell whether t_z^* is positive or negative, i. e. if the dirty good is taxed or subsidized. However, it is likely that t_z^* will be lower than the Pigouvian tax, since the Pigouvian tax always is positive. Thereby, it seems as if the results from the Bovenberg et al. studies becomes more likely when we assume weak separability in the externality.

5. The Effective Marginal Tax

between these two views. The differences in results appears because of differences in definitions and normalizations.

Edwards et al. define the total tax as the total tax payments the government can receive from one person:

(41) $$t(y) = T(y) + t_z z(p_z, y - T(y), y)$$

Differentiation of this expression gives the total, or effective, marginal tax rate:

(42) $$\tau'(y) = T'(y) + t_z\left(\frac{\partial z}{\partial B}(1 - T'(y)) + \frac{\partial z}{\partial y}\right)$$

In their paper, Edwards et al. show that this effective marginal tax becomes zero for the high ability individual and strictly positive for the low ability type. Thus, the Stiglitz' results holds even in a non-linear optimal income tax model with commodity taxes, if both direct and indirect taxes are accounted for. However, as Edwards et al. point out, this result implies that the decisions of the highest earner cannot be entirely undistorted as in the Stiglitz' model. The question raised in this paper is, what happens to the results regarding the effective marginal tax rate in the presence of a consumption externality? We will answer this question under weak separability and a non-binding self-selection constraint, and under additive separability when the self-selection constraint binds.

The effective marginal tax rate when the self-selection constraint does not bind

When the self-selection constraint does not bind, we can derive an effective marginal tax for both individual types, by substituting the MRS, equation (29) with λ equal to zero, into equation (42) and using equation (35). The resulting effective marginal tax rate becomes:

(43) $$\tau'(y^h) = -\frac{1}{\mu}(V_E^1 + \delta V_E^2)\left(\frac{\partial z^h}{\partial B^h} + \frac{\partial z^h}{\partial y^h}\right)$$

From equation (43) it is evident that the effective marginal tax will be positive as long as the dirty good is a substitute for leisure. When leisure and the dirty good are complements the effective marginal tax can be negative, zero or positive, depending on the size of $\partial z/\partial y$ relative to $\partial z/\partial B$ in absolute value. This result is remarkable since the marginal income tax is zero and the commodity tax positive, for both individual types under these assumptions. The explanation for this extraordinary result could be that, even though the Pigouvian tax is able to fully internalize the externality and the non-linear income tax achieves the wanted redistribution, this does not imply that the tax system is able to separate efficiency and equity considerations.

Binding self-selection constraint, utility is additively separable in externality

When utility is additively separable in the externality, the effective marginal tax for the high ability type becomes (see appendix C):

$$\text{(44)} \qquad \tau'(y^2) = \frac{-\frac{1}{\mu}(V_E^1 + \delta V_E^2)\left(\frac{\partial z^2}{\partial B^2} + \frac{\partial z^2}{\partial y^2}\right)}{1 - t_z^s \frac{\partial z}{\partial B^2}}$$

According to equation (44), the marginal tax becomes positive whenever leisure and the dirty good are substitutes. This is also the case when leisure and the dirty good are complements, if the income effect is less than one but greater in absolute magnitude than the complementarity effect, otherwise the effective marginal tax becomes negative. Consequently, the presumptions about the sign of the income and substitution effects, influence the sign of the effective marginal tax in the same way as they influenced the sign for the marginal income tax (when the government did not have access to a commodity tax).

The effective marginal tax for the low ability individual, in the "additive case", becomes (see appendix C):

$$\text{(45)} \qquad \tau'(y^1) = \lambda^*\left(\frac{V_y^{im}}{V_B^{im}} - \frac{V_y^1}{V_B^1}\right) + t_z^p\left(\frac{\partial z^1}{\partial y^1} + \frac{\partial z^1}{\partial B^1}\right) - t_z^p T'(y^1)\frac{\partial z^1}{\partial B^1}$$

As for the high ability type, the results concerning the effective marginal tax are in line with those for the marginal income tax (in the absence of a commodity tax). I. e. in short, the tax is larger than in the no externality case when leisure and the dirty good are substitutes and indeterminable when there is a complementary relationship between leisure and the dirty good.

Accordingly, in spite of the externality, the results from the Edwards et al. paper, that in the presence of commodity taxes the government will set the optimal tax structure so that the effective marginal tax mirrors the marginal income tax (in the absence of commodity taxes) still holds. This is also true for the case when utility is weakly separable in the externality, and for which reason this case is not presented here.

6. Summary and Concluding Remarks

This paper has explored the interactions between a consumption externality and an optimal non-linear income tax, with and without commodity taxation. We find that the consumption externality clearly influences the marginal income tax when the government does not have access to a commodity tax. In addition, the externality affects the marginal tax rates for the two individual types, in our model, in the same directions, whereby we can conclude that, the marginal tax not only has a redistributive purpose, but also a corrective aim under these circumstances. The essence of our results is that the sign of the marginal income tax, in the presence of an externality, becomes different from the Stiglitz "zero on the top, positive at the bottom" result. Moreover, we find that this difference, compared to the conventional results, will depend on the substitution/ complementary relationship between leisure and the dirty good. This will be true whether we assume additive or weak separability regarding the individual preferences over the externality, and whether or not the self-selection constraint binds.

When the government can apply an indirect dirt tax, the marginal income tax becomes zero for both individual types, if the incentive compatibility condition does not bind. Consequently, the dirt tax becomes equal to the first-best Pigouvian tax. There is one surprising result in this case though, the effective marginal tax, defined in Edwards et al., becomes different from zero. Thereby, we conclude that the tax system is not able to separate efficiency and equity considerations even under these, very strict, assumptions.

When the self-selection constraint binds, the commodity (or dirt) tax will be different from the first-best Pigouvian tax. The dirt tax will be higher than the first-best Pigouvian tax as long as leisure and the dirty good are complements and utility is additively separable in the externality, and lower than the Pigouvian tax if leisure and the dirty good are substitutes. Accordingly, in this second case there might be a dirt subsidy instead of a dirt tax. If utiltity is weakly separable in the externality we cannot tell, in general, whether the dirty good is taxed or subsidized. However, this implies, that it becomes more likely that the dirt tax will be lower than the Pigouvian tax, since the first-best Pigouvian tax always is positive.

The results regarding the dirt tax causes the marginal income tax to be positive when the dirt tax is lower than the Pigouvian tax. However, the marginal income tax can be positive even if the dirt tax is set at a higher level than the Pigouvian tax, if the income effect is sufficiently small, otherwise the marginal income tax will be negative for the high ability individual an indeterminable for the low ability one. The results regarding the marginal income tax in the presence of a commodity tax will be the same whether utility is additively or weakly separable in the externality.

Finally, we establish that the government will choose a tax structure where the effective marginal tax mirrors the marginal income tax in the absence of commodity taxes. A result in accordance with the results in Edwards et al.

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Appendix A

The government's problem when utility is weakly separable in the externality.

(A.1) $\max_{y^1, B^1, y^2, B^2, p_z} \quad V^1[G^1(p_z, B^1, y^1), E]$

subject to

(A.2) $V^2[G^2(p_z, B^2, y^2), E] \geq \bar{V}^2$

(A.3) $V^2[G^2(p_z, B^2, y^2), E] \geq V^{im}[G^2(p_z, B^1, y^1), E]$

(A.4) $N_1(y^1 - B^1) + N_2(y^2 - B^2) + t_z(N_1 z^1 + N_2 z^2) \geq R$

The characterization of the solution to the government's optimizing problem:

(A.5) $$V_B^1 = \lambda V_B^{im} + \frac{\partial E}{\partial z^1}\frac{\partial z^1}{\partial B^1}\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] + \mu N_1(1 - t_z \frac{\partial z^1}{\partial B^1})$$

(A.6) $$V_y^1 = \lambda V_y^{im} + \frac{\partial E}{\partial z^1}\frac{\partial z^1}{\partial y^1}\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] - \mu N_1(1 + t_z \frac{\partial z^1}{\partial y^1})$$

(A.7) $$(\delta + \lambda)V_B^2 = \frac{\partial E}{\partial z^2}\frac{\partial z^2}{\partial B^2}\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] + \mu N_2(1 - t_z \frac{\partial z^2}{\partial B^2})$$

(A.8) $$(\delta + \lambda)V_y^2 = \frac{\partial E}{\partial z^2}\frac{\partial z^2}{\partial y^2}\left[\lambda(V_E^{im} - V_E^2) - (V_E^1 + \delta V_E^2)\right] - \mu N_2(1 + t_z \frac{\partial z^2}{\partial y^2})$$

(A.9) $$V^1_{p_z} + (\delta+\lambda)V^2_{p_z} - \lambda V^{im}_{p_z} - \left(\frac{\partial E}{\partial z^1}\frac{\partial z^1}{\partial p_z} + \frac{\partial E}{\partial z^1}\frac{\partial z^1}{\partial p_z}\right)\left[\lambda(V^{im}_E - V^2_E) - (V^1_E + \delta V^2_E)\right]$$

$$+\mu\left(N_1 z^1 + N_2 z^2 + t_z(N_1\frac{\partial z^1}{\partial p_z} + N_2\frac{\partial z^2}{\partial p_z})\right) = 0$$

Appendix B

Transformation of the MRS for the low ability type when utility is additively separable in the externality, i. e. transformation of equation (22):

(22) $$\frac{V^1_y}{V^1_B} = \frac{\lambda V^{im}_y - N_1\frac{\partial z^1}{\partial y^1}(V_E + \delta V_E) - \mu N_1}{\lambda V^{im}_B - N_1\frac{\partial z^1}{\partial B^1}(V_E + \delta V_E) + \mu N_1}$$

Stiglitz' transformation:

$$\frac{V^1_y}{V^1_B}\left[\lambda V^{im}_B - N_1\frac{\partial z^1}{\partial B^1}(V_E + \delta V_E) + \mu N_1\right] = \lambda V^{im}_y - N_1\frac{\partial z^1}{\partial y^1}(V_E + \delta V_E) - \mu N_1$$

$$\frac{V^1_y}{V^1_B}\left[\frac{\lambda V^{im}_B}{\mu N_1} - \frac{1}{\mu}\frac{\partial z^1}{\partial B^1}(V_E + \delta V_E) + 1\right] = \frac{\lambda V^{im}_y}{\mu N_1} - \frac{1}{\mu}\frac{\partial z^1}{\partial y^1}(V_E + \delta V_E) - 1$$

$$\frac{V^1_y}{V^1_B}\left[\frac{\lambda V^{im}_B}{\mu N_1} - \frac{1}{\mu}\frac{\partial z^1}{\partial B^1}(V_E + \delta V_E) + 1\right] + \frac{1}{\mu}\frac{\partial z^1}{\partial y^1}(V_E + \delta V_E) + 1 = \frac{\lambda V^{im}_y}{\mu N_1}\frac{V^{im}_B}{V^{im}_B}$$

$$\frac{V^{im}_y}{V^{im}_B} = \frac{\frac{V^1_y}{V^1_B}\left[\frac{\lambda V^{im}_B}{\mu N_1} - \frac{1}{\mu}\frac{\partial z^1}{\partial B^1}(V_E + \delta V_E) + 1\right] + \frac{1}{\mu}\frac{\partial z^1}{\partial y^1}(V_E + \delta V_E) + 1}{\frac{\lambda V^{im}_B}{\mu N_1}}$$

Make use of equation (11): $\Rightarrow -\frac{V_y^{im}}{V_B^{im}} < -\frac{V_y^1}{V_B^1}$, that is $\frac{V_y^{im}}{V_B^{im}} > \frac{V_y^1}{V_B^1}$

$$\frac{V_y^1}{V_B^1} < \frac{\frac{V_y^1}{V_B^1}\left[\frac{\lambda V_B^{im}}{\mu N_1} - \frac{1}{\mu}\frac{\partial z^1}{\partial B^1}\left(V_E + \delta V_E\right) + 1\right] + \frac{1}{\mu}\frac{\partial z^1}{\partial y^1}\left(V_E + \delta V_E\right) + 1}{\frac{\lambda V_B^{im}}{\mu N_1}}$$

$$\frac{V_y^1}{V_B^1}\frac{\lambda V_B^{im}}{\mu N_1} < \frac{V_y^1}{V_B^1}\frac{\lambda V_B^{im}}{\mu N_1} + \frac{V_y^1}{V_B^1}\left[1 - \frac{1}{\mu}\frac{\partial z^1}{\partial B^1}\left(V_E + \delta V_E\right)\right] + \frac{1}{\mu}\frac{\partial z^1}{\partial y^1}\left(V_E + \delta V_E\right) + 1$$

$$\frac{V_y^1}{V_B^1}\left[1 - \frac{1}{\mu}\frac{\partial z^1}{\partial B^1}\left(V_E + \delta V_E\right)\right] > -1 - \frac{1}{\mu}\frac{\partial z^1}{\partial y^1}\left(V_E + \delta V_E\right)$$

$$\frac{V_y^1}{V_B^1} > \frac{-1 - \frac{1}{\mu}\frac{\partial z^1}{\partial y^1}\left(V_E + \delta V_E\right)}{1 - \frac{1}{\mu}\frac{\partial z^1}{\partial B^1}\left(V_E + \delta V_E\right)}$$

which is inequality (23)

When utility is weakly separable in the externality equation (24) is transformed into inequality (27) according to the same method. The transformation method is also applicable when there is a commodity tax present in the model, i. e. the transformation of equation (30) into inequality (32).

Appendix C

Derivation of effective marginal tax for high ability individuals:

Substitution of equation (38) into equation (29) gives equation (31):

$$\frac{V_y^2}{V_B^2} = \frac{-1 - t_z^s \frac{\partial z^2}{\partial y^2}}{1 - t_z^s \frac{\partial z^2}{\partial B^2}} \qquad \text{where } t_z^s = \lambda^* (z^1 - z^{im}) \Big/ \left(N_1 \frac{\partial z^1}{\partial p_z}\Big|_{\bar{u}} + N_2 \frac{\partial z^2}{\partial p_z}\Big|_{\bar{u}} \right)$$

The effective marginal tax rate for the high ability type is defined as:

$$\tau'(y^2) = T'(y^2) + t_z^* \left(\frac{\partial z^2}{\partial B^2}(1 - T'(y^2)) + \frac{\partial z^2}{\partial y^2} \right)$$

$$\tau'(y^2) = 1 + \frac{V_y^2}{V_B^2} - t_z^* \frac{\partial z^2}{\partial B^2} \frac{V_y^2}{V_B^2} + t_z^* \frac{\partial z^2}{\partial y^2}$$

$$\tau'(y^2) = 1 + \frac{V_y^2}{V_B^2} - (t_z^s + t_z^p) \frac{\partial z^2}{\partial B^2} \frac{V_y^2}{V_B^2} + (t_z^s + t_z^p) \frac{\partial z^2}{\partial y^2}$$

Substitute equation (31) into the effective marginal tax:

$$\tau'(y^2) = \frac{1 - t_z^s \frac{\partial z^2}{\partial B^2} - 1 - t_z^s \frac{\partial z^2}{\partial y^2} + (t_z^s + t_z^p) \left(\frac{\partial z^2}{\partial B^2} + t_z^s \frac{\partial z^2}{\partial y^2} \frac{\partial z^2}{\partial B^2} \right) + (t_z^s + t_z^p) \frac{\partial z^2}{\partial y^2} (1 - t_z^s \frac{\partial z^2}{\partial B^2})}{1 - t_z^s \frac{\partial z^2}{\partial B^2}}$$

$$\tau'(y^2) = \frac{t_z^p \left(\frac{\partial z^2}{\partial B^2} + \frac{\partial z^2}{\partial y^2} \right)}{1 - t_z^s \frac{\partial z^2}{\partial B^2}}$$

since $t_z^p = -\frac{1}{\mu}(V_E + \delta V_E)$ the resulting effective marginal tax becomes:

$$\tau'(y^2) = \frac{-\frac{1}{\mu}(V_E + \delta V_E) \left(\frac{\partial z^2}{\partial B^2} + \frac{\partial z^2}{\partial y^2} \right)}{1 - t_z^s \frac{\partial z^2}{\partial B^2}} \qquad \text{equation (44)}$$

In order to arrive at an interpretable expression of the effective marginal tax rate for the low ability individuals, we begin with the MRS (equation (21) in the presence of a commodity tax):

$$\frac{V_y^1}{V_B^1}=\frac{\lambda V_y^{im}-N_1\frac{\partial z^1}{\partial y^1}(V_E+\delta V_E)-\mu N_1(1+t_z^*\frac{\partial z^1}{\partial y^1})}{\lambda V_B^{im}-N_1\frac{\partial z^1}{\partial B^1}(V_E+\delta V_E)+\mu N_1(1-t_z^*\frac{\partial z^1}{\partial B^1})}$$

$$\frac{V_y^1}{V_B^1}\left(\lambda V_B^{im}-N_1\frac{\partial z^1}{\partial B^1}(V_E+\delta V_E)+\mu N_1(1-t_z^*\frac{\partial z^1}{\partial B^1})\right)\frac{1}{V_B^{im}}=\frac{\lambda V_y^{im}}{V_B^{im}}-\frac{N_1}{V_B^{im}}\frac{\partial z^1}{\partial y^1}(V_E+\delta V_E)-\frac{\mu N_1}{V_B^{im}}(1+t_z^*\frac{\partial z^1}{\partial y^1})$$

$$\frac{\mu N_1}{V_B^{im}}(1+t_z^*\frac{\partial z^1}{\partial y^1})+\frac{N_1}{V_B^{im}}\frac{\partial z^1}{\partial y^1}(V_E+\delta V_E)-\frac{V_y^1}{V_B^1}\frac{N_1}{V_B^{im}}\frac{\partial z^1}{\partial B^1}(V_E+\delta V_E)+\frac{V_y^1}{V_B^1}\frac{\mu N_1}{V_B^{im}}(1-t_z^*\frac{\partial z^1}{\partial B^1})=\lambda\left(\frac{V_y^{im}}{V_B^{im}}-\frac{V_y^1}{V_B^1}\right)$$

$$\frac{\lambda V_B^{im}}{\mu N_1}\left(\frac{V_y^{im}}{V_B^{im}}-\frac{V_y^1}{V_B^1}\right)=1+t_z^*\frac{\partial z^1}{\partial y^1}+\frac{V_y^1}{V_B^1}(1-t_z^*\frac{\partial z^1}{\partial B^1})+\frac{1}{\mu}(V_E+\delta V_E)\left(\frac{\partial z^1}{\partial y^1}-\frac{V_y^1}{V_B^1}\frac{\partial z^1}{\partial B^1}\right)$$

$$\lambda^*\left(\frac{V_y^{im}}{V_B^{im}}-\frac{V_y^1}{V_B^1}\right)=\tau'(y^1)+\frac{1}{\mu}(V_E+\delta V_E)\left(\frac{\partial z^1}{\partial y^1}-\frac{V_y^1}{V_B^1}\frac{\partial z^1}{\partial B^1}\right)$$

$$\tau'(y^1)=\lambda^*\left(\frac{V_y^{im}}{V_B^{im}}-\frac{V_y^1}{V_B^1}\right)-\frac{1}{\mu}(V_E+\delta V_E)\left(\frac{\partial z^1}{\partial y^1}-\frac{V_y^1}{V_B^1}\frac{\partial z^1}{\partial B^1}\right)$$

$$\tau'(y^1)=\lambda^*\left(\frac{V_y^{im}}{V_B^{im}}-\frac{V_y^1}{V_B^1}\right)+t_z^p\left(\frac{\partial z^1}{\partial y^1}+\frac{\partial z^1}{\partial B^1}\right)-t_z^pT\ '(y^1)\frac{\partial z^1}{\partial B^1}\qquad\text{equation (45)}$$