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Far Out on the Yield Curve

Annika Alexius

Department of Economics
Uppsala University
P.O. Box 513
SE-751 20 Uppsala
Sweden
Fax: +46 18 471 14 78

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Abstract

Data on short investments in Swedish long-term bonds as the bonds mature contains unusually rich information about the relationship between duration and the first and second moments of bond returns. We identify three different channels through which duration affects bond returns. The liquidity preference hypothesis yields a direct link between duration and returns, which however disappears once indirect effects through the variance of returns and the price of risk are taken into account. The risk premia obtained from a multivariate GARCH-M model extended to allow the variance to depend on duration are of the same size as observed excess returns. Finally, duration appears to affect the relationship between bond returns and the risk free interest rate. One additional year of duration implies that the $\beta-$coefficient increases by 0.66.

Key words: Bond returns, duration, multivariate GARCH.
JEL classification E43, G12.

*Department of Economics, Uppsala University, Box 513, SE-751 20 Uppsala, Sweden. Tel: +46 18 4711564. Fax +46 18 4711478. E-mail: annika.alexius@nek.uu.se.

1 Introduction

The typical empirical study of the term structure of interest rates tests the unbiased version of the rational expectations hypothesis using data on T-bills with maturities shorter than one year. At least from a monetary policy perspective, it would be desirable to shift the focus of attention much further out on the yield curve as consumption and investment are primarily affected by long-term interest rates. The term structure characteristics of long-term bonds are however relatively neglected, possibly due to data problems. By analyzing returns to short investments in long-term bonds, these data problems are circumvented and the behavior of interest rates the far out on the yield curve can be investigated. In particular, studies of the relationship between long-term bond yields and the duration of the bond have mainly been limited to tests of the liquidity preference hypothesis.

This paper studies the effects of duration on the first and second moments on returns to short investments in six Swedish long-term bonds over time, as the bonds mature. The data set contains unusually rich information about the relationship between the mean and variance of bond returns on one hand and the duration of the bonds on the other. Moreover, information about the coupon payments and expiration date of each bond is available, which makes it easy to calculate exact returns. The main findings can be summarized as follows. First, returns to investments in long-term bonds increase monotonically with duration. We would like to know how and why. The relationship between returns and duration appears to be quadratic, as suggested by theory. To what extent do these excess returns constitute rewards for carrying interest rate risk? It turns out that *the variance* of bond returns increases with the square of the duration of the bond. Furthermore, this interest rate risk is priced by the market and appears to be the only reason for the fact that average returns to bond investments increase with the duration of the bonds. Using a multivariate GARCH-M model where duration is allowed to affect the variance of bond returns, risk premia of the same magnitude and shape as observed excess returns are obtained. There are no significant direct effects of duration on returns over and above the GARCH-M effects. A third finding is that the relationship between bond returns and the risk free short-term interest rates, i.e. the β-coefficient in the standard REH test, can be modelled as a linear function the duration of the bond. One additional year of duration increases the β-coefficient by 0.66. We document this phenomenon and discuss possible reasons for it.

There are a few previous studies of the term structure characteristics of long-term bonds. Elton, Gruber, and Mai (1996) investigate bonds with maturitites up to 13 and 20 years, respectively. The former reject the local expectations hypothesis and find evidence of time-varying term premia without a clear relationship to maturity. They however conclude that the sensitivity to short interest risk is increasing in maturity of the bond, a finding that is related to our discussion about the relationship between duration and the β-coefficient. Klemkovsky and Pilotte (1992) confirm that ex ante term premia on bonds are related to the level and variance of the risk free rate. Again, longer bonds react more to each of the factors. Hooker (1999) shows that excess returns are likely to be increasing in the maturity of the bonds when the yield curve is steep and interest rates are high, but not when the yield curve is flat and interest rates are low. Boudoukh et al. (1999) do not reject the set of inequality restrictions implied by the liquidity preference hypothesis using data on portfolios of T-bills and bonds with maturities ranging from one to above 120 months.

Sarkar and Ariff (2002) show that both ten-year interest rates and 20-year interest rates are negatively related to the volatility of the short-term interest rate.

The rest of the paper is organized as follows. Section 2 describes the data. In Section 3, we study the direct effects of duration on returns by testing the local REH and the liquidity preference hypothesis. Section 4 is devoted to the multivariate GARCH-M model where duration enters into the variance equation. Section 5 discusses how duration affects the relationship between bond returns and the short-term interest rate. Section 6 incorporates all three effects of duration on bond returns into a single, nested specification. Section 7 concludes.

2 The data

Daily data on interest rates on six long-term bonds (loans 1033, 1034, 1035, 1037, 1038, 1040) from January 1993 to 2000 are collected from Hansson and partners. Information about the coupon payments and expiration dates is available from the same source. Bond prices are calculated as follows, using the last observation of each month:

$$P_t^i = \frac{Par}{(1+r_t^i)^{T_t^i}} + \sum_{t=1}^{T_t^i} \frac{C^i}{(1+r_t^i)^t}, \tag{1}$$

where P_t^i is the price of bond i at time t, r_t^i is its yield to maturity, C^i

its coupon payment and T_t^i is the time varying maturity. Typically, approximations are used to obtain both the size of the coupon payments and the maturity of the bond. One advantage of the present data set is that exact information about all the terms in (1) is available for each observation. For instance, loan 1033 expires on May 5, 2003, and has an annual coupon payment of 10.25 percent. T_t is 124 months for the first observation and five months for the last, in December 2002. Monthly returns are calculated from the price change and the coupon payments:

$$R_t^i = \frac{P_{t+1}^i - P_t^i}{P_t^i} + D_i \frac{C^i}{P_t^i} \tag{2}$$

where D_i is a dummy variable that takes on a unity value if the bond has a coupon payment in period t and zero otherwise. Table 1 contain some preliminary descriptive statistics. It is clear that unconditional average excess returns of the bonds against the monthly interest rate are positive throughout the data set. The bond with the longest maturity, loan 1034, also has the highest excess return. The second highest return is however found for loan 1033, which has the shortest remaining time to maturity of the bonds. The longest bond also has the highest average variance. Notably, all bond returns are much higher and also much more variable than the monthly interest rate. Excess returns to long-term bonds are may be unusually large given that interest rates have fallen on average over the sample period.

As interest rates are often found to be integrated of order one, the issue of non-stationarity has to be considered. The short-term interest rate falls from 1993 to 1995 and appears to be stationary thereafter (see Figure 1). As only one of the six bonds is issued before 1995 (loan 1033), we have chosen

Table 1: Summary statistics

loan	maturity	coupon	mean	average term premium	st. error	# obs
1033	5 may -03	10.25	0.0175	0.0139 (7.440)	0.0175	91
1034	20 April -09	9.00	0.0215	0.0175 (4.585)	0.0361	91
1035	9 Feb. -05	6.00	0.0158	0.0120 (5.240)	0.0218	91
1037	15 Aug. -07	8.00	0.0163	0.0130 (4.504)	0.0247	79
1038	25 Oct. -06	6.50	0.0132	0.010 (4.760)	0.0170	71
1040	5 may -08	6.50	0.0128	0.009 (3.237)	0.0211	60
r^{1m}		0	0.003921	0	0.0024	91

to study only the period from 1995 and on. ADF unit root tests confirm that the interest rates and returns are stationary for this sample period (see Table 2). Figures two and three shows the bond returns. They are clearly stationary and their variances appear to fall over time, as the bonds mature.

3 The local rational expectations hypothesis and the liquidity preference hypothesis

We first test several standard term structure hypotheses using the panel of returns to short investments in long-term bonds. According to the local expectations hypothesis, expected returns to investments in interest bearing assets with otherwise identical characteristics (e.g. the same default risk) should be equal. Assuming a simple form of rational expectations, $E_t R_t^i = R_t^i + \varepsilon_t$, an unconditional test of the local REH boils down to testing whether $[\alpha, \beta]$ in (3) equals $[0, 1]$.By controlling for other variables like duration and/or conditional variance, several conditional tests of the same hypothesis will be performed throughout the paper. We have:

$$R_{it} = \alpha_i + \beta_i r_t^{1m} + \varepsilon_{it}, \qquad (3)$$

where R_{it} is the monthly return to investments in bond i, r_t^{1m} is the safe monthly interest rate and ε_{it} is an error term. A typical test of the local expectations hypothesis investigates whether annual expected returns to investments to e.g. ten-year bonds equal the one-year interest rate. Because we have data on six specific bonds over time, we study how expected returns to investments in e.g. bond 1033 are related to the risk free short-term interest rate. Loan 1033 is a ten-year bond in the beginning of the sample and only a two-year bond towards the end of the sample.

The residuals ε_{it} are not significantly autocorrelated but heteroscedastic across bonds and over time. We therefore use robust standard errors as suggested by White (1980)[1].

Table 3 shows the results from the REH tests. Monthly bond returns are significantly positively related to the risk free monthly interest rate. In stark contrast to the typical finding of $\beta-$coefficients that are below unity or even negative (see Campbell and Shiller, 1991), the point estimates in Table 3 are all well above unity. $\beta-$coefficients of 12.88 and 13.25 are obtained for the two longest bonds, and only a coefficient is below 3. The standard errors are however large and only one of the six $\beta-$coefficients (loan 1038) is significantly above unity. The intercept terms are individually as well as

[1] As the analysis proceeds, it will be clear that the variances follow GARCH(1,1) processes and depend on the duration of the bonds. The latter effect is implicitly incorporated here as duration has the same effect as a time trend, which is included in the correction of the standard errors. Allowing for GARCH-effects has negligable effects on the results in Table 2.

jointly insignificant. The full local expectations hypothesis $[\alpha_i, \beta_i] = [0, 1]$ cannot be rejected in four of the six cases, while the less restrictive hypothesis that β is equal to one is not rejected in five of the six cases. Rather than moving one to one with the short-term interest rate as expected from the REH, bond returns amplify movements in the risk free rate.

These results are robust to variations in the empirical specification. Several different models are estimated throughout this paper and the results still involve $\beta-$coefficients above unity, some but not all of them significantly so. Bond returns can easily be calculated for other holding periods since the original data are daily. If returns to quarterly investments are calculated rather than monthly, (weighted) R^2 more than doubles to 0.25-0.3, but the qualitative results are unchanged as the $\beta-$coefficients are still above unity and the local expectations hypothesis still cannot be rejected in most cases.

Does the local expectations hypothesis hold better between returns to investments in the different bonds? The relationship between stochastic bond returns must be estimated using instrumental variables since the independent variable R_{jt} is correlated with the error term ε_{it}:

$$R_{it} = \alpha_i + \beta_i R_{jt} + \varepsilon_{it}, \; i \neq j \tag{4}$$

According to the Sargan test and partial R^2, the first lag of each bond return and the safe interest rate are valid and relevant instruments. Table 4 shows the estimates of β_i and the hypothesis tests of $\beta_i = 1$ for all combinations of bonds i and j. The results are straightforward. The $\beta-$coefficients do not differ significantly from the unity value expected from the local expectations hypothesis. Although some of the point estimates exceed unity here as well, about 90 percent of them fall in the range 0.5-1.0. Hence, the bond returns are related to each other as the local REH predicts. It is the relationship between the long-term bond returns and the risk free short-term interest rate that stand out as the estimated $\beta-$coefficients are much larger than unity.

While our data set is not ideal for testing the local REH given that the duration of each bond varies over the sample, it provides an unusual opportunity to study the effects of varying the duration of the bonds. First we focus on the relationship between the level of returns to bond investments and the duration of the bond. The liquidity preference hypothesis states that expected returns to bond investments increase with duration, which presumably explains why the term structure of interest rates typically slopes upward.

For each observation, we calculate the Macaulay (1938) duration of the bond.[2] Obviously, if a separate dummy variable is created for each duration measured in months, the number of variables to be estimated grows infeasibly large and there is a maximum of six observations on each maturity. We have settled for biannual dummies except for the shortest and longest intervals that cover bonds with durations below three years and above 12 years, respectively. This results in 48 to 144 observations in each interval, at least two bonds with positive entries in each interval, and seven coefficients to be estimated. One argument for using shorter intervals is that term pre-

[2] Macaulays measure of duration is calculated as $D_t = \left[\frac{C}{(1+y_t)} + \frac{2C}{(1+y_t)^2} + ... + \frac{K(1+C)}{(1+y_t)^K}\right]\frac{1}{P_t}$, where C is the annual coupon payment, y_t is the yield to maturity, K is the maturity, and P_t is the price of the bond.

mia have been documented to display a hump-shaped behavior, increasing for maturities of up to six months and then decreasing again (Fama, 1984). However, only one of the bonds have a duration below the year within the sample, so there are very few observations in this interval anyway.

Table 5 contains the results from estimating (5) on the panel of bonds, now including the seven duration dummies:

$$R_{it} = \alpha_i + \beta_i r_t^{1m} + \gamma^\tau D_{it}^\tau + \varepsilon_{it}, \tag{5}$$

where D_{it}^τ are maturity dummies that take on a unity value if the duration of the bond falls in the given interval at t and are set to zero at other times.

Turning first to the lower half of the table containing the duration dummies, we see that average returns to bond investments increase monotonically in the duration of the bond. Because the point estimate of the effects increase with duration in every single case ($\gamma^\tau > \gamma^{\tau-1}$), the econometric issues involved in testing multiple inequality constraints discussed by Boudoukh et al (1999) and others are not relevant here. The final column contain $p-$values of one-sided tests of the hypothesis that each point estimate γ^τ equals $\gamma^{\tau-1}$ The first entry that is significantly positive is γ^{6-7}. Six to seven-year bonds hence yield on average 1.0 percent more per month than the shortest bonds. Figure 4 shows the point estimates and the 95 percent confidence intervals. The dummy variables are significantly positive for durations above 6-7 years. The documented term premia are large but not extremely so relative to other studies. For instance, Hooker (1999) find excess returns to investments in five-year bonds relative to the monthly interest rate of almost 8 percent per year. The individual β-coefficients are still insignificantly above unity as the

REH is not rejected. However, the joint hypothesis that all β-coefficients equal unity is rejected. A final observation that we will return to in Section5 is that the point estimates of β are highest for the longest bonds.

Because the relationship between long-term bond returns and duration is monotonically increasing, the term premia can alternatively be modelled as a continuous function of the duration of the bonds. As discussed more in detail in Section 4, there are theoretical reasons for expecting this relationship to be quadratic. The term premia shown in Figure 4 also indicate that a quadratic function is appropriate. However, we do not a priori exclude a linear term:

$$R_{it} = \alpha_i + \beta_i r_t^{1m} + \gamma_i D_t + \delta_i D_t^2 + \varepsilon_{it}, \tag{6}$$

where D_t is the duration of bond i, measured in years. Several economically interesting hypotheses can be tested on (6). Above all, we want to investigate whether the effects of duration on returns are equal for all the six bonds ($\gamma_i = \gamma, \delta_i = \delta$). The p-value for this joint hypothesis is 0.856, implying that i.e. a six year bond behaves in a similar manner irrespectively of the number of the loan. Table 6 contains the results from estimating (6). When no restrictions are imposed, 24 coefficients are estimated and only a single one is significant. The restriction that each of the coefficients are equal for all bonds is not rejected for any single parameter. However, the joint hypothesis that all coefficients simultaneously are independent of the bond is rejected with a p-value of 0.029. We settle for estimating a fixed effect model where β, γ, and δ are common to all bonds but α_i is allowed to differ. The results are presented below in Table 6. The quadratic effect of duration on returns is significantly positive, while the linear effect is insignificantly negative. Although the estimated parameters are small, the implied term premia are quite large. Moving from a three-year bond to a four-year bond increases average returns by 1.78 percent a year. A 12-year bond has a term premium over the risk free interest rate of 20 percent per year.

To sum up this section, we have found that average returns to investments in long-term bonds increase monotonically with the duration of the bonds. This finding is consistent with the liquidity preference hypothesis. The relationship between returns and duration can be estimated either using discrete duration dummies or a continuous, quadratic function. The REH is not rejected in the unconditional tests. When we control for the direct effect of duration on average returns, the individual β-coefficients are insignificantly above unity but the joint test reveals that returns overreact to movements in the short-term interest rate.

4 A multivariate GARCH-M model where the variances depend on duration

Why should investments in bonds have higher expected returns the longer the maturity of the bond? In the classical formulation of the liquidity preference hypothesis (Hicks 1946, Kessel 1965), returns to bond investments are increasing in maturity because short-term bonds are assumed to be closer substitutes to money than long-term bonds and therefore carry smaller premia. Here, we concentrate on the relationship between duration and the variance of bond returns. Several papers indicate the existence of such a relationship. In the Cox, Ingersoll and Ross (1985) one-factor model of the term structure of interest rates, instantaneous returns to bond investments are increasing in the interest elasticity of the bond, e.g. its duration. Furthermore, they show that the variance of returns increases with the square of duration. Schaefer and Schwartz (1987) estimate a linear relationship between the standard deviation of bond returns and duration in the context of bond option pricing. Brown (2000) suggests that a quadratic relationship between duration and the variance of bond returns can be expected from economic theory. He derives a simple approximation of the theoretical relationship between duration and variance. It turns out that the variance of asset returns increases approximately with the square of the duration of the asset:

$$Var\left(R_{jt}\right) \approx D_{jt}^{*2} Var\left(\Delta i_t^j\right). \tag{7}$$

Here, $Var\left(R_{jt}\right)$ denotes the variance of the return to investments in a bond of duration j, D_{jt}^{*} is its modified duration[3], and $Var\left(\Delta i_t^j\right)$ is the variance of changes in the yield to maturity of a bond with duration j. To the extent that $Var\left(\Delta i_t^j\right)$is unrelated to the duration of the bond, the variance of bond returns increases with duration squared .We have little a priori information about the relationship between $Var\left(\Delta i_t^j\right)$ and duration. Long-term bond rates could be expected to be more stable than shorter term bond rates (see the literature on excess volatility of long-term interest rates, for instance Kuttner 2001). However, the empirical evidence indicates that horizontal shifts dominate among movements in the yield curve (REF: Fuhrer

[3]The modified duration is defined as $D_t^* = -\frac{D_t}{(1+y_t)} = \frac{dP_t}{dy_t}\frac{1}{P_t}$, where P_t is the price of the bond and y_t is the yield to maturity.

and Moore?). In the present data set, the variance of changes in interest rates displays a positive but insignificant relationship to the duration of the bonds.[4] Hence, if anything, the variance of bond returns can be expected to increase more than quadratically with duration. Cox, Ingersoll and Ross (1985) actually also show that the variance is a function of the square of the duration of the bond.[5]

Nyborg, et al (2001) model the variance of daily bond returns as a linear function of the duration of the bond. Their main finding is that the variance increases with 0.0277 for each additional year of duration, implying a coefficient of 0.83 for monthly returns (given the maintained hypothesis of random walk behavior i.e. no mean reversion in returns). Schaeffer and Schwarts (1987) also run a simple regression to show that the standard deviation of returns is proportional to the duration of the bond.

GARCH-M models have frequently been used to describe bond returns (see for instance Dungey et al., 2000). These models imply that the conditional variance of returns to bond investments is an autoregressive process, and that expected returns are higher when the conditional variance is high. In other words, the conditional variance is autocorrelated and this risk is

[4] We have estimated GARCH models of changes in interest rates, Δi_t^j. The variance equations $h_{ii,t} = \theta_{i1} + \theta_{i2}\varepsilon_{i,t-1}^2 + \theta_{i3}h_{ii,t-1}^2 + \theta_{i4}D_{i,t} + \theta_{i5}D_{i,t}^2$

yield estimates of θ_{i4} and θ_{i5} that are insignificant but positive.

[5] In the general equilibrium model of interest rates in continous time of Cox, Ingersoll and Ross (1985), the variance of the rate of return is $O(\Delta t^2)$, where Δt is remaining time to maturity $(T-t)$. The $O-$function is not specified.

priced by the market. A GARCH(1,1)-M model actually fits the present data on bond returns reasonably well at a first glance. The GARCH parameters are significant and the sum of the two parameters is typically slightly below unity. The price of risk is always positive but significant only in four of the six univariate cases. However, visual inspection of the square residuals either from the GARCH-M regressions or the simple REH tests of Section 3 indicates that the variances actually fall over time, with the duration of the bonds. To investigate the effects of duration on the variances of returns across the different bonds, a multivariate GARCH-model is required. Furthermore, the error terms from the univariate models are correlated, implying that there are efficiency gains from using a multivariate model.

We estimate a multivariate GARCH(1,1)-M model where the variances of the bond returns is a possibly quadratic function of the duration of the bonds. In the mean equation (8), returns to investments in each of the i bonds has a bond specific intercept α_i, a bond specific relationship to the safe monthly interest rate β_i, and a bond specific price of risk η_i that captures the effects of the conditional variances on mean returns:

$$R_{it} = \alpha_i + \beta_i r_t^{1m} + \eta_i E_{t-1} h_{iit} + \varepsilon_{it} \tag{8}$$

The covariance matrix of the error terms is denoted H_t :

$$E_{t-1}\left(\varepsilon_{it}\varepsilon_{jt}\right) = H_t \tag{9}$$

The diagonal elements of H_t, h_{iit}, are assumed to follow univariate GARCH(1,1) processes with additional terms that allows duration to affect the conditional variance. As discussed above, duration is expected to enter in quadratic form only. In order to study the shape of this relationship, we allow for a linear term as well:

$$h_{iit} = \theta_{i1} + \theta_{i2}\varepsilon^2_{i,t-1} + \theta_{i3}h^2_{ii,t-1} + \theta_{i4}D_{i,t} + \theta_{i5}D^2_{i,t} \tag{10}$$

Specifying the non-diagonal elements of H_t is a main issue in multivariate GARCH models. Following Bollerslev (1990), the non-diagonal elements of H_t have the simple form

$$h_{ijt} = \rho_{ij}\sqrt{h_{iit}}\sqrt{h_{jjt}}, \tag{11}$$

where ρ_{ij} are time invariant conditional correlations. Alternatively, the conditional covariances could be assumed to follow autoregressive processes as in the BEKK specification of Engle and Kroner (1995). However, the constant conditional correlations (CCC) model appears appropriate here since the cross products of the standardized residuals display no significant autocorrelation. The covariance matrix H_t hence has the form

$$H_t = D_t R D_t, \ D_t = diag\sqrt{h_{iit}} \tag{12}$$

where R contains the correlations, ρ_{ij}.

Estimating the unrestricted multivariate GARCH model with linear and quadratic effects of duration on the variance is not straight forward. First, it is obvious that different starting values yield different results. To avoid local optima, the model is estimated using simulated annealing (SA). SA is a global search algorithm originally designed for thermodynamics. Because it moves downhill as well as uphill, it is better able to deal with likelihood functions with local optima, ridges and/or plateaus than standard numerical or derivate based optimization algorithms. In the words of Goffe et al. (1994), these procedures can be described as a blind man trying to reach the peak of a mountain using solely the information that the ground under his feet is still upward sloping. He will reach the summit only if the mountain has one single peak and no flat sections. Likelihood functions are asymptotically well behaved under certain conditions but this is not necessarily the case in small samples. Practitioners have long varied the starting values to find the global optimum. This is however difficult when there are many parameters. Hence, SA is clearly a useful tool in the present case. To calculate robust standard errors, we use the BHHH algorithm in RATS given the parameters from SA as starting values. As the parameters estimated by SA are optimum values, the BHHH returns practically identical estimates.[6]

Neither standardized residuals or squared standardized residuals display significant autocorrelation according to the Lagrange Multiplier test, or higher order autocorrelation according to the Portmanteau test. Portmanteau tests are also used investigate whether the cross products of standardized residuals ($\varepsilon_{it}\varepsilon_{jt}/\sqrt{h_{iit}h_{jjt}}$) are autocorrelated, i.e. whether the assumption of constant conditional correlations ρ_{ij} is reasonable. Remaining autocorrelation in the cross products would indicate that a more elaborate structure of the non-diagonal elements of the covariance matrix such as the dynamic correlation coefficients of Engle (2002) or the BEKK specification would be appropriate.

Few of the original 79 parameters of the model are individually significant, see Table 7. As loans with different numbers (”names”) can be expected to

[6] The idea to use first SA and than a standard algorithm with a variety of options for calculating standard robust errors is borrowed from Nilsson (2002).

behave in a similar manner once differences in duration are accounted for, we test whether the various parameters differs significantly between the bonds. The most restrictive restriction that is not rejected by the data is that all parameters except the constant θ_{i1} in (13) are equal for all bonds. This implies that a four year bond behaves in a similar manner irrespectively of the name of the bond. Furthermore, the linear effects of duration on the variance are individually as well as jointly insignificant. After imposing the relevant restrictions, we have the following specification of the conditional variances:

$$h_{iit} = \theta_{i1} + \theta_2 \varepsilon^2_{i,t-1} + \theta_3 h^2_{ii,t-1} + \theta_5 D^2_{i,t} \tag{13}$$

Table 8 shows the results from the preferred model where the conditional variance of the bonds are modelled as quadratic functions of duration. Focusing first at the price of risk for each bond, we see that all η_i coefficients are positive. The point estimates vary between 0.70 and 3.72. However, only one of the prices of risk is individually significant. We then test whether these coefficients differ significantly from each other and from zero, i.e $\eta_i = \eta (= 0)$. The former restriction is not rejected while the latter restriction is. Hence, we re-estimate the model with the restriction that the price of risk is the same for all bonds. Using similar testing procedures for the remaining parameters, the only parameter that is allowed to differ between the bonds is θ_{i1}. Furthermore, θ_{i4} are jointly insignificant. Hence, the conditional variance depend only quadratically on the duration of the bonds.

Turning to the effect on the variance of duration, θ_5, the point estimates vary between 5.48*$10^{-7}$and 1.900*10^{-6}. Three of these coefficients are individually significant. Again, the restriction that the effect of one additional year of duration is the same for all five bonds is not rejected (p=0.095). Imposing this restriction, we arrive at 1.351*10^{-6} with a p-value of 6.795*10^{-6}.

Conditional tests of the REH can be performed within the multivariate GARCH as well. The joint hypothesis that all β_i-coefficients in (8) are equal to unity is clearly rejected with a p-value of 0.000. Hence, the REH holds unconditionally only for the present data and not once the effects of duration through the variance are taken into account.

For comparison with Nyborg et al (2001) we have also estimated the model given a linear relationship between duration and the variance of the bonds. Here we find a point estimate of the linear effect θ_{i4} of 0.0000988, i.e. a much smaller coefficient than documented by Nyborg et al (2001). In the linear model, the variance does not significantly affect returns, i.e. the price of risk is insignificant.

In Section 3, excess returns to short investments in bonds of different durations (two year intervals) were estimated. These term premia can be compared to the risk premia obtained from the GARCH-M model. Figure 5 shows the two sets of excess returns. They have similar magnitudes as well as similar shapes - the discrete term premia estimated without any restrictions could well stem from a continuous model with a quadratic effect of duration.

5 Duration and the reaction of bond returns to changes in the short-term interest rate

The results in Tables 3 to 6 indicate that returns to investments in bonds with longer maturity have higher β-coefficients, i.e. react more to a given change in the short-term interest rate. We first want to establish whether this casual observation actually holds by allowing the relationship between bond returns and the risk free interest rate to depend on duration:

$$R_{it} = \alpha_i + \beta_i r_t^{1m} + \vartheta_i D_t r_t^{1m} + \varepsilon_{it} \tag{14}$$

Table 10 shows the results. It is clear that the relationship between bond returns and the short-term interest rate can be modelled as a linear function of the duration of the bonds. The standard β-coefficients are all insignificant and many of them are negative once duration is taken into account. The sector specific estimates of ϑ_i, the effect of one additional year of maturity on the relationship between bond returns and the short-term interest rate varies between 0.47 and 1.26. The restriction that the ϑ_i-coefficient is the same for all bonds is not rejected. The estimated common effect implies that one additional year of duration increases the response of bond returns to movements in the monthly interest rate with 0.649.

Bond returns hence react more to changes in the short-term interest rate the longer the duration of the bond. Duration is a measure of the sensitivity of the bond price to interest rate changes:

$$D_t^i = -\frac{\Delta P_t / P}{\Delta i_t / (1 + i_t)} \tag{15}$$

Rearranging yields

$$\Delta P_t/P = R_t^j = -D_t^j \Delta i_t^j/(1+i_t^j) \tag{16}$$

Hence, returns to bond investments should react more to changes in the yield to maturity of the bond in question the longer the duration of the bond. For (16) to be consistent with the local REH for bonds of different durations, bond rates have to move less in response to changes in the short-term rate the longer the duration of the bond:

$$E_t R_t^j = i_t^s = -D_t^i E_t \Delta i_t^j/(1+E_t^j i_t^j) \approx -D_t^i E_t \Delta i_t^j \Rightarrow \frac{d\Delta i_t^j}{di_t^s} = -\frac{1}{D_t^i} \tag{17}$$

More precisely, the change in the long-term interest rate as the short-term interest rate moves should be the inverse of the duration of the long-term bond. For the local REH to hold, the long end of the yield curve must fluctuate much less than the short end. The observed behavior where bond returns react more the longer the duration of the bond stems from the empirical fact that the entire yield curve typically shifts in a parallel manner.[7]

The observed behavior of the β-coefficent is in a sense consistent with the finding of Klemkovsky and Pilotte (1992) that the factor loadings of short-term interest risk is increasing in the duration of the bond. Because they study data on excess returns, they implicitly impose a unit restriction on the effect of short-term interest rate changes and do not detect possible deviations from this part of the REH hypothesis.

[7] In fact, long-term interest rates move more than one for one with changes in the short-term rate for the present data (this evidence is not reported but available on request).

6 Nested models of duration and bond returns

We have analyzed three different channels through which the duration of a bond affects returns to bond investments, their variance, and/or their relationship to the short-term interest rate. First, there could be a direct effect on expected returns through the liquidity preference hypothesis or the idea that bonds with shorter durations are closer substitutes to money. This is captured by the parameter δ in (6). Second, longer duration implies a higher variance of returns, which in turn increases expected returns if this interest rate risk is priced by the market. This is captured by the parameters θ_5 and η in (13) and (8).[8] Finally, duration appears to affect the relationship between bond returns and the safe short-term rate. Longer bonds returns react more to changes in the short-term interest rate than shorter bond returns. This is captured by the parameter ϑ in (14). Hence, the models estimated in Section 3, 4, and 5 are nested in the more general specification (18):

$$R_{it} = \alpha_i + \beta_i r_t^{1m} + \delta D_{it}^2 + \vartheta D_{it} r_t^{1m} + \eta E_{t-1} h_{iit} + \varepsilon_{it}, \tag{18}$$

where

$$E_{t-1}(\varepsilon_{it}\varepsilon_{jt}) = H_t \tag{19}$$

[8] Possible linear effect of duration either directly on returns or on the variance are excluded here since they were found to be insignificant in previous regressions. Furthermore, only the constant terms are allowed to differ between the bonds. These restrictions were no rejected in previous regressions. By reducing the number of parameters to be estimated, ””

$$E_{t-1}\left(\varepsilon_{it}\varepsilon_{jt}\right)=D_tRD_t,\ D_t=diag\sqrt{h_{iit}} \tag{20}$$

and H_t is defined as in (12). Hence, the conditional variances include a quadratic effect of duration:

$$h_{iit}=\theta_{i1}+\theta_2\varepsilon_{i,t-1}^2+\theta_3h_{ii,t-1}^2+\theta_5D_{it}^2. \tag{21}$$

By setting δ, ϑ, and/or η, and θ_5 to zero, the significance of the different effects of duration on bond returns can be compared. The results appear in Table 11.[9]

Only the quadratic effect of duration on the variance of bond returns, θ_5, remain stable in size and significance throughout Table 11. The price of risk, η, varies between 2.174 and 5.998 and is insignificant even at the ten-percent level in Model G (but significant in the other models). The direct effect of duration on returns, δ, becomes insignificant and negative when the variance is allowed to depend on duration. Duration affects the β-coefficient significantly when there is no direct effect of duration.

All three effects of duration yield significant improvements when added one at a time to the basic model without duration, i.e. models B, C, and D are better than model A. Incorporating a second channel is also useful in two out of three cases as models E and G are significant improvements but model F is not. Adding a third effect of duration (model H) result in significant

[9] In several cases, there are differences between previous models and those referred to in Table x. Several restrictions are imposed from start in order to reduce the number of parameters to be estimated. For instance, no linear effects of duration are allowed and only α_i, β_i, and θ_{i1} are allowed to differ between the bonds.

improvements only relative to model F that excluded effects through the variance and price of risk.

A solid conclusion from Table 11 is that duration definitely belongs in the conditional variance equation. Furthermore, this interest rate risk appears to be priced by the market. There does not appear to be an additional direct effect of duration on average returns once the indirect effect through the variance and price of risk is taken into account. Finally, the results suggest that returns to short investments in long-term bonds react more to movements in the risk free short-term interest rate the longer the duration of the bond.

7 Conclusions

Time series data on returns to short investments in six specific long-term bonds contain unusually rich information about the relationship between bond returns and duration. First, we test two standard term structure models: the local rational expectations hypothesis and the liquidity preference hypothesis. The former is not rejected in the unconditional tests as excess returns are insignificant and the β-coefficients do not differ from unity. In contrast to the typical finding of β-coefficients below unity or even below zero, all our point estimates of the response of long-term bond returns to movements in the riskfree short-term interest rate are above unity.

Once other variables such as duration are added to the empirical model, the local expectations hypothesis is frequently rejected as positive excess returns and slope coefficients significantly above unity are documented. Instead, the liquidity preference hypothesis cannot be rejected for this data set as returns to short investments in long-term bonds increase monotonically with the duration of the bonds. The estimated liquidity premia are quite large. For instance, short investments in 6-7 year bonds yield ten percentage points (annually) above the risk free rate. The increase in return is significant only for durations above six years. Because the point estimates using discrete, biannual duration dummies indicate a smooth, increasing relationship between returns and duration, it can be modelled as a continuous function. The relationship also appears to be quadratic, which is consistent with several theoretical models.

A possible explanation for the finding that returns increase with the duration of the bond is that the *variance* of returns is higher the longer the duration of the bond. The term premia can then be interpreted as rewards for carrying interest rate risk. This hypothesis is tested using a multivariate GARCH-M model. The conditional variances are well described as quadratic functions of duration. Neither the effect of duration on the variance nor the price of risk differ significantly between the bonds, implying that all the bonds behave similarly once differences in duration are taken into account. The term premia obtained are of similar magnitudes the term premia estimated using discrete duration dummies.

Casual observation suggest that the response of returns to movements in the risk free rate increases with the duration of the bond. Estimating a model where the relationship between bond returns and the short-term interest rate is explicitly allowed to depend on duration confirms that this is indeed the case. One additional year of duration increases the β-coefficient with 0.65. This deviation from the local rational expectations hypothesis apparently arises because bond yields do not react less to movements in the short-term rate the longer the duration of the bond, as would be required for the hypothesis to hold. If anything, interest rates react more to changes in the short-term rate the longer the bond, implying that the market expects movements in the short-term interest rate to be not only permanent but magnified in the future.

Using likelihood ratio tests to compare the different channels through which duration may enter into the empirical analysis of long-term bond returns, it is clear that (i) duration has a quadratic effect on the conditional variance (ii) this interest risk is priced by the market (iii) there is a monotonically increasing direct effect of duration on returns but it disappears once the effects through the variance and price of risk are taken into account (iv) the duration of the bond also appears to affect the β-coefficient or the response of returns to bond investments to movements in the risk free short-term rate. Hence, the relationship between returns to investments in bonds and the duration of the bonds is more complex and more important than what has previously been shown.

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Table 2: Unit root tests. Sample 1995:6-2002:12

loan	Interest rates	Bond returns
1033	-3.008	-3.470
1034	-2.925	-8.731
1035	-2.734	-7.448
1037	-2.713	-9.052
1038	-2.594	-6.478
1040	-2.991	-6.783
r1m	-3.734	

ADF unit root test without deterministic trends. Sample: 1995:6-2002:8
1 percent critical value -3.505. 5 percent critival value -2.894.

Table 3: Bond returns and the monthly interest rate

loan	α_i	β_i	H$_0$:$\beta_i = 1$	H$_0$:$[\alpha_i, \beta_i] = [0, 1]$
1033	0.003	3.743	[0.095]	[0.032]
	0.504	(2.282)		
1034	0.013	1.211	[0.961]	[0.573]
	0.856	(4.336)		
1035	0.002	3.362	[0.335]	[0.490]
	0.195	(1.373)		
1037	0.000	4.773	[0.398]	[0.693]
	0.013	(1.089)		
1038	-0.028	12.882	[0.040]	[0.041]
	-1.510	(2.229)		
1040	-0.029	13.252	[0.107]	[0.067]
	-1.217	(1.746)		
$\forall i$	0.000	4.275	[0.082]	[0.228]
	(0.0076)	(3.631)		
H_0	$\alpha_i = \alpha$	$\beta_i = \beta$	$\alpha_i = \alpha, \beta_i = \beta$	
$p(H_0)$	[0.447]	[0.499]	[0.767]	

Estimated equation: $R_{it} = \alpha_i + \beta_i r_t^{1m} + \varepsilon_{it}$
t-values within parenthesis.
p-values within brackets.

Table 4: REH tests across the long-term bonds, estimated $\beta-$coefficients

loan	1034	1035	1037	1038	1040
1033	0.454 (1.168)	0.790 (1.098)	0.559 (3.126)	0.970 (3.248)	0.834 (2.473)
$p(H_0)$	[0.141]	[0.282]	[0.026]	[0.783]	[0.863]
1034		2.004 (2.592)	1.065 (2.111)	0.845 (1.277)	0.818 (1.189)
$p(H_0)$		[0.033]	[0.883]	[0.705]	[0.965]
1035			0.735 (4.300)	0.599 (2.040)	0.629 (1.889)
$p(H_0)$			[0.016]	[0.394]	[0.138]
1037				1.111 (4.887)	0.805 (3.062)
$p(H_0)$				[0.163]	[0.481]
1038					0.858 (3.251)
$p(H_0)$					[0.596]

Estimated equation: $R_{it} = \alpha_i + \beta_i r_t^{1m} + \varepsilon_{it}$
H_0 is the REH, i.e. α=0 and β=1.
t-values within parenthesis.
p-values within brackets.

Table 5: Panel regressions of bond returns against the monthly interest rate and duration

loan	α_i	β_i	H_0: $\beta_i = 1$		
1033	0.001	3.423	[0.063]		
	(0.141)	(2.661)			
1034	-0.022	6.476	[0.173]		
	(-1.534)	(1.613)			
1035	-0.005	3.548	[0.141]		
	(-0.740)	(2.054)			
1037	0.002	4.773	[0.392]		
	(0.013)	(1.084)			
1038	-0.030	10.453	[0.088]		
	(-1.719)	(1.894)			
1040	-0.045	14.193	[0.104]		
	(-1.733)	(1.755)			
$\forall i$	-0.006	5.309	[0.000]	H_0: $\beta_i = \beta$	[0.001]
	(-1.682)	(5.980)			
duration				γ^τ	$p(\gamma^\tau = \gamma^{\tau-1})$
1-3 y				0.001	
				(0.211)	
4-5 y				0.005	[0.069]
				(1.513)	
6-7 y				0.010	[0.051]
				(3.003)	
8-9 y				0.015	[0.046]
				(3.954)	
10-11 y				0.021	[0.150]
				(3.451)	
>12 y				0.026	[0.351]
				(2.153)	

Estimated equation: $R_{it} = \alpha_i + \beta_i r_t^{1m} + \gamma^\tau D_{it}^\tau + \varepsilon_{it}$
t-values within parenthesis.
p-values within brackets.

Table 6: Bond returns and continous duration.

loan	α_i	β_i	γ_i	δ_i
1033	(0.007)	1.654	-0.001	0.000
	(0.740)	(0.786)	(-0.456)	(1.015)
1034	0.107	1.599	-0.023	0.001
	(0.432)	(0.127)	(-0.514)	(0.569)
1035	0.021	1.519	-0.008	0.001
	(0.623)	(0.392)	(-0.829)	(0.957)
1037	0.084	3.088	-0.018	0.001
	(0.661)	(0.316)	(-0.670)	(0.734)
1038	-0.031	11.987	-0.001	0.000
	(-0.706)	(2.365)	(-0.143)	(0.395)
1040	0.068	13.330	-0.028	0.002
	(0.635)	(1.412)	(-1.226)	(1.390)
$\forall i$		3.515	$-4.33*10^{-5}$	$2.12*10^{-5}$
		(3.168)	(-0.279)	(1.860)
H_0	$\alpha_i = \alpha$	$\beta_i = \beta$	$\gamma_i = \gamma$	$\delta_i = \delta$
p-value	[0.879]	[0.382]	[0.167]	[0.866]

t-values in paranthesis
Estimated equation: $R_{it} = \alpha_i + \beta_i r_t^{1m} + \gamma_i D_t + \delta_i D_t^2 + \varepsilon_{it}$
t-values within parenthesis.
p-values within brackets.

Table 7: The unrestricted multivariate GARCH-M

loan	α_i	β_i	η_i	θ_{i1}	θ_{i2}	θ_{i3}	θ_{i4}	θ_{i5}
1033	0.010	1.865	1.189	4.812	0.251	-0.518	4.160	5.280
	(4.242)	(2.592)	(1.875)	(1.305)	(1.395)	-(1.080)	(0.412)	(1.676)
1034	-0.003	4.932	2.840	8.958	-0.163	0.333	-14.83	11.53
	(-0.522)	(2.582)	(0.779)	(0.408)	(-1.738)	(2.525)	(-0.421)	(1.771)
1035	0.003	2.454	2.355	3.525	0.002	0.563	11.68	4.390
	(1.081)	(2.511)	(1.332)	(1.293)	(0.034)	(2.099)	(0.824)	(1.226)
1037	0.006	4.355	1.530	37.97	0.269	0.425	-28.86	11.47
	(1.409)	(2.876)	(1.204)	(1.056)	(2.980)	(1.580)	(-1.452)	(0.988)
1038	-0.009	9.845	0.701	2.917	-0.025	1.035	51.59	-1.194
	(-0.921)	(3.475)	(0.362)	(2.716)	(-0.369)	(7.920)	(2.189)	(-1.181)
1040	0.004	10.398	3.726	40.78	0.097	0.283	-3.664	1.235
	(0.861)	(3.237)	(1.455)	(1.962)	(1.281)	(1.164)	(-0.820)	(1.385)
Scaling*				$*10^{-6}$			$*10^{-7}$	$*10^{-7}$

Estimated equations: $R_{it} = \alpha_i + \beta_i \underline{r_t^{1m}} + \eta_i E_{t-1} h_{iit} + \varepsilon_{it}$,
$E_{t-1}(\varepsilon_{it}\varepsilon_{jt}) = D_t R D_t,\ D_t = diag\sqrt{h_{iit}}$
$h_{iit} = \theta_{i1} + \theta_{i2}\varepsilon_{i,t-1}^2 + \theta_{i3} h_{ii,t-1}^2 + \theta_{i4} D_{i,t} + \theta_{i5} D_{i,t}^2$.
* Parameter estimates in these columns are multiplied with scaling factors (given in a separate row due to space limitations)

Table 8: The restricted GARCH-M model

loan	α_i	β_i	η	θ_{i1}	θ_2	θ_3	θ_5
1033	0.009	1.355		$7.660*10^{-6}$			
	(3.274)	(1.569)		(1.021)			
1034	-0.005	6.291		$3.403*10^{-6}$			
	(-0.613)	(3.676)		(1.227)			
1035	0.004	3.038		$1.211*10^{-6}$			
	(1.153)	(3.214)		(0.681)			
1037	-0.002	5.436		$5.119*10^{-5}$			
	(-0.331)	(3.583)		(1.360)			
1038	-0.018	9.904		$3.657*10^{-5}$			
	(-1.772)	(3.390)		(1.884)			
1040	-0.014	11.337		$4.036*10^{-5}$			
	(-0.917)	(3.916)		(1.859)			
$\forall i$			2.174		0.154	0.448	$1.496*10^{-6}$
			(1.926)		(6.411)	(5.093)	(3.375)

Estimated eqations: $R_{it} = \alpha_i + \beta_i r_t^{1m} + \eta E_{t-1} h_{iit} + \varepsilon_{it}$,
$E_{t-1}(\varepsilon_{it}\varepsilon_{jt}) = H_t$,
$h_{ii,t} = \theta_{i1} + \theta_2 \varepsilon_{i,t-1}^2 + \theta_3 h_{ii,t-1}^2 + \theta_5 D_{i,t}^2$

Table 9: Estimated conditional correlations

loan	1033	1034	1035	1037	1038
1034	-0.0396				
1035	0.166	0.667			
1037	0.218	-0.057	0.143		
1038	0.394	-0.148	0.064	0.151	
1040	0.197	0.081	0.755	0.431	0.765

$h_{ijt} = \rho_{ij}\sqrt{h_{iit}}\sqrt{h_{jjt}}$ are the non-diagonal elements of the covariance matrix.

Table 10: Duration and the effect of the monthly interest rate on bond returns

loan	α_i	β_i	ϑ_i
1033	0.010	-0.345	0.479
	(1.809)	(-0.186)	(3.656)
1034	0.006	-5.020	0.836
	(0.189)	(-0.319)	(1.028)
1035	0.004	-0.221	0.512
	(0.406)	(-0.057)	(1.789)
1037	-0.027	-4.523	0.643
	(-1.778)	(-0.473)	(1.141)
1038	-0.027	6.714	0.801
	(-1.778)	((1.339)	(3.091)
1040	-0.038	5.557	1.260
	(-1.290)	(0.595)	(2.473)
$\forall i$			0.649
			(5.098)
H_0:$\vartheta_i = \vartheta$	$p-$value	[0.659]	

Estimated equation: $R_{it} = \alpha_i + \beta_i r_t^{1m} + \vartheta_i D_t r_t^{1m} + \varepsilon_{it}$

Table 11: Nested models of the effects of duration on returns

	δ	ϑ	η	θ_5	Log L	$p(\chi^2(j))$
Model A					1117.980	
Model B	1.951*10^{-5}				1128.153	20.346
	(4.619)					[0.000]
Model C			2.174	1.496*10^{-6}	1132.591	29.222
			(1.926)	(3.375)		[0.000]
Model D		0.645			1125.974	15.989
		(5.098)				[0.000]
Model E	-2.806*10^{-5}		5.998	1.756*10^{-6}	1135.207	5.232
	(-0.547)		(1.656)	(3.752)		[0.022]
Model F	2.583*10^{-5}	0.253			1128.314	0.322
	(2.189)	(0.474)				[0.570]
Model G		0.442	4.076	1.554	1135.436	5.690
		(4.520)	(1.735)	(5.857)		[0.017]
Model H	-3.100*10^{-5}	0.723	3.576	1.528	1135.727	60.582
	(-1.0679	(1.482)	(1.647)	(2.518)		[0.446]

In the final column, models B, C, and D are compared to model A. Models E and G are compared to model C, model F to model B, and model H to model G.. Estimated equations: $R_{it} = \alpha_i + \beta_i r_t^{1m} + \delta D_{it}^2 + \vartheta D_{it} r_t^{1m} + \eta E_{t-1} h_{iit} + \varepsilon_{it}$,$h_{ii,t} = \theta_{i1} + \theta_2 \varepsilon_{i,t-1}^2 + \theta_3 h_{ii,t-1}^2 + \theta_5 D_{it}^2$.

Figure 1: The one-month interest rate 1993-2002

Figure 2: Returns to investments in bonds 1033, 1034, and 1035

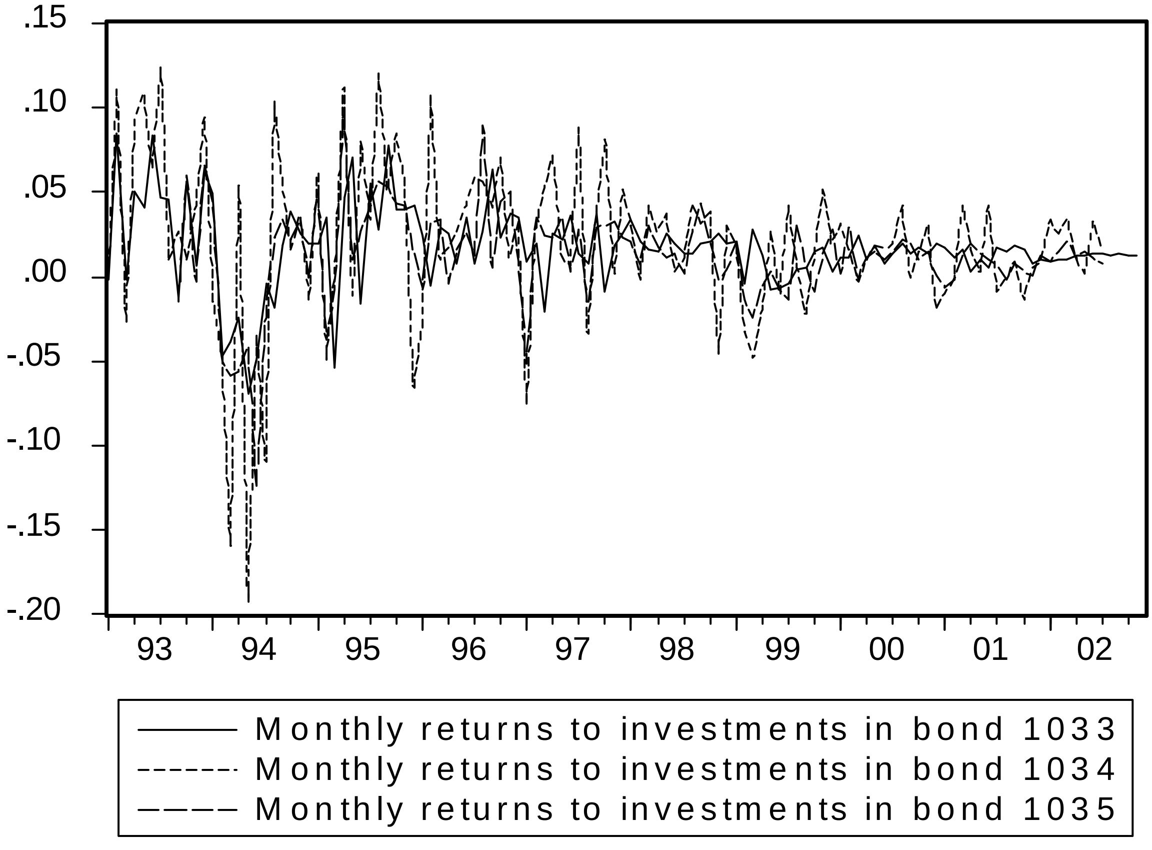


Figure 3: Returns to investments in bonds 1037, 1038, and 1040

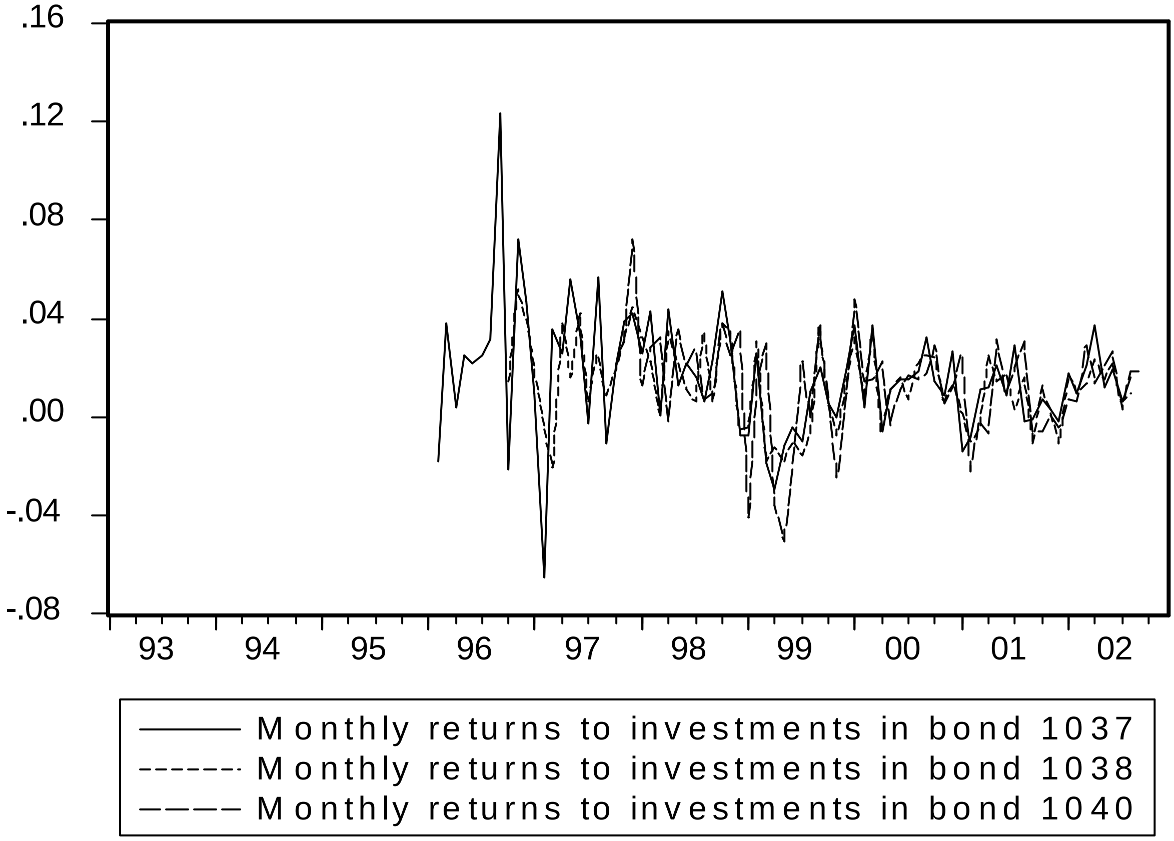


Figure 4: Liquidity premia estimated from discrete duration dummies and 95% confidence intervals

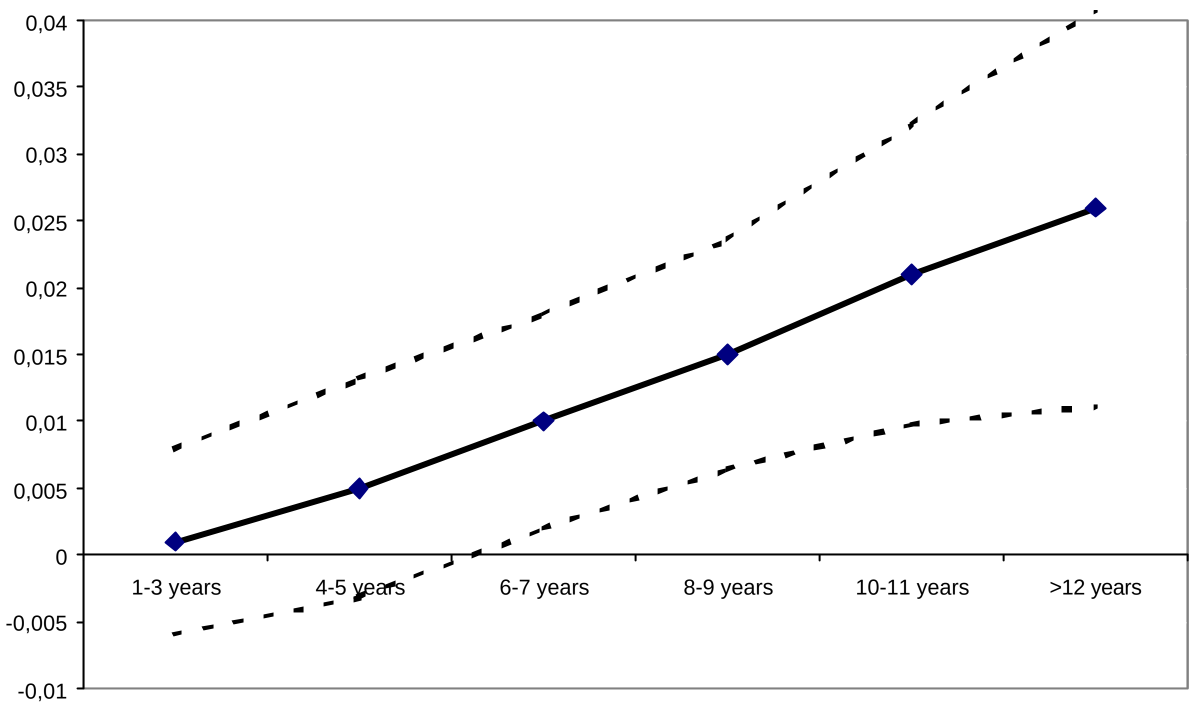
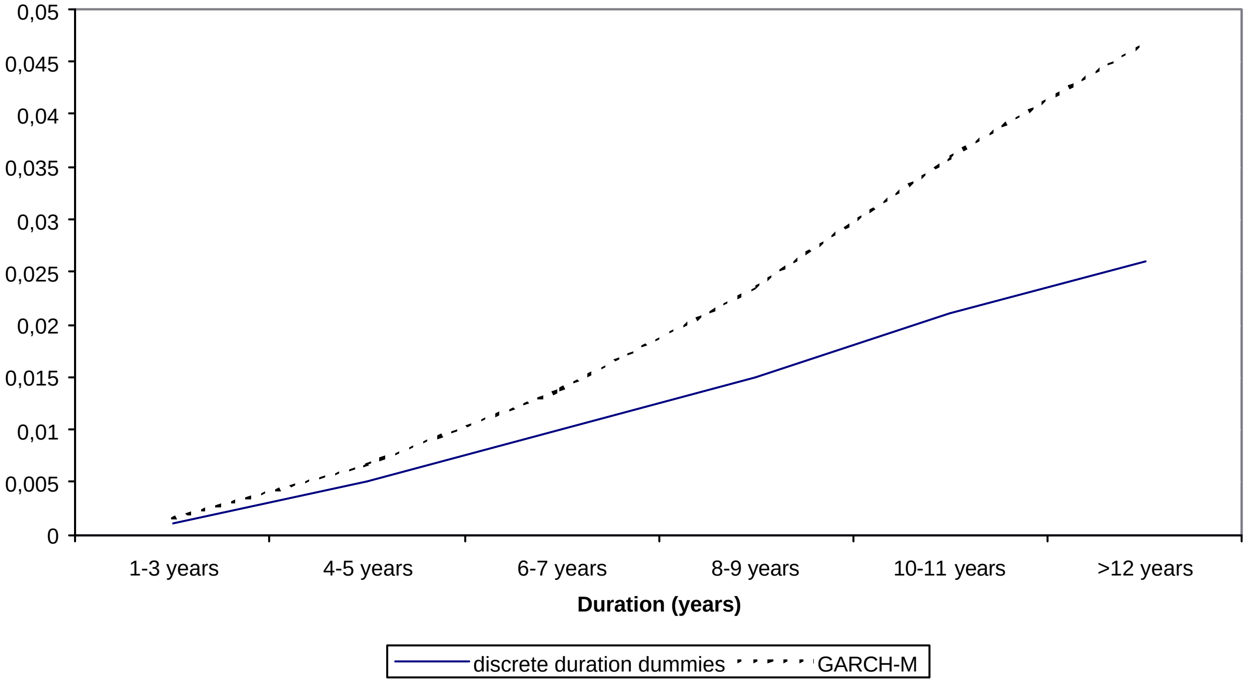


Figure 5: Liquidity premia estimated from discrete duration dummies and from the GARCH-M



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