Optimal Hedging Strategies for the U.S. Cattle Feeder

Mikhail A. Noussinov and Raymond M. Leuthold

Multiproduct optimal hedging for simulated cattle feeding is compared to alternative hedging strategies using weekly price data for 1983–95. Out-of-sample means and variances of hedged feeding margins using estimated hedge ratios for four commodities suggest that there is no consistent domination pattern among the alternative strategies, leaving the hedging decision up to the agent's degree of risk aversion. However, all hedging strategies significantly reduce the feeding margin's means and variances compared to no hedging, with variance reduction always exceeding 50%. Hedging results appear quite sensitive to the data set and its size.

Key Words: cattle feeding, hedge ratios, hedging strategies, multiproduct hedging, optimal hedging

The beef industry represents a major economic activity in the United States economy. Within that industry, the cattle feeding segment is characterized by wide swings in profits. According to Jones, net returns from finishing yearling steers in Kansas ranged from a loss of $120 to a profit of $178 per head during the period from 1981 through 1994.

This high profit volatility is due principally to multiple market price risks. Cattle feeders bear the risk of unfavorable price movements both in the input markets, comprised of feeder cattle, feed (corn and soybean meal), and interest costs, and in the output market, fed cattle. Trapp and Cleveland found that volatility in the market prices for both fed and feeder cattle explained 65.5% of the production margin volatility, compared to 22% attributable to production risks. The high volatility in market prices raises an obvious need for price risk management.

Numerous studies have shown that hedging some or all of the inputs and output in cattle feeding provides considerable success in managing these price risks, resulting typically in improved and less variable feeding margins. These studies usually employed either a naive hedge (one-to-one), or hedge ratios, computed individually for each commodity. No one has estimated multiple hedge ratios for the cattle feeding production complex as a whole, taking the covariances among

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different cash and futures prices into account. While the theory of optimal hedging has been widely discussed in the literature and applied to several production processes, it has not been applied to the cattle feeding industry.

This research evaluates hedging strategies for U.S. cattle feeders, dealing in multiple commodities and futures contracts, utilizing recent developments in multiproduct hedging theory. While both multiproduct and optimal single-commodity hedging have been widely studied, only a few papers have compared the two.

The primary objective of this research is to analyze whether the multiproduct hedging theory has any value for cattle feeders, and can be practically implemented to reduce price risks associated with cattle feeding. First, we simulate a custom feedlot using actual cash and futures prices over the period 1983–95. Then we estimate hedge ratios for the simulated feedlot using optimal hedging theory, as well as various alternative hedging and marketing strategies commonly employed, such as the naive hedge, no hedge, single-commodity hedge, and proportional hedge.

Finally, we investigate whether the optimal multiproduct hedge generates better results as compared to the alternative strategies. Applying estimated hedge ratios out-of-sample, a standard mean-variance framework is used to compare one-period optimal hedges against several alternative hedging strategies.

Previous Studies

Earlier Studies: Hedging Individual Commodities

Numerous studies have assessed the economic effectiveness of hedging cattle with futures in the United States. A comprehensive review of the major findings in the early literature was provided by Leuthold and Tomek in 1979. They noted that livestock hedging studies typically simulate a feedlot operation, with naive hedging being primarily used. Subsequently, several articles have refined this approach, increasing the types of strategies.

Shafer, Griffin, and Johnson, and Leuthold and Mokler were the first to introduce simultaneous hedging for the cattle feeding industry. The hedges consist of taking long positions in input commodities (feeder animal and feed, usually corn) and a short position in the fed cattle, thus locking in a profit margin. Leuthold and Mokler clearly demonstrated that hedging feed, feeders, and fed animals simultaneously is more efficient than no hedge and an output-only hedge. Kenyon and Clay showed similar results for hogs, and also demonstrated the importance of margin requirements and interest expenses. Gorman et al. reported that even when hedging fed cattle only, the operator could limit losses by almost 50%. Finally, Caldwell, Copeland, and Hawkins, and Carter and Lyons provided details peculiar to hedging in U.S. markets for a Canadian cattle producer.

Recent Studies: Optimal Hedging

While the optimal hedging theory for a producer dealing in multiple futures and cash commodity markets was introduced in the late 1970s and early 1980s, it has never been applied to the cattle feeding industry. In one of the first papers, Anderson and Danthine argued that the hedging and production decisions should be made jointly, or simultaneously. The theory states that optimal positions depend on the covariances among the cash and futures prices and the covariances of the futures prices themselves, with cash and futures positions then being determined simultaneously.

Peterson and Leuthold used a portfolio framework to find optimal cash and futures positions for a commercial feedlot operator. Myers and Thompson elaborated on a generalized approach for estimating optimal hedge ratios for a single-commodity case. They emphasized that the hedging decision should be made using a hedge ratio conditioned on the information available when the hedge is placed. Finally, Fackler and McNew expanded the approach of the latter study into multiple cash and futures positions, laying the framework for our analysis.

Fackler and McNew clarified this theory to a multicommodity case and applied it to Central Illinois soybean processors, demonstrating significant advantages for multicommodity optimal hedging relative to simpler strategies. The authors estimated the optimal futures positions using a likelihood model for the joint cash and futures price process. They also demonstrated that hedging and production decisions could be separated for a production process using inputs and outputs in fixed proportions. The optimal futures positions were determined based on the optimal cash position, using the seemingly unrelated regression (SUR) approach. Fackler and McNew's study provides a persuasive argument for applying this methodology to the cattle feeding industry. Cattle feeding, however, differs significantly from the soybean industry in that the production process and time lags are much longer.

Optimal Hedging Framework

The optimal hedging theory outlined here follows the work of Fackler and McNew. Assume that agents maximize the following objective function:

$$(1) \qquad \varphi = E(\pi) - \frac{\lambda}{2} \, \text{Var}(\pi),$$

where $E(\cdot)$ and $\text{Var}(\cdot)$ are the expectation and variance operators, λ is a measure of the agent's degree of risk aversion, and π is a profit function.

U.S. cattle producers have at least five futures markets available to them for price risk management, including contracts on fed and feeder cattle, corn, soybean meal, and T-bill futures. For the simplest case of one holding period, with the hedge placed at the beginning and lifted at the end, the company's profit function can be written as:

$$(2) \qquad \pi = \mathbf{P}'\mathbf{Q} - \mathbf{C}(\mathbf{Q}) + (\mathbf{F} - \mathbf{f})\mathbf{Z},$$

where $\mathbf{Q}$ and $\mathbf{Z}$ are $\{m \times 1\}$ and $\{p \times 1\}$ vectors representing commodity and futures positions, respectively; $\mathbf{P}$ and $\mathbf{F}$ are random vectors of period-end cash and futures prices, respectively (with negative signs representing purchases for commodities or short positions for futures contracts); $\mathbf{f}$ is a vector of futures prices at the beginning of the period; and $\mathbf{C}(\mathbf{Q})$ is a cost function.[1]

Using the mean-variance framework, the firm's problem can now be represented as follows:

$$(3) \qquad \max_{\mathbf{Q},\mathbf{Z}} \varphi = \mathbf{Q}'E(\mathbf{P}) - \mathbf{C}(\mathbf{Q})$$

$$- \frac{\lambda}{2}\left(\mathbf{Q}' \sum_{PP} \mathbf{Q} + 2\mathbf{Q}' \sum_{PF} \mathbf{Z} + \mathbf{Z}' \sum_{FF} \mathbf{Z} \right).$$

First-order necessary conditions for this system express the optimal futures position $\mathbf{Z}$ in terms of the optimal cash position $\mathbf{Q}$. A hedged position can be expressed in terms of the fraction of the cash commodity offset in the futures market. This implies that each cash commodity is offset with the corresponding futures contract, so the $\{m \times 1\}$ vector of hedge ratios $\mathbf{H}$ can be written as:

$$(4) \qquad \mathbf{H} = -\left[diag(\mathbf{Q}) \right]^{-1}\mathbf{Z} = \left[diag(\mathbf{Q}) \right]^{-1} \sum_{FF}^{-1} \sum_{FP} \mathbf{Q}.$$

In the case when all commodities are always held in fixed proportions, as it often is in the cattle feeding industry, the vector $\mathbf{Q}$ is always proportional to some vector $\mathbf{A}$, determined by the technological process. This allows us to rewrite the hedge ratios vector as:

$$(5) \qquad \mathbf{H} = \left[diag(\mathbf{A}) \right]^{-1} \sum_{FF}^{-1} \sum_{FP} \mathbf{A}.$$

The theory discussed here assumes that the firm establishes both cash and futures positions at time zero, and has no opportunity to adjust those positions later. From equation (5), the optimal hedge ratios vector depends on the term:

$$(6) \qquad \mathbf{M} = \sum_{FF}^{-1} \sum_{FP}.$$

To estimate $\mathbf{M}$, we consider a general case of a multivariate normally distributed random variable. Following Fackler and McNew, we express a log-likelihood

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[1] $\mathbf{C}(\mathbf{Q})$ represents cash goods production relations. It is interpreted as the cost of producing the cash positions $\mathbf{Q}$, valued at the period-end. Assuming the production possibilities to be certain, $\mathbf{C}(\mathbf{Q})$ becomes nonstochastic.

function, and then maximum-likelihood estimates of **M** can be obtained using SUR. Assuming that the futures prices are unbiased, $E(\mathbf{F}) = \mathbf{f}$, the estimation model is written:

(7) $$\mathbf{P}_t = \Delta \mathbf{F}_t \mathbf{M} + \mathbf{X}_{1t} \boldsymbol{\beta}_1 + e_t,$$

where Δ is a difference operator, $\Delta \mathbf{F} = \mathbf{F} - \mathbf{f}$, **M** is a $\{p \times m\}$ matrix of coefficients, and **X** is a vector of exogenous variables. In the case when cash commodities are always held in fixed proportions ($\mathbf{Q} \sim \mathbf{A}$), one can use production margin ($\mathbf{P}_t^C = \mathbf{P}_t \mathbf{A}$) as a dependent variable in a single-equation estimation:

(8) $$\mathbf{P}_t^C = \Delta \mathbf{F}_t \mathbf{h} + \mathbf{X}_{1t} \boldsymbol{\beta}^* + e_t,$$

where $\boldsymbol{\beta}^* = \boldsymbol{\beta}_1 \mathbf{A}$. This may lead to efficiency loss from using ordinary least squares (OLS) rather than SUR; however, if the same set of regressors is used in every equation, SUR becomes equivalent to OLS, and no efficiency loss occurs.

Methodology and Data

Feeding Scenario

For this research, we analyze only the final stage of the production sequence for cattle—when a yearling is fattened in a feeding program lasting about four months (120 days). During this period, a 700-pound steer bought by a feedlot operator consumes 42 bushels of corn and 100 pounds of soybean meal, the latter serving as a high-protein additive. Assuming a gain of 3.3 pounds a day, the animal's terminal weight is 1,100 pounds, which conforms to the USDA Choice requirements.

In this model, the feedlot operator buys one lot of feeder cattle every week to begin a feeding process. One month prior to that, in order to fix the feeding margin, the operator hedges both the inputs (feeder cattle, corn, and soybean meal) and output (fed cattle).[2] Then, hedges are lifted at the same time as when respective cash transactions are made. That is, hedges on inputs are held for one month, until the inputs are actually purchased, and the fed cattle hedge is held for five months, until fed cattle are sold.[3] Since this is a one-period model, the futures positions are not adjusted during the life of the hedge. It is also assumed that the operator does not make any transactions with the lot of cattle while on feed, and that cattle are on feed for the full four months regardless of the market situation.

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[2] Hedging borrowing costs is theoretically appealing; however, we found it to be impractical in this particular scenario because of the short hedging period combined with relatively low variability of short-term interest rates.

[3] It is these lags that make multicommodity hedging in the cattle feeding complex uniquely different from the soybean complex, as in Fackler and McNew.

For example, on January 1st, the operator makes a decision about feeding to begin on February 1st. The operator buys March futures on feeder cattle, corn, and soybean meal, and sells August live (fed) cattle futures. The operator is presumed to hedge in all cases, even if the feeding margin is negative. Then, at the beginning of February, animals and feed are purchased as respective hedges are lifted, and feeding begins.[4] Finally, four months after feeding commenced, the fed animals are sold and the fed cattle hedge is lifted. The same process is then repeated beginning on January 8, 15, 22, etc. Futures contracts are selected in a manner to ensure that each will be the nearby contract, but not in its maturity month, when the established hedge is lifted either one or five months later. The same contracts are used throughout the whole "decision-making" month.

Empirical Implementation

For this particular analysis, the second term on the right-hand side of equation (8) can be expanded to include the lagged values of cash and futures prices in the commodities involved and monthly dummies to account for seasonal effects. This yields the following model:

$$(9) \qquad \mathbf{P}_t^C = \Delta \mathbf{F}_t \mathbf{h} + \sum_{j=1}^{q} \mathbf{p}_{t-j} \mathbf{a}_j + \sum_{j=1}^{q} \mathbf{f}_{t-j} \mathbf{b}_j + \sum_{j=1}^{12} \mathbf{D}_{jt} \phi_j + e_t,$$

where $\mathbf{P}_t^C$ is the feeding margin; $\Delta \mathbf{F}_t = \mathbf{F}_t - \mathbf{f}_{t-1}$ is a vector of gains/losses from futures positions; the subsequent summation terms represent $\{4 \times 1\}$ vectors of lagged cash (**p**) and futures (**f**) prices, and monthly dummy variables (**D**), respectively; and e is a random normally distributed error term. The coefficients **h**, **a**, and **b** represent vectors also, and ϕ_j is a scalar. The appropriate lags for cash and futures prices will be determined by the standard minimum Bayesian information criterion (BIC) (Harvey).

Alternatives to optimal hedging include either hedging each commodity individually, or hedging all of them in the same proportion. For individual commodity hedging, the model can be stated as follows:

$$(10) \qquad P_t^{(\alpha)} = \Delta F_t^{(\alpha)} h^{(\alpha)} + \sum_{j=1}^{q_\alpha} p_{t-j}^{(\alpha)} a_j^{(\alpha)} + \sum_{j=1}^{q_\alpha} f_{t-j}^{(\alpha)} b_j^{(\alpha)}$$

$$+ \sum_{j=1}^{12} D_{jt} \phi_j^{(\alpha)} + e_t,$$

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[4] Peterson and Leuthold experimented with buying corn at the beginning of and during the feeding period, but offered no evidence that such behavior affected the results.

where $P_t^{(\alpha)}$ is cash price of α, with the rest of the terms becoming scalars instead of vectors, and α representing either feeder cattle, corn, soybean meal, or fed cattle. In this case, all equations are estimated independently.

Finally, in the case of hedging all commodities in the same proportion,[5] the model above is used to estimate the hedge ratio for the "commodity" $\alpha = f$ of feeding services, with the terms written for the cash and futures gross feeding margin. Thus, equation (10) takes the form of a single-commodity hedge, with the commodity being feeding services. Naive hedge ($h = 1$) and no hedge ($h = 0$) can be considered as special cases of this model.

Testing Hedging Effectiveness

To verify whether multiproduct hedging has advantages over simpler approaches, all hedging alternatives are compared within the mean-variance framework. A strategy is considered superior to another if, in addition to variance reduction in the hedged production margin, the strategy also provides a higher conditional mean for the hedged margin.

Specifically, hedge ratios from all the models, computed every week, are used to hedge out-of-sample the next period's feeding margin. Based on these hedged feeding margin series, one can compute the resulting means and variances for comparison. The ratios are computed using either all prior data available, or only the previous four or six years of data.

In addition, another measure of hedging effectiveness is the variance reduction of the hedged position compared to an unhedged position. This traditional measure is computed using the same margins as above, as follows:

$$(11) \qquad\qquad \alpha = 1 - \frac{\mathrm{Var}(\mathit{hedged})}{\mathrm{Var}(\mathit{unhedged})} .$$

While this measure provides a convenient and useful gauge of risk reduction, using it as the only grounds for comparison presents a limited picture, since it does not take into consideration the corresponding reduction in the hedged margin's means. It supplements our evaluation of hedging effectiveness.

Data

The sample covers the period from January 1, 1983 through December 31, 1994. Since the largest trading volume of cash fed cattle is typically observed on

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[5] The proportional model was originally formulated for similar-commodity complexes, such as the soybean complex or crude oil complex. It is introduced here for its simplicity and to provide an additional benchmark for comparison (see Fackler and McNew).
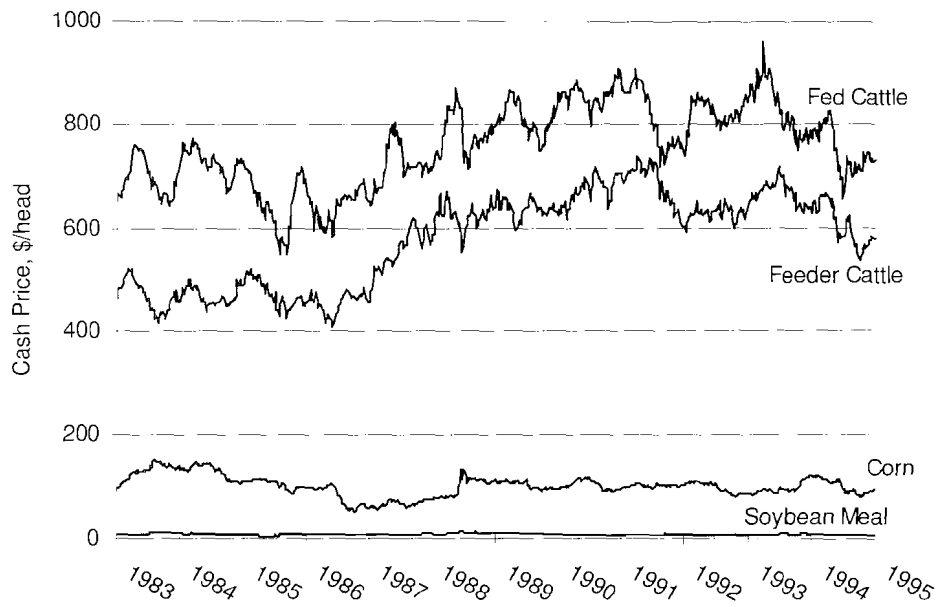
Figure 1. Cash prices for feeder cattle, corn, soybean meal, and fed cattle (dollars per head)
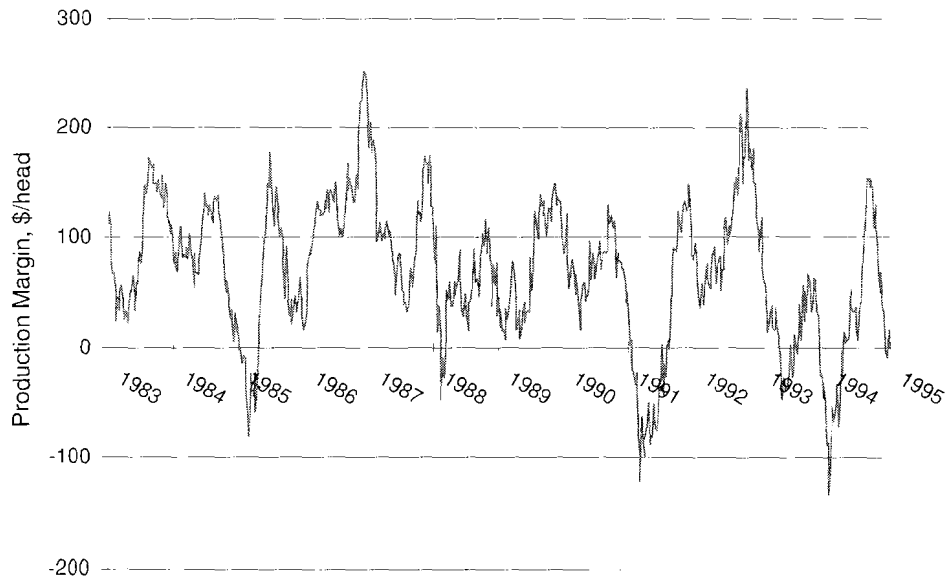

Figure 2. Production margin (dollars per head)

Wednesdays, Wednesday price data are used in the sample. The sample contains 626 observations.[6]

All prices are multiplied by the amounts of the respective commodities necessary to produce one animal, thus expressing the price data on a per head basis. For example, the fed cattle price, expressed in $/cwt, is multiplied by 11.0 to yield the price of a fed animal. With prices expressed in these units, vector **A** in equation (5) becomes {-1, -1, -1, 1}, facilitating direct hedge ratio computation. The producer's gross feeding margin is then computed for each of the 626 feeding observations as the difference between the price of fed animal and the cost of inputs (i.e., cost of feeder animal, corn, and soybean meal), with fixed production costs being ignored.

Corn and soybean meal cash prices are from Central Illinois, national benchmarks. Fed cattle cash prices represent Texas-Oklahoma, and feeder cattle prices are from Oklahoma City, both widely available and acceptable representations. Futures prices for all four products come from the appropriate exchanges in Chicago. Assuming fixed transportation costs for the period of this study, the spatial data distribution can be ignored by incorporating the transportation costs into the cost function $C(Q)$.

Cash prices for feeder cattle, corn, soybean meal, and fed cattle, expressed in dollars per head, are presented in figure 1. The figure clearly shows that the primary cost component for a producer is the cost of the feeder animal, which is on average 5.8 times more than the cost of the next most important input, corn. Finally, the cost of soybean meal is less than 2% of the cost of the feeder animal. All prices exhibit high variability, especially feeder and fed cattle.

The producer's gross feeding margin is shown in figure 2. This is perhaps the most convincing argument for the importance of hedging for cattle feeders. Exhibiting highly cyclical behavior, the margin varies from -$133 to $250 per head, with an average of $70 per head. While the graph serves its purpose of demonstrating high variability of the gross feeding margin, the absolute values of the margin do not accurately reflect the profits actually earned by producers because they do not include other production costs.

Results

Empirical Ratio Estimation

Utilizing equation (9) on the full sample of 626 weekly feeding simulations, the estimated coefficients are summarized in table 1. Based on the minimum Bayesian information criterion, the optimal number of lags for these multiproducts is one

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[6] For a feeding scenario planned in December 1994, cattle were put on feed in January 1995, and were sold in May 1995, with the June 1995 fed cattle futures contract used for hedging. Therefore, the actual data sample extends through the first half of 1995.

Table 1. Estimated Parameter Values for the Multiproduct Optimal Hedging Model (1983–94)

Variable	Coefficient	Std. Dev.	t-Ratio
Current Futures:			
Feeder Cattle	-0.663	0.078	-8.500
Corn	-1.036	0.261	-3.969
Soybean Meal	-2.174	2.679	-0.811
Fed Cattle	1.011	0.034	29.394
Lag-1 Futures:			
Feeder Cattle	0.137	0.143	0.958
Corn	-1.063	0.387	-2.747
Soybean Meal	-5.506	3.588	-1.535
Fed Cattle	0.076	0.111	0.685
Lag-1 Cash:			
Feeder Cattle	-0.424	0.067	-6.328
Corn	0.830	0.308	2.695
Soybean Meal	6.612	2.979	2.220
Fed Cattle	0.030	0.034	0.882
Monthly Dummies:			
MO_{01}	175.553	21.224	8.272
MO_{02}	158.498	21.125	7.503
MO_{03}	137.339	20.894	6.573
MO_{04}	142.616	20.830	6.847
MO_{05}	142.517	21.870	6.517
MO_{06}	146.915	21.870	6.718
MO_{07}	173.112	21.927	7.895
MO_{08}	177.197	22.377	7.919
MO_{09}	183.503	23.145	7.928
MO_{10}	170.207	23.500	7.243
MO_{11}	201.980	21.952	9.201
MO_{12}	191.753	21.679	8.845

week. Notice the low t-ratio value for the current soymeal futures hedge ratio, which is not significantly different from zero.

Most notable is the hedge ratio for feeder cattle (0.663) being significantly less than one, based on the 97.5% confidence interval. Therefore, the cattle feeder who implements the multiproduct optimal hedging model will be substantially less than

Table 2. Alternative Estimated Hedge Ratios for Feeder Cattle, Corn, Soybean Meal, and Fed Cattle (1983–94)

Model / Commodity	Hedge Ratio	Standard Error	t-Ratio ($H_0 = 0$)	t-Ratio ($H_0 = 1$)
Multiproduct Hedge:				
Feeder Cattle	0.663	0.078	8.500	4.321
Corn	1.036	0.261	3.969	0.138
Soybean Meal	2.174	2.679	0.811	0.438
Fed Cattle	1.011	0.034	29.735	0.324
Proportional Hedge:	0.970	0.033	29.394	0.909
Single-Commodity Hedge:				
Feeder Cattle	0.911	0.043	21.186	2.070
Corn	0.978	0.029	33.724	0.759
Soybean Meal	0.983	0.031	31.710	0.548
Fed Cattle	1.095	0.033	33.182	2.879

fully hedged in feeder cattle. This result can be attributed to the positive correlation between feeder and fed cattle prices, and to the fact that some of the feeder cattle price risks are being offset in the fed cattle market positions. Based on their confidence intervals, ratios for corn and fed cattle do not differ significantly from one, hence being statistically indistinguishable from naive, one-to-one hedges.

In order to compare these optimal ratios, all alternative hedge ratios are shown in table 2. Along with repeating optimal ratios from the multiproduct optimal model, the table lists ratios from the single-commodity hedging and proportional hedging models.

For the single-commodity hedges, the minimum BIC procedure yielded lags of $q = 4$ for feeder cattle and $q = 1$ for the other commodities. The single-commodity hedge ratios for both corn and soybean meal do not differ significantly from one, based on 95% confidence intervals. Meanwhile, the hedge ratio of 0.911 for feeder cattle is significantly less than one, while the ratio of 1.095 for fed cattle is significantly greater than one. This suggests that a feeder interested in hedging feeder cattle alone will be slightly less than fully hedged, while someone hedging fed cattle alone will be a little more than fully hedged.

For comparison, the proportional hedging model generates a ratio of 0.97, which is not significantly different from one. Therefore, if the hedger decides to hedge everything in the same proportion, the operator will be slightly less than fully hedged in all commodities.

Table 2 reports the main difference among the models: while traditional models tend to have hedge ratios very close to one, the multiproduct optimal hedging model

suggests that the producer can be less than fully hedged in feeder cattle, the principal input in the production process. Also, the hedge ratio for soybean meal exceeds two, but is not significant, suggesting that the other hedges and market positions carry the price risk in meal.

Next, we examine the optimal hedge ratios continuously over time. Starting with a data sample covering the first five years, and then extending the sample by one observation every week, we compute new hedge ratios based on the available information. Each ratio is computed based on all information available at the moment of computation, resulting in a weekly series of hedge ratios over the period January 1, 1988 through January 1, 1995. The ratios for feeder cattle, corn, and fed cattle are shown in figure 3, panels A–C, respectively. Since the hedge ratio for soybean meal is not significant, it is not included in this illustration.

Figure 3 shows that for feeder and fed cattle, hedge ratios computed according to the multiproduct optimal model are consistently lower than those provided by the single-commodity hedging. The proportional hedge ratio, identical in each graph, has an upward trend, asymptotically approaching one toward the end of the sample. The multiproduct ratio for corn (figure 3, panel B) is higher than both the single-commodity ratio and the proportional ratio. It is also less stable, but it asymptotically approaches one toward the end of the sample. Finally, for the fed cattle ratios, the single-commodity hedge ratio is consistently above one (at the level of approximately 1.1), whereas the multiproduct optimal hedge ratio is below one for most of the sample, with an upward trend.

To determine if the ratios are equal to each other, an F-test compares the unrestricted system [equation (9)] with the restricted system [equation (10)], with α denoting the cattle feeding services (essentially assuming $h_{FC} = h_C = h_{SM} = h_{LC}$). The test statistic is distributed as $F(3, 602)$ and has a value of 7.22, which exceeds the 0.99 percentile of 3.78. Therefore, the hypothesis of hedging all commodities in the same proportion should be rejected relative to the optimal model.

For comparison, hedge ratios for all the alternative models were also computed based on the most recent four and six years of data rather than on an expanding set of data. These hedge ratios computed on shorter and fixed sample sizes are much less stable compared to those computed on an expanding sample. Specifically, the ratios computed based on a four-year-long sample exhibit more variability than the ratios computed on a six-year-long sample, which, in turn, are still more volatile than the ratios computed on the growing sample. This suggests that sample size can have an important effect on the hedge ratio size and stability.

Effectiveness Testing

The variance reduction resulting from hedging is analyzed simultaneously with the corresponding change in profits, using out-of-sample testing. Hedge ratios for all four commodities computed for alternative models in the previous section were used to hedge one-step-ahead margins, using the actual prices, from 1989 through 1994,
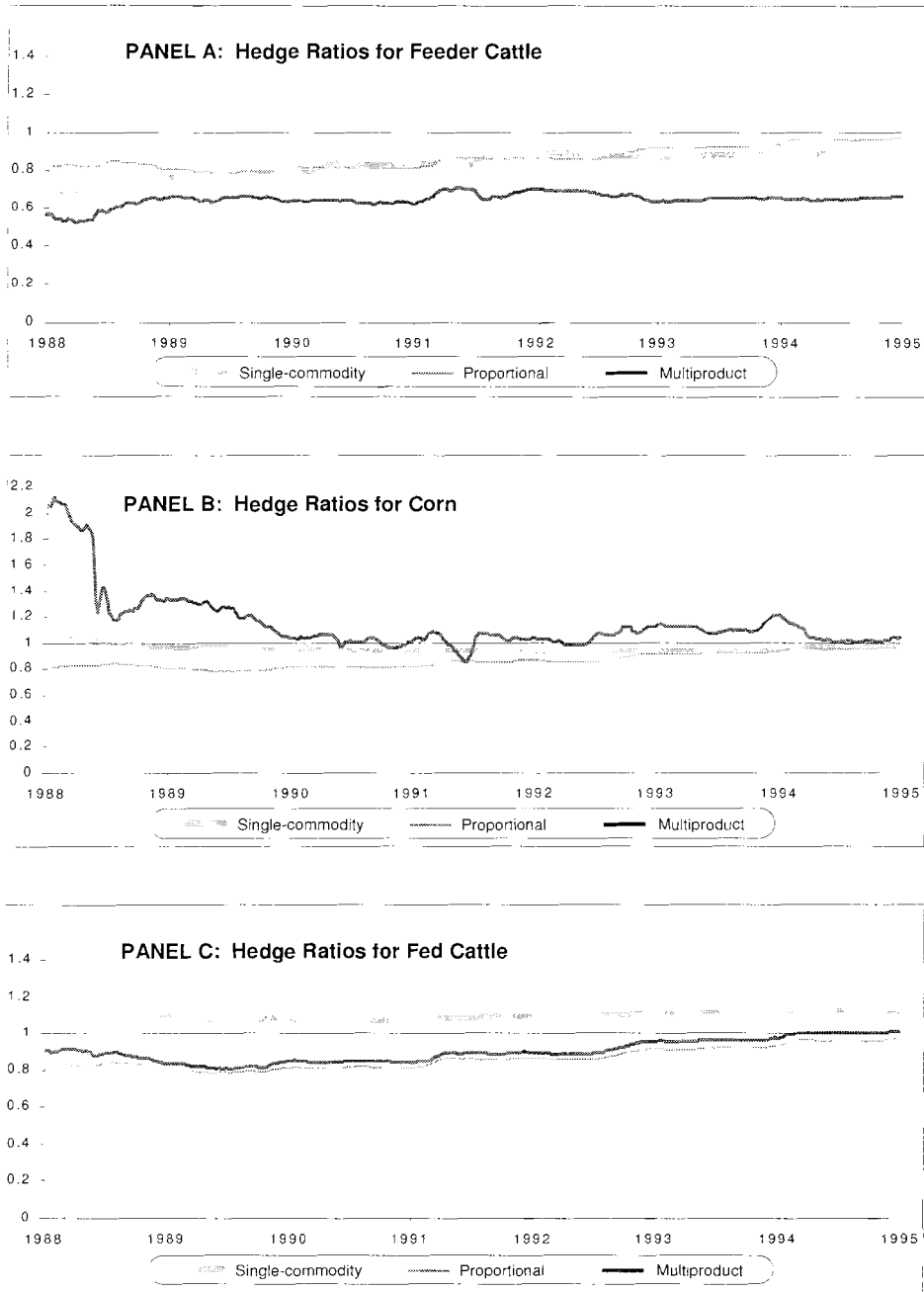
**Figure 3. Hedge ratios for feeder cattle, corn, and fed cattle
(computed on all prior data available)**

Table 3. Gains/Losses of Gross Feeding Margin (P^C) and Changes in Values of Futures Positions on Per Head Basis, 1989–94 (n = 293)

Description	P^C	ΔF_{FC}	ΔF_C	ΔF_{SM}	ΔF_{LC}
Minimum	-133.8	-46.0	-21.53	-1.58	-123.2
Maximum	236.1	48.5	13.44	1.95	122.1
Median	60.7	4.4	-0.53	-0.09	20.6
Mean	53.2	3.6	-0.78	-0.05	15.9
Standard Deviation	68.9	14.5	5.05	0.43	39.7

Note: Units represent dollars per animal. Changes in futures represent value per head when hedge is lifted minus value when hedge is placed.

for a total of 293 observations. These hedges were held either one month for the three inputs, or five months for fed cattle. Values for the hedged production margin were calculated by adding gains or losses from the futures positions (ΔFh) to the producer's gross feeding margin (P^C) (ignoring fixed costs). For all resulting series of differently hedged production margins, means and standard errors were computed, and resulting values are shown in figure 4.

Clearly, production margin means are lower for all hedging strategies relative to no hedging. For the testing period 1989–94, the mean of ΔF_{LC} is almost \$16 per head, resulting in a \$16 loss for the naive short hedge, and even higher losses for other hedges where fed cattle hedge ratios are above one.[7] These losses from fed cattle futures positions are partially offset by gains in long feeder cattle futures positions (see table 3).[8]

For the margins hedged using ratios based on the most recent four years of data, all strategies are efficient in the mean-variance framework, with the exception of proportional hedging, which is dominated by the multiproduct optimal hedging strategy (figure 4). The naive hedge turns out to be the least risky strategy, followed by single-commodity hedge, multiproduct optimal hedge, and no hedge.

For the margins hedged using ratios based on the most recent six years of data, naive hedge dominates single-commodity hedge, with the rest of the strategies being efficient. Again, our results indicate that the naive hedge is the least risky strategy, followed by multiproduct optimal hedge, proportional hedge, and no hedge.

Finally, for the margins hedged using ratios based on the growing sample, all strategies are efficient in the mean-variance framework, with none of them

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[7] The downward price bias (upward price trend) in the cattle futures market has been noted previously by Elam and Wayoopagtr and by Kolb. Elam and Njukia estimate this downward bias costs \$12–\$13 per head, consistent with the results of our study. This cost often exceeds the returns from cattle feeding.

[8] Recall that feeder cattle positions are held for only one month, while fed cattle positions are held for five months. These apparent biases in fed and feeder cattle futures prices undoubtedly help generate the lower returns from hedging relative to the no-hedging strategy.
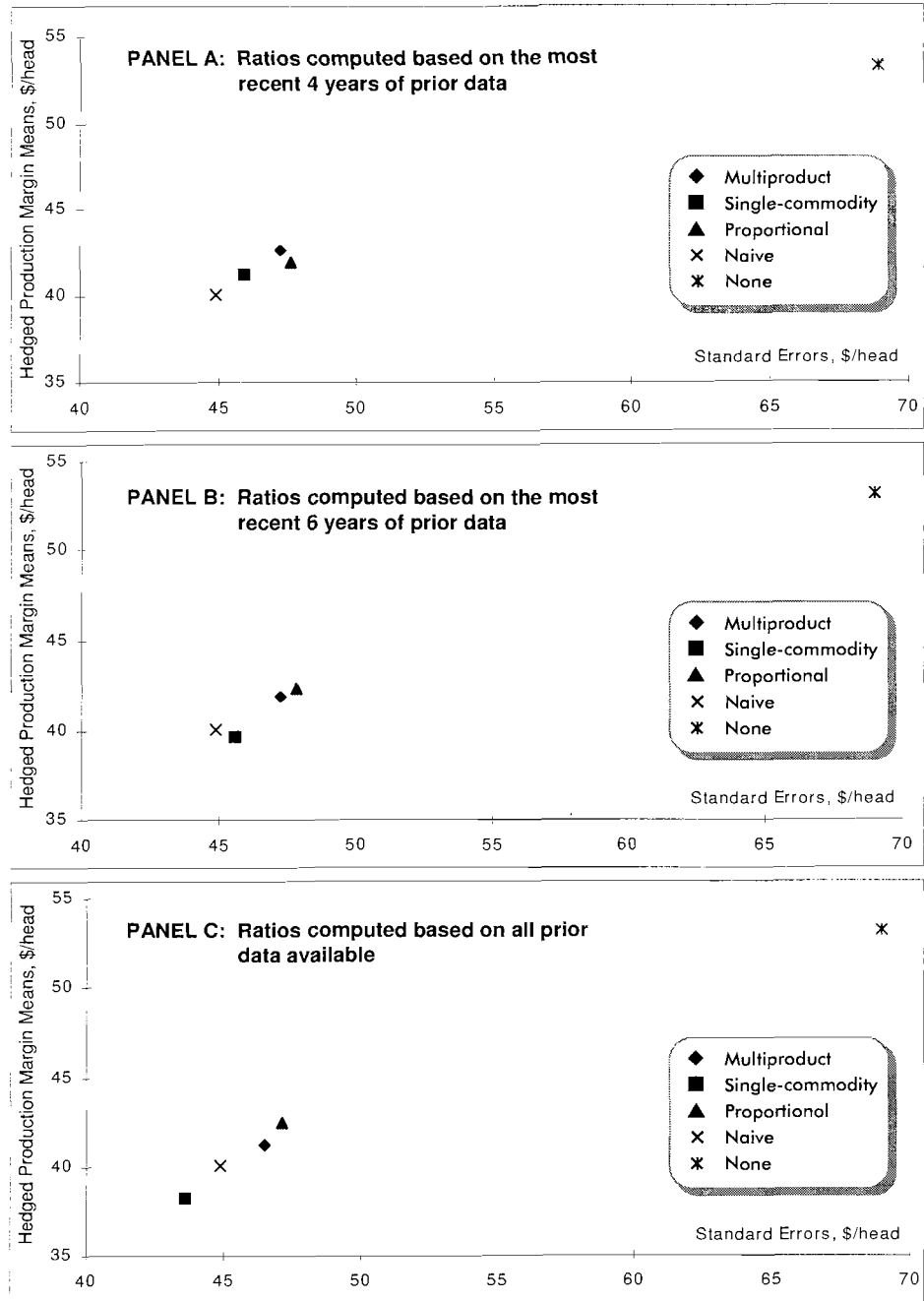
Figure 4. Means vs. standard errors for production margins hedged during 1989–94, for five different hedging strategies

dominating the others. The single-commodity hedging strategy is the least risky, followed by naive hedge, multiproduct optimal hedge, proportional hedge, and the no-hedge strategy.

There is no consistent domination pattern in figure 4, suggesting high dependency on data and sample size. Even though the models can be ranked in terms of their riskiness, these ratings are inconsistent, and vary depending on the size of data sample, and likely which part of the cattle cycle is used for testing. Clearly, when testing out-of-sample, multiproduct hedging did not have the lowest risk, as might be expected.

Whether each commodity should be hedged using only its own corresponding futures contract can be determined by analyzing matrix **M** in equation (6). Single-commodity hedging implies that **M** is diagonal; therefore, the role of cross-hedges in risk reduction for the multiproduct optimal hedging model can be determined by the hypothesis of $M_{ij} = 0$ for $i \neq j$. The LM multipliers test (Bera and Jarque) has a test statistic associated with this hypothesis equal to $\lambda_{LM} = 203.4$, with a $\chi^2(6)$ distribution. The corresponding 0.995 percentile is 18.55; therefore, we clearly reject the null hypothesis of **M** being diagonal. Hence, cross-hedges play a significant role in the optimal hedge risk reduction.

Variance reduction in hedged positions compared to the unhedged position were computed according to equation (11). All models provide more than 50% reduction in profit margin variance, with the range being from 51.9% to 60.1%. Rankings of these models can be seen in figure 4. For the most part (other considerations absent), the hedging decision by a U.S. cattle feeder will depend on the feeder's degree of risk aversion.

The Model's Implications and Limitations

Figure 4 represents a classical efficient frontier, with no-hedge being the most risky strategy, and single-commodity or naive hedges being the least risky. As the usage of futures increases from zero for the unhedged position to fully hedged positions, an efficient frontier of futures and cash portfolios is traced out. Clearly, the level of usage of futures is left to the cattle feeder depending on his or her degree of risk aversion.

These conclusions should be treated with caution. First, we had to make some assumptions about agent behavior. One of the most significant underlying assumptions was that the operator would always hedge, while in reality hedgers do not always hedge expected negative profits. We also did not allow for position adjustment—that is, hedge ratio being nonstationary—during the period of the hedge. Finally, this framework does not allow multiperiod hedging, a practice widely followed in the industry. These are all topics for further research.

Second, we assumed (as is standard in the optimal hedging literature) that the futures prices are unbiased. That assumption may be violated in the case of fed and feeder cattle futures over past data, resulting in lower hedged returns in this

study. A more accurate model would incorporate the "speculative" term, allowing for potential price bias. Such a model would at the same time allow for evaluation of the original objective function over alternative risk-aversion parameters. However, in the case of optimal multicommodity hedging, this model would be very complex, and is beyond the scope of this study. Additionally, in further analysis, one needs to be aware of potential autocorrelation when data overlap (as in this study).

Finally, there are several uncontrolled and unpredictable factors, such as weather (which affects pasture conditions, grain crops, and feed costs), animal performance, cattle marketings, and consumer demand. Another example of an uncontrolled factor can be animal health: livestock in good health and condition gain weight more rapidly and efficiently, increasing profit opportunities. These considerations call for a more in-depth model of hedging, which would allow the hedge ratios to have a stochastic nature, and at the same time integrate biological, human, and geophysical aspects of the production process. Thus, there are several possible extensions to this study.

Summary and Conclusions

This study compares multiproduct optimal hedging to alternative hedging strategies as applied to the price risks faced by a midwestern cattle feeder. Previous studies have suggested that hedging inputs and output as individual commodities helps reduce the producer's margin variability, and that multiproduct optimal hedging could lead to further improvements. A one-period multiproduct optimal hedging model, derived from a mean-variance framework, is used to estimate hedge ratios and analyze production margin variability.

Hedge ratios were estimated for alternative models using weekly data from a simulation of a custom feeding feedlot. For each model, a series of hedge ratios was estimated using either the prior four years, six years, or all prior data available at the moment of estimation. The ratios demonstrate less variability as the length of the underlying sample increases, meaning the ratios based on the most recent four years of data are the least stable. The hypothesis of all hedge ratios being equal to each other (the proportional hedging model) is rejected. The results show single-commodity hedge ratios all being quite close to one, but substantially different results exist for multiproduct hedging. In this latter case, the feeder cattle hedge ratio is only 0.66, significantly less than one, and the soybean meal hedge ratio is not significantly different from zero.

Calculated hedge ratios are tested out-of-sample for one-step-ahead hedging of feeding margins over a sample of 293 feeding periods. Utilizing the mean-variance framework, there is no consistent domination pattern among the alternative strategies. For the ratios computed based on all prior data available, all strategies are efficient, leaving the hedging decision up to the agent's degree of risk aversion. Nevertheless, all hedging strategies significantly reduce the production margin's

means and variances compared to no hedge, with variance reduction always exceeding 50%.

Clearly, following a regimented hedging plan substantially reduces cattle feeding price margin risks. These results potentially could be enhanced considerably if one were able to predict the future cattle feeding margin and then place hedges selectively depending upon the predicted trend. Figure 2 shows the production margin as highly cyclical and autocorrelated. Forecasting the feeding margin five months forward is left for future research.

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