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Modeling Timber Supply, Fuel-Wood, and Atmospheric Carbon Mitigation

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Abstract

There is general agreement that global warming is occurring and that the main contributor to this probably is the buildup of green house gasses, GHG, in the atmosphere. Two main contributors are the utilization of fossil fuels and the deforestation of many regions of the world. This paper examines a number of current issues related to mitigating the global warming problem through forestry. We use discrete time optimal control to model a simplified carbon cycle. The burning of fossil fuels increases atmospheric carbon while the burning of fuel-wood along with its forest source maintain an atmospheric carbon level. The standing timber in the forests is a carbon sink, as are wood buildings and structures, and fossil fuel in the ground. Through time the buildings and structures decay and release carbon to the atmosphere. We also present a numerical example to help illustrate the characteristics of the model. The conclusions are that the forest sector can have a significant impact.

INTRODUCTION

There is general agreement that global warming is occurring and that it probably has been occurring for a few centuries. Most but not all would agree that the main contributor to this is the buildup of green house gasses ,GHG, in the atmosphere that has resulted from the utilization of fossil fuels and the deforestation of many regions of the world. Connected with the discussion of this topic is the discussion of the role that forest management can play in mitigating or reversing this trend. (See for example Sedjo and Toman, 2001). Much of this discussion has been prompted by the Kyoto Protocol and the reports of later sessions of the Conference of the Parties of the United Nations Framework Convention on Climate Change. These explicitly recognize afforestation, reforestation and deforestation activities as having significant impacts on atmospheric carbon.

Forests can contribute to the mitigation of GHG buildup in several ways. They can act as carbon sinks as standing trees, they can supply fuel-wood to reduce fossil fuel consumption, and they can provide building materials for long-lived wood buildings and structures that are substitutes for building products for such fossil-energy-consumptive materials such as concrete and steel. As standing forests the growth rate of trees can be increased by management practices such as fertilizer applications, and the volume of standing timber can be increased by lengthening the rotation period. The effect of carbon sequestration on the rotation period has been studied in the Faustmann framework by Hoen, 1994; Van Kooten et al., 1995; Romero et al., 1998. All of these agree that the effect of this additional return is to increase the rotation period. These studies, however, do not consider the impact of these changes upon world timber markets. Two studies that take some of these market implications into account are Sedjo and Sohngen, 2000, and Lee and Lyon, 2004.

We use discrete time optimal control to model a simplified carbon cycle. The burning of fossil fuels increases atmospheric carbon while the burning of fuel-wood along with its forest source maintain an atmospheric carbon level. The standing timber in the forests is a carbon sink, as are wood buildings and structures, and fossil fuel in the ground. Through time the buildings and structures decay and release carbon to the atmosphere.

The objective function is the present value of net surplus, which is maximized subject to several constraints. On the demand side there is a demand for Btu’s and for the services of buildings and structures. On the cost side there are costs of harvesting timber, extracting fossil fuels, and converting wood to structures and buildings. The negative impact of atmospheric carbon is modeled through a social cost function, which is strictly increasing in atmospheric carbon, and has increasing marginal costs (increasing and strictly convex). The constraints include the laws of motion for solidwood and fuel-wood, buildings and structures, and fossil fuels. In addition, there is a provision for shifting forest land between solidwood and fuel-wood production.

A numerical example is presented to illustrate the characteristics of the model. While only a few features of the example are calibrated to the real world several key elements of the forest sectors potential contributions. We incrementally shift the social cost function for atmospheric carbon upward, and observe the changes within the model results. At our lowest lever of social costs there are insignificant effects in the forest sector, and at the highest level there are large impacts of and on the forest sector.

The final section contains the summary.

THE MODEL

The net surplus function is given by:

$$(1) \qquad s_j = \int_0^{\hat{B}_j} D^{\hat{B}}(\upsilon)d\upsilon + \int_0^{Q_j^b} D^b(\upsilon)d\upsilon - C^a(z_j^a) - C^c(q_j^c, y_j) - C^H(q_j^s, q_j^f) - C^b(q_j^s)$$

where the sub *j's* are for year *j*,

$\hat{B}$ is Btu's,

$D^{\hat{B}}(\cdot)$ is the demand function for Btu's in inverse form,

$D^b(\cdot)$ is the flow demand for the services of buildings and structures,

q^s is the cubic meters of commercial solidwood harvested and sold,

Q^b is the stock of buildings and structures,

$C^a(\cdot)$ is the social cost function for atmospheric carbon,

z^a is the stock of atmospheric carbon,

$C^c(\cdot)$ is the cost function for the extraction of coal,

q^c is the metric tons of coal (fossil fuel) extracted and consumed,

y is the stock of coal (fossil fuel),

$C^H(\cdot)$ is the harvest cost function,

q^f is the cubic meters of commercial fuel wood harvested and consumed, and

$C^b(\cdot)$ is the cost function for converting solidwood into buildings and structures.

Note that the demand for solidwood is a derived demand. It is derived from the demand for the services of buildings and structures.

We define

$$(2) \qquad \hat{B}_j = a^c q_j^c + a^f q_j^f$$

where a^c and a^f are parameters identifying the relationship between the fuels and their Btu content. We posit

$$z_j^a + z_j^c + z_j^F + z_j^b + z_j^o = Z$$

where z is the stock of carbon on various forms. The superscripts a and c are identified above: z^F is for the forest stock,

z^b is stock in structures (buildings),

z^o is for the other stocks, and
Z is a constant.

From this last equation we get:

(3) $$z^a_{j+1} = -z^c_{j+1} - z^F_{j+1} - z^b_{j+1} - z^o_{j+1} + Z$$

Equation (3) will play an important role below.

There are several alternatives for the law of motion for z^o_{j+1}. One alternative is to assume that the rest of the atmospheric carbon world balances itself, so $\Delta z^0 = 0$. Another alternative is to assume its absorption is proportional to atmospheric carbon,

(4) $$z^o_{j+1} = z^o_j + \alpha^o z^a_j \,.$$

Of course, the second alternative collapses to the first if $\alpha^o = 0$. We use this latter alternative with $\alpha^o > 0$.

For carbon tied up in structures (buildings):

(5) $$z^b_j = \alpha^b Q^b_j$$

where Q^b is the stock of wood in structures (buildings), α^b is carbon per cubic meter of structure wood. We assume the depreciation on the stock of structure wood is proportional to the stock; hence, the law of motion for structure wood is:

(6) $$Q^b_{j+1} - Q^b_j = q^s_j - \delta Q^b_j \quad \text{with } Q^b_0 = Q^{b0} \text{ given;}$$

thus,

(7) $$z^b_{j+1} = \alpha^b Q^b_{j+1} = \alpha^b \left(q^s_j + (1-\delta) Q^b_j \right)$$

The stock of coal (fossil fuel) is decreased by the size of the extraction, so its law of motion can be given by:

(8) $$y_{j+1} - y_j = -q^c_j \quad \text{with } y_0 = y^0 \text{ given,}$$

and

$$z^c_j = \alpha^c y_j \,,$$

(9) $$z^c_{j+1} = \alpha^c y_{j+1} = \alpha^c \left(y_j - q^c_j \right)$$

To identify the harvest for each forestland class we define u^h_{ij} for $h = s, f$ to be the portion of hectares of age group *i* trees harvested in year *j*, with the constraint

(10) $0 \le u_{ij}^h \le 1$ for $h = s, f$ and all i, j

We also define:

u_j^h to be a column vector with typical element u_{ij}^h,

$g^s(i)$ and $g^f(i)$ to be the yield functions for commercial volume a function of age,

g^h to be a column vector of length M with typical element $g_i^h = g^h(i)$, $h = s, f,$

$x_{i,j}^s$ and $x_{i,j}^f$ to be the hectares of forest in the respective types with age i in year j, and

x_j^h to be a column vector of length M with typical element x_{ij}^h, $h = s, f.$

The parameter M is equal to or greater than the index number of the oldest age group in the problem, and $g^s(i)$ and $g^f(i)$ are assumed to be concave and differentiable. With these definitions the harvests are given by

(11) $q_j^h = u_j^{h^T} X_j^h g^h$,

where the super T is for transpose and X_j^h is a diagonal matrix using the elements of x_j^h.

The laws of motion for the forestland classes are given by

(12) $x_{j+1}^h = \left(A + BU_j^h\right)x_j^h + v_j^h e$ for $h = s, f$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad e = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdot & \cdot & \cdot & 1 \\ -1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & -1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & -1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & -1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

A, B, and *U* are *M*-square matrices; U_j^h is a diagonal matrix using the elements of u_j^h; and *e* is a *M*-vector where *M* is equal to or greater than the index number of the oldest age group in the problem. In addition, initial hectares of forest by land class are given, $x_0^h = x^{h0}$. The variable v_j^h is used to move forestland between the two types of trees. We hold the forestland constant but allow for transfers between the two types with $v_j^s + v_j^f = 0$, where $v_j^h < 0$ means hectares are being transferred out of this land class. To keep the problem manageable we only allow harvested hectares to be transferred, so we require

$$(13) \qquad \sum_{i=1}^{M} u_{ij}^h x_{ij}^h \geq -v_j^h \quad \text{or} \quad \vec{1}^T U_j^h x_j^h + v_j^h \geq 0$$

where $\vec{1}^T$ is a row vector of ones.

The stock of carbon in the forest is posited to be proportional to the commercial volume of timber on the forestland area; thus the pre-harvest stock is:

$$(14) \qquad z_j^F = \alpha^w \left(\sum_{i=1}^{M} g^s(i) x_{ij}^s + \sum_{i=1}^{M} g^f(i) x_{ij}^f \right) = \alpha^w \left(g^{s^T} x_j^s + g^{f^T} x_j^f \right)$$

where all variables were defined above except α^w which is the carbon per cubic meter of commercial volume of timber. We can now state the stock of forest carbon as:

$$(15) \qquad z_{j+1}^F = \alpha^w \left(g^{s^T} x_{j+1}^s + g^{f^T} x_{j+1}^f \right)$$

We collect the carbon relationships in Equation (3) by substituting into it Equations (4), (7), (9), and (15). This yields:

$$(16) \qquad z_{j+1}^a = -\alpha^c \left(y_j - q_j^c \right) - \alpha^w \left(g^{s^T} x_{j+1}^s + g^{f^T} x_{j+1}^f \right) - \alpha^b \left(q_j^s + (1-\delta) Q_j^b \right) - \left(z_j^o + \alpha^o z_j^a \right) + Z$$

The objective functional is:

(17) $$W = \sum_{j=0}^{\infty} \rho^j s_j$$

where $\rho = e^{-r}$ with *r* the real rate of interest. The problem can be stated as finding the sequence $\{q_j^c, q_j^s, q_j^f\}_{j=0}^{\infty}$ or the sequence $\{q_j^c, u_j^s, u_j^f\}_{j=0}^{\infty}$ to maximize Equation (17) subject to Equations (2)-(16). We use the Bellman Equation to identify the necessary conditions. The Bellman Equation can be written:

(18) $$V(x_j^s, x_j^f, y_j, z_j^a) = \underset{u_j^s, u_j^f, q_j^c \in K}{Max} \left(s_j + \rho V\left(x_{j+1}^s, x_{j+1}^f, y_{j+1}, z_{j+1}^a, Q_{j+1}^b\right)\right)$$

where *K* is the constraint set. To proceed we use the following Lagrangian function:

(19) $$L = s_j + \rho V\left(x_{j+1}^s, x_{j+1}^f, y_{j+1}, z_{j+1}^a, Q_{j+1}^b\right) + \sum_{h=s,f}\left(-\phi_j^{h^T}\left(u_j^h - 1\right) + \psi_j^{h^T} u_j^h + \zeta_j^h\left(\vec{1}^T U_j^h x_j^h + v_j^h\right)\right)$$

where the Lagrangian multipliers ϕ and ψ are for Equation (10) and ζ is for Equation (13). We substitute Equation (6) for Q_{j+1}^b, Equation (8) for y_{j+1}, Equation (11) for q_j^h, Equation (12) for x_{j+1}^h, and Equation (16) for z_{j+1}^a to generate the necessary conditions. In the derivatives that follow, the derivative of a function or a scalar with respect to a scalar is indicated as a partial derivative, the derivative of a function or a scalar with respect to a vector is a gradient column vector indicated with a *d*, and the derivative of a vector with respect to a vector is a Jacobian matrix indicated with a *d*. In the Jacobian matrix the columns are gradients. For example, the first column is the gradient of the first variable. The first order necessary conditions are:

(20a

$$\frac{dL}{du_j^s} = \left(-\frac{\partial C^H}{\partial q_j^s} - \frac{\partial C^b}{\partial q_j^s}\right)\frac{dq_j^s}{du_j^s} + \rho\left(\frac{dx_{j+1}^s}{du_j^s}\frac{dV}{dx_{j+1}^s} + \frac{\partial V}{\partial z_{j+1}^a}\frac{dz_{j+1}^a}{du_j^s} + \frac{\partial V}{\partial Q_{j+1}^b}\frac{dQ_{j+1}^b}{du_j^s}\right) - \phi_j^s + \psi_j^s + \zeta_j^s x_j^s = 0$$

(20b)

$$\frac{dL}{du_j^f} = \left(D^{\hat{B}}\left(a^c q_j^c + a^f q_j^f\right)a^f - \frac{\partial C^H}{\partial q_j^f}\right)\frac{dq_j^f}{du_j^f} + \rho\left(\frac{dx_{j+1}^f}{du_j^f}\frac{dV}{dx_{j+1}^f} + \frac{\partial V}{\partial z_{j+1}^a}\frac{dz_{j+1}^a}{du_j^f}\right) - \phi_j^f + \psi_j^f + \zeta_j^f x_j^f = 0$$

(20c) $$\frac{\partial L}{\partial q_j^c} = D^{\hat{B}}\left(a^c q_j^c + a^f q_j^f\right)a^c - \frac{\partial C^c}{\partial q_j^c} + \rho\left(-\frac{\partial V}{\partial y_{j+1}} + \frac{\partial V}{\partial z_{j+1}^a}\alpha^c\right) = 0$$

(20d) $$\frac{\partial L}{\partial v_j^s} = \rho\left(\frac{dV}{dx_{j+1}^s} - \frac{dV}{dx_{j+1}^f}\right)^T e + \zeta_j^s - \zeta_j^f = 0$$

(20e) $\zeta_j^h\left(\vec{1}^T U_j^h x_j^h + v_j^h\right) = 0$, $\phi_j^{s^T}\left(u_j^s - 1\right) = 0$, and $\psi_j^s u_j^s = 0$.

By the Envelope Theorem we get:

(21a) $$\frac{dV}{dx_j^s} = \frac{dL}{dx_j^s} = \frac{ds_j}{dx_j^s} + \rho\left(\frac{dx_{j+1}^s}{dx_j^s}\frac{dV}{dx_{j+1}^s} + \frac{\partial V}{\partial z_{j+1}^a}\frac{dz_{j+1}^a}{dx_j^s} + \frac{\partial V}{\partial Q_{j+1}^b}\frac{dQ_{j+1}^b}{dx_j^s}\right) + \zeta_j^s u_j^s$$

(21b) $$\frac{dV}{dx_j^f} = \frac{dL}{dx_j^f} = \frac{ds_j}{dx_j^f} + \rho\left(\frac{dx_{j+1}^f}{dx_j^f}\frac{dV}{dx_{j+1}^f} + \frac{\partial V}{\partial z_{j+1}^a}\frac{dz_{j+1}^a}{dx_j^f}\right) + \zeta_j^f u_j^f$$

(21c) $$\frac{\partial V}{\partial y_j} = \frac{\partial L}{\partial y_j} = -\frac{\partial C^c}{\partial y_j} + \rho\left(\frac{\partial V}{\partial y_{j+1}}\frac{\partial y_{j+1}}{\partial y_j} + \frac{\partial V}{\partial z_{j+1}^a}\frac{\partial z_{j+1}^a}{\partial y_j}\right)$$

(21d) $$\frac{\partial V}{\partial z_j^a} = \frac{\partial L}{\partial z_j^a} = -\frac{\partial C^a}{\partial z_j^a} + \rho\frac{\partial V}{\partial z_{j+1}^a}\frac{\partial z_{j+1}^a}{\partial z_j^a}$$

(21e) $$\frac{\partial V}{\partial Q_j^b} = \frac{\partial L}{\partial Q_j^b} = D^b\left(Q^b\right) + \rho\left(\frac{\partial V}{\partial z_{j+1}^a}\frac{\partial z_{j+1}^a}{\partial Q_j^b} + \frac{\partial V}{\partial Q_{j+1}^b}\frac{\partial Q_{j+1}^b}{\partial Q_j^b}\right)$$

Some of the gradients and Jacobians are:

$$\frac{dq_j^h}{du_j^h} = X_j^h g^h$$

$$\frac{dx_{j+1}^h}{du_j^h} = \frac{dBU_j^h x_j^h}{du_j^h} = \frac{dBX_j^h u_j^h}{du_j^h} = \left(BX_j^h\right)^T = X_j^{h^T} B^T$$

$$\frac{dz_{j+1}^a}{du_j^s} = -\alpha^w \frac{dg^{s^T} x_{j+1}^s}{u_j^s} - \alpha^b \frac{dq_j^s}{du_j^s} = -\alpha^w X_j^{s^T} B^T g^s - \alpha^b X_j^s g^s$$
$$= -\left(\alpha^w X_j^{s^T} B^T + \alpha^b X_j^s\right) g^s$$

$$\frac{dz_{j+1}^a}{du_j^f} = -\alpha^w X_j^{s^T} B^T g^f$$

$$\frac{dx_{j+1}^h}{dx_j^h} = \frac{d\left(A + BU_j^h\right)x_j^h}{dx_j^h} = \left(A + BU_j^h\right)^T$$

$$\frac{dq_j^h}{dx_j^h} = U_j^h g^h$$

$$\frac{dz_{j+1}^a}{dx_j^s} = -\alpha^w \frac{dg^{s^T} x_{j+1}^s}{dx_j^s} - \alpha^b \frac{dq_j^s}{dx_j^s} = -\alpha^w \left(A + BU_j^s\right)^T g^s - \alpha^b U_j^s g^s$$

$$\frac{ds_j}{dx_j^s} = \left(-\frac{\partial C^H}{\partial q_j^s} - \frac{\partial C^b}{\partial q_j^s}\right)\frac{dq_j^s}{dx_j^s} = \left(-\frac{\partial C^H}{\partial q_j^s} - \frac{\partial C^b}{\partial q_j^s}\right)U_j^s g^s$$

$$\frac{ds_j}{dx_j^f} = \left(D^{\hat{B}}\left(a^c q_j^c + a^f q_j^f\right)\alpha^f - \frac{\partial C^H}{\partial q_j^f}\right)\frac{dq_j^f}{dx_j^f} = P_j^f U_j^f g^f$$

$$P_j^f := \left(D^{\hat{B}}\left(a^c q_j^c + a^f q_j^f\right)\alpha^f - \frac{\partial C^H}{\partial q_j^f}\right)$$

$$\frac{dQ_{j+1}^b}{du_j^s} = \frac{dq_j^s}{du_j^s} = X_j^s g^s$$

$$\frac{dQ_{j+1}^b}{dx_j^s} = \frac{dq_j^s}{dx_j^s} = U_j^s g^s$$

In Equations (21a)-(21e) the left-hand side is the shadow value of a state variable and is the same concept as the costate variable in Optimal Control Theory; hence, we define five costate variables, λ_j^s, λ_j^f, λ_j^c, λ_j^a, and λ_j^b to correspond to the left-hand side of Equations (21a)-(21e), respectively. In the manipulations below the stumpage prices of the two types of wood become relevant. Because the demand for solidwood is a derived demand, the expression for its stumpage price is slightly complicated. The market price of solidwood is given by

$$\widehat{P}_j^s = \rho\lambda_{j+1}^b - \frac{dC^b}{dq_j^s}.$$

In this λ_{j+1}^b is the value of a cubic meter of buildings and structures in the next time period, and ρ discounts this to the current time period. From this is subtracted the cost of transforming a cubic meter of solidwood into a cubic meter of buildings and structures. This gives the net value of a unit of solidwood, and will be its market price. Hence, we define the stumpage price of solidwood as:

$$P_j^s := \widehat{P}_j^s - \frac{dC^H}{dq_j^s}.$$

In addition, define the fuel wood stumpage price as:

$$P_j^f := D^{\hat{B}}\left(a^c q_j^c + a^f q_j^f\right)a^f - \frac{\partial C^H}{\partial q_j^f}.$$

Using these definitions and evaluating some of the derivatives (See Appendix A) we can rewrite Equations (20a)-(21d) as:

(20a')

$$\left(-\frac{\partial C^H}{\partial q_j^s} - \frac{\partial C^b}{\partial q_j^s}\right)X_j^s g^s + \rho\left(X_j^{sT} B^T \lambda_{j+1}^s - \lambda_{j+1}^a\left(\alpha^w X_j^{sT} B^T + \alpha^b X_j^s\right)g^s + \lambda_{j+1}^b X_j^s g^s\right) - \phi_j^s + \psi_j^s + \zeta_j^s x_j^s = 0$$

$$\left(\rho\lambda_{j+1}^b - \frac{\partial C^H}{\partial q_j^s} - \frac{\partial C^b}{\partial q_j^s}\right)X_j^s g^s + \rho\left(X_j^{sT} B^T \lambda_{j+1}^s - \lambda_{j+1}^a X_j^{sT}\left(\alpha^w B^T + \alpha^b I\right)g^s\right) - \phi_j^s + \psi_j^s + \zeta_j^s x_j^s = 0$$

$$P_j^s X_j^s g^s + \rho\left(X_j^{sT}\left(B^T \lambda_{j+1}^s - \lambda_{j+1}^a\left(\alpha^w B^T + \alpha^b I\right)g^s\right)\right) - \phi_j^s + \psi_j^s + \zeta_j^s x_j^s = 0$$

(20b')

$$\left(D^{\hat{B}}\left(a^c q_j^c + a^f q_j^f\right)a^f - \frac{\partial C^H}{\partial q_j^f}\right)X_j^f g^f + \rho\left(X_j^{fT} B^T \lambda_{j+1}^f - \lambda_{j+1}^a \alpha^w X_j^{fT} B^T g^f\right) - \phi_j^f + \psi_j^f + \zeta_j^f x_j^f = 0$$

$$P_j^f X_j^f g^f + \rho\left(X_j^{fT} B^T \lambda_{j+1}^f - \lambda_{j+1}^a \alpha^w X_j^{fT} B^T g^f\right) - \phi_j^f + \psi_j^f + \zeta_j^f x_j^f = 0$$

(20c') $D^{\hat{B}}\left(a^c q_j^c + a^f q_j^f\right)a^c - \frac{\partial C^c}{\partial q_j^c} + \rho\left(-\lambda_{j+1}^y + \lambda_{j+1}^a \alpha^c\right) = 0$

(20d') $\rho\left(\lambda_{j+1}^s - \lambda_{j+1}^f\right)e + \zeta_j^s - \zeta_j^f = 0$

The laws of motion for the costate variables:

(21a')

$$\lambda_j^s = \left(-\frac{\partial C^H}{\partial q_j^s} - \frac{\partial C^b}{\partial q_j^s}\right)U_j^s g^s + \rho\left(\left(A + BU_j^s\right)^T \lambda_{j+1}^s - \lambda_{j+1}^a\left(\alpha^w\left(A + BU_j^s\right)^T + \alpha^b U_j^s\right)g^s + \lambda_{j+1}^b U_j^s g^s\right) + \zeta_j^s u_j^s$$

$$\lambda_j^s = P_j^s U_j^s g^s + \rho\left(\left(A + BU_j^s\right)^T \lambda_{j+1}^s - \lambda_{j+1}^a\left(\alpha^w\left(A + BU_j^s\right)^T + \alpha^b U_j^s\right)g^s\right) + \zeta_j^s u_j^s$$

(21b') $\lambda_j^f = P_j^f U_j^f g^f + \rho\left(\left(A + BU_j^f\right)^T \lambda_{j+1}^f - \lambda_{j+1}^a \alpha^w\left(A + BU_j^f\right)^T g^f\right) + \zeta_j^f u_j^f$

(21c') $\lambda_j^y = -\frac{\partial C^c}{\partial y_j} + \rho\left(\lambda_{j+1}^y - \lambda_{j+1}^a \alpha^c\right)$

(21d') $\lambda_j^a = -\frac{\partial C^a}{\partial z_j^a} - \rho \lambda_{j+1}^a \alpha^o$

(21e') $\lambda_j^b = D^b\left(Q_j^b\right) + \rho\left[-\lambda_{j+1}^a \alpha^b (1-\delta) + \lambda_{j+1}^b (1-\delta)\right]$

Because our primary purpose is to find Faustmann type results we now examine the stationary state. The Faustmann framework consists of doing the same thing over and over again, which is a stationary state concept.

POLICY IMPLICATIONS

A market based economy will not automatically produce the above necessary conditions since atmospheric carbon is a public 'bad' or an externality to the firms; however, as is usual in cases such as these taxes and subsidies can be used to achieve them. In this discussion we assume perfect certainty so we can concentrate on the issues at hand. Equations (21a'), (21b'), (21c'), and (21e') for the shadow values of solidwood forests, fuel-wood forests, coal (fossil fuel) in the ground, and buildings and structures, respectively, reveals that all of them are impacted by shadow value of atmospheric carbon; therefore, some tax or subsidy would be required. In the absence of externalities the shadow values are identified in the firms as the present value of the return stream to the marginal unit of the stock. In Equation (21a') the shadow value of solidwood forests needs to be subsidized for sequestering carbon (the term containing λ^a) and for producing wood for buildings and structures (the term containing λ^b). Equation (21b') is similar except the term for buildings and structures is missing. The shadow value of coal (fossil fuel) in the ground, Equation (21c') can be handled in one of two ways. Either coal in the ground can be subsidized as indicated by the term containing λ^a , or coal burning can be taxed by the appropriate amount to achieve the same effect. In addition, since buildings and structures are a carbon sink they also require a subsidy as indicated by the term containing λ^a .

The necessary conditions also must be taken into account. Equations (20a'), (20b'), and (20c') all are impacted by the above discussion of shadow values. Equation (20a') the necessary condition for solidwood requires a subsidy for λ^a . The λ^b in this equation will be taken care of in the market place if buildings and structures are subsidized as discussed above for their role as a carbon sink. The production of fuel-wood also requires a subsidy as indicated by the presence of λ^a in Equation (20b'). The necessary condition for the burning of coal (fossil fuel) indicates that there is a cost of releasing carbon into the atmosphere, as indicated by the term containing λ^a . This can be handled by a tax on the burning of coal (fossil fuel).

STATIONARY STATE

We assume the system evolves to a stationary state (ss) or something that is approximately a stationary state. The problem here is that either coal (fossil fuel) extraction must go to zero or the stock of coal must be infinite to get a stationary state. By an approximate ss we mean a state that will last long enough that the discounted value of the distant future flows that are ignored is nil.

In the stationary state the flows, prices, and the shadow values are constant through time; hence, we examine Equations (4), (5), (6), (7), (8), (9), (12), (15), (16), and (20a')-(21d') with constant flows, prices, and shadow values. Because of these conditions we leave the sub-*j* off from the variables. Starting with Equation (21c') we have

(21c'ss)
$$\begin{aligned} \lambda^y &= -\frac{\partial C^c}{\partial y} + \rho\left(\lambda^y - \lambda^a \alpha^c\right) \\ \lambda^y &= -\left(\frac{\partial C^c}{\partial y} + \lambda^a \alpha^c\right)\frac{1}{1-\rho} \end{aligned} .$$

Note that the shadow value of coal (fossil fuel) in the ground, λ^y, includes its value as a repository for carbon, $\lambda^a \alpha^c$. For (21d') we get:

(21d'ss)
$$\begin{aligned} \lambda^a &= -\frac{\partial C^a}{\partial z^a} - \rho \lambda^a \alpha^o \\ &= -\frac{\partial C^a}{\partial z^a}\frac{1}{1+\rho\alpha^o} \end{aligned}$$

Equation (21e') yields:

(21e'ss)
$$\begin{aligned} \lambda^b &= D^b\left(Q^b\right) + \rho\left[-\lambda^a \alpha^b (1-\delta) + \lambda^b (1-\delta)\right] \\ &= \frac{D^b\left(Q^b\right) - \rho\lambda^a\alpha^b(1-\delta)}{1-\rho(1-\delta)} \end{aligned} .$$

This value of λ^a is then used in Equations ((20a'), (20b'), (20c'), (21a'), and (21b'). Equation (21c') gives the optimal marginal condition for coal (fossil fuel) utilization, and its stationary state version is:

(20c'ss)
$$D^{\hat{B}}\left(a^c q^c + a^f q^f\right)a^c - \frac{\partial C^c}{\partial q^c} + \rho\left(-\lambda^y + \lambda^a \alpha^c\right) = 0 .$$

This states that at the margin the value of burning a ton of coal (fossil fuel), $D^{\hat{B}}(\cdot)a^c$ must be equal to the marginal extraction cost of that ton, $\frac{\partial C^c}{\partial q^c}$, plus the value of the coal in the ground as a repository for carbon, $\rho\lambda^y$, plus the shadow value of marginal degradation of the atmosphere, $\lambda^a\alpha^c$.

The next task is to solve for λ^s using Equation (21a'). In the stationary state there will be no pressure to switch hectares between land classes; hence, by Equation (20e) we will have $\zeta^s = 0$. Substitute λ^a from this last result into (21a') and set $\zeta^s = 0$ yields:

$$\lambda^s = P^s U^s g^s + \rho\left(\left(A + BU^s\right)^T \lambda^s - \lambda^a\left(\alpha^w\left(A + BU^s\right)^T + \alpha^b U^s\right)g^s\right)$$
$$\left(I - \rho\left(A + BU^s\right)^T\right)\lambda^s = P^s U^s g^s - \rho\lambda^a\left(\alpha^w\left(A + BU^s\right)^T + \alpha^b U^s\right)g^s$$
$$\lambda^s = \left(I - \rho\left(A + BU^s\right)^T\right)^{-1}\begin{bmatrix} P^s U^s g^s + \rho\lambda^a\alpha^w\left(-\left(A + BU^s\right)^T\right)g^s \\ -\rho\lambda^a\alpha^b U^s g^s \end{bmatrix}$$
$$\lambda^s = \left(I - \rho\left(A + BU^s\right)^T\right)^{-1}\begin{bmatrix} P^s U^s + \lambda^a\alpha^w I - \lambda^a\alpha^w I \\ +\rho\lambda^a\alpha^w\left(-\left(A + BU^s\right)^T\right) \\ -\rho\lambda^a\alpha^b U^s \end{bmatrix} g^s$$
$$\lambda^s = \left(I - \rho\left(A + BU^s\right)^T\right)^{-1}\begin{bmatrix} P^s U^s - \lambda^a\alpha^w I \\ +\lambda^a\alpha^w\left(I - \rho\left(A + BU^s\right)^T\right) \\ -\rho\lambda^a\alpha^b U^s \end{bmatrix} g^s$$

This simplifies to:

(21a'ss) $$\lambda^s = \left(I - \rho\left(A + BU^s\right)^T\right)^{-1}\left[P^s U^s - \lambda^a\alpha^w I - \rho\lambda^a\alpha^b U^s\right]g^s + \lambda^a\alpha^w g^s$$

This result for λ^s is then substituted into Equation (20a'):

$$P^s X^s g^s + \rho\left(X^{sT} B^T \lambda^s + X^s\left(-\lambda^a\left(\alpha^w B^T + \alpha^b I\right)\right)g^s\right) - \phi^s + \psi^s = 0$$
$$P^s X^s g^s + \rho\begin{pmatrix} X^{sT} B^T \begin{bmatrix} \left(I - \rho\left(A + BU^s\right)^T\right)^{-1}\left[P^s U^s - \lambda^a\alpha^w I - \rho\lambda^a\alpha^b U^s\right]g^s \\ +\lambda^a\alpha^w g^s \end{bmatrix} \\ +X^s\left(-\lambda^a\left(\alpha^w B^T + \alpha^b I\right)\right)g^s \end{pmatrix}$$
$$-\phi^s + \psi^s = 0$$

Continuing

$$P^s X^s g^s + \rho \begin{pmatrix} X^{s^T} B^T \left(I - \rho\left(A + BU^s\right)^T\right)^{-1} \left[P^s U^s - \lambda^a \alpha^w I - \rho \lambda^a \alpha^b U^s\right] g^s + \lambda^a \alpha^w X^{s^T} B^T g^s \\ + X^s \left(-\lambda^a \left(\alpha^w B^T + \alpha^b I\right)\right) g^s \end{pmatrix}$$

$$-\phi^s + \psi^s = 0$$

$$P^s X^s g^s + \rho \begin{pmatrix} X^{s^T} B^T \left(I - \rho\left(A + BU^s\right)^T\right)^{-1} \left[P^s U^s - \lambda^a \alpha^w I - \rho \lambda^a \alpha^b U^s\right] g^s \\ - \lambda^a \alpha^b X^s g^s \end{pmatrix} - \phi^s + \psi^s = 0$$

$$P^s X^s \left[I + \rho \left(B^T \left(I - \rho\left(A + BU^s\right)^T\right)^{-1} \left(U^s - \frac{\lambda^a \alpha^w I}{P^s} - \frac{\rho \lambda^a \alpha^b U^s}{P^s} \right) - \frac{\lambda^a \alpha^b}{P^s} I \right) \right] g^s$$

$$-\phi^s + \psi^s = 0$$

Define $\omega^s := \frac{-\lambda^a \alpha^w}{P^s}$ and $\omega^b := \frac{-\lambda^a \alpha^b}{P^s}$; therefore

(20a'ss) $$P^s X^s \left(I + \rho B^T \left(I - \rho\left(A + BU^s\right)^T\right)^{-1} \left(U^s + \omega^s I + \rho \omega^b U^s\right) + \rho \omega^b I \right) g^s - \phi^s + \psi^s = 0$$

To examine Equation (20a'ss) let k^s be the optimal rotation age for solidwood, and examine the elements in rows $k^s - 1$, and k^s. In this we assume there is no pressure to redistribute hectares between the land classes, so $\zeta = 0$. The first part of necessary condition (20e) for the stationary state is:

$$\zeta^h \left(\vec{1}^T U^h x^h + v^h\right) = 0$$

which implies for $v^h = 0$ with $U^h \neq 0$ that $\zeta^h = 0$. By Equation (20d') this means

(20d'ss) $$\begin{aligned} \rho\left(\lambda^s - \lambda^f\right) e &= 0 \\ \rho\left(\lambda_1^s - \lambda_1^f\right) &= 0 \\ \lambda_1^s - \lambda_1^f &= 0 \end{aligned}$$

where the sub-ones are for row one. That is hectares of one-year-old solidwood trees and one-year-old fuel wood trees have the same value, and this implies that the land rental is the same for the two land classes. We have

$$\lambda_1^s = P^s \omega^s \left(\frac{1}{\left(1 - \rho^k\right)} \left(g_1^s + \rho g_2^s + \rho^2 g_3^s + \cdots + \rho^{k-1} g_k^s\right) - g_1^s \right) + \frac{\rho^{k-1}}{\left(1 - \rho^k\right)} P^s \left(1 + \rho \omega^b\right) g_k^s$$

and

$$\lambda_1^f = P^f \omega^f \left(\frac{1}{\left(1 - \rho^k\right)} \left(g_1^f + \rho g_2^f + \rho^2 g_3^f + \cdots + \rho^{k-1} g_k^f\right) - g_1^f \right) + \frac{\rho^{k-1}}{\left(1 - \rho^k\right)} P^f g_k^f .$$

In these the sub-k 's are for the land classes s and f , respectively. The conventional Faustmann shadow values are a special case of these. If $\lambda^a = 0$ then $\omega^s = 0$, $\omega^b = 0$, and $\omega^f = 0$, and these last two equations collapse respectively to:

$$\lambda_1^s = \frac{\rho^{k-1}}{\left(1-\rho^k\right)} P^s g_k^s ,$$

and

$$\lambda_1^f = \frac{\rho^{k-1}}{\left(1-\rho^k\right)} P^f g_k^f .$$

We also assume all of age group k^s is harvested, $u_k^s = 1$(row k), and no other age group is harvested. From Equation (20e) we have:

$$\phi^{s^T}\left(u^s - 1\right) = 0 \text{ and } \psi^s u^s = 0 .$$

Thus, we have $\phi_k^s \geq 0$, but $\psi_k^s = 0$. For $k^s - 1$ the signs are reversed, $\phi_{k-1}^s = 0$ and $\psi_{k-1}^s \geq 0$. Row $k^s - 1$:

(20a'*k-1*)

$$\frac{(1-\rho)}{\left(1-\rho^k\right)}\omega^s\left(\rho g_1^s + \rho^2 g_2^s + \rho^3 g_3^s + \cdots + \rho^{k-1} g_{k-1}^s\right) + \frac{\rho^k - \rho}{\left(1-\rho^k\right)}\left(1+\omega^s + \rho\omega^b\right)g_k^s + \left(1+\rho\omega^b\right)g_{k-1}^s \leq 0$$

For row k^s we have:

(20a'*k*)

$$\frac{(1-\rho)}{\left(1-\rho^k\right)}\omega^s\left(\rho g_1^s + \rho^2 g_2^s + \rho^3 g_3^s + \cdots + \rho^{k-1} g_{k-1}^s\right) + \frac{\rho^k(1-\rho)}{\left(1-\rho^k\right)}\left(1+\omega^s + \rho\omega^b\right)g_k^s + \left(1+\rho\omega^b\right)g_k^s - \rho\left(1+\omega^s+\rho\omega^b\right)g_{k+1}^s \geq 0$$

Note that if $\lambda^a = 0$ then $\omega^s = 0$ and $\omega^b = 0$, so these two equations collapse respectively to:

$$\left(\frac{\rho^{k-1}}{1-\rho^{k-1}}\right)g_{k-1}^s \leq \left(\frac{\rho^k}{1-\rho^k}\right)g_k^s$$

and

$$\left(\frac{\rho^k}{1-\rho^k}\right)g_k^s \geq \left(\frac{\rho^{k+1}}{1-\rho^{k+1}}\right)g_{k+1}^s$$

These two identify the standard discrete time Faustmann rotation relation. See Olli Tahvonen, Canadian Economic Journal 2004 for a statement of the Faustmann rotation in a discrete time problem.

Because the structure of Equations (20a') and (20b') and Equations (21a') and (21b') are very similar we need not repeat all of the manipulations. Instead we can modify these last few results, changing the super s to f and drop α^b and ω^b. This yields

(21b'ss) $$\lambda^f = \left(I - \rho\left(A + BU^f\right)^T\right)^{-1}\left[P^f U^f - \lambda^a \alpha^w I\right] g^f + \lambda^a \alpha^w g^f$$

and

(20b'ss) $$P^f X^f \left(I + \rho B^T \left(I - \rho\left(A + BU^f\right)^T\right)^{-1}\left(U^f + \omega^f I\right)\right) g^f - \phi^f + \psi^f = 0$$

where $\omega^f := \dfrac{-\lambda^a \alpha^w}{P^f}$.

For row $k^f - 1$ we have:

(20b'*k-1*) $$\frac{(1-\rho)}{\left(1-\rho^k\right)}\omega^f\left(\rho g_1^f + \rho^2 g_2^f + \rho^3 g_3^f + \cdots + \rho^{k-1} g_{k-1}^f\right) + \frac{\rho^k - \rho}{\left(1-\rho^k\right)}\left(1+\omega^f\right) g_k^f + g_{k-1}^f \le 0$$

For row k^f we have:

(20b'*k*) $$\frac{(1-\rho)}{\left(1-\rho^k\right)}\omega^f\left(\rho g_1^f + \rho^2 g_2^f + \rho^3 g_3^f + \cdots + \rho^{k} g_{k}^f\right) + \left(1 + \frac{\rho^k(1-\rho)}{1-\rho^k}\left(1+\omega^f\right)\right) g_k^f - \rho\left(1+\omega^f\right) g_{k+1}^f \ge 0$$

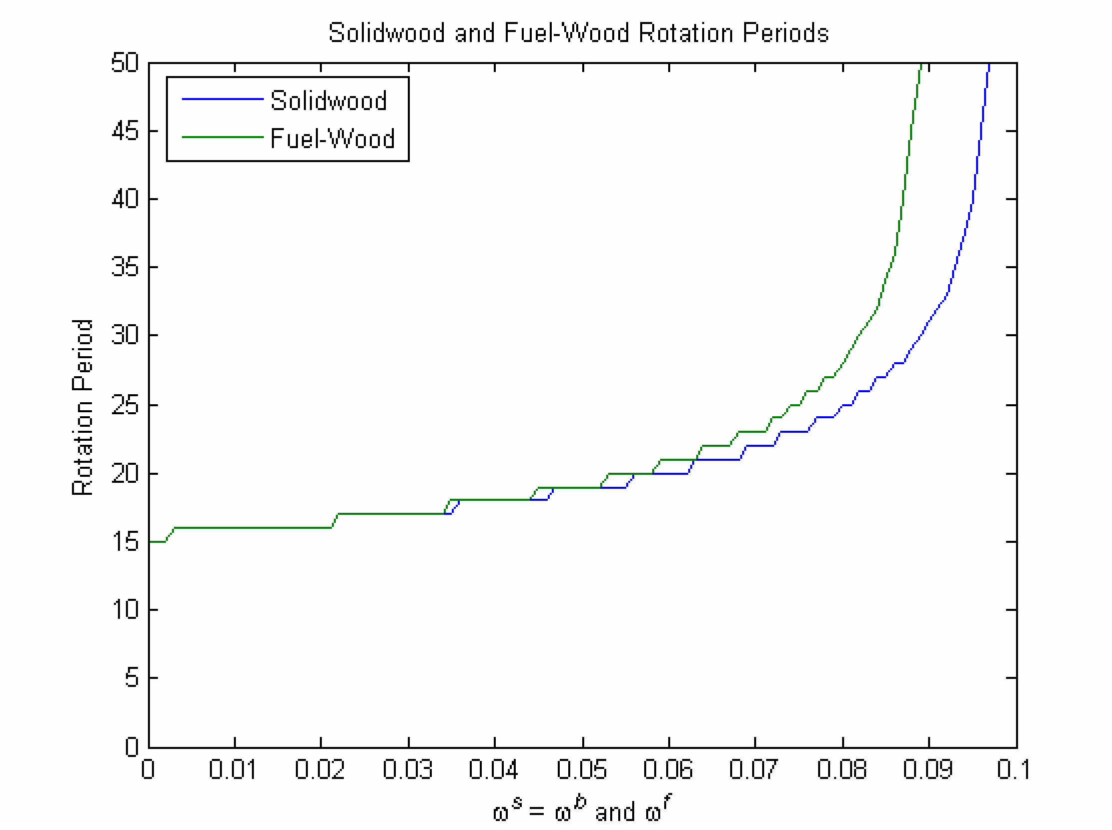
Note that if $\lambda^a = 0$ then $\omega^f = 0$, so these two equations collapse respectively to:

$$\left(\frac{\rho^{k-1}}{1-\rho^{k-1}}\right) g_{k-1}^f \le \left(\frac{\rho^k}{1-\rho^k}\right) g_k^f$$

and

$$\left(\frac{\rho^{k}}{1-\rho^{k}}\right) g_{k}^f \ge \left(\frac{\rho^{k+1}}{1-\rho^{k+1}}\right) g_{k+1}^f$$

As in the solidwood case these two identify the standard discrete time Faustmann rotation relation.



The effect of ω^s, ω^b, and ω^f on the optimal rotation periods is not easy to see qualitatively; however, for the numerical example given below as the omegas increase the rotation periods also increase. In addition, the rotation periods are very sensitive to the omegas over a very narrow range. Figure 1 depicts these results using the yield functions and parameter values of the numerical example. The values used yield $\omega^s = \omega^b$, and a Faustmann rotation for both types of wood equal to 15. The rotation periods are equal because we use the same yield function and parameter values for both types of wood. It does not take a very large increase in ω^i to cause the rotation period to increase to 50 which is the largest value examined. Note that the effect of ω^b on the solidwood rotation is to decrease it, as is evidenced by the solidwood rotation line lying under the fuel-wood rotation line in the graph. An examination of the rotation equations given above reveals that if $\omega^s = \omega^f$ and $\omega^b = 0$ then the two rotation periods will be equal, since the two yield functions are identical.

NUMERICAL EXAMPLE

To illustrate the characteristics of the model we present a numerical example. We compare stationary state solution levels of the endogenous variables of the model for shifts of the social cost function for atmospheric carbon. We anticipate that as this social cost function shifts upward the extraction of coal (fossil fuel) will decrease, the utilization of fuel-wood will increase as production is shifted away from solidwood, and the rotation period for both types of wood will increase. The results are consistent with these anticipations, but there are some surprises with respect to the rotation period.

We now identify the equations we use to represent the functions in the model

$$
\begin{aligned}
&D^{\hat{B}}\left(\hat{B}\right)=a_0-b_0\hat{B} && a_0,b_0>0\\
&D^{b}\left(q^s\right)=a_1-b_1Q^b && a_1,b_1>0\\
&C^{a}\left(z^a\right)=ca_1z^a+.5ca_2\left(z^a\right)^2 && ca_2>0\\
&C^{b}\left(q^s\right)=cb_1q^s && cb_1>0\\
&C^{c}\left(q^c\right)=cc_1q^c && cc_1>0\\
&C^{H}\left(q^s,q^f\right)=cs_1q^s+cf_1q^f && cs_1,cf_1>0
\end{aligned}
.
$$

These functions and variables were defined above as:

$D^{\hat{B}}(\cdot)$ is the demand function for Btu’s in inverse form,

$D^{b}(\cdot)$ is the flow demand for the services of buildings and structures,

q^s is the cubic meters of commercial solidwood harvested and sold,

Q^b is the stock of buildings and structures,

$C^a(\cdot)$ is the social cost function for atmospheric carbon,

z^a is the stock of atmospheric carbon,

$C^c(\cdot)$ is the cost function for the extraction of coal,

q^c is the metric tons of coal (fossil fuel) extracted and consumed,

y is the stock of coal (fossil fuel),

$C^H(\cdot)$ is the harvest cost function,

q^f is the cubic meters of commercial fuel wood harvested and consumed, and

$C^b(\cdot)$ is the cost function for converting solidwood into buildings and structures.

The parameter values are:

$$
\begin{aligned}
&a_0=9,\ b_0=0.00000001,\ a_1=6,\ b_1=0.0001,\ ca_1=-100,\cdots,20\\
&ca_2=0.0000001,\ cb_1=0.02,\ cc_1=0.02,\ cs_1=0.5,\ cf_1=0.5
\end{aligned}
$$

Other parameters in the model are:

$\rho=e^{-r}=0.95$ discount factor,

$a^c=30.8$ is millions of Btu’s per metric ton of coal (fossil fuel),

$a^f=8.33$ is millions of Btu’s per cubic meter of fuel-wood,

$\alpha^c=0.7$ is tons of carbon per ton of coal (fossil fuel),

$\alpha^w=0.27$ is metric tons of carbon per cubic meter of commercial wood in standing forests,

$\alpha^b=0.27$ is metric tons of carbon per cubic meter of building and structure wood,

$\delta = 0.1$ is rate the of depreciation of carbon in building and structure wood,
$\alpha^o = 0.02$ portion of atmospheric carbon absorbed by the unmodeled sector per unit time

The yield functions for both solidwood and fuel-wood have the equation:

$$q^h(j) = \exp(d_1 + d_2 / (j - d_3)) \qquad d_1, d_2, d_3 => 0, \qquad j > d_3$$

where *j* is age. The parameter values are:

$$d_1 = 6.52,\ d_2 = 6.5889,\ d_3 = 7$$

The Faustmann rotation for this yield function and parameter values are the same as those in Sedjo and Lyon for the Emerging Region.

Some of the parameter values are averages of their real world counterparts such as the carbon and Btu content of coal (fossil fuel) and wood, but there is no claim that these and the other values calibrate the model to the real world. Instead, this section gives an illustrative example. In this the social cost function for atmospheric carbon is shifted upward by increasing ca_1 from *-100* to *20* in steps of *10* to create 13 scenarios. These changes cause the shadow value of atmospheric carbon, λ^a, to increase in absolute value approximately 55 fold from *-0.36* to *-19.75*. At the same time the utilization of coal decreased about 785 fold from $2{,}869*10^4$ to *3,657* metric tons per year. Corresponding to this is a decrease in percent of total of Btu's derived from coal (fossil fuels) from *100* percent to *26* percent as total Btu's decrease 204 fold from $88{,}277*10^4$ to $432.32*10^4$ million Btu's.

In the forest sector the percent of total forest hectares in solidwood production decreases for *100* percent to $0.3*10^{-7}$. Corresponding to this is a decrease in solidwood production from *407,910* to *0.01* cubic meters per year, and an increase in fuel-wood production from *0* to *383,790* cubic meters per year.

Everything to this point is as expected. As the social cost of a 'social bad' is incrementally increased it is anticipated that the use of those things that propagate the bad will be successively reduced and the use of those things that mitigate the bad will be successively increased. Tables 1, 2, and 3 support these statements and give additional information about the results.

Table 1 Scenario Results, Coal and th eAtmosphere

ca_1	z^a	λ^a	λ^y	q^c	P_{Btu}	*Percent Coal Btu's*	*Millions of Btu's*
20	1.2812e+006	-19.753	276.54	3.6565e+004	8.96	26.1	4.3232e+006
10	8.4840e+007	-18.139	253.95	2.4241e+006	8.23	96.4	7.7480e+007
0	1.6841e+008	-16.527	231.37	4.8138e+006	7.49	98.4	1.5067e+008
-10	2.5198e+008	-14.915	208.81	7.1959e+006	6.76	99.1	2.2362e+008
-20	3.3553e+008	-13.300	186.21	9.5976e+006	6.03	99.5	2.9717e+008

-30	4.1911e+008	-11.689	163.65	1.1983e+007	5.30	99.7	3.7027e+008
-40	5.0267e+008	-10.076	141.06	1.4376e+007	4.56	99.8	4.4352e+008
-50	5.8622e+008	-8.462	118.46	1.6736e+007	3.84	99.9	5.1578e+008
-60	6.6980e+008	-6.850	95.89	1.9135e+007	3.11	100	5.8937e+008
-70	7.5328e+008	-5.229	73.20	2.1532e+007	2.37	100	6.6319e+008
-80	8.3671e+008	-3.602	50.43	2.3916e+007	1.63	100	7.3735e+008
-90	9.2021e+008	-1.984	27.77	2.6300e+007	0.90	100	8.1003e+008
-100	1.0037e+009	-0.363	5.08	2.8690e+007	0.16	100	8.8277e+008

Table 2 Scenario Results, Solidwood and Fuel-Wood

ca_1	k_s	k_f	ω^s	ω^f	P_s	P_f
20	24	24	0.0778	0.0722	68.78	74.11
10	24	24	0.0776	0.0723	63.34	68.02
0	24	24	0.0777	0.0723	57.67	61.92
-10	24	24	0.0778	0.0724	51.97	55.84
-20	24	24	0.0778	0.0725	46.33	49.72
-30	24	24	0.0778	0.0726	40.73	43.63
-40	24	24	0.0781	0.0728	34.98	37.53
-50	24	24	0.0782	0.0728	29.31	31.51
-60	24	na	0.0772	na	24.04	25.38
-70	22	na	0.0685	na	20.68	19.23
-80	20	na	0.0558	na	17.48	13.11
-90	18	na	0.0367	na	14.66	6.95
-100	16	na	0.0078	na	12.62	0.86

Table 3 Scenario Results, Solidwood and Fuel-Wood Continued

ca_1	q^s	q^f	Q^b	λ^b
20	0.01	383,790	0.002	72.94
10	45,684	338,110	4,566	67.22
0	94,800	288,990	9,484	61.25
-10	145,440	238,350	14,439	55.26
-20	195,940	187,850	19,308	49.32
-30	241,520	142,270	24,120	43.42
-40	293,740	90,053	29,170	37.36
-50	344,760	39,033	34,082	31.40
-60	383,790	0	38,379	25.86
-70	397,590	0	39,759	22.31
-80	408,770	0	40,878	18.94
-90	414,210	0	41,421	15.98
-100	407,910	0	40,791	13.83

The surprising result is that the effect of the shifts of the social cost function for atmospheric carbon had such a small impact on the optimal rotations. Figure 1 above shows that these rotations are very sensitive to the omegas; however, Table 2 reveals that as the shadow value of atmospheric carbon, λ^a , increases, initially ω^s increases causing the solidwood rotation to increase from 16 to 24. While this is occurring fuel-wood production is zero because the price of fuel-wood, P_f , is too low to warrant its production. Once fuel-wood production begins increases in the absolute value of λ^a and increases in wood prices, P_s and P_f , balance the effects of each other, keeping ω^s and ω^f approximately constant. They are at lease constant enough to hold the rotation period constant at 24 for the eight highest atmospheric social cost scenarios.

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