Tests for the Role of Risk Aversion on Input Use

Selected Paper
American Agricultural Economics Association Annual Meeting
Chicago, IL, August 5 - August 8, 2001

February 2001[1]

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[1] Journal paper No. J-18743 of the Iowa Agriculture and Home Economics Experiment Station, Ames, Iowa. Project No. 3463, and supported by Hatch Act and State of Iowa funds. We appreciate the help of Bruce Babcock, Bill Pan, and Dave Huggins in locating data.

Tests for the Role of Risk Aversion on Input Use

Abstract—Agricultural inputs can create negative externalities. For risk averting agents, risk will alter production decisions while the existence of institutions to insure against adverse states of nature will likely restore decisions toward levels under risk neutrality. In this paper, conditions are identified on a stochastic technology to test $H_{0,\leq}^{rn}$: that risk averters choose smaller input levels than risk neutral agents, and $H_{0,\leq}^{ra}$: that an increase in risk aversion reduces input use. A robust statistical method (Klecan, McFadden, and McFadden) to test for dominance is adapted to stochastic production relations. It is found that $H_{0,\leq}^{rn}$ is likely true for nitrogen application on Iowa corn. Weaker evidence is found in favor of hypothesis $H_{0,\leq}^{ra}$.

Keywords: dominance tests, incomplete risk markets, pollution, stochastic technology

JEL Classification: D8, C1, C8, Q0.

Tests for the Role of Risk Aversion on Input Use

Production uncertainty is pervasive in many agricultural environments. The market solution, risk markets, often fail because the costs of maintaining such markets cannot be covered in expectation, because the markets are deterred by regulation or even prohibited, or because of problems regarding information asymmetry. Thus, firms often have to bear the full brunt of the uncertainty when making decisions. Governments have many reasons to be interested in how the risk environment that faces growers affects factor use. Among these, we focus on two strongly interrelated reasons; that risk market incompleteness may impede the overall level of factor use efficiency and that the level of input use often generates externalities.

As an example, field crop production tends to be a risky enterprise while inputs such as pesticides and nitrogen are known to run off into the water supply. It is often difficult to regulate these non-point pollution sources.[2] Chambers and Quiggin (1996) propose to do so through an insurance scheme that pays out on adverse states of nature.[3] But if regulatory schemes such as this are to work, it is necessary to understand the effect of risk aversion and risk market incompleteness on pollution. If it is found that the existence or level of risk aversion reduces the use of polluting inputs, then risk management instruments are quite likely to complement with the inputs in the production process. And so, as pointed out by Chambers and Quiggin (1996), government actions to facilitate firm-level risk management through legislation or subsidies may exacerbate a pollution problem.

The intent of our paper is to align more closely findings from the expected utility modeling framework with empirical methods. In the theoretical dimension, our work extends findings by MacMinn and Holtmann and especially by Ramaswami. The results we develop rely on the diffidence theorem due to Gollier and Kimball.

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[2] A detailed exposition of this problem, and of regulatory solutions, is provided in Chambers and Quiggin (1996) and in Quiggin and Chambers.

[3] Here, payouts are a function of adverse outcomes that a firm has no control over. Insurance on rainfall levels or flood levels rather than on crop shortfall and property damage are examples.

In order to understand the role of risk aversion on choice for a stochastic production technology, we need to characterize the technology. To date the most influential empirically implementable characterization has been the Just and Pope (1978) production function.[4] While a major econometric innovation, its two-moment construction limits its ability to explain behavior in the expected utility framework. One concern, identified in Rothschild and Stiglitz, is that an increase in variance does not capture all attributes of risk that a von Neumann & Mortgenstern (vNM) risk averter faces. A second concern, due to Chambers and Quiggin (2001), is that the Just-Pope technology imposes inflexibility on how inputs and states of nature interact. In the manner of Ramaswami, we will work with a very general representation of a stochastic technology. As a consequence, we are in a position to establish and test the exact conditions on the technology such that the existence and level of vNM risk aversion have determinate effects on input use.

Our empirical methods are adapted from the small literature on discerning partial orderings among a set of empirical distributions. Recognizing that a quest for stochastic dominance among empirical distributions requires an accommodation of the sampling error problem, Tolley and Pope developed a non-parametric test on a pair of such distributions. The test statistic is generated by drawings from the pooled observations. McFadden rigorized the permutation test approach to generate a species of Kolmogorov-Smirnov test for the non-comparability of two distributions in the senses of first-order and second-order dominance. Among the limitations of McFadden's test are the requirement that empirical distributions have equal sample sizes, the assumption that the distribution drawings are independent, and the need to calculate significance levels through Monte Carlo simulations. A test developed by Kaur, Prakasa Rao, and Singh (KRS), based on the intersection-union concept, allows for unbalanced sample sizes and

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[4] For example, Saha, Shumway, and Talpaz employ the specification in a study of behavior by Kansas wheat growers. Smale *et al.* apply it when inquiring into the effects of diversity in wheat varieties on yield risk.

simplifies the work needed to establish the test significance level. However, the KRS test does assume independence in drawings. Anderson also provides a convenient test because the test statistic is known to be distributed according to a variant of the Pearson Goodness-of-Fit statistic with known asymptotic distribution. But the test assumes independence.

We have balanced data, and our main concern with the aforementioned tests is with the need for independence. The production relation we seek to understand is that of nitrogen on corn yield. Independence is likely not a valid assumption for nitrogen-conditioned corn yield distributions because weather and other factors are likely common drivers for all distributions. We choose an extension to McFadden's approach. By assuming stationarity, α-mixing, and generalized exchangeability, Klecan, McFadden, and McFadden (KMM) account for dependence between distributions and rely on Monte Carlo analysis to derive the test distributions. Their test is an intersection union test that relies on multiple interval comparisons.[5]

In section 2 we ascertain an empirically testable condition on a stochastic payoff function such that the effect of risk aversion on input choice is determinate. We also identify conditions such that an increase in risk aversion has a determinate effect on choice. The data to be studied is discussed in section 3. We then present the statistical methods and apply them to the data. The results are analyzed, and the paper concludes.

Model and Theory

In accord with standard notation for models of expected utility maximization, let $U(z)$ be a twice continuously differentiable, increasing and concave utility function. Concavity need not be strict,

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[5] As with other intersection – union tests, such as in KRS, the test does not exploit the covariance structure of the distributions to be compared. This buys computational efficiency at the cost of lower statistical efficiency. Alternative approaches, such as in Davidson and Duclos or Dardanoni and Forcina, include the structure of the covariance matrix in their test statistics. According to Goldberger, taking into account the covariance structure leads to efficiency gains relative to multiple comparison procedures "mainly when these estimates are negatively correlated" (Dardanoni and Forcina, page 58). This is not the case with the data we will analyze.

and we denote the class of all utility functions satisfying our assumptions as $U \in U_2$. In a two-period model, argument *z* is the payoff and is itself a twice continuously differentiable function; $z = z(b,x)$. Here, *b* is a time point 0 action chosen from the closed interval $B \in \overline{\mathbb{R}^+}$. Variable *x* is an index measuring a random factor that is realized at time point 1. The index over time point 1 states of nature is chosen so that more is better, i.e., $z_x(b,x) > 0$ where the subscript denotes a derivative. And at no additional loss of generality, we assume that $x \in [0, 1]$. The variable has an absolutely continuous distribution, $G(x)$, with strictly positive support on all $x \in (0, 1)$. The time point 0 problem is then to $\max_{b \in B} \int_0^1 U[z(b,x)]\, dG(x)$, and the first-order condition is

(1) $$\int_0^1 U_z[z(b,x)]\, z_b(b,x)\, dG(x) = 0.$$

Strict concavity of $z(b,x)$ in *b* ensures a unique solution, and we make this assumption.

Risk-Neutral vs. Risk Averse

For the risk neutral agent the problem simplifies to $\max_{b \in B} \int_0^1 z(b,x)\, dG(x)$ with first-order condition $\int_0^1 z_b(b,x)\, dG(x) = 0$. We denote the optimal argument by b^{rn} so that $\int_0^1 z_b(b^{rn},x)\, dG(x)$. $= 0$. Because the objective function is concave in the action, all risk averters choose b^{ra} such that $b^{ra} \le b^{rn}$ if

(2) $$\int_0^1 U_z[z(b^{rn},x)]\, z_b(b^{rn},x)\, dG(x) \le 0.$$

And all risk averters choose action b^{ra} such that $b^{ra} \ge b^{rn}$ if the inequality in (2) is reversed.

Integrating the left-hand expression by parts yields

$$(3) \quad \begin{aligned} &U_z[z(b^{rn},0)]\int_0^1 z_b(b^{rn},x)\,dG(x) + \int_0^1 U_{zz}[z(b^{rn},s)]\,z_x(b^{rn},s)\int_s^1 z_b(b^{rn},x)\,dG(x)\,ds \\ &= \int_0^1 U_{zz}[z(b^{rn},s)]\,z_x(b^{rn},s)\int_s^1 z_b(b^{rn},x)\,dG(x)\,ds, \end{aligned}$$

where optimality at the risk-neutral solution has been employed.[6] Some work, provided in Appendix A, establishes that the left-hand side of Eqn. (3) is negative (positive) for all risk averters such that $z_x(b,x) > 0$ if and only if

$$(4) \qquad \int_s^1 z_b(b^{rn},x)\,dG(x) \;\geq(\leq)\; 0 \quad \forall\, s \in [0,1].$$

Our results can be summarized as follows

PROPOSITION 1. *Let* $z_x(b,x) > 0$ *and* $z_{bb}(b,x) < 0$. *Then* $b^{ra} \leq(\geq)\, b^{rn}$ *for all* $U \in U_2$ *if and only if the payoff relationship adheres to condition* (4) *above.*

Given that $\int_0^1 z_b(b^{rn},x)\,dG(x) = 0$, monotonicity of $\int_s^1 z_b(b^{rn},x)\,dG(x)$ in *s* would be a sufficient condition for the proposition to hold. However, monotonicity cannot hold because $z_b(b^{rn},x = s)$ cannot be uniform in sign unless it has zero value on all sets of non-null measure.

Now let us assume that $z(b,x)$ takes on the specific functional form $pJ(b,x) - wb$ where $J(b,x)$ is a production function, *p* is the output price, and *w* is the factor price. Then condition (4) becomes $\int_s^1 J_b(b^{rn},x)\,dG(x)/\overline{G}(s) \geq(\leq)\, w/p \;\; \forall\, s \in [0,1]$ where $\overline{G}(s) = 1 - G(s)$. Noting that $\int_0^1 J_b(b^{rn},x)\,dG(x) = w/p$, condition (4) for the payoff function in question may be re-written as

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[6] This integration by parts might be said to be 'from above'. The usual integration by parts is 'from below', in which case we have $-\int_0^1 U_{zz}[z(b^{rn},s)]\,z_x(b^{rn},s)\int_0^s z_b(b^{rn},x)\,dG(x)\,ds$ in place of the expression in (3). But the expressions are equal because $\int_0^1 z_b(b^{rn},x)\,dG(x)\,ds = 0$.

(5) $$\frac{\int_s^1 J_b(b^{rn},x)\,dG(x)}{\overline{G}(s)} \geq (\leq) \int_0^1 J_b(b^{rn},x)\,dG(x) \quad \forall\, s \in [0,1].$$

Ramaswami has identified a criterion that is essentially the same as that in (5), and it is clear that if $\int_s^1 J_b(b^{rn},x)\,dG(x)/\overline{G}(s)$ is nondecreasing (nonincreasing) in *s* then relation (5) assuredly holds. The latter sufficient condition requires that $J_b(b^{rn},s) \leq (\geq) \int_s^1 J_b(b^{rn},x)\,dG(x)/\overline{G}(s)$. That is, $J_b(b^{rn},s) \leq (\geq) E[J_b(b^{rn},x) \,|\, x \geq s]$ where *E* is the expectation operator and conditions on the expectation are given after the vertical bar symbol. The condition requires that mean marginal product, conditional on the source of randomness being in the upper interval $[s, 1]$, is no smaller (no larger) than marginal product evaluated at the lower bound of that upper interval. As we shall show later, with a little re-working of (5), an alternative and distinct sufficient condition is that $J_b(b^{rn},s) \geq (\leq) E[J_b(b^{rn},x) \,|\, x \leq s]$.

In the empirical analysis to follow, we will test for condition (5), and we will abstract from the particular point of evaluation. When (5) is true in the $\geq$ direction, then we state that hypothesis $H_{0,\leq}^{rn}$ is accepted and so we conclude that $b^{ra} \leq b^{rn}$.[7] When (5) is true in the $\leq$ direction, then we state that hypothesis $H_{0,\geq}^{rn}$ is accepted, i.e., we accept $b^{ra} \geq b^{rn}$. In its discrete form, condition (5) may be re-written as

(6) $$\frac{\int_s^1 J(b+\delta,x)\,dG(x)}{\overline{G}(s)} - \int_0^1 J(b+\delta,x)\,dG(x) \geq (\leq) \frac{\int_s^1 J(b,x)\,dG(x)}{\overline{G}(s)} - \int_0^1 J(b,x)\,dG(x) \quad \forall\, s \in [0,1],\ \forall\, \{b, b+\delta\} \in \mathrm{B},\ \delta \geq 0.$$

Because it has been assumed that $J_x(b,x) > 0$, quantity $G(s)$ measures the *s*th quantile of the

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[7] The direction of the inequality in $H_{0,\leq}^{rn}$ is perhaps confusing. The direction $\leq$ is chosen to be consistent with hypothesized inequality $b^{ra} \leq b^{rn}$ rather than the hypothesized inequality in (5).

input-conditioned random production variable $J(b,x)$. In this light, we can write (6) as

$$\text{(7)}\qquad \begin{aligned} &E[J(b+\delta,x)\,|\,x \geq s] - E[J(b+\delta,x)] \geq(\leq) E[J(b,x)\,|\,x \geq s] - E[J(b,x)] \\ &\forall\, s \in [0,1],\ \forall\, \{b, b+\delta\} \in \mathrm{B},\ \delta \geq 0. \end{aligned}$$

For future reference, we denote $R(b,s) = E[J(b,x)\,|\,x \geq s] - E[J(b,x)]$ so that (7) becomes

$$\text{(8)}\qquad R(b+\delta,s) \geq(\leq) R(b,s) \quad \forall\, s \in [0,1],\ \forall\, \{b, b+\delta\} \in \mathrm{B},\ \delta \geq 0.$$

We may interpret (5), (7), and (8) as follows. The expected marginal value product conditional on being in the upper *s*th quantile, $s \in [0,1]$, exceeds (is less than) the unconditional expected marginal value product.[8]

More Risk Averse

The contrast between risk neutrality and risk aversion is rather stark. A more general scenario is to contrast agents that are ordered by their degree of risk aversion. In this case, it will be shown that the critical functional is

$$\text{(9)}\qquad M(b,x) = E[J(b,x)\,|\,x \leq s].$$

The important attribute is whether $M(b,s)$ has a cross-derivative that is uniform in sign.

PROPOSITION 2. *Let* $z(b,x) = pJ(b,x) - wb$ *where p and w are positive constants, and where* $J_b(b,x) > 0 > J_{bb}(b,x)$. *Then, for all* $U \in U_2$, *an increase in the degree of risk aversion*

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[8] As a point of reference for Proposition 1, a stochastic order related to that described by (7) above has been studied in a non-economic context by Fernandez-Ponce, Kochar, and Muñoz-Perez. Their order, the right-spread order, is the quantile variant of the mean residual life order which is a central concept in the statistical theory of reliability. Other related stochastic structures have arisen in studies by Jewitt and by Landsberger and Meilijson on the economics of welfare under partial insurance. To our knowledge, no empirical tests for any of this set of orders have been developed.

reduces (increases) optimum b if

$$M(b+\delta, s_2) - M(b+\delta, s_1) \geq (\leq) M(b, s_2) - M(b, s_1) \quad (10)$$
$$\forall s_1, s_2 \in [0,1],\ s_2 \geq s_1,\ \forall \{b, b+\delta\} \in \mathrm{B},\ \delta \geq 0.$$

The proof is provided in Appendix B. As with comparison (5), in the empirical analysis we will test for condition (10), and we will abstract from the particular point of evaluation. When (10) is true in the $\geq$ direction, then we state that hypothesis $H_{0,\leq}^{ra}$ is accepted and so we conclude that $b^{ra}(\rho_2) \leq b^{ra}(\rho_1)$ where the $\rho_i = -U_{zz}^{\rho_i}(z)/U_z^{\rho_i}(z)$ are coefficients of risk aversion for utility function $U^{\rho_i}(z)$, and where $\rho_2 \geq \rho_1$. In earlier work, MacMinn and Holtmann demonstrated that if $J(b,x)$ has a positive cross derivative then an increase in risk aversion reduces optimum *b*. As Proposition 2 shows, it suffices that the less structured expression $E[J(b,s)|x \leq s]$ possess the property.

To see how Propositions 1 and 2 relate, note that inequality (7) may be written as

$$M(b+\delta, s=1) - M(b+\delta, s) \geq (\leq) M(b, s=1) - M(b, s) \quad (11)$$
$$\forall s \in [0,1],\ \forall \{b, b+\delta\} \in \mathrm{B},\ \delta \geq 0.$$

Thus, as should be the case, condition (7) is less restrictive than (10). In Proposition 1 we compare an arbitrary risk averse agent with a fixed agent in the equivalence class of risk neutral agents. It is because the degree of risk aversion for one agent is fixed that the fixed value $s = 1$ arises in (11) and (7). The comparison in Proposition 2 is more general, and therefore more demanding, in that neither agent is fixed at a point along an ordering of degrees of risk aversion. This added generality is purchased at the expense of the more restrictive condition (10) relative to (7).[9] If an econometric test accepts (10) in a given direction of inequality, then it is likely to

[9] We have not been able to ascertain whether condition (10) is necessary for all $U \in U_2$.

accept (7) in the same direction. But it should be no surprise if a version of (7) holds when the corresponding version of (10) is not supported by data. A rejection of (7) is, however, possible even if (10) is accepted. This would occur if there exists overwhelming evidence in favor of (10) on $s_2 < 1$ while the data in the neighborhood of $s_2 = 1$ does not support (10).

Data

We test for the direction of inequality (7) using Iowa corn yield data that was collected from four different Iowa farms during 1987-1991 using ten different nitrogen application levels from 0-300 lbs./acre. The data has previously been used by Babcock and Hennessy. The overall sample consists of 600 observations and we have subsamples of 60 observations per nitrogen application rate. The data comes from four distinct regional locations over a time period of five years. Table 1 gives the means and standard deviations of yield observations by site.

A statistical analysis of state-level Iowa corn yields shows a clear time trend in the data due to technical progress. Hence, we postulate a deterministic component of yield depending on time *t*, as well as location effects and a stochastic component. To account for changes in yield due to technical progress, the data is corrected by estimating a linear time trend for Iowa corn yields from Iowa average yield over the period 1973-1994 (Iowa Dept. of Agriculture). Corn yields are estimated to increase by 1.526 bu./acre/year. The parameter to this linear time trend has a t-value of 2.326 and the regression R^2 is 0.213.

Denote observation *i* at site *j* for year *t* at a nitrogen application level *b* as $x_{ijt}(b)$. The average yield across years and sites is formed as $\bar{x}_{\bullet\bullet\bullet}(b)$, and the estimated mean yield for any given year is given as $\bar{x}_{\bullet\bullet t}(b) = \bar{x}_{\bullet\bullet\bullet}(b) + 1.526t$ where the linear time trend *t* is centered at *t* = 0 in 1989. The observations are then realized as the sum of the mean and a residual component, $x_{ijt}(b) = \bar{x}_{\bullet\bullet t}(b) + \epsilon_{ijt}$. Using this series of residuals, we tested for equality in mean across sites, and we rejected the hypothesis of equality at the 1% level with an *F* statistic $F_{1,595} = 8.97$. Therefore, we proceeded by correcting for the differences across sites. The resulting yield deviations, as functions of the level of nitrogen applied, are estimated as $\hat{\epsilon}_{ijt}(b)$ where

$$\hat{\epsilon}_{ijt}(b) = x_{ijt}(b) - \bar{x}_{\bullet j \bullet}(b) - 1.526t \tag{12}$$

and where $\bar{x}_{\bullet j \bullet}(b)$ is the site *j* mean given nitrogen level *b*. The mean of the $\hat{\epsilon}_{ijt}(b)$ is zero by construction, and the resulting standard deviations are shown in Table 2. It appears that the level of yield variability increases with the quantity of nitrogen applied. This gives a first indication that the character of the distribution, in terms of higher moments, changes as *b* changes, and so there is circumstantial evidence to hypothesize that the level of risk aversion may dampen the level of input choice. But a more formal analysis is required.

Empirical Procedures

We have shown that a risk averse agent will tend to decrease factor use relative to risk neutrality if $R(b,s)$ is increasing in *b*, while factor use will be ordered by degree of risk aversion if $M(b,s)$ has a cross-derivative that is uniform in sign. Using the corn-yield data, we wish to find empirical evidence for or against these attributes. Equation (7) compares two corrected conditional means. In varying the point *s* of truncation for the conditional mean we are comparing two distributions that, without practical loss of generality, can be assumed to spread over a finite interval and can thus be transformed to take values on the [0,1] interval. An examination of Eqn. (7) or Eqn. (10) shows that we are in fact comparing two quantile distributions when we study $R(b,s)$ across *s* at two different levels of *b* or when we study $M(b,s)$ across *s* at two different levels of *b*. In the case of $R(b,s)$, the quantile can be read immediately as $G(s)$. For $M(b,s)$, with $s_2 \geq s_1$ then the quantile is $G(s_1)/G(s_2)$.

When comparing quantiles, the randomness arises because the empirical distribution constructed from the data consists of random draws from the true underlying distribution. Tests for comparing quantile distributions have been discussed in the literature in relation to stochastic dominance (e.g., Tolley and Pope, and also KMM). Recently, as in Anderson and in Maasoumi and Heshmati, such tests have been applied to compare the Lorenz curve representations of income distributions.

We implement the test proposed by KMM because this test can accommodate dependence between the distributions being compared. Two types of regularity conditions are required for the test. These are that: (a) the observations from each yield distribution are strictly stationary and α-mixing[10]; and (b) the random variables satisfy the generalized exchangeability property.[11] The α-mixing assumption is required to establish asymptotic convergence via the appropriate version of the strong law of large numbers (White and Domowitz). Because the underlying problem of establishing order dominance, or not, among distributions is innately symmetric, the generalized exchangeability property is convenient in that it admits symmetric treatment of the data when seeking to approximate test statistic confidence intervals. The conditions are very general and it seems reasonable to apply the test to our data. Concern might arise with regard to condition (b), and so we present the correlation matrix of the data in table 2. It is clear that the general structure is very stable for input levels other than the extreme values $b = 0\,\text{lb./acre}$ and $b = 300\,\text{lb./acre}$.

The test proposed by KMM is an extension of the Kolmogorov-Smirnov test to multivariate distributions. For a pair of random variables $\{\omega_1, \omega_2\}$ with respective cumulative distribution functions $F_1(\omega)$ and $F_2(\omega)$, it compares the distributions with respect to their first-degree stochastic maximality property. A set of distributions is defined as first-degree maximal if no

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[10] A process $X_n, n = 1,2, ...$ is α-mixing if there exists a sequence $\alpha(k)$ such that $\text{Lim}_{k\to\infty}\alpha(k) = 0$ and $\alpha(k) \geq |P(B \cap A) - P(B)P(A)|$ for each event A regarding the behavior of the process up to time n and each event B regarding the behavior of the process after time $n + k$ (see KMM). As such the α-mixing property can be viewed as the requirement of asymptotic convergence toward independence as the lag along dimension n increases.

[11] An example of generalized exchangeable random variables $\Omega_1, \dots, \Omega_K$ is $\Omega_k = \alpha_k + \beta_k(Z_0\sqrt{\rho} + Z_k\sqrt{1-\rho})$ where $Z_0, \dots, Z_K$ are independent random variables with mean zero and variance one, where $Z_1, \dots, Z_K$ are identically distributed, and where the α_k, β_k, and $\rho \in [0, 1)$ are parameters (see, again, KMM).

distribution in the set is first-degree weakly stochastically dominated by another distribution in the set, i.e., neither $F_1(\omega) \overset{FSD}{\geq} F_2(\omega)$ nor $F_2(\omega) \overset{FSD}{\geq} F_1(\omega)$. Therefore, the random variables $\{X_k, k = 1, 2, \ldots, n\}$ are first-degree stochastically maximal if $d^* = \min_{i \neq j} \max_\omega \{F_i(\omega) - F_j(\omega)\} > 0$ over the index set. That is, first-degree maximality requires that all distributions in the maximal set cross. Comparing the distributions for first-degree stochastic maximality is a two-sided test. By contrast, tests on $F_i(\omega) \overset{FSD}{\geq} F_j(\omega);\ i, j = 1, \ldots, n$ are one-sided tests. These one-sided tests are also computed in the procedure.

Identify now empirical distributions constructed from *N* observations by an appended subscripted *N*, i.e., $F_{iN}(\omega)$. The test statistic should then test for the sign of

$$(13) \qquad d^*_{2N} = \min_{i \neq j} \max_\omega \{F_{iN}(\omega) - F_{jN}(\omega)\} > 0.$$

KMM derive the statistical properties of the test statistic and provide a computational algorithm to test for both first-degree stochastic dominance and maximality. Since the distribution of d^*_{2N} is not analytically tractable, their procedure is based on Monte Carlo simulations that calculate the critical value of the test statistic. In this sense the test is exact. Upon subjecting the test to Monte Carlo experiments over varying hypotheses and sample sizes, KMM conclude that their statistic for first-order dominance performs reasonably well even at sample sizes as small as 50.

Turning to our application, the empirical distributions yield the random variables to compare in Proposition 1 as

$$(14) \qquad R_N(b,s) = E_N[\hat{\epsilon}_{ijt}(b) \mid \hat{\epsilon}_{ijt}(b) \geq s] - E_N[\hat{\epsilon}_{ijt}(b)]$$

where the subscripted *N* on the expectation operator identifies a mean generated from the data. If $R(b+\delta, s) \geq R(b,s)$ for all *s*, then the distribution of $R(b+\delta, s)$ in a sense dominates the distribution of $R(b,s)$. For each nitrogen application level we observe an ordered sample of

$R_N(b,s_1), R_N(b,s_2), \ldots, R_N(b,s_{60})$, where we use the subscript N to denote the observed sample statistics. For the test we use the empirical distribution functions of $R_N(b,s)$ which is denoted by $F_N[R(b,s)]$. Applying this notation to equation (13) results in the inequality to be tested for,

$$(15) \qquad d^*_{2N} = \min\{\max_s[\mathfrak{L}(N,b,b+\delta,s)],\ \max_s[-\mathfrak{L}(N,b,b+\delta,s)]\} > 0.$$

where $\mathfrak{L}(N,b,b+\delta,s) = F_N[R(b+\delta,s)] - F_N[R(b,s)]$.

For the test of the condition underpinning Proposition 2 we proceed similarly and form the sample equivalent to $M(b,s_2) - M(b,s_1)$ as

$$(16) \qquad W_N(b,s_1,s_2) = E_N[\epsilon_{ijt}(b) \mid \epsilon_{ijt}(b) \geq s_2] - E_N[\epsilon_{ijt}(b) \mid \epsilon_{ijt}(b) \geq s_1].$$

Equation (16) gives effectively the observations to be compared for b and $b+\delta$, so that each test entails $\Sigma_{i=1}^{60}(60-i) = 1770$ quantile comparisons. The test statistic d^*_{2N} for (16) then follows in the way that (15) was constructed from (14).

Results

The results for testing Proposition 1 are summarized in tables 3 and 4. Table 3 shows *p*-values for the test statistic that the row first-degree stochastically dominates the column. For instance the *p*-value in the cell (column 1, row 3) is the *p*-value testing the hypothesis that $R(b=50,s)$. $\overset{FSD}{\geq} R(b=0,s)$. The value is 0.0764, and therefore the hypothesis that $R(b=50,s)$ dominates $R(b=0,s)$ is not rejected at the 5% level. Similarly the *p*-value in the cell (column 2, row 1) tests the hypothesis that $R(b=25,s) \overset{FSD}{\geq} R(b=0,s)$. It assumes a value of zero and hence this hypothesis is rejected.

To aid in interpreting the general implications of the test outcomes reported in the tables, we shade the cells in which the hypothesis of *FSD* is rejected at a significance level of 0.05. In the upper right triangle, 9 of 45 tests are not rejected whereas in the lower triangle 14 out of 45 are rejected. The overall picture indicates that in most cases $R(b+\delta,s) \overset{FSD}{\geq} R(b,s)$, $\delta > 0$, i.e., for most distribution comparisons, inequality (7) should be of the direction $\geq$ and then we accept $H_{0,\leq}^{rn}$ with inference that $b^{ra} \leq b^{rn}$. Exceptions occur notably for the distributions arising from factor levels $b = 250$ and $b = 300$, which we could interpret as the upper end of suitable nitrogen levels. Actual nitrogen application rates on commercial Iowa corn vary between 125-200 lb./acre and, of course, application rates that are of no commercial relevance should be of little policy relevance. Also, at high values exceptions occur for small δ, i.e., close to the diagonal of the table. Then shifts in distributions according to increased fertilizer levels may not be clear because the intra-distribution spread might not be clearly distinguished from inter-distribution spread.

Table 4 reports the *p*-value for the hypothesis of not stochastic maximal, i.e., that one of the two distributions compared in a test is first-degree stochastically dominated by the other. Consistent with the results in Table 3, where we found support for $H_{0,\leq}^{rn}$ in most cases, we cannot reject the 'not first-degree stochastically maximal' hypothesis in most instances. There are 9 exceptions among 45 tests at the 5% level of significance, and so in only 9 comparisons do we conclude that the pair of distributions compared is first-degree stochastic maximal. And 7 of these 9 exceptions coincide with distribution comparisons where Table 3 indicated a test outcome which was out of step with the generally supported hypothesis $H_{0,\leq}^{rn}$. This suggests that the shaded boxes in the lower left triangle of Table 3 represent incomparabilities in the partial order of distributions rather than reversals of the intuitive dominance relation. A rejection of 2 tests among a set of 45 comparisons should not be surprising at the 5% significance level.

The results of the test for the stochastic structure underlying Proposition 2 are given in tables 5 and 6. Consistent with the observation that we could nest Proposition 1 as a special case of Proposition 2, the test outcomes do not as clearly support $H_{0,\leq}^{ra}$, i.e., that the technology conditions are sufficient to assert that the privately optimal use of fertilizer decreases as risk aversion increases. However, the general picture is supportive of $H_{0,\leq}^{ra}$. Notice, though, that while more cells are shaded in Table 5 than in Table 3, the set of shaded cells in Table 5 need not contain the set of shaded cells in Table 3. This is because, in contrast to the theoretical comparison where violation of Proposition 1 implies violation of Proposition 2, the statistical comparison allows for violations. And violations at $s_2 = 1$ may be covered over by a preponderance of non-violations on $s_2 \in [0, 1)$.

Discussion

This paper has provided quite strong evidence that, for stochastic corn production technologies in Iowa, optimizing risk averters use less nitrogen than do optimizing risk neutral producers. Weaker evidence was found in favor of the claim that there is a monotone decreasing relation between the degree of risk aversion and the optimal nitrogen application rate. We did not directly test whether nitrogen is a 'risk increasing input' because we would then have to settle on a formal definition of that attribute. Suppose that we define a risk increasing input as one that falls with the choice-taker's risk aversion index. Then we find that nitrogen is likely risk increasing, and so our results corroborate the conclusions of Just and Pope (1979), of Love and Buccola, and of Nelson and Preckel.

We complete our study by making two points concerning the interpretation of our findings. The first is of an empirical nature, and is relevant for the data we have studied. The Agricultural Markets Transition Act, signed into law in the U.S.A. in 1996, replaced a price-contingent subsidy of the form $A \max[p^t - p, 0]$ where A and p^t are parameters and p is the market price of certain crops. The crops in question included corn. Instead of this price-contingent subsidy, growers were to receive a fixed annual transition payment, which we will denote by T. This subsidy was to decline towards zero over seven years. Chavas and Holt have provided evidence in support of the assertion that U.S. corn and soybean growers exhibit risk preferences that are decreasing absolute risk averse (DARA). And so the gradual elimination of transition payments would, *ceteris paribus*, induce an increase in the degree of risk aversion. The evidence in support of our Proposition 2 would suggest that a drawback in the use of nitrogen would occur as the transition payments are phased out.

Our findings also have relevance when agents are exposed to multiple sources of risk. Work by Gollier and Pratt has considered the effect of introducing an actuarially adverse wealth shock. For example, suppose that the stochastic payoff $z(b,x)$ changes to $z(b,x) + y$ where *y* is random with nonpositive expected value and where the random variables are independent. Then, for a given value of *x*, construct the indirect utility function $v[z(b,x)] = E\{u[z(b,x) + y]|x\}$, i.e., the expectation conditional on *x*. A study of the impact of the introduction of wealth risk *y* on welfare and optimal actions is equivalent to a study of the impact of preference function mapping $u(\cdot) \rightarrow v(\cdot)$ on welfare and optimal actions. Thus motivated, Gollier and Pratt studied the properties of risk vulnerability. For utility functions $u(\cdot)$ and $v(\cdot)$ mentioned above, and assuming that *y* has nonpositive expected value, $u(\cdot)$ is said to be risk vulnerable if $v(\cdot)$ has a larger coefficient of risk aversion than $u(\cdot)$, pointwise, over the relevant domain of *x*. While an exact characterization of the $u(\cdot)$ that are risk vulnerable is rather involved, simpler necessary and also simpler sufficient conditions are identified by Gollier and Pratt. DARA is necessary. Concerning sufficiency, if $-u^{(4)}(\cdot)/u^{(3)}(\cdot) \geq -u^{(3)}(\cdot)/u^{(2)}(\cdot) \geq -u^{(2)}(\cdot)/u^{(1)}(\cdot)$, which is called the standardness condition, then vulnerability is assured. Here, the superscripted term in parentheses identifies the order of differentiation. Hyperbolic Absolute Risk Averse risk preferences together with DARA are sufficient for adherence to standardness. Returning to Proposition 2, our test for the impact of an increase in risk aversion converts immediately to sufficient conditions on the technology such that the introduction of an independent, actuarially unfair, background risk induces a reduction in optimal input use when risk attitudes are standard.

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Appendix A

Proof of Proposition 1. The forward implication follows immediately from noting that if $\int_s^1 z_b(b^{rn},x)\, dG(x)$ is uniform in sign over $s \in [0, 1]$, then expression (3) has the opposite sign. To demonstrate necessity, we seek a violation such that a) the expressions in (2) and (3) are strictly negative, while b) the expression in (4) is strictly positive at a point $s^+ \in (0, 1)$ but is strictly negative outside a small metric neighborhood around that point. Suppose that $\int_s^1 z_b(b^{rn},x)\, dG(x)$ > 0 at $s = s^+ \in (0, 1)$. We have assumed that $z_b(b,x)$ is integrable, and so we know that $\int_s^1 z_b(b^{rn},x)\, dG(x)$ is continuous in *s*. Therefore there exists an interval $[\underline{s}, \bar{s}]$ of strictly positive measure with $s^+ \in (\underline{s}, \bar{s})$ such that $\int_s^1 z_b(b^{rn},x)\, dG(x) > 0 \;\; \forall\, s \in (\underline{s}, \bar{s})$. Choose a set of strictly positive measure, $A \in [\underline{s}, \bar{s}]$, and a real number $\epsilon_1 > 0$ such that $\int_s^1 z_b(b^{rn},x)\, dG(x) \geq \epsilon_1 > 0$ $\forall\, s \in A$. Find an $\epsilon_2 > 0$ such that $z_x(b^{rn},x) > \epsilon_2 \;\; \forall\, x \in [0, 1]$. Choose $U_{zz}[z(b^{rn},s)] = \epsilon_3 < 0$ $\forall\, s \in A$ and $U_{zz}[z(b^{rn},s)] = 0 \;\; \forall\, s \in \{[0, 1] \backslash A\}$. Now we can write

$$\begin{aligned}
&\int_0^1 U_{zz}[z(b^{rn},s)]\, z_x(b^{rn},s) \int_s^1 z_b(b^{rn},x)\, dG(x)\, ds \\
&= \int_0^{\underline{s}} U_{zz}[z(b^{rn},s)]\, z_x(b^{rn},s) \int_s^1 z_b(b^{rn},x)\, dG(x)\, ds \\
&+ \int_{\{[\underline{s},\bar{s}]\backslash A\}} U_{zz}[z(b^{rn},s)]\, z_x(b^{rn},s) \int_s^1 z_b(b^{rn},x)\, dG(x)\, ds \\
&+ \int_A U_{zz}[z(b^{rn},s)]\, z_x(b^{rn},s) \int_s^1 z_b(b^{rn},x)\, dG(x)\, ds \\
&+ \int_{\bar{s}}^1 U_{zz}[z(b^{rn},s)]\, z_x(b^{rn},s) \int_s^1 z_b(b^{rn},x)\, dG(x)\, ds \\
&= \int_A \epsilon_3 z_x(b^{rn},s) \int_s^1 z_b(b^{rn},x)\, dG(x)\, ds \;<\; \int_A \epsilon_3 \epsilon_2 \int_s^1 z_b(b^{rn},x)\, dG(x)\, ds \\
&\leq \int_A \epsilon_3 \epsilon_2 \epsilon_1\, ds \;<\; 0.
\end{aligned} \tag{A1}$$

Thus, when condition (4) is violated then there exist concave utility functions such that optimal choice under risk aversion is larger than b^{rn}. ∎

Appendix B

Proof of Proposition 2. As demonstrated in Mas-Colell, Whinston, and Green (1995, Proposition 6.C.2), agent β with utility function $Y(z)$ is more risk averse than agent α with utility function $U(z)$ if and only if there exists an increasing and concave transformation $V(U)$ of U, i.e., $V_U(U) \geq 0 \geq V_{UU}(U)$ so that $Y(z) \equiv V[U(z)]$. For agent α, the equilibrium choice, b^α, satisfies

$$\text{(B1)} \qquad \int_0^1 U_z[z(b^\alpha,x)]\, z_b(b^\alpha,x)\, dG(x) \;=\; 0.$$

By concavity of the objective function, we have that $b^\beta \leq b^\alpha$, where b^β optimizes for agent β, if

$$\text{(B2)} \qquad \int_0^1 V_U[U(z)]\, U_z[z(b^\alpha,x)]\, z_b(b^\alpha,x)\, dG(x) \;\leq(\geq)\; 0.$$

We will prove the result in the ≤ direction. The other result can be demonstrated by symmetric reasoning. Integrating the left-hand expression in (B2) by parts, we have the equivalent expression

$$\text{(B3)} \qquad \begin{aligned} & V_U(U[z(b^\alpha, x=1)]) \int_0^1 U_z[z(b^\alpha, x)]\, z_b(b^\alpha,x)\, dG(x) \\ & - \int_0^1 V_{UU}(U[z(b^\alpha,x=s)])\, U_z[z(b^\alpha,x=s)]\, z_x(b^\alpha,x=s) \int_0^s U_z[z(b^\alpha, x)]\, z_b(b^\alpha,x)\, dG(x)\, ds \\ & = \; - \int_0^1 V_{UU}(U[z(b^\alpha,x=s)])\, U_z[z(b^\alpha,x=s)]\, z_x(b^\alpha,x=s) \int_0^s U_z[z(b^\alpha, x)]\, z_b(b^\alpha,x)\, dG(x)\, ds \\ & \leq \; 0. \end{aligned}$$

Therefore, we need to find conditions such that $\int_0^s U_z[z(b^\alpha, x)]\, z_b(b^\alpha,x)\, dG(x)\, ds \leq 0 \;\; \forall\, s \in [0, 1]$ where $U_z[z(b^\alpha, x)]$ is nonincreasing in x. A further integration by parts yields the equivalent requisite condition

$$U_z[z(b^\alpha, x = s)] \int_0^s z_b(b^\alpha, x)\, dG(x)$$

(B4)

$$- \int_0^s U_{zz}[z(b^\alpha, x = t)]\, z_x(b^\alpha, x = t) \int_0^t z_b(b^\alpha, x)\, dG(x)\, dt \;\le\; 0 \quad \forall s \in [0, 1].$$

And so it would suffice to demonstrate that $\int_0^s z_b(b^\alpha,x)\, dG(x) \le 0 \;\; \forall s \in [0, 1]$. But this is unlikely to be true for all such *s*. In particular, if $b^\alpha < b^{rn}$ then $\int_0^1 z_b(b^\alpha,x)\, dG(x) > 0$ because $z_{bb}(b,x) < 0$.

To find a more satisfactory condition, note that in our case the statement $\int_0^s z_b(b^\alpha,x)\, dG(x) \le 0$ may be written as $\int_0^s J_b(b^\alpha,x)\, dG(x)/G(s) \le w/p$. By condition (10), we have that the expression $\int_0^s J_b(b^\alpha,x)\, dG(x)/G(s)$ is monotone increasing in *s*. Define $\hat{s}$ such that $\int_0^{\hat{s}} J_b(b^\alpha,x)\, dG(x)/G(s) = w/p$. Clearly, because $\int_0^s J_b(b^\alpha,x)\, dG(x)/G(s)$ is increasing in *s*, we have that $\int_0^s z_b(b^\alpha,x)\, dG(x) \le 0 \;\; \forall s \in [0, \hat{s}]$ and $\int_0^s z_b(b^\alpha,x)\, dG(x) \ge 0 \;\; \forall s \in (\hat{s}, 1]$. And so (B4) is true for $\forall s \in [0, \hat{s}]$. For $s \in (\hat{s}, 1]$ we have from (10), i.e., $\int_0^s J_b(b^\alpha,x)\, dG(x)/G(s)$ increasing in *s*, that $J_b(b^\alpha, x = s) \ge \int_0^s J_b(b^\alpha,x)\, dG(x)/G(s)$. Because we are on the interval $s \in (\hat{s}, 1]$, we have $\int_0^s J_b(b^\alpha,x)\, dG(x)/G(s) \ge w/p$ and therefore $J_b(b^\alpha, x = s) \ge w/p$. Note now that the condition $\int_0^s U_z[z(b^\alpha, x)]\, z_b(b^\alpha,x)\, dG(x)\, ds \le 0 \;\; \forall s \in [0, 1]$ may be re-written as

$$\int_0^s U_z[z(b^\alpha, x)]\, z_b(b^\alpha,x)\, dG(x) \;=\; \int_0^1 U_z[z(b^\alpha, x)]\, z_b(b^\alpha,x)\, dG(x)$$

(B5)

$$- \int_s^1 U_z[z(b^\alpha, x)]\, z_b(b^\alpha,x)\, dG(x)$$

$$= \; - \; p \int_s^1 U_z[z(b^\alpha, x)]\, [J_b(b^\alpha,x) - w/p]\, dG(x)\, ds \;\le 0 \quad \forall s \in (\hat{s}, 1],$$

where the first-order condition for agent α has been invoked. We can conclude that $\int_0^s U_z[z(b^\alpha, x)]\, z_b(b^\alpha,x)\, dG(x)\, ds \le 0 \;\; \forall s \in [0, 1]$, and so the proposition is demonstrated. ∎

Table 1. Mean and Variance of Yield Residuals by Site

Moment	Site 1	Site 2	Site 3	Site 4
Mean	122.1	123.5	131.4	117.2
Std. Dev.	34.58	44.38	45.17	32.94

Table 2. Standard Deviations and Correlations of Yield Residuals by Nitrogen level

N-level	0	25	50	75	100	125	150	200	250	300
Std. Dev.	19.67	21.05	25.32	28.66	31.08	35.58	30.64	35.99	31.28	32.59
Correlation Matrix										
0		0.75	0.55	0.54	0.30	0.38	0.43	0.47	0.46	0.49
25			0.63	0.76	0.68	0.72	0.72	0.72	0.72	0.73
50				0.74	0.65	0.70	0.76	0.75	0.73	0.73
75					0.76	0.81	0.80	0.80	0.75	0.74
100						0.87	0.85	0.81	0.80	0.82
125							0.90	0.90	0.86	0.91
150								0.88	0.87	0.90
200									0.84	0.92
250										0.92

Table 3. p Values for First-Stochastic Dominance, Proposition 1

N-level	0	25	50	75	100	125	150	200	250	300
0		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
25	0.3759		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
50	0.0764	0.2362		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
75	0.6561	0.8493	1.0000		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
100	1.0000	1.0000	1.0000	1.0000		0.0000	0.5203	0.0000	0.8831	0.3188
125	0.9569	0.9952	1.0000	1.0000	0.0000		0.9996	0.3080	0.9995	1.0000
150	0.6966	0.8667	1.0000	0.8436	0.0000	0.0000		0.0000	0.0000	0.0000
200	0.9547	1.0000	1.0000	1.0000	0.0188	0.0000	1.0000		0.9998	0.9994
250	0.9635	0.9971	1.0000	0.6504	0.0000	0.0000	0.0306	0.0000		0.0000
300	0.9566	0.9956	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	

Table 4. p Values for 'Not First-Degree Stochastic Maximal', Proposition 1

N-level	0	25	50	75	100	125	150	200	250	300
0										
25	0.0999									
50	0.0031	0.0362								
75	0.3752	0.7068	1.0000							
100	1.0000	1.0000	1.0000	1.0000						
125	0.9134	0.9899	1.0000	1.0000	0.0000					
150	0.4394	0.7372	1.0000	0.6908	0.1755	0.9994				
200	0.9072	0.9999	1.0000	1.0000	0.0000	0.0313	1.0000			
250	0.9266	0.9953	1.0000	0.3683	0.7726	0.9982	0.0000	0.9991		
300	0.9107	0.9917	1.0000	0.9999	0.0464	1.0000	0.0000	0.9985	0.0000	

Table 5. p Values for First-Stochastic Dominance, Proposition 2

N-level	0	25	50	75	100	125	150	200	250	300
0		1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.115		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	1.000	1.000		0.000	0.000	0.000	0.000	0.000	1.000	0.9999
75	1.000	1.000	1.000		0.998	0.943	0.003	1.000	1.000	1.000
100	1.000	1.000	1.000	0.005		0.000	0.004	1.000	1.000	1.000
125	1.000	1.000	1.000	0.088	1.000		0.059	1.000	1.000	1.000
150	1.000	1.000	1.000	1.000	1.000	0.973		1.000	1.000	1.000
200	1.000	1.000	0.550	0.000	0.013	0.000	0.000		1.000	1.000
250	0.999	1.000	0.000	0.000	0.000	0.000	0.000	0.000		0.112
300	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	

Table 6. p Values for 'Not First-Degree Stochastic Maximal', Proposition 2

N-level	0	25	50	75	100	125	150	200	250	300
0										
25	1.0000									
50	1.0000	1.0000								
75	1.0000	1.0000	1.0000							
100	1.0000	1.0000	0.9999	0.9972						
125	1.0000	1.0000	1.0000	0.8834	1.0000					
150	1.0000	1.0000	1.0000	1.0000	1.0000	0.9456				
200	1.0000	1.0000	0.2789	0.9998	1.0000	1.0000	1.0000			
250	0.9974	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
300	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0.0114	