Assessing Social Costs of Inefficient Procurement Design*

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Abstract

This paper considers the social costs implied by inefficient allocation of contracts in a first price, sealed bid procurement auction with asymmetric bidders. We adopt a constrained (piecewise linear) strategy equilibrium concept and estimate the structural parameters of the bidders' distribution of costs. We estimate social costs defined as the predicted cost difference between the winning firm and the most efficient bidding firm. We also compare the expected procurement costs under two different auction formats. The data is collected from procurement auctions of road painting in Sweden during 1993-99. The results indicate that the social costs of inefficient contract allocation is about 1.7 per cent of total potential social cost and that an efficient second price auction would lower the expected procurement cost by 2.8 per cent.

Key words: Procurement auctions, inefficiency, constrained strategy equilibrium, simulation.

JEL classification: D44, H57, C15.

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1 Introduction

Each year the Swedish Road Adminstration (SRA) procures goods and services to maintain large parts the public road net. The procurements are decentralized to seven autonomous regions and organized as first price, sealed bid auctions with a relatively moderate number of potential contractors. At each separate procurement, the bidding contractors differ in size, location, and workload, and they consequently face different costs to complete the contract put out for tender. These differences are observable for all participants in the market.

Commonly known asymmetries across bidders implies that the first price auction procedure may fail to allocate the contract to the most efficient (low cost) firm. That is, the first price auction is not efficient if the bidders draw their values (costs) from asymmetric distributions (Hansen 1984, Milgrom & Weber 1982).An intuitive definition of the social costs associated with inefficient allocation is thus the cost difference between the contracted and the most efficient bidding firm. However, changing the identity of the contracted firm in one auction may alter the identities of winning firms in proceeding auctions due to dynamic effects caused by capacity constraints. In a complete analysis such effects should be accounted for. In this paper, we only consider the first order, static effects of inefficient contract allocation. Further, if participation is endogenous, changing the auction design may also change the decision to participate. We believe that this is of less importance in this case since the firms with relatively low cost draws are likely to participate irrespective of the auction format. The endogenous participation decision is probably more important in the right tail of the cost distribution. Hence, we abstract from this feature.

There is also an issue about expected procurement cost, i.e., the cost financed by taxation in the case of public procurements. Assuming private values, and risk neutral, *ex ante* symmetric bidders, the standard auctions are all revenue equivalent and efficient (Vickrey 1961). In the asymmetric case, the revenue equivalence breaks down and the revenue ranking becomes ambiguous (Maskin

& Riley 2000a). Hence, it is an empirical question of which auction format minimizes the procurement costs. Therefore we also perform a Monte Carlo analysis that simulates the procurement costs under different auction formats. One should also note that we are only concerned about first order social costs. There is of course a second order effect stemming from the cost of financing the public contract. Changing the auction mechanism may also change the expected procurement costs, and thereby the cost of funding.

In recent research, the assumption on symmetric bidders in first price auctions has been relaxed by several authors (Pesendorfer 2000, Maskin & Riley 2000a, Lebrun 1999). In the independent private value model with continuous types, the results illustrate that the Nash equilibrium can be stated as a solution to a system of first order ordinary differential equations. However, the empirical work on structural estimation with asymmetric bidders has been obstructed by the computational difficulties associated with the solution of the Nash equilibrium with asymmetric bidders (Bajari 2000, Bajari & Ye 2000, Marshall, Meurer, Richard & Stromquist 1994) Also, the Nash equilibrium itself has been the subject of some criticism due to the high degree of rationality that is imposed on the players and their mathematical capabilities. This criticism has motivated the development of an alternative solution concept where the players strategies are constrained to some predefined set of feasible strategies (Armantier, Florens & Richard 2000). These feasible strategies could be interpreted either as Nash equilibrium approximations or as rules-of-thumb.

This paper evaluates the social costs induced by the inefficient allocation procedure the first price, sealed bid auction constitutes if the bidders are *ex ante* asymmetric. We adopt a constrained strategy equilibrium approach in which the players are constrained to simplified, piecewise linear strategies. This allows us to estimate the structural elements of the bidders' private values, i.e., the parameterized distribution of the bidders' costs. Conditionally on these distributions, we evaluate the social costs of inefficient contract allocation in the sample. We finally perform some simulations to assess the expected procurement costs for a second price procurement design that is efficient in the presence of asymmetries.

Our preliminary findings are that about 17 per cent of the contracts are inefficiently allocated and that the implied social costs is about 1.7 per cent of potential costs. Switching to an efficient second price auction would in addition yield a 2.6 per cent decrease in procurement costs.

The paper is organized as follows. In section 2, we briefly discuss concept of constrained strategy equilibrium in the asymmetric first price, sealed bid auction with independent private values. The appendix gives the corresponding discussion on the Nash equilibrium. Section 3 gives a description of the procurement market and the available data. The econometric model is presented in section 4, and the results are presented in section 5. Section 6 concludes.

2 Constrained strategy equilibrium

This section presents the Constrained Strategy Equilibrium (CSE) in a first price auction with independent private values and *ex ante* asymmetric bidders. This brief presentation draws heavily from the original work by Armantier et al. (2000). A discussion on the corresponding Nash equilibrium is relegated to appendix.

The auction format considered here is a standard first price, sealed bid procurement auction. The format implies that the bidders simultaneously submit sealed bids and the low bidder wins the contract and is compensated by her bid. At a single game (auction), the set of *players* are denoted by $\mathbb{N}$, and the players are indexed by $i = 1, \ldots, n$. The continuous set of the players' *types* are denoted by $\mathbb{T} \in \mathbb{R}^n$ with elements $\mathbf{t} = (t_1, \ldots, t_n)$. The players' types are drawn from $\mathbb{T} = \prod_{i=1}^{n} \mathbb{T}_i$, using a joint distribution $F(\mathbf{t})$ with support on $[\underline{t}, \overline{t}]$. The set of *unconstrained feasible strategies* is denoted by $\mathbb{S} = \prod_{i \in \mathbb{N}} \mathbb{S}_i$ with elements $\mathbf{s} = (s_1, \ldots, s_n)$, (*an unconstrained strategy profile*), which map types into the set of *feasible actions* $\mathbb{X} = \prod_{i=1}^{n} \mathbb{X}_i$ with elements $\mathbf{x} = (x_1, \ldots, x_n)$.

The vector of *individual von Neumann-Morgenstern utility functions* is denoted by $\hat{U}(\mathbf{s},\mathbf{t}) = (\hat{U}_1(\mathbf{s},\mathbf{t}),\ldots,\hat{U}_n(\mathbf{s},\mathbf{t}))$, such that

$$\hat{U}_i(\mathbf{s},\mathbf{t}) \quad = \quad (s_i(t_i) - t_i)\mathbf{1}(s_i(t_i) < s_j(t_j), \forall j \neq i) \tag{1}$$

The structure of the game is thus defined by $\Gamma = (\mathbb{N}, \mathbb{T}, F, \mathbb{S}, \hat{U})$, which is assumed common knowledge.

Player i's type, t_i, is interpreted as the player's cost to complete the contract, which is drawn *independently* from a marginal distribution $F_i(t_i)$. Hence, the joint distribution of types is

$$F(\mathbf{t}) \quad = \quad \prod_{i=1}^{n} F_i(t_i) \tag{2}$$

A strategy s_i is interpreted as a *bid function* that transforms cost t_i into bid x_i. Given the distribution of types F and a strategy profile $\mathbf{s}$, the induced joint distribution of bids is denoted by $G(\mathbf{x};\mathbf{s},F)$ with marginal distributions $G_i(x_i;\mathbf{s},F)$.

An individual's type t_i is assumed private information. Hence, letting the distribution of $y_i = \min_{j\neq i}\{s_j(t_j)\}$, i.e., the lowest of the rivals' bids, be denoted by $G^*_{-i}(y_i;\mathbf{s}_{-i},F)$, where $\mathbf{s}_{-i} = (s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_n)$, a bidder's expected utility conditional on the strategy profile can be written as

$$U_i(s_i;\mathbf{s}_{-i},F) \quad = \quad \int_{\mathbb{T}_i} (s_i(t_i) - t_i)(1 - G^*_{-i}(s_i(t_i);\mathbf{s}_{-i},F))dF_i(t_i) \tag{3}$$

As discussed in the appendix, the Nash equilibrium is defined as the solution to a system of ordinary differential equations. The solutions can not be expressed in closed form unless we impose severe restrictions on the distribution of types. Hence, solving the unconstrained Nash equilibrium may be considered a rather complicated procedure in the eyes of the bidders. Not only do the bidders need to know the cost parameters of the competitors, they would also need to solve a system of ordinary differential equations. As an alternative solution concept, Armantier et al. (2000) propose a Constrained Strategy Equilibrium (CSE) concept. This concept only considers a constrained set of feasible strategies, e.g. polynomials of low order or piece-wise linear splines, and lets the bidders maximize their expected utilities over a few parameters that define the strategies. (For a discussion of “Rules of Thumb” and NE approximation, see Armantier, Florens & Richard (1999)).

Consider a compact set of constrained feasible strategies $\mathbb{S}_i^{(P)} \subset \mathbb{S}_i$, where $\mathbb{S}_i$ is the unconstrained set of feasible strategies and (P) reflects the “dimensionality” of the constrained set. A *constrained strategy equilibrium* (CSE) is a set of strategies

$$s^{(P),CSE} = \{s_i^{(P),CSE}\}_{i\in\mathbb{N}} \in \mathbb{S}^{(P)} = \textstyle\prod_{i=1}^{n} \mathbb{S}_i^{(P)} \tag{4}$$

where the $s_i^{(P),CSE}$'s satisfy the “mutually best response” condition in the strategic form game, i.e.,

$$U(s_i^{(P),CSE}; s_{-i}^{(P),CSE}, F) \geq U(s_i^{(P)}; s_{-i}^{(P),CSE}, F), \quad \forall s_i^{(P)} \in \mathbb{S}^{(P)}, \forall i \in \mathbb{N} \tag{5}$$

Since $\mathbb{S}_i$ is compact, the constrained strategies can be parameterized such that

$$s_i^{(P)}(t_i; F) \quad = \quad s^{(P)}(t_i; \mathbf{d}_i, \mathbf{d}_{-i}, F) \tag{6}$$

where $\mathbf{d} = \{\mathbf{d}_i\}_{i\in\mathbb{N}} \in \mathbb{R}^{mn}$ denotes the collection of the parameters. The optimal constrained strategy for firm i, conditional on the strategies of the other firms is then defined by the optimal parameters $\mathbf{d}_i$ as a function of the other firms' parameters $\mathbf{d}_{-i}$

$$\mathbf{d}_i^*(\mathbf{d}_{-i}, F) \quad = \quad \arg\max_{\mathbf{d}_i} U_i(s_i^{(P)}(t; \mathbf{d}_i, \mathbf{d}_{-i}, F); s_{-i}^{(P),CSE}(t; \mathbf{d}, F), F) \tag{7}$$

The CSE profile is then defined by the solution to the fixed point problem

$$\mathbf{d}_i^{CSE}(F) = \mathbf{d}_i^*(\mathbf{d}_{-i}^{CSE}(F), F) \tag{8}$$

and $s^{(P),CSE}(t_i; \mathbf{d}_i, \mathbf{d}_{-i}, F) = s^{(P)}(t_i; \mathbf{d}_i^{CSE}, \mathbf{d}_{-i}^{CSE}, F)$.

The calculation of the CSE involves a number of numerical issues. Dropping the CSE superscript, bidder i's expected utility can be expressed as

$$U_i(s_i^{(P)}; s_{-i}^{(P)}, F) = \int_{\underline{t}}^{\bar{t}} (s_i^{(P)}(t_i) - t_i)(1 - G_{-i}^*(s_i^{(P)}(t_i)))dF_i(t_i) \tag{9}$$

The first numerical problem stems from the fact that the distribution of the rivals low bid G_{-i}^* is a non-trivial transformation of the rivals' type distributions. It is, however, straightforward to make random draws from that distribution, given the vector of CSE parameters $\mathbf{d}_{-i}$.

Let $y_i = \min_{j \neq i}\{s^{(P)}(t_j; \mathbf{d}_j, F)\}$, then we can make a random draw $(\tilde{t}_i, \tilde{y}_i)$ from the joint distribution (t_i, y_i) by simply making a random draw $t = (t_i, t_{-i})$ from the joint distribution F, transform the t_{-i} using the defined strategy $s_{-i}^{(P)}(t_{-i}; \mathbf{d}_{-i}, F)$, and setting $\tilde{y}_i = \min_{j \neq i}\{s^{(P)}(\tilde{t}_j; \mathbf{d}_j, \theta)\}$. [1] Using R such random draws, indexed by the superscript r, an estimator of the expected utility is then

$$U_i(s_i^{(P)}; s_{-i}^{(P)}, F) \approx \frac{1}{R}\sum_{r=1}^{R}(s_i^{(P)}(\tilde{t}_i^r) - \tilde{t}_i^r)\mathbf{1}(s_i^{(P)}(\tilde{t}_i^r) < \tilde{y}^r) \tag{10}$$

This approximation is not smooth in $\mathbf{d}_i$ since the indicator function equals either 0 or 1 depending on its argument. In order to use standard numerical methods to solve the maximization problem, we substitute the indicator function for a smooth c.d.f. kernel estimator $K_h(x)$ where h denotes the bandwidth that determines the "smoothness" of the kernel.[2] Hence, the approximative expected

[1] In the empirical analysis we use Halton draws instead of standard random number generators. This should reduce the variance of the integral estimator.

[2] As $h \to 0$, $K_h() \to \mathbf{1}()$

utility function can be written as

$$U_i(s_i^{(P)}; s_{-i}^{(P)}, F) \quad \approx \quad \frac{1}{R}\sum_{r=1}^{R}(s_i^{(P)}(\tilde{t}_i^r) - \tilde{t}_i^r)K_h(\tilde{y}_i^r - s_i^{(P)}(\tilde{t}_i^r)) \tag{11}$$

Taking the first derivatives w.r.t. $\mathbf{d}_i$ yields the first order conditions, a system of mn non-linear equations (m is the number of parameters in the constrained strategies) that are smooth in $\mathbf{d}$,

$$\begin{aligned} 0 \quad = \quad & \frac{1}{R}\sum_{r=1}^{R}\frac{\partial s_i^{(P)}(\tilde{t}_i^r)}{\partial \mathbf{d}_i}\Big\{K_h(\tilde{y}^r - s_i^{(P)}(\tilde{t}_i^r)) - \\ & (s_i^{(P)}(\tilde{t}_i^r) - \tilde{t}_i^r)K_h'(\tilde{y}_i^r - s_i^{(P)}(\tilde{t}_i^r))\Big\}, \forall i \in \mathbb{N} \end{aligned} \tag{12}$$

In general, the $\frac{\partial s_i^{(P)}(\tilde{t}_i^r)}{\partial \mathbf{d}_i}$ is a simple function of $\tilde{t}_i^r$ and $\mathbf{d}_i$. Choosing a kernel K such that the derivative K' can be expressed in closed form, further decreases the computational burden.

In Figures 1 and 2, we illustrate the constrained strategy equilibriums for a set of asymmetric firms and their relation to the Nash equilibrium. We use a set of three asymmetric bidders, where the distribution of types is assumed truncated normal $N(\mu_i, 3)$ where $\mu = (14, 14.5, 15)$ and $5 \leq t \leq 25$. The Nash equilibrium is numerically calculated using a Runge-Kutta algorithm combined with the search algorithm discussed in appendix with $\varepsilon = 1e - 10$. Due to numerical issues, the solutions presented in the figure are poor in the right tail hand of the cost distributions. The horizontal axis shows the costs and the vertical axis shows the optimal bid.

The constrained strategy set here consists of 3-piecewise linear splines with fixed nodes.[3] The CSE could be interpreted as an approximative solution to the NE equilibrium, which is highlighted in figure 2. Including more splines would decrease the "distance" between the NE and the CSE. However, at this stage, we will adopt a 3-piece spline.

[3] We discuss the construction of the splines in the empirical section.

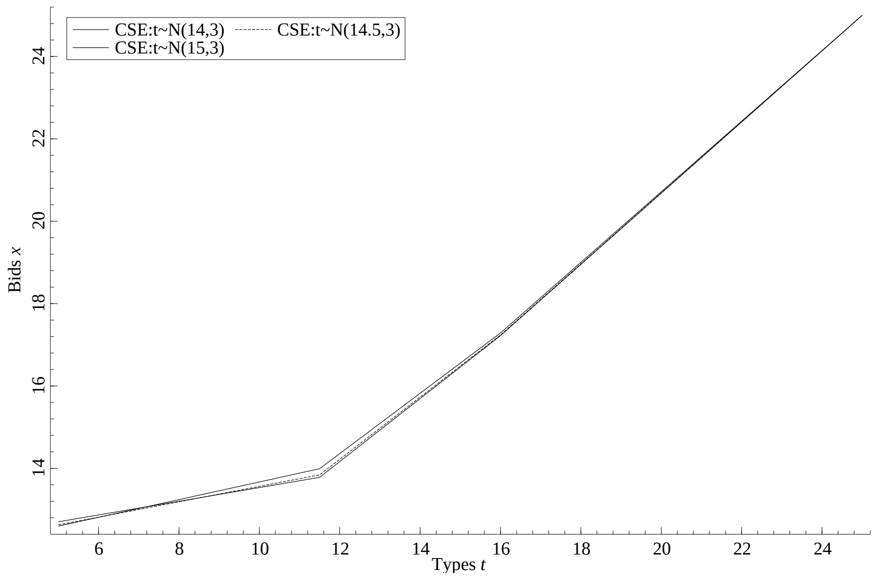

Figure 1: Constrained strategy equilibrium with three asymmetric bidders and 3-piecewise linear splines.

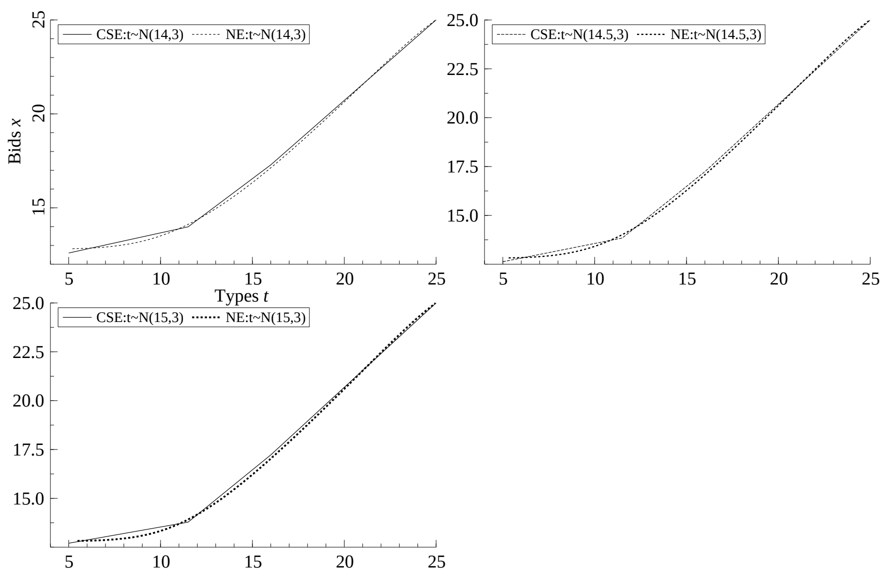

Figure 2: CSE and NE with three asymmetric bidders.

3 Market characteristics and Data

Every year, normally in spring, the Swedish Road Administration (SRA henceforth) contracts firms for maintenance of road markings in Sweden.[4] The firms are contracted through a first-price sealed-bid auction. The SRA's seven regional offices, where a regional office is responsible for the procurement in its region, conduct the auctions. A region consists of three or four provinces. In each province, there are normally three types of road marking contracts subjected to procurement bidding: thermoplastic, spray-plastic, and hand-applicated road markings. Each contract is sold in a separate auction. The local office requires in general that a bidding firm submit its sealed bids for the desired contracts simultaneously for all the provinces in their region. Also, each local office decides on its own when to announce the contracts for the procurement of road markings in its region, when the sealed bids must at the latest be tendered, and when to announce the results. Therefore, the outcome of the auctions in one specific region may or may not be common knowledge before the firms must submit their bids in another region. During the studied period 1993-1999, twelve firms have been active in bidding. Some of them are nationwide, meaning that they can operate in every province in Sweden, whereas other firms operate in just a few provinces. The contracted firms normally fulfill their undertaking during the summer (May-September).

The local office invites firms to tender bids for contract of road markings by advertising the project and, on request, by sending out an inquiry document to potential bidding firms. Among other things, this document contains information about the demanded quantity of marking color, technical requirements of the road marking material, required thickness and width of road markings, instructions how the bids are to be evaluated, and the latest day to tender bids. The tendered bids are denoted in price per kilo or price per meter. A bidding firm is also usually required to enclose a description of its organization, the type

[4] On average the Swedish Road Administration yearly 1993-1999 spent about 100 Million SEK on the procurement of different types of road markings.

of material, and the road marking equipment it intends to use.

To collect the data we went to the SRA's local offices and got extracts from minutes of the results from the auctions. We have focused upon the procurement of thermoplastic road markings, which represents about 50 % of the SRA's total cost for procurement of road markings. The data set covers all procurement auctions of thermoplastic road markings in every province for the period 1993-1999. The data set consists of 138 auctions with a total number of 621 bids.[5] All bids are tendered in terms of price per unit (kilo or meter).[6] Further, the data set contains information about the date and province of each auction, the quantity of tons of road marking material demanded, the number of firms that has received the inquiry document and the identity of all bidders. We have converted all bid data into real terms (1999 price level), using a branch price index.[7]

The data that we have collected refers solely to contract characteristics, such as size, location, etc. However, in the analysis, it is important to capture any difference across the firms that may explain the variation in bids. At this stage we focus on two different dimensions. Since road markings require heavy equipment (trucks, boilers, etc.), a firm's distance to the contract site may influence its costs. Further, the production also requires some special skills of the work force. Hence, given that a firm has limited access to these specialists, a firm's costs may be characterized by increasing marginal costs. We construct

[5] In a couple of auctions we lack some of the rejected bids.

[6] A contract for thermoplastic road marking implies painting both thick and thin lines. For a given quantity of road marking material, the thicker the line, or the broader the line, the lower the contractor's cost per kilo. In the inquiry document the local office very roughly specifies on what types of road marking lines the demanded quantity material is to be distributed. As a general rule, the bidding firms do not submit their bids in terms of a total sum for carrying out the contract, but in terms of price per kilo or price per meter for each type of road marking line. Once all the bids are submitted, the SRA computes each firm's competitive bid as a weighted average of its prices for the various types of lines. The weight put on each price is given in the inquiry document, i.e. it is common knowledge prior to the deadline. The firm that submits the lowest weighted average bid, gets the contract. For the years 1993-1995, the tendered bid has been a weighted average price per kilo, and since 1996 a firm's bid is expressed as a weighted average price per meter. Having knowledge of the relationship between price per meter and the quantity road marking material that is needed to produce one meter road marking of a given type of line, we have converted all bids into price per kilo.

[7] The branch index is "Entreprenadindex E84 2162 Vägmarkeringar" which includes wage rates, material costs, etc.

two variables that measures these dimensions.

From the minutes, we observe the bidding firms' headquarter locations (and production plants in the relevant cases). We assume that a firm must transport its equipment from this location to the contract site. We do not have access to the true distance. Therefore we construct a pseudo distance defined as follows. First, we create a proportional map of Sweden in the xy -plane. Next, we calculate the coordinates of each province's centra and denote these (x_t^s, y_t^s). For each firm i, we locate its headquarter(s) on the map and denote the coordinates as (x_i^f, y_i^f). The distance, $DIST_{it}$, between the firm i and the contract t site is then measured as $\sqrt{(x_t^s - x_i^f)^2 + (y_t^s - y_i^f)^2} * scale$, where $scale$ is a scaling factor to convert the pseudo distance into (approximative) kilometers. If a firm has more than one production plant, we automatically define the distance as the distance between the closest plant and the contract province.

The second variable measures a firm's utilization rate at any given point in time. This variable is constructed in two steps. First, we calculate the total contract size (measured as tons) each firm has won in any single year. Firm i's capacity, CAP_i, is then defined as the maximum total contract size firm i has won in any year. Next, a firm's utilization rate at the letting of contract t, $UTIL_{it}$, is defined as the ratio between the sum of previously won contracts (in tons) in auctions within the same year but prior to the letting of auction t and the firm's capacity. We measure the potential maximum utilization rate, $PUTIL_{it}$, as the ratio between the sum of won and unannounced contracts (in tons) at the letting date of contract t and the firm's capacity.[8] One should note that the SRA is not the single buyer of these services. Also municipalities procure road markings. Therefore, a firms utilization rate may be hidden since we only observe a part of the market for any firm. It is also the case that some of the firms are involved in other types of construction projects which may

[8] A firm has "won a contract at time t" if the winner (the firm) of the contract has been publicly announced. A firm's "open contracts" are those on which the firm has submitted bids, but no winner has been announced. In many cases, several contracts has the same letting date. In those cases, we have included the bids submitted to the other contracts in the open contracts.

Table 1: Descriptive statistics on contracts and winning bids

	Mean (a)	**Std** (b)	**Min** (c)	**Max** (d)	**Valid** (e)
TON	160.29	115.34	10.00	600.00	133.00
NYLAGG	80.85	93.81	1.00	500.00	74.00
NBIDS	4.64	1.22	1.00	7.00	138.00
BID	14.76	0.98	11.85	17.53	133.00
DIST	110.23	104.93	5.43	652.68	138.00
UTILR	0.07	0.14	0.00	0.65	138.00
PUTILR	1.81	4.59	0.00	39.80	138.00

Table 2: Descriptive statistics on bidders

	Mean (a)	**Std** (b)	**Min** (c)	**Max** (d)	**Valid** (e)
BID	15.57	1.29	11.85	21.23	597.00
DIST	166.46	141.30	5.43	806.50	621.00
UTILR	0.13	0.22	0.00	1.00	615.00
PUTILR	1.41	2.49	0.00	39.80	615.00

affect the firm's utilization rate. However, at this stage we abstract from these features. We also construct an indicator variable $INCUM_{it}$ that equals 1 if firm i was the incumbent bidder on contract t. Furthermore, we construct variables that control for some of the competitors costs. For each firm we construct the variables $CMIND$ and $CMINU$ that are defined as the minimum distance and utilization rate of the bidding competitors.[9]

Below we give some descriptive statistics of the sample. There are 138 observed auctions in the sample. Table 1 presents some statistics on these contracts. The firm specific variables, $\{BID, DIST, UTILR, PUTILR\}$, refer to the contracted firm. Table 2 presents some descriptive statistic on the bidding firms, i.e., the statistics are based on the sample including all 621 bidding firms.

The sample statistics indicates that the winning firm is closer and has a lower utilization rate than a bidding firm on average. We also look at the firms' bidding patterns. In Table 3, we present the activity of each firm in each region, measured as the fraction of the total number of contracts in that

[9]Note that we use the bidding, not the potential, competitors' observable costs.

Table 3: Firm activity rate across regions

	CLE	EAB	EKC	FOG	JOC	NCC	NOR	PRO	SAN	SVA	TIE	Total
NORR	1.00	1.00	0.00	0.82	0.64	0.91	0.00	0.82	0.00	0.00	0.45	11.00
MITT	1.00	0.79	0.13	0.50	0.63	0.75	0.00	0.88	0.00	0.04	0.00	24.00
STOC	1.00	0.43	0.00	0.50	0.79	0.36	0.00	0.86	0.07	0.50	0.00	14.00
MALA	1.00	1.00	0.15	0.59	0.85	1.00	0.00	1.00	0.00	0.00	0.00	27.00
VAST	0.88	0.32	0.03	0.00	0.56	0.88	0.03	0.91	0.00	0.00	0.00	34.00
SYDO	0.95	0.57	0.00	0.00	0.76	0.76	0.00	0.81	0.05	0.05	0.00	21.00
SKAN	1.00	0.14	0.00	0.14	0.57	0.86	0.00	1.00	0.00	0.00	0.00	7.00

Table 4: Market shares across years

	CLE	EAB	EKC	FOG	JOC	NCC	PRO	SAN	SVA
93	0.59	0.14	0.00	0.00	0.11	0.10	0.06	0.00	0.00
94	0.35	0.12	0.00	0.05	0.06	0.08	0.33	0.00	0.00
95	0.17	0.21	0.00	0.06	0.15	0.10	0.24	0.07	0.00
96	0.25	0.09	0.00	0.17	0.22	0.08	0.18	0.00	0.01
97	0.22	0.16	0.00	0.37	0.00	0.03	0.21	0.00	0.00
98	0.10	0.19	0.00	0.07	0.27	0.13	0.23	0.00	0.00
99	0.11	0.37	0.26	0.08	0.00	0.14	0.02	0.00	0.02
Average	0.27	0.18	0.03	0.10	0.12	0.09	0.19	0.01	0.01

region during the sample period a firm has submitted bid on. The order of the rows corresponds roughly to the geographic location from north to south of the regions. Column *Total* reports the total number of contracts procured during the sample period 1993-1999. The results in Table 3 indicate that some firms are active over the whole country, whereas other firms are concentrated to specific regions.[10]

The market shares in terms of won contracts (in tons) in our sample is presented in Table 4.[11] Only firms that has a non-zero market share in any year are included in the table. The bottom line (*Total*) reports the average market share during the sample period. The results from Table 4 show that the market shares vary substantially across years and firms. The largest firm, CLE, has an average market share of 27 per cent, followed by PRO, and EAB.

[10]Firm EKC entered the market in 1999.

[11]A few contracts where the ton variable is missing are deleted from the sample.

4 Econometric specifications

This section presents the econometric specifications of a reduced form analysis and the structural form analysis based on CSE. The reduced form analysis is mainly done to reveal the "amount of asymmetry" across firms and the relevant explanatory variables to these asymmetries.

4.1 Reduced form analysis

In order to assess the level of asymmetry across bidders and decide upon the relevant set of explanatory variables, we estimate a reduced form model of the level of bids. We account for potential selection bias via a selection model procedure (Heckit). We have seen in the solution of the NE that a firm's bid is a function of all observable characteristics of the rival firms. However, in order to reduce the number of explanatory variables we will only consider the bidding firm's observable characteristics and some observable characteristics that summarize the bidding rivals' characteristics.

Specifically, we assume that the decision of submitting a bid is governed by a latent variable as follows

$$SUBM_{it}^{*} = \mathbf{z}_{it}'\beta^{s} + \mathbf{d}_{it}^{s}{}'\delta^{s} + \varepsilon_{it}^{s} \tag{13}$$

where $\mathbf{z}_{it}$ is a vector of firm and contract specific variables, $\mathbf{d}_{it}^{s}$ is a set of firm and region dummies and $\varepsilon_{it}^{s} \sim N(0,1)$. Hence, firm i submits a bid on contract t iff $SUBM_{it}^{*} > 0$ and this event is indicated by the observable variable $SUBM_{it} = 1$. The parameters of equation (13) are estimated using maximum likelihood (i.e., a standard Probit model).

The bid level, conditional on submission, is parameterized as

$$\ln b_{it} = \mathbf{x}_{it}'\beta^{b} + \mathbf{d}_{it}^{b}{}'\delta^{b} + \varepsilon_{it}^{b} \tag{14}$$

where $\mathbf{x}_{it}$ is a vector of firm and contract specific variables, and $\varepsilon_{it}^{b} \sim N(0, \sigma_b^2)$.

A set of dummy variables is collected in $\mathbf{d}_{it}^{b}$,which includes firm, region, and time dummies. If the error terms in these equations are correlated, then the parameters of the bid level equation are biased. This correlation can be accounted for by adding the regressor $\hat{\lambda}_{it} = \frac{\phi(\mathbf{z}'_{it}\hat{\beta}^{s}+\mathbf{d}_{it}^{s\,\prime}\hat{\delta}^{s})}{\Phi(\mathbf{z}'_{it}\hat{\beta}^{s}+\mathbf{d}_{it}^{s\,\prime}\hat{\delta}^{s})}$ to the bid level model. Hence, the estimated reduced form is

$$\ln b_{it} = \mathbf{x}'_{it}\beta^{b} + \mathbf{d}_{it}^{b\,\prime}\delta^{s} + \beta_{\lambda}\hat{\lambda}_{it} + e_{it} \tag{15}$$

where $e_{it} \sim N(0, \sigma_t^2)$. The coefficient, $\beta_\lambda = \sigma_b \rho$, where ρ is the correlation coefficient of $(\varepsilon^s, \varepsilon^b)$. The parameters are initially estimated using OLS on different sub-samples of active firms with White's heteroscedasticity consistent standard errors.

4.2 Structural form analysis

In the structural form analysis, we assume that the costs per kilo are normal distributed where the mean is a linear function of a set of observable firm and contract specific variables and a set of region and firm dummies. The vector of observables includes the (log) size of contract measured in tons, the (log) distance between the firm's headquarter and the contract province ($LNDIST$), the firm's utilization rate at the time of the contract letting ($UTILR$), the potential utilization rate ($PUTIL$), and a dummy for incumbent bidding firm ($INCUM$). The variance of the stochastic term is assumed constant across firms and contracts. Thus, the cost of firm i for contract t is

$$\begin{aligned} t_{it} &= z'_{it}\gamma + \xi_{it} \\ \xi &\sim N(0, \sigma^2) \end{aligned} \tag{16}$$

If the cost draws t_{ti} were observable the estimation would be completely standard. For example, we could use a OLS estimator to estimate the vector of model parameters $\Omega = (\gamma, \sigma)$.[12] However, the costs are unobserved so this simple estimator is not feasible. Therefore, a feasible estimator of the model with unobserved costs is then defined as the solution to the fixed point problem (Florens, Protopopescu & Richard 1997)

$$\hat{\gamma} = \left(\sum_{it} z_{it} z_{it}'\right)^{-1} \sum_{it} z_{it} \hat{t}_{it} \tag{17}$$

$$\hat{\sigma} = \frac{1}{T} \sum_{t=1}^{T} \frac{1}{n_t} \sum_{i=1}^{n_t} (\hat{t}_{it} - z_{it}' \hat{\gamma})^2 \tag{18}$$

where $\hat{t}_{it} = s^{-1}(x_{it}; \mathbf{d}(z_{...}, \hat{\gamma}, \hat{\sigma}))$ is the estimated types conditional on the CSE equilibrium based on the distribution of types given by $\hat{\gamma}$, $\hat{\sigma}$, and z_{it}.

The constrained strategy set is defined as a 3-piecewise linear spline such that

$$s_{ti}^{(3)}(t; \mathbf{d}_{ti}) = \bar{t} + \sum_{p=0}^{2} d_{tip} \mathbf{1}(t < \tau_p)(t - \tau_p) \tag{19}$$

where τ_p denotes the high endpoint of the pth segment, i.e., the nodes.[13] The nodes are constructed from the values of $z_{ti}'\hat{\gamma}$ and $\hat{\sigma}$ such that the first node is one standard deviation below the overall mean of types in the auction, i.e., $\tau_0 = \frac{1}{n_t} \sum_{i=0}^{n_t} z_{ti}'\hat{\gamma} - \hat{\sigma}$, the second node lies $\frac{\hat{\sigma}}{2}$ above the overall mean, and the third node at the upper boundary of the support of the types, i.e., $\tau_3 = \bar{t}$.[14] This formulation implies that $s_{ti}(\bar{t}) = \bar{t}$ as suggested by the Nash equilibrium. In the optimization algorithm we continuously check that the slopes are positive and that the bid function stays above the 45°-line. These restrictions implies restrictions on the coefficients $\mathbf{d}_{ti}$. If the restriction is violated, we restart the

[12] In the theoretic model the support of types is assumed on $[\underline{t}, \bar{t}]$. In practice these truncations appears so far out in the tails that they do not affect the estimator where we assume untruncated support.

[13] We have also investigated higher order polynomials and splines, but we have no reliable results to present in this version.

[14] In previous versions we tried to have the same nodes for all auctions. This implied that we observed a relative small number of random draws on the first segment, which caused the optimization of the strategy parameters to collapse (no effects of changing the slope on this segment for high-cost firms). This was due to the formulation of the splines where the first element of the strategy parameter vector only affects the slope in the first segment. Another formulation of the splines might reduce this problem.

process with new start values.

Thus, we solve the CSE fixed point problem over the $3n_t$ constrained parameters $\mathbf{d}_t = (d_{t10}, \ldots, d_{tn_tP})$. The first order conditions are

$$\begin{aligned} 0 &= \frac{1}{R}\sum_{r=1}^{R} \frac{\partial s_i^{(3)}(\tilde{t}_i^r)}{\partial \mathbf{d}_i'} \Big\{ K_h(\tilde{y}^r - s_i^{(3)}(\tilde{t}_i^r)) - \\ & (s_i^{(3)}(\tilde{t}_i^r) - \tilde{t}_i^r) K_h'(\tilde{y}_i^r - s_i^{(3)}(\tilde{t}_i^r)) \Big\}, \; i = 1, \ldots, n \end{aligned} \tag{20}$$

where

$$\frac{\partial s_i^{(3)}(t_i)}{\partial d_{tip}'} = \sum_{p=0}^{2} \mathbf{1}(t_i < \tau_p)(t_i - \tau_p) \tag{21}$$

We have chosen to use a logarithmic kernel $K_h(x) = \frac{1}{1+\exp(-x/h)}$ and $K_h^{'}(x) = \frac{1}{h}\frac{\exp(-x/h)}{(1+\exp(-x/h))^2}$ with bandwidth $h = \frac{\bar{t}-\underline{t}}{100}$.

(Armantier & Richard 2000) propose the following algorithm to implement the feasible estimator.

Step 1 Initialize the iterative process to find the fixed point by guessing some values for the bidders' types $\hat{t}^0$, e.g., the type $\hat{t}^0_{ti}$ equals some linear transformation of the the observed action x_{ti}. Set $k = 0$.

Step 2 Estimate the structural parameters $\hat{\Omega}^k = (\hat{\gamma}^k, \hat{\sigma}^k)$ using the chosen estimator, e.g. the OLS estimator in (17) using the vector of guessed types $\hat{t}^k$. Check convergence if $k > 0$. In order to assess convergence, we adopt a relative difference criteria. In practice, we stop the iteration if $\left\| \frac{\hat{\Omega}^{k+1}-\hat{\Omega}^k}{\hat{\Omega}^k} \right\| < 1e-4$ and $\left\| \frac{\hat{t}^{k+1}-\hat{t}^k}{\hat{t}^k} \right\| < 1e-4$, where $\|x\|$ denotes the maximum of the absolute values of x. Hence, we check convergence w.r.t. the structural parameters and the implied types. If convergence, STOP and report $\hat{\Omega}^k$, otherwise continue to Step 3.

Step 3 Use $\hat{\Omega}^k$ to construct the distribution parameters of the bidders' costs in each auction using $\hat{\theta}^k_{ti} = (z_{ti}'\hat{\gamma}^k, \sigma^k)$.

Step 4 Solve the CSE using the procedures presented in section 2, yielding the

CSE strategies $s^{(3)}(t_t; \mathbf{d}_t^k, \hat{\Omega}^k)$ and update the "guess" of the types as $\hat{t}_t^{k+1} = s^{-1}(x_t; \hat{d}_t^k, \hat{\Omega}^k)$.[15]

Since we iterate over different values of Ω, we want the CSE solutions, i.e., $(\mathbf{d}_t)_{t=1}^T$, to be smooth in Ω. This is achieved by using *common random numbers*. This means that outside the iterative algorithm we draw a set of uniform random numbers, which are transformed to normal variables as a function of the Ω's and the z_{ti}'s.

At this stage we ignore the effect of MC simulations on the standard errors of the estimates and treat the simulated types as perfectly observable. This will under-estimate the true covariance matrix.[16]

The *expected social cost of inefficient allocation of contracts*, denoted by S, can be defined as the expected difference between the contracted firm's contract cost and the lowest contract cost in the set of bidding firms.[17] In an efficient auction, this difference is always zero. We approximate this expectation using the average of a large number of simulated outcomes of the observed procurements. In the simulations, we adopt the estimated distributions of bidders cost and the CSE profile.

$$S_t = \frac{1}{R}\sum_{r=1}^{R}(t_{tc}^r - \min\{t_{ti}^r\}_{i\in\mathbb{N}_t})TON_t \tag{22}$$

$$S = \sum_{t=1}^{T} S_t \tag{23}$$

where the subscript c holds the index of the contracted firm in replication r and $\mathbb{N}_t$ denotes the set of bidding firms on contract t. The *predicted social cost* in the sample, denoted by $\hat{s}$, is calculated using the predicted types and observed

[15]Note that Ω parameterizes the distribution function $F()$.

[16]The covariance matrix could be evaluated using bootstrap methods. This is, however, not done in this version.

[17]Note that we weight the bids measured in SEK per kilo by the reported size of the contract in tons.

winner, i.e.,

$$\hat{s} \quad = \quad \sum_{t=1}^{T} (\hat{t}_{tc} - \min\{\hat{t}_{ti}\}_{i \in \mathbb{N}_t}) TON_t \tag{24}$$

The *expected excessive procurement cost*, denoted by P, is defined as the difference between the total procurement cost given the prevailing first price auction procedure, and the potential total cost under a second price auction procedure,

$$P_t \quad = \quad \frac{1}{R} \sum_{r=1}^{R} (x_{tc}^{r} - \min\{t_{ti}^{r}\}_{i \in \mathbb{N}_t \backslash c}) TON_t \tag{25}$$

$$P \quad = \quad \frac{1}{T} \sum_{t=1}^{T} P_t \tag{26}$$

The *predicted excessive procurement cost* in the sample is calculated as

$$p \quad = \quad \sum_{t=1}^{T} (x_{tc} - \min\{\hat{t}_{ti}\}_{i \in \mathbb{N}_t \backslash c}) TON_t \tag{27}$$

i.e., the difference between the observed bid and the predicted second low cost of the bidding firms.

5 Results from auctions of road marking contracts

Some of the firms included in the data set were not present in the market until 1999, and some firms participated only in a few auctions. In order to identify the firm specific dummies, and to get convergence in the iterative process we have, at this stage, been forced to drop these firms from the final reduced form data set. The reduced form data set thus contains 133 auctions including about 570 bids. In the structural analysis, dropping a firm invalidates the use of the auctions where the dropped firm has been bidding. Further, we drop auctions

Table 5: Estimates on submission and bid levels of active firms. Dependent variable: $\ln Bid$

Coeff.	**Probit** (a)	**All bids** (b)	**Winning bids** (c)	**Non-winning bids** (d)
const	$4.08**$	$2.72**$	$2.60**$	$2.73**$
	(0.83)	(0.04)	(0.07)	(0.04)
lnton	$0.16*$	$-0.02**$	-0.01	$-0.02**$
	(0.08)	(0.00)	(0.01)	(0.00)
nbids		-0.01	-0.00	$-0.01*$
		(0.00)	(0.01)	(0.00)
lndist	$-0.64**$	$0.02**$	$0.02*$	$0.02*$
	(0.12)	(0.01)	(0.01)	(0.01)
utilr	0.55	0.01	$-0.08**$	0.01
	(0.34)	(0.02)	(0.03)	(0.02)
putilr	$0.05*$	$-0.01**$	$-0.00**$	-0.01
	(0.02)	(0.00)	(0.00)	(0.00)
lncmind	-0.07	0.01	0.02	0.01
	(0.14)	(0.01)	(0.01)	(0.01)
cminu	$2.79**$	$0.09*$	$0.17**$	0.07
	(0.92)	(0.04)	(0.05)	(0.05)
incum	$1.28**$	$-0.01*$	-0.02	0.01
	(0.34)	(0.01)	(0.01)	(0.01)
lambda		$0.06**$	-0.03	$0.07**$
		(0.02)	(0.02)	(0.02)

Note: In column (b)-(d), White's standard errors are presented in parenthesis. '*' and '**' denotes significance on 10 and 1 per cent level (double sided), respectively.

with less than 3 bidders and contracts in the province *Gotland*.[18] Thus, the final data set used in the structural analysis thus contains 90 auctions including 444 bids in total.

5.1 Results from reduced form analysis

In Table 5, we present the estimated parameters of equation (13) in column (a) and (14) in columns (b)-(d). In this initial analysis, we have used OLS to estimate the parameters of the bid level functions. We also present estimates on bid levels based on winning bids.

The results in Table 5 exhibit some interesting features. The probit model indicates that the probability of submitting a bid decreases with distance ($LNDIST$),

[18] *Gotland* is an island located some distance from the main land. There is one single firm operating on this island and the costs for shipping equipments for the other firms is probably much larger as compared to the transportation costs on the main land.

and increases with the potential utilization rate ($PUTILR$) and the minimum utilization rate of competitors ($CMINU$). We also notice that incumbent firms are more likely to submit a bid. The contract size ($LNTON$) is positively significant on 10 per cent level, which indicates that larger contracts receive more attention from the potential suppliers. There are also significant differences across firms and regions (not reported in the table) indicating that firms are potentially asymmetric. The results are in general consistent with economic intuition, with the exception that a firms potential utilization rate was expected to be negatively correlated with the submission decision due to capacity constraints. This could be perhaps explained by a U-shaped cost function w.r.t. the utilization rate, where most firms are situated on the downward sloping interval. However, it is interesting to find that the minimum utilization rate of competitors enters the decision significantly. One potential interpretation of this is that a firm acknowledges that the competitors are "busy" and therefor expects them to submit higher bids. This would increase the firms expected possibility to win the contract.[19]

The parameters of the bid level model based on all bids (578 observations) exhibit in general the expected signs; larger contracts receive lower bids per unit (bids are in SEK per kilo), whereas distance and the competitors minimum utilization rate increase the bid levels. The number of bidders, and the distance of the competitors do not enter in a significant way, although the signs are the expected. The exception is (again) the potential utilization rate, which enters in a negatively significant way, i.e., higher utilization rates implies *lower* bid levels, all else equal. Finally, the β_λ parameter ($LAMBDA$) is significant positive, indicating that the error terms in the submission and bid level functions are positively correlated. One (counter-intuitive) interpretation of this is that a firm that is more likely to submit a bid is also more likely to submit a high bid.

Column (c) presents the results based on the winning bids (124 observations) and column (d) relates to the non-winning bids (454) bids. Some interesting

[19] This is unfortunately inconsistent with the result that higher utilization rate implies lower bids (see below).

results emerge from these estimates. The utilization rate and the potential utilization rate reduce the bid level for winning firms. Furthermore, a higher utilization rate of competitors implies higher bids. In column (d) we present the corresponding estimates based on the non-winning bids. They main differences between the results in (c) and (d) are *i*) contract size and number of bidders enters with a negative significant coefficient in non-winning bids, *ii*) the utilization rate, the potential utilization rate, and the competitors' utilization rates are not significant for non-winning bids, and *iii*) the estimated parameter β_λ is insignificant for winning bids (almost negatively significant), but significantly positive for non-winning bids. Hence, there is a negative (insignificant) correlation between submission and the level of the winning bid, but a positive correlation between submission and non-winning bids.

We conclude from the results in this section that the firms' bids are related to firm specific characteristics such as distance to contract site, the potential utilization rate, incumbency, and perhaps even the utilization rate (significant for winning bids). These will therefore be included in the structural analysis as control variables for the observable shifts in the firms' cost distributions. We will also include the (log of) contract size to control for economies of scale.

5.2 Results from structural analysis

The data set used for the structural CSE analysis consists of 90 auctions and 444 bids. We assume that the cost draws made for each firm in each auction are conditionally independent. Hence, we abstract from the more plausible situation with synergies across auctions. The fixed point problem in the CSE algorithm is generally solved using initially 1000 Halton draws.[20]

In Table 6, we present the results from the estimations based on the nested fixed point algorithm. We have chosen to include those variables that turned out significant in the reduced form analysis. For comparison, we also present es-

[20]If the algorithm fails to find a zero point of the vector of FOCs, we sequentially increase the number of random draws to 3000 and 4500 Halton draws, respectively. In equilibrium all auctions converge with 1000 draws.

timates based on a simplified reduced form model where the dependent variable is the observed bid in SEK per kilo.[21]

In Table 6, coefficients relating to the firm specific variables distance and incumbency are significant whereas the coefficients relating to the poorly measured utilization rate are insignificant. Further, some of the firm and regional specific dummies are significant indicating that there are some more unobserved heterogeneity across firms and regions.

Comparison across columns, indicates that the costs are more "sensitive" to variations in firm specific variables than bids. E.g. the effect of distance is higher on costs than on bids, which seems intuitive since the bids can not fully compensate for cost increase due to strategic considerations. For example, the effect of being the incumbent firm reduces the cost per kilo by 0.42 SEK, whereas the bid is only reduced by 0.23 SEK. This effect is also almost exclusively observed for the firm specific dummies. Finally, the estimated standard deviation of the stochastic cost draw is 1.16.

Based on the estimated distribution of the costs, we can assess the importance of inefficiently allocated contracts. We find that 16 (out of 90) contracts may be allocated to another than the low cost firm. This inefficiency is assessed to about 1.8 per cent out of potential social costs of completing the contracts. The expected value of the social costs is simulated to about 0.04 per cent.[22] Furthermore, if a second price auction mechanism was used, the predicted procurement costs for the observed sample would be reduced by approximately 2.8 per cent and by 1.1 per cent in expectations with 4.5 per cent inefficiently allocated contracts on average.

Finally, the average markup, i.e., the per cent increase in bids above costs, is about 3.5 per cent for non-winning bids and 4.5 per cent for the winning bids. This means that the winning firm actually pads their costs more than the average non-winning firm.

[21] We have not controlled for selection bias in this version. Further, in contrast to the results presented in Table 5, we have not used the logarithmical bid here.

[22] Based on $R = 2000$ replications of the data set,

Table 6: Estimates on structural parameters.

Var	**Costs**	**Bids**
CONST	15.002**	16.675**
	(0.844)	(0.602)
LNTON	-0.568**	-0.674**
	(0.119)	(0.085)
Firm specific variables:		
LNDIST	0.570**	0.408**
	(0.108)	(0.077)
UTILR	-0.466	-0.206
	(0.323)	(0.230)
PUTILR	-0.028	-0.015
	(0.068)	(0.049)
INCUM	-0.421**	-0.231*
	(0.161)	(0.115)
Firm dummies:		
NCC	0.699**	0.510**
	(0.226)	(0.161)
JOC	0.721**	0.690**
	(0.197)	(0.140)
CLE	0.090	0.114
	(0.205)	(0.146)
FOG	-0.588*	-0.011
	(0.252)	(0.180)
SAN	-0.438	-0.389
	(0.866)	(0.617)
PRO	0.446*	0.388**
	(0.211)	(0.150)
Regional dummies:		
STOC	-0.260	-0.161
	(0.271)	(0.193)
SKAN	-0.252	-0.161
	(0.538)	(0.383)
SYDO	-0.344*	-0.257*
	(0.194)	(0.138)
NORR	0.582	1.282**
	(0.407)	(0.290)
VAST	-0.168	0.015
	(0.181)	(0.129)
MITT	-0.498*	-0.478**
	(0.216)	(0.154)
Sigma	1.159	0.827

Note: Reference firm is EAB, reference region is MALA. Standard errors in parenthesis. * and ** denotes significance on 10 and 1 per cent levels (double sided)

6 Conclusions

This paper investigates the social costs induced by inefficient allocation of contracts in a first price, sealed bid auction with *ex ante* asymmetric bidders. We adopt a constrained strategy equilibrium (CSE) approach where the players are constrained to simplified strategies in order to estimate the structural elements of the players private values. Conditional on these estimates, we investigate the importance of inefficiently allocated contracts.

The empirical analysis is based on procurements of road marking services in Sweden during 1992 through 1999. We observe bids that are assumed to be generated via profit maximizing firms with private and conditionally independent costs.

The market is spatially dispersed with relatively high transportation costs. The potential suppliers, i.e., the bidding firms, are located at various places and consequently the costs for transportation varies across firms and contracts. These asymmetries are observable for all participants in the market. Hence, the firms are assumed *ex ante* asymmetric. This asymmetry implies that the first price sealed bid auction design no longer guarantees that the low cost firm receives the contract. If the low cost firm fails to win the contract we say that there is an *social cost due to inefficient procurement design.* We find that, for the present sample, the social costs of inefficient allocations are a little less than 2 per cent of total costs.

The second price auction is efficient even if the bidders are asymmetric. The expected procurement costs in the second price auction is different from the expected costs in the first price auction (Hansen (1984) and Milgrom & Weber (1982)). In simulations based on the estimated structural parameters, we find that a second price format reduces the predicted procurement cost in the sample by about 2.8 per cent of total procurement costs.

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A Nash equilibrium in asymmetric private value auctions

The Nash Equilibrium (NE) is defined as the strategy profile $\mathbf{s}^{NE} = (s_1^{NE}, \ldots, s_n^{NE}) \in \mathbb{S}$, that satisfies

$$U(s_i^{NE}(t_i); \mathbf{s}_{-i}^{NE}, F) \geq U(s_i(t_i); \mathbf{s}_{-i}^{NE}, F), \qquad (28)$$
$$\forall t_i \in \mathbb{T}_i, \forall s_i \in \mathbb{S}_i, \forall i \in \mathbb{N}$$

Maskin & Riley (2000*b*) proves that, *i*) if the bidders' utilities are monotonic in t_i, and *ii*) if the bidders' are risk neutral or risk averse, then the there exists a NE in monotonic strategies. This implies that an existing NE strategy $s_i^{NE}(t)$ is invertible for all $i \in \mathbb{N}$ and $t \in \mathbb{T}$. Denoting the inverse by $s_i^{-1,NE}(x_i)(= t_i)$, and assuming risk neutrality and independence w.r.t types (which we have done in (2) and (3)), we can define the NE as the solution to the fixed point problem

$$s_i^{NE}(t_i; s_{-i}^{NE}, F) = \arg\max_{x_i}(x_i - t_i) \prod_{j \neq i} (1 - F_j(s_j^{-1,NE}(x_i))), \; \forall i \in \mathbb{N} \qquad (29)$$

The first order conditions (FOC) of this optimization problem are, for all

$i \in \mathbb{N}$ (suppressing the dependence on F and the superscript NE),

$$\begin{aligned} 0 &= \prod_{j \neq i}(1 - F_i(s_j^{-1}(x_i))) \\ &\quad -(x_i - t_i)\sum_{j \neq i} f_i(s_j^{-1}(x_i))\frac{ds_j^{-1}(x_i)}{dx_i}\prod_{k \neq j}(1 - F_i(s_k^{-1}(x_i))) \end{aligned} \tag{30}$$

Rearranging (30) in terms of $\frac{ds_j^{-1}(x_i)}{dx_i}$ illustrates that the FOCs constitute a system of first order ODEs,

$$\begin{aligned} \frac{\partial s_i^{-1}(x_i)}{\partial x_i} &= \frac{1 - F_i(s_i^{-1}(x_i))}{(n-1)f_i(s_i^{-1}(x_i))} \times \\ &\quad (\sum_{j \neq i} \frac{1}{x_i - s_j^{-1}(x_i)} - \frac{n-2}{x_i - s_i^{-1}(x_i)}), \forall i \in \mathbb{N} \end{aligned} \tag{31}$$

Hence, the issue of existence of a Nash equilibrium in this game is equivalent to the existence of a solution to the system of ODEs. Assuming that

A.1. F the distribution of types, is continuous, and

A.2. F and its probability density function $f = F'$ are bounded away from zero over the support of $[\underline{t}, \bar{t}]$,

Lebrun (1999) and Bajari (2000) prove that there exists a unique Nash equilibrium in pure strategies that is continuous and strictly increasing in t_i. The solution is characterized by the system of ODEs in (31) with $2 \times n$ boundary conditions such that $\forall i \in \mathbb{N}$, and for some $\xi^* \in \mathbb{X}_i(F)$

$$s_i^{-1}(\xi^*) = \underline{t} \tag{32a}$$

$$s_i^{-1}(\bar{t}) = \bar{t} \tag{32b}$$

The boundary conditions state that G_i, the induced marginal distributions of bids, have connected supports on $\mathbb{X}(F) = [\xi^*, \bar{t}] \in \mathbb{R}, \forall i \in \mathbb{N}$, which depends on the distribution F. It can also be shown (see e.g. ((Bajari & Ye 2000)) that the limit of the first derivative at the upper bound is given by $\frac{\partial s^{-1}(x)}{\partial x} \underset{x \to \bar{t}^-}{\to} \frac{n}{n-1}$.

Under some circumstances we can solve a system like (31) either analytically or by standard numerical ODE solving procedures. However, the system in (31) has some features that makes it hard to solve analytically as well as numerically. The system is characterized by a singularity at $x_i = \bar{t}$ since the system tends towards $\frac{0}{0}$ as $x_i \to \bar{t}^-$. Therefore, one can not use the upper boundary to solve the system of ODEs. If the constant ξ^* were known, we could solve the system using standard numerical methods (e.g. a Runge-Kutta algorithm).

Bajari (2000) shows that the inverse bid function is monotonic in ξ in the sense that if $\xi < \xi'$ then $s_i^{-1}(t;\xi) > s_i^{-1}(t;\xi'), \forall t \in \mathbb{T}$. This property can be used to find a ξ arbitrarily close to ξ^*. Let $s^\xi(t) = (s_i(t;\xi))_{i\in\mathbb{N}}$ denote the collection of solutions of (31) conditional on the boundary condition $s_i^{-1}(\xi) = \underline{t}\ \forall i$ where $\xi \in \mathbb{X}_i(F)$. If $\mathbb{F}$ is the set of all functions such that $\mathbb{F} \equiv \{f : f \in C^1, f : \mathbb{T} \to \mathbb{T}, f(t) \geq t, \forall t \in \mathbb{T}\}$, then

i) $s_i^\xi(t) \in \mathbb{F}$, $\forall \xi \geq \xi^*$, $\forall i = 1 \to n_t$, and

ii) $s_i^\xi(t) \notin \mathbb{F}$, $\forall \xi < \xi^*$, and some $i = 1 \to n_t$

Hence, an algorithm that seeks over $\xi \in \mathbb{X}_i(F)$ and checks if the elements of the implied strategy profile s^ξ belongs to $\mathbb{F}$, and satisfies $s_i(\bar{t}) = \bar{t}$, could be implemented to find an arbitrarily small interval $\xi^* \pm \varepsilon, \varepsilon > 0$ that contains the true value ξ^*.[23]

In an attempt to illustrate the NE, Figure 3 gives the approximative shape of the solutions to the ODEs. We use a set of three asymmetric bidders, where the distribution of types is assumed truncated normal $N(\mu_i, 9)$ where $\mu = (14, 14.5, 15)$ and $5 \leq t \leq 20$. The bid functions are approximated using a Runge-Kutta algorithm combined with the search algorithm discussed above with $\varepsilon = 1e - 10$. Due to numerical issues, the solutions presented in the figure are rather poor in the right tail hand of the cost distributions. The horizontal axis shows the costs and the vertical axis shows the optimal bid.

[23] Bajari & Ye (2000) also propose an alternative algorithm based on a convergent sequence of best responses.

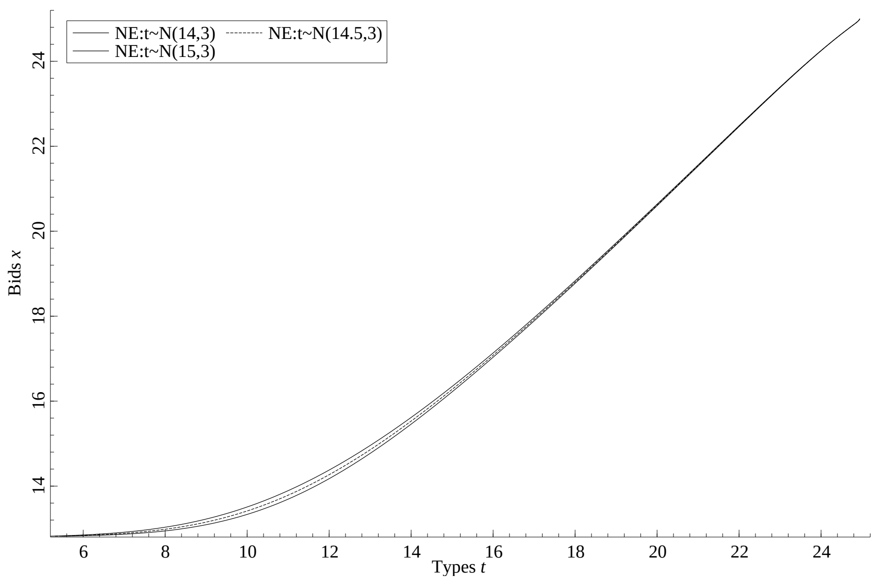

Figure 3: Nash Equilibrium strategies with three asymmetric bidders.

In this example, the value of ξ^* is approximately 12.8, and the connected support of bids is consequently $\mathbb{X}(F) \approx [12.8, 25]$. It can also be noted that the Firm 1, the “low cost” firm, bids higher than the other firms conditionally on the cost draw. Lebrun (1999) and Pesendorfer (2000) provide proofs that if F_i is statistically dominated by F_j, then G_i is statistically dominated by G_j. In our example with identical variances across firms, this means that the firm with the lowest expected costs is also the most probable winning firm. It is also notable that in contrast to the complex structure of the system of ODEs, the solutions seem to be quite smooth and “well-behaved”.

The issue of inefficiency is also illustrated in the figure. Let L denote the firm with the most preferable cost distribution, i.e., $N(14, 9)$ and let H denote the firm with the least preferable distribution $N(15, 9)$. Let t_L and $s_L(t_L)$ denote the cost draw and profit maximizing bid of firm L, respectively. Define t_H and $s_H(t_H)$ correspondingly. For H to "beat" L in the auction, it is required that $s_H(t_H) < s_L(t_L)$, that is, $t_H < s_H^{-1}(s_L(t_L))$, where $s^{-1}()$ denotes the inverse strategy function. Since $s_H^{-1}(x) > s_L^{-1}(x) \, \forall x$, t_H is not necessarily less than

t_L, i.e., H may win the auction even though $t_H > t_L$. In the figure, this is illustrated by the *horizontal distance* between the strategy functions of firm L and H.