Testing for Cointegration in Misspecified Systems – A Monte Carlo Study of Size Distortions[♣]

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Abstract

When dealing with time series that are integrated of order one, the concept of cointegration becomes crucial for the specification of a model. Using the best available tests, one can reduce the probability of estimating econometric models that are misspecified. This paper investigates the small sample performance of four well-known cointegration tests when a system has been misspecified by leaving out one relevant explanatory variable from a system with one cointegrating vector. In a Monte Carlo study, the size distortions of the Augmented Engle-Granger (Engle and Granger, 1987), Johansen's (1988) maximum eigenvalue, Johansen's (1991) trace and the Boswijk (1989) Wald tests are examined. The Johansen trace test adjusted by the finite sample correction of Reinsel and Ahn (1988) is found to have the most robust performance when lag length in the test equations is chosen according to traditional information criteria.

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1. Introduction

In empirical work it is not uncommon that the estimated models are inspired by, rather than derived from, theory. This may generate several problems since there may be a number of conflicting theories in the area so that exactly which variables to include in the estimation could be less than obvious. Needless to say, using this approach, e.g. in “atheoretical” vector autoregressions (VARs), it is possible that we could end up with a misspecified model.

The omission of relevant explanatory variables is known to generate a number of problems. For example, it may be the reason why we get unexpected results such as the price puzzle when studying monetary policy in VARs. The price puzzle is the common observation that in a VAR of output, prices, interest rates, money and possibly some other variables, contractionary shocks of monetary policy lead to persistent price increases. Sims (1992) argues this could be due to a misspecification of the model, such as omitting a leading indicator used by the central bank.

When dealing with variables that are integrated of order one, I(1), we have more concerns than usual about omitted variables since the choice of variables to include in a system will affect possible cointegrating relations. Since the introduction of the term cointegration by Granger (1981), and further development by Engle and Granger (1987), several ways of testing for the presence of cointegration have been proposed. If, after applying a test, we reach the conclusion that there is no cointegration, we say that the variables have no long run equilibrium relationship. No cointegration might lead us to estimate a VAR in differences. However, if the variables are cointegrated, mistakenly estimating the VAR in differences means not just throwing away information – it is misspecified.[1] Conversely, if we act as if we have cointegration when there is none, the model will also suffer from misspecification and we have to consider unpleasant problems such as spurious regression.[2] Hence, getting the properties of the system right is an important matter in order to get estimation and inference as correct as we possibly can.

[1] See for instance Engle and Granger (1987).
[2] See for instance Phillips (1986).

The purpose of this paper is to investigate the behaviour of four different cointegration tests in small samples when a relevant explanatory variable has been omitted from a system. The question to be answered is how likely it is that the different cointegration tests – the Augmented Engle-Granger (Engle and Granger, 1987), Johansen's (1988) maximum eigenvalue, Johansen's (1991) trace and the Boswijk (1989) Wald tests – reach the correct conclusion of no cointegration when we have a system where the variables are related but the model has been misspecified.

There is a fairly extensive literature on the subject of cointegration tests and their behaviour regarding size and power under different circumstances, such as Banerjee et al (1986), Haug (1996), Bewley and Yang (1998) and Pesavento (2000). This study further clarifies the problems and advantages of well-known cointegration tests by investigating their size distortions in a new and empirically reasonable situation. The focus on the four included tests is based on the fact that the Augmented Engle-Granger test and Johansen's tests are by far the most frequently used tests in empirical macroeconomics whilst the very good results of the Boswijk Wald test in a number of Monte Carlo studies recommend it. Other tests that have been shown to have good properties in some aspects, such as the tests proposed by Stock and Watson (1988), Hansen (1990), Bewley and Yang (1995) are not addressed. Though it would be interesting to know the properties of those tests, it is beyond the scope of this paper.

The paper is organised as follows. Section two describes the cointegration tests to be considered and their properties in some previous studies. In section three a Monte Carlo experiment is performed and the results are discussed. Section four empirically applies the tests considered in the paper to real macro data and, finally, section five concludes.

2. Testing for cointegration

2.1 Four cointegration tests

Among the four tests to be considered in this study, we will first look at the Augmented Engle-Granger (AEG) test. Initially a static OLS regression of the form in equation (1) is run.

$$y_t = a + \boldsymbol{\beta}'\mathbf{x}_t + \upsilon_t \quad (1)$$

The residuals from this regression are then tested for the presence of a unit root using an Augmented Dickey-Fuller test (Said and Dickey, 1984), as shown in regression (2) and the test statistic is simply the *t*-statistic on $\hat{\rho}$ as given in (3).

$$\Delta\hat{\upsilon}_t = \rho\hat{\upsilon}_{t-1} + \sum_{i=1}^{f}\gamma_i \Delta\hat{\upsilon}_{t-i} + \omega_t \quad (2)$$

$$AEG = \hat{\rho}/\hat{\sigma}_{\hat{\rho}} \quad (3)$$

If the test statistic, which follows a non-standard distribution, is small enough the null hypothesis of a unit root is rejected and we conclude that we have found a cointegrating relationship.

Second, consider two different tests based on the methodology developed by Johansen (1988). Consider a VAR of order *p*, as given by equation (4).

$$\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \cdots + \mathbf{A}_p\mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t \quad (4)$$

where $\mathbf{y}_t$ is a nx1 vector of non-stationary I(1) variables and $\boldsymbol{\varepsilon}_t$ is a nx1 vector of innovations. We can rewrite the VAR as

$$\Delta\mathbf{y}_t = \boldsymbol{\Pi}\mathbf{y}_{t-1} + \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\Delta\mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t \quad (5)$$

where

$$\boldsymbol{\Pi} = \sum_{i=1}^{p}\mathbf{A}_i - \mathbf{I} \text{ and } \boldsymbol{\Gamma}_i = -\sum_{j=i+1}^{p}\mathbf{A}_j \quad (6)$$

If the coefficient matrix $\mathbf{\Pi}$ has reduced rank $r<n$, then there exist nxr matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ each with rank r such that $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$ and $\boldsymbol{\beta}'\mathbf{y}_t$ is stationary. r is the number of cointegrating relationships, the elements of $\boldsymbol{\alpha}$ are known as the adjustment parameters in the vector error correction model and each column of $\boldsymbol{\beta}$ is a cointegrating vector. If $\mathbf{\Pi}$ has full rank all variables are stationary. It can be shown that for given r, the maximum likelihood estimator of $\boldsymbol{\beta}$ defines the combination of $\mathbf{y}_{t-1}$ that yields the r largest canonical correlations of $\Delta\mathbf{y}_t$ with $\mathbf{y}_{t-1}$ after correcting for lagged differences and deterministic variables when present.[3] Johansen proposes two different likelihood ratio tests to test the significance of these canonical correlations and thereby the reduced rank of the $\mathbf{\Pi}$ matrix: the trace test and maximum eigenvalue test. These are shown respectively in equations (7) and (8).

$$J_{trace} = -T\sum_{i=r+1}^{n}\ln\left(1-\hat{\lambda}_i\right) \tag{7}$$

$$J_{\max} = -T\ln\left(1-\hat{\lambda}_{r+1}\right) \tag{8}$$

where $\hat{\lambda}_i$ is the i:th largest canonical correlation. The trace statistic, J_{trace}, tests the null hypothesis of the number of cointegrating vectors being less than or equal to r against the alternative hypothesis of r+1 or more. The maximum eigenvalue statistic, $J_{\max}$, tests the null hypothesis of the number of cointegrating vectors being less than or equal to r against the alternative hypothesis of r+1. If the test statistic is large enough we reject the null for the alternative. Note that neither of the test statistics for these likelihood ratio tests follows chi square distributions. Asymptotic critical values can be found in Johansen and Juselius (1990) and are given by most econometric software packages. Cheung and Lai (1993), however, show that there is over-rejection of the null hypothesis of no cointegration when the test statistics are compared to the asymptotic critical values. One way of dealing with this problem is to apply the finite sample correction proposed by Reinsel and Ahn (1988) in which the test statistic is adjusted by a factor of $(T-np)/T$ and then compared to the asymptotic critical values. Cheung and

[3] For a detailed description of the procedure, see for instance Johansen (1995).

Lai (1993) find that this method performs well, even if there appears to be some bias, and the method will be used for finite sample corrections in this paper.

Finally, we will look at a Wald-type test in an error correction model, an approach suggested by Boswijk (1989). This test is a multivariate generalization of a model used in Banerjee et al (1986). Consider the error correction model given in equation (9) below.

$$\Delta y_t = \boldsymbol{\delta}_0' \Delta \mathbf{x}_t + \lambda\left(y_{t-1} - \boldsymbol{\theta}' \mathbf{x}_{t-1}\right) + \sum_{j=1}^{m} \left(\varphi_j \Delta y_{t-j} + \boldsymbol{\delta}_j' \Delta \mathbf{x}_{t-j}\right) + \xi_t \tag{9}$$

This could equivalently be written as equation (10).

$$\Delta y_t = \boldsymbol{\delta}_0' \Delta \mathbf{x}_t + \boldsymbol{\pi}' \mathbf{z}_{t-1} + \sum_{j=1}^{m} \left(\varphi_j \Delta y_{t-j} + \boldsymbol{\delta}_j' \Delta \mathbf{x}_{t-j}\right) + \xi_t \tag{10}$$

where

$$\mathbf{z}_t' = \left(y_t, \mathbf{x}_t'\right) \tag{11}$$

$$\boldsymbol{\pi}' = \lambda\left(1, -\boldsymbol{\theta}'\right) \tag{12}$$

If λ in equation (9) is zero, we have no error correction mechanism and no cointegration. Looking at the definition of $\boldsymbol{\pi}$ we can conclude that $\lambda = 0$ implies $\boldsymbol{\pi} = \mathbf{0}$, and thus we have a way of testing for cointegration. The test is, as previously mentioned, of Wald-type and the test statistic is given by equation (13).

$$W = \hat{\boldsymbol{\pi}}' \left[\hat{V}\left(\hat{\boldsymbol{\pi}}\right)\right]^{-1} \hat{\boldsymbol{\pi}} \tag{13}$$

where $\hat{\boldsymbol{\pi}}$ is the OLS estimator of $\boldsymbol{\pi}$ and $\hat{V}\left(\hat{\boldsymbol{\pi}}\right)$ is the estimated OLS covariance matrix. If the null is rejected we have found a cointegrating relationship. The distribution of the test statistic is, however, not chi squared but rather a generalization of the squared Dickey-Fuller *t*-statistic.

2.2 Previous studies

Previous simulation studies have investigated the power and size of these tests for various data generating processes (DGPs). In an early study, Banerjee et al (1986) compared the Cointegrating regression Durbin-Watson test (Sargan and Bhargava, 1983) to a *t*-test on the error correction term in a dynamic model. The latter test is, as previously mentioned, the predecessor of the Boswijk Wald test. Using a simple DGP with zero or one cointegrating vector they found that the *t*-test was more powerful than the Cointegrating regression Durbin-Watson test, but slightly oversized at the five percent level. In most of the later simulation studies, the AEG test has been a frequent guest. Using the same DGP as Banerjee et al (1986), Kremers et al (1992) compared the AEG test to the above mentioned *t*-test and found the AEG test to be less powerful. Boswijk and Franses (1992) compared the AEG, the Boswijk Wald and the Johansen maximum eigenvalue tests for two different DGPs. They found the Boswijk Wald test to outperform the others in terms of size and power. Furthermore, the AEG test turned out to perform badly, with low power and large size distortions, for one of the DGPs – an ARMA model with explanatory variables.

A comprehensive study where nine different tests – both single equation and system – were compared, was conducted by Haug (1996). For a simple DGP with zero or one cointegrating vector, the Stock and Watson (1988) and Phillips and Ouliaris (1990) $\hat{P}_z$ tests were found to perform best in terms of power when the regressors were endogenous. With exogenous regressors, the Phillips and Ouliaris (1990) $\hat{Z}_\alpha$ test performed best. In the study it was also found that the AEG test and Hansen's (1992) L_c test showed the overall least size distortions. A general observation in the study is that single equation tests have smaller size distortions, but also have lower power than system based tests. The recommendation by Haug based on the study is to use the Stock and Watson and AEG tests as a combination. Worth pointing out is that the in empirical work less used Stock and Watson test was preferred over the widely used Johansen maximum eigenvalue test and that the Boswijk Wald test was not considered at all in the study.

Focusing on system tests only, Bewley and Yang (1998) compared the Stock and Watson test to the Johansen maximum eigenvalue test and the Bewley and Yang (1995) test. For DGPs with zero to two cointegrating vectors, they were unable to find any test that dominated over a wide range of parameters. In general, though, the Stock and Watson test and Johansen maximum eigenvalue test were more powerful than the Bewley and Yang test. However, size distortions were found to be severe for the Stock and Watson test in some cases.

The studies referred to so far have all been pure Monte Carlo studies. Pesavento (2000) on the other hand compares, among others, the AEG test, the Johansen maximum eigenvalue test and the Boswijk Wald test in a study investigating properties both analytically and in large and small samples. Using a DGP with one cointegrating vector, the overall conclusion is that the Boswijk Wald test performs better than the other tests in term of power and no worse in term of size distortions.

With these results from previous studies in mind, we now turn to the Monte Carlo simulations in this paper.

3. A Monte Carlo experiment

Initially, a system with one cointegrating vector is generated. The Phillips' (1991) triangular representation of the system is given in equations (14) and (15) below.

$$y_t = \boldsymbol{\alpha}'\mathbf{x}_t + \omega_t \tag{14}$$

$$\Delta\mathbf{x}_t = \boldsymbol{\eta}_t \tag{15}$$

where $\mathbf{x}_t$ is a kx1 vector, $\boldsymbol{\alpha}' = \begin{pmatrix}1 & \cdots & 1\end{pmatrix}$, $\omega_t \sim NID\left(0, \sigma^2_\omega\right)$, $\boldsymbol{\eta}_t \sim NID(\mathbf{0}, \boldsymbol{\Sigma})$, $\boldsymbol{\Sigma} = diag\begin{pmatrix}1 & \cdots & 1 & \sigma_{\eta_k}\end{pmatrix}$ and $E\left(\omega_t \boldsymbol{\eta}'_{t-z}\right) = \mathbf{0}'\ \ \forall z$.

The next step is estimation of equations (1), (5) and (10) and performing the related tests. When these equations are estimated, however, the model is intentionally incorrectly specified; from the $\mathbf{x}_t$ vector, one variable, $x_{k,t}$, is excluded. The exclusion of $x_{k,t}$ turns the system into one without cointegration and accordingly we want the cointegration tests to reach this conclusion with a probability of one minus the chosen significance level. If we do not reach this conclusion, we will end up with a misspecified model and estimation of equations such as (1) will be spurious regressions, just like Granger and Newbold's (1974) regressions with independent random walks in their seminal article on the topic. Given the data generating process in equations (14) and (15) which has one cointegrating vector, we throughout the study assume that we are interested in finding out whether there is zero or one cointegrating vector.

The performance of the tests is evaluated both by setting the lag length in equation (2) to $f = (0 \quad 1 \quad 2 \quad 3)$, in equation (5) to $p = (0 \quad 1 \quad 2 \quad 3)$ and in equation (10) to $m = (0 \quad 1 \quad 2 \quad 3)$ as well as determining lag length based on the Akaike (1974) and Schwarz (1978) criteria. In the experiment the following parameters are also varied: the system size is set to $k+1 = (3 \quad 4 \quad 5 \quad 6)$; the sample size is set to $T = (50 \quad 100 \quad 200)$; the standard deviation of the error term of the omitted variable is set to $\sigma_{\eta_k} = (0.5 \quad 1 \quad 2)$, as is the standard deviation of the error term of the dependent variable, σ_{ω}. For each combination of parameters, 15 000 replications are performed. The Matlab programming language is used for simulation and the routine RANDN generates pseudo-random normal innovations.

3.1 Results in brief

Results from the simulations are given below, where Table 1 and 2 summarise the overall behaviour of the tests when the lag length of the test has been determined using the Akaike and Schwarz criteria. Table 1 gives the average size over the parameter space considered for each system size. This should be compared to the nominal size of five percent and gives us an idea of the tests general size properties. However, since averages are used, we could be concerned about outliers ruining the results for tests that perform well in general but badly in a few cases. Therefore, Table 2 presents how often the tests reach an "acceptable" size of five plus/minus two and a half percent over the same parameter space and system size. Table 3 presents rejection frequencies for the case when $T = 200$ and $\sigma_{\omega} = 1$. The reported values are rejection frequencies of the null hypothesis and should, just like the values in Table 1, be compared to the nominal size of five percent. A complete overview of the results is given in Tables A1 to A4 in the appendix and is discussed in more detail below.

Table 1. Fraction of rejection of the null hypothesis when lag length in the tests was chosen using information criteria. Averages over all 27 combinations of parameters for each system size.

System size	$AEG(A)$	$AEG(S)$	$W(A)$	$W(S)$	$J_{\max}(A)$	$J_{\max}(S)$	$J_{trace}(A)$	$J_{trace}(S)$
3	0.212	0.279	0.113	0.157	0.155	0.213	0.153	0.209
4	0.243	0.315	0.155	0.227	0.220	0.283	0.214	0.270
5	0.279	0.354	0.190	0.281	0.277	0.306	0.267	0.285
6	0.306	0.384	0.218	0.320	0.335	0.313	0.327	0.290
All	**0.260**	**0.333**	**0.169**	**0.246**	**0.247**	**0.279**	**0.241**	**0.263**

System size	$J_{\max}^{RA}(A)$	$J_{\max}^{RA}(S)$	$J_{trace}^{RA}(A)$	$J_{trace}^{RA}(S)$
3	0.140	0.204	0.138	0.199
4	0.191	0.264	0.178	0.248
5	0.224	0.273	0.200	0.243
6	0.236	0.263	0.201	0.222
All	**0.198**	**0.251**	**0.179**	**0.228**

System size refers to the number of variables in the data generating process, i.e. k+1.
A and S in parentheses indicates the usage of the Akaike and Schwarz criteria respectively.
Superscript RA means that the Reinsel and Ahn finite sample correction has been applied to the test statistic.

Table 2. Number of times when the empirical size was 5 ± 2.5 percent when lag length in the tests was chosen using information criteria. 27 combinations of parameters for each system size.

System size	$AEG(A)$	$AEG(S)$	$W(A)$	$W(S)$	$J_{\max}(A)$	$J_{\max}(S)$	$J_{trace}(A)$	$J_{trace}(S)$
3	4/27	2/27	13/27	11/27	8/27	9/27	8/27	9/27
4	3/27	2/27	8/27	8/27	4/27	8/27	4/27	8/27
5	2/27	2/27	4/27	6/27	4/27	7/27	4/27	6/27
6	1/27	2/27	2/27	5/27	4/27	5/27	4/27	4/27
All	**10/108**	**8/108**	**27/108**	**30/108**	**20/108**	**29/108**	**20/108**	**27/108**

System size	$J_{\max}^{RA}(A)$	$J_{\max}^{RA}(S)$	$J_{trace}^{RA}(A)$	$J_{trace}^{RA}(S)$
3	12/27	9/27	12/27	9/27
4	9/27	9/27	9/27	9/27
5	9/27	9/27	9/27	9/27
6	9/27	11/27	12/27	12/27
All	**39/108**	**38/108**	**42/108**	**39/108**

System size refers to the number of variables in the data generating process, i.e. k+1.
A and S in parentheses indicates the usage of the Akaike and Schwarz criteria respectively.
Superscript RA means that the Reinsel and Ahn finite sample correction has been applied to the test statistic.

The first thing to note from Table 1 is that the average size distortion is large; the average size of the tests is between 0.169 and 0.333, which should be compared to the nominal size of five percent. The smallest average size distortions can be found when the Akaike information criterion is used to choose lag length for the Boswijk Wald test and the Johansen trace test adjusted by the finite sample correction of Reinsel and Ahn;

the Wald test has the best result with an average size distortion one percentage point smaller than the adjusted trace test. It is, however, clear from Table 2 that the adjusted trace test outperforms the Wald test, and all other tests, in terms of how often the tests have an "acceptable" size. The slightly worse performance in average size for the adjusted trace test is found to mainly be due to some large size distortions when the variance of the error term is high and the variance of the omitted variable is low. It appears as if the Boswijk Wald test could be a competitive alternative in smaller systems, but that the adjusted Johansen trace test is more robust in general. This will stand as the general conclusion from the Monte Carlo study, but let us now have a look at the details of the study by investigating more closely how the different tests respond to changes in parameters.

Table 3. Fraction of rejection of the null hypothesis when T = 200 and σ_{ω} = 1 and lag length in the tests was chosen using information criteria

System size	σ_{η_k}	$AEG(A)$	$AEG(S)$	$W(A)$	$W(S)$	$J_{\max}(A)$	$J_{\max}(S)$	$J_{trace}(A)$	$J_{trace}(S)$
3	0.5	0.166	0.253	0.090	0.152	0.117	0.233	0.113	0.224
4		0.180	0.292	0.118	0.235	0.189	0.538	0.178	0.507
5		0.210	0.349	0.152	0.324	0.324	0.661	0.305	0.610
6		0.231	0.389	0.186	0.433	0.497	0.700	0.458	0.629
3	1	0.081	0.120	0.062	0.074	0.068	0.117	0.067	0.111
4		0.087	0.137	0.071	0.113	0.098	0.187	0.091	0.168
5		0.096	0.159	0.082	0.173	0.146	0.203	0.134	0.176
6		0.109	0.189	0.088	0.233	0.189	0.210	0.160	0.174
3	2	0.071	0.098	0.060	0.067	0.066	0.067	0.065	0.066
4		0.075	0.103	0.071	0.077	0.068	0.066	0.070	0.065
5		0.083	0.117	0.071	0.076	0.067	0.068	0.067	0.064
6		0.088	0.120	0.075	0.077	0.070	0.068	0.072	0.070

System size	σ_{η_k}	$J_{\max}^{RA}(A)$	$J_{\max}^{RA}(S)$	$J_{trace}^{RA}(A)$	$J_{trace}^{RA}(S)$
3	0.5	0.107	0.223	0.104	0.215
4		0.170	0.528	0.154	0.496
5		0.294	0.646	0.270	0.590
6		0.466	0.679	0.415	0.594
3	1	0.061	0.111	0.060	0.106
4		0.085	0.178	0.077	0.157
5		0.129	0.188	0.112	0.161
6		0.162	0.185	0.126	0.142
3	2	0.063	0.064	0.061	0.063
4		0.061	0.060	0.061	0.057
5		0.061	0.060	0.055	0.053
6		0.057	0.057	0.051	0.050

System size refers to the number of variables in the data generating process, i.e. k+1.
A and S in parentheses indicates the usage of the Akaike and Schwarz criteria respectively.
Superscript RA means that the Reinsel and Ahn finite sample correction has been applied to the test statistic.

3.2 The AEG test

Looking at the results in the appendix, it can be noted that the AEG test has large size distortions, regardless of system size, when few lags are used. This is not completely unexpected since the omission of one of the explanatory variables introduces an ARIMA(0,1,1) structure in the estimated error term. The size of the AEG test is monotonically decreasing with respect to lag length, which in general means that adding lags makes the test get closer to the correct size of five percent. Further it is found that the size is almost in all cases monotonically decreasing with σ_{η_k} and increasing with σ_ω. This makes sense since a smaller σ_{η_k}, ceteris paribus, makes the MA-root in the error term closer to minus unity, as does a larger σ_ω. The largest size distortions can, hence, as expected be found when σ_ω is large and σ_{η_k} is small. In those cases, the AEG test even has the property to spuriously reject the null more often with increasing sample size. With respect to system size, the AEG test shows no clear tendency in size.

When lag length is chosen using information criteria, the AEG test is increasing in size with σ_ω but decreasing with respect to σ_{η_k} and the sample size. Unlike the case when lag length was fixed, size is also found to be increasing with system size. It can be noted that the size distortions for the AEG test were fairly moderate given that two or three lags are used. The rather depressing results using both information criteria though suggests that we in practice are likely to choose a lag length too small to give the AEG test acceptable size properties. This problem is not surprisingly worse when the more conservative Schwarz criterion is used.

3.3 The Johansen trace and maximum eigenvalue tests

The two Johansen tests behave in similar ways with respect to parameters changed. Without finite sample correction, it is almost always the case that the size improves, i.e. decreases, with the sample size. This comes as no surprise when asymptotic critical values are used. Only in a few cases when σ_ω is large and σ_{η_k} is small do the Johansen tests show the same behaviour as the AEG test to reject the null more often with increasing sample size. When few lags are used in the smallest systems, size is fairly close to nominal in a few cases even when the sample size is smaller than 200 observations – the sample size that otherwise seems required in order to make the Johansen tests perform reasonably well. Looking at the two largest systems, the results are less flattering; size distortions in the Johansen tests tend to increase when the system size increases, sometimes to a multiple of the desired five percent. As can be seen in Table A1 to A4, size is monotonically increasing with lag length, except in the cases when $\sigma_{\omega} = 2$ and $\sigma_{\eta_k} = 0.5$ or 1 when size decreases with lag length. The tests show no systematic changes in size with respect to either σ_{η_k} or σ_{ω}.

Switching to information criteria to select lag length, the behaviour of the size of the tests changes; however the general pattern is the same regardless of whether the Akaike or Schwarz criterion is used. Size is increasing with respect to σ_{ω} and decreasing with respect to σ_{η_k}; with respect to sample size the effect on test size is ambiguous. However, system size increases the size of the test and in too many cases the size distortions must be described as severe.

Over all, both the maximum eigenvalue and the trace tests do not perform very well in terms of size and part of this is likely due to the asymptotic critical values being inappropriate approximations in the sample sizes considered here. The permanent over-rejection of the null of no cointegration found here is in line with the findings of Cheung and Lai (1993), as is the fairly general observation in this study that an increasing system size increases the size. The size properties generally improve markedly when the Reinsel and Ahn correction is used to deal with these problems. When the finite sample correction is applied, we find that size is increasing with σ_{ω} and decreasing with respect to σ_{η_k} when the lag length is small. Furthermore, there is a tendency for size to increase with the sample size and decrease with lag length, whereas there is no obvious effect with respect to system size. Determining the lag length using information criteria, both tests respond similarly to changes in σ_{ω} and σ_{η_k} as when lag length is chosen arbitrarily. In contrast, increasing the sample size has a tendency to decrease size in the small systems and increase it when the system size is larger. Summarising the results for the Johansen tests, it is clear that the finite sample corrected test statistics reduce size distortions and are to be preferred.

3.4 The Boswijk Wald test

The Boswijk Wald test shows very good results when three lags are used – the size distortions are then very modest in most cases, though a few "outliers" can be found when σ_{ω} is large and σ_{η_k} is small. When few lags are used, size in most cases decreases with σ_{η_k} and increases with σ_{ω}. This pattern does not, however, appear when three lags are being used, regardless of error term variances and system size. When lag length is determined by the two information criteria, the general pattern seems to be that size increases in σ_{ω} and decreases with σ_{η_k} and sample size. However, apart from fairly good properties in the small systems, the size properties are not very impressive for the Boswijk Wald test anymore – the empirical size is way above nominal. Similar to the results for the AEG test, it is obvious that both the Akaike and Schwarz information criteria choose a lag length too small to make the test behave well in terms of size.

4. An empirical application

In this section, the tests are applied to a model that resembles the DGP considered in the Monte Carlo study. Herbertson and Zoega (1999) suggest, using the national-income account identity and the life-cycle theory of consumption together, that the current account should be a function of the age structure. A nation largely at work should have current account surpluses whereas a nation with proportionately more young and old people should have deficits. The intuition is that the young are saving for retirement while the old are running down past and future savings.

Based on the above arguments, Herbertson and Zoega initially estimated equation (16) on a panel consisting of 84 countries from 1960 to 1990.

$$CA_{it} = \alpha + \beta(1 - D_{it}) + \varepsilon_{it} \tag{16}$$

where CA_{it} and D_{it} are the current account[4] and dependency ratio[5] in country *i* at time *t*. The purpose of the exercise was to see if there is any reason to expect demography to play a big role in the determination of current account surpluses. However, if the variables in equation (16) are I(1) and not cointegrated, the regression is likely to yield spurious results. Hence, there is reason to be concerned about the time series properties of this regression, especially since equation (16) looks rather incomplete in its specification.[6] It could be the case that we have a misspecified system of the kind investigated in the previous section in this paper; D_{it} is likely to be exogenous and if an omitted variable is exogenous as well – and if all variables are I(1) – then the DGP is very similar to the one in the Monte Carlo study.

In order to see how the different cointegration tests perform in an empirical situation, they are applied to this model using Swedish data. Being a small open economy, the effects discussed by Herbertsson and Zoega are fairly likely to be present for Sweden. In terms of equation (16), the question of interest is then whether the Swedish CA_t and D_t are cointegrated, which of course also would imply that CA_t and $(1-D_t)$ are cointegrated. Initially, we have to investigate whether the CA_t and D_t are I(1) or I(0). If they turn out to be stationary in levels, the concept of cointegration is irrelevant. Yearly data from 1960 to 1990 on Swedish current account and dependency ratio was supplied by Central bank of Sweden and Statistics Sweden. The results from applying the ADF test to the two series are reported in Table 4.

Table 4. Results from unit root tests

	$ADF(0)$	$ADF(1)$	$ADF(2)$	$ADF(3)$
Dependency ratio	-0.383	-3.728**	-1.885	-2.121
Current account	-2.165	-2.222	-2.268	-2.319

Number in parentheses is lag length used in the estimation, i.e. f.
* *significant at the 5% level*

The test results indicate that a unit root process generated the current account as the null hypothesis of a unit root cannot be rejected for the series using any lag length. However,

[4] The current account is measured as a fraction of GDP
[5] The dependency ratio is defined as the fraction of the population that is younger than 15 years of age or older than 64.
[6] Herbertsson and Zoega later add another variable – the government budget surplus – to this equation. It is of course still worth paying attention to the matter of cointegration in this initial step.

it is not immediately clear whether the dependency ration is I(1) or I(0). The null hypothesis can be rejected using one lagged difference, but not when lag length is chosen using the Akaike or Schwarz information criteria. The conclusion drawn is therefore that both series contain unit roots.

Turning to the question of cointegration between the two variables, the results from applying the four cointegration tests are given in Table 5. As is often the case when using real data the results are ambiguous. Contradicting results should not be a surprise though given the rather varying rejection frequencies we found for the different tests in the Monte Carlo study.

Table 5. Results from tests of cointegration between CAB and dependency ratio

	Lags			
	0	1	2	3
AEG	-2.723	-2.817	-2.822	-3.042
W	4.976	5.181	5.551	5.781
$J_{\max}$	9.237	18.067*	14.851*	15.427*
J_{trace}	9.544	18.376*	15.875*	16.432*
$J^{RA}_{\max}$	8.621	15.658*	11.881	11.313
J^{RA}_{trace}	8.908	15.925*	12.700*	12.050

Lags refer to lag length used in the estimation, i.e. f, p-1 and m.
* *significant at the 5% level*

The AEG test and the Boswijk Wald test do not find any support for cointegration in the data; the null hypothesis of no cointegration cannot be rejected using any lag length. The two Johansen tests on the other hand reject the null hypothesis for all lags except zero when no finite sample correction is applied. When the Reinsel and Ahn correction is made to the test statistics, the null of no cointegration is rejected using one lag for the maximum eigenvalue test and when using one and two lags for the trace test. Letting information criteria decide the lag length – even though we have seen that this strategy need not produce the best results – it turns out that the Akaike criterion chooses two lagged differences as optimal for the Johansen tests, whereas the Schwarz criterion chooses one. Hence, the Johansen tests unambiguously support cointegration using the Schwarz criterion but give a mixed result when lag length is chosen according to the Akaike criterion.

Since we never know the true data generating process in an empirical application, it then boils down to whether the above results are due to *1):* good size properties for the AEG and Boswijk Wald tests in combination with size distortions for some versions of the Johansen tests or *2):* low power for the AEG and Boswijk Wald tests in combination with good power properties for the Johansen tests. In Monte Carlo studies performed in this and other papers, it has been shown that especially the Boswijk Wald test has good size properties when the lag length is sufficiently large. Furthermore, the Boswijk Wald test also has power properties as good as, or better than, the Johansen maximum eigenvalue test as pointed out by Boswijk and Franses (1992) and Pesavento (2000). This supports the idea that the rejections for the Johansen tests are likely to be spurious and an outcome of size distortions, even though the finite sample corrected version of Johansen's trace test has been shown to be robust. This conclusion is supported by the fact that the estimated cointegrating vector from the Johansen test contradicts theory – the parameter on D_t is highly significant, but has the wrong sign. In conclusion, it seems doubtful that these results should be interpreted to be in favour of cointegration between CA_t and D_t. Hence, one should think at least twice before running a regression like equation (16) on the Swedish data given the above results.

5. Conclusions

When dealing with time series that are I(1), the concept of cointegration becomes crucial for the specification of the model. Using the best available tests, one can reduce the probability of estimating econometric models that are misspecified. Getting the long- and short-run dynamics of a system right will improve estimation and, hence, our understanding of economic relationships.

In this study, the small sample properties of four tests for cointegration – the AEG, Johansen's maximum eigenvalue, Johansen's trace, and Boswijk Wald – have been investigated in a Monte Carlo study. Misspecifying a system by omitting one of the explanatory variables is found to generate large size distortions of the tests in some cases. This is especially likely when the variance of the omitted variable is small and the variance of the dependent variable is large. In this situation the likelihood of running a spurious regression when trying to estimate a cointegrating vector using the first step of the AEG test increases, or a vector error correction model may mistakenly be employed when we in fact should estimate a VAR without any error correction terms.

The Johansen trace test adjusted by the finite sample correction of Reinsel and Ahn (1988) is found to have the most robust performance when lag length in the test equations is chosen according to traditional information criteria. Without the Reinsel and Ahn correction, the two Johansen tests perform worse regardless of specification and we can note that for sample sizes generally used by macroeconomists, they are likely to have considerable size distortions when a system has been misspecified. The AEG and, especially, the Boswijk Wald test perform rather well when the lag length is sufficient in the test equations. However, both the Akaike and Schwarz information criteria tend to choose too few lags, yielding fairly large size distortions for both these tests. Though the Wald test was found to be competitive in smaller systems, this must be seen as a practical limitation of the AEG and the Wald test.

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Appendix

Table A1. Fraction of rejection of the null hypothesis when there is one x-variable in estimated equation and two in DGP, i.e. k+1 = 3

σ_ω	σ_{η_k}	T	$AEG(0)$	$AEG(1)$	$AEG(2)$	$AEG(3)$	$W(0)$	$W(1)$	$W(2)$	$W(3)$
0.5	0.5	50	0.373	0.117	0.060	0.049	0.149	0.059	0.048	0.047
		100	0.400	0.122	0.065	0.052	0.171	0.058	0.045	0.044
		200	0.416	0.118	0.061	0.056	0.191	0.069	0.052	0.050
	1.0	50	0.137	0.062	0.045	0.045	0.060	0.048	0.047	0.045
		100	0.132	0.062	0.049	0.049	0.062	0.048	0.048	0.051
		200	0.129	0.055	0.044	0.049	0.068	0.050	0.051	0.048
	2.0	50	0.076	0.057	0.046	0.044	0.048	0.051	0.049	0.050
		100	0.068	0.050	0.045	0.042	0.048	0.049	0.049	0.047
		200	0.064	0.048	0.045	0.050	0.051	0.049	0.049	0.050
1.0	0.5	50	0.786	0.348	0.147	0.091	0.419	0.156	0.072	0.052
		100	0.842	0.420	0.192	0.103	0.490	0.190	0.087	0.055
		200	0.863	0.458	0.212	0.131	0.539	0.220	0.104	0.070
	1.0	50	0.369	0.117	0.064	0.051	0.147	0.058	0.044	0.046
		100	0.405	0.121	0.062	0.049	0.167	0.062	0.049	0.048
		200	0.412	0.117	0.062	0.056	0.189	0.066	0.048	0.047
	2.0	50	0.140	0.068	0.047	0.043	0.060	0.052	0.051	0.054
		100	0.135	0.062	0.048	0.046	0.059	0.047	0.047	0.048
		200	0.128	0.053	0.043	0.047	0.068	0.050	0.051	0.048
2.0	0.5	50	0.980	0.735	0.401	0.231	0.737	0.422	0.214	0.125
		100	0.993	0.857	0.588	0.365	0.801	0.529	0.309	0.182
		200	0.995	0.894	0.674	0.475	0.851	0.599	0.376	0.236
	1.0	50	0.781	0.352	0.149	0.087	0.416	0.151	0.070	0.050
		100	0.843	0.420	0.188	0.109	0.491	0.186	0.089	0.060
		200	0.863	0.456	0.212	0.125	0.533	0.221	0.105	0.068
	2.0	50	0.361	0.116	0.060	0.047	0.142	0.057	0.047	0.046
		100	0.398	0.112	0.060	0.047	0.168	0.058	0.047	0.046
		200	0.416	0.118	0.059	0.051	0.188	0.065	0.048	0.045

σ_ω	σ_{η_k}	T	$J_{\max}(0)$	$J_{\max}(1)$	$J_{\max}(2)$	$J_{\max}(3)$	$J_{trace}(0)$	$J_{trace}(1)$	$J_{trace}(2)$	$J_{trace}(3)$
0.5	0.5	50	0.133	0.069	0.073	0.089	0.130	0.071	0.076	0.093
		100	0.157	0.066	0.060	0.065	0.150	0.066	0.063	0.069
		200	0.173	0.067	0.057	0.057	0.163	0.065	0.056	0.059
	1.0	50	0.063	0.065	0.076	0.094	0.062	0.067	0.080	0.100
		100	0.065	0.057	0.063	0.070	0.062	0.059	0.067	0.073
		200	0.067	0.055	0.056	0.060	0.064	0.054	0.057	0.059
	2.0	50	0.053	0.062	0.080	0.095	0.054	0.067	0.085	0.103
		100	0.053	0.060	0.064	0.072	0.053	0.059	0.066	0.075
		200	0.053	0.056	0.056	0.060	0.054	0.057	0.058	0.060
1.0	0.5	50	0.384	0.149	0.091	0.089	0.372	0.143	0.089	0.090
		100	0.459	0.181	0.097	0.075	0.447	0.172	0.093	0.075
		200	0.511	0.204	0.105	0.079	0.500	0.193	0.101	0.077
	1.0	50	0.132	0.070	0.071	0.091	0.125	0.069	0.075	0.096
		100	0.156	0.069	0.060	0.065	0.148	0.068	0.062	0.067
		200	0.167	0.063	0.054	0.057	0.159	0.062	0.056	0.057
	2.0	50	0.061	0.066	0.077	0.090	0.060	0.069	0.082	0.098
		100	0.066	0.059	0.062	0.069	0.063	0.061	0.063	0.070
		200	0.066	0.057	0.059	0.061	0.066	0.056	0.059	0.062
2.0	0.5	50	0.718	0.390	0.209	0.151	0.713	0.375	0.205	0.147
		100	0.787	0.504	0.286	0.178	0.785	0.494	0.275	0.171
		200	0.838	0.571	0.345	0.215	0.839	0.560	0.334	0.208
	1.0	50	0.386	0.146	0.093	0.091	0.373	0.141	0.093	0.094
		100	0.461	0.173	0.091	0.074	0.449	0.165	0.090	0.072
		200	0.503	0.201	0.100	0.072	0.495	0.194	0.096	0.070
	2.0	50	0.132	0.069	0.075	0.086	0.125	0.068	0.078	0.092
		100	0.155	0.062	0.058	0.062	0.150	0.060	0.060	0.065
		200	0.170	0.067	0.057	0.057	0.164	0.065	0.057	0.059

σ_{ω}	σ_{η_k}	T	$J_{\max}^{RA}(0)$	$J_{\max}^{RA}(1)$	$J_{\max}^{RA}(2)$	$J_{\max}^{RA}(3)$	$J_{trace}^{RA}(0)$	$J_{trace}^{RA}(1)$	$J_{trace}^{RA}(2)$	$J_{trace}^{RA}(3)$
0.5	0.5	50	0.119	0.049	0.040	0.037	0.115	0.048	0.040	0.040
		100	0.148	0.056	0.046	0.044	0.141	0.055	0.049	0.047
		200	0.169	0.063	0.049	0.048	0.159	0.061	0.050	0.048
	1.0	50	0.053	0.042	0.042	0.042	0.052	0.044	0.044	0.044
		100	0.060	0.047	0.049	0.048	0.058	0.048	0.051	0.048
		200	0.065	0.051	0.050	0.049	0.061	0.049	0.050	0.048
	2.0	50	0.044	0.043	0.042	0.040	0.045	0.045	0.044	0.044
		100	0.048	0.049	0.050	0.050	0.048	0.049	0.049	0.049
		200	0.051	0.050	0.049	0.049	0.052	0.051	0.049	0.051
1.0	0.5	50	0.363	0.114	0.050	0.039	0.348	0.108	0.049	0.038
		100	0.448	0.161	0.076	0.051	0.437	0.153	0.074	0.050
		200	0.506	0.193	0.095	0.067	0.496	0.184	0.092	0.064
	1.0	50	0.117	0.047	0.039	0.040	0.108	0.048	0.041	0.043
		100	0.148	0.057	0.046	0.044	0.140	0.058	0.046	0.046
		200	0.163	0.058	0.048	0.047	0.156	0.057	0.048	0.048
	2.0	50	0.051	0.045	0.042	0.040	0.050	0.047	0.045	0.043
		100	0.060	0.049	0.046	0.047	0.058	0.049	0.048	0.048
		200	0.064	0.052	0.051	0.051	0.063	0.051	0.052	0.052
2.0	0.5	50	0.701	0.334	0.141	0.076	0.695	0.317	0.133	0.074
		100	0.781	0.482	0.251	0.140	0.778	0.471	0.238	0.131
		200	0.835	0.561	0.328	0.198	0.837	0.551	0.317	0.191
	1.0	50	0.364	0.112	0.054	0.041	0.349	0.109	0.053	0.043
		100	0.449	0.155	0.072	0.050	0.437	0.148	0.069	0.050
		200	0.498	0.192	0.091	0.060	0.490	0.185	0.087	0.058
	2.0	50	0.115	0.048	0.040	0.040	0.110	0.046	0.042	0.041
		100	0.148	0.052	0.043	0.041	0.142	0.051	0.044	0.044
		200	0.166	0.061	0.050	0.048	0.161	0.061	0.048	0.050

σ_ω	σ_{η_k}	T	$AEG(A)$	$AEG(S)$	$W(A)$	$W(S)$	$J_{\max}(A)$	$J_{\max}(S)$	$J_{trace}(A)$	$J_{trace}(S)$
0.5	0.5	50	0.246	0.315	0.109	0.137	0.139	0.134	0.137	0.131
		100	0.134	0.218	0.073	0.114	0.105	0.149	0.102	0.144
		200	0.082	0.121	0.064	0.078	0.069	0.119	0.067	0.114
	1.0	50	0.127	0.134	0.078	0.067	0.086	0.066	0.086	0.064
		100	0.099	0.120	0.069	0.066	0.074	0.066	0.073	0.063
		200	0.070	0.101	0.060	0.067	0.066	0.067	0.065	0.064
	2.0	50	0.090	0.084	0.072	0.054	0.076	0.055	0.080	0.056
		100	0.069	0.068	0.059	0.050	0.061	0.053	0.062	0.053
		200	0.061	0.062	0.054	0.051	0.058	0.053	0.060	0.054
1.0	0.5	50	0.458	0.611	0.194	0.315	0.297	0.374	0.292	0.362
		100	0.251	0.383	0.117	0.205	0.199	0.380	0.193	0.372
		200	0.166	0.253	0.090	0.152	0.117	0.233	0.113	0.224
	1.0	50	0.245	0.308	0.111	0.133	0.131	0.132	0.131	0.127
		100	0.136	0.225	0.075	0.113	0.104	0.147	0.103	0.141
		200	0.081	0.120	0.062	0.074	0.068	0.117	0.067	0.111
	2.0	50	0.129	0.137	0.081	0.068	0.086	0.063	0.088	0.062
		100	0.099	0.122	0.065	0.061	0.076	0.067	0.076	0.065
		200	0.071	0.098	0.060	0.067	0.066	0.067	0.065	0.066
2.0	0.5	50	0.731	0.861	0.396	0.580	0.601	0.708	0.596	0.702
		100	0.550	0.719	0.285	0.456	0.490	0.728	0.486	0.726
		200	0.499	0.581	0.262	0.359	0.317	0.602	0.310	0.597
	1.0	50	0.453	0.611	0.194	0.308	0.299	0.373	0.294	0.362
		100	0.249	0.387	0.112	0.204	0.186	0.378	0.185	0.370
		200	0.164	0.252	0.089	0.148	0.114	0.231	0.111	0.225
	2.0	50	0.243	0.306	0.106	0.130	0.134	0.134	0.131	0.127
		100	0.127	0.212	0.073	0.111	0.098	0.144	0.099	0.140
		200	0.080	0.117	0.057	0.074	0.071	0.119	0.070	0.116

σ_ω	σ_{η_k}	T	$J_{\max}^{RA}(A)$	$J_{\max}^{RA}(S)$	$J_{trace}^{RA}(A)$	$J_{trace}^{RA}(S)$
0.5	0.5	50	0.112	0.118	0.109	0.115
		100	0.093	0.140	0.092	0.134
		200	0.063	0.115	0.061	0.111
	1.0	50	0.065	0.054	0.067	0.054
		100	0.066	0.060	0.064	0.058
		200	0.062	0.065	0.060	0.061
	2.0	50	0.059	0.045	0.061	0.047
		100	0.055	0.049	0.056	0.048
		200	0.055	0.051	0.056	0.052
1.0	0.5	50	0.268	0.351	0.261	0.339
		100	0.180	0.369	0.175	0.361
		200	0.107	0.223	0.104	0.215
	1.0	50	0.107	0.117	0.104	0.109
		100	0.092	0.138	0.092	0.133
		200	0.061	0.111	0.060	0.106
	2.0	50	0.065	0.053	0.067	0.052
		100	0.068	0.062	0.067	0.059
		200	0.063	0.064	0.061	0.063
2.0	0.5	50	0.573	0.690	0.566	0.684
		100	0.474	0.720	0.468	0.718
		200	0.302	0.595	0.297	0.590
	1.0	50	0.272	0.351	0.263	0.338
		100	0.171	0.367	0.168	0.358
		200	0.104	0.223	0.101	0.217
	2.0	50	0.109	0.116	0.105	0.111
		100	0.087	0.136	0.086	0.132
		200	0.064	0.115	0.063	0.112

Table A2. Fraction of rejection of the null hypothesis when there are two x-variables in estimated equation and three in DGP, i.e. k+1 = 4

σ_ω	σ_{η_k}	T	$AEG(0)$	$AEG(1)$	$AEG(2)$	$AEG(3)$	$W(0)$	$W(1)$	$W(2)$	$W(3)$
0.5	0.5	50	0.390	0.108	0.047	0.037	0.167	0.058	0.047	0.047
		100	0.424	0.112	0.053	0.039	0.208	0.063	0.046	0.046
		200	0.448	0.113	0.052	0.048	0.239	0.072	0.050	0.050
	1.0	50	0.141	0.057	0.037	0.032	0.065	0.048	0.051	0.050
		100	0.128	0.049	0.037	0.032	0.066	0.047	0.047	0.048
		200	0.132	0.051	0.041	0.045	0.075	0.051	0.050	0.050
	2.0	50	0.078	0.052	0.036	0.032	0.051	0.051	0.051	0.049
		100	0.065	0.045	0.037	0.034	0.049	0.049	0.049	0.046
		200	0.062	0.043	0.038	0.045	0.052	0.050	0.050	0.049
1.0	0.5	50	0.807	0.336	0.132	0.071	0.512	0.168	0.078	0.052
		100	0.874	0.425	0.176	0.091	0.603	0.223	0.096	0.063
		200	0.891	0.476	0.205	0.121	0.656	0.274	0.117	0.073
	1.0	50	0.384	0.110	0.053	0.037	0.171	0.061	0.048	0.046
		100	0.428	0.114	0.054	0.041	0.204	0.066	0.048	0.047
		200	0.442	0.116	0.054	0.049	0.233	0.073	0.054	0.051
	2.0	50	0.136	0.058	0.036	0.033	0.065	0.050	0.050	0.048
		100	0.128	0.050	0.036	0.032	0.064	0.046	0.048	0.047
		200	0.125	0.049	0.039	0.043	0.076	0.055	0.053	0.053
2.0	0.5	50	0.979	0.696	0.332	0.175	0.842	0.462	0.207	0.115
		100	0.996	0.877	0.587	0.346	0.904	0.634	0.359	0.202
		200	0.997	0.921	0.706	0.498	0.929	0.710	0.457	0.280
	1.0	50	0.807	0.335	0.128	0.069	0.506	0.161	0.074	0.050
		100	0.868	0.420	0.174	0.091	0.603	0.221	0.094	0.058
		200	0.890	0.477	0.209	0.122	0.651	0.268	0.120	0.073
	2.0	50	0.388	0.117	0.053	0.040	0.176	0.062	0.049	0.047
		100	0.426	0.111	0.053	0.038	0.200	0.061	0.046	0.044
		200	0.443	0.114	0.056	0.051	0.236	0.073	0.053	0.053

σ_ω	σ_{η_k}	T	$J_{\max}(0)$	$J_{\max}(1)$	$J_{\max}(2)$	$J_{\max}(3)$	$J_{trace}(0)$	$J_{trace}(1)$	$J_{trace}(2)$	$J_{trace}(3)$
0.5	0.5	50	0.151	0.089	0.117	0.167	0.137	0.093	0.134	0.197
		100	0.172	0.077	0.075	0.088	0.157	0.076	0.081	0.097
		200	0.194	0.070	0.060	0.065	0.174	0.067	0.061	0.069
	1.0	50	0.070	0.083	0.123	0.182	0.070	0.094	0.143	0.213
		100	0.068	0.060	0.076	0.090	0.065	0.065	0.084	0.103
		200	0.069	0.058	0.063	0.069	0.066	0.061	0.067	0.074
	2.0	50	0.063	0.086	0.128	0.178	0.065	0.097	0.147	0.216
		100	0.055	0.067	0.078	0.094	0.058	0.072	0.084	0.102
		200	0.053	0.057	0.066	0.070	0.054	0.060	0.068	0.075
1.0	0.5	50	0.434	0.174	0.135	0.167	0.403	0.168	0.148	0.187
		100	0.540	0.199	0.112	0.099	0.511	0.186	0.111	0.107
		200	0.598	0.225	0.111	0.086	0.564	0.207	0.105	0.084
	1.0	50	0.144	0.093	0.117	0.170	0.132	0.096	0.134	0.201
		100	0.173	0.078	0.077	0.092	0.159	0.073	0.082	0.102
		200	0.188	0.073	0.066	0.070	0.169	0.069	0.065	0.073
	2.0	50	0.072	0.087	0.123	0.178	0.072	0.092	0.147	0.212
		100	0.065	0.065	0.078	0.094	0.065	0.069	0.083	0.105
		200	0.066	0.056	0.061	0.066	0.065	0.059	0.064	0.071
2.0	0.5	50	0.792	0.406	0.231	0.209	0.761	0.384	0.237	0.228
		100	0.881	0.571	0.319	0.203	0.872	0.542	0.299	0.201
		200	0.910	0.654	0.395	0.244	0.903	0.627	0.364	0.220
	1.0	50	0.425	0.161	0.131	0.162	0.389	0.156	0.137	0.185
		100	0.533	0.190	0.103	0.092	0.506	0.179	0.102	0.099
		200	0.592	0.222	0.108	0.081	0.561	0.202	0.102	0.077
	2.0	50	0.150	0.095	0.118	0.169	0.136	0.101	0.136	0.202
		100	0.166	0.075	0.075	0.087	0.152	0.073	0.082	0.098
		200	0.188	0.075	0.064	0.069	0.174	0.073	0.067	0.073

σ_{ω}	σ_{η_k}	T	$J^{RA}_{\max}(0)$	$J^{RA}_{\max}(1)$	$J^{RA}_{\max}(2)$	$J^{RA}_{\max}(3)$	$J^{RA}_{trace}(0)$	$J^{RA}_{trace}(1)$	$J^{RA}_{trace}(2)$	$J^{RA}_{trace}(3)$
0.5	0.5	50	0.116	0.041	0.035	0.034	0.101	0.039	0.032	0.033
		100	0.155	0.053	0.043	0.041	0.136	0.051	0.041	0.041
		200	0.184	0.060	0.046	0.044	0.162	0.055	0.045	0.044
	1.0	50	0.050	0.037	0.038	0.037	0.045	0.037	0.040	0.037
		100	0.055	0.042	0.042	0.045	0.053	0.041	0.043	0.044
		200	0.063	0.047	0.048	0.048	0.061	0.048	0.049	0.048
	2.0	50	0.043	0.038	0.043	0.035	0.042	0.039	0.041	0.038
		100	0.046	0.046	0.044	0.044	0.045	0.046	0.043	0.043
		200	0.048	0.047	0.048	0.049	0.049	0.048	0.050	0.050
1.0	0.5	50	0.382	0.097	0.044	0.032	0.338	0.083	0.037	0.030
		100	0.519	0.159	0.068	0.049	0.483	0.139	0.062	0.045
		200	0.588	0.207	0.089	0.060	0.553	0.182	0.079	0.057
	1.0	50	0.112	0.045	0.036	0.033	0.095	0.041	0.036	0.032
		100	0.156	0.053	0.042	0.043	0.138	0.049	0.044	0.044
		200	0.180	0.061	0.050	0.047	0.159	0.056	0.047	0.048
	2.0	50	0.051	0.042	0.039	0.036	0.048	0.039	0.040	0.037
		100	0.054	0.045	0.043	0.044	0.053	0.045	0.045	0.045
		200	0.060	0.046	0.047	0.046	0.057	0.048	0.047	0.048
2.0	0.5	50	0.754	0.274	0.090	0.049	0.709	0.233	0.078	0.042
		100	0.873	0.522	0.238	0.117	0.859	0.476	0.206	0.107
		200	0.907	0.636	0.360	0.200	0.899	0.602	0.321	0.175
	1.0	50	0.373	0.090	0.044	0.032	0.328	0.078	0.039	0.027
		100	0.512	0.150	0.061	0.041	0.478	0.133	0.055	0.041
		200	0.583	0.204	0.089	0.058	0.550	0.179	0.078	0.053
	2.0	50	0.112	0.044	0.035	0.032	0.101	0.041	0.037	0.033
		100	0.148	0.052	0.043	0.040	0.135	0.048	0.041	0.042
		200	0.180	0.063	0.047	0.046	0.165	0.061	0.049	0.047

σ_ω	σ_{η_k}	T	$AEG(A)$	$AEG(S)$	$W(A)$	$W(S)$	$J_{\max}(A)$	$J_{\max}(S)$	$J_{trace}(A)$	$J_{trace}(S)$
0.5	0.5	50	0.298	0.354	0.145	0.167	0.178	0.151	0.171	0.137
		100	0.161	0.269	0.101	0.174	0.150	0.172	0.142	0.157
		200	0.083	0.133	0.070	0.114	0.096	0.191	0.092	0.172
	1.0	50	0.138	0.142	0.097	0.072	0.108	0.070	0.115	0.071
		100	0.102	0.118	0.076	0.067	0.078	0.068	0.077	0.065
		200	0.078	0.107	0.067	0.075	0.071	0.069	0.069	0.066
	2.0	50	0.095	0.088	0.085	0.058	0.101	0.063	0.107	0.066
		100	0.068	0.066	0.065	0.050	0.063	0.055	0.068	0.058
		200	0.058	0.061	0.059	0.052	0.055	0.053	0.057	0.054
1.0	0.5	50	0.572	0.705	0.303	0.463	0.406	0.434	0.385	0.403
		100	0.303	0.462	0.176	0.377	0.371	0.537	0.357	0.509
		200	0.180	0.292	0.118	0.235	0.189	0.538	0.178	0.507
	1.0	50	0.296	0.351	0.149	0.170	0.173	0.145	0.168	0.133
		100	0.165	0.271	0.100	0.170	0.152	0.173	0.145	0.159
		200	0.087	0.137	0.071	0.113	0.098	0.187	0.091	0.168
	2.0	50	0.134	0.138	0.093	0.072	0.110	0.072	0.115	0.073
		100	0.099	0.118	0.073	0.066	0.076	0.065	0.076	0.065
		200	0.075	0.103	0.071	0.077	0.068	0.066	0.070	0.065
2.0	0.5	50	0.801	0.914	0.574	0.784	0.738	0.791	0.713	0.760
		100	0.635	0.812	0.437	0.697	0.746	0.880	0.738	0.871
		200	0.540	0.645	0.336	0.557	0.537	0.874	0.521	0.867
	1.0	50	0.573	0.702	0.301	0.455	0.398	0.426	0.373	0.389
		100	0.298	0.465	0.173	0.377	0.366	0.530	0.352	0.504
		200	0.184	0.301	0.118	0.232	0.192	0.529	0.180	0.507
	2.0	50	0.301	0.355	0.151	0.175	0.179	0.151	0.174	0.137
		100	0.161	0.266	0.097	0.165	0.148	0.166	0.141	0.153
		200	0.088	0.134	0.072	0.116	0.103	0.187	0.100	0.173

σ_ω	σ_{η_k}	T	$J_{\max}^{RA}(A)$	$J_{\max}^{RA}(S)$	$J_{trace}^{RA}(A)$	$J_{trace}^{RA}(S)$
0.5	0.5	50	0.120	0.116	0.108	0.102
		100	0.126	0.155	0.114	0.137
		200	0.087	0.182	0.078	0.160
	1.0	50	0.065	0.050	0.061	0.045
		100	0.061	0.055	0.060	0.053
		200	0.063	0.063	0.061	0.061
	2.0	50	0.060	0.043	0.059	0.042
		100	0.052	0.046	0.051	0.045
		200	0.050	0.048	0.052	0.049
1.0	0.5	50	0.334	0.382	0.299	0.338
		100	0.343	0.516	0.323	0.481
		200	0.170	0.528	0.154	0.496
	1.0	50	0.118	0.113	0.103	0.096
		100	0.130	0.156	0.117	0.138
		200	0.085	0.178	0.077	0.157
	2.0	50	0.067	0.051	0.065	0.049
		100	0.061	0.054	0.060	0.053
		200	0.061	0.060	0.061	0.057
2.0	0.5	50	0.678	0.753	0.636	0.708
		100	0.727	0.872	0.712	0.858
		200	0.518	0.869	0.494	0.862
	1.0	50	0.328	0.373	0.289	0.327
		100	0.336	0.509	0.317	0.476
		200	0.175	0.520	0.157	0.495
	2.0	50	0.116	0.113	0.108	0.101
		100	0.125	0.148	0.116	0.135
		200	0.091	0.179	0.086	0.164

Table A3. Fraction of rejection of the null hypothesis when there are three x-variables in estimated equation and four in DGP, i.e. k+1 = 5

σ_ω	σ_{η_k}	T	$AEG(0)$	$AEG(1)$	$AEG(2)$	$AEG(3)$	$W(0)$	$W(1)$	$W(2)$	$W(3)$
0.5	0.5	50	0.412	0.108	0.042	0.032	0.180	0.060	0.048	0.051
		100	0.456	0.111	0.048	0.035	0.227	0.067	0.049	0.047
		200	0.481	0.119	0.055	0.051	0.262	0.074	0.052	0.050
	1.0	50	0.153	0.058	0.034	0.025	0.063	0.050	0.049	0.051
		100	0.143	0.053	0.037	0.029	0.064	0.047	0.047	0.049
		200	0.133	0.044	0.033	0.041	0.078	0.051	0.051	0.048
	2.0	50	0.089	0.054	0.033	0.027	0.051	0.049	0.052	0.051
		100	0.066	0.041	0.031	0.027	0.046	0.044	0.045	0.048
		200	0.057	0.038	0.033	0.039	0.050	0.049	0.051	0.052
1.0	0.5	50	0.829	0.334	0.119	0.059	0.556	0.167	0.073	0.056
		100	0.901	0.448	0.168	0.083	0.678	0.246	0.093	0.059
		200	0.921	0.516	0.215	0.126	0.738	0.309	0.126	0.071
	1.0	50	0.416	0.110	0.044	0.031	0.184	0.063	0.049	0.052
		100	0.457	0.113	0.049	0.038	0.225	0.064	0.044	0.045
		200	0.481	0.114	0.055	0.048	0.269	0.077	0.055	0.050
	2.0	50	0.152	0.055	0.032	0.026	0.070	0.050	0.052	0.052
		100	0.145	0.049	0.034	0.029	0.069	0.047	0.047	0.047
		200	0.137	0.048	0.036	0.039	0.076	0.049	0.048	0.047
2.0	0.5	50	0.984	0.675	0.297	0.157	0.893	0.440	0.181	0.105
		100	0.997	0.888	0.575	0.321	0.948	0.689	0.371	0.195
		200	0.998	0.943	0.735	0.523	0.968	0.790	0.513	0.306
	1.0	50	0.828	0.336	0.120	0.063	0.557	0.165	0.072	0.054
		100	0.901	0.443	0.171	0.083	0.676	0.242	0.096	0.063
		200	0.924	0.503	0.207	0.124	0.739	0.305	0.129	0.071
	2.0	50	0.418	0.110	0.044	0.031	0.182	0.060	0.048	0.049
		100	0.456	0.116	0.048	0.035	0.221	0.062	0.048	0.046
		200	0.482	0.120	0.054	0.049	0.266	0.075	0.051	0.050

σ_ω	σ_{η_k}	T	$J_{\max}(0)$	$J_{\max}(1)$	$J_{\max}(2)$	$J_{\max}(3)$	$J_{trace}(0)$	$J_{trace}(1)$	$J_{trace}(2)$	$J_{trace}(3)$
0.5	0.5	50	0.149	0.123	0.199	0.339	0.141	0.145	0.259	0.450
		100	0.177	0.092	0.100	0.130	0.161	0.093	0.112	0.159
		200	0.199	0.075	0.073	0.080	0.173	0.075	0.076	0.090
	1.0	50	0.080	0.120	0.217	0.351	0.081	0.141	0.273	0.463
		100	0.068	0.080	0.102	0.136	0.068	0.089	0.125	0.169
		200	0.069	0.062	0.069	0.080	0.064	0.064	0.078	0.092
	2.0	50	0.071	0.121	0.221	0.361	0.077	0.147	0.283	0.481
		100	0.060	0.077	0.101	0.138	0.058	0.085	0.115	0.170
		200	0.052	0.059	0.071	0.084	0.055	0.064	0.081	0.097
1.0	0.5	50	0.444	0.200	0.214	0.325	0.392	0.216	0.261	0.415
		100	0.587	0.212	0.133	0.139	0.532	0.203	0.145	0.164
		200	0.662	0.248	0.123	0.098	0.611	0.219	0.120	0.103
	1.0	50	0.152	0.130	0.203	0.336	0.146	0.149	0.264	0.445
		100	0.175	0.089	0.101	0.133	0.156	0.091	0.115	0.165
		200	0.203	0.075	0.071	0.081	0.176	0.076	0.076	0.089
	2.0	50	0.081	0.128	0.214	0.356	0.080	0.146	0.277	0.468
		100	0.069	0.078	0.100	0.132	0.068	0.085	0.122	0.163
		200	0.068	0.062	0.072	0.084	0.064	0.065	0.076	0.093
2.0	0.5	50	0.811	0.385	0.292	0.361	0.744	0.392	0.346	0.453
		100	0.923	0.596	0.327	0.234	0.904	0.546	0.317	0.251
		200	0.950	0.715	0.428	0.262	0.939	0.671	0.391	0.241
	1.0	50	0.446	0.195	0.214	0.332	0.397	0.215	0.263	0.421
		100	0.585	0.208	0.134	0.141	0.529	0.195	0.145	0.159
		200	0.659	0.239	0.118	0.093	0.604	0.212	0.117	0.101
	2.0	50	0.153	0.123	0.200	0.342	0.143	0.144	0.260	0.445
		100	0.175	0.085	0.094	0.127	0.157	0.090	0.116	0.159
		200	0.206	0.078	0.072	0.082	0.179	0.078	0.079	0.094

σ_{ω}	σ_{η_k}	T	$J_{\max}^{RA}(0)$	$J_{\max}^{RA}(1)$	$J_{\max}^{RA}(2)$	$J_{\max}^{RA}(3)$	$J_{trace}^{RA}(0)$	$J_{trace}^{RA}(1)$	$J_{trace}^{RA}(2)$	$J_{trace}^{RA}(3)$
0.5	0.5	50	0.089	0.037	0.031	0.027	0.075	0.031	0.027	0.023
		100	0.146	0.051	0.039	0.040	0.123	0.045	0.038	0.038
		200	0.183	0.056	0.047	0.044	0.155	0.053	0.043	0.041
	1.0	50	0.045	0.037	0.033	0.031	0.038	0.032	0.031	0.027
		100	0.051	0.044	0.042	0.041	0.048	0.041	0.042	0.038
		200	0.059	0.046	0.045	0.042	0.054	0.044	0.043	0.044
	2.0	50	0.038	0.037	0.034	0.032	0.034	0.032	0.031	0.028
		100	0.044	0.042	0.038	0.038	0.041	0.040	0.038	0.038
		200	0.044	0.042	0.046	0.044	0.045	0.045	0.045	0.046
1.0	0.5	50	0.345	0.072	0.032	0.027	0.271	0.057	0.027	0.020
		100	0.552	0.139	0.059	0.043	0.481	0.114	0.050	0.037
		200	0.647	0.211	0.085	0.054	0.590	0.175	0.073	0.049
	1.0	50	0.096	0.037	0.032	0.029	0.079	0.034	0.030	0.025
		100	0.145	0.050	0.041	0.038	0.121	0.044	0.038	0.037
		200	0.188	0.055	0.045	0.042	0.161	0.054	0.044	0.043
	2.0	50	0.044	0.037	0.033	0.030	0.037	0.032	0.030	0.028
		100	0.053	0.042	0.041	0.039	0.048	0.040	0.042	0.038
		200	0.060	0.048	0.047	0.046	0.053	0.044	0.044	0.044
2.0	0.5	50	0.732	0.180	0.056	0.033	0.616	0.137	0.045	0.023
		100	0.912	0.504	0.191	0.088	0.884	0.419	0.153	0.073
		200	0.946	0.685	0.365	0.184	0.933	0.626	0.306	0.146
	1.0	50	0.346	0.069	0.034	0.026	0.271	0.055	0.027	0.020
		100	0.546	0.139	0.058	0.042	0.474	0.114	0.050	0.037
		200	0.643	0.204	0.082	0.054	0.583	0.169	0.070	0.046
	2.0	50	0.096	0.037	0.031	0.029	0.076	0.031	0.027	0.023
		100	0.145	0.046	0.038	0.037	0.118	0.044	0.035	0.034
		200	0.191	0.060	0.046	0.045	0.160	0.055	0.047	0.044

σ_ω	σ_{η_k}	T	$AEG(A)$	$AEG(S)$	$W(A)$	$W(S)$	$J_{\max}(A)$	$J_{\max}(S)$	$J_{trace}(A)$	$J_{trace}(S)$
0.5	0.5	50	0.341	0.390	0.171	0.185	0.210	0.149	0.224	0.141
		100	0.206	0.331	0.133	0.213	0.176	0.177	0.166	0.161
		200	0.101	0.163	0.079	0.171	0.144	0.199	0.133	0.173
	1.0	50	0.155	0.159	0.108	0.072	0.151	0.080	0.165	0.081
		100	0.118	0.136	0.081	0.065	0.076	0.068	0.079	0.068
		200	0.079	0.112	0.073	0.079	0.072	0.069	0.068	0.064
	2.0	50	0.110	0.103	0.100	0.059	0.142	0.071	0.160	0.077
		100	0.070	0.068	0.064	0.046	0.066	0.060	0.066	0.058
		200	0.056	0.057	0.060	0.051	0.053	0.052	0.056	0.055
1.0	0.5	50	0.662	0.768	0.383	0.538	0.459	0.444	0.430	0.392
		100	0.369	0.551	0.248	0.552	0.522	0.587	0.479	0.532
		200	0.210	0.349	0.152	0.324	0.324	0.661	0.305	0.610
	1.0	50	0.346	0.395	0.178	0.189	0.220	0.152	0.228	0.146
		100	0.209	0.329	0.129	0.210	0.177	0.175	0.162	0.156
		200	0.096	0.159	0.082	0.173	0.146	0.203	0.134	0.176
	2.0	50	0.154	0.156	0.115	0.079	0.153	0.081	0.165	0.080
		100	0.116	0.138	0.082	0.071	0.076	0.069	0.078	0.068
		200	0.083	0.117	0.071	0.076	0.070	0.068	0.067	0.064
2.0	0.5	50	0.863	0.948	0.670	0.870	0.781	0.811	0.736	0.744
		100	0.705	0.865	0.569	0.857	0.876	0.923	0.860	0.904
		200	0.581	0.705	0.421	0.726	0.755	0.950	0.732	0.939
	1.0	50	0.662	0.771	0.377	0.537	0.461	0.446	0.434	0.397
		100	0.376	0.555	0.244	0.549	0.518	0.585	0.477	0.529
		200	0.206	0.342	0.148	0.318	0.312	0.659	0.295	0.604
	2.0	50	0.351	0.395	0.173	0.187	0.217	0.153	0.224	0.143
		100	0.212	0.329	0.130	0.206	0.174	0.175	0.164	0.157
		200	0.101	0.168	0.078	0.177	0.150	0.206	0.139	0.179

σ_ω	σ_{η_k}	T	$J^{RA}_{\max}(A)$	$J^{RA}_{\max}(S)$	$J^{RA}_{trace}(A)$	$J^{RA}_{trace}(S)$
0.5	0.5	50	0.099	0.089	0.087	0.075
		100	0.140	0.146	0.120	0.123
		200	0.125	0.183	0.110	0.155
	1.0	50	0.064	0.045	0.058	0.038
		100	0.056	0.051	0.053	0.048
		200	0.061	0.059	0.056	0.054
	2.0	50	0.056	0.038	0.053	0.034
		100	0.047	0.044	0.045	0.041
		200	0.045	0.044	0.046	0.045
1.0	0.5	50	0.317	0.345	0.253	0.271
		100	0.478	0.552	0.421	0.481
		200	0.294	0.646	0.270	0.590
	1.0	50	0.106	0.096	0.090	0.079
		100	0.140	0.145	0.119	0.121
		200	0.129	0.188	0.112	0.161
	2.0	50	0.062	0.044	0.056	0.037
		100	0.057	0.053	0.052	0.048
		200	0.061	0.060	0.055	0.053
2.0	0.5	50	0.662	0.732	0.562	0.616
		100	0.856	0.912	0.829	0.884
		200	0.733	0.945	0.705	0.933
	1.0	50	0.317	0.346	0.253	0.271
		100	0.475	0.546	0.413	0.474
		200	0.285	0.643	0.261	0.583
	2.0	50	0.106	0.096	0.089	0.077
		100	0.138	0.145	0.115	0.118
		200	0.132	0.191	0.118	0.160

Table A4. Fraction of rejection of the null hypothesis when there are four x-variables in estimated equation and five in DGP, i.e. k+1 = 6

σ_ω	σ_{η_k}	T	$AEG(0)$	$AEG(1)$	$AEG(2)$	$AEG(3)$	$W(0)$	$W(1)$	$W(2)$	$W(3)$
0.5	0.5	50	0.423	0.104	0.039	0.025	0.181	0.056	0.049	0.053
		100	0.486	0.113	0.042	0.028	0.258	0.072	0.052	0.050
		200	0.518	0.114	0.049	0.044	0.284	0.072	0.046	0.044
	1.0	50	0.162	0.054	0.027	0.020	0.063	0.051	0.054	0.057
		100	0.145	0.047	0.030	0.023	0.075	0.053	0.051	0.053
		200	0.139	0.043	0.031	0.033	0.077	0.050	0.050	0.051
	2.0	50	0.090	0.049	0.023	0.020	0.054	0.052	0.052	0.060
		100	0.068	0.041	0.029	0.024	0.057	0.055	0.055	0.056
		200	0.061	0.038	0.031	0.033	0.051	0.050	0.050	0.049
1.0	0.5	50	0.829	0.319	0.102	0.050	0.564	0.158	0.071	0.061
		100	0.919	0.448	0.159	0.074	0.736	0.258	0.100	0.064
		200	0.939	0.524	0.211	0.110	0.790	0.328	0.129	0.076
	1.0	50	0.429	0.107	0.041	0.024	0.185	0.064	0.051	0.056
		100	0.491	0.110	0.041	0.029	0.256	0.069	0.053	0.053
		200	0.511	0.114	0.048	0.040	0.291	0.074	0.048	0.046
	2.0	50	0.166	0.059	0.028	0.021	0.064	0.053	0.056	0.056
		100	0.143	0.049	0.030	0.024	0.074	0.048	0.049	0.052
		200	0.138	0.042	0.031	0.033	0.076	0.048	0.046	0.048
2.0	0.5	50	0.983	0.634	0.252	0.120	0.896	0.402	0.158	0.092
		100	0.998	0.896	0.560	0.299	0.972	0.730	0.380	0.198
		200	0.999	0.954	0.746	0.507	0.984	0.834	0.553	0.318
	1.0	50	0.836	0.317	0.104	0.050	0.569	0.159	0.074	0.062
		100	0.920	0.453	0.165	0.077	0.731	0.266	0.106	0.066
		200	0.942	0.532	0.209	0.111	0.789	0.326	0.126	0.071
	2.0	50	0.434	0.108	0.037	0.023	0.189	0.064	0.055	0.055
		100	0.479	0.115	0.044	0.030	0.246	0.073	0.051	0.050
		200	0.507	0.110	0.048	0.040	0.287	0.074	0.049	0.046

σ_ω	σ_{η_k}	T	$J_{\max}(0)$	$J_{\max}(1)$	$J_{\max}(2)$	$J_{\max}(3)$	$J_{trace}(0)$	$J_{trace}(1)$	$J_{trace}(2)$	$J_{trace}(3)$
0.5	0.5	50	0.156	0.179	0.348	0.617	0.158	0.236	0.495	0.801
		100	0.181	0.105	0.138	0.197	0.162	0.120	0.180	0.274
		200	0.202	0.078	0.084	0.101	0.172	0.081	0.097	0.126
	1.0	50	0.089	0.184	0.363	0.631	0.095	0.245	0.520	0.827
		100	0.077	0.100	0.143	0.210	0.079	0.117	0.194	0.300
		200	0.072	0.072	0.090	0.107	0.070	0.079	0.105	0.132
	2.0	50	0.084	0.183	0.367	0.648	0.096	0.253	0.532	0.832
		100	0.062	0.097	0.143	0.210	0.069	0.119	0.193	0.307
		200	0.059	0.070	0.088	0.109	0.060	0.079	0.106	0.134
1.0	0.5	50	0.425	0.234	0.351	0.596	0.387	0.297	0.485	0.773
		100	0.606	0.222	0.168	0.204	0.531	0.219	0.205	0.273
		200	0.700	0.249	0.131	0.115	0.629	0.226	0.139	0.134
	1.0	50	0.157	0.179	0.347	0.622	0.158	0.234	0.493	0.805
		100	0.180	0.105	0.136	0.195	0.161	0.120	0.181	0.279
		200	0.210	0.079	0.084	0.103	0.174	0.083	0.097	0.122
	2.0	50	0.091	0.178	0.363	0.640	0.102	0.239	0.521	0.826
		100	0.073	0.092	0.137	0.209	0.076	0.115	0.188	0.293
		200	0.068	0.068	0.083	0.101	0.070	0.076	0.101	0.126
2.0	0.5	50	0.768	0.394	0.413	0.612	0.681	0.452	0.547	0.782
		100	0.945	0.593	0.342	0.284	0.911	0.539	0.365	0.354
		200	0.971	0.756	0.449	0.278	0.959	0.693	0.408	0.277
	1.0	50	0.429	0.241	0.357	0.597	0.379	0.296	0.489	0.782
		100	0.611	0.220	0.164	0.198	0.535	0.224	0.201	0.270
		200	0.694	0.249	0.132	0.112	0.623	0.226	0.139	0.134
	2.0	50	0.158	0.185	0.355	0.614	0.161	0.239	0.501	0.803
		100	0.174	0.097	0.135	0.202	0.156	0.116	0.178	0.279
		200	0.205	0.084	0.085	0.106	0.177	0.085	0.099	0.130

σ_{ω}	σ_{η_k}	T	$J_{\max}^{RA}(0)$	$J_{\max}^{RA}(1)$	$J_{\max}^{RA}(2)$	$J_{\max}^{RA}(3)$	$J_{trace}^{RA}(0)$	$J_{trace}^{RA}(1)$	$J_{trace}^{RA}(2)$	$J_{trace}^{RA}(3)$
0.5	0.5	50	0.075	0.030	0.026	0.023	0.055	0.022	0.019	0.014
		100	0.136	0.044	0.038	0.034	0.105	0.040	0.032	0.030
		200	0.178	0.052	0.044	0.041	0.143	0.047	0.040	0.039
	1.0	50	0.038	0.030	0.028	0.025	0.029	0.022	0.021	0.018
		100	0.049	0.040	0.039	0.038	0.043	0.037	0.036	0.036
		200	0.059	0.045	0.047	0.046	0.051	0.044	0.046	0.043
	2.0	50	0.034	0.033	0.029	0.026	0.027	0.024	0.021	0.022
		100	0.041	0.039	0.036	0.037	0.039	0.040	0.035	0.038
		200	0.047	0.046	0.046	0.045	0.046	0.046	0.044	0.046
1.0	0.5	50	0.278	0.050	0.026	0.020	0.194	0.035	0.016	0.012
		100	0.549	0.118	0.050	0.037	0.436	0.089	0.040	0.030
		200	0.679	0.196	0.075	0.050	0.594	0.155	0.067	0.046
	1.0	50	0.073	0.031	0.026	0.024	0.057	0.021	0.020	0.015
		100	0.136	0.043	0.037	0.037	0.106	0.037	0.032	0.030
		200	0.185	0.054	0.042	0.040	0.142	0.047	0.041	0.038
	2.0	50	0.039	0.029	0.028	0.024	0.029	0.023	0.022	0.020
		100	0.046	0.037	0.035	0.037	0.041	0.036	0.034	0.033
		200	0.057	0.043	0.044	0.043	0.050	0.040	0.043	0.042
2.0	0.5	50	0.618	0.104	0.037	0.023	0.450	0.075	0.026	0.013
		100	0.930	0.446	0.144	0.063	0.873	0.326	0.104	0.052
		200	0.968	0.712	0.350	0.161	0.951	0.613	0.269	0.124
	1.0	50	0.277	0.048	0.025	0.022	0.191	0.036	0.017	0.013
		100	0.553	0.120	0.046	0.032	0.438	0.090	0.040	0.027
		200	0.672	0.198	0.074	0.052	0.588	0.155	0.065	0.044
	2.0	50	0.076	0.033	0.026	0.025	0.059	0.024	0.020	0.016
		100	0.130	0.041	0.035	0.035	0.099	0.035	0.032	0.032
		200	0.182	0.055	0.045	0.043	0.147	0.049	0.040	0.041

σ_ω	σ_{η_k}	T	$AEG(A)$	$AEG(S)$	$W(A)$	$W(S)$	$J_{\max}(A)$	$J_{\max}(S)$	$J_{trace}(A)$	$J_{trace}(S)$
0.5	0.5	50	0.372	0.413	0.182	0.190	0.308	0.156	0.343	0.158
		100	0.254	0.383	0.172	0.253	0.186	0.181	0.173	0.162
		200	0.105	0.190	0.084	0.228	0.184	0.202	0.160	0.172
	1.0	50	0.166	0.171	0.124	0.074	0.263	0.089	0.296	0.095
		100	0.123	0.141	0.094	0.076	0.083	0.077	0.086	0.079
		200	0.090	0.121	0.077	0.078	0.073	0.072	0.072	0.070
	2.0	50	0.112	0.107	0.121	0.065	0.258	0.084	0.294	0.096
		100	0.078	0.074	0.080	0.057	0.066	0.062	0.075	0.069
		200	0.061	0.061	0.061	0.051	0.060	0.059	0.062	0.060
1.0	0.5	50	0.708	0.794	0.401	0.557	0.511	0.425	0.513	0.387
		100	0.431	0.632	0.330	0.673	0.584	0.606	0.518	0.531
		200	0.231	0.389	0.186	0.433	0.497	0.700	0.458	0.629
	1.0	50	0.380	0.416	0.189	0.193	0.312	0.157	0.343	0.158
		100	0.256	0.390	0.169	0.251	0.185	0.180	0.171	0.161
		200	0.109	0.189	0.088	0.233	0.189	0.210	0.160	0.174
	2.0	50	0.172	0.175	0.126	0.075	0.259	0.091	0.293	0.102
		100	0.122	0.138	0.091	0.076	0.079	0.073	0.084	0.076
		200	0.088	0.120	0.075	0.077	0.070	0.068	0.072	0.070
2.0	0.5	50	0.898	0.964	0.665	0.886	0.776	0.768	0.731	0.681
		100	0.774	0.902	0.683	0.936	0.929	0.945	0.897	0.911
		200	0.596	0.758	0.506	0.832	0.889	0.971	0.876	0.959
	1.0	50	0.720	0.803	0.408	0.562	0.510	0.429	0.498	0.379
		100	0.440	0.641	0.333	0.673	0.589	0.611	0.523	0.535
		200	0.235	0.395	0.183	0.434	0.497	0.694	0.463	0.623
	2.0	50	0.384	0.422	0.194	0.198	0.319	0.158	0.350	0.161
		100	0.255	0.381	0.167	0.241	0.181	0.174	0.168	0.156
		200	0.103	0.188	0.086	0.227	0.185	0.205	0.162	0.177

σ_ω	σ_{η_k}	T	$J^{RA}_{\max}(A)$	$J^{RA}_{\max}(S)$	$J^{RA}_{trace}(A)$	$J^{RA}_{trace}(S)$
0.5	0.5	50	0.082	0.075	0.064	0.055
		100	0.135	0.136	0.107	0.105
		200	0.157	0.178	0.127	0.143
	1.0	50	0.057	0.038	0.048	0.029
		100	0.052	0.049	0.046	0.043
		200	0.060	0.059	0.052	0.051
	2.0	50	0.056	0.034	0.049	0.027
		100	0.043	0.041	0.042	0.039
		200	0.048	0.047	0.047	0.046
1.0	0.5	50	0.240	0.278	0.171	0.194
		100	0.522	0.549	0.417	0.436
		200	0.466	0.679	0.415	0.594
	1.0	50	0.081	0.073	0.065	0.057
		100	0.135	0.136	0.108	0.106
		200	0.162	0.185	0.126	0.142
	2.0	50	0.057	0.039	0.047	0.029
		100	0.049	0.046	0.045	0.041
		200	0.057	0.057	0.051	0.050
2.0	0.5	50	0.516	0.618	0.381	0.450
		100	0.909	0.930	0.853	0.873
		200	0.875	0.968	0.854	0.951
	1.0	50	0.239	0.277	0.169	0.191
		100	0.527	0.553	0.421	0.438
		200	0.468	0.672	0.418	0.588
	2.0	50	0.087	0.076	0.070	0.059
		100	0.130	0.130	0.102	0.099
		200	0.160	0.182	0.130	0.147