Optimal Utilitarian Taxation and Horizontal Equity*

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Abstract

We impose a horizontal equity restriction on the problem of finding the optimal utilitarian tax mix. The horizontal equity constraint requires that individuals with the same ability have to pay the same amount of taxes regardless of their preferences for leisure. Contrary to normal findings, we find that a good that is complementary to leisure need not be discouraged by the tax system, and that a good that normally should be discouraged by the tax system need not be taxed at a positive rate even if the economy is composed of only two private commodities plus leisure. Similarly, the marginal effective tax rate need not be equal to zero at the top when the tax mix obeys the horizontal equity constraint.

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1 Introduction

A well-known problem with income taxes is that they punish hard-working people. Nozick (1974) for example asks why somebody who prefers looking at the sunset should pay less taxes than somebody who has to earn money in order to attain his pleasures. This question is not only important on its own normative ground, but also because tax systems that violate general conceptions of equity will be replaced if enough citizens call them into question.

As a matter of fact, such questions of equity have to a large extent been neglected in the optimal taxation literature, where one of the standard assumptions is that all individuals share the same preferences.[1] At the same time, philosophers and social choice scholars have been investigating redistributive schemes where individuals are held responsible for certain inequalities.[2] In particular, it is often advocated that an individual ought to bear the consequences of the characteristics which he has chosen himself. This line of reasoning, which originates from Dworkin (1981a, b), is especially relevant for optimal income taxation if the utility of leisure is heterogeneous across individuals. In such case, the government may not want to compensate people for income differences that are due to differences in tastes. However, since it is generally assumed that the government can only observe the income of an individual, it is impossible to find an income tax scheme that only compensates for differences in abilities. Indeed, in the public debate it is frequently pointed out that transfers to hard-working low-skilled persons are also benefiting more highly skilled but also more epicurean individuals. In the eyes of the government, they are alike since their pre-tax incomes are similar. We investigate if and how the government can use the tax instruments typically

[1] Possible exceptions are provided by Cuff (2000) and Boadway *et al.* (2002) for the finite case, while Tarkiainen and Tuomala (1999) develop a computational approach to tackle the problem of two-dimensional population in the continuous case. All the quoted authors neglect the problem of the optimal structure of commodity taxation and work with models where leisure is additive separable from other consumption goods. Sandmo (1993) examines the utilitarian case for a linear income tax under the assumption that differences in earnings are explained by differences in preferences over work and consumption; he also has a brief section in which both market abilities and preferences for leisure are allowed to vary. Ebert (1988) provides conditions on preference orderings and utility functions which allow us to transform the problem of optimal (utilitarian) income taxation for a two-dimensional population into a one-dimensional problem.

[2] See Fleurbaey and Maniquet (2002) for a review.

observed in developed countries—linear commodity taxes and a non-linear income tax—to escape this dilemma.

Related to the principle of responsibility for certain inequalities is the horizontal equity principle of equal treatment of equals. Indeed, an interpretation of the horizontal equity principle is that if two individuals differ only in tastes, then the government ought to treat them identically. The literature contains several suggestions of the status and definition of horizontal equity.[3] Musgrave (1959) argues that in the ability-to-pay approach to taxation, "horizontal and vertical equity are but different sides of the same coin." However, there are several reasons for taxing people with the same ability differently. Besides the conflicts arising from the government's lack of information, Stiglitz (1982) demonstrates that the horizontal equity requirement does not follow from the maximization of a traditional utilitarian or more general social welfare function (which does not consider relations between individual outcomes), and, more strongly, that it may also be inconsistent with Pareto optimality. In a recent contribution, Kaplow and Shavell (2001b) prove formally that any non-welfarist method of policy assessment,[4] such as the concern for horizontal equity, violates the Pareto principle.

In view of this expected conflict between horizontal and vertical equity, it is often argued that the former should take precedence over the latter. In line with this, Atkinson and Stiglitz (1976, 1980) suggest the imposition of a horizontal equity constraint on the maximization

[3] The relevance of this concept has recently been questioned in a series of articles by Kaplow (1989, 1995, 2000) and Kaplow and Shavell (2000, 2001a), where it is claimed that usually provided indices of deviations from horizontal equity are developed without knowing what they should try to measure and why, but are merely stipulated or supported by ad hoc appeals to intuition. Moreover, Kaplow refers to and seems to share Westen's (1982) view, according to which "Equality will cease to mistify—and cease to skew moral and political discourse—when people come to realize that it is an empty form having no substantive content of its own. (...) The endurance of the principle of equality—that likes should be treated alike—is due to the fact that it is empty of content. For the principle to have meaning, it must incorporate some external values, but once these external values are found, the principle of equality is superfluous." According to these authors, to pay attention to the unequal treatment of equals can be useful at most from a practical perspective. Even though the measures offered by horizontal equity indices are not of independent normative significance, they can be useful in order to alert about circumstances in which something is amiss and social welfare, as the notion is conventionally understood, is reduced.

[4] The term "non-welfarist" refers to any conception of social welfare that gives weight to factors other than the satisfaction of the individuals' preferences.

of a social welfare function.[5] Feldstein (1976) instead suggests to balance the fulfilment of horizontal equity against the utilitarian principle of welfare maximization. Regarding the definition of horizontal equity, the proposed measures are as a rule either based on tax payments or on utilities. Still Johnson and Mayer (1962) emphasize the number of inequities, though this measure could be weighted by the corresponding differences in tax payments. Atkinson and Stiglitz (1976, 1980) refer to Pigou, who observed that persons with different tastes who pay the same amount of taxes will not necessarily suffer equal burdens in terms of foregone utility. Thus they argue that commodity taxes should be set to maintain the parity of utilities for persons who, according to the government's value judgments, attained equal utility before the imposition of taxes. Feldstein (1976) on his part proposes measures based on actual utilities as experienced by the individuals. His measures include the after-tax variance of utilities for people who attained equal utilities before taxes were levied. Plotnick (1981) compares "preordered" and standard Lorenz curves and derives an index which is sensitive to the magnitude of the changes in income rankings produced by redistribution. Auerbach and Hassett (1999) propose a new income-based measure, suitable for applied work. Their idea is to incorporate horizontal equity in a social welfare function by weighting each inequity between two groups by the distance (e.g. in pre-tax income) between the groups in a way that has been used in the econometric literature on kernel density functions. Other articulations of the concept of horizontal equity include Rosen (1978), King (1983), Berliant and Strauss (1985), and Balcer and Sadka (1986).

In this paper we stay close in spirit to the interpretation of the concept given by Bossert (1995) in terms of "equal transfers for equal circumstances",[6] and require that individuals of the same ability must pay the same amount of taxes irrespective of their preferences.[7] This

[5] Actually, as recognized by Kaplow (1989), this approach helps avoiding at least some of the objections related to the concept of horizontal equity.

[6] This terminology comes from the division of the sources of individual outcomes into wills, resources and circumstances. According to this division, the individual is responsible for his wills, whereas the circumstances are factors outside his control. Differences in circumstances can be compensated by reallocating the resources.

[7] A similar argument was put forward by Allingham (1974) who also recognizes that the idea that taxation

can be justified by the observation that people with the same ability share the same opportunity set, and while differences in this set can in some moral sense be deemed "irrelevant" and therefore call for compensation, differences in preferences may be regarded as morally "relevant", suggesting that compensation is ruled out for such differences. According to this reasoning the individuals are fully responsible for their preferences.

Our approach is to introduce the principle of equal transfers for equal circumstances as a constraint on the maximization of a utilitarian social welfare function. Although we have to admit that the choice of a tax-based rather than a utility-based measure is to some extent arbitrary, it is simple and also sufficient for focusing on the moral difficulties raised by the fact that the government can only observe income differences and not differences in abilities or preferences.[8]

We find that the imposition of the horizontal equity requirement modifies the rule for optimal commodity taxes. Contrary to normal findings a good that is complementary to leisure need not be discouraged by the tax system, and perhaps more peculiar, a good that should be discouraged by the tax system in the absence of the horizontal equity condition need not be taxed at a positive rate once this condition is imposed, even if the economy is composed by only two private commodities plus leisure.[9] In essence the trade-off between the effects on revenue from the commodity tax and the desire to encourage individuals to reveal their true characteristics is amended by the requirement to uphold horizontal equity. When this requirement is taken into account the popular prescription to loosen the incentive compatibility constraint by taxing goods that are complementary to leisure is only one part of the story. We also derive effective marginal tax rates for individuals with different

should be just in the sense of depending on ability (but not on preferences) may be traced at least as far back as Guicciardini and Bodin in the sixteenth century.

[8] The policies analysed in this paper differ in two important respects from policies that can be derived from conceptions of justice based on equality of opportunity. In contrast to what is suggested by Roemer (1998, 2002) the government in our model does not seek to equalize outcomes for comparable people with different abilities.

[9] It is easy to show that in the standard optimal taxation problem with two private commodities plus leisure, where the indirect tax structure collapses to the definition of only one commodity tax rate, the concept of discouragement (encouragement) becomes the same as "being taxed at a positive rate" ("being subsidized").

characteristics and compare them with the tax rates derived in ordinary optimal taxation models. Also in this case we find that ordinary prescriptions have to be amended in order to satisfy the horizontal equity principle; particularly interesting, the popular result of having no distortion at the top (of the skill distribution) can be violated.

2 The Model

In our model economy there are three goods (two private consumption goods c and z plus leisure), and three types of individuals. The individuals are characterized by their skill or ability (w^H or w^L) (reflected, by assumption of perfect competition, in the unitary wage rate they are paid) and by their taste for leisure (α^H or α^L), where superscript H (L) denotes a high (low) ability or taste for leisure. There are π^1 low skilled, low taste for leisure individuals (type 1 with w^L and α^L), π^2 high skilled, high taste for leisure individuals (type 2 with w^H and α^H), and π^3 high skilled, low taste for leisure individuals (type 3 with w^H and α^L). Preferences are represented by the utility function $u(c, z, \alpha^i l)$, where α^i is the particular preference parameter of an individual of type i and l is the supply of labor.

Production is linear and uses labor as the only input; units are chosen to make all producer prices equal to one, good z is chosen as *numéraire* and is set untaxed, so that consumer prices are represented by the vector $(1+t, 1) = (q, 1)$. In addition to the commodity tax, t, the individuals also have to pay a non-linear tax $T(Y)$ on income Y. Thus disposable income B equals $Y - T(Y)$ and the total tax liability amounts to $\tau(Y) = T(Y) + tc$. By using the relation $\frac{Y}{w} = l$, the indirect utility of type i is $V^i\left(q, B^i, \frac{\alpha^i}{w^i}Y^i\right)$, where the superscript on the indirect utility function is for notational convenience only (all types share the same utility function).[10] Henceforth $\left(\frac{\alpha}{w}\right)^{(i)}$ will denote the ratio of the preference parameter to the productivity parameter for the representative individual of type i. The indirect utility function has the following properties: $V_q < 0$, $V_B > 0$, $V_3 < 0$ (the subscripts denote

[10] For given B and Y the conditional indirect utility $V(q, B, Y, w, \alpha)$ is defined as $\max_{c,z}\left\{u\left(c, z, \frac{\alpha}{w}Y\right) \mid qc + z = B\right\}$; optimizing agents will then maximize their own $V(q, B, Y, w, \alpha)$ subject to the link between pre-tax earnings and post-tax earnings available for goods expenditure implied by the direct tax schedule.

partial derivatives; in particular, V_3 denotes the partial derivative with respect to the third argument). In order to satisfy the single-crossing condition (indifference curves cross only once), we will also assume $V_{33} < 0$ (labor is annoying at increasing rates) and $V_{B3} < 0$ (an increase in private consumption is valued more, the less "experienced hours" (αl) the person is working, i.e. normality of private consumption and "experienced" leisure).

To establish that single crossing holds, we will calculate the slopes of the indifference curves in (pre-tax income, disposable income)-space, henceforth referred to as (Y,B)-space, of the three types. The slope of such an indifference curve is given by

$$\left.\frac{\partial B}{\partial Y}\right|_{V^i=k} = -\left(\frac{\alpha}{w}\right)^{(i)} \frac{V_3\left(q, B, \left(\frac{\alpha}{w}\right)^{(i)} Y\right)}{V_B\left(q, B, \left(\frac{\alpha}{w}\right)^{(i)} Y\right)}. \tag{1}$$

It turns out that the slopes of the indifference curves can be ranked according to the ratio of the preference parameter α to ability w. Comparing two types i and j, the slope of the indifference curves of type i are steeper than those of type j if

$$-\left(\frac{\alpha}{w}\right)^{(i)} \frac{V_3\left(q, B, \left(\frac{\alpha}{w}\right)^{(i)} Y\right)}{V_B\left(q, B, \left(\frac{\alpha}{w}\right)^{(i)} Y\right)} > -\left(\frac{\alpha}{w}\right)^{(j)} \frac{V_3\left(q, B, \left(\frac{\alpha}{w}\right)^{(j)} Y\right)}{V_B\left(q, B, \left(\frac{\alpha}{w}\right)^{(j)} Y\right)}. \tag{2}$$

With the assumptions $V_{33} < 0$ and $V_{B3} < 0$, this inequality is satisfied if $\left(\frac{\alpha}{w}\right)^{(i)} > \left(\frac{\alpha}{w}\right)^{(j)}$. Since, at every ($Y$,$B$)-bundle, the type specific ratio $\frac{\alpha}{w}$ determines the slope of the indifference curve for each type of individual, the indifference curves of two individuals of different types can cross only once.

2.1 A Comparison with Related Models

Compared with the related models developed by Cuff (2000) and Boadway et al. (2002), the distinguishing feature of our model is the introduction of an additional, taxable commodity. Cuff uses a model with three types of individuals and a two goods economy (private consumption plus leisure), where high skilled individuals have low taste for leisure, while there are low skilled individuals with both high and low taste for leisure. In order to make the results comparable to those of Besley and Coate (1992, 1995), she uses individual utility functions that are affine in consumption.

Boadway et al. (2002) instead use a model with four types of individuals (and the same two goods economy), in which low skilled individuals with low taste for leisure are indiscernible from the new type of high skilled individuals with high taste for leisure. The individuals' utility functions are quasi-linear, but in their case affine in labor. More precisely, they assume $U = u(z) - \alpha^i l$, and $\frac{\alpha^L}{w^L} = \frac{\alpha^H}{w^H}$. Thus the "intermediate" types are indistinguishable because they share the same map of indifference curves in (Y,B)-space.

Dealing with a two goods model (consumption and leisure), the quoted papers are confined to studying the shape of the optimal income tax schedule and cannot examine the potential role of commodity taxation. Notice however that if $\frac{\alpha^L}{w^L} = \frac{\alpha^H}{w^H}$ commodity taxes could not be employed in order to screen between low skilled, low taste for leisure individuals and high skilled, high taste for leisure ones even if individual utility functions are not separable between leisure and other goods.

Suppose now, as in our model, that we have an economy composed of three goods (two private consumption goods c and z plus leisure), where there are three types of individuals: type 1 with low ability and low taste for leisure, type 2 with high ability and high taste for leisure, and type 3 with high ability and low taste for leisure. Suppose also, as in Boadway et al. (2002), that type 1 and type 2 are indiscernible because $\frac{\alpha^L}{w^L} = \frac{\alpha^H}{w^H}$, and that they share the same map of indifference curves in (Y,B)-space. Finally, assume as we proposed in the introduction, that the concern for horizontal equity (hereafter HE) translates into the requirement that the total tax liability of individuals of type 2 and type 3 must be the same, and that the social planner maximizes a utilitarian social welfare function subject to the self-selection constraints (stating that no individual would gain by masquerading as an individual of a different type), and a budget-balance constraint. In this situation we can have two cases, depending on whether taxation is merely redistributive or if the government has an exogenous amount of expenditure to finance. In the former case the solution involves no taxation at all and the optimal outcome is the laissez-faire one. This is due to the fact that, since there is necessarily bunching[11] between type 1 and type 2, the HE requirement $\tau^2 = \tau^3$ implies that we must have $\tau^1 = \tau^2 = \tau^3$. Everybody must pay the same amount of taxes and, since there is no public expenditure to finance, no taxation is involved. In the latter case, suppose that the government has to collect a fixed amount of revenue. Following the same reasoning as before, each type of individuals must pay the same amount of taxes. Since income and commodity taxation are distortive and cause deadweight losses, the optimal policy is a uniform lump sum tax without any excess burden.

As we mentioned above, Boadway et al. (2002) are concerned with the limit situation where high skilled individuals with high taste for leisure cannot be distinguished from low skilled individuals with low taste for leisure. If we relax this assumption, two different cases are possible:

$$\begin{aligned} \text{assumption 1} \quad &: \qquad \frac{\alpha^L}{w^L} < \frac{\alpha^H}{w^H}, \\ \text{assumption 2} \quad &: \qquad \frac{\alpha^L}{w^L} > \frac{\alpha^H}{w^H}. \end{aligned}$$

Suppose first that assumption 1 holds; then we face the following chain of inequalities:

$$\frac{\alpha^L}{w^H} < \frac{\alpha^L}{w^L} < \frac{\alpha^H}{w^H}.$$

This means that at every point in (Y,B)-space, the slope of the indifference curve of a low skilled, hard working individual is shallower than the one of a high skilled, epicurean individual, and that for this pair of individuals the ordinary ranking of the indifference curves based on their slopes is reversed.

If instead assumption 2 holds, the chain of inequalities is the following:

[11] According to Weymark (1986), bunching is said to occur if individuals with different characteristics receive the same commodity bundle. In fact as long as $\frac{\alpha^L}{w^L} = \frac{\alpha^H}{w^H}$, individuals of type 1 and of type 2 will not only receive the same bundle in (Y,B)-space, but they will also spend their disposable income across goods in exactly the same way, and this is true even if preferences are not separable between leisure and other goods.

$$\frac{\alpha^L}{w^H} < \frac{\alpha^H}{w^H} < \frac{\alpha^L}{w^L}.$$

This case reflects more closely the standard one since there is no individual with high ability that has indifference curves in (Y,B)-space that are steeper than the ones of the low skilled individuals. The ordinary ranking of the indifference curves that one gets in a one-dimensional model persists. Since some of the mechanisms in the model are only present under assumption 2, and also since this assumption is perhaps more realistic, we will only present the solution of the model under this assumption. This omission of assumption 1 saves space without impairing the reader's understanding of the model because the results under the two assumptions are very similar.

2.2 Some Considerations about the Pattern of the Binding Self-selection Constraints

In conventional optimal taxation models, the inability to observe the types of the individuals raises a familiar problem. The government may wish to redistribute resources from high skilled to low skilled individuals (since utility normally increases with the wage rate). Not knowing who is who, however, all it can do is to tax higher incomes more heavily than lower incomes. This may create an incentive for a high skilled individual to reduce his labor supply and earn the same gross income as a low skilled individual. Thus, having imposed the single crossing condition, the binding self-selection constraint that thwarts the government in its attempts to redistribute among individuals runs downwards from high skilled (high earning) individuals towards low skilled (low earning) ones. In a finite-class economy this is generalized by saying that an optimal allocation results in a simple monotonic chain to the left (Guesnerie and Seade, 1982), which means that each pair of successive bundles are L-linked[12] by a downwards binding incentive-compatibility constraint. However, as long as

[12] Using the terminology of Guesnerie and Seade (1982), a corner (or chosen bundle) is linked to another if they both belong to the optimal set of some individual h; or equivalently if there is an indifference curve of h which passes through both corners and is the highest h can reach on his budget set. Individual h is said to link these corners. A corner C_i is W-linked (W for winner) if some h links C_i to some other corner C_j, and is allocated C_i. A corner C_i is L-linked (L for loser) if some h links C_i to C_j, and is allocated C_j.

individuals differ both according to their market ability and according to their preferences for leisure, this is no longer necessarily true even if (as in our case) the single crossing condition still holds. We cannot tell *a priori* which one of the pair of self-selection constraints that is going to be binding. Under certain circumstances, both constraints could even be binding at the same time (generating what Brito et al. (1990) call a "self-selection cycle"). Notice that the mentioned properties are a common feature of all models that introduce heterogeneity along more than one dimension (see Balestrino, Cigno, and Pettini, 1999, 2002; Cremer, Pestieau, and Rochet, 2001). Finally, notice also that although the single crossing condition holds in our model, it will not generally do so in models with more than one differentiating characteristic of the individuals.

3 The Optimal Tax Mix

In this section we will solve the model for the optimal tax mix[13] for a utilitarian planner under the case where the "ordinary" ranking holds[14] (i.e. assumption 2 in the previous section). In order to untangle the different mechanisms in the model we will first present the results that we obtain without imposing the HE constraint. Then we go on to present the results of the full model including this constraint.

3.1 Without the Horizontal Equity Constraint

Without the HE constraint the planner's problem is the following:

$$\max_{B^1,B^2,B^3,Y^1,Y^2,Y^3,t} \pi^1 V^1\left(q, B^1, \frac{\alpha^L}{w^L}Y^1\right) + \pi^2 V^2\left(q, B^2, \frac{\alpha^H}{w^H}Y^2\right) + $$

$$+\pi^3 V^3\left(q, B^3, \frac{\alpha^L}{w^H}Y^3\right), \tag{3}$$

[13] In accordance with standard practice in the optimal taxation literature, we will simply assume that a solution exists and characterize the optimal tax mix conditional on this assumption.

[14] The same procedure can also be followed to deal with the case where the ranking is transposed.

subject to the budget constraint:[15]

$$\pi^1 \left(Y^1 - B^1 + tc^1\right) + \pi^2 \left(Y^2 - B^2 + tc^2\right) + \pi^3 \left(Y^3 - B^3 + tc^3\right) \geq 0, \qquad (\gamma)$$

and the following self-selection constraints:[16]

$$V^2 \left(q, B^2, \frac{\alpha^H}{w^H} Y^2\right) \geq \widehat{V^{2(1)}} \left(q, B^1, \frac{\alpha^H}{w^H} Y^1\right), \qquad \left(\lambda_2^d\right)$$

$$V^3 \left(q, B^3, \frac{\alpha^L}{w^H} Y^3\right) \geq \widehat{V^{3(2)}} \left(q, B^2, \frac{\alpha^L}{w^H} Y^2\right), \qquad \left(\lambda_3^d\right)$$

where a "hat" above the function V^i indicates that the indirect utility is evaluated at a point where type i is mimicking another type and we follow the convention to denote by $\widehat{V^{j(i)}}$, the indirect utility of an individual of type j trying to masquerade as an individual of type i. The subscripts on the Lagrange multipliers indicate the type of the potential mimicker, and the superscripts indicate the direction of the incentive compatibility constraint: "u" for upwards and "d" for downwards (according to the ranking given by the slopes of the indifference curves). Since single crossing holds, we only need to take the self-selection constraints linking pairs of adjacent individuals into account.

Due to the way we have chosen to represent heterogeneity in tastes, the model can also be interpreted as one with three types of agents, all with the same utility function but with three different market abilities, reflected in the unitary competitive wage they are paid. Since our utilitarian solution belongs to the family of Pareto efficient solutions with redistribution from high- to low-skilled agents, it must share the properties of the more general solutions found in the papers by Guesnerie and Seade (1982) and Edwards et al. (1994).

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[15] We are assuming that taxation serves a merely redistributive purpose.

[16] In shaping the self-selection constraints we are implicitly exploiting the circumstance that the utilitarian solution belongs to the family of "normal cases", i.e. entails redistribution from high- to low skilled agents.

3.1.1 The Indirect Tax Structure

By applying Roy's identity, making use of the first order conditions for B^1, B^2, and B^3, and also the Slutsky equation, the first order condition for t can be written (see Appendix A):

$$t\sum_{i=1}^{3}\pi^i\frac{\partial\widetilde{c^i}}{\partial q}=\frac{1}{\gamma}\left[\lambda_2^d\widehat{V_B^{2(1)}}\left(c^1-\widehat{c^{2(1)}}\right)+\lambda_3^d\widehat{V_B^{3(2)}}\left(\underbrace{c^2-\widehat{c^{3(2)}}}_{A}\right)\right]. \tag{4}$$

Notice that in eq. (4) the term A inside brackets (which is referred to types of agents that are both high skilled) is non-zero as long as c_3^i, the derivative of the demand of agents of type i for commodity c with respect to the third argument in the individual utility function, is different from zero. On the other hand, $c_3 \neq 0$ simply means that the consumption of the taxed commodity is positively related to labor (if $c_3 > 0$) or negatively related to labor (if $c_3 < 0$).

Without the HE constraint the sign of the right hand side of eq. (4), and therefore of t, is determined once the relation between the taxed commodity and leisure is known. This will be discussed in detail after we have introduced the HE restriction.

3.1.2 The Marginal Effective Tax Rates

Now consider the marginal effective tax rate (METR). Since there are only two commodities in this model and one of them is chosen as *numéraire* and set untaxed, the effective tax rate is defined as

$$\tau(Y)=T(Y)+tc\left[q,Y-T\left(Y\right),\frac{\alpha}{w}Y\right], \tag{5}$$

where $T\left(Y\right)=Y-B$ represents the income tax liability. By differentiation of (5) we get the METR

$$\tau'=T'+t\left[\frac{\partial c}{\partial B}\left(1-T'\right)+c_3\frac{\alpha}{w}\right], \tag{6}$$

As usual we can derive an expression for the marginal income tax rate faced by an individual by considering his optimal choice of labor supply. The first order conditions of the problem max $V\left(q, B, \frac{\alpha}{w}Y\right)$ subject to $B = Y - T(Y)$ entail that the following condition must hold:

$$T' = 1 + \frac{\alpha}{w}\frac{V_3}{V_B}. \tag{7}$$

Substituting (7) into (6) allow us to rewrite the expression for the METR in a more convenient way:

$$\tau' = 1 + tc_3\frac{\alpha}{w} + \frac{\alpha}{w}\frac{V_3}{V_B}\left(1 - t\frac{\partial c}{\partial B}\right). \tag{8}$$

As expected, the METR turns out to be positive for individuals of type 1 and of type 2, but zero for individuals of type 3. This is shown formally in Appendix B.

3.2 With the Horizontal Equity Constraint

Since we have defined HE as a requirement that individuals of the same skill type should pay the same amount of taxes, the formal HE constraint states that

$$Y^3 = t\left(c^2 - c^3\right) + Y^2 - B^2 + B^3, \tag{9}$$

i.e. that total taxes paid by an individual of type 2 equal total taxes paid by an individual of type 3. The planner's problem then becomes:

$$\max_{B^1,B^2,B^3,Y^1,Y^2,t} \pi^1 V^1\left(q, B^1, \frac{\alpha^L}{w^L}Y^1\right) + \pi^2 V^2\left(q, B^2, \frac{\alpha^H}{w^H}Y^2\right) +$$

$$+\pi^3 V^3\left(q, B^3, \frac{\alpha^L}{w^H}\left[t\left(c^2 - c^3\right) + Y^2 - B^2 + B^3\right]\right) \tag{10}$$

subject to the budget constraint:

$$\pi^1\left(Y^1 - B^1 + tc^1\right) + \pi^2\left(Y^2 - B^2 + tc^2\right) + \pi^3\left(Y^2 - B^2 + tc^2\right) \geq 0, \tag{γ}$$

and the following self-selection constraints[17] :

$$V^1\left(q,B^1,\frac{\alpha^L}{w^L}Y^1\right)\geq\widehat{V^{1(2)}}\left(q,B^2,\frac{\alpha^L}{w^L}Y^2\right), \qquad (\lambda_1^u)$$

$$V^2\left(q,B^2,\frac{\alpha^H}{w^H}Y^2\right)\geq\widehat{V^{2(1)}}\left(q,B^1,\frac{\alpha^H}{w^H}Y^1\right), \qquad (\lambda_2^d)$$

$$V^2\left(q,B^2,\frac{\alpha^H}{w^H}Y^2\right)\geq\widehat{V^{2(3)}}\left(q,B^3,\frac{\alpha^H}{w^H}\left[t\left(c^2-c^3\right)+Y^2-B^2+B^3\right]\right), \qquad (\lambda_2^u)$$

$$V^3\left(q,B^3,\frac{\alpha^L}{w^H}\left[t\left(c^2-c^3\right)+Y^2-B^2+B^3\right]\right)\geq\widehat{V^{3(2)}}\left(q,B^2,\frac{\alpha^L}{w^H}Y^2\right). \qquad (\lambda_3^d),$$

where we have substituted the HE constraint into the indirect utility function of type 3 individuals.

Notice that the way we have chosen to incorporate the constraint (9) implies that every variation in B^2, B^3, Y^2 and t must be accompanied by a proper variation in Y^3, the pre-tax income of type 3 individuals, in order to match the HE requirement. By differentiating the HE constraint (9) we get the following useful results:

$$\frac{dY^3}{dB^2}=\frac{t\frac{\partial c^2}{\partial B^2}-1}{1+tc_3^3\frac{\alpha^L}{w^H}}, \qquad (11)$$

$$\frac{dY^3}{dB^3}=\frac{1-t\frac{\partial c^3}{\partial B^3}}{1+tc_3^3\frac{\alpha^L}{w^H}}, \qquad (12)$$

$$\frac{dY^3}{dY^2}=\frac{1+tc_3^2\frac{\alpha^H}{w^H}}{1+tc_3^3\frac{\alpha^L}{w^H}}, \qquad (13)$$

$$\frac{dY^3}{dq}=\frac{c^2-c^3+t\left(\frac{\partial c^2}{\partial q}-\frac{\partial c^3}{\partial q}\right)}{1+tc_3^3\frac{\alpha^L}{w^H}}. \qquad (14)$$

[17] Since single crossing holds, we only need to take the self selection constraints linking pairs of adjacent individuals into account.

3.2.1 The Indirect Tax Structure

Next we will derive the formula for the commodity tax rate that maximizes the planner's problem. Equations (11)–(14) are used when deriving the commodity tax rule. The first order condition of the planner's problem with respect to t is:

$$\pi^1 V_q^1 + \pi^2 V_q^2 + \pi^3 \left\{ V_q^3 + V_3^3 \frac{\alpha^L}{w^H} \left[c^2 - c^3 + t \left(\frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right) \right] \frac{1}{1 + tc_3^3 \frac{\alpha^L}{w^H}} \right\} +$$

$$+\gamma \left[\pi^1 c^1 + \left(\pi^2 + \pi^3 \right) c^2 + t \left(\frac{\partial c^1}{\partial q} \pi^1 + \frac{\partial c^2}{\partial q} \left(\pi^2 + \pi^3 \right) \right) \right] + \lambda_1^u \left(V_q^1 - \widehat{V_q^{1(2)}} \right) + \lambda_2^d \left(V_q^2 - \widehat{V_q^{2(1)}} \right) +$$

$$-\lambda_2^u \left\{ V_q^2 - \widehat{V_q^{2(3)}} - \widehat{V_3^{2(3)}} \frac{\alpha^H}{w^H} \left[c^2 - c^3 + t \left(\frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right) \right] \frac{1}{1 + tc_3^3 \frac{\alpha^L}{w^H}} \right\} +$$

$$+\lambda_3^d \left\{ V_q^3 + V_3^3 \frac{\alpha^L}{w^H} \left[c^2 - c^3 + t \left(\frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right) \right] \frac{1}{1 + tc_3^3 \frac{\alpha^L}{w^H}} - \widehat{V_q^{3(2)}} \right\} = 0. \tag{15}$$

If we consider the "normal" case when redistribution goes from high- to low skilled individuals we are able to show that an optimum is not compatible with the Lagrange multiplier λ_1^u being different from zero. The argument proceeds along the same lines as in Proposition 1 in Brito et al. (1990), which states that at any efficient allocation, individuals of one type will always view the bundles of individuals of other types who have a larger total tax liability as strictly inferior to their own. To show that a similar result holds also in our model, suppose that there is a solution to the planner's problem such that the constraint λ_1^u is binding. Then the planner could improve upon the suggested allocation by simply letting the low skilled agents reach the point intended for those whom they are willing to mimic. Since redistribution is assumed to go from high- to low skilled individuals, this would imply that the low skilled agents would switch from a bundle where the total tax liability is negative (they are net receivers) to a bundle where it is positive. Leaving the value of the maximand of the planner's problem unaffected (since the low skilled agents were supposed to be indifferent between the two points), such a switch would be socially desirable since it weakens the budget constraint of the planner while not tightening the incentive-compatibility constraints. Notice in particular that we couldn't have invoked something similar to Proposition 1 by Brito et al. (1990) if the imposition of the additional (HE) constraint to the standard problem of a utilitarian planner had reversed the direction of redistribution among agents. But it is easy to see that this can never happen since then, in a purely redistributive context, the objective to maximize the sum of utilities of different agents would be better achieved by an optimal tax policy involving no taxation at all (*laissez-faire* outcome). The nature of what we called the HE requirement does not change the circumstance that, with a utilitarian objective function, if some kind of fiscal policy is in place, then low skilled agents pay a strictly lower amount of taxes than high skilled ones.

On the other hand it is not possible to avoid taking the other self selection constraints into account. In particular, and contrary to what would have happened hadn't we imposed the HE restriction, we cannot disregard the constraint λ_2^u. This is because individuals belonging to type 2 and 3 are adjacent and since we require they should pay the same total tax liability, it is possible that either of them would like to mimic the other type. Notice however that the HE constraint (9) rules out bunching of these two types of high skilled individuals. In fact, bunching would mean that they earn the same gross income and have the same income tax liability but, in order to pay the same total taxes, it would be necessary that they also pay the same amount of commodity taxes and this could happen only in the special case when $c_3 = 0$. It also follows that, apart from this special case, constraints λ_2^u and λ_3^d cannot be binding at the same time.

By applying Roy's identity, making use of the first order conditions for B^1, B^2 and B^3 and also the Slutsky equation (see Appendix A), eq. (15) can be written:

$$\gamma t \left[\pi^1 \frac{\partial \widetilde{c^1}}{\partial q} + \left(\pi^2 + \pi^3\right) \frac{\partial \widetilde{c^2}}{\partial q} \right] + \left(\pi^3 + \lambda_3^d\right) V_3^3 \frac{\alpha^L}{w^H} \frac{t}{1 + t c_3^3 \frac{\alpha^L}{w^H}} \left(\frac{\partial \widetilde{c^2}}{\partial q} - \frac{\partial \widetilde{c^3}}{\partial q} \right) +$$

$$-\lambda_2^u \widehat{V_3^{2(3)}} \frac{\alpha^H}{w^H} \frac{t}{1 + t c_3^3 \frac{\alpha^L}{w^H}} \left(\frac{\partial \widetilde{c^2}}{\partial q} - \frac{\partial \widetilde{c^3}}{\partial q} \right) =$$

$$= \lambda_2^d \widehat{V_B^{2(1)}} \left(c^1 - \widehat{c^{2(1)}} \right) + \lambda_2^u \widehat{V_B^{2(3)}} \left(c^3 - \widehat{c^{2(3)}} \right) + \lambda_3^d \widehat{V_B^3} \left(c^2 - \widehat{c^{3(2)}} \right). \tag{16}$$

Let's try to explain the difference between our eq. (16) and the structure one usually gets when deriving optimal rules for commodity taxation, i.e. eq. (4). Basically, the only differences between eq. (16) and eq. (4) are found on the left-hand sides.[18] The standard formula for optimal commodity taxation (in a framework where it supplements a non-linear income tax schedule) balances the gains of weakening the self-selection constraints against the effects on revenue from a marginal increase in the commodity tax rate, whilst the revenue effect is evaluated only indirectly through the change in the pattern of hicksian commodity demand since the marginal increase in the tax rate is performed in a compensated way. Having this in mind it becomes easy to interpret eq. (16). Inside the square brackets, $\pi^3 \frac{\partial \tilde{c^2}}{\partial q}$ replaces $\pi^3 \frac{\partial \tilde{c^3}}{\partial q}$ since the variables referring to individuals of type 3 are replaced by the HE constraint in the government's budget constraint.

The second term on the left-hand side of eq. (16) measures the impact of the compensated increase in the commodity tax rate on the indirect utility of individuals of type 3. The impact is socially evaluated, i.e. weighted by the numerosity of type 3 individuals, and the magnitude of the Lagrange multiplier of the constraint that prevents individuals of this type from mimicking individuals of type 2. Moreover, the impact differs from zero since the substitution of the HE constraint into the social objective function implies adjustments in the gross income of the high skilled and hard working type 3 individuals to accommodate every variation in one of the variables entering the indirect utility function of the high skilled and more epicurean type 2 individuals. The last term on the left-hand side of eq. (16) provides an evaluation of the effect generated by a compensated increase in t in terms of weakening (or tightening) the (eventually) binding incentive-compatibility constraint requiring that agents of type 2 are not tempted to mimic agents of type 3.

It should be clear that the last two terms on the left-hand side of eq. (16) appear as a

[18] Actually, the difference between the right-hand sides of eq. (16) and (4) also concerns the pattern of the binding self-selection constraints (but not the number since, as we previously noticed, the constraints λ_2^u and λ_3^d in (16) cannot be binding at the same time).

consequence of the HE requirement. The intuition is that when we perform the "standard" experiment of marginally increasing by dq the commodity tax rate while at the same time reducing by $dT^i = -c^i dq < 0$, $i = 1, 2, 3$ the income tax liabilities of the three types of individuals at their original earnings (i.e. in a compensated way), the need to meet the HE constraint requires adjustments in the gross income of individuals of type 3 that make the reform non-neutral for their welfare. The "residual" net effect on their welfare is given by $dV^3 = V_3^3 \frac{\alpha^L}{w^H} \left[t \left(\frac{\partial \widetilde{c^2}}{\partial q} - \frac{\partial \widetilde{c^3}}{\partial q} \right) \right] \frac{1}{1+tc_3^3 \frac{\alpha^L}{w^H}} dq$ (see eq. (20)). However, this "residual" does not only affect the welfare of the high skilled, low taste for leisure individuals, but will also have an impact on the welfare that individuals belonging to different types would obtain by mimicking individuals of type 3. This is relevant since in principle nothing prevents that high skilled, high taste for leisure individuals can like to hide their true type and select the bundle intended for the high skilled, low taste for leisure ones. Therefore, if the Lagrange multiplier λ_2^u is different from zero, the described tax change affects the indirect utility of an individual of type 2 who is mimicking an individual of type 3 not only by the standard channel working through the different way in which the mimicker spend income across goods ($\widehat{V_B^{2(3)}} \left(c^3 - \widehat{c^{2(3)}} \right)$), but also by the adjustment of the mimickers income that is required to restore the HE constraint, and it is precisely this latter effect that is captured by the term $\widehat{V_3^{2(3)}} \frac{\alpha^H}{w^H} \frac{t}{1+tc_3^3 \frac{\alpha^L}{w^H}} \left(\frac{\partial \widetilde{c^2}}{\partial q} - \frac{\partial \widetilde{c^3}}{\partial q} \right)$.

The just mentioned intuitive explanation can be analytically restated in the following way. Consider the effects of a small increase dq in the tax rate on commodity c accompanied by reductions $dT^i = -c^i dq < 0$, $i = 1, 2, 3$, in the income tax liabilities of the three types of individuals at their original earnings. This reform has no effect on the welfare of individuals of either type 1 or 2 since by use of Roy's identity:

$$dV^i = V_q^i dq + V_B^i dB^i = -V_B^i (c^i dq + dT^i) = -V_B^i \left(c^i - c^i \right) dq = 0, \qquad i = 1, 2. \tag{17}$$

Now look at the impact of this "compensated" reform on the welfare of the individuals belonging to type 3:

$$
\begin{aligned}
dV^3 &= V_q^3 dq + V_B^3 dB^3 + V_3^3 \frac{\alpha^L}{w^H}\left[c^2 - c^3 + t\left(\frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q}\right)\right]\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}dq + \\
&\quad + V_3^3\frac{\alpha^L}{w^H}\left(1 - t\frac{\partial c^3}{\partial B^3}\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}dB^3 + V_3^3\frac{\alpha^L}{w^H}\left(t\frac{\partial c^2}{\partial B^2} - 1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}dB^2.
\end{aligned} \tag{18}
$$

Substituting $dB^2 = c^2 dq$ and $dB^3 = c^3 dq$ into eq. (18) and making use of Roy's identity, one gets

$$
dV^3 = V_3^3 \frac{\alpha^L}{w^H}\left[t\left(\frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q}\right) + t\left(c^2\frac{\partial c^2}{\partial B^2} - c^3\frac{\partial c^3}{\partial B^3}\right)\right]\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}dq, \tag{19}
$$

which by use of the Slutsky equation can be written

$$
dV^3 = V_3^3 \frac{\alpha^L}{w^H}\left(\frac{\partial \widetilde{c^2}}{\partial q} - \frac{\partial \widetilde{c^3}}{\partial q}\right)\frac{t}{1+tc_3^3\frac{\alpha^L}{w^H}}dq. \tag{20}
$$

We can refer to (20) as to the non-sterilized effect on utility of a compensated (in the standard meaning of the term) increase in the commodity tax rate that arises because of the requirement to uphold the HE constraint. In order to evaluate how the indirect utility of an agent of type two who mimics an agent of type 3 is affected by a compensated increase in t, we must also consider the quantity

$$
d\widehat{V^{2(3)}} = \widehat{V_3^{2(3)}} \frac{\alpha^H}{w^H}\left(\frac{\partial \widetilde{c^2}}{\partial q} - \frac{\partial \widetilde{c^3}}{\partial q}\right)\frac{t}{1+tc_3^3\frac{\alpha^L}{w^H}}dq, \tag{21}
$$

which is related to the change in labor supply needed to pick up the point intended for agents of type 3.

Denoting by $d\widetilde{V^3}$ and $d\widetilde{\widehat{V^{2(3)}}}$ (where the "tildes" help to remember that the effects are produced by "compensated" marginal variations in the commodity tax rate) the quantities $\frac{dV^3}{dq}$ and $\frac{d\widehat{V^{2(3)}}}{dq}$ from eq. (20) and (21), we can rewrite eq. (16) as

$$
\gamma t\left[\pi^1 \frac{\partial \widetilde{c^1}}{\partial q} + \left(\pi^2 + \pi^3\right)\frac{\partial \widetilde{c^2}}{\partial q}\right] + \left(\pi^3 + \lambda_3^d\right) d\widetilde{V^3} - \lambda_2^u d\widetilde{\widehat{V^{2(3)}}} =
$$

$$= \lambda_2^d \widehat{V_B^{2(1)}} \left(c^1 - \widehat{c^{2(1)}} \right) + \lambda_2^u \widehat{V_B^{2(3)}} \left(c^3 - \widehat{c^{2(3)}} \right) + \lambda_3^d \widehat{V_B^{3(2)}} \left(c^2 - \widehat{c^{3(2)}} \right). \tag{22}$$

In order to further clarify the mechanisms at work, consider the case where the constraint λ_2^u is not binding and check the sign of $d\widetilde{V^3}$. It is easy to recognize that, since V_3^3 is negative, the "compensated" effect of the conjectured reform on the indirect utility of individuals of type 3 is positive if and only if $\left(\frac{\partial \widetilde{c^2}}{\partial q} - \frac{\partial \widetilde{c^3}}{\partial q}\right) \frac{t}{1+tc_3^3 \frac{\alpha^L}{w^H}} < 0$; in turn, this requirement means that one of the following four conditions holds:

$$i) \qquad t > 0, \qquad \left| \frac{\partial \widetilde{c^2}}{\partial q} \right| > \left| \frac{\partial \widetilde{c^3}}{\partial q} \right|, \qquad and \qquad 1 + tc_3^3 \frac{\alpha^L}{w^H} > 0,$$

$$ii) \qquad t > 0, \qquad \left| \frac{\partial \widetilde{c^3}}{\partial q} \right| > \left| \frac{\partial \widetilde{c^2}}{\partial q} \right|, \qquad and \qquad 1 + tc_3^3 \frac{\alpha^L}{w^H} < 0,$$

$$iii) \qquad t < 0, \qquad \left| \frac{\partial \widetilde{c^3}}{\partial q} \right| > \left| \frac{\partial \widetilde{c^2}}{\partial q} \right|, \qquad and \qquad 1 + tc_3^3 \frac{\alpha^L}{w^H} > 0,$$

$$iv) \qquad t < 0, \qquad \left| \frac{\partial \widetilde{c^2}}{\partial q} \right| > \left| \frac{\partial \widetilde{c^3}}{\partial q} \right|, \qquad and \qquad 1 + tc_3^3 \frac{\alpha^L}{w^H} < 0,$$

(where $|\bullet|$ denotes absolute values).

Regarding the conditions that make $d\widetilde{V^3} < 0$, we only give the one of them which will be useful later on:

$$i') \qquad t < 0, \qquad \left| \frac{\partial \widetilde{c^2}}{\partial q} \right| > \left| \frac{\partial \widetilde{c^3}}{\partial q} \right| \qquad and \qquad 1 + tc_3^3 \frac{\alpha^L}{w^H} > 0.$$

Conditions $i)$ and $iii)$ are characterized by the quite reasonable assumption that $1+tc_3^3 \frac{\alpha^L}{w^H}$ is bigger than zero, and denote situations starting from which a marginal increase in the commodity tax rate raises the total tax liability of an individual of type 3 (high skilled, low taste for leisure) more than the one of an individual of type 2 (high skilled, high taste for leisure). Then the total tax liability of the type 3 individuals needs to be reduced and this is achieved by a (utility enhancing) reduction in their income tax liability (i.e. gross income Y^3).

Conditions ii) and iv) instead characterize situations where a marginal increase in the commodity tax rate weighs less upon an individual of type 3 than it does upon an individual of type 2 since relatively more revenue is extracted from type 2 individuals. In order to match the HE constraint it is now necessary to raise the total tax liability of the type 3 individuals. However, in this case the goal is achieved, counterintuitively, by a reduction in gross income Y^3. If $1 + tc_3^3 \frac{\alpha^L}{w^H} < 0$, a marginal increase in Y^3 actually reduces total tax revenue even if the marginal unit of income is taxed at a 100 percent rate. The explanation is that demand is recomposed across consumption goods in a way which greatly reduces indirect tax receipts. To raise the total tax payment of individuals of type 3, one has to lower their gross income and rely on the positive net effect working through the change in their consumption pattern.

However, conditions ii) and iv) are mainly a curiosity, and can never hold at an interior optimum. It can be proved that they are conflicting with one of the first order conditions of the planner's problem. To show this, take the derivative of the budget constraint of an individual of type 3 ($qc^3 + z^3 = B^3$) with respect to disposable income, B:

$$qc_B^3 + z_B^3 = 1 \Longrightarrow 1 - tc_B^3 = c_B^3 + z_B^3. \tag{23}$$

If commodities c and z are normal, from (23) we have $1 - tc_B^3 > 0$. A necessary condition for eq. (C3) in Appendix C to be satisfied (remember that we are assuming $\lambda_2^u = 0$) is that $\frac{1-tc_B^3}{1+tc_3^3\frac{\alpha^L}{w^H}} \left(= \frac{dY^3}{dB^3}\right) > 0$ (since $V_B^3 > 0$ and $V_3^3 < 0$), but if $1 - tc_B^3 > 0$ this is only compatible with $1 + tc_3^3 \frac{\alpha^L}{w^H} > 0$.

Finally, it is interesting to look at conditions that make our modified commodity tax rule, eq. (16), collapse into the "classical" one given by eq. (4). For this purpose we rewrite eq. (16) as

$$\gamma t\left(\pi^1\frac{\partial\widetilde{c^1}}{\partial q}+\pi^2\frac{\partial\widetilde{c^2}}{\partial q}+\pi^3\frac{\partial\widetilde{c^3}}{\partial q}\right)+\underbrace{\left[\gamma t\pi^3+\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\frac{t}{1+tc_3^3\frac{\alpha^L}{w^H}}\right]\left(\frac{\partial\widetilde{c^2}}{\partial q}-\frac{\partial\widetilde{c^3}}{\partial q}\right)}_{B}-\lambda_2^u d\widetilde{\widehat{V^{2(3)}}}=$$

$$=\lambda_2^d\widehat{V_B^{2(1)}}\left(c^1-\widehat{c^{2(1)}}\right)+\lambda_2^u\widehat{V_B^{2(3)}}\left(c^3-\widehat{c^{2(3)}}\right)+\lambda_3^d\widehat{V_B^3}\left(c^2-\widehat{c^{3(2)}}\right). \tag{24}$$

In order to come back to eq. (4), λ_2^u must be equal to zero and the term labelled by B on the left-hand side of eq. (24) must vanish. To satisfy the latter requirement, one of the two following conditions must hold:

$$\frac{\partial\widetilde{c^2}}{\partial q}=\frac{\partial\widetilde{c^3}}{\partial q}, \tag{25}$$

$$\gamma t\pi^3=-\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\frac{t}{1+tc_3^3\frac{\alpha^L}{w^H}}. \tag{26}$$

Condition (25) says that, if the compensated response to a marginal price increase is the same for agents of type 2 as it is for agents of type 3, then there is no additional scope for the use of commodity taxation besides the standard one of weakening the binding self-selection constraints.

Regarding condition (26), it turns out to be more significant if put in the following way:

$$\left(\pi^3+\lambda_3^d\right)\left|V_3^3\right|\frac{\alpha^L}{w^H}=\gamma\pi^3\left(1+tc_3^3\frac{\alpha^L}{w^H}\right). \tag{27}$$

When interpreting condition (27) it is important to remember that $V_3^3\frac{\alpha^L}{w^H}$ measures the marginal disutility of gross income for an agent of type 3 ($V_3^3\alpha^L$ being the marginal disutility of labor for such an agent), while γ measures the marginal cost of public funds, and $1+tc_3^3\frac{\alpha^L}{w^H}$ the net effect on the government's revenue coming from a marginal increase in the gross income Y^3, when the marginal unit of income is taxed away at a 100 percent rate, and taking the reallocation of demand across consumption goods into account.

The left-hand side of eq. (27) represents the costs of raising an additional unit of revenue by increasing the income tax liability of agents of type 3: in fact, a marginal increase in the gross income of these agents, holding constant their disposable income, has a total direct negative impact on their indirect utility measured by $\pi^3 V_3^3 \frac{\alpha^L}{w^H}$ (since there are π^3 agents of type 3), and as such it negatively affects the objective function of the government; moreover, since this policy change also tightens the self-selection constraint that prevents agents of type 3 from envying the bundle of agents of type 1, there is another social cost captured by $\lambda_3^d V_3^3 \frac{\alpha^L}{w^H}$.

The right-hand side of eq. (27) represents instead the benefits of this policy measure: this change raises total additional funds and, when evaluated at the shadow price for public funds, the social value of this increase is $\gamma\pi^3 \left(1 + tc_3^3 \frac{\alpha^L}{w^H}\right)$.

Condition (27) therefore says that we are also back to the standard formula for optimal commodity taxation if the social benefits of a marginal increase in the gross income Y^3 are exactly offset by its social costs.

Having interpreted eq. (16), the rule for the optimal commodity tax, and also having clarified some of the details of this rule, we are ready to state its novel properties in a proposition. Following the proposition, we will discuss the intuition behind some of the most interesting cases.

Proposition 1 *At the constrained utilitarian optimum (with redistribution from high to low skilled individuals), it is not always true that under the assumption* $\frac{\alpha^H}{w^H} < \frac{\alpha^L}{w^L}$*:*
(a) a commodity that is complementary to leisure should be discouraged whereas a commodity that is complementary to labor should be encouraged;
(b) a commodity that is normally expected to be encouraged should be subsidized whereas a commodity that is normally expected to be discouraged should be taxed at a positive rate.

Proof. *See Appendix D.*

Proposition 1 clearly differs from the popular prescription[19] in the literature on optimal taxation telling that goods complementary to labor should be encouraged while goods complementary to leisure[20] should be discouraged by the commodity tax system (whereas

[19] See for example Edwards et al. (1994) and Stiglitz (1998).

[20] Complementary to labour (leisure) is here used as a short for and corresponds to the definition by Pollak (1969) of negatively related to leisure (labour).

"encouraged" and "discouraged" are both intended in the Mirrleesian sense).[21] In the standard[22] counterpart of eq. (4) with many types of individuals and many commodities, this is reflected in that the right-hand side, which provides a social evaluation of the gains from a marginal (compensated) increase in one of the commodity tax rates in terms of relaxing the binding incentive compatibility constraints, is positive (negative) if the commodity which price is marginally increased by the tax is complementary to labor (leisure).

We have already observed that, when people differ along more than one dimension, we cannot invoke Proposition 6 by Guesnerie and Seade (1982), and say that the budget set will be a simple monotonic chain to the left. In our model, the HE constraint together with the assumption of redistribution from high- towards low skilled individuals (what we referred to as the "normal" case) imply that the corner intended for the low skilled type 1 individuals is L-linked (see footnote 12) to the corner intended for the high skilled but more epicurean individuals of type 2. Regarding the relation between individuals of type 2 and 3, the corner intended for type 2 can be L-linked to the corner intended for the high skilled hard working type, or the corner intended for the high skilled hard working type 3 can be L-linked to the one intended for type 2. This remark is part of the story why it is not necessarily expected that a commodity that is complementary to labor should be encouraged or a commodity that is complementary to leisure should be discouraged by the indirect tax system.

The feature just mentioned is shared by all models where the Pareto-efficient tax structure doesn't entail a simple monotonic chain to the left, but in our model a second peculiarity stands out which is strictly related to the introduction of the HE constraint. Whereas in the standard model with two private consumption goods, the term by which the commodity tax

[21] In a general context where there are n commodities and m individuals, the *index of discouragement* of commodity i is defined by Mirrlees (1976) as $d_i = \sum_{h=1}^{m} \sum_{j=1}^{n} \frac{\partial \widetilde{x_i^h}}{\partial q_j} t_j \left(\sum_{h=1}^{m} x_i^h \right)^{-1}$, where q and t denote consumer prices and commodity tax rates, x_i^h is the demand for commodity i by individual h and a tilde denotes hicksian demand. The index is an approximate measure of the change in compensated demand due to the tax system; positive values of the index mean that the commodity is encouraged by the indirect tax system, while negative values correspond to discouragement.

[22] Standard is here meant to describe a situation where individuals differ only with regard to their skill level and wages are exogenous.

rate t is multiplied is always negative (because of the concavity of the expenditure function), here it is not possible to rule out the possibility that, due to the presence of two additional factors ($d\widetilde{V}^3$ and $d\widetilde{V^{2(3)}}$), the aforesaid term turns out to be positive. In that case we have the "anomalous" prescriptions that a commodity that according to the right-hand side of eq. (16) should be encouraged, should in fact be taxed at a positive rate, whereas a commodity that according to the right-hand side of eq. (16) should be discouraged, should be subsidized.

Referring once more to the case when $\lambda_2^u = 0$, let us try to explain why a commodity that is complementary to labor and that according to the right-hand side of eq. (16) should be encouraged can nonetheless be taxed at a positive rate. Starting with a positive value of the commodity tax rate, even if a decrease in the excise is beneficial in terms of (compensated) revenue and weakening of the self-selection constraints, the reduction is not realized because the benefits are more than offset by the cost in terms of reduced indirect utility of individuals of type 3 descending from the demand to keep the HE constraint satisfied, which would require an increase in their gross income Y^3 (cf. condition i)). Similarly, a commodity that is complementary to leisure and that also should be discouraged according to the right-hand side of eq. (16) can nevertheless be subsidized. Starting with a negative value of the commodity tax rate, even if an increase in the excise would be beneficial in terms of (compensated) revenue and weakening of the self-selection constraints, there will be no increase because of the damaging effect on the indirect utility of individuals of type 3 coming from the increase in their gross income needed to maintain the HE constraint satisfied (cf. condition i')).

3.2.2 The Marginal Effective Tax Rates

We now turn to the problem of evaluating the METR faced by the individuals at the optimal allocation. Using eq. (8) we can characterize the METR in the following proposition.

Proposition 2 *Under the assumption $\frac{\alpha^H}{w^H} < \frac{\alpha^L}{w^L}$, the constrained utilitarian optimum with redistribution from high to low skilled individuals is characterized by:*
(a) a marginal effective tax rate faced by type 1 (low skilled, low taste for leisure) that is positive;
(b) a zero marginal effective tax rate faced by either type 2 (high skilled, high taste for leisure)

or type 3 (high skilled, low taste for leisure), and a marginal effective tax rate different from zero for the other type.

Proof. *See Appendix E.*

For future reference, we note that the METR faced by type 2 and type 3 are (see Appendix C):

$$\tau_2' = \frac{\lambda_3^d \widehat{V_B^{3(2)}} \left(\frac{\widehat{V_3^{3(2)}}}{\widehat{V_B^{3(2)}}} \alpha^L - \frac{V_3^2}{V_B^2} \alpha^H \right)}{\gamma w^H \left(\pi^2 + \pi^3\right) + \frac{1}{1+tc_3^3 \frac{\alpha^L}{w^H}} \left[\left(\pi^3 + \lambda_3^d\right) V_3^3 \alpha^L - \lambda_2^u \widehat{V_3^{2(3)}} \alpha^H \right]}, \tag{28}$$

$$\tau_3' = \frac{\lambda_2^u \widehat{V_B^{2(3)}}}{\left(\pi^3 + \lambda_3^d\right) V_B^3} \left(1 + \frac{\widehat{V_3^{2(3)}}}{\widehat{V_B^{2(3)}}} \frac{\alpha^H}{w^H} \frac{1 - t\frac{\partial c^3}{\partial B^3}}{1 + tc_3^3 \frac{\alpha^L}{w^H}} \right) \left(1 + tc_3^3 \frac{\alpha^L}{w^H} \right). \tag{29}$$

The positive METR faced by the low skilled individuals is not surprising if we notice that the corner intended for the low skilled individuals is L-linked with another corner (the one intended for high skilled, high taste for leisure individuals) by a downwards incentive compatibility constraint and that this kind of distortion is the standard one which makes it possible to relax the binding constraint in such a circumstance.

Regarding the high skilled individuals, we have already noticed that in this particular framework it is not possible that λ_2^u and λ_3^d are binding at the same time. Suppose first that $\lambda_3^d \neq 0$ while the other self-selection constraint is slack ($\lambda_2^u = 0$). From eq. (29) we have $\tau_3' = 0$: the corner intended for type 3 (high skilled, low taste for leisure) individuals is not L-linked to any other corner and they are "on average" undistorted at the margin. From eq. (28), instead, we have that type 2 (high skilled, high taste for leisure) individuals face a METR different from zero. Considering the pattern of the binding self-selection constraints (only downwards), we would expect that the sign of the METR faced by this type of individuals were positive since this is the standard prescription for weakening the binding constraints in such a case. However, still looking at eq. (28), this is not obvious since, if the numerator has a positive sign ($\frac{\widehat{V_3^{3(2)}}}{\widehat{V_B^{3(2)}}} \alpha^L - \frac{V_3^2}{V_B^2} \alpha^H > 0$), the sign of the denominator is instead ambiguous, implying that it is impossible to decide the sign of τ_2' *a priori*. Notice that, from the f.o.c. for B^3 (see Appendix C, eq. (C3)), we have as a necessary condition for an optimum with $\lambda_2^u = 0$ that $1+tc_3^3\frac{\alpha^L}{w^H} > 0$. If this is the case, then it could be that $\tau_2' < 0$. Basically, when $\lambda_2^u = 0$, the standard formula for the METR faced by individuals of type 2 is amended by the additional term $\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}\left(\pi^3 + \lambda_3^d\right)V_3^3\alpha^L$ and the magnitude and sign of this term could reverse the overall sign of the expression. The reason why this is the case is again connected to the requirement of HE. The point is that the sign of the METR faced by individuals of type 2 determines the sign of the variation in their total tax liability as they increase the labor supply; this in turn means that, if agents of type 2 should marginally increase their labor supply, it would be necessary to adjust the gross income of individuals of type 3 to have equal total tax payments, and this would obviously affect their indirect utility in a manner that the government must take into account. More formally, suppose that individuals of type 2 choose to marginally increase their gross income Y^2. In order to be induced to do so, their disposable income has to be increased by their marginal valuation of foregone leisure $-\frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}$. From eq. (13) and (11) we have that the variation in the gross income Y^3 needed to keep constraint (9) satisfied is

$$dY^3 = \frac{-\frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}\left(t\frac{\partial c^2}{\partial B^2} - 1\right) + 1 + tc_3^2\frac{\alpha^H}{w^H}}{1 + tc_3^3\frac{\alpha^L}{w^H}} = \frac{\tau_2'}{1 + tc_3^3\frac{\alpha^L}{w^H}}. \tag{30}$$

Other things being equal, $dY^3 > (<)\, 0$ is welfare damaging (beneficial) for individuals of type 3 since it means less (more) leisure, and therefore the only way to get a solution entailing $\tau_2' < 0$ is that the encouragement of labor supply provided by the marginal subsidy is coupled by an increase in the indirect utility of individuals of type 3, obtained by a reduction in their labor supply, which in turn, from (30), is possible only if $1 + tc_3^3\frac{\alpha^L}{w^H} > 0$.

Suppose instead that $\tau_2' > 0$. This means that, if individuals of type 2 are willing to marginally increase their labor supply, their total tax liability will increase as well. In order not to violate the HE constraint, an increase in the total tax liability paid by individuals of type 3 is also required, and this is obtained by increasing Y^3 (remember that, when $\lambda_2^u = 0$, $1 + tc_3^3\frac{\alpha^L}{w^H} > 0$ is a necessary condition for the existence of a solution of the planner's problem). This variation ($dY^3 > 0$) is detrimental to their welfare and tightens the binding self-selection constraint λ_3^d. Eq. (28) says that in this case the METR faced by individuals of type 2 should be increased, as compared to the case where the additional term $\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}\left(\pi^3+\lambda_3^d\right)V_3^3\alpha^L$ is absent, in order to prevent them from being tempted to increase their labor supply.

Suppose now that $\lambda_2^u \neq 0$ whereas it is the incentive compatibility constraint that prevents individuals of type 3 to mimic those of type 2 that is slack ($\lambda_3^d = 0$). From eq. (28) we have $\tau_2' = 0$: now it is the corner intended for individuals with high taste for leisure that is not L-linked to any other corner and they should therefore be "on average" undistorted at the margin. According to eq. (29), the METR faced by the high skilled, low taste for leisure individuals is instead different from zero, coherent with the rule prescribing that an individual should be "on average" distorted at the margin when the bundle intended for him is L-linked to another bundle by individuals of a different type. However, whereas we would normally expect a marginal ("on average") subsidy when the self-selection constraint is binding upwards, in this case the sign is ambiguous.

With obvious notation we can express the METR faced by type 3 individuals in a more compact form as (rewrite eq. (E9) in Appendix E)

$$\tau_3' = \frac{\lambda_2^u\left(1+tc_3^3\frac{\alpha^L}{w^H}\right)}{\left(\pi^3+\lambda_3^d\right)V_B^3}\frac{d\widehat{V^{2(3)}}}{dB^3}. \tag{31}$$

From eq. (31) we see that the sign of the METR faced by individuals of type 3 is determined by the product $\left(1+tc_3^3\frac{\alpha^L}{w^H}\right)\frac{d\widehat{V^{2(3)}}}{dB^3}$. We have the following four cases:

$$i^*) \qquad 1+tc_3^3\frac{\alpha^L}{w^H} > 0 \quad and \quad \frac{d\widehat{V^{2(3)}}}{dB^3} > 0 \quad \Longrightarrow \quad \tau_3' > 0;$$

$$ii^*) \qquad 1+tc_3^3\frac{\alpha^L}{w^H} < 0 \quad and \quad \frac{d\widehat{V^{2(3)}}}{dB^3} > 0 \quad \Longrightarrow \quad \tau_3' < 0;$$

$$iii^*) \qquad 1+tc_3^3\frac{\alpha^L}{w^H} > 0 \quad and \quad \frac{d\widehat{V^{2(3)}}}{dB^3} < 0 \quad \Longrightarrow \quad \tau_3' < 0;$$

$$iv^*) \qquad 1 + tc_3^3 \frac{\alpha^L}{w^H} < 0 \quad and \quad \frac{d\widehat{V^{2(3)}}}{dB^3} < 0 \qquad \Longrightarrow \qquad \tau_3' > 0.$$

Case i^*): since $1+tc_3^3 \frac{\alpha^L}{w^H} > 0$, $\frac{dB^3}{dY^3} = \frac{1+tc_3^3 \frac{\alpha^L}{w^H}}{1-t\frac{\partial c^3}{\partial B^3}} > 0$ (remember that we proved $1-t\frac{\partial c^3}{\partial B^3} > 0$); this means that a marginal increase in the labor supply of type 3 individuals (in this case the ones being potentially mimicked) would be coupled with an increase in their disposable income in order to satisfy constraint (9). However we also have that $\frac{d\widehat{V^{2(3)}}}{dB^3} > 0$, i.e. the mimickers would profit by the change. In order to prevent this and weaken the binding self-selection constraint it is appropriate to have individuals of type 3 facing a positive distortion at the margin, discouraging them from increasing their labor supply.

Case ii^*): since $1+tc_3^3 \frac{\alpha^L}{w^H} < 0$, $\frac{dB^3}{dY^3} = \frac{1+tc_3^3 \frac{\alpha^L}{w^H}}{1-t\frac{\partial c^3}{\partial B^3}} < 0$; this means that a marginal increase in the labor supply of individuals of type 3 would be coupled with a reduction in their disposable income in order to maintain constraint (9) satisfied. However we also have that $\frac{d\widehat{V^{2(3)}}}{dB^3} > 0$, i.e. the mimickers would be damaged by the change. In order to exploit this opportunity and weaken the binding self-selection constraint, it is appropriate to have individuals of type 3 facing a negative distortion (a subsidy) at the margin, encouraging them to increase their labor supply.

Case iii^*): again $1 + tc_3^3 \frac{\alpha^L}{w^H} > 0$ and $\frac{dB^3}{dY^3} = \frac{1+tc_3^3 \frac{\alpha^L}{w^H}}{1-t\frac{\partial c^3}{\partial B^3}} > 0$: a marginal increase in the labor supply of individuals of type 3 would be coupled with an increase in their disposable income in order to maintain constraint (9) satisfied. However, opposite to case i^*), we also have that $\frac{d\widehat{V^{2(3)}}}{dB^3} < 0$, i.e. the mimickers would be harmed by the change. In order to exploit the chance to relax the binding self-selection constraint it is appropriate to have individuals of type 3 facing a negative METR, encouraging them to increase their labor supply.

Case iv^*): again $1 + tc_3^3 \frac{\alpha^L}{w^H} < 0$ and $\frac{dB^3}{dY^3} = \frac{1+tc_3^3 \frac{\alpha^L}{w^H}}{1-t\frac{\partial c^3}{\partial B^3}} < 0$: this means that a marginal increase in the labor supply of individuals of type 3 would be coupled with a reduction in their disposable income in order to keep constraint (9) satisfied. However, opposite to case ii^*) we also have that $\frac{d\widehat{V^{2(3)}}}{dB^3} < 0$, i.e. the mimickers would profit by the change. In order to prevent this and weaken the binding self-selection constraint it is appropriate to have individuals of type 3 facing a positive METR, discouraging them from increasing their supply of labor.

4 Concluding Remarks

Our aim with this paper has not been to make an ethical case for the horizontal equity principle. As argued by Kaplow (1995), although horizontal equity is intuitively appealing, there is need for studies that try both to justify this principle and to derive a precise measure of equity from the justification. Our intentions have rather been to investigate how the preferred tax mix might change if we were to take horizontal equity seriously.

The investigation has made clear that the horizontal equity principle may seriously affect the incentives for income and commodity taxation. The basic intuition and policy implications from models with heterogeneity in ability may not carry over into models where preferences for leisure are heterogeneous as well. Contrary to normal findings, our results indicate that a good that is complementary to leisure need not be discouraged by the tax system, and a good that is normally expected to be discouraged need not be taxed even if the economy is composed by only two private goods and leisure. As expected, the direction of redistribution is a crucial factor for the marginal effective tax rates, but the introduction of the horizontal equity restriction complicates matters here as well. It is for instance possible to have a marginal tax instead of a subsidy "on average" for the high ability, hard working-type even though the self-selection constraint relating them to the high ability, epicurean-type is binding upwards.

Before concluding, we note some possible objections against our model. Since the individuals are held responsible for their preferences, a higher taste for leisure can in our model be interpreted as laziness. Obviously there are several alternative interpretations. Cuff (2000) discusses the alternative to interpret a high taste for leisure as a kind of disability. Possibly, her interpretation leads to very different implications. Whereas it is intuitive to argue that compensation for laziness should be ruled out, it is—at least in the framework of responsibility and compensation—less obvious that people who are for some other reason unable to work as hard as others, should not receive any compensation for this disability.

The discussion clearly touches upon the concept of free will and whether preferences are to be treated as given or as acquired. It would be presumptuous to try to answer such questions in passing—we confine ourselves to saying that besides the benefits associated with a focus on one of the polar cases, our findings are also relevant as long as the taste for leisure among some individuals is to some extent interpreted more as laziness than as a disability.

Moreover, the economic analysis of the family (Becker, 1991; Cigno, 1991) can be used to explain why certain individuals are less prone to work longer hours. If production of certain goods and services can take place at home, individuals who are relatively more productive at home than at work will act as if they had a greater taste for leisure. Home production could mean the producing of substitutes to services available on the market as in Kleven *et al.* (2000), or it could be child rearing as in Balestrino *et al.* (2002). Apart from explaining why the labor market has a greater appeal to certain individuals, these studies also narrow down the set of goods that are candidates for relatively higher taxes. Almost fifty years ago, Corlett and Hague (1953) suggested that efficiency could be improved by taxing goods that are complementary to leisure more heavily. Yet their rule has not come to much use since the relation between most goods and leisure is wrapped in mystery. This lack of information is of course just as problematic in models like ours. Therefore models with home production provide promising inputs to extensions of our model which aim at more practically oriented policy implications.

To conclude, although our model is very simple and stylized, we hope that we have called attention to the relevance and some potential consequences of a horizontal equity restriction for tax policy. Without doubt there are prospects for more research in this relatively unexplored area of tax theory.

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5 Appendix A: Derivation of eq. (4)

The first order conditions for B^1, B^2, B^3, and t are as follows:

$$\left(B^1\right): \qquad \pi^1 V_B^1 + \pi^1 \gamma \left(t \frac{\partial c^1}{\partial B^1} - 1 \right) - \lambda_2^d \widehat{V_B^{2(1)}} = 0, \tag{A1}$$

$$\left(B^2\right): \qquad \pi^2 V_B^2 + \pi^2 \gamma \left(t \frac{\partial c^2}{\partial B^2} - 1 \right) + \lambda_2^d V_B^2 - \lambda_3^d \widehat{V_B^{3(2)}} = 0, \tag{A2}$$

$$\left(B^3\right): \qquad \pi^3 V_B^3 + \pi^3 \gamma \left(t \frac{\partial c^3}{\partial B^3} - 1 \right) + \lambda_3^d V_B^3 = 0, \tag{A3}$$

$$(t): \qquad \pi^1 V_q^1 + \pi^2 V_q^2 + \pi^3 V_q^3 + \lambda_2^d \left(V_q^2 - \widehat{V_q^{2(1)}} \right) + \lambda_3^d \left(V_q^3 - \widehat{V_q^{3(2)}} \right) +$$

$$+\gamma \left[\pi^1 c^1 + \pi^2 c^2 + \pi^3 c^3 + t \left(\frac{\partial c^1}{\partial q} \pi^1 + \frac{\partial c^2}{\partial q} \pi^2 + \frac{\partial c^3}{\partial q} \pi^3 \right) \right] = 0. \tag{A4}$$

Applying Roy's identity, eq. (A4), the first order condition for t, becomes:

$$-\pi^1 c^1 V_B^1 - \pi^2 c^2 V_B^2 - \pi^3 c^3 V_B^3 - \lambda_2^d c^2 V_B^2 - \lambda_3^d c^3 V_B^3 + \lambda_2^d \widehat{c^{2(1)}} \widehat{V_B^{2(1)}} + \lambda_3^d \widehat{c^{3(2)}} \widehat{V_B^{3(2)}} +$$

$$+\gamma \left[\pi^1 c^1 + \pi^2 c^2 + \pi^3 c^3 + t \left(\frac{\partial c^1}{\partial q} \pi^1 + \frac{\partial c^2}{\partial q} \pi^2 + \frac{\partial c^3}{\partial q} \pi^3 \right) \right] = 0. \tag{A5}$$

From (A1), (A2) and (A3) we have respectively:

$$-\pi^1 V_B^1 = \pi^1 \gamma \left(t \frac{\partial c^1}{\partial B^1} - 1 \right) - \lambda_2^d \widehat{V_B^{2(1)}}, \tag{A6}$$

$$-\left(\pi^2 + \lambda_2^d\right) V_B^2 = \gamma \pi^2 \left(t \frac{\partial c^2}{\partial B^2} - 1 \right) - \lambda_3^d \widehat{V_B^{3(2)}}, \tag{A7}$$

$$-\left(\pi^3 + \lambda_3^d\right) V_B^3 = \pi^3 \gamma \left(t \frac{\partial c^3}{\partial B^3} - 1 \right). \tag{A8}$$

Substituting in eq. (A5), we get:

$$\gamma\pi^1 c^1 \left(t\frac{\partial c^1}{\partial B^1} - 1\right) + \gamma\left[c^2\pi^2\left(t\frac{\partial c^2}{\partial B^2} - 1\right) + c^3\pi^3\left(t\frac{\partial c^3}{\partial B^3} - 1\right)\right] +$$

$$-c^1\lambda_2^d\widehat{V_B^{2(1)}} - c^2\lambda_3^d\widehat{V_B^{3(2)}} + \lambda_2^d\widehat{c^{2(1)}}\widehat{V_B^{2(1)}} + \lambda_3^d\widehat{c^{3(2)}}\widehat{V_B^{3(2)}} +$$

$$+\gamma\left[\pi^1 c^1 + \pi^2 c^2 + \pi^3 c^3 + t\left(\frac{\partial c^1}{\partial q}\pi^1 + \frac{\partial c^2}{\partial q}\pi^2 + \frac{\partial c^3}{\partial q}\pi^3\right)\right] = 0 \tag{A9}$$

By using the Slutsky equation and by some further manipulation, one finally gets eq. (4).

6 Appendix B: The Marginal Effective Tax Rates

The first order conditions of the planner's problem with respect to gross incomes Y^1, Y^2 and Y^3 are:

$$(Y^1): \qquad \pi^1 V_3^1 \frac{\alpha^L}{w^L} = \lambda_2^d \widehat{V_3^{2(1)}} \frac{\alpha^H}{w^H} - \pi^1\gamma\left(1 + tc_3^1\frac{\alpha^L}{w^L}\right), \tag{B1}$$

$$(Y^2): \qquad \left(\pi^2 + \lambda_2^d\right) V_3^2 \frac{\alpha^H}{w^H} = \lambda_3^d \widehat{V_3^{3(2)}} \frac{\alpha^L}{w^H} - \pi^2\gamma\left(1 + tc_3^2\frac{\alpha^H}{w^H}\right), \tag{B2}$$

$$(Y^3): \qquad \left(\pi^3 + \lambda_3^d\right) V_3^3 \frac{\alpha^L}{w^H} = -\pi^3\gamma\left(1 + tc_3^3\frac{\alpha^L}{w^H}\right). \tag{B3}$$

To find the METR faced by agents of type 1, divide (B1) by (A6) and multiply the result by $\pi^1\gamma\left(t\frac{\partial c^1}{\partial B^1} - 1\right) - \lambda_2^d\widehat{V_B^{2(1)}}$. This gives

$$\frac{\alpha^L}{w^L}\frac{V_3^1}{V_B^1}\left[\lambda_2^d\widehat{V_B^{2(1)}} - \pi^1\gamma\left(t\frac{\partial c^1}{\partial B^1} - 1\right)\right] = \lambda_2^d\widehat{V_3^{2(1)}}\frac{\alpha^H}{w^H} - \pi^1\gamma\left(1 + tc_3^1\frac{\alpha^L}{w^L}\right). \tag{B4}$$

Using notation $\overline{w^1} = \frac{w^L}{\alpha^L}$, $\overline{w^2} = \frac{w^H}{\alpha^H}$, $\overline{w^3} = \frac{w^H}{\alpha^L}$ and $\overline{\Omega^{1,2}} = \frac{\overline{w^1}}{\overline{w^2}}$ we get

$$\tau_1' = \frac{\lambda_2^d\widehat{V_B^{2(1)}}}{\gamma\pi^1}\frac{1}{\overline{w^1}}\left(\frac{\widehat{V_3^{2(1)}}}{\widehat{V_B^{2(1)}}}\overline{\Omega^{1,2}} - \frac{V_3^1}{V_B^1}\right). \tag{B5}$$

Since assumption 2 implies that $\overline{\Omega^{1,2}} < 1$, and single-crossing holds, the METR faced by type 1 is positive.

Similarly, for the METR faced by agents of type 2, divide (B2) by (A7) and multiply the result by $\gamma\pi^2\left(t\frac{\partial c^2}{\partial B^2} - 1\right) - \lambda_3^d \widehat{V_B^{3(2)}}$. This gives

$$\frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}\left[-\gamma\pi^2\left(t\frac{\partial c^2}{\partial B^2} - 1\right) + \lambda_3^d \widehat{V_B^{3(2)}}\right] = -\gamma\pi^2\left(1 + tc_3^2\frac{\alpha^H}{w^H}\right) + \lambda_3^d \widehat{V_3^{3(2)}}\frac{\alpha^L}{w^H}.$$

Manipulating this expression we get

$$\gamma\pi^2\left[1 + tc_3^2\frac{\alpha^H}{w^H} + \frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}\left(1 - t\frac{\partial c^2}{\partial B^2}\right)\right] = \lambda_3^d \widehat{V_B^{3(2)}}\left(\frac{\widehat{V_3^{3(2)}}}{\widehat{V_B^{3(2)}}}\frac{\alpha^L}{w^H} - \frac{V_3^2}{V_B^2}\frac{\alpha^H}{w^H}\right). \tag{B6}$$

Using the definition of METR ($\tau' = 1 + tc_3\frac{\alpha}{w} + \frac{\alpha}{w}\frac{V_3}{V_B}\left(1 - t\frac{\partial c}{\partial B}\right)$) and rearranging, we get

$$\tau_2' = \frac{\lambda_3^d \widehat{V_B^{3(2)}}}{\gamma\pi^2 w^H}\left(\frac{\widehat{V_3^{3(2)}}}{\widehat{V_B^{3(2)}}}\alpha^L - \frac{V_3^2}{V_B^2}\alpha^H\right), \tag{B7}$$

which again gives a positive value for τ_2'.

Finally, to obtain the METR faced by type 3, divide (B3) by (A8) and multiply the result by $\pi^3\gamma\left(t\frac{\partial c^3}{\partial B^3} - 1\right)$. This gives

$$\frac{\alpha^L}{w^H}\frac{V_3^3}{V_B^3}\left[\pi^3\gamma\left(1 - t\frac{\partial c^3}{\partial B^3}\right)\right] = -\pi^3\gamma\left(1 + tc_3^3\frac{\alpha^L}{w^H}\right).$$

Using the definition of METR ($\tau' = 1 + tc_3\frac{\alpha}{w} + \frac{\alpha}{w}\frac{V_3}{V_B}\left(1 - t\frac{\partial c}{\partial B}\right)$) and rearranging, we get

$$\tau_3' = 0.$$

7 Appendix C: Derivation of eq. (16)

The first order conditions for B^1, B^2 and B^3 are as follows:

$$(B^1): \qquad \pi^1 V_B^1 + \pi^1\gamma\left(t\frac{\partial c^1}{\partial B^1} - 1\right) + \lambda_1^u V_B^1 - \lambda_2^d \widehat{V_B^{2(1)}} = 0 \tag{C1}$$

$$\left(B^2\right): \qquad \pi^2 V_B^2 + \pi^3 V_3^3 \frac{\alpha^L}{w^H}\left(t\frac{\partial c^2}{\partial B^2} - 1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}} + \gamma\left(\pi^2+\pi^3\right)\left(t\frac{\partial c^2}{\partial B^2}-1\right)+$$

$$-\lambda_1^u \widehat{V_B^{1(2)}} + \left(\lambda_2^d + \lambda_2^u\right) V_B^2 - \lambda_3^d \widehat{V_B^{3(2)}} - \lambda_2^u \widehat{V_3^{2(3)}} \frac{\alpha^H}{w^H}\left(t\frac{\partial c^2}{\partial B^2} - 1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+$$

$$+\lambda_3^d V_3^3 \frac{\alpha^L}{w^H}\left(t\frac{\partial c^2}{\partial B^2} - 1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}} = 0 \tag{C2}$$

$$\left(B^3\right): \qquad \pi^3\left[V_B^3 + V_3^3\frac{\alpha^L}{w^H}\left(1 - t\frac{\partial c^3}{\partial B^3}\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}\right]+$$

$$-\lambda_2^u\left[\widehat{V_B^{2(3)}} + \widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left(1 - t\frac{\partial c^3}{\partial B^3}\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}\right] + \lambda_3^d\left[V_B^3 + V_3^3\frac{\alpha^L}{w^H}\left(1 - t\frac{\partial c^3}{\partial B^3}\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}\right] = 0 \tag{C3}$$

Applying Roy's identity, eq. (15), the first order condition for t, becomes:

$$-\pi^1 c^1 V_B^1 - \pi^2 c^2 V_B^2 - \pi^3 c^3 V_B^3 - \lambda_2^d c^2 V_B^2 - \lambda_2^u c^2 V_B^2 - \lambda_3^d c^3 V_B^3 +$$

$$+\gamma\left\{\pi^1 c^1 + \left(\pi^2+\pi^3\right)c^2 + t\left[\frac{\partial c^1}{\partial q}\pi^1 + \frac{\partial c^2}{\partial q}\left(\pi^2+\pi^3\right)\right]\right\}+$$

$$+\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\left[c^2 - c^3 + t\left(\frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q}\right)\right]\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}} + \lambda_2^d \widehat{c^{2(1)}}\widehat{V_B^{2(1)}} + \lambda_2^u \widehat{c^{2(3)}}\widehat{V_B^{2(3)}}+$$

$$-\lambda_2^u \widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left[c^2 - c^3 + t\left(\frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q}\right)\right]\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}} + \lambda_3^d \widehat{c^{3(2)}}\widehat{V_B^{3(2)}} = 0. \tag{C4}$$

From (C1), (C2) and (C3) we have respectively:

$$-\pi^1 V_B^1 = \pi^1\gamma\left(t\frac{\partial c^1}{\partial B^1} - 1\right) - \lambda_2^d \widehat{V_B^{2(1)}}, \tag{C5}$$

$$-\left(\pi^2+\lambda_2^d+\lambda_2^u\right)V_B^2=\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\left(t\frac{\partial c^2}{\partial B^2}-1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+\gamma\left(\pi^2+\pi^3\right)\left(t\frac{\partial c^2}{\partial B^2}-1\right)+$$

$$-\lambda_3^d\widehat{V_B^{3(2)}}-\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left(t\frac{\partial c^2}{\partial B^2}-1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}, \tag{C6}$$

$$-\left(\pi^3+\lambda_3^d\right)V_B^3=\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\left(1-t\frac{\partial c^3}{\partial B^3}\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+$$

$$-\lambda_2^u\left[\widehat{V_B^{2(3)}}+\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left(1-t\frac{\partial c^3}{\partial B^3}\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}\right]. \tag{C7}$$

Substituting in eq. (C4), we get:

$$\gamma\pi^1c^1\left(t\frac{\partial c^1}{\partial B^1}-1\right)-c^1\lambda_2^d\widehat{V_B^{2(1)}}+\left(\pi^3+\lambda_3^d\right)c^2V_3^3\frac{\alpha^L}{w^H}\left(t\frac{\partial c^2}{\partial B^2}-1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+$$

$$+\gamma c^2\left(\pi^2+\pi^3\right)\left(t\frac{\partial c^2}{\partial B^2}-1\right)-c^2\lambda_3^d\widehat{V_B^{3(2)}}-c^2\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left(t\frac{\partial c^2}{\partial B^2}-1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+$$

$$+\left(\pi^3+\lambda_3^d\right)c^3V_3^3\frac{\alpha^L}{w^H}\left(1-t\frac{\partial c^3}{\partial B^3}\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}-c^3\lambda_2^u\left[\widehat{V_B^{2(3)}}+\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left(1-t\frac{\partial c^3}{\partial B^3}\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}\right]+$$

$$+\gamma\left[\pi^1c^1+\left(\pi^2+\pi^3\right)c^2+t\left(\frac{\partial c^1}{\partial q}\pi^1+\frac{\partial c^2}{\partial q}\left(\pi^2+\pi^3\right)\right)\right]+\lambda_2^d\widehat{c^{2(1)}}\widehat{V_B^{2(1)}}+\lambda_2^u\widehat{c^{2(3)}}\widehat{V_B^{2(3)}}+$$

$$-\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left[c^2-c^3+t\left(\frac{\partial c^2}{\partial q}-\frac{\partial c^3}{\partial q}\right)\right]\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+$$

$$+\lambda_3^d\widehat{c^{3(2)}}\widehat{V_B^{3(2)}}+\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\left[c^2-c^3+t\left(\frac{\partial c^2}{\partial q}-\frac{\partial c^3}{\partial q}\right)\right]\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}=0. \tag{C8}$$

By using the Slutsky equation and by some further manipulation, one finally gets eq. (16).

8 Appendix D: Proof of Proposition 1

In eq. (16) the following four main cases are possible:

$$
\begin{aligned}
&\text{Case 1:} \quad RHS > 0, \quad t > 0, \quad \gamma\pi^1 \frac{\partial \widetilde{c^1}}{\partial q} + \gamma \frac{\partial \widetilde{c^2}}{\partial q} \sum_{i=2}^{3} \pi^i + \left(\pi^3 + \lambda_3^d\right) \frac{d\widetilde{V^3}}{t} - \lambda_2^u \frac{d\widetilde{\widehat{V^{2(3)}}}}{t} > 0; \\
&\text{Case 2:} \quad RHS > 0, \quad t < 0, \quad \gamma\pi^1 \frac{\partial \widetilde{c^1}}{\partial q} + \gamma \frac{\partial \widetilde{c^2}}{\partial q} \sum_{i=2}^{3} \pi^i + \left(\pi^3 + \lambda_3^d\right) \frac{d\widetilde{V^3}}{t} - \lambda_2^u \frac{d\widetilde{\widehat{V^{2(3)}}}}{t} < 0; \\
&\text{Case 3:} \quad RHS < 0, \quad t > 0, \quad \gamma\pi^1 \frac{\partial \widetilde{c^1}}{\partial q} + \gamma \frac{\partial \widetilde{c^2}}{\partial q} \sum_{i=2}^{3} \pi^i + \left(\pi^3 + \lambda_3^d\right) \frac{d\widetilde{V^3}}{t} - \lambda_2^u \frac{d\widetilde{\widehat{V^{2(3)}}}}{t} < 0; \\
&\text{Case 4:} \quad RHS < 0, \quad t < 0, \quad \gamma\pi^1 \frac{\partial \widetilde{c^1}}{\partial q} + \gamma \frac{\partial \widetilde{c^2}}{\partial q} \sum_{i=2}^{3} \pi^i + \left(\pi^3 + \lambda_3^d\right) \frac{d\widetilde{V^3}}{t} - \lambda_2^u \frac{d\widetilde{\widehat{V^{2(3)}}}}{t} > 0,
\end{aligned}
$$

where RHS is used to denote the right hand side of eq. (16).

For (a), note that the sign of the right hand side is not determined *a priori* by the relation of the taxed commodity to labor or leisure. If the taxed commodity is complementary to labor we have that $c^1 > \widehat{c^{2(1)}}$, $c^3 < \widehat{c^{2(3)}}$, and $c^2 > \widehat{c^{3(2)}}$. Thus, if $\lambda_3^d = 0$ and $\lambda_2^u > 0$, case 3 encompasses a "non-ordinary" sub-case with a commodity complementary to labor which in spite of this is taxed at a positive rate, besides the standard case of subsidizing a commodity which is complementary to labor. If the taxed commodity is instead complementary to leisure we have that $c^1 < \widehat{c^{2(1)}}$, $c^3 > \widehat{c^{2(3)}}$, and $c^2 < \widehat{c^{3(2)}}$. Thus, case 2 encompasses a "non-ordinary" sub-case with a commodity complementary to leisure which in spite of this is subsidized. Those two "non-ordinary" sub-cases demonstrate that a good that is normally expected to be discouraged (encouraged) in order to loosen the self-selection constraints, may actually be encouraged (discouraged) in a model where the agents differ along more than one dimension.

For (b), consider in case 1 the sub-case of a commodity complementary to labor that should be encouraged, according to the right hand side of eq. (16); nevertheless, due to a positive value of $\left(\pi^3 + \lambda_3^d\right) \frac{d\widetilde{V^3}}{t} - \lambda_2^u \frac{d\widetilde{\widehat{V^{2(3)}}}}{t}$ which is greater than the absolute value of $\gamma\pi^1 \frac{\partial \widetilde{c^1}}{\partial q} + \gamma \frac{\partial \widetilde{c^2}}{\partial q} \sum_{i=2}^{3} \pi^i$, the commodity is taxed. Similarly, consider in case 4 the sub-case of a commodity complementary to leisure that should be discouraged, according to the right hand side of eq. (16). The commodity is nonetheless subsidized since also in this instance the requirement to uphold horizontal equity implies that the term that multiplies t on the left hand side of eq. (16) turns out to be positive. However, whilst in the former sub-case this requires a high and positive value of $\left(\pi^3+\lambda_3^d\right) d\widetilde{V^3} - \lambda_2^u d\widetilde{\widehat{V^{2(3)}}}$ (since we are looking conditions compatible with a tax), in the latter sub-case this requirement means a high and negative value of the aforesaid term (since we are looking for conditions compatible with a subsidy). Apart from this difference, from eq. (16) we get that $\left|\frac{\partial \tilde{c^2}}{\partial q}\right| > \left|\frac{\partial \tilde{c^3}}{\partial q}\right|$ must be satisfied in both of these sub-cases.

■

9 Appendix E: Proof of Proposition 2

Considering the "normal" case when redistribution is directed towards the low skilled individuals and $\lambda_1^u = 0$, the first order conditions of the planner's problem with respect to gross incomes Y^1 and Y^2 are:

$$(Y^1): \qquad \pi^1 V_3^1 \frac{\alpha^L}{w^L} = \lambda_2^d \widehat{V_3^{2(1)}} \frac{\alpha^H}{w^H} - \pi^1 \gamma \left(1 + t c_3^1 \frac{\alpha^L}{w^L}\right), \tag{E1}$$

$$(Y^2): \qquad \pi^2 V_3^2 \frac{\alpha^H}{w^H} + \pi^3 V_3^3 \frac{\alpha^L}{w^H}\frac{dY^3}{dY^2} + \gamma\left(\pi^2+\pi^3\right)\left(1 + t c_3^2 \frac{\alpha^H}{w^H}\right) + \lambda_2^d V_3^2 \frac{\alpha^H}{w^H} + \lambda_2^u V_3^2 \frac{\alpha^H}{w^H} +$$

$$-\lambda_2^u \widehat{V_3^{2(3)}} \frac{\alpha^H}{w^H}\frac{dY^3}{dY^2} + \lambda_3^d V_3^3 \frac{\alpha^L}{w^H}\frac{dY^3}{dY^2} - \lambda_3^d \widehat{V_3^{3(2)}} \frac{\alpha^L}{w^H} = 0. \tag{E2}$$

Making use of eq. (13), eq. (E2) becomes:

$$\left(\pi^2+\lambda_2^u+\lambda_2^d\right) V_3^2 \frac{\alpha^H}{w^H} = -\left(\pi^3+\lambda_3^d\right) V_3^3 \frac{\alpha^L}{w^H} \frac{1 + t c_3^2 \frac{\alpha^H}{w^H}}{1 + t c_3^3 \frac{\alpha^L}{w^H}} - \gamma\left(\pi^2+\pi^3\right)\left(1 + t c_3^2 \frac{\alpha^H}{w^H}\right) +$$

$$+\lambda_2^u \widehat{V_3^{2(3)}} \frac{\alpha^H}{w^H} \frac{1 + t c_3^2 \frac{\alpha^H}{w^H}}{1 + t c_3^3 \frac{\alpha^L}{w^H}} + \lambda_3^d \widehat{V_3^{3(2)}} \frac{\alpha^L}{w^H}. \tag{E3}$$

For (a), divide (E1) by (C5) and multiply the result by $\pi^1\gamma\left(t\frac{\partial c^1}{\partial B^1}-1\right)-\lambda_2^d\widehat{V_B^{2(1)}}$. This gives

$$\frac{\alpha^L}{w^L}\frac{V_3^1}{V_B^1}\left[\lambda_2^d\widehat{V_B^{2(1)}}-\pi^1\gamma\left(t\frac{\partial c^1}{\partial B^1}-1\right)\right]=\lambda_2^d\widehat{V_3^{2(1)}}\frac{\alpha^H}{w^H}-\pi^1\gamma\left(1+tc_3^1\frac{\alpha^L}{w^L}\right). \tag{E4}$$

Using notation $\overline{w^1}=\frac{w^L}{\alpha^L}$, $\overline{w^2}=\frac{w^H}{\alpha^H}$, $\overline{w^3}=\frac{w^H}{\alpha^L}$ and $\overline{\Omega^{1,2}}=\frac{\overline{w^1}}{\overline{w^2}}$ we get

$$\frac{\lambda_2^d\widehat{V_B^{2(1)}}}{\gamma\pi^1}\frac{1}{\overline{w^1}}\left(\frac{\widehat{V_3^{2(1)}}}{\widehat{V_B^{2(1)}}}\overline{\Omega^{1,2}}-\frac{V_3^1}{V_B^1}\right)=\tau_1'. \tag{E5}$$

Since assumption 2 implies that $\overline{\Omega^{1,2}}<1$, and single-crossing holds, the METR faced by type 1 is positive.

For (b), we first need expressions for the METR faced by type 2 and 3. Starting with type 2, we divide (E3) by (C6) and multiply the result by $\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\left(t\frac{\partial c^2}{\partial B^2}-1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+\gamma\left(\pi^2+\pi^3\right)\left(t\frac{\partial c^2}{\partial B^2}-1\right)-\lambda_3^d\widehat{V_B^{3(2)}}-\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left(t\frac{\partial c^2}{\partial B^2}-1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}$. This gives

$$\frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}\left[\begin{array}{c}-\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\frac{t\frac{\partial c^2}{\partial B^2}-1}{1+tc_3^3\frac{\alpha^L}{w^H}}+\\ -\gamma\left(\pi^2+\pi^3\right)\left(t\frac{\partial c^2}{\partial B^2}-1\right)+\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\frac{t\frac{\partial c^2}{\partial B^2}-1}{1+tc_3^3\frac{\alpha^L}{w^H}}\end{array}\right]+\frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}\lambda_3^d\widehat{V_B^{3(2)}}=$$

$$=-\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\frac{1+tc_3^2\frac{\alpha^H}{w^H}}{1+tc_3^3\frac{\alpha^L}{w^H}}-\gamma\left(\pi^2+\pi^3\right)\left(1+tc_3^2\frac{\alpha^H}{w^H}\right)+$$

$$+\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\frac{1+tc_3^2\frac{\alpha^H}{w^H}}{1+tc_3^3\frac{\alpha^L}{w^H}}+\lambda_3^d\widehat{V_3^{3(2)}}\frac{\alpha^L}{w^H}. \tag{E6}$$

Manipulating this expression we get

$$\left[1+tc_3^2\frac{\alpha^H}{w^H}+\frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}\left(1-t\frac{\partial c^2}{\partial B^2}\right)\right]\left[\frac{\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}-\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}}{1+tc_3^3\frac{\alpha^L}{w^H}}+\gamma\left(\pi^2+\pi^3\right)\right]=$$

$$=\lambda_3^d\widehat{V_B^{3(2)}}\left(\frac{\widehat{V_3^{3(2)}}}{\widehat{V_B^{3(2)}}}\frac{\alpha^L}{w^H}-\frac{V_3^2}{V_B^2}\frac{\alpha^H}{w^H}\right). \tag{E7}$$

Using eq. (8) and rearranging, we get

$$\tau_2' = \frac{\lambda_3^d \widehat{V_B^{3(2)}} \left(\frac{\widehat{V_3^{3(2)}}}{\widehat{V_B^{3(2)}}} \alpha^L - \frac{V_3^2}{V_B^2} \alpha^H \right)}{\gamma w^H \left(\pi^2 + \pi^3\right) + \frac{1}{1+tc_3^3 \frac{\alpha^L}{w^H}} \left[\left(\pi^3 + \lambda_3^d\right) V_3^3 \alpha^L - \lambda_2^u \widehat{V_3^{2(3)}} \alpha^H \right]}. \tag{E8}$$

To obtain the METR faced by type 3, note that from eq. (C7) we have that

$$-1 = \frac{V_3^3}{V_B^3} \frac{\alpha^L}{w^H} \frac{1 - t\frac{\partial c^3}{\partial B^3}}{1 + tc_3^3 \frac{\alpha^L}{w^H}} - \frac{\lambda_2^u}{\left(\pi^3 + \lambda_3^d\right) V_B^3} \left(\widehat{V_B^{2(3)}} + \widehat{V_3^{2(3)}} \frac{\alpha^H}{w^H} \frac{1 - t\frac{\partial c^3}{\partial B^3}}{1 + tc_3^3 \frac{\alpha^L}{w^H}} \right).$$

Multiplying by $1 + tc_3^3 \frac{\alpha^L}{w^H}$ and rearranging terms gives

$$\frac{V_3^3}{V_B^3} \frac{\alpha^L}{w^H} \left(1 - t\frac{\partial c^3}{\partial B^3}\right) + 1 + tc_3^3 \frac{\alpha^L}{w^H} = \left(1 + tc_3^3 \frac{\alpha^L}{w^H}\right) \frac{\lambda_2^u}{\left(\pi^3 + \lambda_3^d\right) V_B^3} \left(\widehat{V_B^{2(3)}} + \widehat{V_3^{2(3)}} \frac{\alpha^H}{w^H} \frac{1 - t\frac{\partial c^3}{\partial B^3}}{1 + tc_3^3 \frac{\alpha^L}{w^H}} \right). \tag{E9}$$

Using eq. (8), eq. (E9) can be written

$$\tau_3' = \frac{\lambda_2^u \widehat{V_B^{2(3)}}}{\left(\pi^3 + \lambda_3^d\right) V_B^3} \left(1 + \frac{\widehat{V_3^{2(3)}}}{\widehat{V_B^{2(3)}}} \frac{\alpha^H}{w^H} \frac{1 - t\frac{\partial c^3}{\partial B^3}}{1 + tc_3^3 \frac{\alpha^L}{w^H}} \right) \left(1 + tc_3^3 \frac{\alpha^L}{w^H}\right). \tag{E10}$$

Since we already noted that it is not possible that λ_3^d and λ_2^u, which enter the expressions for τ_2' and τ_3' multiplicatively, are both binding or slack at the same time, part (b) of the proposition follows from eq. (E8) and (E10).

■