A Model of Union Formation[*]

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Abstract

In an analysis of the formation of unions within a single firm, this paper addresses conditions under which encompassing unions form. It is shown that a production function satisfying decreasing marginal productivity leads to the formation of encompassing unions. This result holds for different ways of dividing the surplus within the union. The effects of changes in heterogeneity, e.g. increased demand for skilled labor, are also analyzed. In the most reasonable setup, a change in heterogeneity does not affect the decision whether to form a union or not. This contrasts with the result in Jun (1989).

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1 Introduction

Unions are an important labor market institution in many countries, more so in Western Europe than in the United States. Even though the share of organized labor ranges from around 90% in Scandinavia to little more than 10% in the US, there seem to be few models that address the questions why and when unions form. Moreover, labor markets are undergoing constant and rapid change. As pointed out by Nickell and Bell (1995), the demand for skilled relative to unskilled labor has increased. At least in some sectors of the labor market, heterogeneity among workers also seems to have increased. This in turn can affect unions. Hirsch (1982), for example, found empirical evidence that heterogeneity of the labor force affects union membership. Legislation is also changing to make the labor market more flexible. Legislative amendments are likely to affect the formation and organization of unions. In an empirical study, Freeman and Pelletier (1990) found that legislation favorable to unions tends to increase union density.

The main objective of this paper is to analyze the circumstances under which identical workers form an encompassing union. In earlier models, Horn and Wolinsky (1988) and Jun (1989) studied union formation when there are two (groups of) workers. Here we allow for an arbitrary number of workers. We also examine how changes in the relative demand between skill groups, heterogeneity and legislation affect the pattern of unionization.

In the model developed here, the agents consist of a set of workers and a firm. Workers can form unions and bargain either jointly or separately with the firm. The timing is as follows (in two steps): first unions form and then they bargain with the firm. Union formation can be modeled in different ways. Here, the main model relies on the exclusive membership setup in Shin and Yi (1991). In this setup, a worker cannot join a union without the consent of all members of the union. In an extension of the model we also study the case of open membership. In the second step, the unions' payoffs are given by the bargaining game in Westermark (1998).

We study conditions on the production function that lead workers of the same type to form an encompassing union. As an example, consider a production function that satisfies decreasing marginal productivity. Then, in any coalition-proof Nash equilibrium, all workers join the same union. The intuition is that if, say, two workers contemplate standing together, the increase in production due to the two workers is more than twice as large as the marginal contribution of each of the workers. This implies that the wage is higher when bargaining together, since workers and the firm split the surplus equally. There are also some other production functions that lead to the formation of an encompassing union.

The effects of changes in both relative demand for skilled/unskilled workers and heterogeneity depend on how the wage structure within unions is determined. We study a model where the workers bargain noncooperatively over the wage differential. We consider two cases where the outside option differ. In the first case, workers are unemployed if they do not agree on the wage differential. In the second they are employed and gets the stand alone wage. The second setup seems to be the most reasonable. In the first case, an increase in demand for skilled labor or in heterogeneity leads to a breakup of unions in some firms. The most productive worker can get more by bargaining alone. In the other case changes in heterogeneity or demand for skilled labor do not affect the formation of unions. Also, if the game in the first setup is played repeatedly, changes in demand for skilled labor or in heterogeneity do not lead to the breakup of unions. Thus, the main conclusion is that changes in heterogeneity do not affect the formation of unions.

The theoretical model builds primarily on three earlier studies by Horn and Wolinsky (1988), who analyzed bargaining between a firm and two workers, Gül (1989), who examined noncooperative foundations for the Shapley value, and Shin and Yi (1991), whose exclusive membership game serves as a basis for our union-formation game. This paper differs from the paper by Horn and Wolinsky in that we allow for an arbitrary number of workers. A further difference is that the model can be used to analyze the consequences of heterogeneity.

The model is developed in Section 2. Existence of equilibrium and conditions that make similar workers join together in an encompassing union are examined. Some extensions of the model are introduced in Section 3. Heterogeneity is analyzed in Section 4 and Section 5 concludes.

2 The Model

The set of agents consists of a set of workers, denoted W, with generic element i, and a firm, denoted F. Let $\mathcal{W}$ be the set of subsets of W, including the empty set.

Production opportunities are given by the production function $f : \mathcal{W} \to \Re$. Let $\bar{u}_i$ represent the utility for worker i when not working, measured in units of production. The production function and utilities when not working satisfy the following condition.

Definition 1 The production function f and $(\bar{u}_i)_{i \in W}$ satisfy *restricted strict superadditivity* if

$$\text{for all } C, D \subseteq W \text{ such that } C \cap D = \emptyset,\ f(C \cup D) - f(C) > \sum_{j \in D} \bar{u}_j. \quad (1)$$

The economic interpretation of this condition is the following. Suppose the firm employs the workers in C. Then, adding the set D to the firm leads to an increase in output larger than the sum of the losses of utility for the workers in D.

2.1 The Union Formation Game

A union structure is a partition of the set of workers. Each set in the partition is called a union. We use this convention to simplify notation, although it implies that workers bargaining separately are also called unions. Let n denote the number of sets in the partition. In the union formation game, each worker simultaneously announces a set of workers with whom he would like to form a union. Formally, the strategy space for worker i is $S_i = \{T \subseteq W \mid i \in T\}$, with generic element s_i. The strategy space is then $S = \times_{i \in W} S_i$, with generic element s. Let s_{-i} denote $(s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_{|W|})$. Also, let $s_T = (s_i)_{i \in T}$ and $s_{-T} = (s_i)_{i \in W \setminus T}$. The outcome, given a strategy profile s, is a partition of the set of workers, denoted $C(s) = \{C_1, \ldots, C_n\}$. Let $u_i(s)$ denote the payoff for worker i, given the strategy profile s. The payoffs are given by the bargaining game described below.

A union cannot form unless all its' members agree on who should be accepted. Formally, a set C is in $\mathcal{C}(s)$ if for all $i, j \in C$ we have $s_j = s_i \subseteq C$ and for all $k \in W \setminus C$ and all $i \in C$ we have $s_k \neq s_i$. A worker can unilaterally leave a union, but he cannot break it up completely, as illustrated by the following example. Suppose that there are three workers and that $s_i = \{1, 2, 3\}$ for all $i \in W$. The outcome is $\mathcal{C}(s) = \{\{1, 2, 3\}\}$. Now, if 1 deviates and announces $s_1^0 = \{1\}$, the outcome is $\mathcal{C}(s_1^0, s_2, s_3) = \{\{1\}, \{2, 3\}\}$.

The equilibrium concepts we use when studying the union formation game are Nash equilibrium and coalition-proof Nash equilibrium (CPNE).

Definition 2 A strategy profile s is a Nash equilibrium if

$$\text{for all } i \text{ and all } s_i^0 \in S_i,\ u_i(s) \geq u_i(s_i^0; s_{-i}):$$

To define a coalition-proof Nash equilibrium we need to introduce the following concept.

Definition 3 An internally consistent improvement for $T \subseteq W$ on s is defined by induction.

If $|T| = 1$, i.e. $T = \{i\}$ for some $i \in W$, then s_i^0 is an internally consistent improvement for i on s if it is an improvement on s; that is, $u_i(s_i^0; s_{-i}) > u_i(s)$.

If $|T| > 1$, then s'_T is an internally consistent improvement for T on s if

i) $u_i(s'_T, s_{-T}) > u_i(s)$ for all $i \in T$;

ii) there is no $K \subset T$ $(K \neq \emptyset)$ for which there is an internally consistent improvement on (s'_T, s_{-T}).

Definition 4 A strategy profile s is a coalition-proof Nash equilibrium if there is no $T \subseteq W$ that has an internally consistent improvement on s.

The CPNE concept allows for a situation where several agents deviate simultaneously, although with some restrictions. When considering union formation, it seems reasonable to allow for multilateral deviations by workers, since it is probably not difficult for subgroups of workers to meet and discuss forming a union.

2.2 The Bargaining Game

In the bargaining game, the players consist of all the unions that formed in the union formation game and the firm. The firm is denoted C_F. In this section, we denote $C(s)$ by C. With some abuse of notation, the player set is then $\{C_F, C_1, ..., C_n\}$. Let U denote the set of possible union structures

We assume that the payoffs for any union $C_i \in C$ is given by the payoffs in the non-cooperative bargaining game in Westermark (1998), where unions are treated as individual players. Then, for all $C_i \in C$ we define $\tilde{u}_{C_i} = \sum_{j \in C_i} \tilde{u}_j$. Also, recall that c denotes the bargaining cost incurred on the players. The cost is split equally between the participants in the bargaining.

For all $C \in U$, let $U_{C_i}(C)$ denote the expected utility for player $C_i \in C \cup C_F$. Recall that n denotes the number of unions. For all $C \in U$, the payoff for a union $C_i \in C$, is

$$U_{C_i}(C) = \frac{f(W) - f(W \setminus C_i) + t_{C_i}}{2} - \frac{c}{2}. \qquad (2)$$

Also,

$$U_{C_F}(C) = f(W) - \sum_{i=1}^{n} \frac{f(W) - f(W \setminus C_i) + t_{C_i}}{2} - n\frac{c}{2}. \qquad (3)$$

One problem is how the agreed upon payoff for the union should be divided among the union members. The assumption that all members of the union get the same wage accords with the observation that unions often attempt to equalize wages. It has been pointed out by Blau and Kahn (1996) that wages have a higher variance for non-union than for union workers. One explanation for this is that unions are political organizations; see Freeman and Medoff (1984).

To see that political decisions can lead to equalization of wages, consider the following example. Let θ denote the productivity of a worker and let $\bar{\theta}$ denote the mean productivity of the workers in the union. Suppose union members vote on wage profiles that compensate them for some of the productivity difference, $a \cdot \left(\theta - \bar{\theta}\right) + \bar{\theta}$, provided that this profile is feasible for all $a \in [0;1]$. Then, if the median productivity is lower than the average, the median voter chooses to vote for $a = 0$. Thus, given the strategy profiles in the union formation game leading to the coalition structure C, the equal division wage for worker $k \in C_i$ is

$$w_k^{ed}(C_i) = \frac{f(W) - f(W \setminus C_i) + t_{C_i} - c}{2|C_i|} \qquad \forall k \in C_i;$$

where $C_i \in \mathcal{C}$.

An alternative assumption is that union members bargain over the wages. Let $< Z; d >$ denote a bargaining problem, where d is the disagreement point and Z is the feasible set. Let φ denote a bargaining solution. Since utility is transferable, the feasible set for the workers in union C_i is

$$Z_{C_i} = \{x \in \mathbb{R}^{|C_i|} \mid \sum_{k \in C_i} x_k \leq \frac{f(W) - f(W \setminus C_i) + t_{C_i} - c}{2}\}.$$

Let $d_{C_i} \in \mathbb{R}^{|C_i|}$ denote the disagreement point for the workers in union C_i. Let φ denote a bargaining solution. Given that the payoffs are distributed according to φ, the wage for worker $k \in C_i$ is

$$w_k^{\varphi}(C_i) = \varphi_k(Z_{C_i}; d_{C_i}).$$

Also, the stand alone wage is denoted $w_k(k)$.

2.3 Equilibrium

Now consider union formation again and assume that payoffs are given by expressions 2 and 3. It is easily seen that a Nash equilibrium exists. Consider the following strategy profile. Let all workers announce $s_i = i$. Then $C(s) = \{\{1\}, \{2\}, \ldots, \{n\}\}$. Consider any s' such that $s'_i \neq i$ for some i and $s'_j = j$ for all $j \neq i$. Then $C(s) = C(s')$ and thus no worker can gain from a unilateral deviation; see Lemma 8.1 in Shin and Yi (1991). The reason is that, given the strategy profile s, a unilateral deviation cannot change the outcome. At least two workers have to deviate in order for the outcome to change.

To study CPNE in the union formation game we need to determine the payoff distribution within unions. No equilibrium need exist as can be seen by the following example.

Example 1 Suppose $|W| = 3$ and $f(W) = 4$, $f(S) = 3$ whenever $|S| = 2$ and $f(S) = 1$ whenever $|S| = 1$. Furthermore, for all $i \in W$, we assume $\bar{u}_i = 0$. Then, ignoring c, the surplus for a union if all workers form a union is 2, if two workers form a union $\frac{3}{2}$. Let $\wedge$ denote the solution that gives rise to the wages described below. The stand alone wage is $\frac{1}{2}$ for all workers. Let $w_i^\wedge(W) = \frac{2}{3}$ for all $i \in W$, $w_1^\wedge(12) = 0.8$ and $w_2^\wedge(12) = 0.7$, $w_2^\wedge(23) = 0.8$ and $w_3^\wedge(23) = 0.7$, $w_1^\wedge(13) = 0.79$ and $w_3^\wedge(13) = 0.71$. Then no CPNE exists.

Too see this, first consider a strategy profile s that leads to an encompassing union. Then let 1 and 2 deviate and announce $s_1^0 = s_2^0 = \{12\}$. Clearly, $w_1^\wedge(12) > w_1^\wedge(123)$ and $w_2^\wedge(12) > w_2^\wedge(123)$. Also, since $w_1^\wedge(12) > w_1^\wedge(1)$ and $w_2^\wedge(12) > w_2^\wedge(2)$ none of the deviating workers has an incentive to deviate from the profile $(s_1^0; s_2^0; s_3)$. Then $s_1^0 = s_2^0 = \{12\}$ is an internally consistent improvement.

Now consider a strategy profile s that gives rise to the coalition structure $\{\{12\}\{3\}\}$. Let worker 2 and 3 deviate by choosing $s_2^0 = s_3^0 = \{23\}$. Clearly, $w_2^\wedge(23) > w_2^\wedge(12)$ and $w_3^\wedge(23) > w_3^\wedge(3)$. The deviation $s_2^0; s_3^0$ is an internally consistent improvement, since $w_2^\wedge(23) > w_2^\wedge(12) > w_2^\wedge(2)$ and $w_3^\wedge(23) > w_3^\wedge(3)$. By a similar argument, no profile where 2 workers form a union can be an equilibrium.

Lastly, consider a strategy profile s that such that all workers stand separately. Then let 1 and 2 deviate and announce $s_1^0 = s_2^0 = \{12\}$. Clearly, $w_1^\wedge(12) > w_1^\wedge(1)$ and $w_2^\wedge(12) > w_2^\wedge(2)$. Also, since $w_1^\wedge(12) > w_1^\wedge(1)$ and $w_2^\wedge(12) > w_2^\wedge(2)$ none of the deviating workers has an incentive to deviate from the profile $(s_1^0; s_2^0; s_3)$. We conclude that no CPNE exists.

Given some sharing rule φ, define for all $S \subseteq W$, $\Delta_i^\varphi(S; d) = w_i^\varphi(S) - d_i$

for all $i \in S$. Say that φ satisfies the proportional property if

$$\text{for all } i, j \in W, \text{ for all } S \subseteq W, \text{ if } i, j \in S \text{ then } \Delta\varphi_i(S;d) = k_{ij}\,\Delta\varphi_j(S;d).$$

Thus worker j always gets, for example, three times as much of the surplus as worker i if they are in the same union. Under our assumptions, the Nash bargaining solution satisfies the proportional property for $k_{ij} = 1$ for all $i, j \in W$. The weighted Nash bargaining solution can also be accommodated by other choices of k_{ij}.

Theorem 1 *A CPNE exists in the union formation game if φ satisfies the proportional property.*

Proof. Step 1: Normalization.

Let $\Delta^1\varphi_1(W;d) = \Delta\varphi_1(W;d)$. For all $i \in W$, define $k_{i1} = \frac{\Delta\varphi_i(W;d)}{\Delta\varphi_1(W;d)}$ and let $\Delta^1\varphi_i(W;d) = \frac{1}{k_{i1}}\Delta\varphi_i(W;d)$. Then $\Delta^1\varphi_i(W;d) = \Delta^1\varphi_1(W;d)$. Also, let $\Delta^1\varphi_i(S;d) = \frac{\Delta\varphi_i(S;d)}{\Delta\varphi_i(W;d)}\Delta^1\varphi_i(W;d)$. Then, for $i, j \in S$, by the proportional property we have

$$\frac{\Delta^1\varphi_i(S;d)}{\Delta^1\varphi_j(S;d)} = \frac{\frac{\Delta\varphi_i(S;d)}{\Delta\varphi_i(W;d)}\Delta^1\varphi_i(W;d)}{\frac{\Delta\varphi_j(S;d)}{\Delta\varphi_j(W;d)}\Delta^1\varphi_j(W;d)} = \frac{\Delta\varphi_i(S;d)}{\Delta\varphi_j(S;d)}\frac{\Delta\varphi_j(W;d)}{\Delta\varphi_i(W;d)} = 1.$$

Step 2: Construction of strategy profile s^{max}.

Let $\Delta^1\varphi(S;d) = \Delta^1\varphi_i(S;d)$ for all $i \in S$. Find $\arg\max_{S\subseteq W}\{\Delta^1\varphi(S;d)\}$. If there is more than one such set, pick one of the largest, S^{max1}. For all $k \in S^{max1}$, let $s_k = S^{max1}$.

Now find $\arg\max_{S\subseteq W\setminus S^{max1}}\{\Delta^1\varphi(S;d)\}$. If there is more than one such set, pick one of the largest, S^{max2}. For all $k \in S^{max2}$, let $s_k = S^{max2}$.

Repeat this until $\cup_{j=1}^{m} S^{max\,j} = W$. Denote this strategy profile by s^{max}.

Step 3: Equilibrium.

We claim that s^{max} constitutes a coalition-proof Nash equilibrium. No set of workers in S^{max1} has an internally consistent improvement on s^{max}, since they already get the highest possible payoff. This follows since, for $i \in S, T$,

$$\Delta\varphi_i(S;d) > \Delta\varphi_i(T;d) \text{ iff } \Delta\varphi_1(S;d) > \Delta\varphi_1(T;d):$$

Hence, if a set of workers has an internally consistent improvement on s^{max} it must consist of workers in $W \setminus S^{max1}$.

However, no set of workers in S^{max2} has an internally consistent improvement on s^{max} together with some other set of workers in $W \setminus S^{max1}$, since they already get the highest possible wage in $W \setminus S^{max1}$. Hence, if a set of workers has an internally consistent improvement on s^{max} it must consist of workers in $W \setminus \{S^{max1} \cup S^{max2}\}$.

By repeating this argument for all S^{maxj} no set of workers has an internally consistent improvement on the strategy profile s^{max}. Thus, s^{max} constitutes a coalition-proof Nash equilibrium. ■

The intuition behind the proof is as follows. First, normalize all wages according to the wage of, say, worker 1. Proportionality implies that if the (net) nonnormalized wage is higher for a worker in one union than in another, then the (net) normalized wage is also higher. Next, find the coalition that gives the highest normalized wage. The workers in this coalition form a union. By proportionality, all workers in this coalition get the highest possible wage. Then, among the remaining workers, find the coalition that gives the highest wage. This coalition forms a union. Repeat this until all workers are members of unions. The resulting union structure is an equilibrium. This follows, since a deviating coalition cannot consist of workers in the union where the workers get the highest wage. Among the remaining workers, no worker in the coalition that gets the next highest wage wants to deviate. Then a deviating coalition cannot consist of workers in this coalition either. By repeating this argument for all unions, we have an equilibrium.

The Shapley Value

Now assume that the surplus in a union is distributed according to the Shapley value. The surplus that accrues to a union is the benefits from bargaining with the firm. Thus, the value of a coalition C_i is then

$$v(C_i) = \frac{f(W) - f(W \setminus C_i) + t_{C_i} - c}{2}. \qquad (4)$$

Note that if $|C_i| = 1$, $v(C_i)$ is equal to the stand alone wage for the worker in C_i. If the union C_i forms, then the payoff for a union member $k \in C_i$ is

$$w_k^{Sh}(C_i) = \sum_{\substack{S \subseteq C_i \\ k \notin S}} K_S^{C_i} [v(S \cup k) - v(S)]:$$

where $K_S^{C_i} = \frac{|S|!(|C_i| - |S| - 1)!}{|C_i|!}$. Rearranging this and using expression 4 gives

$$w_k^{Sh}(C_i) = \sum_{\substack{S \subseteq C_i \\ k \notin S}} K_S^{C_i} \frac{f(W \setminus S) - f(W \setminus \{S \cup k\})}{2} - \frac{c}{|C_i|} + t_k. \qquad (5)$$

Thus, each worker gets a weighted average of his marginal contributions when joining a union $S \subseteq C_i$. For example, suppose an encompassing union forms. Then the payoff for a worker k is

$$w_k^{Sh}(W) = \sum_{\substack{S \subseteq W \\ k \notin S}} K_S^W \frac{f(W \setminus S) - f(W \setminus \{S \cup k\})}{2} - \frac{c}{|W|} + t_k.$$

Note that this coincides with the solution presented in Stole and Zweibel (1996) when workers bargain separately and sign non-binding contracts.

Existence of a CPNE equilibrium cannot be guaranteed. The following production function and reservation utilities is an example that has no CPNE. Assume that $|W| = 4$ and that $f(W) = 41$, $f(123) = 35$, $f(124) = 40$, $f(134) = 32$, $f(234) = 40$, $f(12) = 20$, $f(13) = 30$, $f(14) = 20$, $f(23) = 25$, $f(24) = 20$, $f(34) = 20$, $f(1) = 10$, $f(2) = 19$, $f(3) = 0$ and $f(4) = 19$. Also, assume that $\bar{u}_1 = \bar{u}_2 = \bar{u}_3 = \bar{u}_4 = 0$.

2.4 Encompassing Unions

Now suppose all workers are identical and that the payoffs are distributed equally among union members. Any symmetric and weakly Pareto optimal solution gives rise to this distribution of payoffs within the union. We assume $f(\emptyset) = 0$. Let $\bar{u} = \bar{u}_i$ for all $i \in W$. If the following condition holds,

$$\frac{f(W)}{2|W|} \geq \frac{f(W) - f(W \setminus i)}{2} \tag{6}$$

then there exists a Nash equilibrium in the union formation game where all workers are members of the same union.

Condition 6 implies that the wage when everyone stands together is higher than when a worker stands alone. Since the only deviation a worker can make when an encompassing union has formed is to stand alone the union structure with an encompassing union is a Nash equilibrium. The bargaining cost also makes an encompassing union more attractive. Alone, a worker has to pay $\frac{c}{2}$ himself. If he is a member of an encompassing union, the cost $\frac{c}{2}$ is split up equally among union members.

Suppose the following condition holds

$$\forall C \subseteq W \text{ we have } \frac{f(W)}{|W|} \geq \frac{f(W) - f(W \setminus C)}{|C|}. \tag{7}$$

This condition implies that the wage is highest when all workers bargain together. Condition 7 is implied by decreasing marginal productivity. Then any CPNE union structure consists of an encompassing union, as is shown below.

Theorem 2 *If φ is symmetric, efficient and 7 holds, then a CPNE union structure consists of an encompassing union.*

Proof. Symmetry and efficiency imply $\varphi = \varphi^{ed}$. We have

$$w_i^{ed}(W) = \frac{f(W)}{2|W|} + \frac{\bar{u}}{2} - \frac{c}{2|W|}$$

and, for $i \in C$,

$$w_i^{ed}(C) = \frac{f(W) - f(W \setminus C)}{2|C|} + \frac{\bar{u}}{2} - \frac{c}{2|C|}.$$

Clearly, for all $C \subseteq W$, expression 7 implies that, for all $i \in C$, $w_i^{ed}(W) > w_i^{ed}(C)$. Then the strategy profile s^* where, for all $i \in W$, $s_i^* = W$ is a CPNE. Since wages are smaller in any union $C \subseteq W$, there is no C that has an internally consistent improvement on s^*.

Now consider a strategy profile s' such that there exists a union $C \in C(s)$ with $C \subseteq W$. Let all workers deviate by playing $s_i'' = W$. Since for all $C \subseteq W$ and all $i \in C$ we have $w_i^{ed}(W) > w_i^{ed}(C)$, all workers gain. Also, by 7, there is no profitable deviation from s'' for any set of workers. Thus s'' is an internally consistent improvement of W on s'. Hence, a strategy profile s

such that there exists a $C \in \mathcal{C}(s)$ with $C \subset W$ cannot be a CPNE. ■

The intuition behind this result is as follows. Since the wage is highest for the encompassing union and since multilateral deviations are allowed, in any structure which has more than one union, all workers deviate together and form an encompassing union. Given that an encompassing union has formed, no set of workers wants to deviate, since that would reduce their wages.

Recall that all unions are selected with the same probability. Other selection probabilities are also reasonable. For example, the probability that a union is selected to bargain with the firm could depend on the number of workers in the union. Let $p^p_{C_i}$ denote the probability that union C_i is selected as proposer, and assume c small. The payoff for a union C_i is

$$p^p_{C_i}[f(W) - f(W \setminus C_i) - \mu_{C_i}] + \mu_{C_i} - p^p_{C_i} c.$$

Assume that payoffs in the union formation game are given by this equilibrium. Then, as long as the probability that the union is selected as proposer is the same for all unions, the condition is the same as condition 7. If the probability that a union is selected as proposer varies among unions, then the condition corresponding to 7 is

$$\forall C \subset W \text{ we have } \frac{p^p_W}{|W|}[f(W) - \mu_W] \geq \frac{p^p_C}{|C|}[f(W) - f(W \setminus C) - \mu_C]:$$

The Shapley Value

Now suppose workers are paid wages according to the Shapley value as in section 2.3 and relax the assumption of identical workers. The payoff for a worker is given by expression 5. Suppose that the production function satisfy decreasing marginal productivity. We have the following result.

Lemma 1 If $\varphi = Sh$ and f satisfies decreasing marginal productivity, then an encompassing union forms.

Proof. Decreasing marginal productivity implies that, for all $S, T \subset W$ such that $S \supset T$ and all $k \in T$ we have

$$\frac{f(W \setminus S) - f(W \setminus \{S \cup k\})}{2} > \frac{f(W \setminus T) - f(W \setminus \{T \cup k\})}{2}.$$

Recall that the wage for the worker k is given by

$$w_k^{Sh}(C_i) = \sum_{\substack{S \subset C_i \\ k \in S}} K_S^{C_i} \frac{f(W \setminus S) - f(W \setminus \{S \cup k\})}{2} - \frac{c}{|C_i|} + u_k. \qquad (8)$$

Consider adding a worker j to some union $C_i \subset W$. Let $C_i' = C_i \cup j$. Let $S \subset C_i$ and $k \in C_i$ be arbitrary. Consider union C_i. The probability that coalition S is selected is $K_S^{C_i} = \frac{|S|!(|C_i| - |S| - 1)!}{|C_i|!}$ and the contribution is $\frac{f(W \setminus S) - f(W \setminus \{S \cup k\})}{2}$.

Consider the union C_i'. The probability that coalition S is selected is

$$K_S^{C_i'} = \frac{|S|!(|C_i'| - |S| - 1)!}{|C_i'|!} = \frac{|C_i| - |S|}{|C_i'|} \frac{|S|!(|C_i| - |S| - 1)!}{|C_i|!}$$

and the contribution is $\frac{f(W \setminus S) - f(W \setminus \{S \cup k\})}{2}$. The probability that $S \cup j$ is selected is

$$K_{S \cup j}^{C_i'} = \frac{|S \cup j|!(|C_i'| - |S \cup j| - 1)!}{|C_i'|!} = \frac{|S| + 1}{|C_i'|} \frac{|S|!(|C_i| - |S| - 1)!}{|C_i|!}$$

and the contribution is $\frac{f(W \setminus \{S \cup j\}) - f(W \setminus \{S \cup k \cup j\})}{2}$. Also, we have

$$w_k^{Sh}(C_i') = \sum_{\substack{S \subset C_i \\ k \in S}} \Bigg\{ K_S^{C_i'} \frac{f(W \setminus S) - f(W \setminus \{S \cup k\})}{2}$$

$$+ K^{C'_i}_{S\cup j}\frac{f(W\setminus\{S\cup j\}) - f(W\setminus\{S\cup k\cup j\})}{2}\Bigg\} - \frac{c}{|C_i|+1} + t_k:$$

Note that

$$K^{C'_i}_{S} + K^{C'_i}_{S\cup j} = \frac{|C_i| - |S|}{|C'_i|}K^{C_i}_S + \frac{|S|+1}{|C'_i|}K^{C_i}_S = K^{C_i}_S:$$

By decreasing marginal productivity we have

$$\frac{f(W\setminus\{S\cup j\}) - f(W\setminus\{S\cup k\cup j\})}{2} > \frac{f(W\setminus S) - f(W\setminus\{S\cup k\})}{2}:$$

Then, since k and S are arbitrary, for all $k \in C_i$, we have $w^{Sh}_k(C'_i) > w^{Sh}_k(C_i)$. By induction, for all D_i and C_i such that $|D_i| > |C_i|$ and all $k \in C_i$, we have $w^{Sh}_k(D_i) > w^{Sh}_k(C_i)$. By an argument similar to the proof of Theorem 1 an encompassing union is the only CPNE union structure. ■

Thus, if a worker is added to a union, expression 5 implies that the payoff for all the workers in the union increases. Then, encompassing unions is a CPNE when the production function satisfy decreasing marginal productivity. Also, it is the unique CPNE, since any other union structure has the workers deviating and forming an encompassing union.

3 Extensions

3.1 Open Membership

We now turn to the case where workers are allowed to join a union without the consent of its members. This seems to be most reasonable when the workers are identical. The union formation game is modified as follows. The strategy space for worker i is $S_i = \{a_1, a_2, \ldots, a_r\}$ where $r \geq |W|$. A coalition C forms if, for all $i, j \in C$, we have $s_i = s_j$ and, for all $k \notin C$ and all $i \in C$, we have $s_k \neq s_i$. Then a worker can unilaterally join a union, as can be seen by the following example. Suppose there are five workers. Assume that $s_1 = s_2 = s_3 = a_2$, $s_4 = a_4$ and $s_5 = a_1$. Then $C(s) = \{\{1, 2, 3\}, \{4\}, \{5\}\}$. Now suppose worker 5 announces $s_5^0 = a_2$ instead. Then $C(s_1; s_2; s_3; s_4; s_5^0) = \{\{1, 2, 3, 5\}, \{4\}\}$.

In this setup, no coalition-proof Nash equilibrium need exist, as can be seen from the next example with identical workers.

Example 2 If the workers are identical, wages in a union depend only on the size of the union. Let $\hat{w}(i)$ denote the wage in a union of size i. Now suppose that $|W| = 5$ and that $\hat{w}(3) > \hat{w}(4) > \hat{w}(2) > \hat{w}(1) > \hat{w}(5)$. First, suppose that a strategy profile s has a union structure where no union consists of exactly three workers. Then let workers $1, 2$ and 3 deviate and announce $s_1^0 = s_2^0 = s_3^0 = a_i$ where $a_i \neq s_4$ and $a_i \neq s_5$. By construction, the deviating workers strictly gain. Also, none of the workers $1, 2$ and 3 wants to deviate from $(s_1^0; s_2^0; s_3^0; s_4; s_5)$, since they get the highest possible wage. Thus, workers $1, 2$ and 3 have an internally consistent improvement on s. Second, suppose that a strategy profile s has a union structure with a union that consists of exactly three workers, say workers 1, 2 and 3. Then worker 4 gains by deviating and announcing $s_4^0 = s_1$, since $\hat{w}(4) > \hat{w}(2) > \hat{w}(1)$. Thus, worker 4 has an internally consistent improvement on s. This implies that no CPNE union structure exists.

Next, consider conditions on the production function for the formation of an encompassing union. Consider the following condition. The condition for an encompassing union to be the only Nash equilibrium coalition structure is

$$\forall C', C \subseteq W \text{ s.t. } C' \subset C \quad \frac{f(W) - f(W \setminus C)}{|C|} \geq \frac{f(W) - f(W \setminus C')}{|C'|}. \qquad (9)$$

This condition implies that the wage increases with the size of the union, taking the bargaining cost into account. It implies condition 7, which follows by setting $C = W$.

Lemma 2 *If φ is symmetric, efficient and 9 hold, then a NE union structure consists of an encompassing union.*

Proof. To see this, recall that the wage with equal sharing, is

$$w_i^{ed}(C) = \frac{f(W) - f(W \setminus C)}{2|C|} + \frac{\bar{u}}{2} - \frac{c}{2|C|}.$$

By condition 9 above, we have $w_i^{ed}(C) > w_i^{ed}(D)$ whenever $|C| > |D|$, for all $i \in D$.

First, consider a union structure with more than one union. Let $C' \in \mathcal{C}(s)$ denote one of the largest unions. Then, for all $i, j \in C'$, $s_i = s_j$. Consider some $k \in W \setminus C'$. Suppose $k \in D' \in \mathcal{C}(s)$. For all $i \in C'$, let $s_k' = s_i$. Since $|C'| \geq |D'|$, we have $w_k^{ed}(C' \cup \{k\}) > w_k^{ed}(D')$. Thus, no strategy profile where there are more than one union can be a NE union structure.

Second, suppose $W \in \mathcal{C}(s)$. Then, since $w_i^{ed}(W) > w_i^{ed}(\{i\})$ no worker gains by deviating from s. ■

Consider a union structure with more than one union. Find the largest possible union. Then the workers in that union have the highest wage, compared with the wage in the other unions. Moreover, if another worker joins the union, the wage increases. Since it is possible for a worker to join a union unilaterally in the open membership setup, he will choose to do so. Hence any initial structure with more than one union cannot be an equilibrium. The union structure where all workers form an encompassing union is a NE, since the wage is lower in all other possible unions.

The condition on the production function for an encompassing union to be a CPNE union structure is the same as in the case of exclusive membership.

3.2 Endogenous Labor Choice

In this section, we make the additional assumption that the firm can choose the level of employment after unions have formed. Thus, we have a game with two time periods. We restrict attention to identical workers and assume that payoffs within a union is distributed according to a symmetric and weakly Pareto optimal solution. Thus, if workers are identical, all workers that are members of the same union receive equal wages.

The player set is $W \cup C_F$. In the first period, each worker i simultaneously announces a set of workers it wants to form a union with. Formally, the actions available to worker i in period 1 is $A_{1i} = \{T \subseteq W \mid i \in T\}$, with generic element a_{1i}. Let $A_1 = \times_{i \in W} A_{1i}$ and let $a_1 = (a_{1i})_{i \in W}$. For each $a_1 \in A_1$ there exist a union structure, denoted $C(a_1)$, defined by the exclusive membership mechanism, as described in Section 2. Now consider the actions available to the firm, given $a_1 \in A_1$. For all $a_1 \in A_1$, we assume that the actions available to the firm is $A_2(a_1) = \{T \subset C(a_1) \cup \emptyset\}$ with generic element a_2. Thus, the firm is allowed to choose a subset of $C(a_1)$ and then it hires all workers in the selected unions. Hiring none of the workers is allowed. Let

Σ denote the strategy space of this game, with generic element σ. Let H_t denote the set of possible histories at time t. Note that, $H_0 = \emptyset$. A strategy in period t is a function of all possible histories up to period t. Given σ, let $U(\sigma) = \{i \in \sigma_2(a_1)$ where $a_1 = \sigma_1(\emptyset)\}$ denote the set of workers the firm employs. For some $\sigma \in \Sigma$, the payoffs for any $j \in C_i \in C(\sigma_1(\emptyset))$ is then

$$u_j(\sigma) = \frac{1}{|C_i|}\frac{f(U(\sigma)) - f(U(\sigma)\setminus C_i) + \bar{u}_{C_i} - c}{2}.$$

Also,

$$u_F(\sigma) = f(U(\sigma)) - \sum_{C_i \in \sigma_2(a_1)\setminus\emptyset} \frac{f(U(\sigma)) - f(U(\sigma)\setminus C_i) + \bar{u}_{C_i} - c}{2}.$$

Any worker $j \in W$ that is not hired by the firm gets $\bar{u}_j$. Let Γ denote this extensive game.

Say that a strategy profile $\sigma \in \Sigma$ is a Perfectly CPNE in the game Γ if it is a CPNE in any proper subgame of Γ. This does not coincide with the concept used in Bernheim, Peleg and Whinston (1987). However, using their concept does not affect our result.

Given some $\sigma \in \Sigma$, since the firm is the only player in the last period, a strategy profile $\sigma \in \Sigma$ that is a Perfectly CPNE requires that the firm must choose to bargain with the subset of unions that gives the firm the highest profits. Also, there cannot be an ICI where the firm deviates together with some workers to a profile where the firm's profits are not maximal, given the resulting union structure. Then the firm can deviate further and choose to hire the unions that gives the firm highest profit.

A motivation for letting the firm choose employment after unions have formed is that the choice of union structure is a more long-run decision than the firm's choice of employment. We examine some of the consequences of introducing a labor choice into the model by studying a three-worker example, related to the analysis in Horn and Wolinsky (1988).

Example 3 Consider an example with three identical workers. Let φ be any symmetric and weakly Pareto optimal solution. We assume $\bar{u}_i = 0$ for $i = 1, 2$ and 3. Let the production function be defined as follows

$$f(S) = \begin{cases} 4 & \text{for } |S| = 3 \\ 2\epsilon & \text{for } |S| = 2 \\ \epsilon & \text{for } |S| = 1 \end{cases}$$

where $0 < \epsilon < 2$. Thus, all workers produce four units, two workers produce 2ϵ units and one worker produces ϵ units. This implies that the workers are nearly perfect complements if ϵ is small.

Consider the payoffs. Suppose all workers are hired and the workers form separate unions. The wages are then $w_i(i) = 2 - \epsilon - \frac{c}{2}$ for all $i = 1, 2, 3$. Now suppose two of the workers, say 1 and 2, join together in a union, and all three workers are hired. Then $w_1^{\varphi}(12) = w_2^{\varphi}(12) = 1 - \frac{\epsilon}{4} - \frac{c}{4}$. If all workers form an encompassing union wages are $w_i^{\varphi}(123) = \frac{2}{3} - \frac{c}{6}$ for $i = 1, 2, 3$. If two workers are hired, the wages for the hired workers are $\frac{\epsilon}{2} - \frac{c}{2}$ if no union forms or $\frac{\epsilon}{2} - \frac{c}{4}$ if the hired workers formed a union. Profit is $\epsilon - \frac{c}{2}$ if the hired workers formed a union and $\epsilon - c$ otherwise. If one worker is hired, the wage is $\frac{\epsilon}{2} - \frac{c}{2}$ and profit is $\frac{\epsilon}{2} - \frac{c}{2}$.

Suppose $c < \frac{1}{4}$. First, suppose $\epsilon = \frac{1}{4}$. The profits of the firm for different hiring levels and union structure is given by the following table.

Table 1. Profits when $\alpha = \frac{1}{4}$.

Union structure	1 hired	2 hired	3 hired
(ijk)	not feasible	not feasible	$2 - \frac{c}{2}$
$(ij)(k)$	$\frac{1}{8} - \frac{c}{2}$	$\frac{1}{4} - \frac{c}{2}$	$\frac{3}{8} - c$
$(i)(j)(k)$	$\frac{1}{8} - \frac{c}{2}$	$\frac{1}{4} - c$	$-\frac{5}{4} - \frac{3c}{2}$

Consider a strategy profile σ leading to a union structure where all workers bargain by themselves. Then the firm hires two workers and wages for the hired workers are $\frac{1}{8} - \frac{c}{2}$. Consider the deviation where two workers j, k announce $\sigma'_{1j}(?) = \sigma'_{1k}(?)$, leading to j and k forming a union. Then the firm hires all workers. Note that both workers that deviate gains, since $w'_j(jk) = w'_k(jk) = \frac{15}{16} - \frac{c}{4} > \frac{1}{8} - \frac{c}{2}$. Also, since $w'_i(123) < w'_i(jk)$, none of the workers j, k has a further deviation. Thus, this deviation is an ICI upon σ. Consider a strategy profile σ leading to a union structure where all workers bargain together. Then the firm hires all workers. A deviation by two workers to form a separate union is again an ICI upon σ, since, for $i \in W$, $w'_i(123) < w'_i(jk)$. Also, a strategy profile that leads to a union structure where two workers form a union is a Perfectly CPNE, since any deviation from this profile leads to lower wages.

Now suppose $\alpha = \frac{5}{4}$. The profits of the firm for different hiring levels and union structures when $\alpha = \frac{5}{4}$ is given by the following table.

Table 2. Profits when $\varepsilon = \frac{5}{4}$.

Union structure	1 hired	2 hired	3 hired
(ijk)	not feasible	not feasible	$2 - \frac{c}{2}$
$(ij)(k)$	$\frac{5}{8} - \frac{c}{2}$	$\frac{5}{4} - \frac{c}{2}$	$\frac{15}{8} - c$
$(i)(j)(k)$	$\frac{5}{8} - \frac{c}{2}$	$\frac{5}{4} - c$	$\frac{7}{4} - \frac{3c}{2}$

The firm always hire three workers. Then $w_j'(W) = \frac{2}{3} - \frac{c}{6}$ and for all $j \in W$, $w_j(j) = \frac{3}{4} - \frac{c}{2}$. Furthermore, when $|S| = 2$ we have $w_j'(S) = \frac{11}{16} - \frac{c}{4}$ for $j \in S$. Consider any strategy profile σ where a union consisting of more than one worker has formed. Consider the deviation where the worker j is in this union. Let $\sigma'_{1j}(\perp) = \{j\}$. Since $w_j(j) > w_j'(S)$ for any S such that $|S| > 1$, this deviation is an ICI upon σ. Also, since $w_k(k) > w_k'(S)$ for all $k \in W$ for any S such that $|S| > 1$, any strategy profile where the workers bargain separately is a Perfectly CPNE.

The reason underlying the result when $\varepsilon = \frac{1}{4}$ is that workers have very strong bargaining power when negotiating by themselves. They can more or less shut down production by going on strike. The firm would then incur losses; hence the firm chooses to hire less than three workers. All players get low payoffs. By joining together, two workers could reduce their bargaining power. Increasing employment from two to three workers makes it possible to utilize the increasing returns of the production function. Everyone would gain, compared with the situation where workers bargain separately. When $\varepsilon = \frac{5}{4}$, this problem does not occur. Thus, the only equilibrium union structure is where each worker stands alone.

As shown by Horn and Wolinsky, if two workers are close complements, they do not form an encompassing union. The reason is that under strong complementarity, when an individual worker withholds his labor, the firm loses a lot of output. Hence he can negotiate a high wage. On the other hand, when the workers are substitutes, output is less affected if one worker withholds his labor; hence this worker gets a low wage if he bargains by himself. In other words, when workers are substitutes, joining together is profitable for the workers. Horn and Wolinsky also claimed (without proof) that the results extend to the case with more than two workers. Contrary to the claim in Horn and Wolinsky, an increase in complementarity (ε decreases from $\frac{5}{4}$ to $\frac{1}{4}$) leads to formation of a union.

4 Demand Shifts and Changes in Heterogeneity

To study changes in relative demand and heterogeneity, we consider a model with two types of workers, 1 and 2.

Let the production function be defined as

	2 works	2 does not work
1 works	$\mu(1+\theta)$	θ
1 does not work	1	0

where μ represents the degree of complementarity between the two workers, and $\theta \geq 1$ the productivity difference.[1] Thus, worker 1 is the most productive worker. We assume $0 < \delta_2 \leq \delta_1$. An increase in demand for skilled workers or an increase in heterogeneity is modeled as an increase in θ.

In the paper by Jun (1989), who studies if two workers form a union or not, equal sharing is obtained when a union is formed. In the paper, workers first decide whether to form a union and then bargain over the wage differential before production starts. The reason equal sharing is obtained in Jun's paper does not seem very sensible, however. In Jun's model, in the case where wages are unequal in a union, workers bargain over the wage differential. When bargaining, the outside option of the high productivity worker is binding. Then bargaining gives him the outside option (the stand alone wage). At an earlier stage, the high productivity worker can choose to stand by himself (and get production started immediately). Because the worker is impatient he does so, since he gets the same wage earlier. Thus no union forms.

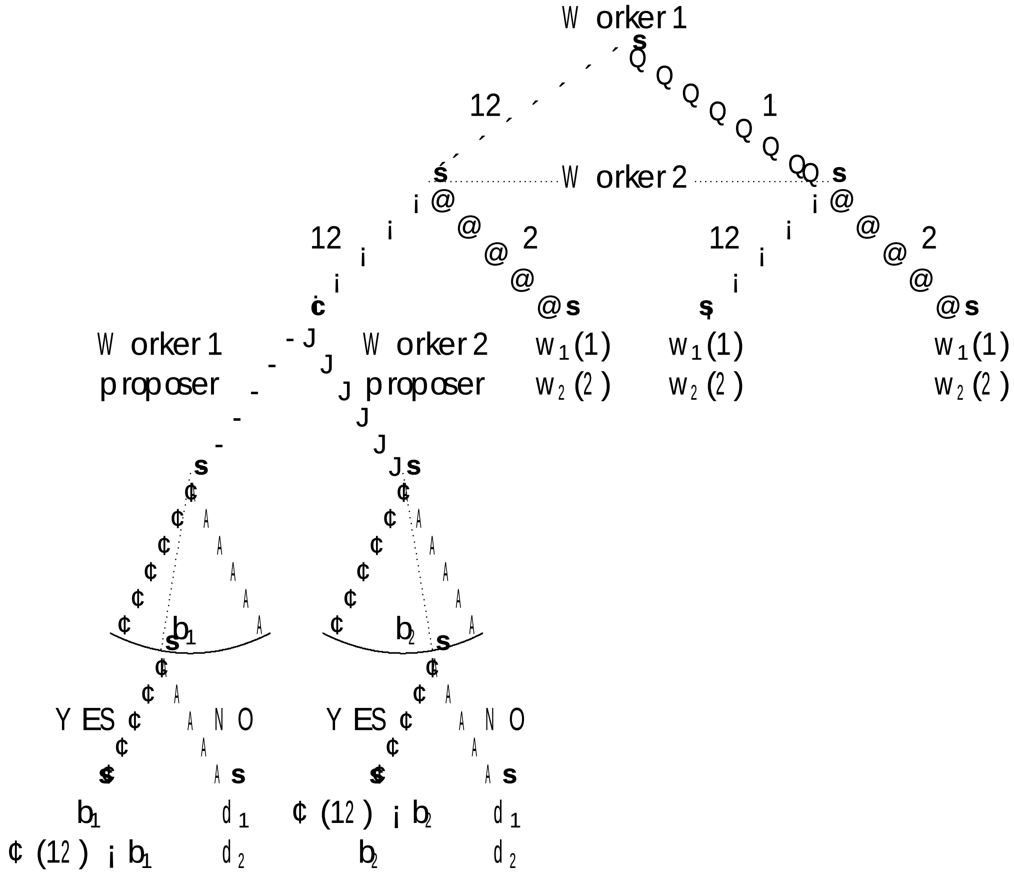
To study this, we study a game where a union first is formed. If a union is formed then noncooperative bargaining over the wage differential takes place. Thus, first the two workers simultaneously announce a coalition of workers. If both workers do not agree to form a union, they both receive their stand alone wages. If they agree to form a union, one of the workers is selected as proposer with equal probability. The selected proposer then

[1] Horn and Wolinsky (1988) use

	U works	U does not work
S works	$y + x$	x
S does not work	x	0

makes a proposal, denoted b_i for $i = 1, 2$, how to divide the surplus within the union. The respondent then answers yes or no. If the proposal is accepted, the allocation is implemented. If the proposal is rejected, each worker i gets a payoff d_i. The figure below describes the game.

Figure 1: The Union Formation Game.



We analyze two cases where we assume $d_i = \delta u_i$ and $d_i = w_i(i)$ for $i = 1, 2$. The motivation for the first setup is that production cannot start until the workers have agreed on how to organize themselves. We study subgame perfect equilibria that are also CPNE of this game. The perfectly CPNE concept presented in Section 3.2 gives results similar to those presented below. However, in a repeated game, studied in the next section, problems with existence arise if the perfectly CPNE concept is used. Recall that a strategy profile is a subgame perfect equilibrium (SPE) if it is a Nash equilibrium in any proper subgame of the game.

The strategy space is denoted Σ^b with generic element σ. Let

$$\Delta(12) = \frac{\mu(1+\theta) - \delta_1 - \delta_2}{2} + \delta_1 + \delta_2$$

be the (gross) surplus for the union.

We have the following result.

Lemma 3 Suppose $d_i = \delta_i$ for $i = 1, 2$. If $\mu < 1 - \frac{1}{(1+\theta)} |(\theta - \delta_1) + (\delta_2 - 1)|$ then any $\sigma \in SPE \cap CPNE$ has a union structure consisting of an encompassing union.

Proof. Recall that the stand alone wages are

$$w_1(1) = \frac{\mu(1+\theta) - 1 - \delta_1}{2} + \delta_1$$

and

$$w_2(2) = \frac{\mu(1+\theta) - \theta - \delta_2}{2} + \delta_2 :$$

First, consider the proposals made by workers if an encompassing union forms.

If a worker i is selected as proposer, he offers the other worker his continuation payoff. Any such bid is accepted. Thus, $b_1 = \Delta(12) - \delta_2$ and $b_2 = \Delta(12) - \delta_1$. The payoff for worker 1 if he is the proposer is then $\frac{\mu(1+\theta) - \delta_1 - \delta_2}{2} + \delta_1$ and the payoff for worker 2 if he is the proposer is then

$\frac{^1(1+\mu)\,¡\,ŧ_1\,¡\,ŧ_2}{2} + ŧ_2$. N ow consider the continuation payo¤ after an encompassing union has formed, but before the proposer is selected. T he continuation payo¤ for worker 1 is $v_1 = \frac{^1(1+\mu)\,¡\,ŧ_1\,¡\,ŧ_2}{4} + ŧ_1$ and for worker 2 $v_2 = \frac{^1(1+\mu)\,¡\,ŧ_1\,¡\,ŧ_2}{4} + ŧ_2$.

N ow consider the union formation game. Clearly, a strategy pro…le ¾ 2 SPE where an encompassing union forms can only be a CPN E if $v_1 \;¸\; w_1(1)$ and $v_2 \;¸\; w_2(2)$. T his implies

$$1 + \frac{(1\;¡\;ŧ_2)}{(1+\mu)} + \frac{ŧ_1\;¡\;\mu}{(1+\mu)} \;¸\; ^1 \qquad (10)$$

and

$$1 + \frac{(\mu\;¡\;ŧ_1)}{(1+\mu)} + \frac{(ŧ_2\;¡\;1)}{(1+\mu)} \;¸\; ^1 : \qquad (11)$$

T heses conditions are both satis…ed if $^1 \;·\; 1\;¡\;\frac{1}{(1+\mu)}\,j(\mu\;¡\;ŧ_1) + (ŧ_2\;¡\;1)j$. I f an encompassing union has formed no worker wants to deviate, since $v_1 \;¸\; w_1(1)$ and $v_2 \;¸\; w_2(2)$.

A lso, if the inequality is strict and if ¾ do not lead to the formation of an encompassing union, both workers has an ICI upon ¾. T he reason is the following. Expressions 10 and 11 imply that the following deviation is an ICI upon ¾. B oth workers announce f12g and choose $b_1 = ¢(12)\;¡\;ŧ_2$ and $b_2 = ¢(12)\;¡\;ŧ_1$. T hese bids are accepted. T hen expressions 10 and 11 imply that both workers gain strictly. ■

A lso, given that a union has formed, suppose bargaining over the wage differential instead is constructed as in a standard R ubinstein bargaining game (with in…nitely many bargaining rounds). T hen workers alternate proposing and payo¤s are discounted. I f workers get the utility of leisure as long as no agreement has been reached, a similar result appears when the discount factor is close to one.

Now suppose θ changes. The effect on the possible values of δ that allows for the formation of an encompassing union is ambiguous, depending on whether $(\theta - \alpha_1) + (\alpha_2 - 1)$ is positive or negative. If $\theta + \alpha_2 > 1 + \alpha_1$ the net surplus accruing to worker 1 is larger than for worker 2. Then

$$\frac{\partial \left(1 - \frac{1}{(1+\theta)}\, |(\theta - \alpha_1) + (\alpha_2 - 1)|\right)}{\partial \theta} = \frac{\alpha_2 - \alpha_1 - 2}{(1+\theta)^2} < 0.$$

If $\theta + \alpha_2 < 1 + \alpha_1$ then

$$\frac{\partial \left(1 - \frac{1}{(1+\theta)}\, |(\theta - \alpha_1) + (\alpha_2 - 1)|\right)}{\partial \theta} = \frac{2 + \alpha_1 - \alpha_2}{(1+\theta)^2} > 0.$$

Note that, if $\theta + \alpha_2 < 1 + \alpha_1$ and $1 - \frac{1}{(1+\theta)} |(\theta - \alpha_1) + (\alpha_2 - 1)| < \delta < 1$, then the sum of stand alone wages is smaller than $\Delta(12)$. Also, it is the low productive worker that gains by leaving the union and standing by himself.

The most reasonable case is clearly when $\theta + \alpha_2 > 1 + \alpha_1$. This holds for example when the utility from leisure is proportional to productivity, i.e. $\alpha_1 = k\theta$ and $\alpha_2 = k$ such that $k < 1$. Then, when θ increases, the cutoff level for δ decreases. An increase in the demand for skilled labor or heterogeneity leads to the breakup in unions form some firms. The high productivity worker gains by leaving the union and standing by himself.

If an encompassing union forms the expected wages are $\frac{\delta(1+\theta) - \alpha_1 - \alpha_2}{4} + \alpha_1$ and $\frac{\delta(1+\theta) - \alpha_1 - \alpha_2}{4} + \alpha_2$. Then, if $\alpha_1 = \alpha_2$ we have equal division, as in Jun (1989).

Now assume $d_i = w_i(i)$ for $i = 1, 2$. We have the following result.

Corollary 1 Suppose $d_i = w_i(i)$ for $i = 1; 2$. If $\mu < 1$ then any $\sigma \in SPE \cap CPNE$ has a union structure consisting of an encompassing union.

Proof. If a worker i is selected as proposer, he offers the other worker his continuation payoff. Thus, $b_1 = \frac{\theta - u_1}{2} + u_1$ and $b_2 = \frac{1 - u_2}{2} + u_2$. Now consider the continuation payoff after an encompassing union has formed, but before the proposer is selected. The continuation payoff for worker 1 is $v_1 = \frac{1}{2} w_1(1) + \frac{1}{2} \left(\frac{\theta - u_1}{2} + u_1 \right)$ and for worker 2 it is $v_2 = \frac{1}{2} w_2(2) + \frac{1}{2} \left(\frac{1 - u_2}{2} + u_2 \right)$. For an encompassing union to form the following must hold

$$v_1 \geq w_1(1)$$

and

$$v_2 \geq w_2(2):$$

Clearly, both conditions hold if $\mu \leq 1$. If $\mu < 1$, by a similar argument as in 3, any $\sigma \in SPE \cap CPNE$ has a union structure consisting of an encompassing union. ■

Thus, in this setup, changes in heterogeneity does not affect the formation of an encompassing union.

4.1 Repeated game

The second setup in the previous section seems to be the most reasonable. Furthermore, if we consider the first setup and assume that the union formation game is played repeatedly, an encompassing union can be formed.

Let δ denote the common discount factor for the workers. Note that a worker can guarantee himself the stand alone wage in the game described above. Also, note that, if $\mu < 1$ then $w_1(1) + w_2(2) < \Delta(12)$. Thus, the sum of stand alone wages is smaller than the sum of the wages when an encompassing union forms. Then, using a folk theorem (Osborne and Rubinstein (1995)), a profile where an encompassing union forms in all periods and where the discounted payoff for each worker i is strictly larger than $\frac{1}{1-\delta} w_i(i)$ is a subgame perfect equilibrium, for a sufficiently large δ. To see this, consider any $\alpha_1 > w_1(1)$ and $\alpha_2 > w_2(2)$ such that $\alpha_1 + \alpha_2 = \Delta(12)$. Consider a strategy profile constructed as follows. Both workers start by announcing $\{12\}$ and proposing $b_1 = \alpha_1$ and $b_2 = \alpha_2$ when each worker is selected as proposer. These bids are accepted. As long as there is no deviation from this profile, the workers continue using announcing $\{12\}$, proposing $b_1 = \alpha_1$ and $b_2 = \alpha_2$ and accepting these proposals. If there is a deviation by worker 2 then worker 1 punishes worker 2 by announcing $\{1\}$ in T time periods and thus forcing worker 2 to his minmax payoff. After the T periods of punishments, worker 1 and 2 announces $\{12\}$, proposing $b_1^0 > \alpha_1$ and $b_2^0 < \alpha_2$ and accept these proposals. A similar argument is used when worker 1 deviates. Theorem 151.1 in Osborne and Rubinstein (1995) implies that this strategy profile is a SPE for δ close to one. Also, if an encompassing union form in all periods, the discounted payoff sum to $\frac{1}{1-\delta}\Delta(12)$ for both workers.

Thus, for any $\mu < 1$, there is a SPE such that an encompassing union forms in all time periods. Also, note that this profile is also a CPNE. This follows since a strategy profile being a SPE implies that no single worker has an ICI upon the strategy profile. Since the discounted payoff sum to $\frac{1}{1-\delta}\Delta(12)$ for both workers which is the highest possible sum of discounted payoffs, both workers cannot gain strictly by a deviation. Also, consider any

SPE such that an encompassing union not form in all periods. Then the discounted payoff sum for both workers is strictly smaller than $\frac{1}{1-\delta}\cdot(12)$. Then, for δ close to one this profile cannot be a CPNE. This follows since, by the folk theorem, there is a SPE strategy profile where an encompassing union form in all time periods and gives a strictly higher discounted payoff for both workers.

4.2 Legislative Advantages for Unions

What happens if union bargaining power is stronger than when workers bargaining separately? Such a situation could occur because unions have certain legislative advantages. This could be modeled in different ways. For example, suppose the union has a better outside option when bargaining with the firm, compared to when workers stand alone. Let ψ denote the outside option for the union, where $\psi > u_1 + u_2$. Then the total surplus for the union is

$$\frac{\mu(1+\theta)+\psi}{2}.$$

Assuming that $d_i = w_i(i)$ for $i = 1;2$. Then, by using the same argument as in Corollary 1, an encompassing union forms if $1 + \frac{\psi-(u_1+u_2)}{(1+\theta)} \geq \mu$. Thus a union forms for a larger set of values of μ if ψ increases. This is in line with the findings of Freeman and Pelletier (1990), who studied the effects of changes in industrial relations legislation on union density in the UK. They found that legislative amendments have an important effect on union density.

5 Conclusions

We have studied the conditions under which similar workers join together in an encompassing union. If workers are of the same type, then the set of production functions leading to a CPNE with an encompassing union contains the set of production functions which satisfy decreasing marginal productivity.

In a bargain between a firm and two workers, Horn and Wolinsky (1988) showed that, if two workers are substitutes, they form a union, whereas if the workers are complements, they bargain separately. In our model we obtained the same result as Horn and Wolinsky with two workers, and were able to show that the result does not necessarily extend to the case of more than two workers. The intuition behind this result is that, if complementarity is high, the workers have strong bargaining power when bargaining separately. Thus, the workers get high wages. The firm then incurs losses if it hires all three workers and thus it chooses to hire less than three workers. This leads to low payoffs for all workers. By forming a union, the workers can reduce their bargaining power and induce the firm to hire all workers. This makes it possible to capture the gain from the increasing returns in production.

Effects of changes in demand between skilled and unskilled labor and in heterogeneity were also analyzed. If workers are unemployed if they do not agree upon the wage differential within the union, then if demand for skilled labor increases, a smaller range of production functions gives rise to an encompassing union (the production function needs to have "enough" decreasing marginal productivity). However, if the game in this case is played repeatedly, an increase in demand for skilled labor does not (necessarily) lead to the breakup of unions. If workers get the stand alone wage if they do not agree upon the wage differential within the union, changes in heterogeneity or demand for skilled labor do not affect the formation of an encompassing union. This is contrary to the result in the paper by Jun (1989). The reason is that an increase in heterogeneity raises not only the stand alone wage for the most productive worker but also the surplus to be shared.

We also examined the consequences of legislative amendments that affect the bargaining power of unions. Greater union bargaining power increases the likelihood that an encompassing union forms.

References

Bernheim, B.D., Peleg, B. and Whinston, M.D. (1987), "Coalition-Proof Nash Equilibria: I. Concepts", Journal of Economic Theory, 42, 1-12.

Blau, F.D. and Kahn, L.M. (1996), "International Differences in Male Wage Inequality: Institutions versus Market Forces", Journal of Political Economy, 104, 791-837.

Freeman, R.B. and Medoff, J.L. (1984), "What do Unions do ?", Basic Books, New York.

Freeman, R.B. and Pelletier, J. (1990), "The Impact of Industrial Relations Legislation on British Union Density", British Journal of Industrial Relations, 28, 141-164.

Fudenberg, D. and Tirole, J. (1991), "Game Theory, MIT Press", Cambridge.

Gül, F. (1989), "Bargaining Foundations of the Shapley Value", Econometrica, 57, 81-95.

Hirsch, B.T. (1982), "The Interindustry Structure of Unionism, Earnings, and Earnings Dispersion", Industrial and Labor Relations Review, 36, 22-39.

Horn, H. and Wolinsky, A. (1988), "Worker Substitutability and Patterns of Unionization", Economic Journal, 98, 484-97.

Jun, B.H. (1989), "Non-cooperative Bargaining and Union Formation", Review of Economic Studies, 56, 59-76.

Nickell, S. and Bell, B. (1995), "The Collapse in Demand for the Unskilled and Unemployment Across the OECD", Oxford Review of Economic Policy, 11(1), 40-62.

Shin, H.-S. and Yi, S.-S. (1991), "Endogenous Formation of Coalitions, Part 1: Theory", mimeo, Harvard University.

Stole, L. and Zweibel, J. (1996), "Organizational Design and Technology Choice under Intrafirm Bargaining", American Economic Review, 86, 195-222.

Westermark, A. (1998), "A Firm-Worker Bargaining Model", Working paper 1998:1, Uppsala University.