Extremism, Campaigning and Ambiguity[*]

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Abstract

This paper studies a model of how political parties use resources for campaigning to inform voters. We show existence of equilibrium under mild assumptions for an arbitrary number of parties. The main result is that if the parties are more extreme, then they spend less resources on campaigning (on average), compared with moderate parties. The reason is the following. Consider voters that are informed by one party only, say party 1. If both parties move closer to each other, then the actual and expected platform moves closer to the indifferent voters peak. By concavity of preferences, the increase in payoff of voting for the party that informed is bigger than the increase in payoff of voting for the other party. Thus, the previously indifferent voter now strictly prefers party 1. The effect makes parties gain more votes by informing when parties are moderate. Since spending increases, voters are (on average) more informed when parties are moderates.

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1 Introduction

In political science, an important issue is information transmission between political representatives and the electorate. This issue has several aspects. One is the question whether parties send truthful information to voters, or not. This issue has been studied by Banks (1990) and others. Banks finds that, if the realized platform of a party is far away from the median of the voter distribution, voters are able to infer the true platform of that party. If the platform is close to the median of the voter distribution, this is not the case. Martinelli (1997) has studied whether voters can learn from parties that have private information during the electoral process. Schultz (1996) studies a situation where parties posses more information about the true state of the world compared with voters. He finds that polarization leads to non-revealing sequential equilibria. However, in none of these papers is it costly to send messages to voters. An important aspect in campaigning is that it is costly to send information to voters. This motivates the study of a model with costly information transmission.

The main aim of this paper is to analyze how parties use resources for campaigning to inform voters. We describe how the resources spent on campaigning depend on how close parties are to each other and how this in turn affects voters.

We study a general model and are able to show existence of equilibrium. In the model there is an exogenously given number of parties. Each party has a predetermined ideology that is drawn from some distribution. This ideology could be determined by the history of the group, for example. The parties care about the number of votes as well as the consumption of some private good. Each party has access to resources that can be used for campaigning. Initially, voters do not know the platform of the parties. To affect the voters the parties use campaigning to inform the voters about the policy of the party. The parties are assumed to use only truthful messages. If a voter is informed by some party, it is assumed that he knows the platform of the party with certainty. This, combined with risk aversion, makes informed voters on average more positive to the party. Given the platforms and the strategies of the parties, voters update their beliefs and then vote sincerely for the parties.

Then we study a symmetric model with two parties and find that, the farther away parties are from each other (on average), the less information is supplied (on average) in equilibrium. Note that informing a voter eliminates the risk of voting for that party. Then, consider voters that are informed by one party only, say party 1, and assume that the platform of party 1 is close to the median voter. For the indifferent voter, the expected platform of party 2 is closer to the voters' peak than the actual platform of party 1. Since voters are risk-averse, the voter would otherwise strictly prefer to vote for party 1. If both parties move closer to each other, then the actual and expected platform moves closer to the indifferent voters peak. Since voter preferences are concave, the closer to the peak a platform is, the flatter preferences are. This implies that the increase in payoff of voting for the party that informed is bigger than the increase in payoff of voting for the other party. Thus, the previously indifferent voter now strictly prefers party 1. The effect makes parties gain more votes by informing when parties are moderate. Since spending increases, voters are (on average) more informed when parties are moderates. Thus, extremism leaves more voters uninformed. Also, the uninformed voters are going to be more uncertain if the parties are further away from each other. The reason is that the variability of spending goes down.

One of the influences of this paper is Harrington and Hess (1996). In Harrington and Hess campaigning is explicitly modeled. Parties are assumed to have a fixed ideology. Parties can use resources either to move their platform closer to the opponent (positive campaigning) or to move their opponents platform further away from the party's own platform (negative campaigning). However, there is no explicit model of why expenditures can affect voter's perceptions of the parties. Thus, the model of influencing voters is modeled as a black box.

The paper by Chappell (1994) has a more sophisticated model of voter behavior. In the model campaigning is assumed to be truthful. There are two parties that can choose either to spend an endowment on campaigning or not. Thus, only two possible levels of campaigning are allowed. Existence of equilibrium cannot be proven even in this simple setup. In contrast, in the model presented here, equilibria generally exist.

In section 2 the model is described and in section 3 existence of equilibrium is analyzed. In section 4 we study how spending depends on how extreme parties are and how this affects voters. Finally, section 5 concludes.

2 The Model

Let $Y \subseteq \Re$ denote the policy space. There is a finite set of parties, denoted by P, and a continuum of voters. Let $X_k \subseteq Y$ denote the set of possible platforms for party k. For all $k \in P$, X_k is compact. Let $X = \times_{k \in P} X_k$ denote the set of platform profiles. Also, for all $S \subset P$, let $X_S = \times_{k \in S} X_k$. For each $k \in P$, the platform is drawn from a probability distribution $g_k : X_k \to \Re_+$. Let $p_k \in X_k$ denote the platform that is drawn. Let $p = (p_k)_{k \in P}$ and for all $S \subset P$, let $p_S = (p_k)_{k \in S}$. The platforms are not necessarily known to voters. Instead, they are perceived as uncertain. However, voters (and parties) know the distribution from which the platforms are drawn. Let $g(p) = \prod_{k \in P} g_k(p_k)$ and for all $S \subset P$, let $g_S(p_S) = \prod_{k \in S} g_k(p_k)$. Thus, the probability that a platform p_k is drawn for party k is independent of the platforms drawn for the parties other than k.

The parties can use resources to inform voters of their platform by campaigning. The parties truthfully reveal their platforms. Thus, a voter that is informed by party k knows p_k with certainty. A motivation for that assumption is that parties repeatedly take part in elections. By observing the actions of the parties in parliament, voters can infer whether parties have told the truth. The cost in terms of loss of reputation by sending untruthful messages deter parties from using such messages. Empirical evidence seems to justify this assumption. See for example Budge and Hofferbert (1990).

Each party k has access to some resource $\omega_k \geq 0$, which can be used either for informing voters or for consumption. Let $\omega_{max} = \max_{k \in P} \omega_k$. Let $c_k \in [0; \omega_k]$ denote the resources party k spends on campaigning and v_k the vote share received by party k. A party is concerned about getting as many votes as possible, as well as the consumption of some private good. Party k has the following utility function, defined over pairs $(c_k; v_k)$,

$$u_k(c_k; v_k) = \omega_k - c_k + \gamma v_k;$$

where $\gamma > 0$ is a weight reflecting the importance of power.[1] A motivation for the assumption that parties care about the number of votes is that the power of a party depends positively on the number of seats it controls. Another motivation is that the distribution of the electorate may be uncertain. Then v_k is the probability of winning.

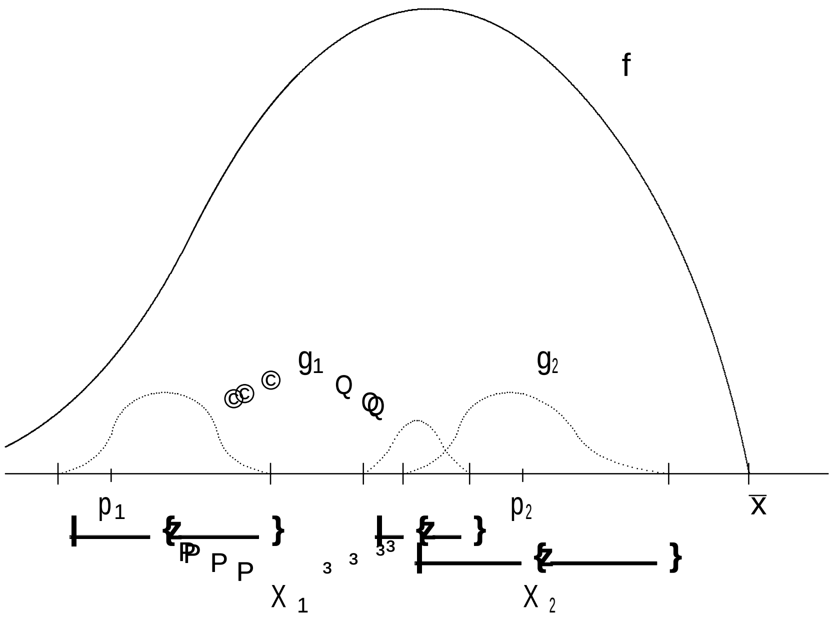
Voters vote sincerely, i.e. vote for the party that gives them the highest expected utility, given their beliefs concerning the platforms. Since there is a continuum of voters, strategic voting is not an issue. Let $Pr_k(p_k)$ denote the voters (common) posterior belief that the true platform of party k is p_k. Posterior beliefs are determined in equilibrium.

The preferences for voters are single peaked. Let x^i denote the ideal point of voter i. All voters with the same ideal point have the same preferences. Then the population of voters can be described by the distribution of the voters' ideal points. Let the density function be denoted by $f : Y \to \Re_+$ and the cumulative distribution function by $F : Y \to [0; 1]$. Let $\underline{x}$ denote the lower bound of the voter distribution, if a lower bound exists. Otherwise, let $\underline{x} = -\infty$. Let $\overline{x}$ denote the upper bound of the voter distribution, if an upper bound exists. Otherwise, let $\overline{x} = \infty$. The figure below describes the model if there are two parties.

[1] The results also hold if $u_k(r_k; v_k)$ is concave, increasing in both arguments, $u_{12} > 0$ and $v_k = \mu_k(c_k)$ where μ_k is increasing and concave in c_k. Then, if $r_k = \omega_k - c_k$, it follows that the function $W(c_k) = u_k(r_k; \mu_k(c_k))$ is concave in c_k.

Let $V : \Re \to \Re$ be a concave function that is symmetric around zero and continuously differentiable. The function $V : \Re \to \Re$ is assumed to represent voters' preferences. We assume that V is concave, symmetric around zero and continuously differentiable.

Figure 1: An example with two parties.



Consider party k and suppose that voter i is not informed by party k. Then, the expected utility for the voter, when voting for party k, is given by

$$\int_{z \in X_k} V\left(z - x^i\right) Pr_k(z)\,dz.$$

If party k has informed the voter the payoff when voting for k is

$$V\left(p_k - x^i\right).$$

2.1 Campaigning

The timing is the following. First, the platforms are revealed to the parties. Then the parties choose campaign expenditures. Parties have complete information. Campaign expenditures and initially, platforms, are unobservable by voters. Otherwise, voters could infer the true platform of a party by observing the resources spent, since they generally depend on the platform of the party.

To see this, consider an example with two political parties. Suppose the strategy of party 2 assigns a different spending level to each platform profile. Consider a voter informed about the policy of party 1 and knowing the spending level party 2 but not the platform. The voter can, by observing the cost, then infer the exact platform of party 2. The parties can observe the other parties' expenditure choices, as well as the platforms of the other parties.

Voters are informed about the policy of party k with probability $\rho(c_k)$ where $\rho : \Re_+ \to [0;1]$. We assume $\rho(0) \geq 0$, $\rho(y) < 1$ and $\rho'(y) \geq 0$ for all $y \in [0;\omega_{max}]$. Also, for all $y > \omega_{max}$ we have $\rho(y) = \rho(\omega_{max})$. This assumption is made for technical convenience only. Also, ρ is concave. A motivation for $\rho(0) > 0$ is that a voter could receive information from other sources such as newspapers and television broadcasts. Since the population is large, $\rho(c_k)$ is also the fraction of the population informed by party k.

Messages cannot be directed to specific groups of voters. Also, the probability that a voter is reached by party k is assumed to be independent of the probability that he is reached by party $j \neq k$.

3 Equilibrium

In this section voters update their beliefs when observing signals from parties. Since voters need to compute expected utility of voting for parties they have not been informed by, we need to restrict the strategies for the parties to integrable functions. Let $L_2[X]$ denote the set of measurable functions on X such that $\|f\| = \left(\int_X |f|^2\right)^{\frac{1}{2}}$ is bounded. Let the set of strategies for party k be

$$T_k = \{f \in L_2[X] \mid 0 \leq f(x) \leq \omega_k \text{ for a.e. } x\}.$$

Thus, the strategy space for each party consist of all integrable functions that are smaller than the endowment almost everywhere and bounded in the L_2 norm. A strategy for party k is denoted c_k. Let $T = \prod_{k \in P} T_k$.

Let $\lambda_k(A)$ denote the Lebesgue measure of $A \subseteq X_k$. For all $S \subseteq P$, let $\lambda_S(A)$ denote the product measure of a set $A \subseteq X_S$. Also, let $\lambda(A)$ denote the product measure of a set $A \subseteq X$.

Convexity and compactness of the strategy sets is shown in the following Lemma.

Lemma 1 *For all $k \in P$, the strategy space T_k is convex and compact.*

Proof. Step 1: Convexity.

Let $c_k; d_k \in T_k$. Then there exists a set $X^c \subseteq X$ such that $\lambda(X^c) = 1$, where for all $x \in X^c$ we have $c_k(x) \leq \omega_k$. Similarly, there exists a set $X^d \subseteq X$ such that $\lambda(X^d) = 1$, where for all $x \in X^d$ we have $d_k(x) \leq \omega_k$. Let $e_k = \alpha c_k + (1 - \alpha) d_k$ for some $\alpha \in [0; 1]$. Note that $\lambda(X^c \cap X^d) = 1$. Clearly, for all $x \in X^c \cap X^d$ we have $0 \leq e_k(x) \leq \omega_k$.

It remains to show that $e_k \in L_2[X]$. Since c_k and d_k are measurable, e_k is measurable. Also, $\int e_k^2 = \alpha^2 \int c_k^2 + 2\alpha(1-\alpha)\int c_k d_k + (1-\alpha)^2 \int d_k^2$. Since $\int c_k^2 < \infty$ and $\int d_k^2 < \infty$ and, moreover, $\int c_k d_k \le \int \omega_k^2$, we have $\int e_k^2 < \infty$. Thus $e_k \in L_2[X]$.

Step 2: Compactness.

Let $B = \{f \mid \|f\| \le \omega_k\}$ be the ball of diameter ω_k. This ball contains T_k. By Alaoglus' Theorem, (Royden, (1988)), it is compact. Thus, we need to show that T_k is closed. Let $\{c_k^n\}_{n=1}^{\infty}$ be a sequence of functions such that $c_k^n \to c_k$ and, for all n, $c_k^n \in T_k$. Then, as a corollary of the Riesz-Fischer Theorem (Aliprantis and Burkinshaw (1990) p. 206), there exist a set $\hat{X} \subseteq X$ such that $\lambda(\hat{X}) = 1$ and a subsequence $\{c_k^{n_l}\}_{l=1}^{\infty}$ that converges pointwise on $\hat{X}$. Suppose $c_k \notin T_k$. Then there exists a set $U = \{z \in X \mid c_k(z) > \omega_k\}$ such that $\lambda(U) > 0$. Note that $\lambda(U \cap \hat{X}) > 0$ and $\int_{U \cap \hat{X}} c_k > \lambda(U \cap \hat{X})\omega_k$. Since $\{c_k^{n_l}\}_{l=1}^{\infty}$ converges pointwise on $\hat{X}$, we have $\int_{U \cap \hat{X}} c_k^{n_l} \to \int_{U \cap \hat{X}} c_k > \lambda(U \cap \hat{X})\omega_k$. This contradicts the fact that $c_k^{n_l} \in T_k$ for all l. ■

3.1 Voters Always Reached with Positive Probability

In this section, we only allow $\rho(0) > 0$. This assumption is relaxed in the next section.

Consider the updating of voter beliefs. Given the information voters get they use the information to revise their beliefs about the parties. Clearly, if a party informs a voter, then the voter knows the platform of that party with certainty. However, if a voter receives information from party k but not from party j, then the voter knows that party j takes the platform p_k as given when choosing his strategy. For each possible platform p_j, given the strategy profile of party j, there is some probability that the voter is not informed. Given some platform p_j, if the probability that the voter is informed by j is lower than the average probability of being informed by j, then the voter increases the weight on this platform p_j. This follows, since being uninformed is more likely when p_j is true. A similar argument shows that, given p_j, if the probability that the voter is informed by j is higher than the average probability of being informed by j, then the voter decreases the weight on the platform p_j. Thus, the voter can use Bayes rule to update beliefs over the types of party j. This process of revising beliefs would in general be different for different p_k, since the strategy of party j in general varies with p_k.

Let $c = (c_k)_{k\in P}$. Since $\rho(0) > 0$, there is a positive probability that voters are informed by the parties in S when p_S is true. Then Bayes rule can be applied and we have the following result. For all $S \subset P$, all $k \notin S$, all $z \in X_k$, $p_S \in X_S$ and all $c \in T$, the posterior density, denoted $Pr_k(z \mid S; c, p_S)$, is

$$Pr_k(z \mid S; c, p_S) = \frac{\int_{y\in X_{P\setminus S\setminus\{k\}}} \prod_{j\in S} \rho(c_j(p_S; z; y)) \prod_{l\in P\setminus S} [1-\rho(c_l(p_S; z; y))]\, g(p_S; z; y)\, dy}{\int_{x\in X_{P\setminus S}} \prod_{j\in S} \rho(c_j(p_S; x)) \prod_{l\in P\setminus S} [1-\rho(c_l(p_S; x))]\, g(p_S; x)\, dx}. \quad (1)$$

If the expression above can be used to define a probability measure, the corresponding measure is denoted $Pr_k(S; c, p_S)$. If $S = \emptyset$, then this is a well defined probability measure, since c is measurable on X. Then, when $S \neq \emptyset$, this is also a well defined probability measure (Billingsley (1986), section 33). When studying convergence of measures, we use the weak* topology.

To see how expression 1 is derived, let $p(S \mid c, x)$ denote the probability that a voter is informed by exactly the parties in S, given c and $x \in \mathcal{X}$. Bayes rule gives

$$\Pr_k(z \mid S; c, p_S) = \frac{\int_{y \in \mathcal{X}_{P\setminus\{S\cup k\}}} p(S \mid c, (p_S; z; y))\, g(p_S; z; y)\, dy}{\int_{x \in \mathcal{X}_{P\setminus S}} p(S \mid c, (p_S; x))\, g(p_S; x)\, dx}. \qquad (2)$$

Since the electorate is large the share of the population of voters that is informed about the platforms for the parties in S, for a given $x \in \mathcal{X}$, is

$$p(S \mid c, x) = \prod_{j \in S} \rho(c_j(x)) \prod_{l \in P\setminus S} [1 - \rho(c_l(x))]. \qquad (3)$$

Combining expressions 2 and 3 gives expression 1.

The following Lemma shows that, for all $S \subset P$, if $c^n \to c$ then there exists a subsequence of $\{c^n\}_{n=1}^{\infty}$ such that, for almost all $p_S \in \mathcal{X}_S$, the subsequence converges pointwise a.e. on $\mathcal{X}_{-S}$.

Note that, if $\{c^n\}_{n=1}^{\infty}$ is a sequence such that $c^n \to c$, then, by the Riesz-Fischer Theorem, there exists a set $\tilde{\mathcal{X}} \subseteq \mathcal{X}$ such that $\lambda(\tilde{\mathcal{X}}) = 1$ and a subsequence, $\{c^{n_l}\}_{l=1}^{\infty}$, that converges pointwise on $\tilde{\mathcal{X}}$.

Lemma 2 *Let $\{c^n\}_{n=1}^{\infty}$ be a sequence such that $c^n \to c$. For all $S \subset P$, there exists a set $\tilde{\mathcal{X}}_S \subseteq \mathcal{X}_S$ such that $\lambda_S(\tilde{\mathcal{X}}_S) = 1$, a set $\tilde{\mathcal{X}}_{-S}(p_S) = \{y \in \mathcal{X}_{-S} \mid (p_S; y) \in \tilde{\mathcal{X}}\}$ such that, for all $p_S \in \tilde{\mathcal{X}}_S$, $\lambda_{-S}(\tilde{\mathcal{X}}_{-S}(p_S)) = 1$ and a subsequence $\{c^{n_l}\}_{l=1}^{\infty}$ such that, for all $p_S \in \tilde{\mathcal{X}}_S$, all $x \in \tilde{\mathcal{X}}_{-S}(p_S)$, $c^{n_l}(p_S; x)$ converges to $c(p_S; x)$.*

Proof. Let $\{c^n\}_{n=1}^{\infty}$ be a sequence such that $c^n \to c$.

Then, let $\tilde{\mathcal{X}}_{-S}(p_S) = \{x \in \mathcal{X}_{-S} \mid (p_S; x) \in \tilde{\mathcal{X}}\}$. By Theorem 22.4 in Aliprantis and Burkinshaw (1990) there exists a set $\mathcal{Y}_S \subseteq \mathcal{X}_S$ such that $\lambda_S(\mathcal{Y}_S) = 1$ and, for all $p_S \in \mathcal{Y}_S$, $\tilde{\mathcal{X}}_{-S}(p_S)$ is measurable.

Step 1: Finding measurable sets $\tilde{X}_S$ and $\tilde{X}_S^c$ such that $\mu_S(\tilde{X}_S) + \mu_S(\tilde{X}_S^c) = 1$, and, for all $p_S \in \tilde{X}_S$, we have $\mu_{-S}(\tilde{X}_{-S}(p_S)) = 1$ and all $p_S \in \tilde{X}_S^c$, we have $\mu_{-S}(\tilde{X}_{-S}(p_S)) < 1$.

a) Finding $\tilde{X}_S$.

Consider the sets $\hat{X}_S = \{p_S \in Y_S \mid \mu_{-S}(\tilde{X}_{-S}(p_S)) = 1\}$ and $\hat{X}_S^c = \{p_S \in Y_S \mid \mu_{-S}(\tilde{X}_{-S}(p_S)) < 1\}$. If $\hat{X}_S$ is measurable, let $\tilde{X}_S = \hat{X}_S$. Suppose $\hat{X}_S$ is not measurable. Let μ^* denote the outer measure associated with μ. By Theorem 12.11 in Aliprantis and Burkinshaw (1990) there exists a measurable set $E \supseteq \hat{X}_S^c$ such that $E \subseteq Y_S$ where $\mu^*(E) = \mu^*(\hat{X}_S^c)$. Then, let $\tilde{X}_S = Y_S \setminus E \subset Y_S \setminus \hat{X}_S^c = \hat{X}_S$. Since $\tilde{X}_S \subseteq \hat{X}_S$ we have, for all $p_S \in \tilde{X}_S$, $\mu_{-S}(\tilde{X}_{-S}(p_S)) = 1$.

Thus, $\tilde{X}_S \subseteq Y_S$ is a set of platforms for the parties in S such that, given any $p_S \in \tilde{X}_S$, $\{c^t\}_{t=1}^{\infty}$ converges pointwise, for almost all $x \in X_{-S}$.

b) Finding $\tilde{X}_S^c$.

If $\hat{X}_S^c$ is measurable, let $\tilde{X}_S^c = \hat{X}_S^c$. Suppose $\hat{X}_S^c$ is not measurable. As in a), there exists a measurable set $E' \supseteq \hat{X}_S$ such that $E' \subseteq Y_S$ where $\mu^*(E') = \mu^*(\hat{X}_S)$. Let $\tilde{X}_S^c = Y_S \setminus E' \subset Y_S \setminus \hat{X}_S = \hat{X}_S^c$. Since $\tilde{X}_S^c \subseteq \hat{X}_S^c$ we have, for all $p_S \in \tilde{X}_S$, $\mu_{-S}(\tilde{X}_{-S}(p_S)) < 1$.

Thus, $\tilde{X}_S^c \subset Y_S$ is a set of platforms for the parties in S such that, given any $p_S \in \tilde{X}_S^c$, $\{c^t\}_{t=1}^{\infty}$ converges pointwise on a set $Z_{-S} \subset X_{-S}$ where $\mu_{-S}(Z_{-S}) < 1$.

c) Proving that $\mu_S(\tilde{X}_S) + \mu_S(\tilde{X}_S^c) = 1$.

Note that since $\mu_S(Y_S) = 1$ we have $\mu_S(\tilde{X}_S) = \mu^*(Y_S \setminus E) = 1 - \mu^*(E)$. Also, we have $\mu_S(\tilde{X}_S^c) = \mu^*(Y_S \setminus E') = 1 - \mu^*(E')$. Also, by definition of the outer measure, we have $\mu^*(E) = \mu^*(\hat{X}_S^c) = 1 - \mu^*(\hat{X}_S) = 1 - \mu^*(E')$.

Combining these conditions gives $\mu_S(\tilde{X}_S) + \mu_S(\tilde{X}_S^c) = 1$. Also, from a) and b), we have $\tilde{X}_S \cup \tilde{X}_S^c \subseteq Y_S$.

Step 2: Proving that $\mu_S(\tilde{X}_S) = 1$.

Suppose that $\mu_S(\tilde{X}_S^c) > 0$. Let $X^1 = \cup_{p_S \in Y_S}(\{p_S\} \times (X_{-S} \setminus \tilde{X}_{-S}(p_S)))$. If X^1 is measurable, let $\hat{X} = X^1$. If X^1 is not measurable, there exists a measurable set $E'' \supseteq X^1$ where $\mu^*(E'') = \mu^*(X^1)$. Moreover, since $X^1 \subseteq X \setminus \tilde{X}$ and $\tilde{X}$ is measurable we have $\mu(E'') = \mu^*(X^1) \le \mu^*(X \setminus \tilde{X}) = \mu(X \setminus \tilde{X})$. Let $\hat{X} = E''$. Then we get $\mu(X \setminus \hat{X}) = 1 - \mu(\hat{X}) \ge \mu(\tilde{X})$.

Consider

$$\mu(\hat{X}) = \int_{z \in \hat{X}} d\mu(z) \ge \int_{p_S \in \tilde{X}_S} \mu_{-S}(X_{-S} \setminus \tilde{X}_{-S}(p_S)) d\mu_S(p_S)$$

$$+ \int_{p_S \in \tilde{X}_S^c} \mu_{-S}(X_{-S} \setminus \tilde{X}_{-S}(p_S)) d\mu_S(p_S) = \int_{p_S \in \tilde{X}_S^c} \mu_{-S}(X_{-S} \setminus \tilde{X}_{-S}(p_S)) d\mu_S(p_S):$$

where the inequality follows from Fubini's Theorem, and the second equality since, for all $p_S \in \tilde{X}_S$, we have $\mu_{-S}(X_{-S} \setminus \tilde{X}_{-S}(p_S)) = 0$. Since, for all $p_S \in \tilde{X}_S^c$, we have $\mu_{-S}(X_{-S} \setminus \tilde{X}_{-S}(p_S)) > 0$, if $\mu_S(\tilde{X}_S^c) > 0$ then $\mu(\hat{X}) > 0$. Since $1 - \mu(\hat{X}) \ge \mu(\tilde{X})$, this contradicts the definition of $\tilde{X}$. Thus, we have $\mu_S(\tilde{X}_S) = 1$. ■

In reality, parties very seldom change their relative ranking. For example, democrats always are to the left of the republicans. Thus, we assume the following.

Separability assumption For all $j, k \in P$ such that $j \neq k$, for all $y \in X_j$ we have either $z < y$ for all $z \in X_k$ or $z > y$ for all $z \in X_k$.

Now consider a voter who is indifferent between two parties j and k. Given $p_S \in X_S$ and $c \in T$, let $x_{jk}(S; c, p_S)$ denote the peak of the voter who is indifferent between voting for party j and party k when informed about the parties in S, if such a voter exists.

The following Lemma shows that, for all $S \subset P$, if $c^n \to c$ and V is strictly concave, then there exists a subsequence, $\{c^{n_l}\}_{l=1}^{\infty}$, such that $x_{jk}(S; c^{n_l}, p_S)$ converges to $x_{jk}(S; c, p_S)$ for almost all $p_S \in X_S$.

To give a hint of the intuition behind the proof assume that $y < x^i < z$ for all $y \in X_j$ and all $z \in X_k$. Consider voters informed by the parties in some $S \subseteq P$. Then, given that we have found some voter i with peak x^i that is indifferent between two parties, moving closer to the party with the highest platform always makes the voter like that party more. Also, moving away from the party with the lowest platform makes the voter lose. This implies uniqueness of the indifferent voter. Also, a sequence $\{c^{n_l}\}_{l=1}^{\infty}$ that converges pointwise almost everywhere affects the voter payoff continuously almost everywhere. This in turn implies that the indifferent voter changes continuously. Note that strict concavity is not used in this informal description of the proof. Strict concavity is needed when it is not the case that $y < x^i < z$ for all $y \in X_j$ and all $z \in X_k$.

Lemma 3 *Let $S \subseteq P$ and $j, k \in P$ such that $j \neq k$ be given. Let $\{c^n\}_{n=1}^{\infty}$ be a sequence such that $c^n \to c$. Suppose $x_{jk}(S; c^n, p_S)$ exists for all n. If the Separability assumption is satisfied and V is strictly concave, then there exists a set $\tilde{X}_S \subseteq X_S$ such that $\lambda_S(\tilde{X}_S) = 1$ and a subsequence $\{c^{n_l}\}_{l=1}^{\infty}$ such that for all $p_S \in \tilde{X}_S$, $x_{jk}(S; c^{n_l}, p_S)$ converges to $x_{jk}(S; c, p_S)$.*

Proof. Step 1: The indifferent voter is unique.

Fix c. Suppose $j, k \in S$. The indifferent voter is given by $x_{jk}(S; c, p_S) = x^i$ where x^i is chosen such that

$$\int_{z \in X_k} V\left(z - x^i\right) \Pr_k(z \mid S; c, p_S)\,dz = \int_{y \in X_j} V\left(y - x^i\right) \Pr_j(y \mid S; c, p_S)\,dy. \quad (4)$$

Suppose without loss of generality that, for all $z \in X_k$ and all $y \in X_j$ we have $z > y$. Differentiating both sides of expression 4 with respect to x^i gives the change in utility when voting for k as

$$-\int_{z \in X_k} V'\left(z - x^i\right) \Pr_k(z \mid S; c, p_S)\,dz$$

and the change in utility when voting for j as

$$-\int_{y \in X_j} V'\left(y - x^i\right) \Pr_j(y \mid S; c, p_S)\,dy.$$

Consider any x^i, y and z. Since $z > y$ we know that $z - x^i > y - x^i$. Strict concavity implies that $V'(w) > V'(w')$ whenever $w < w'$. This implies that

$$\int_{z \in X_k} V'\left(z - x^i\right) \Pr_k(z \mid S; c, p_S)\,dz < \int_{y \in X_j} V'\left(y - x^i\right) \Pr_j(y \mid S; c, p_S)\,dy.$$

Then any voter h where $x^h > x^i$ strictly prefers party j. A similar argument works for any other S.

Step 2: Continuity in c.

By Fubini's Theorem (Royden p. 307) there exists a set X^I_S where $\lambda_S(X^I_S) = 1$ such that, for all $p_S \in X^I_S$, the function $c_j(p_S; y)$ is integrable on X_{-S}. Since ρ is continuous, $\rho(c_j(p_S; y))$ is integrable. Also, since $\rho(0) > 0$, $\frac{1}{\rho(c_j(p_S; y))}$ is integrable. Then, let $\tilde{X}_{-S}(p_S) = \{x \in X_{-S} \mid (p_S, x) \in \tilde{X}\}$. Also, from Lemma 2, there exists a set $\tilde X_S$ such that $\lambda_S(\tilde X_S) = 1$, a set $\tilde X_{-S}(p_S)$ such that $\lambda_{-S}(\tilde X_{-S}(p_S)) = 1$ and a subsequence, $\{\sigma^l\}_{l=1}^{\infty}$, such that for all $p_S \in \tilde X_S$ and all $y \in \tilde X_{-S}(p_S)$, $\sigma^l(p_S; y)$ converges to $\sigma(p_S; y)$.

Consider any $p_S \in \tilde X_S \cap X^l_S$. From the assumptions on ρ, there exists some $\bar\rho_k \in \Re_+$ such that, for all $z \in X_k$, we have $Pr_k(z | S; \sigma^l; p_S) \leq \bar\rho_k < \infty$. Since $Pr_k(z | S; \sigma^l; p_S)$ is measurable, by the Lebesgue Convergence Theorem (Royden p. 267) we have

$$\int_{z \in X_k} V\left(z - x^i\right) Pr_k(z | S; \sigma^l; p_S)\, dz \to \int_{z \in X_k} V\left(z - x^i\right) Pr_k(z | S; \sigma; p_S)\, dz \tag{5}$$

for all $p_S \in \tilde X_S \cap X^l_S$. Note that expression 5 implies that $x_{jk}(S; \sigma; p_S)$ exists. Since $\int_{z \in X_k} V(z - x^i) Pr_k(z | S; \sigma; p_S)\, dz$ is continuous in x^i, the solution for x^i in 4 is continuous, for all $p_S \in \tilde X_S \cap X^l_S$. ■

Note that, if there does not exist a voter that is indifferent between party j and party k for all l there does not exist a voter that is indifferent between party j and party k when the strategy profile is σ. Too see this, suppose that the expression $\int_{z \in X_k} V(z - x^i) Pr_k(z | S; \sigma^l; p_S)\, dz$ is larger than the expression $\int_{z \in X_j} V(z - x^i) Pr_k(z | S; \sigma^l; p_S)\, dz$ for all l. Then, by expression 5, the first expression must be larger than the second when the strategy profile is σ.

To compute the votes we first define

$$U(k; x^i | S; \sigma; p_S) = \begin{cases} -V(p_k - x^i) & \text{if } k \in S \\ -\int_{z \in X_k} V(z - x^i) Pr_k(z | S; \sigma; p_S)\, dz & \text{otherwise.} \end{cases}$$

Given p_S and c, $U(k; x^i | S; c, p_S)$ is the expected payoff for a voter with peak x^i if he is informed by the parties in $S \subset P$ and votes for party k.

For any $c \in T$, let

$$O_k(S; c, p_S) = \{x^i \in Y \mid U(k; x^i | S; c, p_S) \geq U(j; x^i | S; c, p_S) \forall j \in P\}.$$

Given $S \subset P$, $p_S \in X_S$ and $c \in T$, $O_k(S; c, p_S)$ is the set of ideal points of the voters who, if informed by just the parties in S, would prefer party k. Note that the expression $\int_{x \in O_k(S; c, p_S)} f(x) dx$ then is the mass of voters that prefer party k.

Also, note that the separability assumption implies that $O_k(S; c, p_S)$ is an interval. This follows, since if any two voters prefer to vote for party k, by strict concavity of V, any voter in between these two voters strictly prefer to vote for party k.

Assume that $O_k(S; d; p_S)$ is nonempty. Suppose $\min\{x \in O_k(S; d; p_S)\}$ and $\max\{x \in O_k(S; d; p_S)\}$ exist. Then there are parties $j, l \in P$ such that, given S, there is a voter that is indifferent between voting for party k and voting for parties j and l, respectively. Note that these parties might be different for different S.

Then, for some $j \in P$, we have $x_{jk}(S; d; p_S) = \min\{x \in O_k(S; d; p_S)\}$ and for some $l \in P$ we have $x_{lk}(S; d; p_S) = \max\{x \in O_k(S; d; p_S)\}$. Then

$$\int_{x \in O_k(S; d; p_S)} f(x) dx = \int_{x_{jk}(S; d; p_S)}^{x_{lk}(S; d; p_S)} f(x) dx.$$

If $\min\{x \in O_k(S; d; p_S)\}$ or $\max\{x \in O_k(S; d; p_S)\}$ do not exist, replace $x_{jk}(S; d; p_S)$ or $x_{lk}(S; d; p_S)$ with $\underline{x}$ and $\overline{x}$, respectively. If $O_k(S; d; p_S)$ has measure 0 or is empty, then $\int_{x \in O_k(S; d; p_S)} f(x) dx = 0$.

Then, let $v_k(c_k; c_{-k} \mid \delta; p)$ denote the votes for party k if party k chooses c_k, the other parties choose c_{-k}, and voters update beliefs according to $\delta \in T$, for some $p \in X$. We have

$$v_k(c \mid \delta; p) = \rho(c_k(p)) \sum_{\substack{S \subseteq P \\ k \in S}} \prod_{\substack{j \in S \\ j \neq k}} \rho(c_j(p)) \prod_{l \in P \setminus S} [1 - \rho(c_l(p))] \int_{O_k(S;\delta;p_S)} f(x)\,dx$$

$$+ [1 - \rho(c_k(p))] \sum_{\substack{S \subseteq P \\ k \notin S}} \prod_{j \in S} \rho(c_j(p)) \prod_{\substack{l \in P \setminus S \\ l \neq k}} [1 - \rho(c_l(p))] \int_{O_k(S;\delta;p_S)} f(x)\,dx. \quad (6)$$

Note that, by separability, two parties cannot be perceived as identical by voters.

Let

$$w_k(c \mid \delta; p) = u_k(c_k; v_k(c \mid \delta; p)).$$

Since $v_k(c_k; c_{-k} \mid \delta; p)$ can be rewritten as $\rho(c_k)A(c_{-k} \mid \delta; p) + B(c_{-k} \mid \delta; p)$ and ρ is concave, $w_k(c \mid \delta; p)$ is quasi-concave in c_k. Too see this, fix p and first assume that $A(c_{-k} \mid \delta; p) > 0$.

By concavity of ρ we have

$$\frac{\partial^2 w_k(c_k(p); c_{-k} \mid \delta; p)}{\partial c_k^2(p)} = \gamma A(c_{-k} \mid \delta; p)\frac{\partial^2 \rho(c_k(p))}{\partial c_k^2(p)} - 1 < 0.$$

This implies that w_k is concave.

Second, suppose $A(c_{-k} \mid \delta; p) \leq 0$. Since $\frac{\partial \rho(c_k(p))}{\partial c_k(p)} > 0$ we have

$$\frac{\partial w_k(c_k(p); c_{-k} \mid \delta; p)}{\partial c_k(p)} = \gamma A(c_{-k} \mid \delta; p)\frac{\partial \rho(c_k(p))}{\partial c_k(p)} - 1 < 0.$$

Since the function w_k is decreasing in $c_k(p)$, it is quasi-concave.

An equilibrium is a strategy profile for the parties such that, firstly, each party chooses an optimal strategy, taking the other parties' strategies and voter beliefs as given and secondly, voters revise their beliefs given the strategy profile of the parties.

Definition 1 *An Informational Nash Equilibrium is a $c^* \in T$ such that*

i) *for all k, $c^*_k(z) = \arg\max_{e \in [0;\omega_k]} \omega_k - e + \gamma v_k(e; c^*_{-k} \mid c^*; z)$ for almost all $z \in \mathcal{X}$;*

ii) *for all $z \in \mathcal{X}_k$, all $S \subseteq P$, all $k \notin S$ and all $p_S \in \mathcal{X}_S$ posterior beliefs are given by $Pr_k(z \mid S; c^*; p_S)$.*

(7)

The first condition means that c^* is a best response almost everywhere. The second means that voters use Bayesian updating, according to the profile c^*.

Now consider the best-reply correspondence for party $k \in P$, given that the other parties use the strategy profile c_{-k} and voters update their beliefs according to c. Thus, for all $c \in T$, let

$$h^*_k(c) = \{f_k \in L_2(\mathcal{X}) \mid f_k(z) = \arg\max_{e \in [0;\omega_k]} \omega_k - e + \gamma v_k(e; c_{-k} \mid c; z) \text{ a.e. on } \mathcal{X}\}.$$

Clearly $h^*_k : T \to T_k$. Let $h^* = (h^*_k)_{k \in P}$. Then $h^* : T \to T$.

Since expression $w_k(c \mid d; p)$ is quasi-concave in c_k, convex-valuedness of h^* follows.

The following Lemma shows that h^* has a closed graph.

Lemma 4 *If the Separability assumption is satisfied and V is strictly concave, h^* has a closed graph.*

Proof. Step 1: $v_k(f_k; c_{-k} \mid c, p)$ continuous in $(f_k; c)$ a.e. on X.

Again let $\{c^n\}_{n=1}^{\infty}$ be a sequence such that $c^n \to c$ and let $\{f_k^n\}_{n=1}^{\infty}$ be a sequence such that $f_k^n \in h_k^*(c^n)$ for all n and $f_k^n \to f_k$. Then, by the Riesz-Fischer Theorem, there exists a subsequence $\{c^{n_r}\}_{r=1}^{\infty}$ that converges pointwise a.e. on X. Corresponding to $\{c^{n_r}\}_{r=1}^{\infty}$ there is a subsequence $\{f_k^{n_r}\}_{r=1}^{\infty}$. Again, by the Riesz-Fischer Theorem, there exists a further subsequence $\{f_k^{n_{r'}}\}_{r'=1}^{\infty}$ that converges pointwise a.e. on X. Then, let $\hat{X} \subseteq X$ denote the set of platforms such that $c^{n_{r'}}$ and $f_k^{n_{r'}}$ converge pointwise. Clearly, $\lambda(\hat{X}) = 1$.

As in Lemma 3 there exists a set $X_S^{!}$ where $\lambda_S(X_S^{!}) = 1$ such that, for all $p_S \in X_S^{!}$ and all $j \in P \setminus S$, the function $c_j(p_S; y)$ is integrable on X_{-S}. Let $\tilde{X}(S) = \{(p_S; y) \in X \mid p_S \notin \tilde{X}_S \cap X_S^{!}\}$. From Lemma 2 we have $\lambda(\tilde{X}(S)) = 0$. Let $\tilde{X}^* = \hat{X} \setminus \{\cup_{S \subseteq P} \tilde{X}(S)\}$. Since $\lambda(\tilde{X}(S)) = 0$ for all $S \subseteq P$ and P is finite, $\lambda(\tilde{X}^*) = 1$. Then, since $c^{n_{r'}}$ converge pointwise, $x_{jk}(S; c^{n_{r'}}; p_S)$ converges to $x_{jk}(S; c, p_S)$ by Lemma 3. Since ρ is continuous $v_k(f_k^{n_{r'}}; c_{-k}^{n_{r'}} \mid c^{n_{r'}}; p)$ converges to $v_k(f_k; c_{-k} \mid c, p)$ for all $p \in \tilde{X}^*$ such that $\lambda(\tilde{X}^*) = 1$.

Step 2: $f \in h^*(c)$.

Step 1 implies that

$$f_k(p) = \arg\max_{e \in [0; \omega_k]} \omega_k - e + \nu v_k(e; c_{-k} \mid c, p)$$

for all k and all $p \in \tilde{X}^* \setminus U$ where $U \subset \tilde{X}^*$ and $\lambda(U) = 0$. Suppose $\lambda(U) > 0$. Then, for all $p \in U$,

$$f_k(p) \neq \arg\max_{e \in [0; \omega_k]} \omega_k - e + \nu v_k(e; c_{-k} \mid c, p):$$

Clearly $f_k^{n_{r'}}$ and $c^{n_{r'}}$ converges pointwise on $U \cap \tilde{X}^*$ where $\lambda(U \cap \tilde{X}^*) > 0$ By Egoroff's Theorem, there exists a set $\hat{U} \subseteq U \cap \tilde{X}^*$ such that $\lambda(\hat{U}) > 0$ where

$f_k^{n_{r'}}$ and $c^{n_{r'}}$ converge uniformly on $\hat{U}$. Thus, for all $p \in \hat{U}$, there exists a number $\hat{r}$ such that, for all $r' > \hat{r}$ we have

$$f_k^{n_{r'}}(p) \neq \arg\max_{e \in [0, \omega_k]} \omega_k - e + \gamma v_k(e, c_{-k}^{n_{r'}} \mid c^{n_{r'}}, p);$$

contradicting the definition of $\{f^{n_{r'}}\}_{r'=1}^{\infty}$. Thus, $f \in h^*(c)$. Thus h^* has a closed graph. ■

By using Lemma 4 we are now able to show existence.

Proposition 1 *If the Separability assumption is satisfied and V is strictly concave, there exists an Informational Nash Equilibrium c^*.*

Proof. Note that by Lemmas 1, 4 above and Theorem 14.49 in Aliprantis and Border (1994), a fixed point $c^* = h^*(c^*)$ exists. Then, for all $p \in X$, all $S \subseteq P$ and all $k \in P$, by construction of h^*, the beliefs are given by

$$Pr(z \mid S, c^*, p_S) =$$

$$\frac{\int_{y \in X_{P \setminus S \cup \{k\}}} \prod_{j \in S} \rho(c_j^*(p_S, z, y)) \prod_{l \in P \setminus S} [1 - \rho(c_l^*(p_S, z, y))] g(p_S, z, y) dy}{\int_{v \in X_{P \setminus S}} \prod_{j \in S} \rho(c_j^*(p_S, v)) \prod_{l \in P \setminus S} [1 - \rho(c_l^*(p_S, v))] g(p_S, v) dv}.$$

Also, given these beliefs and by the construction of the mapping h^*, c^* satisfies i) for all $k \in P$. Thus c^* is an Informational Nash Equilibrium. ■

The main idea behind the proof is to start with some arbitrary strategy profile for the parties and let voters revise their beliefs according to this strategy profile. Given this profile and the revised beliefs, each party chooses an optimal strategy. Thus, we construct a mapping from the set of strategy profiles to itself. Then a fixed point theorem can be applied to show existence.

3.2 Allowing $\rho(0) = 0$

Note that, if $\rho(0) = 0$, the probability measures defined in expression 1 need not be well defined.

Let

$$\rho^n(y) = \rho(y) + \frac{1 - \rho(\omega_{max})}{2n}$$

for all $y \in [0, \omega_{max}]$.

Since $\rho(\omega_{max}) < 1$ we have $0 < \rho^n(y) < 1$ for all $y \in [0, \omega_{max}]$. Also, $\frac{\partial \rho^n(y)}{\partial y} = \rho'(y) \geq 0$ for all $y \in [0, \omega_{max}]$. Then ρ^n converges pointwise to ρ as $n \to \infty$. Since ρ is concave, ρ^n is concave.

Say that c is an INE a.e. if the profile c satisfies the requirement ii) in expression 7 almost everywhere. Thus, we require that beliefs are updated according to c^* almost everywhere.

Definition 2 *The profile $c^* \in T$ is an Informational Nash Equilibrium a.e. if*

i) for all k, $c^*_k(z) = \arg\max_{e \in [0, \omega_k]} \omega_k - e + \gamma v_k(e; c^*_{-k} \mid c^*; z)$ for almost all $z \in X$;

ii) for all $z \in X_k$, all $S \subseteq P$, all $k \notin S$ there exists $Z_S \subseteq X_S$ where $\lambda_S(Z_S) = 1$ such that for all $p_S \in Z_S$, posterior beliefs are given by $Pr_k(z \mid S; c^*; p_S)$ when Bayes rule applies.

(8)

The following result shows existence of an INE a.e.

Theorem 1 *If $\rho(0) = 0$ there exists a $\sigma^* \in T$ such that σ^* is an INE a.e.*

Proof. Step 1: The candidate equilibrium profile and beliefs.

Consider a sequence $\{\rho^n\}_{n=1}^{\infty}$. Since ρ^n is concave, for each n, an equilibrium $\sigma^n \in T$ exists by Proposition 1. Let $\{\sigma^n\}_{n=1}^{\infty}$ denote a sequence of such equilibria. Since T is compact there exists a convergent subsequence, $\{\sigma^{n_g}\}_{g=1}^{\infty}$ with limit $\sigma^* \in T$. Also, for all n, all $S \subset P$ and all $k \in S$, there exists a probability measure, $Pr_k(S; \sigma^n; p_S)$ defined as in expression 1. Note that, since X is compact, any such measure is tight. Also, note that, by the Riesz-Fischer Theorem, there exists a further subsequence $\{\sigma^{n_h}\}_{h=1}^{\infty}$ that converges pointwise a.e. on X.

Step 2: Existence of limit probability measures.

Consider the subsequence $\{\sigma^{n_h}\}_{h=1}^{\infty}$. Fix $S \subset P$ and $k \in S$. Take the subsequence of measures corresponding to this subsequence. Denote the elements of this subsequence $Pr_k(S; \sigma^{n_h}; p_S)$. Then, by Helly's Theorem (Billingsley (1986), p. 392) there exists a further subsequence of measures that converges weakly to some well defined measure, $Pr_k^*(S; p_S)$. Let $\{\sigma^{n_{h_l}}\}_{l=1}^{\infty}$ denote the subsequence of equilibria corresponding to the subsequence of measures. Clearly, this sequence is a subsequence of $\{\sigma^{n_h}\}_{h=1}^{\infty}$. Thus, $Pr_k(S; \sigma^{n_{h_l}}; p_S)$ converges weakly to $Pr_k^*(S; p_S)$. By repeating this argument for all $S \subset P$ and all $k \in S$ there exists a subsequence $\{\sigma^{n_{h_q}}\}_{q=1}^{\infty}$ such that, for all $S \subset P$, all $k \in S$, $Pr_k(S; \sigma^{n_{h_q}}; p_S)$ converges weakly to $Pr_k^*(S; p_S)$.

Step 3: Proof that σ^* is a best response, given limit beliefs.

Consider the subsequence $\{\sigma^{n_{h_q}}\}_{q=1}^{\infty}$. This subsequence converges pointwise on $\tilde{X}$, such that $\lambda(\tilde{X}) = 1$. Note that, since $\sigma^{n_{h_q}}(v)$ converges to $\sigma^*(v)$

for any $v \in \tilde{X}$ then $\rho^n(c^{*n\langle q\rangle}(v))$ converges to $\rho(c^*(v))$. Since the sequence of measures converges weakly, for all $S \subset P$, for all $k \notin S$ and all $p_S \in X_S$, we have, by the weak* topology

$$\int_{z\in X_k} V(z - x^i)Pr_k(z \mid S; c^{*n\langle q\rangle}; p_S)dz \to \int_{z\in X_k} V(z - x^i)Pr_k(z \mid S; p_S)dz$$

for all $x^i \in Y$.

When voter beliefs are given by $Pr_k^*(S; p_S)$ for all $j, k \in P$, all $S \subset P$ and all $p_S \in X_S$, let $x^*_{jk}(S; p_S)$ denote the peak of the indifferent voter, when informed by just the parties in $S \subset P$ and when $p_S \in X_S$ is true.

By a continuity argument similar to step 2 in Lemma 3, for all $S \subseteq P$ and all $p_S \in X_S$, $x_{jk}(S; c^{*n\langle q\rangle}; p_S)$ converges to $x^*_{jk}(S; p_S)$ for all $j, k \in P$. Also, for all $v \in \tilde{X}$, since $c^{*n\langle q\rangle}(v)$ converges to $c^*(v)$ and ρ^n converges pointwise to ρ it follows that $\rho^n(c^{*n\langle q\rangle}(v))$ converges to $\rho(c^*(v))$. Then $v_k(c^{*n\langle q\rangle} \mid c^{*n\langle q\rangle}; v)$ converges to $v_k(c^* \mid c^*; v)$ for all $v \in \tilde{X}$. By an argument identical to the argument in Step 2 in Lemma 4, $c^* \in h(c^*)$.

Step 4: Proof that limit beliefs result from c^*.

Consider the sequence $\{c^{*n\langle q\rangle}\}_{q=1}^{\infty}$. For any $S \subset P$, $k \notin S$, let

$$C(S; c^*; p_S) = \{v_{-S} \in X_{-S} \mid c^*_k(p_S, v_{-S}) > 0 \ \forall k \in S\}.$$

Thus, $C(S; c^*; p_S)$ consists of all the platforms of the parties not in S such all parties in S spends a positive amount, given c^* and p_S.

Fix some $S \subset P$ and $p_S \in X_S$.

Case 1: $\lambda_{-S}(C(S; c^*; p_S)) = 0$.

Since $\lambda_{-S}(C(S; c^*; p_S)) = 0$, Bayes rule do not apply and any beliefs can be applied. Then, for all $S \subset P$ and $p_S \in X_S$ such that $\lambda_{-S}(C(S; c^*; p_S)) = 0$, we assume that beliefs are given by $Pr_k^*(S; p_S)$.

Case 2: $\lambda_{-S}(C(S;\sigma^*;p_S)) > 0$.

Let $\tilde{X}_{-S}(p_S) = \{v \in X_{-S} \mid (p_S;v) \in \tilde{X}\}$. Furthermore, as in Lemma 3, we let $\tilde{X}_S = \{p_S \in X_S \mid \lambda_{-S}(\tilde{X}_{-S}(p_S)) = 1\}$. Clearly $\lambda_S(\tilde{X}_S) = 1$. Then, for any $p_S \in \tilde{X}_S$ such that $\lambda_{-S}(C(S;\sigma^*;p_S)) > 0$ we have $Pr_k(z \mid S;\sigma^{n_{h_q}};p_S)$ converges to $Pr_k(z \mid S;\sigma^*;p_S)$, for all $k \notin S$ a.e. on X_{-S}. Then, by Scheffé's Theorem (Billingsley (1968), p. 224), $Pr_k(S;\sigma^*;p_S) = Pr_k^*(S;p_S)$ for all $p_S \in \tilde{X}_S$.

Thus, for all $S \subset P$, all $k \notin S$ and all $p_S \in \tilde{X}_S$ we have that beliefs derived from σ^* are given by limit beliefs. ■

The idea behind the proof is the following. For each n, an equilibrium $\sigma^n \in T$ exists from Proposition 1. Since T is compact there exists a convergent subsequence, $\{\sigma^{n_h}\}_{h=1}^{\infty}$ with limit $\sigma^* \in T$. Then, by repeatedly using Helly's Theorem, there exists a subsequence $\{\sigma^{n_{h_q}}\}_{q=1}^{\infty}$, such that, for all $S \subseteq P$, all $k \notin S$ we have $Pr_k(S;\sigma^{n_{h_q}};p_S)$ converges weakly to some measure $Pr_k^*(S;p_S)$. Then the profile $\sigma^* \in T$ and the beliefs given by the limit measure is an INE a.e. If this where not the case, then for q sufficiently large, $\sigma^{n_{h_q}}$ cannot be an equilibrium.

3.3 Quadratic Preferences

Since the model in general is too complicated to solve analytically we restrict the analysis in the remainder of the paper to quadratic preferences. Then

$$V(z - x^i) = -(z - x^i)^2 :$$

Given some $S \subseteq P$, $c \in T$ and p_S, let $E[p_k | S; c, p_S]$ denote the expected platform of party k and let $VAR[p_k | S; c, p_S]$ denote the variance of the platform of party k. Now consider two parties j and k. Quadratic preferences implies that

$$x_{jk}(S; c, p_S) = \frac{E[p_k | S; c, p_S] + E[p_j | S; c, p_S]}{2} + \frac{VAR[p_k | S; c, p_S] - VAR[p_j | S; c, p_S]}{2\left(E[p_k | S; c, p_S] - E[p_j | S; c, p_S]\right)}. \qquad (9)$$

Too see this, note that the payoff when voting for party j is

$$-E\left[(x_j - x^i)^2 \mid S; c, p_S\right] = -(E[x_j | S; c, p_S] - x^i)^2 - VAR[p_j | S; c, p_S]. \qquad (10)$$

Similarly, the payoff when voting for party k is

$$-E\left[(x_k - x^i)^2 \mid S; c, p_S\right] = -(E[x_k | S; c, p_S] - x^i)^2 - VAR[p_k | S; c, p_S]. \qquad (11)$$

Setting expression 10 equal to 11 and solving for x^i gives expression 9.

4 Extreme versus Moderate Parties

The main issue in this paper is what happens with campaigning and voter ambiguity if parties become more or less extreme. To study this, let f be symmetric with mean 0. We also assume $f'(x) > 0$ whenever $x < 0$. We restrict attention to a situation with two parties where the Separability Assumption is satisfied. Furthermore, let $X_k = \{p_{kL}; p_{kH}\}$ where $p_{kH} = p_{kL} + 2l$, $l > 0$ and $p_{2L} = -p_{1H}$. Let $X = X_1 \times X_2$. Note that this is not a special case of the set of admissible platforms described in section 3 above. The strategy space for party k is then $\Upsilon_k = \{c \mid c : X \to [0,\omega]\}$. Let $\Upsilon = \Upsilon_1 \times \Upsilon_2$. Define $\delta = E\{p_2\} = -E\{p_1\}$ as the mean of the prior distribution for party 2. Thus, a party is either far away from the median voter or close to the median voter, relative to the expected platform. Also, by the Separability Assumption we have $p_{2L} > 0 > p_{1H}$ which implies $\delta > \lambda$. Each platform is drawn with probability $\frac{1}{2}$. This implies that the prior variance of the platforms for both parties is λ^2. The parties have access to the same amount of resources. Then, we let $\omega = \omega_1 = \omega_2$. Let $c_k(p_{1i}, p_{2j})$ be denoted c_{kij}. Given some $p \in X$, and $c, d \in \Upsilon$, the votes for party 1 is

$$v_k(c|d;p) = \rho(c_{1ij})\rho(c_{2ij}) \int_{\underline{x}}^{x_{12}(\{1,2\};d;p)} f(y)dy + \rho(c_{1ij})[1-\rho(c_{2ij})] \int_{\underline{x}}^{x_{12}(1;d;p_1)} f(y)dy$$

$$+ [1-\rho(c_{1ij})]\rho(c_{2ij}) \int_{\underline{x}}^{x_{12}(2;d;p_2)} f(y)dy + [1-\rho(c_{1ij})][1-\rho(c_{2ij})] \int_{\underline{x}}^{x_{12}(\emptyset;d;p_{\emptyset})} f(y)dy. \quad (12)$$

The following Lemma gives posterior variances and expectations when a voter is reached by one party only. Then we are able to compute indifferent voters when a voter is reached by one party only. Since we study symmetric equilibria, we can deduce that the indifferent voter is at 0, when voters are informed by none of the parties.

Lemma 5 Let c be a strategy profile. Suppose $S = \{2\}$. If $p_2 = p_{2i}$ and $c_{2ji} > 0$ for some $j \in \{L, H\}$ then

$$E[p_1 \mid S; c, p_S] = E\{p_1\} + \lambda \frac{\rho(c_{2Hi})[1-\rho(c_{1Hi})] - \rho(c_{2Li})[1-\rho(c_{1Li})]}{\rho(c_{2Li})[1-\rho(c_{1Li})] + \rho(c_{2Hi})[1-\rho(c_{1Hi})]}$$

and

$$VAR[p_1 \mid S; \sigma, p_S] = 4\lambda^2 \frac{\rho(\sigma_{2Hi})\rho(\sigma_{2Li})[1-\rho(\sigma_{1Li})][1-\rho(\sigma_{1Hi})]}{(\rho(\sigma_{2Li})[1-\rho(\sigma_{1Li})]+\rho(\sigma_{2Hi})[1-\rho(\sigma_{1Hi})])^2}$$

for each $i \in \{L, H\}$.

Proof. Updating of voters. Since $\sigma_{ji} > 0$ for some $j \in \{L, H\}$ Bayes rule applies.

Let $S = 2$. Bayes rule gives us

$$Pr_1(p_{1L} \mid 2; \sigma, p_2) = \frac{\rho(\sigma_{2Li})[1-\rho(\sigma_{1Li})]\frac{1}{2}}{\rho(\sigma_{2Li})[1-\rho(\sigma_{1Li})]\frac{1}{2}+\rho(\sigma_{2Hi})[1-\rho(\sigma_{1Hi})]\frac{1}{2}}$$

for $i \in \{L, H\}$. Let $A_1 = \rho(\sigma_{2Li})[1-\rho(\sigma_{1Li})]$ and $B_1 = \rho(\sigma_{2Li})[1-\rho(\sigma_{1Li})]+\rho(\sigma_{2Hi})[1-\rho(\sigma_{1Hi})]$ for $i \in \{L, H\}$. Then $Pr_1(p_{1L} \mid 2; \sigma, p_S) = \frac{A_1}{B_1}$ and

$$E[p_1 \mid 2; \sigma, p_S] = -\delta + \lambda \frac{B_1 - 2A_1}{B_1}.$$

Also,

$$VAR[p_1 \mid 2; \sigma, p_S] = 4\lambda^2 \frac{A_1[B_1 - A_1]}{B_1^2}.$$

Substituting in the expressions for A_1 and B_1 gives the desired result. ■

A similar result holds for party 2.

Now consider the optimal spending choice when a party is far away from the median voter, relative to the voters prior.

Lemma 6 Suppose σ^* is a symmetric IN E. For each $i \in \{L, H\}$ we have $\sigma^*_{1Li} = 0$ and $\sigma^*_{2iH} = 0$.

Proof. Consider party 1. By symmetry, a similar argument holds for party 2. Let σ be an arbitrary symmetric strategy profile. By hypothesis, we have $p_1 = p_{1L}$.

Step 1: $x_{12}(1;\text{ç}p_1) \leq x_{12}(?;\text{ç}p_?)$ and $x_{12}(12;\text{ç}p_{12}) \leq x_{12}(2;\text{ç}p_2)$.

Case 1: $x_{12}(1;\text{ç}p_1) \leq x_{12}(?;\text{ç}p_?)$.

We have

$$x_{12}(1;\text{ç}p_1) - x_{12}(?;\text{ç}p_?) =$$
$$\frac{[p_{1L}^2 - p_{2L}^2 \Pr[p_{2L} | 1;\text{ç}p_1] - p_{2H}^2 \Pr[p_{2H} | 1;\text{ç}p_1]]}{2(p_{1L} - E[p_2 | 1;\text{ç}p_1])}.$$

Since $p_{1L} < p_{2L} < p_{2H}$ and $\Pr[p_{2L} | 1;\text{ç}p_1] + \Pr[p_{2H} | 1;\text{ç}p_1] = 1$ the numerator is nonnegative and since $p_{1L} < E[p_2 | 1;\text{ç}p_1]$ the denominator is negative. Then $x_{12}(1;\text{ç}p_1) - x_{12}(?;\text{ç}p_?) \leq 0$.

Case 2: $x_{12}(12;\text{ç}p_{12}) \leq x_{12}(2;\text{ç}p_2)$.

We have

$$x_{12}(12;\text{ç}p_{12}) - x_{12}(2;\text{ç}p_2) = \frac{p_{1L} - E[p_1 | 2;\text{ç}p_2]}{2} - \frac{V[p_1 | 2;\text{ç}p_2]}{2(E[p_1 | 2;\text{ç}p_2] - p_2)}$$
$$= \frac{(p_{1L} - p_{1H})p_{1H} \Pr[p_{1H} | 1;\text{ç}p_2] + p_2(E[p_1 | 2;\text{ç}p_2] - p_{1L})}{2(E[p_1 | 2;\text{ç}p_2] - p_2)}.$$

Since $(p_{1L} - p_{1H}) < 0$, $p_2 > 0$, $p_{1H} < 0$ and $E[p_1 | 2;\text{ç}p_2] - p_{1L} \geq 0$ the numerator is nonnegative. Also, since $E[p_1 | 2;\text{ç}p_2] < p_2$, the denominator is negative. Then we have $x_{12}(12;\text{ç}p_{12}) - x_{12}(2;\text{ç}p_2) \leq 0$.

Step 2: $\text{ç}^{*}_{1L|} = 0$. Since $x_{12}(1;\text{ç}p_1) - x_{12}(?;\text{ç}p_?) \leq 0$ and $x_{12}(12;\text{ç}p_{12}) - x_{12}(2;\text{ç}p_2) \leq 0$ by Step 1, party 1 loses by informing. Note that this holds for any symmetric strategy profile. Then $\text{ç}^{*}_{1L|} = 0$. A similar argument holds for party 2. ■

Thus, each party spends nothing if the realized platform is far away from the median voter, compared with the prior. The reason is that, when informing voters, the voters become aware of the platform of the party. Even though voters perceive the platform with certainty, the reduction in risk is not sufficient to counter the effect that the platform is far away from the median voter. This result makes it easy to compute expenditures for the party.

Now consider the expenditures when parties have a realization of the platform that is close to the median voter, compared with the prior. The following Lemma shows that the spending level of a party is independent of the realized platform of the other party.

Lemma 7 Suppose Δ^* is a symmetric INE. For each $i, j \in \{L, H\}$ we have $\Delta^*_{1Hi} = \Delta^*_{2jL} = \Delta^*_m \in [0; \omega]$.

Proof. Let Δ be an arbitrary symmetric strategy profile. Consider party 1. By hypothesis we have $p_1 = p_{1H}$.

Step 1: The votes for party 1.

Note that, since $x_{12}(*; \Delta p_*) = 0$, we have $\int_{\underline{x}}^{x_{12}(*; \Delta p_*)} f(y)dy = \frac{1}{2}$.

Case 1: $p_2 = p_{2L}$.

By using expression 12 the votes for party 1 is

$$\rho(\Delta^*_{1HL})\rho(\Delta^*_{2HL})\left(\frac{1}{2}\right) + \rho(\Delta^*_{1HL})[1 - \rho(\Delta^*_{2HL})]\left[\frac{1}{2} + \int_0^{x_{12}(1;\Delta p_1)} f(y)dy\right] +$$

$$[1 - \rho(\Delta^*_{1HL})]\rho(\Delta^*_{2HL})\left[\frac{1}{2} - \int_{x_{12}(2;\Delta p_2)}^{0} f(y)dy\right] + [1 - \rho(\Delta^*_{1HL})][1 - \rho(\Delta^*_{2HL})]\frac{1}{2}:$$

Case 2: $p_2 = p_{2H}$.

Then $\Delta^*_{2HH} = 0$ and $\rho(\Delta^*_{2HH}) = 0$. The votes for party 1 is then

$$\rho(\Delta^*_{1HH})\left[\frac{1}{2} + \int_0^{x_{12}(1;\Delta p_1)} f(y)dy\right] + [1 - \rho(\Delta^*_{1HH})]\frac{1}{2}: \quad (13)$$

Step 2: Optimal spending choice.

By symmetry, we have $x_{12}(2; c, p_2) = -x_{12}(1; c, p_1)$. Then, by symmetry of the voter distribution, we have (in Case 1)

$$\int_0^{x_{12}(1;c,p_1)} f(y)dy = \int_{x_{12}(2;c,p_2)}^{0} f(y)dy.$$

From Case 1 and 2 in Step 1, the first-order condition is

$$\alpha\rho'(c^*_{1Hi}) \int_0^{x_{12}(1;c,p_1)} f(y)dy - 1 = 0 \tag{14}$$

for all $i \in \{L, H\}$. Since expression 14 only depend on c_2 through beliefs, the optimal choice for party 1 is independent of the choice of c_2, given voter beliefs. This implies $c^*_{1HL} = c^*_{1HH}$. Then, let $c^*_m = c^*_{1HL}$ denote the solution to 14. By symmetry, we have $c^*_m = c^*_{2HL} = c^*_{1LL}$. ■

In particular, we focus attention on stable equilibria.

4.1 Stability

To analyze stability, we consider an equilibrium and perturb the equilibrium profile. The players take the new profile as given and reoptimize. This again gives a new profile. Given this profile, the players reoptimize again and so forth. This gives a system of differential equations by using the first-order conditions. Let $\hat{c}$ denote the starting value of c. Let the system of differential equations be denoted $\dot{c}(t)$. For all $c, d \in \Re^4$, let $\|c - d\|_e$ denote the (euclidian) distance between c and d. Let a solution to the system of differential equations be denoted $\phi(t; \hat{c})$.

Definition 3 An equilibrium c^* is locally stable if there exists a $\eta > 0$ such that

$$\left\| c - c^* \right\|_e \leq \eta \Rightarrow \lim_{t \to \infty} \phi(t, c) = c^*.$$

Let $\Delta \subset \Re_{++}$ denote the (open) set of possible parameter values of δ. The following Lemma shows existence. The conditions on ρ and the distribution are technical conditions that guarantees the existence of an interior solution.

Lemma 8 *Suppose $\nu \rho'(0) \int_0^{z(0)} f(y)dy - 1 > 0$, $\nu \rho'(\omega) \int_0^{z(\omega)} f(y)dy - 1 < 0$ and $\rho'(0) < \infty$. There exists a symmetric, locally stable INE a.e. on Δ.*

Proof. Symmetric equilibria.

Step 1: The fixed point mapping.

Let

$$z(b) = \frac{l}{[2 - \rho(b)]}\left(1 - \frac{l[1 - \rho(b)]}{(l - 2\delta + [\delta - l]\rho(b))}\right).$$

Note that, if c is symmetric and if $c_{HL} = c_{LL} = b$ then $z(b) = x_{12}(1; c, p_S) = -x_{12}(2; c, p_S)$.

Consider

$$\nu \rho'(c_m) \int_0^{z(b)} f(y)dy - 1 = 0.$$

This expression defines a mapping $k : [0, \omega] \to [0, \omega]$, such that $k(b) = c_m$. Also, by construction, $k(b)$ is the optimal campaign expenditure level for a party that has a platform close to the median voter, for beliefs resulting from b.

Since

$$\frac{\partial z}{\partial b} = \frac{\rho'(b) l \delta [\delta - l]}{(l - 2\delta + [\delta - l]\rho(b))^2} > 0; \quad (15)$$

$\rho'(\gamma_m) > 0$ and $\rho''(\gamma_m) < 0$ for any $\gamma_m \in (0;\infty)$, it follows that k is single-valued and nondecreasing. The fixed points of the mapping k is the set of symmetric equilibria. Since z is continuously differentiable and ρ is twice continuously differentiable, if $b \in (0;\infty)$ and $k(b) \in (0;\infty)$, then k is differentiable.

Since k is a continuous function, there exists at least one symmetric equilibrium.

Step 2: Using k to find a candidate equilibrium.

Consider any interior fixed point γ_m^* such that

$$k'(\gamma_m^*) = -\frac{\rho'(\gamma_m^*)\, f(z(\gamma_m^*))\frac{\partial z}{\partial \gamma_m^*}}{\rho''(\gamma_m^*)\int_0^{z(\gamma_m^*)} f(y)dy} < 1: \qquad (16)$$

Any fixed point satisfying 16 is a candidate equilibrium. Since $k(0) > 0$ and $k(\infty) < \infty$; Proposition 8.3.1 in Mas-Colell (1989) implies that there exists such a fixed point for almost all $\delta \in \Delta$.

Step 3: First, we show that we can restrict attention to the stability of parties where the realized platform is close to the median voter.

Let $B_\epsilon(c^*) = \{c \mid \|c - c^*\|_e < \epsilon\}$. Suppose we start with some $c \in B_\epsilon(c^*)$ such that, a party with a platform that is far away from the median voter spends a positive amount. Recall $\rho'(0) < 1$.

Consider the best response, given some c. Suppose party 1 is far away from the median voter. Let

$$\Pi_1 = \rho(\gamma_{Lj})[F(x_{12}(12;\gamma p_{12})) - F(x_{12}(2;\gamma p_2))]$$

$$+ [1 - \rho(\gamma_{Lj})][F(x_{12}(1;\gamma p_1)) - F(x_{12}(?;\gamma p_?))]:$$

If party 1 chooses a positive spending level, c^0_{1Lj}, the …rst-order condition is

$$º½^0(c^0_{1Lj})¤_1 ¡ 1 = 0 :$$

Since symmetry of the strategy pro…le is only used in Lemma 6 to show $x_{12}(?; c,p_?) = 0$; Lemma 6 implies that $F(x_{12}(12; c,p_{12}) · F(x_{12}(2; c,p_2))$. Also, we have $x_{12}(1; c,p_1) < 0$.

It can be shown that

$$x_{12}(?; c,p_?) = \frac{l± ¡A^?_2 ¡ A^?_1¢}{¡(± ¡ l)B^? + lA^?_2 + lA^?_1¢}$$

where

$$A^?_1 = (1 ¡ ½(c_{1LL}))(1 ¡ ½(c_{2LL})) + (1 ¡ ½(c_{1LH}))(1 ¡ ½(c_{2LH})) ;$$

$$A^?_2 = (1 ¡ ½(c_{1HH}))(1 ¡ ½(c_{2HH})) + (1 ¡ ½(c_{1LH}))(1 ¡ ½(c_{2LH}))$$

and

$$B^? = \sum_{j2 fL;H g} \sum_{i2 fL;H g} (1 ¡ ½(c_{1ij}))(1 ¡ ½(c_{2ij})) :$$

Since $½(!) < 1$, $B^? > 0$ for any $c · !$ and $A^?_1$, $A^?_2$ and $B^?$ are continuous in c, we have $x_{12}(?; c,p_?)$ continuous in c. Moreover, if $c! \bar{c}$ then $x_{12}(?; c,p_?)$ converges to 0.

Then, for any $² > 0$ there exists an $´$ such that, if $c 2 B_´(\bar{c})$ then $jx_{12}(?; c,p_?)j < ²$.

Since $F(x_{12}(1; c,p_{1L})) · \frac{1}{2}$ we have

$$¤_1 · \frac{1}{2} ¡ F(x_{12}(?; c,p_?)):$$

Since $½^0(0) < 1$, there exists an $^ > 0$ such that

$$o½^0(0) > \frac{1}{\frac{1}{2} ¡ F(x_{12}(?;c,p_?))}$$

holds for all c2 B^(c¤). Repeat this for all states where the platform of some party is far away from the median voter. Let ~ denote the value of ´ such that, for all c 2 B~(c¤), for all states where the platform of some party is far away from the median voter, any such party choose zero spending. Since $x_{12}(?;c,p_?)$ is continuous in c and ½⁰(0) < 1, we have ~ > 0.

Let $B^0_´(c^¤)$ = fcjkc ¡ c¤k_e < ´ and c_{Li} = c_{iH} = 0 for i = L;H g. Thus, $B^0_´(c^¤)$ is the ball around c¤, given that parties far away from the median voter spend zero. If c¤ is locally stable to changes in spending of parties that have platforms close to the median voter, there exists a ¹ > 0 such that, if Á(t; ĉ) 2 $B^0_¹(c^¤)$ then $\lim_{t^0 ! 1}$ Á(t⁰; ĉ) ! c¤ for t⁰ > t.

Now consider the solution to the …rst-order condition, given some ĉ 2 B~(c¤). Let the solution to the …rst-order conditions be denoted ĉ¤. Since votes for parties that have platforms close to the median voter are continuous in c, for a su¢ciently small ´ · ~, denoted ´⁰, if ĉ 2 $B_{´^0}(c^¤)$ then ĉ¤ 2 $B^0_¹(c^¤)$.

Thus, we can restrict attention to the case when parties are close to the median voter and show that this system is stable.

Step 4: The system of di¤erential equations of parties with platforms close to median voter.

Given i; j 2 fL;H g, let $c_{kij}(t)$ denote the di¤erential equation for party k. Using a …rst-order Taylor series expansion at $c^¤_m$ of the …rst-order conditions, gives the system of di¤erential equation as

$$\begin{bmatrix} \frac{d\theta_{1HL}}{dt} \\ \frac{d\theta_{2HL}}{dt} \\ \frac{d\theta_{LL}}{dt} \\ \frac{d\theta_{HH}}{dt} \end{bmatrix} = \begin{bmatrix} a_{mm} & 0 & d_{mm} & b_{mm} \\ 0 & a_{mm} & b_{mm} & d_{mm} \\ d_{em} & b_{em} & a_{em} & 0 \\ b_{em} & d_{em} & 0 & a_{em} \end{bmatrix} \cdot \begin{bmatrix} \theta_{1HL}(t) - \theta^*_m \\ \theta_{2HL}(t) - \theta^*_m \\ \theta_{LL}(t) - \theta^*_m \\ \theta_{HH}(t) - \theta^*_m \end{bmatrix} . \qquad (17)$$

Let

$$E = \frac{\rho'(\theta^*_m)}{(2 - \rho(\theta^*_m))} \frac{l\delta}{((\delta - l)(2 - \rho(\theta^*_m)) + 2l)}$$

and

$$D = \frac{\delta l \rho'(\theta^*_m)(\delta - l)\rho(\theta^*_m)}{((\delta - l)[\rho(\theta^*_m)(2 - \rho(\theta^*_m))] + l\rho(\theta^*_m))^2} .$$

Note that $\frac{\partial x^{12}(\star;\theta,p_\star)}{\partial \theta_{1HH}} = -E$ and $\frac{\partial x^{12}(\star;\theta,p_\star)}{\partial \theta_{LL}} = E$. Furthermore, note that $\frac{\partial x_{12}(1;\theta^*_m;p_1)}{\partial \theta^*_m} = \rho(\theta^*_m)D$ and $\frac{\partial x_{12}(2;\theta^*_m;p_2)}{\partial \theta^*_m} = -\rho(\theta^*_m)D$. Also, let $f^1 = f(x_{12}(2;\theta^*_m;p_2))$.

It can be shown that

$$a_{mm} = \rho''(\theta^*_m)\left[\int_{x_{12}(2;\theta^*_m;p_2)}^{0} f(y)\,dy\right] - \rho'(\theta^*_m) D f^1 \{1 - 2\rho(\theta^*_m)\};$$

$$b_{mm} = \rho'(\theta^*_m)(1 - \rho(\theta^*_m))\left\{f^1(1 - \rho(\theta^*_m))D + f(x_{12}(\star;\theta,p_\star))E\right\}$$

and

$$d_{mm} = \rho'(\theta^*_m)(1 - \rho(\theta^*_m))\left\{\rho(\theta^*_m) f^1 D - f(x_{12}(\star;\theta,p_\star))E\right\} .$$

Moreover,

$$a_{em} = \rho''(\theta^*_m)\int_0^{x_{12}(1;\theta^*_m;p_1)} f(y)\,dy + \rho'(\theta^*_m) f^1 (1 - \rho(\theta^*_m))D;$$

$$b_{em} = -\rho'(\theta^*_m) f^1 (1 - \rho(\theta^*_m))D$$

and

$$d_{em} = \rho'(\theta^*_m) f^{-1} \rho(\theta^*_m) D:$$

Step 5: Stability of system in Step 4.

For $i = 1;2;3;4$, let λ_i denote the eigenvalues of this matrix.

Consider the first eigenvalue. It is given by

$$\lambda_1 = \rho''(\theta^*_m) \int_{x_{12}(2;\theta^*_m;p_2)}^{\infty} f(y)dy + \frac{\rho'(\theta^*_m) D f^{-1} \rho(\theta^*_m)}{2}$$

$$- \frac{\rho'(\theta^*_m) f^{-1} D}{2} \cdot \sqrt{\rho(\theta^*_m)^2 - 4(1 - \rho(\theta^*_m)) \left\{ 2\frac{f(x_{12}(?;\theta p_?))}{f^{-1}} \frac{E}{D} \right\}}:$$

We have two cases.

If $\rho(\theta^*_m)^2 - 4(1 - \rho(\theta^*_m)) \left\{ 2\frac{f(x_{12}(?;\theta p_?))}{f^{-1}} \frac{E}{D} \right\} \geq 0$, then

$$\lambda_1 \leq \rho''(\theta^*_m) \left[\int_{x_{12}(2;\theta^*_m;p_2)}^{\infty} f(y)dy \right] + \frac{1}{2}\rho'(\theta^*_m) D f^{-1} \rho(\theta^*_m):$$

Clearly, by expression 16, λ_1 is negative.

If $\rho(\theta^*_m)^2 - 4(1 - \rho(\theta^*_m)) \left\{ 2\frac{f(x_{12}(?;\theta^*_m;p_?))}{f^{-1}} \frac{E}{D} \right\} < 0$, then the real part of λ_1 is

$$\rho''(\theta^*_m) \left[\int_{x_{12}(2;\theta^*_m;p_2)}^{\infty} f(y)dy \right] + \frac{1}{2}\rho'(\theta^*_m) D f^{-1} \rho(\theta^*_m):$$

Clearly, by expression 16, the real part of λ_1 is negative.

Now consider λ_2. It is given by

$$\lambda_2 = \rho''(\Delta^*_m)\int_{x_{12}(2;\Delta^*_m;p_2)}^{0} f(y)dy + \frac{\rho'(\Delta^*_m)D\gamma^{-1}\rho(\Delta^*_m)}{2}$$

$$+\frac{\left(\rho'(\Delta^*_m)\gamma^{-1}D\right)}{2}\sqrt{\left\{\rho(\Delta^*_m)^2 - 4(1-\rho(\Delta^*_m))\,2\,\frac{f(x_{12}(\star;\Delta p_\star))}{\gamma^{-1}}\frac{E}{D}\right\}}.$$

We have two cases.

If $\rho(\Delta^*_m)^2 - 4(1-\rho(\Delta^*_m))\left\{2\frac{f(x_{12}(\star;\Delta p_\star))}{\gamma^{-1}}\frac{E}{D}\right\} \geq 0$, then

$$\lambda_2 \leq \rho''(\Delta^*_m)\left[\int_{x_{12}(2;\Delta^*_m;p_2)}^{0} f(y)dy\right] + \rho'(\Delta^*_m)D\gamma^{-1}\rho(\Delta^*_m).$$

Clearly, by expression 16, λ_2 is negative.

If $\rho(\Delta^*_m)^2 - 4(1-\rho(\Delta^*_m))\left\{2\frac{f(x_{12}(\star;\Delta p_\star))}{\gamma^{-1}}\frac{E}{D}\right\} < 0$, then the real part of λ_2 is

$$\rho''(\Delta^*_m)\left[\int_{x_{12}(2;\Delta^*_m;p_2)}^{0} f(y)dy\right] + \frac{1}{2}\rho'(\Delta^*_m)D\gamma^{-1}\rho(\Delta^*_m).$$

Clearly, by expression 16, the real part of λ_2 is negative.

Now consider λ_3 and λ_4. We have

$$\lambda_3 = \rho''(\Delta^*_m)\left[\int_{x_{12}(2;\Delta^*_m;p_2)}^{0} f(y)dy\right]$$

and

$$\lambda_4 = \rho''(\Delta^*_m)\left[\int_{x_{12}(2;\Delta^*_m;p_2)}^{0} f(y)dy\right] + \rho'(\Delta^*_m)D\gamma^{-1}\rho(\Delta^*_m).$$

Since $\rho''(\Delta^*_m) < 0$ and $\rho''(\Delta^*_m)\left[\int_{x_{12}(2;\Delta^*_m;p_2)}^{0} f(y)dy\right] + \rho'(\Delta^*_m)D\gamma^{-1}\rho(\Delta^*_m) < 0$, we have $\lambda_3 < 0$ and $\lambda_4 < 0$.

Thus, the real parts of all eigenvalues are negative. Thus, any solution c^*_m found in Step 2 is stable.

Note that $\lambda_4 \geq 0$ for any fixed point such that $k'(b) \geq 1$. Thus, the linear approximation of the differential equations used above is not stable for any such fixed point. ■

Now consider the effects of changes in δ on campaign spending. Let $\Sigma : \Delta \rightrightarrows [0;\omega]$ denote the correspondence that associates to each δ the set of symmetric equilibria.

We have the following Proposition that shows that spending decreases with polarization.

Proposition 2 *Let c^* be any stable symmetric INE, where $c^*_m \in (0;\omega)$. We have $\frac{dc^*_m}{d\delta} < 0$.*

Proof. Note that a stable equilibrium is locally isolated. Consider the effects of a small change in δ. Let $\zeta(c^*_m;\delta) = \frac{l}{[2-\rho(c^*_m)]}\left[1 - \frac{l[1-\rho(c^*_m)]}{(l-2\delta+[\delta-l]\rho(c^*_m))}\right]$. At a fixed point we have

$$0 = \kappa\rho'(c^*_m)\int_0^{\zeta(c^*_m;\delta)} f(y)dy - 1. \tag{18}$$

Consider a linear approximation of Σ at $(c^*_m;\delta)$. The linear approximation is given by $c_m = \alpha_{c^*_m;\delta}\delta + \beta_{c^*_m;\delta}$. By using a linear approximation at an isolated symmetric equilibrium we get

$$\alpha_{c^*_m;\delta} = -\frac{\kappa\rho'(c^*_m)f(\zeta(c^*_m;\delta))\frac{\partial\zeta}{\partial\delta}}{\rho''(c^*_m)\int_0^{\zeta(c^*_m;\delta)} f(y)dy + \rho'(c^*_m)f(\zeta(c^*_m;\delta))\frac{\partial\zeta}{\partial c^*_m}}.$$

Since $z(c^*_m;\delta) = -\frac{1\delta}{1-2\delta+[\delta-1]\rho(c^*_m)}$ we have

$$\frac{\partial z}{\partial \delta} = -\frac{1}{1-2\delta+[\delta-1]\rho(c^*_m)} + \frac{1\delta[\rho(c^*_m)-2]}{[1-2\delta+[\delta-1]\rho(c^*_m)]^2}$$

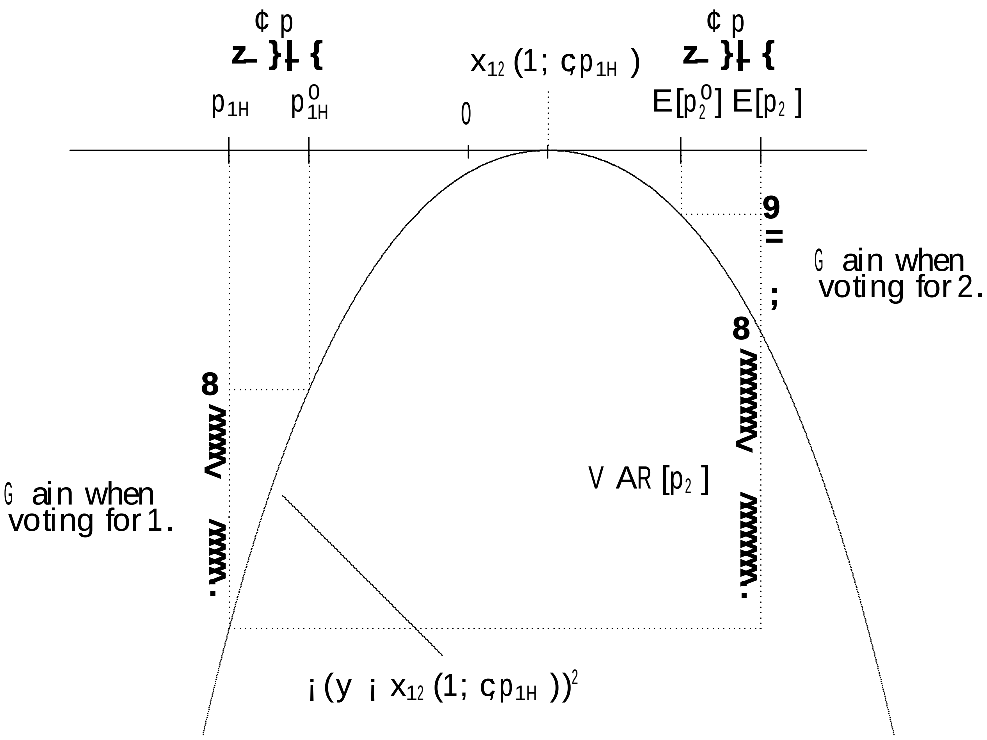
$$= \frac{1[\delta[\rho(c^*_m)-2]-[1-2\delta+[\delta-1]\rho(c^*_m)]]}{[1-2\delta+[\delta-1]\rho(c^*_m)]^2}. \qquad (19)$$

Thus, $\frac{\partial z}{\partial \delta} = -\frac{1^2[1-\rho(c^*_m)]}{[1-2\delta+[\delta-1]\rho(c^*_m)]^2} < 0$.

Stability implies that $\rho''(c^*_m)\int_0^{z(c^*_m;\delta)} f(y)dy + \rho'(c^*_m)f(z(c^*_m;\delta))\frac{\partial z}{\partial c^*_m} < 0$. If c^* is stable, then expressions 16 and 19 imply that $\frac{dc^*_m}{d\delta} = \alpha_{c^*_m;\delta} < 0$. ■

The reason behind the result is the following. Note that, for fixed voter beliefs about c^*_m, the positive effect of reduced uncertainty on voters is unchanged. Consider voters that are informed by, say party 1 only, and assume that $p_1 = p_{1H}$. For the indifferent voter, the expected platform of party 2 is closer to the voters' peak than the actual platform of party 1. Since voters are risk-averse the voter would otherwise strictly prefer to vote for party 1. If both parties move closer to each other, then the actual and expected platform moves closer to the indifferent voters peak by exactly the same distance. By strict concavity, the closer to the peak a platform is, the flatter preferences are. Then, since the actual and expected platform of the two parties move the same distance, the increase in payoff of voting for the party that informed is bigger than the increase for the other party. Thus, the voter with peak at $x_{12}(1;c^*;p_{1H})$ strictly prefers party 1. This makes parties gain more votes by informing if the parties are moderates, when the realized platform is close to zero. This can be seen in the figure below, where the platform profile changes from p to p', by the distance Δp.

Figure 2: Effect on the Indifferent Voter Informed by Party 1 only when Parties Become more Moderate (for fixed voter beliefs).



Since the parties spend nothing when they are far away from zero this implies that both the mean and variability of spending increases when ς^{*}_{m} increases. The increase in spending also reduce the posterior variance associated with each party. This reduces the incentives to inform voters. However, the reduction only partially counteracts the initial increase.

4.2 Voter Uncertainty

Reduction of voter uncertainty in this model comes from two different sources. First, an increase in spending leads to a bigger share of the electorate being informed. Then, since Δ^*_m increase as parties get closer to each other, the share of the electorate that is informed increases. The second source is that uninformed voters revise their beliefs, given the strategy profiles of the parties. How beliefs are revised can be analyzed by using the result in Proposition 2. We have the following corollary of Proposition 2.

Corollary 1 *If c^* is a stable INE then the posterior variance is increasing in δ.*

Proof. Note that, given a symmetric equilibrium c^*, Lemma 5 implies that, for any S such that $k \notin S$ for $k = 1, 2$, we have

$$VAR[p_K \mid S; c^*; p_S] = 4l^2 \frac{[1 - \rho(\Delta^*_m)]}{(2 - \rho(\Delta^*_m))^2}:$$

Then we have $\frac{\partial VAR[p_K \mid S; c^*; p_S]}{\partial \delta} = \frac{\partial VAR[p_K \mid S; c^*; p_S]}{\partial \Delta^*_m}\frac{d\Delta^*_m}{d\delta}$. From Proposition 2 we know that $\frac{d\Delta^*_m}{d\delta} < 0$. Also, we have

$$\frac{\partial VAR[p_K \mid S; c^*; p_S]}{\partial \Delta^*_m} = -\frac{\rho'(\Delta^*_m) 4l^2 \rho(\Delta^*_m)}{(2 - \rho(\Delta^*_m))^3} < 0:$$

The conclusion then immediately follows. ■

The fact that variability of expenditures increases as parties get closer to each other, makes it possible for uninformed voters to make a more precise prediction of the platform of the party. This in turn leads to the posterior uncertainty associated with a particular party to decrease. Thus, the uncertainty associated with the parties decrease, even for the uninformed voters.

5 Conclusions

The model described in this paper analyses political campaigning. We are able to show existence of equilibrium under mild assumptions for an arbitrary number of parties. In general, it is difficult to explicitly solve for an equilibrium. However, in a symmetric example with two parties we can show that voters are more informed when parties are moderate, than when parties are extreme.

The motivation is the following. If a party informs a voter, then the party knows the platforms of that party with certainty. Thus, the risk of voting for that party is eliminated. Consider voters that are informed by one party only, say party 1, and assume that the platform of party 1 is close to the median voter. For the indifferent voter, the expected platform of party 2 is closer to the voters' peak than the actual platform of party 1. Otherwise, since voters are risk-averse, the voter would strictly prefer to vote for party 1. If both parties move closer to each other, then the actual and expected platform moves closer to the indifferent voters peak by the same distance. By strict concavity, the closer to the peak a platform is, the flatter preferences are. Since the actual and expected platform of the two parties move the same distance, the increase in payoff of voting for the party that informed is bigger than the increase in payoff of voting for the other party. Thus, the previously indifferent voter now strictly prefers party 1. The effect makes parties gain more votes by informing when parties are moderate. Since spending increases, voters are more informed when parties are moderates. Thus extremism is bad in the sense that it leaves voters uninformed.

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