BANCA D'ITALIA

Temi di discussione

del Servizio Studi

Estimating state price densities by Hermite polynomials: theory and application to the Italian derivatives market

by Paolo Guasoni

Number 507 - July 2004

ESTIMATING STATE PRICE DENSITIES BY HERMITE POLYNOMIALS: THEORY AND APPLICATION TO THE ITALIAN DERIVATIVES MARKET

by Paolo Guasoni*

Abstract

We study the problem of extracting the state price densities from the market prices of listed options.

Adapting a model of Madan and Milne to a multiple expiration setting, we present an estimation method for the risk-neutral probability at a moving horizon of fixed length. With the exception of volatility, all model parameters can be estimated by linear regression and their number can be chosen arbitrarily, depending on the size of the dataset.

We discuss empirical issues related to the application of this model to real data and show results on listed options on the Italian MIB30 equity index.

JEL classification: G12,G13.

Keywords: option pricing, state-price densities, orthogonal polynomials, risk-neutral valuation, calibration.

Contents

*Università di Pisa, Dipartimento di Matematica and Boston University.

1 Introduction

[1] In modern finance theory usual financial instruments are seen as combi-nations of elementary Arrow-Debreu securities and asset prices can be obtained as expected values of their future payoff under a state-price (or risk-neutral) density Q.

In general, Q is unique only if the market is complete and in this case option prices are exactly determined by the no-arbitrage condition. On the contrary, in a market with incomplete information there are infinitely many risk-neutral probabilities, each of them reflecting a particular attitude to risk.

Knowledge of Q can be relevant for a number of applications, ranging from the pricing of unlisted derivatives (such as OTC contracts) to risk management. From the point of view of a regulator, knowledge of the risk-neutral probability Q can be useful in combination with that of the physical probability P, as it allows the time change of risk aversion in the market to be monitored. In view of these applications, the natural question is whether we can recover the marginal distribution of an underlying asset S under Q from the observation of option prices.

Several methods have been proposed in the literature for this purpose: a first approach consists in modeling the dynamics of the underlying asset under Q so that risk-neutral densities can be written in parametric form. This case encompasses the stochastic volatility models of Heston [3] and Stein and Stein [7], as well as several others with deterministic volatility. In a few simple cases, this method is particularly flexible and easy to implement, but in general it raises a number of issues:

- it is heavily model-dependent since it requires an *a priori* specification of a stochastic process for the asset price;
- for complex processes, the risk-neutral densities do not admit closed-form expressions and numerical solutions of PDEs or simulation algorithms must be employed;
- when several multiple parameters appear in a joint minimization problem it is necessary to devise an estimation algorithm that avoids local minima.

---

[1] The first draft of this paper was completed while the author was affiliated to the Research Department of the Bank of Italy. The author wishes to thank Gur Huberman, Roberto Violi and Giuseppe Grande for stimulating discussions and for carefully reading earlier versions of the paper. This paper benefited from comments of seminar participants at the Bank of Italy and at the III Workshop in Quantitative Finance held in Verona (2002). All remaining errors are the author's responsibility. Special thanks go Borsa Italiana SpA for kindly providing the options dataset.

The views expressed herein are those of the author and not necessarily those of the Bank of Italy. E-mail: guasoni@dm.unipi.it

A second approach directly prescribes a parametric form for risk-neutral densities, without specific assumptions on the underlying process under Q. This method includes all various parametrizations of the "smile", along the lines of Shimko [6], Rosenberg and Engle [2] and several others. Although it has a clear edge for its simplicity and it circumvents the first two issues above, the third problem remains and others arise:

- the choice of a particular functional form is often arbitrary and may pose specification problems;
- in the attempt to span a wide range of densities, several parameters might be necessary, leading to the risk of over-fitting.

Some authors take up a Bayesian approach, solving for the risk-neutral density which is closest to a given prior, under the constraint of pricing correctly all options observed. For example, this was done by Rubinstein [5] to calibrate an *implied binomial tree* from option prices. While this approach is general enough to allow virtually any density form, it is not completely clear what distance criteria should be preferred and what is the impact of the prior on the final result.

The last approach, proposed by Aït-Sahalia and Lo [1], is essentially nonparametric: first the pricing function $C(S, K, r, \delta, \tau)$ is estimated with the kernel regression technique, then the risk-neutral density is obtained via the well-known identity due to Breeden and Litzemberger:

$$\left.\frac{\partial^2 C}{\partial K^2}\right|_{K=x} = e^{-r(T-t)} q(x)$$

where $q(x)$ denotes the marginal density of S_T at x. While this method is the most general and can capture virtually any feature displayed by the data, it works best when a semi-parametric variant is used and it generally requires the aggregation of data across different time observations. This means that it is best suited for large-sample studies, where a single risk-neutral density is assumed to explain prices for a certain period of time.

In this paper we adopt a model suggested by Madan and Milne [4], which is parametric in its implementation while it allows the representation of any risk-neutral density satisfying reasonable integrability conditions. More precisely, the density of the underlying logarithm is expanded in Hermite series after scaling by a normalization factor, which plays the role of volatility. When all other parameters are equal to zero, the model boils down to the standard Black-Scholes case.

Developing further the analysis of this model, we translate in terms of parameter constraints the conditions that the density indeed represents a probability (i.e.

it integrates to one) and that it is risk-neutral. This reduces the scope for inconsistency and over-fitting. We also show how the densities corresponding to two different expirations can be used to estimate the risk-neutral density at a fixed-length horizon. This involves calculating the Hermite expansion of a convolution of two densities and assuming that the underlying process has independent increments.

In this model, option prices are calculated in closed form as the scalar product between the vector of Hermite coefficients and a vector of explicit formulas, depending only on the volatility parameter: we show an efficient method to obtain these formulas recursively, in symbolic form.

For a given value of volatility, the model is linear and can be easily solved with ordinary least squares. The full nonlinear model can also be solved with standard nonlinear regression algorithms, and convergence to the global minimum is guaranteed by the convexity of the functional.

Finally, we show that for a particular two-parameter choice we have a one-to-one correspondence between the Hermite coefficients and skewness and kurtosis. Indeed, the two Hermite coefficients become constant multiples respectively of skewness and excess kurtosis, thereby providing a consistent framework for the estimation of these quantities.

The paper is organized as follows: in section 2 we describe the model in detail and show how the components of option prices can be computed recursively. Then we exploit the same calculations to write the risk-neutrality condition in terms of parameter values and see that one parameter can be eliminated if the density has to integrate to one. The aggregation of data across expirations is covered in Section 3, where we present a method to estimate the risk-neutral measure at intermediate horizons.

Empirical issues, as well as an application to real data from the Italian derivatives market, are the subject of Section 4. We discuss the choice of the set of parameters, which is intimately related to the moments of risk-neutral densities, and show numerical results from our dataset, which consists of intra-day data on prices and volumes of all transactions on MIB30 index options during 1998. In fact, the period under consideration has shown a wide range of market conditions, which provide a challenging stress test for the model. In the last section we briefly comment our results, discussing the benefits and the limits of this methodology.

2 The model

Throughout the paper, S_t denotes the price of the underlying asset at time t, T the expiration date of an option, K its strike price, r the interest rate, and δ the dividend yield.

We represent the random variable S_T as:

$$S_T = S_t e^{(r-\delta-\frac{\sigma^2}{2})(T-t)+\sigma\sqrt{T-t}\psi}$$

where σ is an arbitrary positive constant. Note that this representation does not involve assumptions on the asset price dynamics, but only establishes a one-to-one mapping between a positive random variable S_T and a real-valued random variable ψ. In particular, if S_T is a lognormal then $\psi \sim N(0,1)$. Denoting by $q(x)$ the probability density of ψ under Q, $\tau = T - t$ and $d_2 = (\log \frac{S_t}{K} + (r - \delta - \frac{\sigma^2}{2})\tau)/(\sigma\sqrt{\tau})$, we can rewrite the call price as:

$$\begin{aligned} C(S_t, K, r, \tau, \sigma) =& e^{-r\tau} \int_{-d_2}^{\infty} (S_t e^{(r-\delta-\frac{\sigma^2}{2})\tau+\sigma\sqrt{\tau}x} - K) q(x) dx = \\ =& e^{-\delta\tau} S_t \int_{-d_2}^{\infty} e^{-\frac{\sigma^2}{2}\tau+\sigma\sqrt{\tau}x} q(x) dx - e^{-r\tau} K \int_{-d_2}^{\infty} q(x) dx \end{aligned}$$

We denote the Hermite expansion of $q(x)$ as:

$$q(x) = \phi(x) \sum_{n=0}^{\infty} \theta_n H_n(x) = \phi(x) \sum_{n=0}^{\infty} \theta_n H_n(x)$$

where $\phi(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$ is the standard normal density and $H_n(x) = \frac{1}{\phi} \frac{d^n \phi}{dx^n}\Big|_x$ are the Hermite polynomials. We recall their properties in the following:

Proposition 1. *We have:*

- $\int_{-\infty}^{+\infty} H_i(x) H_j(x) \phi(x) dx = 0$ *for all* $i \neq j$.
- $\int_{-\infty}^{+\infty} H_n(x)^2 \phi(x) dx = n!$ *for all* n.
- *If* $f \in L^2(\mathbb{R}, N(0,1))$*, then* $f(x) = \sum_{n=0}^{\infty} \zeta_n H_n(x)$ *for some set* $\{\zeta_n\}_{n\in\mathbb{N}}$.

In other words, the set $\{H_n\}_{n\in\mathbb{N}}$ is an orthogonal basis of the Hilbert space $L^2(\mathbb{R}, N(0,1))$. We shall assume that $f \in L^2(\mathbb{R}, N(0,1))$, so that convergence holds. The price of a call option can be calculated as:

$$\begin{aligned} C(S_t, K, r, \tau, \sigma) &= e^{-\delta\tau} S_t \int_{-d_2}^{\infty} e^{-\frac{\sigma^2}{2}\tau+\sigma\sqrt{\tau}x} q(x) dx - e^{-r\tau} K \int_{-d_2}^{\infty} q(x) dx = \\ &= e^{-\delta\tau} S_t \int_{-d_2}^{\infty} e^{-\frac{\sigma^2}{2}\tau+\sigma\sqrt{\tau}x} \phi(x) \sum_{n=0}^{\infty} \theta_n H_n(x) dx - e^{-r\tau} K \int_{-d_2}^{\infty} \phi(x) \sum_{n=0}^{\infty} \theta_n H_n(x) dx = \\ &= \sum_{n=0}^{\infty} \theta_n \left(e^{-\delta\tau} S_t \int_{-d_2}^{\infty} e^{-\frac{\sigma^2}{2}\tau+\sigma\sqrt{\tau}x} \phi(x) H_n(x) dx - e^{-r\tau} K \int_{-d_2}^{\infty} \phi(x) H_n(x) dx \right) \end{aligned} \tag{1}$$

To proceed further we need the following lemma, which is the key to most calculations in this section:

Lemma 1. *Let us define:*

$$Y_n(y,\gamma) = \int_y^{+\infty} e^{-\frac{\gamma^2}{2}+\gamma x}\phi(x)H_n(x)dx$$

Then the following relations hold:

$$\begin{cases} Y_n(y,\gamma) = -\gamma Y_{n-1}(y,\gamma) - \phi(y-\gamma)H_{n-1}(y) \\ Y_0(y,\gamma) = 1 - \Phi(y-\gamma) \end{cases}$$

In particular:

$$\int_{-\infty}^{+\infty} e^{-\frac{\gamma^2}{2}+\gamma x}\phi(x)H_n(x)dx = (-\gamma)^n$$

Proof. We prove the Lemma by induction. By definition of H_n and integrating by parts:

$$\begin{aligned} Y_n(y,\gamma) &= \int_y^{+\infty} e^{-\frac{\gamma^2}{2}+\gamma x}\phi(x)H_n(x)dx = \int_y^{+\infty} \frac{1}{\sqrt{2\pi}}e^{-\frac{\gamma^2}{2}+\gamma x}\frac{d^n\phi}{dx^n}dx = \\ &= \int_y^{+\infty}(-\gamma)\frac{1}{\sqrt{2\pi}}e^{-\frac{\gamma^2}{2}+\gamma x}\frac{d^{n-1}\phi}{dx^{n-1}}dx - \frac{1}{\sqrt{2\pi}}e^{-\frac{\gamma^2}{2}+\gamma y}\left.\frac{d^{n-1}\phi}{dx^{n-1}}\right|_y = \\ &= -\gamma Y_{n-1}(y,\gamma) - \phi(y-\gamma)H_{n-1}(y) \end{aligned}$$

Since for $n = 0$ the calculation is trivial, the proof is complete. □

The integrals

$$A_n(y) = \int_y^{\infty} H_n(x)\phi(x)dx \qquad \text{and} \qquad B_n(y,\sigma,\tau) = \int_y^{\infty} e^{-\frac{\sigma^2}{2}\tau+\sigma\sqrt{\tau}x}H_n(x)\phi(x)dx$$

can be computed in closed-form by an application of Lemma 1, with $\gamma = 0$ and $\gamma = \sigma\sqrt{\tau}$ respectively. It follows that $C(S_t, K, r, \tau, \sigma)$ admits an explicit formula in series form:

$$C(S_t, K, r, \tau, \sigma) = \sum_{n=0}^{\infty} \theta_n C_n(S_t, K, r, \tau, \sigma)$$

where

$$C_n(S_t, K, r, \tau, \sigma) = e^{-\delta\tau}S_t B_n(-d_2, \sigma, \tau) - e^{-r\tau}KA_n(-d_2)$$

The first terms of C_n are shown in the appendix: $C_0(S_t, K, r, \tau, \sigma)$ is simply the Black-Scholes formula. This is not surprising, since choosing $\theta_0 = 1$ and $\theta_n = 0$ for $n > 0$, $q(x)$ is a standard normal density.

As mentioned before, by suitable choices of $\{\theta_n\}$ we can represent any function $q(x) \in L^2(\mathbb{R}, N(0,1))$. However, we have two natural conditions on $q(x)$:

- $q(x)$ is a probability density;
- Q is risk-neutral.

It is then important to translate these conditions in terms of parameter constraints in order to improve the estimation precision.

In fact, these properties are characterized by the following:

Proposition 2. *Let $q(x) \in L^2(\mathbb{R}, N(0,1))$ be a positive function. Then we have:*

- *$q(x)$ is a probability density if and only if $\theta_0 = 1$.*
- *$q(x)$ is risk-neutral if and only if $\sum_{n=0}^{\infty} \theta_n(-\sigma\sqrt{\tau})^n = 1$.*

Proof. From the first condition we simply get:

$$1 = \int_{-\infty}^{+\infty} q(x)dx = \int_{-\infty}^{+\infty} \phi(x) \sum_{n=0}^{\infty} \theta_n H_n(x)dx = \sum_{n=0}^{\infty} \theta_n \int_{-\infty}^{+\infty} \phi(x)H_n(x)dx = \theta_0$$

where the last equality follows from the observation that $\int_{-\infty}^{+\infty} \phi(x)H_n(x)dx = 0$ for all $n > 0$. Hence we simply set $\theta_0 = 1$.

The second condition is:

$$E_Q[S_T] = S_t e^{(r-\delta)\tau}$$

In other words:

$$\int_{-\infty}^{+\infty} S_t e^{(r-\delta-\frac{\sigma^2}{2})\tau+\sigma\sqrt{\tau}x} q(x)dx = S_t e^{(r-\delta)\tau}$$

and hence:

$$\int_{-\infty}^{+\infty} e^{-\frac{\sigma^2}{2}\tau+\sigma\sqrt{\tau}x} q(x)dx = 1$$

Observe that:

$$\int_{-\infty}^{+\infty} e^{-\frac{\sigma^2}{2}\tau+\sigma\sqrt{\tau}x} q(x)dx = \int_{-\infty}^{+\infty} e^{-\frac{\sigma^2}{2}\tau+\sigma\sqrt{\tau}x} \phi(x) \sum_{n=0}^{\infty} \theta_n H_n(x)dx =$$

$$= \sum_{n=0}^{\infty} \theta_n \int_{-\infty}^{+\infty} e^{-\frac{\sigma^2}{2}\tau+\sigma\sqrt{\tau}x} \phi(x)H_n(x)dx = \sum_{n=0}^{\infty} \theta_n(-\sigma\sqrt{\tau})^n$$

where the last equality follows from Lemma 1. □

3 Multiple expirations

The previous section outlines a method for extracting the risk-neutral density implied by a cross-section of option prices with the same expiration and different strikes. Since listed options are available for multiple expirations, this procedure can be applied separately to each of them, obtaining densities for different horizons, which approach from day to day. In contrast, risk management practice requires us to look at a time window of fixed length, which generally does not coincide with the expiration date of an option.

In this section we show how the information obtained on the risk-neutral densities on two successive expirations T_1 and T_2 can be used to estimate the density at a certain time T between them. Of course, such an estimation requires some assumptions on the process of the underlying: here we assume that the increment $S_{T_2} - S_{T_1}$ is independent of $S_{T_1} - S_t$, and that the random variable $\log \frac{S_T}{S_{T_1}}$ has the same distribution as $\sqrt{\frac{T-T_1}{T_2-T_1}} \log \frac{S_{T_2}}{S_{T_1}}$. This allows us to write:

$$S_T = S_{T_1} \frac{S_T}{S_{T_1}} \sim S_{T_1} \frac{S_{T_2}}{S_{T_1}} e^{\sqrt{\frac{T-T_1}{T_2-T_1}}}$$

and, by the independence assumption, the density of S_T is obtained by convolution from those of S_{T_1} and $S_{T_2} - S_{T_1}$. This reduces the problem to the computation of the density of $S_{T_2} - S_{T_1}$ in terms of those of S_{T_1} and S_{T_2}. In a similar fashion as the previous section we can write S_{T_2} as:

$$S_{T_2} = S_t e^{(r-\delta)(T_2-t) - \frac{\sigma_1^2}{2}(T_1-t) + \sigma_1\sqrt{T_1-t}\psi_1 - \frac{\sigma_2^2}{2}(T_2-T_1) + \sigma_2\sqrt{T_2-T_1}\psi_2}$$

where the random variables ψ_1 and ψ_2 represent the normalized returns of the underlying respectively in the (t, T_1) and (T_1, T_2) intervals. We denote by $q_1(x)$ and $q_2(x)$ respectively the densities of ψ_1 and ψ_2 under Q. Again, we expand q_1 and q_2 in Hermite series:

$$q_1(x) = \phi(x) \sum_{n=0}^{\infty} \theta_n^1 H_n(x) \qquad q_2(x) = \phi(x) \sum_{n=0}^{\infty} \theta_n^2 H_n(x)$$

The next proposition shows the relation between the Hermite decompositions of $q_1(x)$, $q_2(x)$ and a normalized linear combination $q(x)$. In particular, the relation is linear and is given the coefficients of two of them, those of the third are uniquely determined.

Proposition 3. *Let $q_1(x)$ and $q_2(x)$ be the densities of ψ_1 and ψ_2 as above and denote by $q(x)$ the density of the random variable $(\gamma_1\psi_1 + \gamma_2\psi_2)/\sqrt{\gamma_1^2 + \gamma_2^2}$, where*

$\gamma_1, \gamma_2 > 0$. *Assuming that ψ_1 and ψ_2 are independent, the coefficients θ_n in the Hermite expansion*

$$q(x) = \phi(x) \sum_{n=0}^{\infty} \theta_n H_n(x)$$

are given by:

$$\theta_n = \left(\gamma_1^2 + \gamma_2^2\right)^{-\frac{n}{2}} \sum_{k=0}^{n} \gamma_1^{n-k} \gamma_2^k \theta_{n-k}^1 \theta_k^2$$

Proof. Denoting $\beta = \frac{\sqrt{\gamma_1^2+\gamma_2^2}}{\gamma_2}$ we have:

$$q(y) = \beta \int_{-\infty}^{+\infty} q_1(x) q_2 \left(\beta y - \frac{\gamma_1}{\gamma_2} x\right) dx =$$
$$= \beta \sum_{i,j=0}^{\infty} \theta_i^1 \theta_j^2 \int_{-\infty}^{+\infty} H_i(x) H_j \left(\beta y - \frac{\gamma_1}{\gamma_2} x\right) \phi(x) \phi\left(\beta y - \frac{\gamma_1}{\gamma_2} x\right) dx$$

Integrating by parts we get:

$$\int_{-\infty}^{+\infty} H_i(x) H_j \left(\beta y - \frac{\gamma_1}{\gamma_2} x\right) \phi(x) \phi\left(\beta y - \frac{\gamma_1}{\gamma_2} x\right) dx = \int_{-\infty}^{+\infty} \left.\frac{d^i \phi}{dx^i}\right|_x \left.\frac{d^j \phi}{dx^j}\right|_{\beta y - \frac{\gamma_1}{\gamma_2} x} dx =$$
$$= \left(\frac{\gamma_2}{\gamma_1}\right)^j \int_{-\infty}^{+\infty} \left.\frac{d^{i+j} \phi}{dx^{i+j}}\right|_x \phi\left(\beta y - \frac{\gamma_1}{\gamma_2} x\right) dx$$

Finally:

$$\int_{-\infty}^{+\infty} \left.\frac{d^n \phi}{dx^n}\right|_x \phi\left(\beta y - \frac{\gamma_1}{\gamma_2} x\right) dx = \frac{1}{\beta} \phi(y) H_n(y) \left(\frac{\gamma_1}{\sqrt{\gamma_1^2 + \gamma_2^2}}\right)^n$$

And the proof is complete. □

The above proposition shows how to compute the density of $S_{T_2} - S_{T_1}$ in terms of the densities of S_{T_1} and S_{T_2}. In fact, it is sufficient to substitute $\gamma_1 = \sigma_1 \sqrt{T_1 - t}$ and $\gamma_2 = \sigma_2 \sqrt{T_2 - T_1}$.

For estimation purposes, Hermite polynomials are truncated to a finite number of terms, hence q_1 and q_2 are typically partial sums of degrees n_1 and n_2 respectively. From the proposition above it is evident that the degree of q cannot exceed $n_1 + n_2$, since for higher order terms all the products $\theta_{n-k}^1 \theta_k^2$ vanish.

A further observation, which may be useful in applications, is that the k-th moment of a density $q(x)$ depends only on the first k terms of the Hermite expansion. In practice, this means that truncation can be based on the number of moments that are considered relevant.

4 Application to the Italian derivatives market

4.1 Data

We now turn to the estimation of the model to market data. Our dataset consists of prices and volumes of all transactions on listed options and futures contracts on the MIB30 index during the year 1998. These options are traded on the IDEM (Italian derivatives market), a section of the Italian Exchange. For the interest rates, we used the three-month LIBOR on the Italian lira.

Before we discuss empirical issues let us spend a few words on the institutional features of the market: the MIB30 is a capitalization-weighted index based on a fixed basket of the 30 most liquid and highly capitalized common stocks on the Italian Exchange. Since options and futures are traded simultaneously on the same exchange, there is no time lag between the reporting of option and underlying prices, unlike in the S&P 500 options market.

Options with expiration in the quarterly cycle of March, June, September and December are listed at any time. In addition, the expirations corresponding to the two nearest months outside the cycle are made available. In practice, there are sufficient liquid contracts only for the two nearest months, therefore our analysis is constrained to this time horizon. Liquidity tends to decrease near expiration dates as trading shifts from one contract to the next.

4.2 Methodology

Estimation of the risk-neutral density requires the simultaneous observation of a cross section of options with the same expiration but different strike. In a very liquid market this is achieved considering the last quote on each contract before a fixed time of the day. This procedure is not feasible with our dataset, which does not include quotes; even if applicable it would not exploit all the information embedded in transaction prices, as many options (usually those in-the-money) are thinly traded and bid-ask spreads are very wide.

For each trading day we record the last transaction before noon for each option contract as well as the underlying value at the time of each recorded transaction. This ensures that illiquid contracts, which may be traded few times in an hour, are not associated with the value of the underlying at noon, which may be significantly different from the time of the last transaction.

A critical point is usually the measurement of the underlying value as many authors have pointed out that it is often unreliable, either due to a time lag in reporting or to the unobservability of dividends or both. As mentioned before, the first issue does not arise in our case, while the latter remains.

A possible solution is suggested by Aït-Sahalia and Lo, observing that option prices depend on the underlying price S_T and the dividend yield δ only through the forward price

$$F_T = S_t e^{(r-\delta)\tau}$$

which can be estimated using the model-independent call-put parity:

$$C_t(K) - P_t(K) = e^{-r\tau}(F_T - K)$$

Attempts to apply this idea to our dataset gave disappointing results as asynchronous transactions on calls and puts substantially compromise accuracy. In fact, the estimated underlying price varies widely even in short periods of time owing to the relative illiquidity of the option market.

However, F_T can be estimated from the future price, which is also part of our dataset and is reported synchronously with option prices. When the expiration of the future contract coincides with that of the option, the estimation error is reduced to the difference between the future and the forward prices and to the uncertainty in expected dividends. As the two expirations may differ for two months at most (since future contracts follow the quarterly cycle), the forward price is obtained by discounting the future (we use the three-month LIBOR) but expected dividends in the expiration lag cannot be eliminated and add up to the estimation error.

Summing up, for each trading day we observe the cross sections of those options with the two nearest expiration months. For each expiration, we have a certain number of strikes (typically from 10 to 20) for which a call or a put option, or both, are available. We keep the contract with higher trading volume, which generally coincides with the one out-of-the money (i.e. calls for high strikes and puts for low strikes).

Denoting by $\mathcal{K}$ the set of strikes, for each $K \in \mathcal{K}$ we have an option price P_K, the corresponding underlying value S_K, and a dummy variable F_K, which is equal to 0 for a call option and to 1 for a put option. With this notation we can write the theoretical price Π_K of a call or put option in the single formula:

$$\Pi_K = C(S_K, K, r, \tau, \sigma, \theta) - F_K(e^{-\delta\tau} S_K - e^{-r\tau} K)$$

which is more convenient for estimation purposes than a conditional statement. Then we specify the model as:

$$P_K = \Pi_K + \varepsilon_K$$

where $\{\varepsilon_K\}_{K\in\mathcal{K}}$ are IID random variables. The parameters σ and θ can then be estimated by the least-squares method:

$$\chi^2(\sigma, \theta) = \sum_{K\in\mathcal{K}} (P_K - \Pi_K(F_K, S_K, K, r, \tau, \sigma, \theta))^2$$

$$(\hat{\sigma}, \hat{\theta}) = \arg\min_{\theta,\sigma} \chi^2(\sigma, \theta)$$

As mentioned before, the problem above is nonlinear but only in the parameter σ. This means that it can be solved easily even without nonlinear regression software. In fact, one can define:

$$\phi(\sigma) = \min_{\theta} \chi^2(\sigma, \theta)$$

and minimize ϕ with a standard one-dimensional minimization algorithm (the golden search, for instance), while $\phi(\sigma)$ can be computed explicitly. A natural starting guess for σ is the implied volatility of the at-the-money option.

If nonlinear regression software is available all the parameters can be estimated simultaneously. Since the sum of squares χ^2 is quadratic in $\{\theta_i\}$ the convergence is faster with the Levenberg-Marquardt algorithm than with the ordinary steepest descent method.

At this point, it remains to select an appropriate set of θ_i. Since each cross section consists roughly of 10 to 20 prices it is clear that precision can only be achieved if a very small number of θ is used.

As we remarked earlier, the choice of an appropriate set of θ_i can be guided by considerations on the moments of the risk-neutral density, as the first n terms of the Hermite expansion uniquely determine its first n moments. Since we are constrained by the dataset to a small number of parameters we choose to restrict our attention to the first four moments. This still leaves a total of five parameters, namely σ, θ_1, θ_2, θ_3 and θ_4. Not surprisingly, the simultaneous estimation of all parameters leads to unsatisfactory results as the size of the data is quite limited. Other attempts showed that the elimination of only one parameter would not produce a significant improvement, therefore we opted to leave only three parameters free. As two of them must be chosen out of θ_1, θ_2, θ_3 and θ_4 there are six possible combinations.

The first combination to be ruled out is (θ_2, θ_4) as it can represent only symmetric distributions. The combinations (θ_1, θ_2) and (θ_2, θ_3) can also be dropped since they force negative excess kurtosis in a neighborhood of $(0, 0)$ (which is the typical domain of these estimators). We are thus left with the three combinations (θ_1, θ_3), (θ_1, θ_4), (θ_3, θ_4). While all of them are acceptable with respect to the above considerations, there are a few differences which are worth noting. In fact, expressing skewness and kurtosis with respect to the two parameters, and expanding in a neighborhood of $(0, 0)$ we obtain the following results:

The table above shows that the parametrization (θ_3, θ_4) has two clear advantages over the others. First, skewness and kurtosis are not only linear on the parameters but depend separately on each of them. Contrast this with the other cases, where either skewness or kurtosis depend on higher order terms and boil down to zero for typical parameter values.

A further advantage of the last parametrization lies in the separation between the roles of σ and θ_i. In fact, while most of the variance is generally captured

Table 1: Series expansion of skewness and kurtosis

(x,y)	Skewness	Kurtosis
(θ_1, θ_3)	$-6y + O(x^2y + x^3)$	$3 - 24xy + O(x^3y)$
(θ_1, θ_4)	$-2x^3 + O(x^4)$	$3 + 24y + O(x^2y)$
(θ_3, θ_4)	$-6x$	$3 + 24y$

by σ, the parameters θ_1 and θ_2 may still explain a part of it. When both of them are set equal to zero all the variance must be explained by σ, resulting in a higher accuracy for this estimator.

Finally, if two random variables ψ_1 and ψ_2 have densities q_1 and q_2 with Hermite coefficients $\{1, 0, 0, \theta_3^1, \theta_4^1\}$ and $\{1, 0, 0, \theta_3^2, \theta_4^2\}$, Proposition 3 implies that the random variable $(\gamma_1\psi_1 + \gamma_2\psi_2)/\sqrt{\gamma_1^2 + \gamma_2^2}$ has the following Hermite expansion:

$$\{1, 0, 0, \frac{\theta_3^1 \, {\gamma_1}^3 + \theta_3^2 \, {\gamma_2}^3}{\gamma^3}, \frac{\theta_4^1 \, {\gamma_1}^4 + \theta_4^2 \, {\gamma_2}^4}{\gamma^4}, 0, \frac{\theta_3^1 \, \theta_3^2 \, {\gamma_1}^3 \, {\gamma_2}^3}{\gamma^6}, \frac{\theta_3^2 \, \theta_4^1 \, {\gamma_1}^4 \, {\gamma_2}^3 + \theta_3^1 \, \theta_4^2 \, {\gamma_1}^3 \, {\gamma_2}^4}{\gamma^7}, \frac{\theta_4^2 \, \theta_4^1 \, {\gamma_1}^4 \, {\gamma_2}^4}{\gamma^8}\}$$

where $\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$. In other words, we obtain that the fifth coefficient is nil, regardless of the parameter values. Therefore, the error caused by neglecting the terms after the fourth only involves the coefficients from six to the eight.

4.3 Numerical results

In fact, the choice of (θ_3, θ_4) performs better in empirical tests, as shown by the following table:

Table 2: Estimated standard deviation of option prices (index points)

	First expiration	Second expiration
(θ_1, θ_3)	34.41	115.47
(θ_1, θ_4)	32.55	130.14
(θ_3, θ_4)	31.16	94.48

The table also shows that the standard error is over three times larger for the second expiration, which is less liquid.

Event at this stage parameters still exhibit some instability owing to the small number of strikes available. In practice, the functional χ^2 exhibits large flat regions, where a small perturbation in the data causes a wide fluctuation in the minimizers. This problem can be circumvented adding to the χ^2 functional a small stability term that discourages large fluctuations from the previous value. While this term is generally negligible with respect to the sum of squared errors, it becomes significant in a flat region, leading the estimators to move as little as possible. A convenient choice can be:

$$\tilde{\chi}^2(\sigma, \theta) = \chi^2(\sigma, \theta) + (\alpha|\sigma - \tilde{\sigma}|^2 + \beta|\theta - \tilde{\theta}|^2)$$

for suitable values of the parameters α and β. Here $\tilde{\sigma}$ and $\tilde{\theta}$ are the previous values of the estimators. The choice of the particular functional above can be justified in terms of ease of implementation as it can be embedded in a least-squares framework by adding a further dummy variable λ and an additional set of data with a dummy strike. λ is then set equal to 1 for regular data and to 0 for the additional one. We set:

$$f(\sigma, \theta) = \lambda(P_K - \Pi_K) + (1 - \lambda)(\alpha|\sigma - \tilde{\sigma}|^2 + \beta|\theta - \tilde{\theta}|^2)$$

so that:

$$\tilde{\chi}^2(\sigma, \theta) = \sum_{K \in \mathcal{K} \cup \{\tilde{K}\}} f(\sigma, \theta)^2$$

where the dummy strike $\tilde{K}$ can take any value.

Figures 1 and 2 show the values of volatility (σ), skewness and kurtosis for the first two expirations, estimated with or without the smoothing term in the functional: α and β were both set equal to 10. While the addition of the penalization term greatly reduces the variance of the estimators, virtually eliminating outliers, in principle it may create a bias. Calculating the differences between the two estimators (with or without smoothing) and discarding those values lying outside the centered 95% confidence interval, it turns out that the average biases on volatility, skewness and kurtosis are respectively -1.2%, -71.0% and 4.8% of the parameter averages. In other words, the bias is not serious for volatility and excess kurtosis, while it is significant for skewness.

Figure 3 shows the estimates for a moving horizon of one-month, obtained using the procedure in section 3 from the estimates on the first two expirations. In the last set we have the graphs of the parameter estimates versus the index. In the sample period there were two major events affecting the Italian market: the admission to the core group of countries participating to the EMU and the global crisis of world markets following the default of Russia. The first event caused a strong rally in the Italian index in anticipation of the admission to the EMU, followed by a sharp drop during April. In this case, the option market correctly anticipated the chance of large movements with both implied volatility and kurtosis rising from mid-March and with skewness decreasing over the same period. On the contrary, the Russian crisis, which caused a much larger drop in the index, was not anticipated at all by market participants as volatility began to rise only at the end of August, kurtosis continued to shrink until the end of October, and skewness even rose throughout the crisis.

Table 3: First terms of C_n

n	$C_n(S,K,r,\tau,\sigma)$
0	$e^{-\delta\tau}S\Phi(d_1) - e^{-r\tau}K\Phi(d_2)$
1	$-e^{-\delta\tau}S\sigma\sqrt{\tau}\Phi(d_1)$
2	$d_2e^{-r\tau}K\sigma\sqrt{\tau}\phi(d_2) + e^{-\delta\tau}S\sigma^2\tau\Phi(d_1)$
3	$\sigma\sqrt{\tau}\left(e^{-r\tau}K\left(-1+d_2^2-d_2\sigma\sqrt{\tau}\phi(d_2)\right) - e^{-\delta\tau}S\sigma^2\tau\Phi(d_1)\right)$
4	$e^{-r\tau}K\sigma\sqrt{\tau}\left(d_2^3+\sigma\sqrt{\tau}-{d_2}^2\sigma\sqrt{\tau}+d_2\left(-3+\sigma^2\tau\right)\right)\phi(d_2) + e^{-\delta\tau}S\sigma^4\tau^2\Phi(d_1)$

$$
\begin{aligned}
d_1 &= \frac{\log\frac{S_t}{K} + \left(r-\delta+\frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \\
d_2 &= \frac{\log\frac{S_t}{K} + \left(r-\delta-\frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \\
\phi(x) &= \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} \\
\Phi(x) &= \int_{-\infty}^{x}\phi(t)dt
\end{aligned}
$$

Figure 1: First expiration

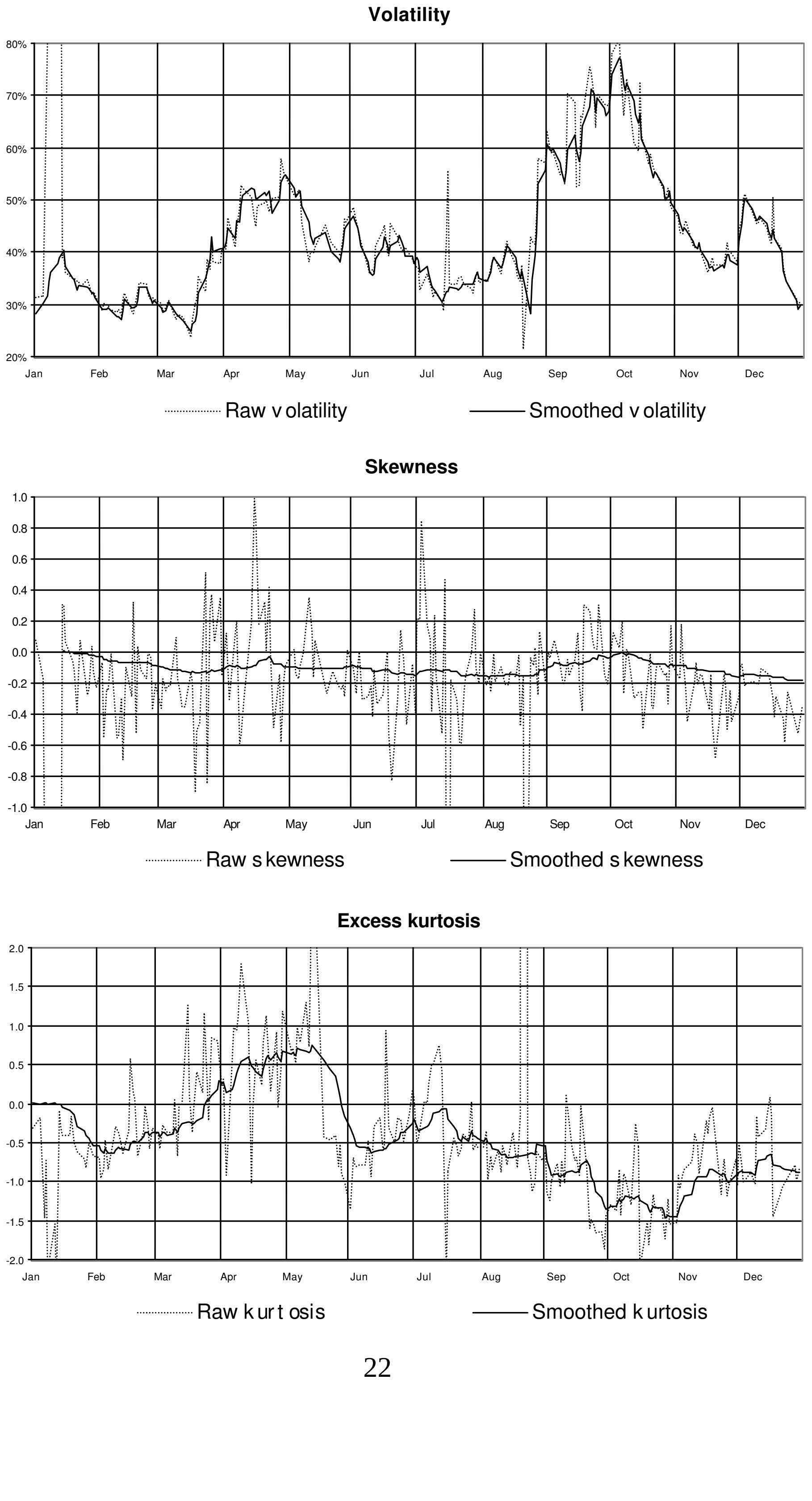


Figure 2: Second expiration

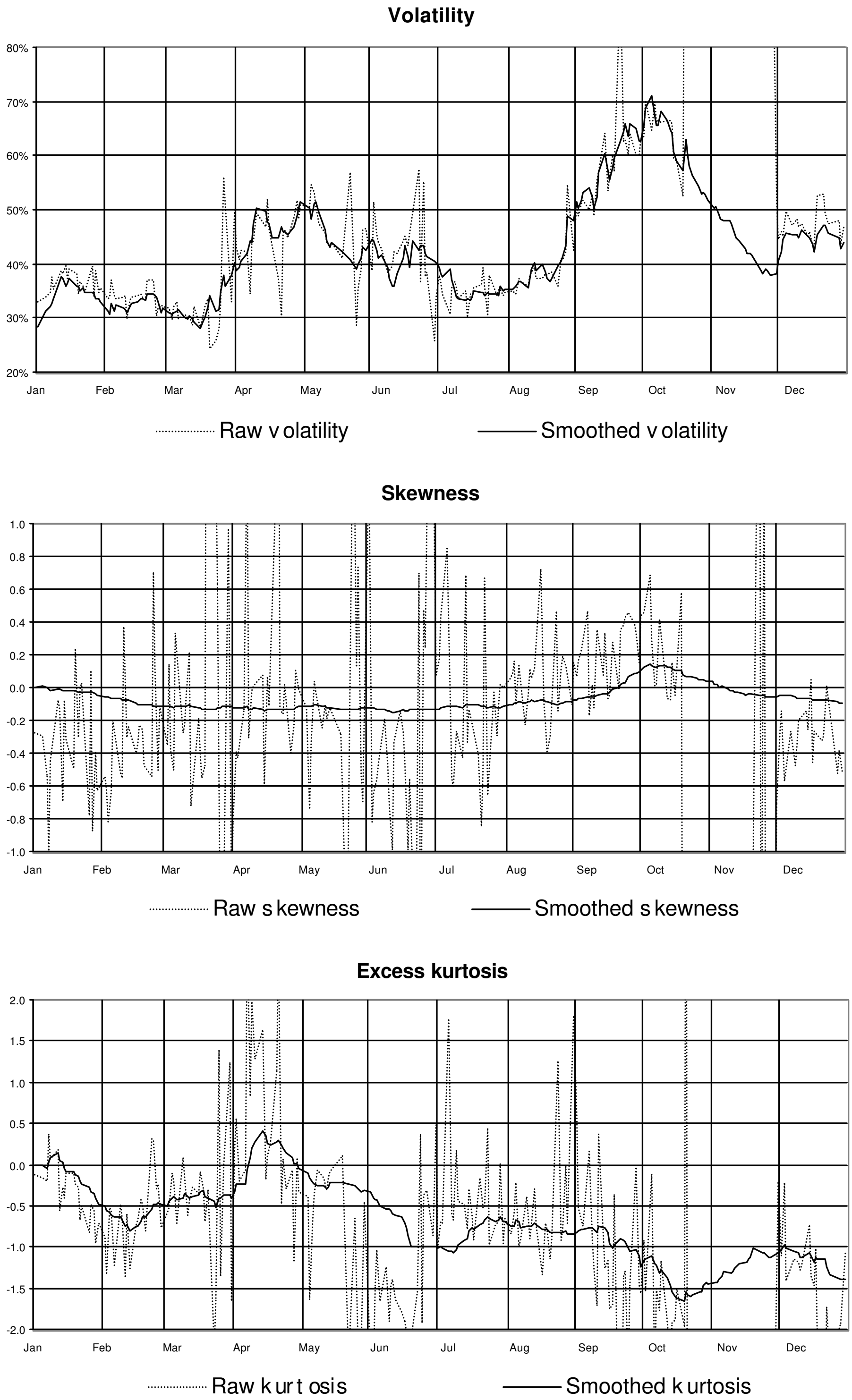
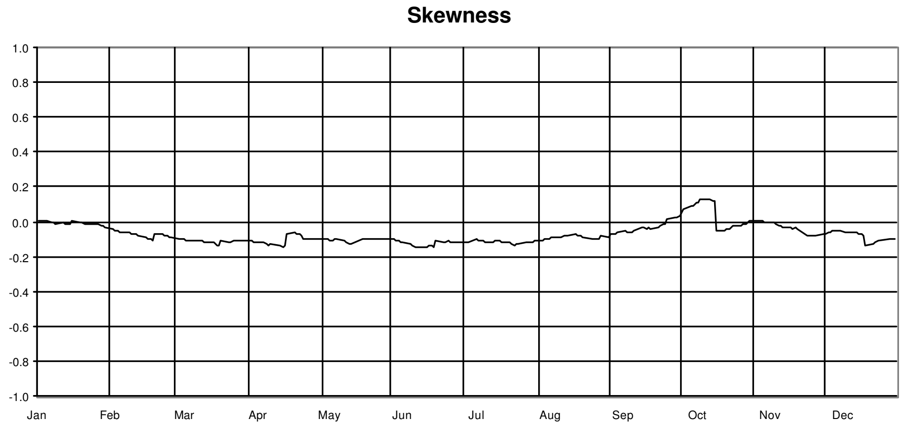
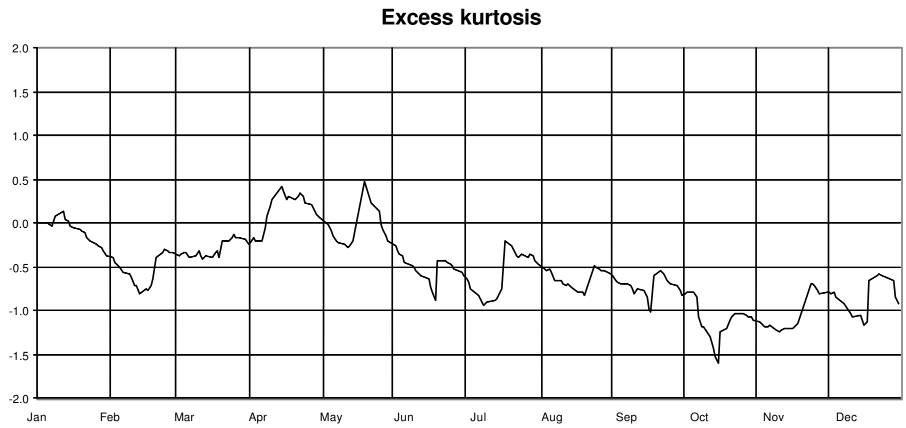


Figure 3: Costant expiration





References

[1] Yacine Aït Sahalia and Andrew Lo. Nonparametric estimation of state-price densities implicit in financial asset prices. *Journal of Finance*, 53:299–547, July 1998.

[2] Robert Engle and Joshua Rosenberg. Empirical pricing kernels. Stern School of Business Working Paper, July 2000.

[3] S. Heston. A closed-form solution for option with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6:327–343, 1993.

[4] Dilip B. Madan and Frank Milne. Contingent claims valued and hedged by pricing and investing in a basis. *Math. Finance*, 4(3):223–245, 1994.

[5] Mark Rubinstein. Implied binomial trees. *Journal of Finance*, July 1994.

[6] David Shimko. Bounds of probability. *Risk*, 6:33–37, 1993.

[7] E. M. Stein and J. C. Stein. Stock price distributions with stochastic volatility: an analytic approach. *Review of Financial Studies*, 4:727–752, 1991.

RECENTLY PUBLISHED "TEMI" (*)

N. 481 – *Bank competition and firm creation*, by E. Bonaccorsi di Patti and G. Dell'Ariccia (June 2003).

N. 482 – *La distribuzione del reddito e della ricchezza nelle regioni italiane*, by L. Cannari and G. D'Alessio (June 2003).

N. 483 – *Risk aversion, wealth and background risk*, by L. Guiso and M. Paiella (September 2003).

N. 484 – *What is this thing called confidence? A comparative analysis of consumer indices in eight major countries*, by R. Golinelli and G. Parigi (September 2003).

N. 485 – *L'utilizzo degli indicatori compositi nell'analisi congiunturale territoriale: un'applicazione all'economia del Veneto*, by P. Chiades, M. Gallo and A. Venturini (September 2003).

N. 486 – *Bank capital and lending behaviour: empirical evidence for Italy*, by L. Gambacorta and P. E. Mistrulli (September 2003).

N. 487 – *A polarization of polarization? The distribution of inequality 1970-1996*, by C. Biancotti (March 2004).

N. 488 – *Pitfalls of monetary policy under incomplete information: imprecise indicators and real indeterminacy*, by E. Gaiotti (March 2004).

N. 489 – *Information technology and productivity changes in the banking industry*, by L. Casolaro and G. Gobbi (March 2004).

N. 490 – *La bilancia dei pagamenti di parte corrente Nord-Sud (1998-2000)*, by L. Cannari and S. Chiri (March 2004).

N. 491 – *Investire in Italia? Risultati di una recente indagine empirica,* by M. Committeri (March 2004).

N. 492 – *Centralization of wage bargaining and the unemployment rate: revisiting the hump-shape hypothesis,* by L. Forni (June 2004).

N. 493 – *Endogenous monetary policy with unobserved potential output,* by A. Cukierman and F. Lippi (June 2004).

N. 494 – *Il credito commerciale: problemi e teorie,* by M. Omiccioli (June 2004).

N. 495 – *Condizioni di credito commerciale e differenziazione della clientela,*byL. Cannari, S. Chiri and M. Omiccioli (June 2004).

N. 496 – *Il debito commerciale in Italia: quanto contano le motivazioni finanziarie?,* by P. Finaldi Russo and L. Leva (June 2004).

N. 497 – *Funzionamento della giustizia civile e struttura finanziaria delle imprese: il ruolo del credito commerciale,* by A. Carmignani (June 2004).

N. 498 – *Does trade credit substitute for bank credit?,* by G. de Blasio (June 2004).

N. 499 – *Monetary policy and the transition to rational expectations,* by G. Ferrero (June 2004).

N. 500 – *Turning-point indicators from business surveys: real-time detection for the euro area and its major member countries,* by A. Baffigi and A. Bassanetti (June 2004).

N. 501 – *La ricchezza delle famiglie italiane e americane,* by I. Faiella and A. Neri (June 2004).

N. 502 – *Optimal duplication of effort in advocacy systems,* by G. Palumbo (June 2004).

N. 503 – *Il pilastro privato del sistema previdenziale. Il caso del Regno Unito,* by F. Spadafora (June 2004).

N. 504 – *Firm size distribution and employment protection legislation in Italy,* by F. Schivardi and R. Torrini (June 2004).

N. 505 – *Social mobility and endogenous cycles in redistribution,* by F. Zollino (July 2004).

N. 506 – *Estimating expectations of shocks using option prices,* by A. Di Cesare (July 2004).

(*) Requests for copies should be sent to:
Banca d'Italia – Servizio Studi – Divisione Biblioteca e pubblicazioni – Via Nazionale, 91 – 00184 Rome (fax 0039 06 47922059). They area available on the Internet www.bancaditalia.it.

"TEMI" LATER PUBLISHED ELSEWHERE

1999

L. GUISO and G. PARIGI, *Investment and demand uncertainty*, Quarterly Journal of Economics, Vol. 114 (1), pp. 185-228, **TD No. 289 (November 1996)**.

A. F. POZZOLO, *Gli effetti della liberalizzazione valutaria sulle transazioni finanziarie dell'Italia con l'estero*, Rivista di Politica Economica, Vol. 89 (3), pp. 45-76, **TD No. 296 (February 1997)**.

A. CUKIERMAN and F. LIPPI, *Central bank independence, centralization of wage bargaining, inflation and unemployment: theory and evidence*, European Economic Review, Vol. 43 (7), pp. 1395-1434, **TD No. 332 (April 1998)**.

P. CASELLI and R. RINALDI, *La politica fiscale nei paesi dell'Unione europea negli anni novanta,* Studi e note di economia, (1), pp. 71-109, **TD No. 334 (July 1998)**.

A. BRANDOLINI, *The distribution of personal income in post-war Italy: Source description, data quality, and the time pattern of income inequality*, Giornale degli economisti e Annali di economia, Vol. 58 (2), pp. 183-239, **TD No. 350 (April 1999)**.

L. GUISO, A. K. KASHYAP, F. PANETTA and D. TERLIZZESE, *Will a common European monetary policy have asymmetric effects?*, Economic Perspectives, Federal Reserve Bank of Chicago, Vol. 23 (4), pp. 56-75, **TD No. 384 (October 2000)**.

2000

P. ANGELINI, *Are banks risk-averse? Timing of the operations in the interbank market*, Journal of Money, Credit and Banking, Vol. 32 (1), pp. 54-73, **TD No. 266 (April 1996).**

F. DRUDI and R: GIORDANO, *Default Risk and optimal debt management,* Journal of Banking and Finance, Vol. 24 (6), pp. 861-892, **TD No. 278 (September 1996)**.

F. DRUDI and R. GIORDANO, *Wage indexation, employment and inflation,* Scandinavian Journal of Economics, Vol. 102 (4), pp. 645-668, **TD No. 292 (December 1996)**.

F. DRUDI and A. PRATI, *Signaling fiscal regime sustainability,* European Economic Review, Vol. 44 (10), pp. 1897-1930, **TD No. 335 (September 1998)**.

F. FORNARI and R. VIOLI, *The probability density function of interest rates implied in the price of options,* in: R. Violi, (ed.) , Mercati dei derivati, controllo monetario e stabilità finanziaria, Il Mulino, Bologna, **TD No. 339 (October 1998)**.

D. J. MARCHETTI and G. PARIGI, *Energy consumption, survey data and the prediction of industrial production in Italy,* Journal of Forecasting, Vol. 19 (5), pp. 419-440, **TD No. 342 (December 1998)**.

A. BAFFIGI, M. PAGNINI and F. QUINTILIANI, *Localismo bancario e distretti industriali: assetto dei mercati del credito e finanziamento degli investimenti,* in: L.F. Signorini (ed.), Lo sviluppo locale: un'indagine della Banca d'Italia sui distretti industriali, Donzelli, **TD No. 347** (**March 1999)**.

A. SCALIA and V. VACCA, *Does market transparency matter? A case study*, in: Market Liquidity: Research Findings and Selected Policy Implications, Basel, Bank for International Settlements, **TD No. 359 (October 1999)**.

F. SCHIVARDI, *Rigidità nel mercato del lavoro, disoccupazione e crescita*, Giornale degli economisti e Annali di economia, Vol. 59 (1), pp. 117-143, **TD No. 364 (December 1999)**.

G. BODO, R. GOLINELLI and G. PARIGI, *Forecasting industrial production in the euro area,* Empirical Economics, Vol. 25 (4), pp. 541-561, **TD No. 370 (March 2000)**.

F. ALTISSIMO, D. J. MARCHETTI and G. P. ONETO, *The Italian business cycle: Coincident and leading indicators and some stylized facts,* Giornale degli economisti e Annali di economia, Vol. 60 (2), pp. 147-220, **TD No. 377 (October 2000)**.

C. MICHELACCI and P. ZAFFARONI, *(Fractional) Beta convergence*, Journal of Monetary Economics, Vol. 45, pp. 129-153, **TD No. 383 (October 2000)**.

R. DE BONIS and A. FERRANDO*, The Italian banking structure in the nineties: testing the multimarket contact hypothesis*, Economic Notes, Vol. 29 (2), pp. 215-241, **TD No. 387 (October 2000)**.

2001

M. CARUSO, *Stock prices and money velocity: A multi-country analysis,* Empirical Economics, Vol. 26 (4), pp. 651-72, **TD No. 264 (February 1996)**.

P. CIPOLLONE and D. J. MARCHETTI, *Bottlenecks and limits to growth: A multisectoral analysis of Italian industry*, Journal of Policy Modeling, Vol. 23 (6), pp. 601-620, **TD No. 314 (August 1997)**.

P. CASELLI, *Fiscal consolidations under fixed exchange rates,* European Economic Review, Vol. 45 (3), pp. 425-450, **TD No. 336 (October 1998)**.

F. ALTISSIMO and G. L. VIOLANTE, *Nonlinear VAR: Some theory and an application to US GNP and unemployment,* Journal of Applied Econometrics, Vol. 16 (4), pp. 461-486, **TD No. 338 (October 1998)**.

F. NUCCI and A. F. POZZOLO, *Investment and the exchange rate*, European Economic Review, Vol. 45 (2), pp. 259-283, **TD No. 344 (December 1998)**.

L. GAMBACORTA, *On the institutional design of the European monetary union: Conservatism, stability pact and economic shocks,* Economic Notes, Vol. 30 (1), pp. 109-143, **TD No. 356 (June 1999)**.

P. FINALDI RUSSO and P. ROSSI, *Credit costraints in italian industrial districts,* Applied Economics, Vol. 33 (11), pp. 1469-1477, **TD No. 360 (December 1999)**.

A. CUKIERMAN and F. LIPPI, *Labor markets and monetary union: A strategic analysis,* Economic Journal, Vol. 111 (473), pp. 541-565, **TD No. 365 (February 2000)**.

G. PARIGI and S. SIVIERO, *An investment-function-based measure of capacity utilisation, potential output and utilised capacity in the Bank of Italy's quarterly model*, Economic Modelling, Vol. 18 (4), pp. 525-550, **TD No. 367 (February 2000)**.

F. BALASSONE and D. MONACELLI, *Emu fiscal rules: Is there a gap?,* in: M. Bordignon and D. Da Empoli (eds.), Politica fiscale, flessibilità dei mercati e crescita*,* Milano*,* Franco Angeli, **TD No. 375 (July 2000)**.

A. B. ATKINSON and A. BRANDOLINI, *Promise and pitfalls in the use of "secondary" data-sets: Income inequality in OECD countries*, Journal of Economic Literature, Vol. 39 (3), pp. 771-799, **TD No. 379 (October 2000)**.

D. FOCARELLI and A. F. POZZOLO, *The determinants of cross-border bank shareholdings: An analysis with bank-level data from OECD countries*, Journal of Banking and Finance, Vol. 25 (12), pp. 2305-2337, **TD No. 381 (October 2000)**.

M. SBRACIA and A. ZAGHINI, *Expectations and information in second generation currency crises models*, Economic Modelling, Vol. 18 (2), pp. 203-222, **TD No. 391 (December 2000)**.

F. FORNARI and A. MELE, *Recovering the probability density function of asset prices using GARCH as diffusion approximations,* Journal of Empirical Finance, Vol. 8 (1), pp. 83-110**, TD No. 396 (February 2001)**.

P. CIPOLLONE, *La convergenza dei salari manifatturieri in Europa,* Politica economica, Vol. 17 (1), pp. 97-125, **TD No. 398 (February 2001)**.

E. BONACCORSI DI PATTI and G. GOBBI, *The changing structure of local credit markets: Are small businesses special?*, Journal of Banking and Finance, Vol. 25 (12), pp. 2209-2237, **TD No. 404 (June 2001)**.

G. MESSINA, *Decentramento fiscale e perequazione regionale. Efficienza e redistribuzione nel nuovo sistema di finanziamento delle regioni a statuto ordinario*, Studi economici, Vol. 56 (73), pp. 131-148, **TD No. 416 (August 2001)**.

2002

R. CESARI and F. PANETTA, *Style, fees and performance of Italian equity funds*, Journal of Banking and Finance, Vol. 26 (1), **TD No. 325 (January 1998)**.

L. GAMBACORTA, *Asymmetric bank lending channels and ECB monetary policy*, Economic Modelling, Vol. 20 (1), pp. 25-46, **TD No. 340 (October 1998)**.

C. GIANNINI, *"Enemy of none but a common friend of all"? An international perspective on the lender-of-last-resort function*, Essay in International Finance, Vol. 214, Princeton, N. J., Princeton University Press, **TD No. 341 (December 1998)**.

A. ZAGHINI, *Fiscal adjustments and economic performing: A comparative study*, Applied Economics, Vol. 33 (5), pp. 613-624, **TD No. 355 (June 1999)**.

F. ALTISSIMO, S. SIVIERO and D. TERLIZZESE, *How deep are the deep parameters?,* Annales d'Economie et de Statistique,.(67/68), pp. 207-226, **TD No. 354 (June 1999)**.

F. FORNARI, C. MONTICELLI, M. PERICOLI and M. TIVEGNA, *The impact of news on the exchange rate of the lira and long-term interest rates,* Economic Modelling, Vol. 19 (4), pp. 611-639, **TD No. 358 (October 1999**).

D. FOCARELLI, F. PANETTA and C. SALLEO, *Why do banks merge?,* Journal of Money, Credit and Banking, Vol. 34 (4), pp. 1047-1066, **TD No. 361 (December 1999)**.

D. J. MARCHETTI, *Markup and the business cycle: Evidence from Italian manufacturing branches,* Open Economies Review, Vol. 13 (1), pp. 87-103, **TD No. 362 (December 1999**).

F. BUSETTI, *Testing for stochastic trends in series with structural breaks*, Journal of Forecasting, Vol. 21 (2), pp. 81-105, **TD No. 385 (October 2000)**.

F. LIPPI, *Revisiting the Case for a Populist Central Banker,* European Economic Review*,* Vol. 46 (3), pp. 601-612, **TD No. 386 (October 2000)**.

F. PANETTA, *The stability of the relation between the stock market and macroeconomic forces*, Economic Notes, Vol. 31 (3), **TD No. 393 (February 2001)**.

G. GRANDE and L. VENTURA, *Labor income and risky assets under market incompleteness: Evidence from Italian data,* Journal of Banking and Finance, Vol. 26 (2-3), pp. 597-620, **TD No. 399 (March 2001)**.

A. BRANDOLINI, P. CIPOLLONE and P. SESTITO, *Earnings dispersion, low pay and household poverty in Italy, 1977-1998,* in D. Cohen, T. Piketty and G. Saint-Paul (eds.), The Economics of Rising Inequalities, pp. 225-264, Oxford, Oxford University Press, **TD No. 427 (November 2001)**.

L. CANNARI and G. D'ALESSIO, *La distribuzione del reddito e della ricchezza nelle regioni italiane*, Rivista Economica del Mezzogiorno (Trimestrale della SVIMEZ), Vol. XVI(4), pp. 809-847, Il Mulino, **TD No. 482 (June 2003)**.

2003

F. SCHIVARDI, *Reallocation and learning over the business cycle,* European Economic Review, , Vol. 47 (1), pp. 95-111, **TD No. 345 (December 1998)**.

P. CASELLI, P. PAGANO and F. SCHIVARDI, *Uncertainty and slowdown of capital accumulation in Europe*, Applied Economics, Vol. 35 (1), pp. 79-89, **TD No. 372 (March 2000).**

P. PAGANO and G. FERRAGUTO, Endogenous growth with intertemporally dependent preferences, Contribution to Macroeconomics, Vol. 3 (1), pp. 1-38, **TD No. 382 (October 2000).**

P. PAGANO and F. SCHIVARDI, *Firm size distribution and growth,* Scandinavian Journal of Economics, Vol. 105(2), pp. 255-274, **TD No. 394 (February 2001)**.

M. PERICOLI and M. SBRACIA, *A Primer on Financial Contagion,* Journal of Economic Surveys, **TD No. 407 (June 2001)**.

M. SBRACIA and A. ZAGHINI, *The role of the banking system in the international transmission of shocks,* World Economy, **TD No. 409 (June 2001)**.

E. GAIOTTI and A. GENERALE, *Does monetary policy have asymmetric effects? A look at the investment decisions of Italian firms,* Giornale degli Economisti e Annali di Economia, Vol. 61 (1), pp. 29-59, **TD No. 429 (December 2001)**.

L. GAMBACORTA, *The Italian banking system and monetary policy transmission: evidence from bank level data*, in: I. Angeloni, A. Kashyap and B. Mojon (eds.), Monetary Policy Transmission in the Euro Area, Cambridge, Cambridge University Press, **TD No. 430 (December 2001).**

M. EHRMANN, L. GAMBACORTA, J. MARTÍNEZ PAGÉS, P. SEVESTRE and A. WORMS, *Financial systems and the role of banks in monetary policy transmission in the euro area,* in: I. Angeloni, A. Kashyap and B. Mojon (eds.), Monetary Policy Transmission in the Euro Area, Cambridge, Cambridge University Press, **TD No. 432 (December 2001)**.

F. SPADAFORA, *Financial crises, moral hazard and the speciality of the international market: further evidence from the pricing of syndicated bank loans to emerging markets*, Emerging Markets Review, Vol. 4 (2), pp. 167-198, **TD No. 438 (March 2002)**.

D. FOCARELLI and F. PANETTA, *Are mergers beneficial to consumers? Evidence from the market for bank deposits*, American Economic Review, Vol. 93 (4), pp. 1152-1172, **TD No. 448 (July 2002)**.

E.VIVIANO, *Un'analisi critica delle definizioni di disoccupazione e partecipazione in Italia,* Politica Economica, Vol. 19 (1), pp. 161-190, **TD No. 450 (July 2002)**.

F. BUSETTI and A. M. ROBERT TAYLOR, *Testing against stochastic trend and seasonality in the presence of unattended breaks and unit roots*, Journal of Econometrics, Vol. 117 (1), pp. 21-53, **TD No. 470 (February 2003)**.

2004

P. CHIADES and L. GAMBACORTA, *The Bernanke and Blinder model in an open economy: The Italian case,* German Economic Review, Vol. 5 (1), pp. 1-34, **TD No. 388 (December 2000)**.

M. PAIELLA, *Heterogeneity in financial market participation: appraising its implications for the C-CAPM*, Review of Finance, Vol. 8, pp. 1-36, **TD No. 473 (June 2003)**.

E. BONACCORSI DI PATTI and G. DELL'ARICCIA, *Bank competition and firm creation*, Journal of Money Credit and Banking, Vol. 36 (2), pp. 225-251, **TD No. 481 (June 2003)**.

R. GOLINELLI and G. PARIGI, *Consumer sentiment and economic activity: a cross country comparison*, Journal of Business Cycle Measurement and Analysis, Vol. 1 (2), **TD No. 484 (September 2003)**.

FORTHCOMING

A. F. POZZOLO, *Research and development regional spillovers, and the localisation of economic activities*, The Manchester School, **TD No. 331 (March 1998)**.

F. LIPPI, *Strategic monetary policy with non-atomistic wage-setters,* Review of Economic Studies, **TD No. 374 (June 2000)**.

P. ANGELINI and N. CETORELLI, *Bank competition and regulatory reform: The case of the Italian banking industry,* Journal of Money, Credit and Banking, **TD No. 380 (October 2000)**.

L. DEDOLA AND F. LIPPI, *The Monetary Transmission Mechanism: Evidence from the industry Data of Five OECD Countries*, European Economic Review, **TD No. 389 (December 2000)**.

M. BUGAMELLI and P. PAGANO, *Barriers to Investment in ICT*, Applied Economics, **TD No. 420 (October 2001)**.

D. J. MARCHETTI and F. NUCCI, *Price Stickiness and the Contractionary Effects of Technology Shocks*, European Economic Review, **TD No. 392 (February 2001)**.

G. CORSETTI, M. PERICOLI and M. SBRACIA, *Correlation analysis of financial contagion: what one should know before running a test*, Journal of International Money and Finance, **TD No. 408 (June 2001)**.

D. FOCARELLI, *Bootstrap bias-correction procedure in estimating long-run relationships from dynamic* *panels, with an application to money demand in the euro area,* Economic Modelling, **TD No. 440 (March 2002)**.

A. BAFFIGI, R. GOLINELLI and G. PARIGI, *Bridge models to forecast the euro area GDP*, International Journal of Forecasting, **TD No. 456 (December 2002)**.

F. CINGANO and F. SCHIVARDI, *Identifying the sources of local productivity growth,* Journal of the European Economic Association, **TD NO. 474 (June 2003)**.

E. BARUCCI, C. IMPENNA and R. RENÒ, *Monetary integration, markets and regulation*, Research in Banking and Finance, **TD NO. 475 (June 2003)**.

G. ARDIZZI, *Cost efficiency in the retail payment networks:first evidence from the Italian credit card system,* Rivista di Politica Economica, **TD NO. 480 (June 2003)**.

L. GAMBACORTA and P. E. Mistrulli, *Does bank capital affect lending behavior?*, Journal of Financial Intermediation, **TD NO. 486 (September 2003)**.