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BUSINESS CYCLE NON-LINEARITIES AND PRODUCTIVITY SHOCKS

by Paolo Piselli

Abstract

The recent empirical evidence documenting the presence of asymmetries in business cycles represents a challenge for the standard equilibrium models of real business cycle. These models successfully explain most first and second moments of the actual time series, but cannot replicate non-linear features of the data, unless a non-linear innovation is introduced. This paper aims at investigating the possible non-linearity in the technology shock, the basic innovation in Real Business Cycle models.

In order to measure the unobservable technology shock, we derive some alternative measures of total factor productivity such as revenue-based and cost-based Solow residual and we also control for cyclical factor utilisation. We test for non-linearities and model a non-linear SETAR model for the productivity shock as a natural extension of the autoregressive linear process, the standard way of representing technology shocks.

Our findings suggest that, although the standard Solow residual turns out to be linear, the other measures of technology shock appear non-linear, as soon as non-technological cyclical components are ruled out.

JEL classification: C22, C52, E32.

Keywords: Solow residual, technology shock, non-linear models, linearity test.

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1. Introduction\textsuperscript{1}

A growing literature has shown that fluctuations in most economic time series can be drawn in terms of non-linear models. This literature on so called “asymmetric business cycles” (Sichel 1993, 1994; DeLong and Summer 1988; Potter 1995), has introduced several tests and models for singling out asymmetries and non-linearities in the different cyclical components: GDP (Hamilton 1989, Potter 1995), unemployment (Neftci 1984, Montgomery et al. 1998), employment (Hussey 1992, Palm and Pfann 1997), consumption (Holly and Stannet 1995), inventories (Sensier 1997), and investment (Arden et al. 1997).

Furthermore, in the last two decades productivity fluctuations have taken center stage in modelling output fluctuations and are now viewed as an essential part of the cycle. The development of Real Business Cycle models (RBC) has pointed out the role of productivity shocks in explaining economic fluctuations (Prescott 1986, Cooley 1995). However, standard general-equilibrium models cannot replicate non-linear features of the data, unless some exogenous non-linear driving force is introduced (Kim et al. 1996, Choi 1998, Eudey and Perli 1999).

This work aims to analyse the non-linearity present in time series of technical progress indicators, by way of explanation of asymmetric business cycle fluctuations. Unlike the recent literature focusing on the presence of non-linearity in the mechanism of propagation of exogenous shocks, owing to an asymmetric response of factor demands to exogenous shocks across the business cycle (Pfann and Palm 1993, Palm and Pfann 1997, Harmermesh and Pfann 1996 for a survey), we investigate the presence of asymmetry in the shock itself.

In the RBC literature the exogenous technological shock is typically modelled through a simple autoregressive process (AR). In this paper, we investigate if a simple non-linear extension of this representation, the Self-Exciting Transition Autoregressive model (SETAR), is appropriate for the data. Moreover, this model, embodying the linear autoregressive model as a nested model allows us to undertake a test procedure against the linear hypothesis.

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Traditionally, the Solow residual (1957) is used as a proxy of technology shocks, but recent literature has pointed out that this measure becomes quite inaccurate when market structure, returns to scale or factor utilisation over the cycle are taken into account. When standard hypotheses fail to hold, the Solow residual embodies many cyclical components, not necessarily correlated with technical progress, and as a result the contribution of productivity shocks to business cycles fluctuations is usually overestimated (Hall 1988, 1990; Caballero and Lyons 1992, Bernanke and Parkinson 1991, Burnside and Eichenbaum 1996, Basu and Fernald 1997 among others).

Our main result is that these “non-technological components” of productivity fluctuations, far larger and more volatile than the actual technology shock, reduce the observed non-linearity, in that the productivity pattern over the cycle comes out non-linear, once these other cyclical components are netted out.

This paper is organised as follows. In section 2 the problem of measuring technology shocks is introduced and section 3 computes total factor productivity change, a proxy for the technological shock under different hypotheses. Section 4 introduces the linearity tests of McLeod and Li (1983) and Ramsey and Rothman (1996). Section 5 outlines the SETAR model. Findings are shown in section 6 and section 7 concludes. An Appendix briefly describes the data and their construction.

2. Measuring Productivity Shocks

2.1 Mismeasurement of technological shock and non-linearity

Since the models of Kydland and Prescott (1982) and Prescott (1986), much of the literature on business cycle fluctuations has been based on an extension of the basic neo-classical growth model with exogenous productivity shocks. In these models, changes in total factor productivity (TFP) underlie the cyclical ups and downs. These changes have typically been measured by Solow’s residual (1957), used as a proxy for technical progress, under the assumptions of constant returns to scale, perfect competition and full utilisation of inputs.

However, since Solow (1964), it had been clear that, in the presence of significant adjustment costs of hiring or firing or accumulating or disposing of capital, firms hoard factors

---

2 Throughout the literature “technology shock” and “exogenous productivity shock” or simply “productivity shock” are often used synonymously. See for instance King and Rebelo(1999) or Salyer et al. (1998, chap. 1).
in order to adjust the factor demand instantaneously to output demand. Consequently, factors are used more intensively in booms than recessions.

While it is unlikely that technological declines will occur during recessions, it is far more plausible, according to the traditional keynesian explanation of procyclical productivity, that the productivity of both labour and capital falls in business slowdowns and contractions because workers and machines have less work to do (Bernanke and Parkinson 1991).

Some RBC models with variable capital utilisation rate show that capital utilisation affects the shock propagation mechanism and that the variability of the Solow residual due to technology shocks is normally far overestimated (Burnside and Eichenbaum 1996, Bils and Cho 1993). Similar results occur when the variation of labour utilisation over the business cycle (labour hoarding) is taken into account (Burnside, Eichenbaum and Rebelo 1993, Sbordone 1996, Basu and Kimball 1997, Marchetti and Nucci 2001b for a recent survey). In general, cyclical movements in capacity utilisation or labour utilisation will cause the measurement of TFP growth to be more variable than actual TFP growth.

Finally, in the standard Solow residual, changes in efficiency due to increasing returns or market power are neglected. Factors are paid their marginal product, and the share of product attributed to the factor is equal to its cost. By contrast, Hall (1988, 1990) finds evidence of increasing returns and market power in the US manufacturing sector. Since Hall’s contribution, many extensions of the basic RBC model have been undertaken and a number of authors have provided evidence that increasing returns and imperfect competition help explain procyclical productivity (Hornstein 1993, Benhabib and Farmer 1994, Devereux et al. 1996, to cite only a few).

Yet, in spite of the remarkable and continuous developments of the basic model, extensions towards non-symmetry or non-linearity have rarely been allowed for, although it is highly unlikely that positive and negative technology fluctuations are similar in amplitude and duration. Technology shocks may be a main cause of productivity shifts, and an expansion may be caused by a positive technology shock, but it is hard to read a recession as stemming from a negative technology shock (Zarnowitz 1992, chap.1). Although the rate at which inventions and discoveries are made may vary over time, the stock of knowledge should not decrease. In other words, technological regression is supposed to occur with a lower probability than the negative growth rates observed in productivity. In fact, correcting the Solow residual for
input utilisation leads to a reduction in the probability of technological regression relative to the standard model (Burnside, Eichenbaum and Rebelo 1996, King and Rebelo 1999).

If technology is expected to fluctuate over the cycle in such an asymmetric way, we can no longer represent it by a linear structure (e.g. Sichel 1993). Actually, this issue has recently been investigated both empirically and within a theoretical framework. Business cycle models based on the stochastic optimal growth paradigm have been extended to catch some non-linear dynamics. Granger and Swanson (1996) extend the model of King et al. (1991) and show that fluctuations in response to the common productivity shock can be asymmetric and better represented with an asymmetric error correction model, where negative and positive shocks adjust differently. Likewise, Sarno (1999) points out how the basic stochastic growth model of Solow-Swan embodies a non-linear adjustment of labour productivity.

Choi (1998) and Eudey and Perli (1999) remark that a typical stochastic growth model will not be able to generate asymmetric behaviour over the business cycle, unless some exogenous non-linear driving force is introduced. To mimic asymmetries of actual data, Choi allows the driving shock to be skewed, while Eudey and Perli add Markov-process-driven expectations to the standard RBC model.

This brief review suggests that a proper investigation of non-linearities in the technology shock is needed. To the best of our knowledge, only Altug et al. (1999) have tested for non-linearity in the Solow residual (allowing for capital utilisation), finding no evidence of non-linearity, but they have not set out any non-linear model for the productivity shock.

2.2 Deriving a proper technology shock

In this section we compute different measures of productivity, starting from Solow’s definition (1957). Consider a firm that produces an output \( Y \) with a Cobb-Douglas production function:

3 Regarding the “puzzle” of too frequent technology regression, it pays to remember that TFP is calculated as a residual and as such can absorb all the unexplained components of the output equation, expressed by the aggregate production function. In other words, as suggested by Hansen and Prescott (1993), technology shocks are (all) changes in the production function, or more generally, the production possibility set of the profit centers, which are exogenous to the profit centers themselves. A typical example is environmental protection laws that force firms to use less damaging production methods and to use some portion of available inputs to maintain some standard of environmental conditions. Actually, when governments impose constraints on firms with regard to the amount of pollution per unit of output, this is a technology shock, since the amount of output that can be produced from given quantities of market inputs diminishes.
(1) \[ Y_t = A_t K_t^\theta N_t^{1-\theta} \]

using capital $K$ and labour $N$ and where $A_t$ is a Hicks-neutral capital technical progress$^4$.

Under perfect competition and constant returns to scale, the observed shares of labour and capital exactly measure the elasticity of the production function with respect to labour and capital so that elasticity can be read directly from the data. In growth rates (small letters) (1) yields:

(2) \[ sr_t = a_t = y_t - \theta k_t - (1 - \theta) n_t \]

The share of labour factor $\theta$ can be obtained easily from the relationship:

(3) \[ (1 - \theta_t) = N_t w_t / p_t Y_t, \]

where $w_t$ is labour cost and $p_t$ is price level, that is we take the arithmetic mean labour share throughout the entire sample, assuming a constant factor share. Once elasticity is known, the rate of productivity growth can be obtained by subtracting the rate of growth of total input from the rate of the growth of output$^5$.

This expression of the Solow residual is uncorrelated with all variables known to be neither causes nor effects of productivity shifts (Hall, 1990 p. 74) under perfect competition and constant returns to scale.

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$^4$ Neutral disembodied technical progress is used, because in this case a technical change leaves the marginal rate of substitution unaltered, so there is no incentive to alter factor proportions unless relative factor prices change (Wallis 1979, p. 64).

$^5$ This procedure for achieving the residual is called “indirect estimation” (e.g. Farmer 1993). Alternatively, one can measure cyclical variation in productivity by estimating equation (2), although the estimation of factor shares raises some problems, due to correlation between residuals, which represents technology shocks and regressors that are the quantity of inputs used in production (Bernanke and Parkinson 1991, Marchetti 1999).
However, if inputs are not always fully used, then one needs effective measures of inputs to construct the Solow residual. In order to allow for variations of factor utilisation over the business cycle, we expand the Cobb-Douglas production function (Burnside, Eichenbaum and Rebelo 1993) to:

\[ Y_t = A_t K_t^\alpha [H_t E_t L_t]^{1-\alpha}, \]

where a working individual stays at work for \( H_t \) hours with an effort of \( E_t \), so that the effective work supplied is \( E_t H_t L_t \). \( L_t \) is the number of employees.

In order to embody input utilisation in (1), we can define \( N_t^* \), the effective labour services, equal to the number of employees \( L_t \) times the number of hours \( H_t \) times the individual hourly effort \( E_t \):

\[ N_t^* = L_t H_t E_t = N_t E_t \]

Effective capital services can be defined as:

\[ K_t^* = K_t U_t, \]

which combines the installed capital stock \( K_t \) and its rate of utilisation \( U_t \). The production function in terms of inputs effectively used is then:

\[ Y_t = A_t K_t^* N_t^{*1-\alpha} \]

The relationship with (2) is (growth rates):
where \( a_t^* \) is the actual technology shock, derived from (7). Expression (8) analyses the above claim, according to which neglected cyclical factors such as factor utilisation levels will cause measurement of TFP growth to be more variable than actual TFP growth. In fact, taking the variance of the left side, we obtain a sum of positive variances on the right side and three normally positive covariances,\(^6\) so that TFP variability under full utilisation of factors will be overestimated. Altogether (8) reminds us that the basic Solow residual embodies other components that move productivity over the cycle, which have nothing to do with technology shocks.

Finally, to tackle the problem of increasing returns or imperfect competition, positing firms as price-takers in the factor market, optimal shares can be obtained by minimizing the total cost function \( C_t \), under the technology constraint represented by the production function (Hall 1988, 1990):

\[
(9) \quad C_t = w_t N_t + r_t K_t
\]

In (9) \( w \) is labour cost and \( r \) is the renormal price of capital. If we define the degree of return to scale \( \gamma \) as the ratio of average cost to marginal cost, we obtain\(^7\):

\[
(10) \quad C = \gamma \frac{dC}{dY} Y_t
\]

Cost minimisation subject to the general production function (2.1) implies the following first order conditions:

---

\(^6\) In our data, correlation between capacity utilisation and labour utilisation is 0.46.

\(^7\) Subscript \( t \) is ruled out for simplicity.
\[
\frac{dY}{dN} = w \frac{dY}{dC/dY^*}; \quad \frac{dY}{dK} = r \frac{dC/dY^*};
\]

from which we obtain:

\[
\frac{dY}{dN} = \frac{\theta_n^c Y}{N} \gamma; \quad \frac{dY}{dK} = \frac{\theta_k^c Y}{K} \gamma;
\]

where \(\theta_n^c = \frac{wN}{C}\) and \(\theta_k^c = \frac{rK}{C}\) represent factor cost shares of total cost. By definition \(\theta_k^c + \theta_n^c = 1\). The Solow residual can be obtained as (growth rates):

\[
s_r c_t = a_t = y_t - \gamma (\theta_n^c n_t - \theta_k^c k_t)
\]

This measure is independent of the degree of competitiveness of the economy and embodies explicitly a coefficient for return to scale.

An expression equivalent to (13), in order to single out the degree of competitiveness, uses mark-up definition \(\mu\) as ratio of price \(P_t\) to marginal cost. By minimizing costs we obtain:

\[
a_t = y_s - \mu (\theta_n^v n_t - \theta_k^v k_t),
\]

where \(\theta_n^v = NW/Py\) and \(\theta_k^v = rK/Py\). Expressions (13) and (14) are equivalent because \(\theta_n^v = \frac{\gamma}{\mu} \theta_n^c\) and \(\theta_k^v = \frac{\gamma}{\mu} \theta_k^c\). The parameters \(\mu\) and \(\gamma\), when they are assumed constant, can be easily estimated knowing the cost shares through (13) and (14).

However, recent studies show that under imperfect competition, mark-ups and returns to scale are likely to be upward-biased due to the use of value-added data as a measure of output rather than gross-output data (Basu and Fernald 1997). Norrbin (1993) even suggests that market power in US data vanishes when gross output is used as the measure of output. In general, as Paquet and Robidoux (1997) note, the use of value-added data yields upper-bound estimates of mark-up and return to scale.
In our analysis, we use value added to measure output, mainly because of lack of gross output and intermediate inputs at the quarterly frequencies. When gross output is used, these have to be included among inputs (Marchetti 1999)\(^8\).

3. Testing for linearity

Before introducing our model, we undertake two tests to investigate general non-linearity in the data, after filtering them through an AR\((p)\) linear process\(^9\). On the basis of these tests, as found by Altug et al. (1999), the technology shock turns out to be linear, regardless of how it is measured. Yet, as is well known, these *portmanteau* tests are less powerful than those against a specific non-linear alternative (Granger and Terasvirta 1993, chap. 6); and when a SETAR model is set out as an alternative non-linear hypothesis, linearity is strongly rejected.

The most common tests are McLeod and Li (1983) and BDS statistics (Brock, Dechert and Scheinkman 1987)\(^10\). We use the McLeod and Li test, but we replace the BDS test with the most recent Time Reversibility (TR) test (Ramsey and Rothman 1996), which is more powerful against a TAR alternative (Rothman 1992).

3.1 The McLeod-Li Test

Granger and Anderson (1978) suggested that the autocorrelation function of the square of a time series could be useful in identifying bilinear non-linear models. They found that when squared residuals of a linear model are autocorrelated (and simple residuals are not), a simple bilinear model can improve forecasts with respect to the ARMA specification. Tong (1995) found that other non-linear models such as threshold autoregressive models could be used in this case. Hence, a test for linearity may be based on the square of the residuals. Let us consider the ARMA\((p,q)\) model for a mean stationary time series \(\{X_t\}\):

8 Value added is normally used with quarterly data (Bernanke and Parkinson 1991, Altug et al. 1999, Caballero and Lyons 1992) whereas annual gross output data are used in Marchetti (1999) and Marchetti and Nucci (2001a).

9 This procedure, quite usual in the field of linearity tests (Lee et al. 1993), is designed to rule out autocorrelation, that is (linear) time dependence, usually present in a time series.

10 BDS has its origin in the empirical literature on testing for low-dimensional chaos in economic and financial data. Suggested among other tests for non-linearity by Lee et al. (1993) this test seems to have reasonable power, but only in samples of at least 200 observations. That is not our case, nor the case of most macroeconomic series, unless monthly data are available for a long time. Nevertheless, this test is used in Altug et al. (1999) and in Stanca (1999), which have only quarterly data.
where \( \mu \) is the series mean and \( B \) is the back shift operator in \( t \). \( a_t \) are \( i.i.d. \) with finite variance. Let \( e_t \) be the estimated residuals. The estimated square correlation of order \( k \) is

\[
(16) \quad r_k^2 = \sum_{j=k+1}^{n} (e_j^2 - \sigma^2)(e_{j-k}^2 - \sigma^2) / \sum_{j=1}^{n} (e_j^2 - \sigma^2)^2
\]

McLeod and Li (1983) prove that for fixed \( m \), \( \sqrt{n}(r_1^2, r_2^2, \ldots, r_m^2) \) is asymptotically multivariate unit normal \( N(0, I_m) \) as \( n \) tends to infinite. Analogously to the Ljung-Box portmanteau statistic, McLeod and Li then propose the statistic:

\[
(17) \quad Q = n(n + 2) \sum_{j=1}^{m} r_j^2 / (n - k),
\]

distributed like \( \chi^2(m) \) if the true innovations are independent.

### 3.2 Time Reversibility Test

Time reversibility is a fundamental concept in dynamics. A process is time-reversible when substituting \((-t)\) for \((t)\) in the equations of motion leaves the result invariant.

Time-irreversible processes do not have this property. Typical irreversible processes in economics may be investment and disinvestment or, more relevant to the present work, the diffusion of technology.

The large body of work on business cycle asymmetry, that implies that expansions are not symmetric with respect to contractions suggests that time reversibility might not be an implicit property of economic fluctuations, although many formulations implicitly assume it.

A fundamental property of time-reversible processes (Ramsey and Rothman 1996) is that
for all $\alpha, \beta, k$, where $E$ is the expectation of the joint distribution. The moments in (3.4) are called generalized autocovariances. In theory, (18) should hold for each $\alpha, \beta, k$, but in practice a weaker definition of reversibility is chosen. A process satisfying (18) for $\alpha + \beta \leq m$ and for $k \leq K$ is said to be reversible of order $m$ and degree $K$. Ramsey and Rothman verify that it is sufficient to take into account bicovariances of the process $(\alpha + \beta = m = 3)$. They also set $K = 5$. The test suggested is:

\begin{equation}
TR_{2,1}(k) = E(X_t^2 X_{t-k}) - E(X_t^2 X_{t-k}^2),
\end{equation}

where $TR=0$ under the hypothesis of reversibility. Bicovariances are estimated by the expressions:

\begin{align}
B_{21}(k) &= \frac{1}{T-k} \sum_t X_t^2 X_{t-k} \\
B_{12}(k) &= \frac{1}{T-k} \sum_t X_t X_{t-k}^2,
\end{align}

so that

\begin{equation}
TR^* = B_{21}(k) - B_{12}(k)
\end{equation}

---

11 When $\alpha = \beta = 1$ the equality is the trivial identity between covariances $E(X_t X_{t-k}) = E(X_t X_{t-k})$ so that covariances do not give any information about potential irreversibility.

12 A similar approach is followed regarding the empirical definition of strict stationarity, limited to “stationarity of order $m$”.

13 They find that $5$ is a compromise between looking at many terms and the decreasing efficiency of estimates when $k$ increases.
Under the null hypothesis that \( X(t) \) is reversible, \( E(TR^*) = 0 \) with a distribution asymptotically normal:

\[
\sqrt{T}(TR^* - TR)/\sigma(TR^*) \sim N(0, 1).
\]

Variance of the process in small samples is computed by Ramsey and Rothman (1996) under the hypothesis of \( X(t) \) \( i.i.d. \):

\[
var(TR^*) = 2(\mu^4 - \mu_2^3 + \mu_2^2)/(T - k) - 2\mu_2^3(T - 2k)/(T - k)^2,
\]

where \( \mu \) is the central moment.

If \( \{X_t\} \) is a stationary series not serially autocorrelated, expression (23) can be calculated on the raw data, using the expression (24) for the variance and the asymptotic normal distribution. As a rule, however, economic time series are autocorrelated instead. A solution is filtering the series through an ARMA model and obtaining a residual from it. This residual will be approximately time reversible (null hypothesis) and not autocorrelated.

4. SETAR model

Threshold Autoregressive Models (TAR) are the simplest generalisation of linear autoregressive models. They were introduced by Tong (1983, 1995 for a survey) and successfully applied to GDP or industrial production series by Potter (1995), Pesaran and Potter (1997), Gallegati and Mignacca (1995) and others.

The basic model can be written as:

\[
y_t = a_0^{(t)} + a_1^{(t)} y_{t-1} + a_2^{(t)} y_{t-2} + ... + a_p^{(t)} + \sigma^{(t)} \epsilon_t
\]

where \( j(t) = 1 \) if \( X_t < r_1; j(t) = 2 \) if \( r_1 < X_t < r_2, \) etc.; \( J(t) \) is the index representing the different regimes \( (j) \), defined by the \( j - 1 \) thresholds \( r_j; X_t \) is the variable, whose value defines the regime where the model is. Normally, this variable coincides with the lagged endogenous
variable \( X_t = y_{t-d} \). In this case the parameter \( d \) is called delay and the model becomes SETAR (Self-Exciting Threshold Autoregressive).

If \( \{r_j\} \) and \( d \) were known, the model could be estimated by separating the data into groups by regime and finding the least squares estimates for the parameters in each regime.

Unfortunately, these parameters are not known and this kind of non-linear model cannot be estimated by standard techniques for non-linear models, because the sum of squares of residuals is not differentiable with respect to these parameters (Tong 1995, p. 387). Yet, by assuming a finite number of discrete values for the parameters of the delay and order of autoregressive lags it is easy to repeat the least squared estimation for each choice (grid search)\(^{14} \). In the case of the threshold parameters, a certain number of thresholds is assumed and the obvious least square estimates of \( \{r_j\} \) are the values associated with the smallest sum of square errors\(^{15} \).

The sequence of SETAR(\( j \)) models, where \( j \) indicates the number of regimes is a sequence of nested models, which embodies the linear model itself (\( j = 1 \)). As a result, searching for the best SETAR model describing the data amounts to testing for a linear specification against a more general non-linear model. The experience within time series models has shown that the number of thresholds ordinarily significant is not greater than 2 (three regimes); so we are going to test the linearity hypothesis against SETAR(2) and SETAR(3). SETAR(1) is then the usual autoregressive model, estimated by OLS:

\[
\hat{\alpha} = (X'X)^{-1} X'Y, \tag{26}
\]

where \( X = (y_{t-1}, y_{t-2}, \ldots, y_{t-p}) \) and \( \alpha = (a_0, a_1, a_2, \ldots, a_p) \) are the parameters.

The generic SETAR(\( j \)) model is estimated instead by minimising also with respect to different values of the threshold(s) \( r_j \) and of the delay parameter \( d \):

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\(^{14} \) For instance, Potter (1995) easily estimates a SETAR model for the US GDP, by making some a priori assumptions on the delay and the number of regimes.

\(^{15} \) Under the assumption of Gaussian errors this would be the Maximum Likelihood estimate.
(27) \[ \min_{\theta} \sum_{t=1}^{n} (y_t - a_0(r, d) + a_1(r, d)y_{t-1} + a_p(r, d)y_{t-p})^2, \]

where \( \theta = (\alpha, r, d) \) with \((r = r_1, \ldots, r_{j-1})\).

The estimation procedure is undertaken sequentially: at first, we estimate \( \hat{\alpha}(r, d) \) for given \((r, d)\) which minimize the variance of errors \( S_j(r, d) \); then, throughout a grid of values for the threshold and the delay, we search for

(28) \[ (\hat{r}, \hat{d}) = \arg \min_{r, d} S_j(r, d) \]

In the traditional literature on non-linear models (Granger and Terasvirta 1993), the non-linear specification is assessed better than the linear one on the basis of the ratio of the variances of errors in the two models \( \frac{\sigma^2_{N(r,c)}}{\sigma^2_L} \) or through some other criterion function, which allows for degrees of freedom such as Akaike’s criterion (AIC). As these measures do not allow for the sample variance so as to make the result depend on the sample selected, more formal tests have been performed for some specific non-linear models: Lukkonen et al. (1988) for STAR models or Tsay (1989) for the specification of TAR models.

When we test for a non-linear specification against the nested linear specification, we have to face the problem of nuisance parameters, namely the fact that the null hypothesis (the linear one), does not embody the parameters which define the non-linear specification under the alternative, on which the asymptotic distribution of the test depends.

The likelihood ratio (LR from here on) under the null will have a \( \chi^2 \) distribution in large samples, but by varying these free parameters, a larger or smaller LR might be found. This problem has been studied by Davies (1977, 1987), Andrews and Ploberger (1994) and Chan and Tong (1990) in TAR models. They estimate an empirical null distribution of LR tests across different values of the threshold, under the assumption that \( d \), the delay of the threshold variable, i.e. the threshold variable itself, is known and \( \varepsilon_t \) is Gaussian.
If \( y_{t-d} \) is unknown, \( d \) becomes a parameter taking values in the discrete set \( D = (1, 2, \ldots p) \). Hansen’s (1996) generalisation allows \( d \) to be unknown and \( \varepsilon_t \) arbitrarily distributed and improves Chan and Tong’s procedure.

In this work, using Hansen’s procedure, we estimate a SETAR model up to two thresholds and we test for the superiority of the non-linear specification. As the number of thresholds increases the effort required to calculate the test exponentially, two thresholds turns out to be a sensible compromise between generalisation and practical use.

Residuals of the SETAR\((j)\) \( \varepsilon_j \) are used to compute the F test, in the form of ratio of variances. If we indicate \( S_j = \varepsilon_j', \varepsilon_j \), the test, in the case of the SETAR\((2)\) against the linear model, has the form

\[
F = n \left( \frac{S_1 - S_2}{S_2} \right)
\]

When errors are independent, this is also a Likelihood Ratio test. The problem is the distribution of this test, because it depends on the nuisance parameters \((r, d)\), which define the non-linear model. Still in the case of \( j = 2 \) for simplicity’s sake\(^{16}\)

\[
F_{1,2} (r, d) = n \left( \frac{S_1 - S_2(r, d)}{S_2(r, d)} \right)
\]

One important testing issue is that each regime must contain a minimum number of observations: Hansen assumes that at least 10% of observations have to belong to each regime. Given that he also restricts the search for the values of thresholds to the set of realisations of the lagged variable, all values of \( y_{t-d} \) between the quantile 0.1 and the quantile 0.9, say \( N \), are probed. If, as is usually assumed, the value of the delay \( d \) is equal to the maximum lag of the autoregression, this search implies at most \( Np \) estimations in the case of the SETAR\((2)\) model and \( N^3p \) in the case of the SETAR\((3)\) model.

As \((\hat{r}, \hat{d})\) minimizes \( S_2 \), this implies

\[16\] We refer the reader to Hansen (1999) for details.
\[ F_{1,2} = \max_{r,d} F_{1,2} (r,d) \]

\( F_{1,2} (r, d) \) is a fairly conventional statistic for \( r \) and \( d \) fixed, distributed as \( \chi^2(p) \); it is equivalent to the test for the exclusion of \( x_1 (r,d) \) from a regression of \( y \) on \( x \) and \( x_1 (r,d) \). That is when the test involves only a single value of \( r,d \).

Since the actual test implies a very large number of values, it looks for the maximum of \( pN \chi^2 (p) \) random variables, and it is larger than \( \chi^2 (p) \)\(^{17} \). Thus, it is helpful to think of the statistic \( F_{1,2} \) as a random function of the argument \( (r,d) \) and \( F_{1,2} \) as the random maximum.

Hansen (1996) uses the theory of empirical processes to develop an asymptotic distribution theory for these statistics. Andrews (1993, 1994) gives a review of this approach. Hansen shows that the asymptotic distribution of the empirical process \( F_{1,2} (r,d) \) is \( T (r,d) \), a random function with argument \( (r,d) \) and \( F_{1,2} \) is the maximum of this random limit function

\[ F_{1,2} \implies T = \max_{r,d} T (r,d) \]

The statistic \( T (r,d) \) on which the test is based is still \( \chi^2 (p) \) for \( r \) and \( d \) fixed, but the distribution of \( T \) depends on the degree of dependence between the random variables \( T (r,d) \) for distinct values of \( r \) and \( d \), which depends on the moments of regression and the threshold variable \( y_{1-d} \). Consequently, the distribution \( T \) cannot be tabulated for general use, but must be calculated for each application.

Hansen (1996) describes an algorithm to calculate the asymptotic distribution, and Hansen (1999) calculates the p-value of \( T \) from the bootstrap distribution \( T_n \) on the basis of the statistical result that bootstrap is a better approximation to finite sample distribution than first order approximation (Davidson and Hinkley 1997). The bootstrap distribution calculates the distribution of \( F_{1,2} \) under the assumption that the data satisfy the SETAR model estimated,

\(^{17} \) Thus, if \( F_{1,2} (r, d) \) is not significant, when compared to \( \chi^2 (p) \), it will certainly not be significant when compared to the correct asymptotic distribution. However, in most applications this will not be a helpful bound, as normally the observed \( F_{1,2} (r, d) \) will be highly significant when compared to the \( \chi^2 (p) \) distribution.
by using the model estimated and adding an error assumed independent, distributed like the residual $e_j$.

5. Results

In this work we reconstruct the technology shock for the Italian industrial sector. Many reasons induce us to calculate the residual for one sector only rather than for the whole economy.

Measures of returns to scale and market power mostly stem from sectoral analysis, mainly of the industrial sector (Caballero and Lyons 1992, Bernanke and Parkinson 1991). In addition, measures of the utilisation of factors or the user cost of capital are not available for the aggregate economy and above all hard to compute.

Finally, the contribution of the different sectors to the aggregate output has changed over time, with an increasing importance of services relative to industry. Given the different dynamics of productivity within these two sectors, aggregate productivity will present some structural change over the sample, which would invalidate the econometric analysis.

As we mean to carry out an analysis of the cyclical behaviour of TFP, data are quarterly\textsuperscript{18}. This represents a novelty in the applied literature on Italian data, as all previous analyses have been based on annual data (Rossi and Toniolo 1993, Atella and Quintieri 1996, Marchetti 1999 and Marchetti and Nucci 2001a).

As table 1 and table 2 show, the conventional Solow residual ($\sigma r_t$), overlooking the other unobserved components which bring about productivity change, in that this is the most commonly employed measure of productivity shock in almost all RBC models. Factors are assumed fully utilised.

As table 1 and table 2 show, the conventional Solow residual turns out to be linear.\textit{Porranian}eau tests do not suggest any neglected non-linearity. Likewise, Hansen’s test in table 2 does not reject linearity when the linear AR model is tested against SETAR(2) and SETAR(3) specifications. Our findings hold regardless of the test distribution: the asymptotic distribution (A) or the bootstrap distribution (B). As the aim of this work is to investigate non-linearity,

\textsuperscript{18} The data set is described in the appendix.
we do not describe the entire (linear) model, nor the corresponding “non-significant” SETAR specification and we refer the reader to the other models for a further discussion.

Thus, in spite of the doubtful identification of the Solow residual with the exogenous productivity shock, most of the RBC models that draw innovation (2) from the standard production function (1), use a linear autoregressive model properly\(^{19}\).

Keeping this basic result in mind, now we calculate a better approximation of technical progress, starting by removing the assumption of full utilisation of factors over the cycle. Then, the residual is computed taking into account both the degree of utilise of the capital factor \(K_t^*\) and the labour factor (Burnside and Eichenbaum 1996).

There are several problems in calculating services of inputs. First, we have to define a measure of capital utilisation. Second, we do not have data on labour effort.

Solow (1964) allowed for the possibility that the capital utilisation rate could vary across the business cycle by measuring capital services as the product of the physical capital stock and the employment rate. In the most recent empirical applications, capital utilisation has been approximated with electricity use since Burnside et al. (1995). Like other measures of capital utilisation it is likely to be sector-specific and not suitable for non-manufacturing sectors.

Burnside and Eichenbaum (1996) and Marchetti and Nucci (2001a), in order to identify capital service flow, assume that a more intense utilisation of installed capital implies faster depreciation. Unfortunately, the official government series on the stock of capital are constructed under the assumption of approximately straight-line depreciation over fixed service lives for each type of capital (see Istat 1995)\(^{20}\). So, in the spirit of Otto (1999)\(^{21}\) we use a measure of capacity utilisation in order to calculate the effective \(K^*\) in (2.6), estimated by the Bank of Italy for the industrial sector on the basis of Wharton method\(^{22}\).

---

\(^{19}\) The problem emerging in these cases is that such a productivity shock is no longer exogenous, but correlated with other variables of the model (e.g. Burnside et al. 1993).

\(^{20}\) Eudey and Perli (1999) compute the Solow residual both with capital depreciating through use and with a capacity utilisation proportional to industrial energy use, without any significant difference in results.

\(^{21}\) This author uses capacity utilisation in seeking to identify the effects of demand shocks in a SVAR model for the Solow residual.

\(^{22}\) Other measures were available, such as that obtained from a quarterly survey of manufacturing firms undertaken by ISAE, but for too short a time range.
Capacity utilisation (fig. 1) moves over the business cycle and is more volatile relative to the stock of physical capital so that the standard deviation of the growth rate of the effective capital, 0.0108, is roughly 6 times greater than the standard deviation of the growth rate of the stock of the physical capital (0.0017). Full utilisation of capital over the cycle is thus too strong an assumption\textsuperscript{23}.

With regard to effective labour input, following Marchetti and Nucci (2001a) we assume that effort $E$ is related to the number of hours worked $H$

\begin{equation}
E = H^z,
\end{equation}

where $z$ defines the elasticity of individual hourly effort with respect to hours per worker: $z = \frac{de}{e}/(dh/h)$, so that the unobserved change in hourly effort $de$ can be expressed as the change in hours per worker, $h$, times the elasticity $z$. In growth rates then (5) becomes

\begin{equation}
\hat{n}_t^* = l_t + h_t + e_t = l_t + (1 + z_t)h_t = n_t + zh_t
\end{equation}

The estimated value by Marchetti and Nucci (2001a) of $z$, assumed constant, is -0.38 with a standard error of 0.20. Whereas hours per worker is procyclical and effective labour is procyclical, hourly effort is not procyclical; that is, increasing hours at the margin would lead to a reduction in the amount of effort\textsuperscript{24}.

For given elasticity, the index of intensity of use was computed by normalizing to one the average of hours worked index over the period considered\textsuperscript{25} (fig. 2).

In table 3 the correlation of the growth rate of value added to the factors is reported. First of all, we point out the strong procyclicality of indexes of factor utilisation, around 0.60, as

\textsuperscript{23} Burnside and Eichenbaum (1996) find a difference of 4.5 times between the two standard deviations.

\textsuperscript{24} The authors underline that in US manufacturing estimates the elasticity is positive rather than negative. They attribute this divergence in results to the rigidities of the Italian labour market. Moreover, it is worth noting that, when effort is constant over the cycle $e_t = 0$ and $n^* = n$.

\textsuperscript{25} See Marchetti e Nucci (2001a, p. 16).
expected according to the keynesian hypothesis and the general empirical evidence that factors are more intensively used in booms than in recessions (Marchetti and Nucci 2001a)\(^{26}\).

Factors themselves are weakly procyclical, capital in particular, but not surprisingly they become much more positively correlated with value added, where their degree of utilisation is allowed for. This confirms that the neglected procyclical factor utilisation might be an important component of procyclicality of the standard Solow residual.

Labour in our specification is measured in Standard Labour Units, a measure consistent with the other aggregates of national accounts, reference variable in the SEC, defined by the ratio of total hours effectively worked to the average hours worked in a full time position, corresponding to average hours in labour contracts.

Consequently, at a first stage, given that effort is also assumed to depend on the hours worked, using this variable as labour input should sufficiently embody the degree of utilisation of the labour factor. Hence, at first the Solow residual is computed only corrected for the capacity utilisation \(sr\ u_t\)

\[
(35) \quad sr\ u_t = y_t - \theta k_t^* - (1 - \theta) n_t,
\]

and later for labour services as well\(^{27}\) \(sr\ u_l t\)

\[
(36) \quad sr\ u_l t = y_t - \theta k_t^* - (1 - \theta) n_t^*.
\]

At this stage, we neglect the possible presence of returns to scale and market power in the sector.

\(^{26}\) The capital utilisation measure might be strongly correlated with GDP, because it is based on the Wharton method. However, a similar result is obtained taking a survey-based index (see footnote 19).

\(^{27}\) Altug \textit{et al.} (1999) assume that the aggregate electricity usage is proportional to capital services, although they do not correct for labor utilisation, after testing for potential non-linearities in worked hours.
Figure 3 charts the three different measures of productivity shocks. In accordance with what we have just claimed, fluctuations are smoother when factor utilisation is taken into account.

The technology shock $sr_u$ is well explained by an AR(3) model (table 4). Lag length is sorted out on the basis of the usual information criteria in order to obtain uncorrelated residuals. To a first approximation, the process of the percentage change of technology is a random walk with drift, positive and significant (0.005) with some uncorrelated (measurement) error. This error produces the negative first-order serial correlation of differences (Prescott 1986).

Despite the fact that the residuals of the model do not present neglected non-linearity (table 5), the SETAR(2) model, one threshold, two regimes, is significant (table 6)\textsuperscript{28}. This is not surprising, in that Hansen’s test is specifically designed to have power against a SETAR alternative, while preliminary linearity tests were developed to be run against a general unspecified alternative.

Hansen’s test (table 7) verifies the hypothesis $H_0 : a_i^1 = a_i^2$ where $i=0,1,2,3$, or that the model is linear. In fact, the linear model is nested in the SETAR one under the hypothesis that coefficients are the same across regimes. If we ignored the nuisance parameters, the test would be distributed like a $\chi^2(4)$.

As we can see, the test rejects linearity both according to the asymptotic distribution (A) and on the basis of the bootstrap distribution (B). If we used the standard $\chi^2(4)$ distribution we would have $19.76[0.006]$. As expected, the standard distribution rejects the null hypothesis more easily. Before running Hansen’s test we checked for homoskedasticy of residuals $E(\epsilon^2|I_{t-1}) = \sigma^2$, as the distribution of Hansen’s test depends on the presence of heteroskedasticity. Homoskedasticy is checked with the usual Wald test, testing for the squares of regressors being significant in the residuals equation. This yields $\chi^2(3) = 1.480[0.687]$.

As we can see, the usual ratio of variances of errors too is in favour of the non-linear model (0.83). Analogously, if we refer to Akaike’s criterion, we get $-9.42$ for the linear model and $-9.72$ in the SETAR case.

\textsuperscript{28} Parameters are asymptotically normally distributed (Chan 1993, theorem 2).
Threshold value amounts to a growth rate of about 1% per quarter, when productivity surpasses this growth rate in \( t - 1 \); in the following period, its dynamic changes. However, only the high growth mean (0.007) is significant; the lower one is only weakly significant.

The testing procedure is to be completed, by testing for a larger number of thresholds, comparing SETAR(1), the linear hypothesis, to SETAR(3). Nevertheless, our data reject the two-threshold hypothesis. We only report the test, recalling that in the case of two thresholds, the conventional \( \chi^2 \) has \( 2p \) degrees of freedom. The test is in the form

\[
F_{1,3} = n \left( \frac{S_3(r, d) - S_1}{S_3(r, d)} \right)
\]

This time, because of the presence of two thresholds, the asymptotic distribution is more difficult to calculate and very different as well from the bootstrap distribution. As a result, we use only the latter as a better approximation in small samples (both under homoskedasticity and heteroskedastic errors).

We can note that the conventional \( \chi^2 \) (8) = 29.19[0.000] would accept the SETAR(3) specification, as well as the ratio of variances (0.79), but this time the presence of two thresholds increases the weight of nuisance parameters, totally reversing the result of the conventional tests.

When we allow for labour services, the autoregressive specification is AR(4) (Table 9). The model is well specified despite some autocorrelation. Linearity tests are not that strongly in favour of linearity and the TR test for \( k=4 \) actually rejects invertibility (Table 10). Hansen’s test verifies the hypothesis \( H_0 : a_i^1 = a_i^2 \) where \( i=0,\ldots, 4 \). The SETAR model is significant (Table 11) and linearity is sharply rejected\(^{30}\) in accordance with the ratio of variances (0.80) and the AIC measure (\(-9.18\) against \(-9.35\) in the non-linear case). It is interesting to note that the presence of non-linearity is more evident than in the model of table 6, where capital utilisation only was explicitly allowed for.

\(^{29}\) The sample is now smaller, as hours series are available only from 1972.

\(^{30}\) Residuals are checked for homoskedasticity: \( \chi^2(4) = 2.32[0.803] \)
in what follows, we also relax the hypotheses of competitive markets and constant returns to scale. There is recent evidence of market power and increasing returns to scale in the Italian industrial sector. Marchetti (1999) estimates market power and returns to scale in the aggregate manufacturing sector and in the subsectors. He finds increasing returns ($\gamma = 1.16/1.24$, depending on which estimation method is employed), but no real market power ($\mu = 1.01$). In a firm-level study, Sembenelli (1996) instead estimated $\gamma = 0.93$ e $\mu = 1.08$.

However, both studies reveal less market power and lower returns to scale in the aggregate than in the subsectors. This result, quite intuitive, supports the common practice of modelling a Cobb-Douglas aggregate production function and of assuming competitive markets in the aggregate economy\textsuperscript{31}.

Let us consider now the Solow residual computed on the basis of cost shares ($sr_{ct}$), using (2.13). The rental price of capital goods is taken from Gaunolis \textit{et al}. (1996; see the appendix for details). In this procedure, the degree of return to scale is fixed at $\gamma = 1.16$, as estimated by Marchetti (1999)\textsuperscript{32}. Factors are weighted for their degree of utilisation.

Solow residual is well fitted by an AR(4) process (table 13). Some trace of non-linearity is singled out by TR test (table 14). The non-linear structure is clearly evident in the SETAR(2) specification (Table 15). The non-linear model performs much better than the linear, both in terms of ratio of variances (0.77) and in terms of AIC (-8.75 against -8.59). Coefficients and regime means are strongly significant, whereas regressors and constant are weakly significant in the linear specification. Hansen’s test definitely favours the non-linear model, when homoskedasticity is assumed $\chi^2(4) = 9.401[0.05]$. However, non-linearity is partially rejected under heteroskedasticity (Table 16).

Altogether, these estimates provide clear evidence that the “true” technology shock is non-linear in accordance with the prior assumption that technology is expected to fluctuate over the cycle in an asymmetric way (see section 2.1).

---

\textsuperscript{31} However, in a seminal contribution, Caballero and Lyons (1992) find increasing returns in the aggregate industrial sector, more than in the subsectors. The explanation suggested by the authors is the presence of externalities due to the market dimension (thick-market externalities).

\textsuperscript{32} The value of this coefficient does not seriously affect our results, at least in the range of values normally measured in this kind of analysis (1.0-1.4).
However, the standard Solow residual, while overestimating the actual productivity shock, turns out to be linear. Two different explanations are equally possible. If most of the cyclical fluctuation of productivity is attributable to cyclical movements in the factor utilisation rates and if these movements are linear and symmetric over the cycle, they are likely to obscure the underlying dynamic of technology. Conversely, if factor hoarding is affected by the business cycle in a non-linear way, linearity might be the result of aggregation.

6. Conclusions

The recent empirical evidence showing asymmetries in business cycles represents a challenge for the standard equilibrium models of RBC. These models successfully explain most first and second moments of actual time series, but cannot replicate non-linear features of the data, unless a non-linear innovation is introduced. This paper investigates the presence of non-linearity in the technology shock, the basic innovation in RBC models.

Traditionally, TFP changes, estimated by the standard Solow residual, proxy for the unobserved technology shock, but recent literature points out that TFP changes can embody many other cyclical factors, bringing about cyclical changes in productivity such that technology shock variability is overestimated.

Here, we measure the technology shock for the Italian industrial sector at quarterly frequency. In order to measure it more accurately, we derive some alternative measures of total factor productivity such as revenue-based and cost-based Solow residuals; we also control for cyclical changes in factor utilisation.

First, we test for general non-linearities. Two tests are used: McLeod and Li and the Time Reversibility test designed by Ramsey and Rothman (1996). They yield weak evidence of non-linearity, as is usual for general tests without a specific alternative, even though the TR is the most powerful of these tests.

Secondly, as the Solow residual is normally modelled as an AR linear process, we model the technology shock as a SETAR non-linear model, an extension of the AR model, where autoregressive coefficients vary, owing to the variable being above or below some threshold values.

Granger and Lee (1999) find that aggregation is inclined to reduce non-linearity. Non-linearity declines markedly after aggregation, even when time series are few.
Third, we introduce a test suggested by Hansen (1996, 1999), based on the theory of empirical processes and non-standard distributions, to face the problem of nuisance parameters when evaluating the non-linear model against the linear alternative.

Our findings indicate that, although the standard Solow residual turns out to be linear, the other measures of technology shocks appear to be non-linear, when non-technological cyclical components are ruled out. Hence, in setting up a RBC model, if one intends to allow for factor hoarding or to relax the hypothesis of competitive markets and constant returns to scale, a linear innovation might be too restrictive an assumption.
Tables and figures
### Tab. 1

**srₜ - LINEARITY TESTS**

<table>
<thead>
<tr>
<th>McLeod-Li test</th>
<th>TR test</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>m= 2</td>
<td>(\chi^2(2))</td>
<td>1.88 [0.657]</td>
<td>k= 1</td>
</tr>
<tr>
<td>m= 3</td>
<td>(\chi^2(3))</td>
<td>1.88 [0.795]</td>
<td>k= 2</td>
</tr>
<tr>
<td>m= 4</td>
<td>(\chi^2(4))</td>
<td>2.20 [0.841]</td>
<td>k= 3</td>
</tr>
<tr>
<td>m= 5</td>
<td>(\chi^2(5))</td>
<td>6.26 [0.425]</td>
<td>k=4</td>
</tr>
<tr>
<td>m= 6</td>
<td>(\chi^2(6))</td>
<td>7.17 [0.440]</td>
<td>k=5</td>
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### Tab. 2

**srₜ - HANSEN’S TEST**

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<thead>
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<th>AR vs SETAR(2)</th>
<th>AR vs SETAR(3)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Max-F</td>
<td>pv-A</td>
<td>pv-B</td>
<td>Max-F</td>
</tr>
<tr>
<td>15.79</td>
<td>0.09</td>
<td>0.24</td>
<td>32.84</td>
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### Tab. 3

**CORRELATION WITH OUTPUT**

<p>| | | | |</p>
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<tr>
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<tr>
<td>(cu)</td>
<td>0.608</td>
<td>(k)</td>
<td>0.044</td>
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<tr>
<td>(h)</td>
<td>0.658</td>
<td>(n)</td>
<td>0.350</td>
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### Tab. 4

**sruₜ - LINEAR MODEL**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t-stat</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const.</td>
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<td>0.001</td>
<td>4.656</td>
<td>0.000</td>
</tr>
<tr>
<td>sru(-1)</td>
<td>-0.338</td>
<td>0.098</td>
<td>-3.440</td>
<td>0.001</td>
</tr>
<tr>
<td>sru(-2)</td>
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<td>-3.378</td>
<td>0.001</td>
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<tr>
<td>sru(-3)</td>
<td>-0.163</td>
<td>0.097</td>
<td>-1.674</td>
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</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t-stat</th>
<th>p-value</th>
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<tbody>
<tr>
<td>R²</td>
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<tr>
<td>SS (res)</td>
<td>0.008</td>
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<tr>
<td>Var (res)</td>
<td>6.25 × 10⁻⁵</td>
<td>J-B norm.</td>
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<td>0.420</td>
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<td>Skewness</td>
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<td>AR(4) LM test</td>
<td>2.284</td>
<td>0.066</td>
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<tr>
<td>Kurtosis</td>
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<td>ARCH(4) F-test</td>
<td>0.997</td>
<td>0.414</td>
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</table>

### Tab. 5

**sruₜ - LINEARITY TESTS**

<table>
<thead>
<tr>
<th>McLeod-Li test</th>
<th>TR test</th>
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<th>k=3</th>
<th>k=4</th>
<th>k=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>m= 2</td>
<td>χ²(2)</td>
<td>4.23</td>
<td>0.233</td>
<td>N(0, 1)</td>
<td>1.38</td>
<td>0.167</td>
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<tr>
<td>m= 3</td>
<td>χ²(3)</td>
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<td>N(0, 1)</td>
<td>-0.78</td>
<td>0.425</td>
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<tr>
<td>m= 4</td>
<td>χ²(4)</td>
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<td>0.344</td>
<td>N(0, 1)</td>
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<td>0.666</td>
</tr>
<tr>
<td>m= 5</td>
<td>χ²(5)</td>
<td>8.47</td>
<td>0.203</td>
<td>N(0, 1)</td>
<td>-0.33</td>
<td>0.741</td>
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<tr>
<td>m= 6</td>
<td>χ²(6)</td>
<td>9.03</td>
<td>0.257</td>
<td>N(0, 1)</td>
<td>-0.60</td>
<td>0.548</td>
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Tab. 6

**srut - SETAR(2) MODEL**

<table>
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<tbody>
<tr>
<td>Const.</td>
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<td>0.001</td>
<td>Const.</td>
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<td>0.004</td>
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<td>0.202</td>
<td>sru(-1)</td>
<td>-0.570</td>
<td>0.267</td>
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<td>sru(-2)</td>
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<td>0.090</td>
<td>sru(-2)</td>
<td>0.135</td>
<td>0.143</td>
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<tr>
<td>sru(-3)</td>
<td>-0.201</td>
<td>0.117</td>
<td>sru(-3)</td>
<td>-0.154</td>
<td>0.184</td>
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</tbody>
</table>

Var (res) 7.47 $10^{-5}$  var(nl)/var(l) 0.83

Tab. 7

**srut - HANSEN’S TEST**

<table>
<thead>
<tr>
<th>AR vs SETAR(2)</th>
<th>Max-F</th>
<th>pv-A</th>
<th>pv-B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>19.76</td>
<td>0.02</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Tab. 8

**srut - HANSEN’S TEST**

<table>
<thead>
<tr>
<th>AR vs SETAR(3)</th>
<th>Max-F</th>
<th>pv-B - hom.</th>
<th>pv-B - het.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>29.19</td>
<td>0.2250</td>
<td>0.1850</td>
</tr>
</tbody>
</table>
### Tab. 9

**sru1 : LINEAR MODEL**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t-stat.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const.</td>
<td>0.006</td>
<td>0.001</td>
<td>5.009</td>
<td>0.000</td>
</tr>
<tr>
<td>sru(-1)</td>
<td>-0.388</td>
<td>0.088</td>
<td>-4.396</td>
<td>0.000</td>
</tr>
<tr>
<td>sru(-2)</td>
<td>-0.252</td>
<td>0.092</td>
<td>-2.732</td>
<td>0.007</td>
</tr>
<tr>
<td>sru(-3)</td>
<td>-0.192</td>
<td>0.091</td>
<td>-2.115</td>
<td>0.037</td>
</tr>
<tr>
<td>sru(-4)</td>
<td>-0.104</td>
<td>0.085</td>
<td>-1.238</td>
<td>0.219</td>
</tr>
<tr>
<td>R²</td>
<td>0.207</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS (res)</td>
<td>0.009</td>
<td>DW</td>
<td>1.960</td>
<td></td>
</tr>
<tr>
<td>Var (res)</td>
<td>$9.71 \times 10^{-5}$</td>
<td>J-B norm.</td>
<td>0.61</td>
<td>0.737</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.20</td>
<td>AR(4) LM test</td>
<td>20.57</td>
<td>0.031</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.80</td>
<td>ARCH(4) F-test</td>
<td>1.821</td>
<td>0.130</td>
</tr>
</tbody>
</table>

### Tab. 10

**sruₜ : LINEARITY TESTS**

<table>
<thead>
<tr>
<th>McLeod-Li test</th>
<th>TR test</th>
</tr>
</thead>
<tbody>
<tr>
<td>m= 2 $\chi^2(2)$</td>
<td>$k=1 \ N(0, 1)$ 1.38 [0.195]</td>
</tr>
<tr>
<td>m= 3 $\chi^2(3)$</td>
<td>$k=2 \ N(0, 1)$ −0.31 [0.750]</td>
</tr>
<tr>
<td>m= 4 $\chi^2(4)$</td>
<td>$k=3 \ N(0, 1)$ 1.15 [0.249]</td>
</tr>
<tr>
<td>m= 5 $\chi^2(5)$</td>
<td>$k=4 \ N(0, 1)$ 2.84 [0.004]</td>
</tr>
<tr>
<td>m= 6 $\chi^2(6)$</td>
<td>$k=5 \ N(0, 1)$ 1.27 [0.202]</td>
</tr>
</tbody>
</table>
### Tab. 11

**srul$_k$: SETAR(2) MODEL**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Const.</td>
<td>0.004</td>
<td>0.001</td>
<td>Const.</td>
<td>0.008</td>
<td>0.002</td>
</tr>
<tr>
<td>srul(-1)</td>
<td>-0.239</td>
<td>0.145</td>
<td>srul(-1)</td>
<td>-0.397</td>
<td>0.104</td>
</tr>
<tr>
<td>srul(-2)</td>
<td>-0.260</td>
<td>0.177</td>
<td>srul(-2)</td>
<td>0.184</td>
<td>0.098</td>
</tr>
<tr>
<td>srul(-3)</td>
<td>0.012</td>
<td>0.205</td>
<td>srul(-3)</td>
<td>-0.171</td>
<td>0.149</td>
</tr>
<tr>
<td>srul(-4)</td>
<td>0.029</td>
<td>0.123</td>
<td>srul(-4)</td>
<td>-0.419</td>
<td>0.102</td>
</tr>
</tbody>
</table>

Var (res) $7.81 \times 10^{-5}$  
$\text{var(nl)/var(l)}$ 0.80  

### Tab. 12

**srul$_k$: HANSEN’S TEST**

<table>
<thead>
<tr>
<th>AR vs SETAR(2)</th>
<th>Max-F</th>
<th>pv-A</th>
<th>pv-B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>23.06</td>
<td>0.01</td>
<td>0.05</td>
</tr>
</tbody>
</table>
### Tab. 13

**srct:** **LINEAR MODEL**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t-stat.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const.</td>
<td>0.002</td>
<td>0.001</td>
<td>1.675</td>
<td>0.097</td>
</tr>
<tr>
<td>src(-1)</td>
<td>-0.386</td>
<td>0.094</td>
<td>-4.095</td>
<td>0.000</td>
</tr>
<tr>
<td>src(-2)</td>
<td>-0.366</td>
<td>0.100</td>
<td>-3.638</td>
<td>0.000</td>
</tr>
<tr>
<td>src(-3)</td>
<td>-0.164</td>
<td>0.100</td>
<td>-1.643</td>
<td>0.104</td>
</tr>
<tr>
<td>src(-4)</td>
<td>-0.091</td>
<td>0.094</td>
<td>-0.967</td>
<td>0.336</td>
</tr>
</tbody>
</table>

R²: 0.198  LogL: 276.2
SS (res): 0.0165  DW: 1.677
Var (res): 0.000167  J-B norm.: 0.638  0.726
Skewness: -0.16  AR(4) LM test: 9.160  0.057
Kurtosis: 2.77  ARCH(4) F-test: 1.253  0.294

### Tab. 14

**srct:** **LINEARITY TEST**

<table>
<thead>
<tr>
<th>McLeod-Li test</th>
<th>TR test</th>
</tr>
</thead>
<tbody>
<tr>
<td>m= 2 χ²(2)</td>
<td>k= 1 N(0, 1) 1.29 [0.196]</td>
</tr>
<tr>
<td>m= 3 χ²(3)</td>
<td>k= 2 N(0, 1) -1.72 [0.084]</td>
</tr>
<tr>
<td>m= 4 χ²(4)</td>
<td>k= 3 N(0, 1) -0.37 [0.704]</td>
</tr>
<tr>
<td>m= 5 χ²(5)</td>
<td>k=4 N(0, 1) -1.71 [0.086]</td>
</tr>
<tr>
<td>m= 6 χ²(6)</td>
<td>k=5 N(0, 1) -1.72 [0.084]</td>
</tr>
</tbody>
</table>
### Tab. 15

**$src_t \, : \, \textbf{SETAR (2) MODEL}$**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Const.</td>
<td>0.003</td>
<td>0.001</td>
<td>Const.</td>
<td>0.023</td>
<td>0.005</td>
</tr>
<tr>
<td>src(-1)</td>
<td>-0.108</td>
<td>0.112</td>
<td>src(-1)</td>
<td>-1.218</td>
<td>0.178</td>
</tr>
<tr>
<td>src(-2)</td>
<td>-0.288</td>
<td>0.104</td>
<td>src(-2)</td>
<td>-0.311</td>
<td>0.136</td>
</tr>
<tr>
<td>src(-3)</td>
<td>-0.150</td>
<td>0.093</td>
<td>src(-3)</td>
<td>0.270</td>
<td>0.220</td>
</tr>
<tr>
<td>src(-4)</td>
<td>-0.018</td>
<td>0.088</td>
<td>src(-4)</td>
<td>-0.722</td>
<td>0.228</td>
</tr>
</tbody>
</table>

Var (res) 0.000129 var(nl)/var(l) 0.77

### Tab. 16

**$src_t \, : \, \textbf{HANSEN’S TEST}$**

<table>
<thead>
<tr>
<th>AR vs SETAR(2)</th>
<th>Max-F</th>
<th>pv-A</th>
<th>pv-B</th>
<th>pv-A-Het</th>
<th>pv-B-Het</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>29.04</td>
<td>0.000</td>
<td>0.025</td>
<td>0.105</td>
<td>0.080</td>
</tr>
</tbody>
</table>
CAPACITY UTILISATION (percentage)

Fig 1

INDEX OF LABOUR UTILISATION

Fig 2
Fig 3

TECHNOLOGY SHOCKS (LOGS): DIFFERENT DEFINITIONS
Appendix

Data

The yearly stock of net capital from Istat is available from 1980 at 1990 prices; the stock for the 1970s has been reconstructed by Annunziato et al. (1992) at 1985 prices; the complete series has been recalculated for the 1970s on the basis of the variation rate applied to Istat data in 1980.

The simplicity of the reconstruction is based on the use, by Confindustria, of the same procedure used by Istat to reconstruct the stock, starting from the same series of investment: both reconstructions use the method of the permanent inventory with the same distribution of withdrawals and the same linear depreciation law.

The quarterly stock of net capital has been calculated by interpolation of the yearly stock on the basis of quarterly investments. The quarterly series of investment for industry has been derived from quarterly aggregate series for the whole economy (the only quarterly series available), calculating the share to be assigned to industry on the basis of the ratio to yearly investment. The interpolation posits quarterly depreciation rate \( \delta_i \), which by assumption remains unchanged during the year \( i \), as described in Levy and Chen (1994)\(^{34}\)

\[
\begin{align*}
(A.1) \quad K_{t,1} &= (1 - \delta_i)K_{t-1} + I_{t,1} \\
(A.2) \quad K_{t,2} &= (1 - \delta_i)K_{t,1} + I_{t,2} \\
(A.3) \quad K_{t,3} &= (1 - \delta_i)K_{t,2} + I_{t,3}
\end{align*}
\]

\(^{34}\) Usually, more standard linear procedures are used (Bernanke and Parkinson 1991, Burnside and Eichenbaum 1996).
where capital in the fourth quarter is the same as the annual capital stock $K_{t,4} = K_t$. Numerical methods allow us to derive the depreciation rate by solving (A.5); given $\delta_t$, quarterly stock in each quarter is drawn from (A.1)-(A.4).

Incomes are from Istat. The share of labour income in value added is calculated by extrapolating to total employment (including self-employment) the average labour cost for payroll employees.

Capacity utilisation, taken from Banca d’Italia, has been computed by the Wharton method, as in Signorini (1994) and expressed in percentage values.

Real wages in industry are calculated deflating per capita wages with the GDP deflator.

Hours worked has been computed monthly by Istat for large firms (Indagine sulle Grandi Imprese) since 1972 with a break due to the change in the survey unit in 1988. We ignore this break and simply rebase the old index according to the ratio in the conjunction year. The figure given is a seasonally adjusted quarterly average.

The rental price of capital goods is taken from Gaunolis et al. (1996). They estimate the user cost of capital by maximising the discounted value of the future stream of investment of a representative firm. In the basic formulation, the rental price of a unit of capital good is equal to

\[(A.6) \quad C = cf + \delta - dp/p \]
which is to say, the cost of finance plus physical depreciation minus price variation. Hence, user cost is

\[
C_u = C \times p_i
\]

or the rental price by the investment deflator.

The user cost has been computed at the aggregate level for the private sector, net of agriculture and energy, allowing for the incidence of the fiscal system: depreciation allowances, investment incentives, deductibility of interest expenses. Liquidity constraints and tax progressivity have been neglected.

All the data, if not otherwise specified, are seasonally adjusted.
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Solow, R. (1964), Draft of the presidential address to the Econometric Society on the short-run relationship between employment and output.


<table>
<thead>
<tr>
<th>No.</th>
<th>Title</th>
<th>Authors</th>
<th>Publication Date</th>
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<tbody>
<tr>
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<tr>
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<tr>
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<tr>
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<td>Bad loans and entry into local credit markets</td>
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<tr>
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<td>511</td>
<td>Information variables for monetary policy in a small structural model of the euro area</td>
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<td>W. Fuchs and F. Lippi</td>
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<tr>
<td>513</td>
<td>Monetary policy and stock prices: theory and evidence</td>
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<td>July 2004</td>
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<td>514</td>
<td>Financial structure and the transmission of monetary shocks: preliminary evidence for the Czech Republic, Hungary and Poland</td>
<td>A. Anzuoni and A. Levy</td>
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<tr>
<td>515</td>
<td>The pricing behaviour of Italian firms: new survey evidence on price stickiness</td>
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<td>July 2004</td>
</tr>
</tbody>
</table>

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