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SOME FURTHER ECONOMICS OF EASTER ISLAND: ADDING SUBSISTENCE AND RESOURCE CONSERVATION

by

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Further economics of Easter Island: Adding subsistence and resource conservation

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ABSTRACT: We extend Brander-Taylor's model of development on Easter Island by adding a resource subsistence requirement to people's preferences, and a conservation incentive in the form of a revenue-neutral, ad valorem tax on resource consumption. Adding subsistence improves plausibility; makes overshoot and collapse of population more extreme, and the steady state less stable; and allows for the possibility that statue building and erection will suddenly stop, in line with the archaeological evidence. We explore a tax rate path which could have almost completely prevented overshoot, and conjecture that the overall strength of this path must rise when the subsistence level rises. ‡

(JEL Q20, N57, J10)

KEYWORDS: subsistence, renewable resources, conservation, Easter Island

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1. Introduction

Brander and Taylor (1998), hereafter BT, applied a "Ricardo-Malthus" model of people using open-access renewable resources to explain the overall interaction between population and resources on Easter Island in the south-eastern Pacific, between about 400 A.D. and the arrival of European explorers in 1722 and 1774. In so doing, they sought to explain a pattern of economic and population growth, resource degradation and subsequent economic decline, which modern archaeological and anthropological thinking believes to be more common than previously thought. The central puzzle that greeted the first Europeans on Easter Island was the presence of many huge statues carved of volcanic stone, with no explanation of how these could have been erected. The island was treeless, and the population of about 3,000 was too small and knew nothing about moving statues.

Recent evidence shows that the island was first settled by a small group of Polynesians in about 400 A.D., and that the island supported a great palm forest then. Population probably peaked at about 10,000 people around 1400 A.D., and declined thereafter.[1] BT modeled the forests and soil together as a single, renewable resource with logistic growth. This is consumed by a human population whose fertility depends in a Malthusian way on resource consumption per capita, making the model essentially a Lotka-Volterra one, with people as "predators" and the resource as "prey". People continuously divide their time between resource harvesting and "manufacturing". The latter means statue erection and other monument building in the case of Easter Island, but in fact it could be anything done with non-harvesting time, as labor is the sole input to manufacturing. People aim simply to maximize per capita utility at each instant. With this model, BT simulated a growth of population, followed by a decline in the resource stock and then a decline in population a few centuries later, which was a plausible fit to the archaeological data just described.

A key question raised by BT was why there was no institutional adaptation to prevent the collapse of the system. There are many examples of open-access systems where common property management regimes can be developed to prevent the type of degradation observed in the BT model (Ostrom, 1990). Key elements that would affect successful institutional reform, as BT noted, include the population's level of agreed-upon understanding of the problem, and the underlying behavioral attributes of the individual agents. Here we extend the BT model

[1] As BT noted, these figures are quite uncertain. Estimated figures for the population maximum range from 7,000 to 20,000, and figures for the date of this maximum vary by a century or so. However, within this range of uncertainty, the qualitative problem we analyze here remains unchanged.

to include both a subsistence requirement and a mechanism for resource conservation, and we explore how these two features might affect the possibility of institutional adaptation. Some alternative new features have been suggested to us, but they appear to add more mathematical complexity than our chosen two, without altering the latter's basic insights. So we have left these alternatives for further work, though we briefly outline them in our conclusions.

Our reason for adding a subsistence requirement is that the Cobb-Douglas utility function used by BT resulted in people dividing their time between resource harvesting and manufacturing in fixed proportions, even though this results in hunger and population decline when resources are scarce. We use a utility function with a minimum subsistence level of resource good consumption, which avoids the implausibility of this fixed-proportion choice. It also makes the model less stable and more prone to overshoot and collapse, and so better able to explain the complete cessation of statue manufacture believed to have happened after about 1500 on Easter Island.

The importance of subsistence in economics is widely recognized (see Sharif (1986) for a survey), and its impact on growth patterns has been explored by authors such as Steger (2000). But his and other analyses made more conventional assumptions than ours: their productive capital stock was physical rather than natural; instantaneous utility depended only on consumption; the rate of population growth was exogenous; intertemporal utility was maximized; and the timescale of interest was much shorter than the 1300 or so years that we consider. We hope our analysis will lead to useful insights for contemporary development economics, but the challenge of bridging the paradigms is considerable. In particular, it will not be easy to choose an appropriate intertemporal utility function when population growth is treated as endogenous, as it probably should be in an economy near subsistence.

Our second new element is some basic modeling of a "resource conservation" institution, which Ricardo-Malthus-like societies might possibly use to avoid overshoot and collapse. For ease of computation we model this as an instantaneous, revenue-neutral, ad valorem tax on resource extraction. We do not explore how cultural mechanisms roughly equivalent to the tax might evolve and work in a simple, Easter-Island-like economy. Our aim is to explore some general characteristics of a tax path which avoids overshoot, and its dependence on people's underlying behavior.

Section 2 and the Appendix derive our theoretical results with both a subsistence level of resource and a revenue-neutral resource tax. Section 3 uses simulations to examine the effects of subsistence alone (with no tax) on stability in general, and manufactures in particular. Section 4 simulates effects of the chosen tax profile on the economy, and the influence of the subsistence requirement on the characteristics of the tax profile. Section 5 concludes and suggests directions for further work.

2. The Theoretical Effect of Subsistence and Taxation on The Easter Island Economy

Our closed, population-resource model is the BT model in all but two respects. First, our representative consumer is assumed to have an instantaneous utility function

$$u(h, m) = (h - h_\mu)^\beta m^{1-\beta}; \quad 0 < \beta < 1, \;\; h_\mu > 0 \tag{1}$$

where h and m are individual consumptions of the resource good and of manufactures. The positive parameter h_μ is a subsistence or minimum required level of resource consumption, a feature which seems appropriate for studying population collapse in a resource-based society like Easter Island. BT omitted this, i.e. they had $u = h^\beta m^{(1-\beta)}$. Total resource and manufactures consumptions are $H = Lh$ and $M = Lm$, where L is the total population at any time (all time arguments are suppressed). Total "manufactures" M are produced with constant returns to scale using only labor, $M = L_M$, where L_M is the total labor time spent on manufactures. So we could equally well think of the "other good" m as including culture and/or leisure as well as tangible artifacts. For all interior solutions, manufactures are taken as the numeraire (i.e. with price = 1) so that the wage rate $w = 1$, and with total time normalized at 1, total wage income is also 1. The price of the resource in terms of manufactures is p.

Our other departure from BT is that all resource consumption is taxed collectively at an ad valorem rate t, with the revenue being all returned to people as a lump-sum subsidy T. In simple societies without formal governments and taxes, the tax would have to be thought of as a disincentive to "excessive" resource consumption enacted by social pressures or taboo, but we do not discuss the much-studied and difficult problem of how such customs might arise or be enforced.

At each moment, the individual consumer myopically maximizes utility (1) subject to the budget constraint

$$(1 + t)ph + m = 1 + T. \tag{2}$$

while the condition for overall revenue neutrality (adhered to collectively, but not perceived by any individual) is

$$tph = T. \tag{3}$$

The resulting interior and corner solutions for per capita manufactures and resource consumptions (see Appendix for calculations) are respectively

$$h = \begin{cases} \frac{\beta(1-ph_\mu)}{(1+t-\beta t)p} + h_\mu & \text{if } p < \frac{1}{h_\mu} \\ \frac{1}{p} & \text{otherwise} \end{cases} \tag{4}$$

$$m = \begin{cases} \frac{(1-ph_\mu)(1-\beta)(1+t)}{1+t-\beta t} & \text{if } p < \frac{1}{h_\mu} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

The tax rate t shifts the balance of the interior solutions from resource consumption towards manufactures, but has no effect on the corner solution, as one would expect intuitively.

Next, following BT, we assume a Schaefer resource harvest function $H = \alpha S L_H$, where $\alpha > 0$ is the harvestability parameter, S is the resource stock and $L_H = L - L_M$ is the labor used in resource harvesting. Under open access conditions, any rent (user cost) of the resource is ignored, and the price of the resource is just the value of labor expended per unit of resource harvested:

$$p = \frac{wL_H}{H} = \frac{1}{\alpha S} \tag{6}$$

(4) and (5) then become

$$h = \begin{cases} \frac{\beta(\alpha S-h_\mu)}{1+t-\beta t} + h_\mu & \text{if } \alpha S > h_\mu \\ \alpha S & \text{otherwise} \end{cases} \tag{7}$$

$$m = \begin{cases} \frac{(1-\frac{h_\mu}{\alpha S})(1-\beta)(1+t)}{1+t-\beta t} & \text{if } \alpha S > h_\mu \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

As in BT, the resource carrying capacity of the society's land is K, the intrinsic rate of resource growth is r, and the rate of change $\dot{S}$ of the resource stock is natural growth $G(S) = rS(1-S/K)$ minus total harvest $H = Lh$ from (7):

$$\dot{S} = rS(1 - S/K) - Lh, \quad r, K > 0 \tag{9}$$

Finally, the population growth rate is a Malthusian fertility term proportional to per capita resource consumption, $F(H, L) = \frac{\phi H}{L}, \phi > 0$, minus a base rate of decline, which for neatness is written as δ (> 0) here rather than $(d - b)$ as in BT.[2] Thus[3]

$$\frac{\dot{L}}{L} = \phi h - \delta. \tag{10}$$

Throughout we assume that the parameters satisfy BT's condition (13)

$$K > \frac{\delta}{\phi\alpha\beta} \tag{11}$$

which guarantees the existence of a steady state at which the human population is positive. We denote this and the corresponding steady state resource population as L_∞ and S_∞, respectively. The steady state is found by setting $\dot{S} = \dot{L} = 0$, though it is not necessarily a stable equilibrium, as we discuss below. The Appendix shows that the steady state resource stock then depends on our new features t, the tax rate, and h_μ, the subsistence level, according to:

$$S_\infty(t, h_\mu) = \begin{cases} \frac{\frac{(1+t-\beta t)(\frac{\delta}{\phi}-h_\mu)}{\beta}+h_\mu}{\alpha} & \text{if } \frac{\delta}{\phi} > h_\mu \\ \frac{\delta}{\phi\alpha} & \text{otherwise.} \end{cases} \tag{12}$$

while the steady state population is

$$L_\infty(t, h_\mu) = \frac{r\phi}{\delta} S_\infty (1 - \frac{S_\infty}{K}) \tag{13}$$

with per capita resource and manufactures consumption being respectively

$$h_\infty = \frac{\delta}{\phi} \quad \text{and} \quad m_\infty(t, h_\mu) = 1 - \frac{\delta}{\phi\alpha S_\infty}. \tag{14}$$

With a little manipulation, equation (12) shows that provided $\frac{\delta}{\phi} > h_\mu$, a positive resource tax will result in a higher steady state resource stock, while a positive subsistence requirement will have the opposite effect. Notice from (7) and (12) that as h_μ increases, the tax has less effect on the instantaneous per capita harvest rate h and the long run equilibrium resource stock S_∞.

Total steady state resource and manufactures $H = Lh$ and $M = Lm$ follow immediately from the above results. Note how the corner ($\delta/\phi \leq h_\mu$) solutions can be found intuitively from the interior solutions: just set the resource preference parameter $\beta = 1$, thus "forcing" all labor to be spent on resource harvesting and none on manufacturing.

[2] Like BT, we do not distinguish between the effects of nutrition on birth rates (via fecundity and social customs) and on death rates (via disease and perhaps infanticide). See Anderies (1999) for a model that makes this distinction.

[3] One might argue instead that $\frac{\dot{L}}{L} = \phi(h - h_\mu) - \delta$ on the (debatable) grounds that it should be only resource consumption above the subsistence level h_μ that contributes to increased fertility. But since both equations are linear in h, all this does is make the algebra slightly simpler, and change the interpretation of some parameters.

3. The Effect of the Subsistence Requirement

Here we are interested only in how the subsistence consumption level h_μ affects the dynamics of the model, and allows for an abrupt disappearance of monument-building ("manufactures" here) from a culture. We treat people as individuals, and so ignore the possibility of collective resource conservation, and set the tax rate $t = 0$ in the formulae in Section 2.

The effect of the subsistence requirement is fairly easy to understand intuitively. BT's results follow directly from the above by setting $h_\mu = 0$, and thus ignoring all the corner solutions. In BT, no matter how low the resource S gets as a result of harvesting, and thus no matter how negative the population growth rate $\phi\beta\alpha S - \delta$ (from (7) and (10) with $h_\mu = t = 0$) becomes as a result of famine, people still spend a fixed proportion $m = 1 - \beta$ of their time on manufactures. This means that if resources per person become scarce, population stops growing fairly quickly, putting less pressure on the resource.

However, for people still to carve statues while babies starve seems unlikely behavior. Our solution allows the more plausible reaction that as the resource stock S falls, equations (7) and (8) force labor to be reallocated from the manufacturing to the resource sector. Once the resource stock falls below h_μ/α, and hence the maximum per capita harvest falls below the subsistence level h_μ, manufacturing effort stops altogether. In a sense, society has then "collapsed", and all human effort is spent on resource harvesting. Even before this, the greater preference for resources caused by $h_\mu > 0$ means that population growth tails off less rapidly as resources per capita decline, harvesting pressure on the resource is greater, and the overshoot of population is greater, as we shall see.

From (12), the end of manufacturing may be temporary if $\delta/\phi \geq h_\mu$, since there is then an interior ($m > 0$) steady state, which may be stable and to which the population may ultimately converge. But if $\delta/\phi < h_\mu$, manufacturing will never reappear: in this case, a finite population can be sustained forever, but it stays in a true subsistence state with all time spent on harvesting. Such distinctions may be useful in categorizing types of interaction between past civilizations and their environments.

As an illustration, Figure 1 shows resource stock S, per capita manufactures m, population L and total manufactures M over the period 400-1900 A.D. for the simulation in BT's section IV, with parameters as in the first column of Table 1, of the ecological-economic history of Easter Island.

Table 1:
Summary of model parameters.

Parameter	Values from BT used in Figures 1, 2, 3, 7, and 8	Values used in Figure 5
Fertility, ϕ	4	
Resource taste, β	0.4	
Base rate of population decline, δ	0.1	
Initial population, $L(0)$	40	
Harvestability, α	0.00001	0.0000078
Resource intrinsic growth rate, r	0.04	0.035
Carrying capacity, K	12000	11000
Subsistence resource consumption, h_μ	0 or 0.015	0.018
Equilibrium resource level, S_∞ (no tax)	4000	4551
Equilibrium population level, L_∞ (no tax)	4267	4269

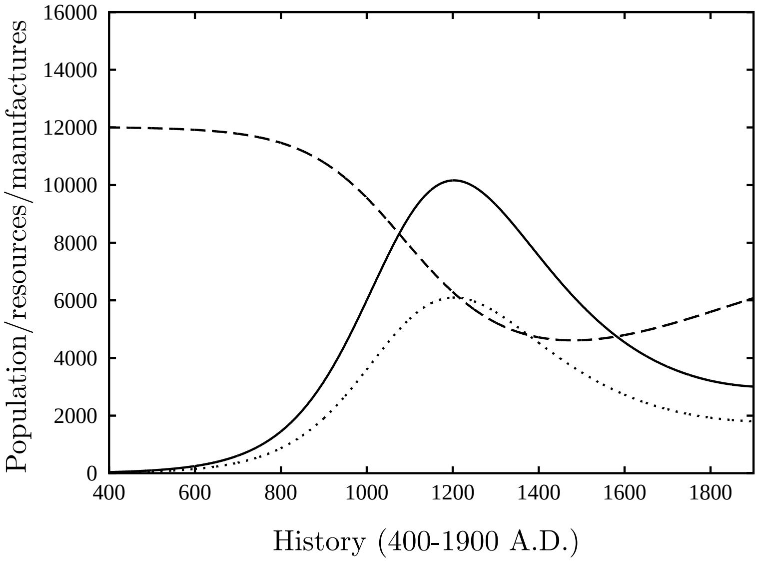


Figure 1:
Population (solid line), resources (dashed line), and manufactures (dotted line) on Easter Island for BT's model.

Figure 2 shows the same simulation, changed only by a positive subsistence requirement $h_\mu = 0.015$. Note firstly, the earlier and much greater population overshoot (to about 16,000 rather than 10,000) which occurs "with subsistence"; secondly, the much more severe falls in population and resource stocks thereafter; and thirdly, the complete halt in manufacturing which lasts from about 1150 till 1650, by which time the resource (but not yet population) has begun to recover.

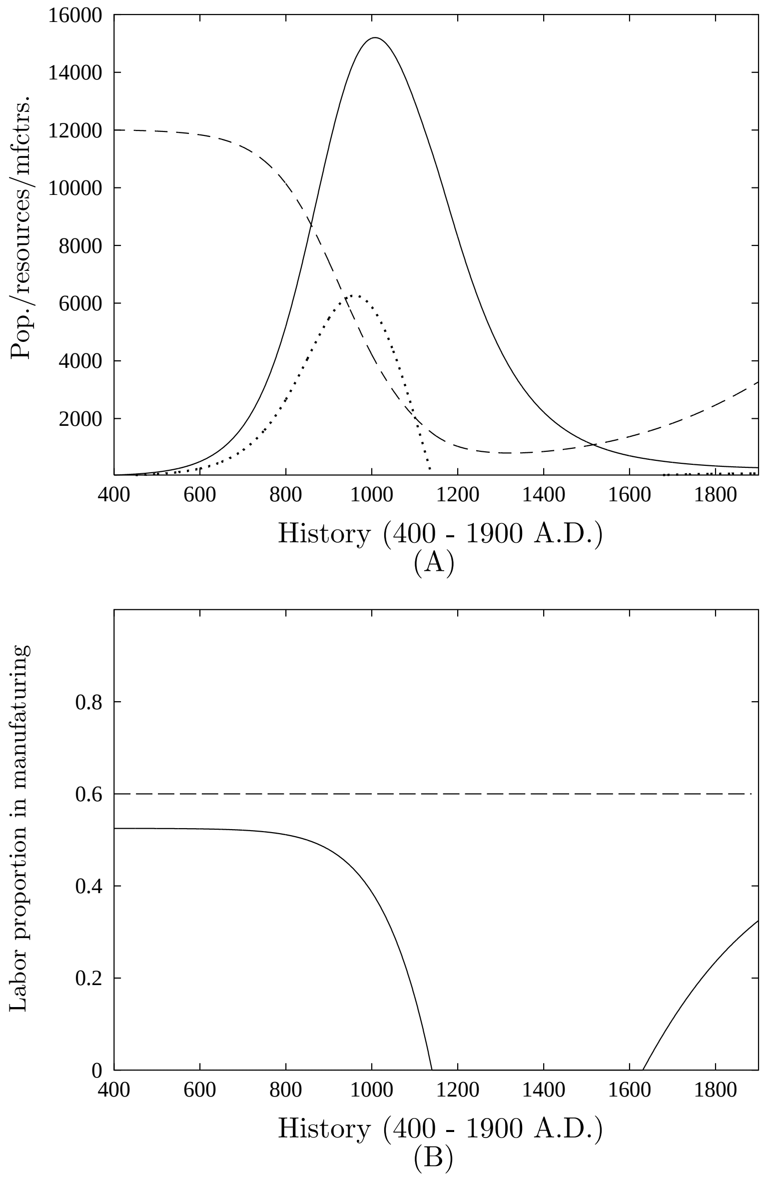


Figure 2:

Figure (A) shows the population (solid line), resource stock (dashed line), and manufacturing output (dotted line) with subsistence $h_\mu = 0.015$. Figure (B) shows the labor allocation to manufacturing over time as a proportion of the total labor force for the BT model (dashed horizontal line), and for $h_\mu = 0.015$ (solid line).

Subsistence also causes differences not just initially, but also in long-term stability behavior. Figure 3 shows just population for the Figure 1 and 2 simulations with a compressed timescale, which extends out as a "prediction" of what would have happened up to 4900 A.D. with no outside intervention. Note how much more rapidly (if "rapidly" is the right word for changes happening over several centuries) the damped oscillations of population converge when there is no subsistence. Indeed, suppose the initial population of Polynesian adventurers had reached Easter Island at least three thousand years earlier, and the BT model had been correct. Then Captain Cook would have found a very stable population and culture, which might have become a historical example of sustainable resource use much admired by modern environmentalists! But under the subsistence model, evidence of quite separate eras of monument-building (and maybe no current building) might have been found. So a subsistence requirement causes not just greater initial overshoot, but a much slower damping of the ecological-economic system.

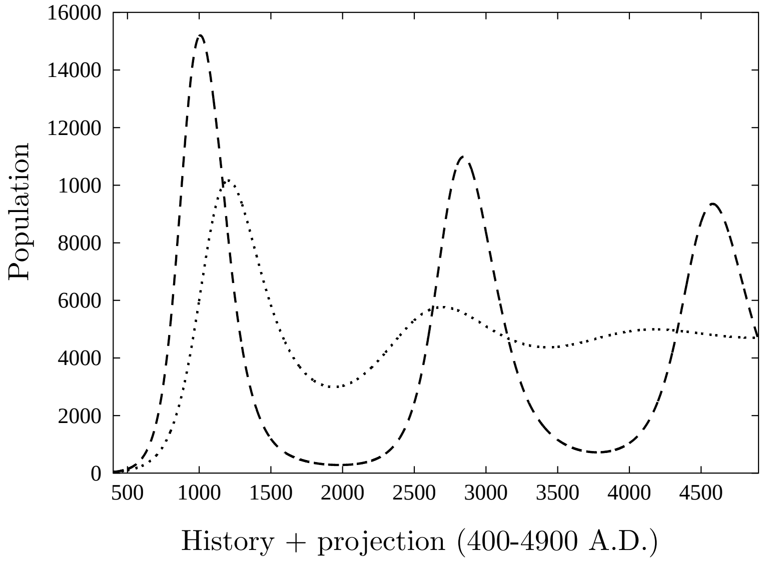


Figure 3:
Population for the BT model (dotted line) and subsistence model (dashed line).

Furthermore, when the subsistence level h_μ rises above a certain threshold $h_\mu^\dagger$, which depends in a complex way on the parameters, the model behavior fundamentally changes. For $h_\mu < h_\mu^\dagger$, the steady state is a stable equilibrium in the (S, L) phase space, towards which the system will converge either monotonically or through a series of damped oscillations. For $h_\mu > h_\mu^\dagger$, the steady state is unstable and the system will converge to a limit cycle circling around the steady state. The transition between the two behavior modes at $h_\mu = h_\mu^\dagger$ is a Hopf bifurcation (Kuznetsov, 1995), and as h_μ rises through $h_\mu^\dagger$, the real parts of the two eigenvalues of the Jacobian of the system change from negative to positive. This change in the real parts of the eigenvalues from negative to positive weakens the natural damping in the model possibly leading to more extreme fluctuations.

Using the pseudo arclength continuation technique found in the software package Auto (Doedel, 1981), we calculate that for the BT set of parameters, $h_\mu^\dagger = 0.0177$. Figure 4, computed from Auto, plots the decline in the steady state population L in (21) as h_μ increases (shown as a continuous fine line), the bifurcation point $h_\mu^\dagger$, and the upper and lower values of the limit cycle which occur for $h_\mu > h_\mu^\dagger$ (dark circles). Because the value of $h_\mu = 0.015$ chosen for Figures 2 and 3 is getting rather close to $h_\mu^\dagger$, the real parts of the system eigenvalues are then quite close to zero, though still negative. This means that though the steady state is still stable, convergence is much slower than with $h_\mu = 0$, as already shown by Figure 3. For values of $h_\mu > h_\mu^\dagger$, the system will never converge to the steady state, and the oscillations will be even more pronounced than those shown in Figure 3.

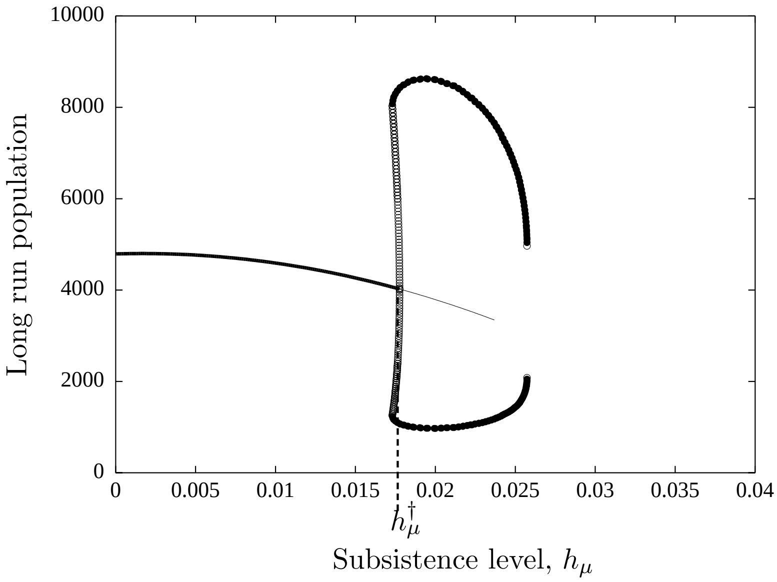
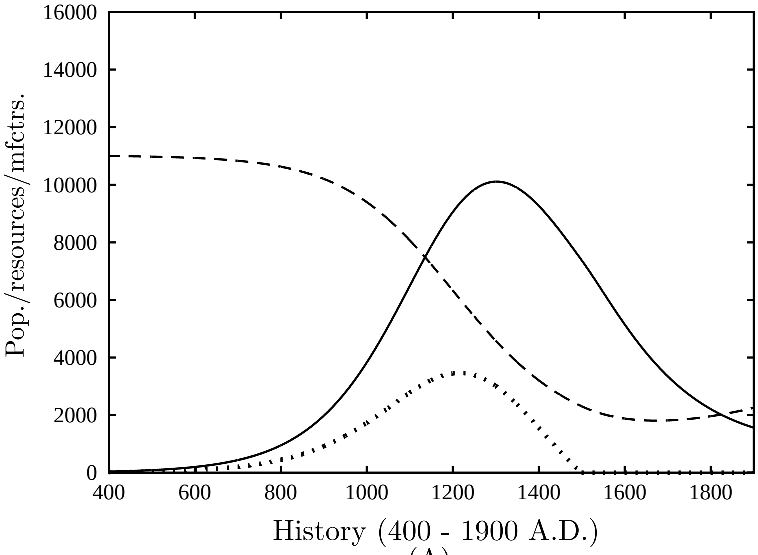
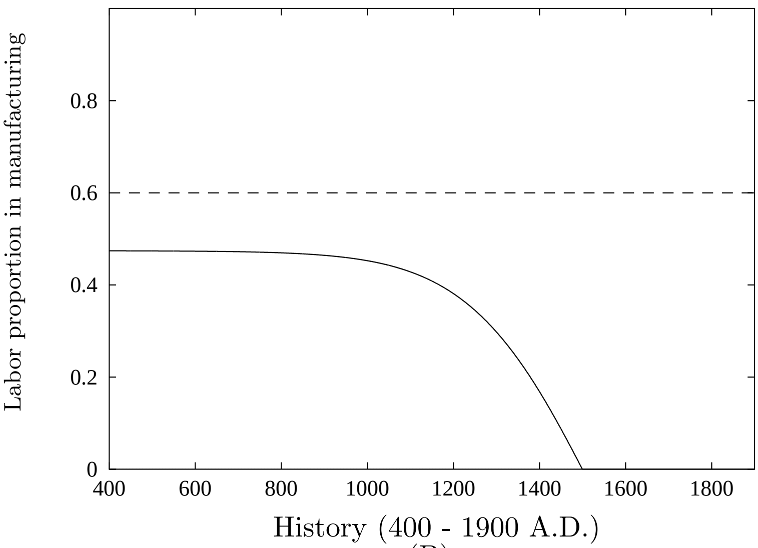


Figure 4:
Bifurcation diagram showing fundamental change in long term behavior as h_μ is varied. Long term behavior changes from a stable equilibrium (solid line showing the value of the long run stable population) to a large amplitude limit cycles (heavy circles showing the maximum and minimum population levels attained over a cycle.

Finally, can our model produce a reasonable fit to what is known of Easter Island history, rough as this is? The parameters in the second column of Table 1, some of which are the same as BT's, while others are "tweaked" but still plausible, give the simulation of resource, population, manufactures and manufactures per capita shown in Figure 5. With the changed parameters the bifurcation threshold can be calculated as $h_\mu^\dagger = 0.0219$, so the chosen subsistence level $h = 0.018$ is below this, but not by much. Also our $\delta/\phi = 0.025 > h_\mu$. The steady state of this simulation is thus a stable, interior one, but convergence in fact takes several thousand years of drastic population-resource cycles. The parameter set was chosen so that three features of Figure 5 all fit BT's summary of the rough data available for Easter Island at least as well their own simulation. Figure 5 gives a good fit to the timing and size of the peak population, the population on European contact around 1720 and 1770, and the complete cessation of manufacturing around 1500. The last of these is is modeled better by Figure 5 than by BT's non-subsistence model in Figure 1, where manufacturing M in 1500 is still more than half the peak reached around 1250. Abrupt halts in manufacturing (i.e. monument-building) have also been noted for other lost civilizations such as the Maya, so the subsistence effect may explain a more general phenomenon.



(A)



(B)

Figure 5:

Figure (A) shows the population (solid line), resource stock (dashed line), and manufacturing output (dotted line) for the subsistence model with the parameter set in column two of Table 1. Figure (B) shows the labor allocation to manufacturing over time as a proportion of the total labor force for the BT model (dashed horizontal line), and corresponding to Figure (A) (solid line).

4. An Institution to Conserve the Resource

BT gave a verbal discussion of whether institutions governing fertility or resource use, which were absent from their formal model, could possibly have been developed in order to save the Easter Island civilization from overshoot and collapse. Here we give a quantitative treatment of how effective one type of institution, a collective incentive to conserve the resource, might be in doing this. But we aim neither to explain how such an incentive could have been evolved, nor to improve the fit of our model to data for Easter Island (or anywhere else).

One reason for avoiding this obvious but daunting challenge is that there is as yet little agreement about the main factors at work, at least as revealed by the secondary sources we have consulted. BT (p133) suggested that it would be difficult for a society like Easter Island to develop effective institutions in a single boom and bust cycle, especially given that the maximum rates of resource degradation are actually quite low (at most 5% over a typical human lifetime in Figure 1), and that the 40 to 60 years needed for a palm tree to mature meant that reafforestation efforts "would almost never have been of direct benefit to the cultivators". In contrast, Ponting (1991, Ch. 1) suggested that the Easter Islanders must have realized how much environmental degradation had occurred, but stressed the role of competition and conflict (including warfare) between rival clans in stopping any collective, island-wide response from developing. Further research into the interaction in resource-based societies between actual and perceived resource degradation, and political structure, would therefore be welcome, though undoubtedly not easy.

Our more limited aim here is to find what kind of institutions could, *if adopted*, avoid overshoot and collapse. This may provide some useful tools for further analyzing the interesting but difficult questions which inevitably arise when one asks if some simple resource-based societies achieved a form of "static sustainability". (This is the idea in Pezzey (1992) of a society keeping both its population and its resources fairly constant, whether by evolutionary accident or by conscious design, while using only renewable resources and constant technology.) One such question (not addressed here) is under what circumstances sustainable societies can persist despite the presence of neighboring, unsustainable ones. Given how modern environmental groups often promote the ethics of resource use that prevailed or still prevail in statically sustainable societies, this topic has some relevance for the formation of modern environmental policies.

An associated question that we do address is how varying the subsistence requirement (i.e. varying a parameter that affects individual behavior) influences the effectiveness of the institutions that can avoid overshoot and collapse. Again this has some relevance for forming modern environmental policies to control resource consumption.

Our modeling is confined to the ad valorem, revenue-neutral tax analyzed in Section 2, and we have not considered any kind of constraint on fertility. The main challenge is to find a path of the tax rate over time that might improve the dynamic evolution of society. But what might such "improvement" mean? Koopmans (1977) emphasized that when population is endogenously determined, as here, agreeing on a dynamic objective raises difficult philosophical questions. Is society's "benefit" at each time seen as population, total utility, or per capita utility? Would it be just the steady state level of one of these variables that is the maximand, or the discounted present value of it, as an anthropologist like Alvard (1998) concludes, along with most growth economists? If the latter, what discount rate should be used? Or should population simply never decline, thus avoiding overshoot (one of several possible definitions of sustainable development for a society)?

Combining an infinite set of time paths of the tax rate with several alternative objectives allows endless possible explorations. So we simplify by assuming that an institution to conserve resources could include three components:

- Some type of threshold event or state that, once crossed, would stir the population to take action;
- Some speed of response, or "relaxation time", over which the population takes action once they have decided to act; and
- Some vision of what the agents want their long term society to look like.

A simple tax schedule that crudely incorporates these components is

$$t = \frac{t_{max}}{1 + exp(-q(1 - \frac{L}{\Lambda}))} \tag{15}$$

where t_{max}, q, and Λ are positive constants. The "design" behind (15) is that the function

$$\frac{1}{1 + exp(-q(1 - \frac{L}{\Lambda}))} \tag{16}$$

"switches on" from near 0 to near 1 as the population L increases. The parameter Λ marks the population level at which expression (16) has a value of 0.5, and thus the halfway point of the switching-on process. The parameter q controls the width of the band around Λ during which the switching on occurs, i.e. how sharply the tax switches on. Figure 6 illustrates the behavior of the tax schedule for $t_{max} = 1$, $\Lambda = 4000$, and three different values of q, 4, 7, and 10, corresponding to increasingly sharp responsiveness, respectively.

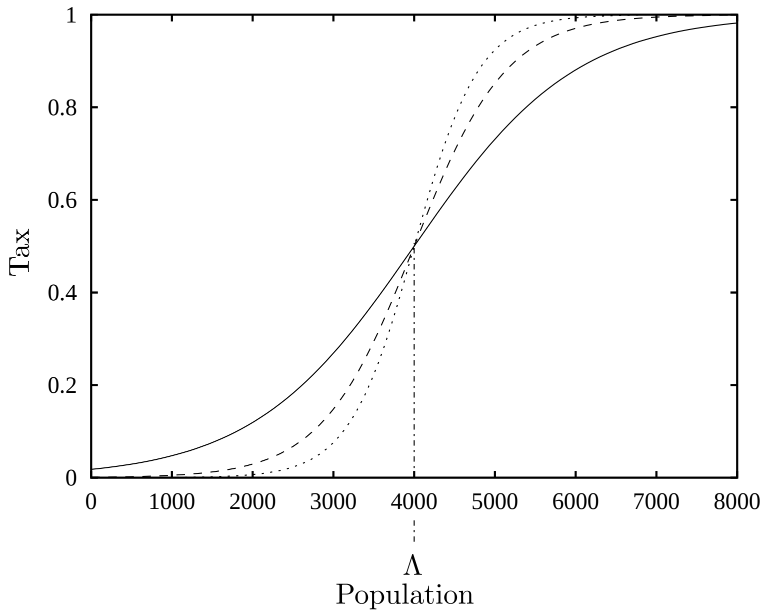


Figure 6:
Tax rate in (15) versus population for $\Lambda = 4000$, $t_{max} = 1$, and three different values of q: 4 (solid line), 7 (dashed line), and 10 (dotted line).

We assume that agents are motivated to take action by population pressure, but a tax schedule based on resource decline works in a similar fashion. The important point is that society responds to the state of the system. For high values of q, the response of an increasing tax rate is rapid, but the population pressure must be higher before this starts to occur. For low values of q, a noticeable tax rate starts at lower population pressures, but increases more slowly over time. Thus by adjusting the two parameters Λ and q, numerous scenarios can be modeled for when and how rapidly society starts responding. Finally, the maximum value that the tax could ever approach is t_{max}, so this measures the overall strength of the tax.

Above we reached no firm conclusion on an ideal dynamic goal for society. However, we identified avoiding any population decline as one possible goal. In practice, we chose the tax path parameters by trial and error in order to curb population overshoot and collapse as much as possible, and Figure 7 shows one outcome of this approach: the effect of the tax path (15) with $t_{max} = 1$, $\Lambda = 4000$, and $q = 6$ on the dynamics of the original BT model (with no subsistence). In choosing our tax path we thus pay little attention to optimizing steady state variables. For example, t_{max}, the asymptotic value of our chosen tax path bears no relation to the value t^* which would maximize steady state population L_∞ in the cases considered.[4] For the standard BT parameters (hence $h_\mu = 0$), $t^* = -0.0667$, whereas for the parameters we will use for Figure 7, t_{max} is about 1.15. Fortunately, for meaningful parameter ranges including BT's parameter set, steady state population turns out to be rather insensitive to the tax rate.

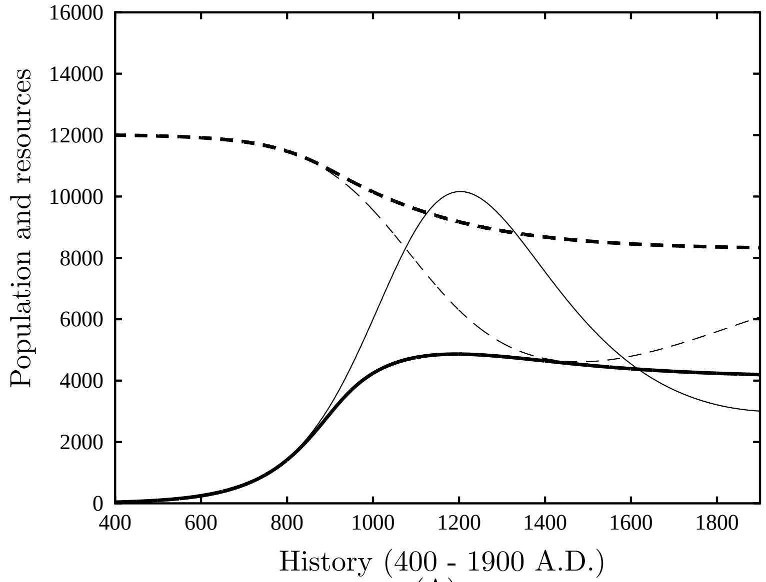
In Figure 7, the tax path is very successful in smoothing out the initial overshoot of population, as shown by comparing in Figure 7(A) the original population and resource trajectories (light lines) with those when the tax regime is in effect (bold lines). Figure 7(B) shows the tax rate (solid) and the per capita resource consumption path (dotted) for both cases. As the population approaches 4000, the tax rate kicks in and drives consumption down earlier than in the case with no tax (bold dotted line versus light dotted line in Figure 7(B)). This prevents per capita consumption of resource goods h from falling much below the zero population growth level of $\frac{\delta}{\phi} = 0.025$.

Figure 8 shows the effect the subsistence requirement ($h_\mu > 0$) has on the system. Figure 8(A) shows the population trajectories for the same parameters as in Figure 7, except for $h_\mu = 0.015$ instead of 0, and now three different values of t_{max}: 0, 1, and 3, shown respectively by the solid, dotted, and solid bold lines. The tax strength $t_{max} = 1$ that nearly stabilized the system with $h_\mu = 0$ is now far too weak to prevent overshoot. Instead, a tax regime with a maximum of about 3 times that level is required to achieve a fairly smooth approach to a long term equilibrium. Some local stability analysis (see Appendix) combined with numerical experiments leads us to conjecture that the value of t_{max} required to avoid overshoot always rises nonlinearly with h_μ. For example, with $h_\mu = 0.02$, a t_{max} of about 6 is needed to avoid overshoot and collapse. So the larger the level of subsistence needed, the more drastic the institutional action required to avoid overshoot and collapse.

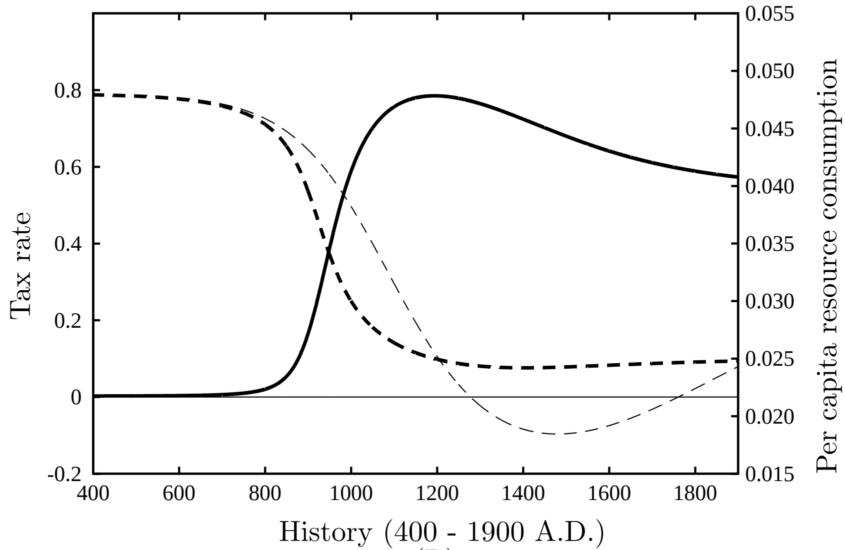
This result can be interpreted in two ways. The first is a fairly straightforward interpretation from the mathematics. Recall from (12) that the tax affects only interior solutions. Increasing h_μ increases the chance of a corner solution with all labor working in the resource sector. The chance of a corner solution simply reduces the size of the area in state space where the tax can act, reducing its effectiveness. This is shown graphically in Figure 8(B), which shows the

[4] By differentiating L_∞, from (13) and (12), with respect to t, one can show that the steady-state-maximizing tax rate is

$$t^* = \frac{\frac{\beta(\alpha K - 2h_\mu)}{2(\delta/\phi - h_\mu)} - 1}{1-\beta}. \tag{17}$$



(A)

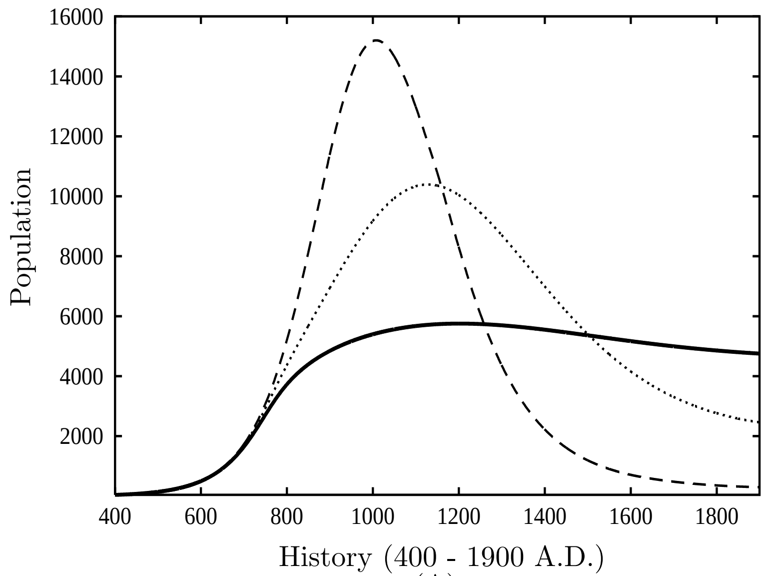


(B)

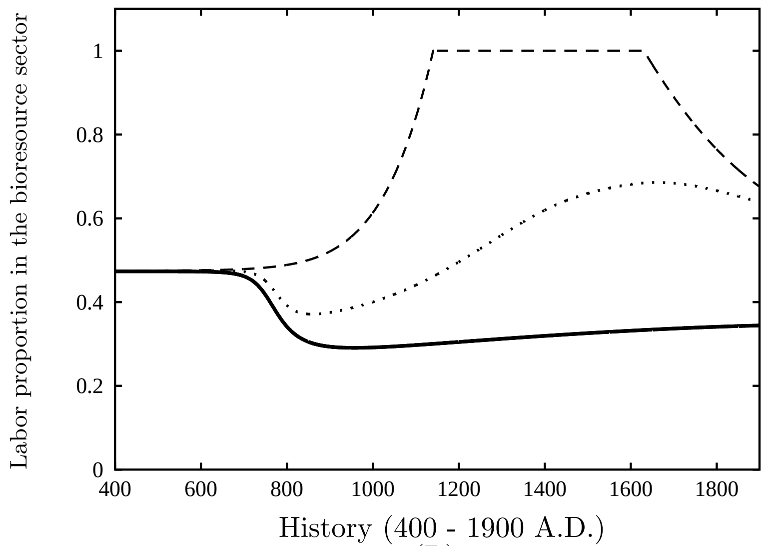
Figure 7:

Figure (A) shows resource (dashed) and population (solid) trajectories without (light) a tax and with (bold) the resource tax schedule with $t_{max} = 1$, $\Lambda = 4000$, and $q = 6$ for the original BT parameters. Figure (B) shows the corresponding tax (solid) and resource consumption (dashed) trajectories without (light) and with (bold) the tax.

fraction of total labor in the resource sector for the three cases above. The solid line shows the case for $t_{max} = 0$, in which resource scarcity progressively drives labor into the resource sector until the entire population is engaged there. The dotted line depicts the case for $t_{max} = 1$. Here when the bioresource is not scarce, the tax effectively moves labor out of the resource sector. However, this effect is temporary as subsequent resource scarcity drives labor back into this sector. Finally, the bold line shows the case for $t_{max} = 3$, in which the effect of the tax is strong



(A)



(B)

Figure 8:
Figure (A) shows population trajectories without (dashed) and with tax ($t_{max} = 1$ (dotted) and $t_{max} = 3$ (bold), with Λ and q as before) for the model with subsistence $h_\mu = 0.015$. Figure (B) shows the corresponding fraction of labor in the resource sector over time.

enough to reduce labor in the resource sector permanently. The key is that the tax must reduce labor in the resource sector sufficiently to prevent the future population growth that will drive labor back into the sector.

The second interpretation of this result has to do with the way a population might attempt to address resource scarcity. A larger subsistence requirement reduces the willingness of consumers to substitute manufactures for resource consumption. It thereby forces society to try to solve resource scarcity problems simply by exploiting the scarce resource more intensively, and this makes collective action to prevent overexploitation more difficult. In this light, this result may have implications for a modern industrial society which responds to food scarcities by forever intensifying agriculture. In the model, society substitutes labor for a declining renewable resource, while a modern society substitute human-made capital and depletable resources (oil, coal, etc.) for a declining renewable resource. In both cases, the effect on the resource base is the same. Ever increasing useful work (human energy or fossil fuel energy) is required to produce food with an accompanying ever decreasing quality of the natural resource base. Our analysis suggests that this tendency may have made institutional reforms more difficult.

Whether or not a tax scheme like (15) would be selected by an evolutionary contest between competing societies is quite another matter. In the case of societies geographically close to each other (clearly not applicable to Easter Island), a society following the no-tax path in Figure 8 might, when it reaches its initial peak in population, be able to conquer a nearby, more sustainable society following the tax path, and thus commandeer fresh resources to prevent its own impending collapse. The effect of h_μ on the model suggests that many populations may have failed to adapt internally to resource scarcity, and thus could survive only by conquering neighboring societies. Such considerations might help explain the history of expansionary versus sustainable development in the pre-history of the world.

5. Conclusions

We have offered a refinement of Brander and Taylor's (1998) innovative analysis of the development of a myopic, Malthusian human society which depends solely on a renewable natural resource with a finite carrying capacity. Adding a minimum subsistence level of resource consumption to the typical individual's utility function improves both psychological realism, and consistency with what is roughly known about the overshoot and collapse of the Easter Island society which erected the famous, giant stone statues. For a plausible choice of parameters, the subsistence requirement results in a development path on which "manufactures" (statue carving and erection) cease suddenly after about 1500 A.D., in accordance with the archaeological data; whereas manufactures go on being produced in significant quantities in Brander and Taylor's model.

More generally, by using bifurcation analysis combined with numerical experiments, we have shown that the subsistence requirement destabilizes development, because it results in a less effective response to declining resources: people try to maintain their resource consumption, rather than allowing population to decline faster, so overshoot and collapse becomes more severe. A hypothetical, collective incentive to conserve the resource (modeled as a particular time path of an ad valorem tax on resource harvest) was shown to exert a restabilizing influence on development, largely preventing overshoot and speeding convergence to the steady state, if a stable one exists. The severity of the tax rate required to achieve this depends positively on the strength of the subsistence requirement, so a higher requirement reduces the chances that collective action could prevent resource degradation. Further work would be worthwhile on the effects of other tax paths, and on the much more complex question of what sort of community objectives over time would thereby be maximized.

Several other specific developments BT's basic model have been suggested during our work on this paper. One, mentioned in our second footnote, would be to include effects of per capita nutrition h independently on the death rate as well as on the birth rate. Another is that depleting resource stocks on Easter Island, specifically of trees, might have caused irreversible soil erosion, thus permanently reducing the carrying capacity K. A third idea is that as well as labor inputs, statue manufacture may have needed substantial inputs of resources, such as tree trunks for use as rollers (Ponting 1991, p5). A simple way to model this would be to make total harvest linearly dependent on manufactures as well as population, $H = \eta M + Lh$ for some constant η. Fourthly, perhaps it was the total stock of statues as much as the flow of their manufacture which gave a religious or cultural reward to the inhabitants, so that the utility function could be modeled as $u(h, \int M(t)dt, m)$. Finally, the model could perhaps be applied to other collapsed civilizations noted in BT's Section V, such as the Mayans, the ancient Mesopotamians, and the Chaco Anasazi. All these developments seem worth exploring in further work, but they appear to involve more complex calculations than, and not to undermine the broad significance of our chosen two features of a subsistence requirement and a resource conservation incentive.

Appendix

Consumer optimization

To derive results (4) for h and (5) for m from the constrained optimization problem defined by (1), (2) and (3), define the Lagrangian

$$\mathcal{L} \equiv (h - h_\mu)^\beta m^{1-\beta} + \lambda[1 + T - (1+t)ph - m] \tag{18}$$

First order conditions for an interior solution are:

$$\frac{\partial \mathcal{L}}{\partial h} = \frac{\beta u}{h - h_\mu} - \lambda(1+t)p = 0 \quad \Rightarrow \quad \frac{u}{\lambda} = \frac{(1+t)p(h-h_\mu)}{\beta} \tag{19}$$

$$\frac{\partial \mathcal{L}}{\partial m} = \frac{(1-\beta)u}{m} - \lambda = 0 \quad \Rightarrow \quad \frac{u}{\lambda} = \frac{m}{(1-\beta)} \tag{20}$$

$$(2) \text{ and } (3) \quad \Rightarrow \quad m = 1 - ph \tag{21}$$

$$(20) \text{ and } (21) \quad \Rightarrow \quad \frac{u}{\lambda} = \frac{1-ph}{1-\beta} \tag{22}$$

$$(19) \text{ and } (22) \quad \Rightarrow \quad \frac{(1+t)p(h-h_\mu)}{\beta} = \frac{1-ph}{1-\beta} \tag{23}$$

which after routine algebra gives (4) and then (5). Since a negative amount of manufactures is physically impossible, the condition for this interior solution to apply is

$$p < \frac{1}{h_\mu} \tag{24}$$

If (24) does not hold, then a corner solution (in which the tax rate plays no part) applies instead:

$$h = \frac{1}{p}, \quad m = 0. \tag{25}$$

Steady state

To derive the steady state for S in (12), L in (13), h and m in (14) corresponding to the first value of $S_\infty(t, h_\mu)$ in (12), setting $\dot{L} = 0$ in (10) gives h immediately. Using (7) then gives

$$\phi \left[\frac{\beta(\alpha S - h_\mu)}{1 + t - \beta t} + h_\mu \right] = \delta \tag{26}$$

$$\Rightarrow \alpha S - h_\mu = \frac{(1 + t - \beta t)(\frac{\delta}{\phi} - h_\mu)}{\beta} \tag{27}$$

from which (12) and (13) follow by straightforward algebra, including setting $\dot{S} = 0$ in (9) (with no check if the resulting steady state is stable). Next,

$$(12) \quad \Rightarrow \quad 1 - \frac{h_\mu}{\alpha S} = 1 - \frac{h_\mu}{(1 + t - \beta t)(\frac{\delta}{\phi} - h_\mu)/\beta + h_\mu} \tag{28}$$

$$= \frac{(1 + t - \beta t)(\frac{\delta}{\phi} - h_\mu)/\beta}{(1 + t - \beta t)(\frac{\delta}{\phi} - h_\mu)/\beta + h_\mu} \tag{29}$$

which with (9) gives m in (14) as required.

Notes on conjecture and local stability analysis

It is very difficult, if not impossible to say anything specific about the nature of trajectories far from steady states. However, local analysis combined with numerical experiments provides support for our global conjecture that the value of t_{max} required to avoid overshoot rises non-linearly with h_μ. Based on this idea, we examine the stability characteristics of the steady state as a function of parameters, exactly as was done numerically in Figure 4. Here we will show analytically that as h_μ increases, t_{max} must increase nonlinearly to have a stabilizing effect. For a 2 dimensional system, the eigenvalues are

$$\lambda = \frac{Tr \pm \sqrt{Tr^2 - 4D}}{2} \tag{30}$$

where Tr is the trace and D is the determinant.

For our model in Section 2, the Jacobian, $J(t, h_\mu)$, at the (nontrivial) fixed point is:

$$J(t, h_\mu) = \begin{pmatrix} r\left(1 - \frac{2S_\infty(t,h_\mu)}{K}\right) - \frac{r\phi\alpha S_\infty(t,h_\mu)}{\delta A(t)}\left(1 - \frac{S_\infty(t,h_\mu)}{K}\right) & -\frac{\delta}{\phi} \\ \phi\frac{r\phi\alpha S_\infty(t,h_\mu)}{\delta A(t)}\left(1 - \frac{S_\infty(t,h_\mu)}{K}\right) & 0 \end{pmatrix} \tag{31}$$

so that

$$Tr \equiv \text{Trace}(t, h_\mu) = r\left(1 - \frac{2S_\infty(t, h_\mu)}{K}\right) - \frac{r\phi\alpha S_\infty(t, h_\mu)}{\delta A(t)}\left(1 - \frac{S_\infty(t, h_\mu)}{K}\right) \tag{32}$$

and

$$D \equiv \text{Det}(t, h_\mu) = \frac{r\phi\alpha S_\infty(t, h_\mu)}{A(t)}\left(1 - \frac{S_\infty(t, h_\mu)}{K}\right) \tag{33}$$

where

$$A(t) \equiv \frac{1 + t(1 - \beta)}{\beta}. \tag{34}$$

The relative values of Tr and D determine the local behavior of the model (Boyce and DiPrima, 1977). Of interest is the sign of the real parts of the eigenvalues (damping) and whether they have nonzero imaginary parts (oscillations). Notice from expression (33) that the $D > 0$ always, so that the equilibrium will be either a source or a sink. After substituting (12) for $S_\infty(t, h_\mu)$ in expression (32) and further manipulation, we arrive at

$$Tr(t, h_\mu) = r\left\{-\frac{S_\infty(t, h_\mu)}{K} + \frac{h_\mu \phi}{\delta}\left(1 - \frac{1}{A(t)}\right)\left(1 - \frac{S_\infty(t, h_\mu)}{K}\right)\right\}. \tag{35}$$

Note that $Tr(0, 0) < 0$ so that with no tax and no subsistence (the BT model) the equilibrium point is asymptotically stable. From this we can compute

$$\frac{\partial Tr}{\partial h_\mu} = r\left\{-\frac{\partial S_\infty(t, h_\mu)}{\partial h_\mu}\left(\frac{1}{K}\right) + \frac{\phi}{\delta}\left(1 - \frac{1}{A(t)}\right)\left(1 - \left(h_\mu \frac{\partial S_\infty(t, h_\mu)}{\partial h_\mu} + \frac{S_\infty(t, h_\mu)}{K}\right)\right)\right\} \tag{36}$$

which since $\frac{\partial S_\infty(t,h_\mu)}{\partial h_\mu} < 0$, $A(t) > 1$, and $\frac{S_\infty(t,h_\mu)}{K} < 1$, is always positive. Thus, increasing h_μ increases Tr and thus has a destabilizing effect. The partial derivative of Tr with respect to t is

$$\frac{\partial Tr}{\partial t} = \frac{r(1-\beta)}{\beta \alpha K}\left[\frac{-\delta}{\phi} + \frac{\phi h_\mu}{\delta}\left(h_\mu + \frac{(\alpha K - h_\mu)}{A^2(t)}\right)\right]. \tag{37}$$

If $h_\mu = 0$, expression (37) implies that

$$\frac{\partial Tr}{\partial t} = \frac{-\delta r(1-\beta)}{\beta \alpha K \phi} < 0. \tag{38}$$

Thus increasing t reduces the trace and tends to stabilize the model. However, increasing h_μ weakens the ability of the tax to stabilize the model. Specifically, for $0 < h_\mu < \dfrac{\delta}{\phi}$, if increasing tax is to stabilize the model, the term in square brackets in (37) must be negative. This is true if

$$t > \frac{\beta\left(\frac{\alpha K - h_\mu}{\left(\frac{\delta}{\phi}\right)^2 \frac{1}{h_\mu} - h_\mu}\right)^{\frac{1}{2}} - 1}{1-\beta}. \tag{39}$$

As h_μ approaches $\dfrac{\delta}{\phi}$, the right hand side of expression (39) becomes arbitrarily large, thus the tax required to have a local stabilizing effect can become arbitrarily large. The value of the tax required to have a local stabilizing effect rises nonlinearly with the subsistence requirement. Numerical experiments suggest that the global behavior of the model is roughly consistent with this local result.

The change in D with respect to t is

$$\frac{\partial D}{\partial t} = \frac{r\phi(1-\beta)}{A(t)\beta K}\left[\left(\frac{\delta}{\phi} - h_\mu\right)(K - 2S_\infty(t, h_\mu)) - \frac{1}{A(t)}(K - S_\infty(t, h_\mu))\right]. \tag{40}$$

The tax can have a stabilizing effect through reducing the determinant if the expression in the square brackets is negative. Define

$$\underline{B}(h_\mu) = \frac{\alpha K + 1 - h_\mu - \sqrt{(\alpha K + 1 - h_\mu)^2 - 8(\alpha K - h_\mu)}}{4\left(\frac{\delta}{\phi} - h_\mu\right)}, \tag{41}$$

and

$$\overline{B}(h_\mu) = \frac{\alpha K + 1 - h_\mu + \sqrt{(\alpha K + 1 - h_\mu)^2 - 8(\alpha K - h_\mu)}}{4\left(\frac{\delta}{\phi} - h_\mu\right)}. \tag{42}$$

Then the tax will have a downward effect on D if

$$t < \frac{\beta \underline{B}(h_\mu) - 1}{1 - \beta} \quad \text{or} \quad t > \frac{\beta \overline{B}(h_\mu) - 1}{1 - \beta}. \tag{43}$$

For the BT parameter set and h_μ equal to 0 and 0.015 as in the examples, this condition is

$$t < 2.18 \quad \text{or} \quad t > 9.44, \quad \text{and} \quad t < 12.20 \quad \text{or} \quad t > 43.06, \tag{44}$$

respectively. In our numerical experiments, the upper bounds on the tax of 1 and 3 for the respective cases are well within the bounds where the tax has a stabilizing effect.

Finally,

$$\frac{\partial D}{\partial h_\mu} = \frac{r\phi}{A(t)} \frac{\partial a}{\partial h_\mu} \left(1 - \frac{2S_\infty(t, h_\mu)}{K}\right). \tag{45}$$

Increasing h_μ will increase or decrease D depending on the sign of the expression in parenthesis. With no tax, the right hand side of (45) is negative, thus increasing h_μ can reduce D. However, the imaginary parts of the eigenvalues depend on both the trace and the determinant. Increasing h_μ increases the trace (makes it less negative) and thus reduces its absolute value which can more than offset the associated reduction in the determinant.

In summary, expressions (36) and (37) indicate that the subsistence requirement and tax have opposing effects on local stability. A nonzero subsistence requirement increases the trace and simultaneously weakens the effect a positive tax can have in countering this effect. Taxes in the range relevant to the model reduce the determinant while h_μ may increase or decrease the the determinant depending on the tax. The effect of the subsistence requirement on the trace is probably more important than its effect on the determinant. Although these results are local, numerical experiments suggest the global behavior of the model is consistent with them.

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