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More is Less: the Tax Effects of Ignoring Flow Externalities

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“More is Less”:
The Tax Effects of Ignoring Flow Externalities

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Abstract

Using a model of nonlinear decay of the stock pollutant, and starting from the same initial conditions, the paper shows that an optimal tax that corrects for both stock and flow externalities may result in a lower tax, fewer cumulative emissions (less decay) and higher output at the steady state than a corrective tax that ignores the flow externality. The “more is less” result emphasizes that setting a corrective tax that ignores the flow externality, or imposing a corrective tax at too low a level where there exists only a stock externality, may affect both transitory and steady state output, tax payments and cumulative emissions. The result has important policy implications for decision makers setting optimal corrective taxes and targeted emission limits whenever stock externalities exist.

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1 Introduction

Pigouvian taxes are widely used to mitigate the externalities that exist in production. Such taxes are favored when there exist many polluters and have been widely applied in Europe to address a large number of environmental externalities (Andersen 1994; OECD 1992). Worldwide, corrective taxes generate billions of dollars annually for governments. In theory, corrective charges should equal the costs imposed on society of a defined level of pollution. In practice, charges are often based on the notion that the current level or flow of pollution (such as the amount of phosphorous discharged into a river) represents the externality imposed on society. However, many pollutants impose both stock and flow externalities such that current and cumulative discharges affect the non-monetary variables of utility or production functions (Baumol and Oates 1988).

Much of the literature on Pigouvian taxes focuses either on flow externalities or stock externalities (Sinclair 1992; Ulph and Ulph 1994; Farzin and Tahvonen 1996; Hoel and Kverndokk 1996; Wirl and Dockner 1996). Where both stock and flow externalities exist and are explicitly considered (Sandal and Steinshamn, 1998), the optimal corrective tax will exceed the shadow cost of pollution for a given level of pollution (Wirl, 1994; Farzin 1996). Thus, we would expect that a corrective tax which ignores flow externalities, when they are present, to result in more pollution and a lower tax payment. By contrast, our paper shows that different taxes result in different time paths and that the specification of the decay function of the stock of pollution has an important affect on both the transitory and steady-state production, taxes and emissions. Moreover, the possibility exists that the optimal tax that corrects for both flow and stock externalities may result in a steady state with higher emissions, but lower accumulated pollution and lower tax payments than a tax that only corrects for the stock externality. A case of more is less. A similar result may also occur whenever there is a propensity to set the corrective tax at too low a level in the presence of a stock externality, whether or not flow externalities exist.

The "more is less" result will only occur if the decay of the pollution stock is a non-monotone function of accumulated pollution. By contrast, most of the economic literature on pollution assume linear decay of the stock of pollution (Sinclair 1992, Ulph and Ulph 1994, Farzin 1996, Hoel and Kverndokk 1996). Although assuming linear decay makes the dynamic modeling of stock pollutants more tractable, only few natural processes, such as radioactive decay, can be characterized in such a manner.[1] The great majority of environmental processes are likely to be nonlinear in the decay of the stock pollutant, and non-linear decay is the only realistic representation of biogeochemical cycles that include both positive and negative feedbacks (Joos et al., 1996).

Several authors, including Pezzey (1996) and Toman and Withagen (2000), who have explored the issues of pollution accumulation and assimilation present the possibility that decay of the stock pollutant may, at different levels of the stock, be initially increasing and then decreasing. For instance, increased concentrations of greenhouse gas (GHG) emissions may initially increase the assimilative capacity of the environment to uptake carbon due to carbon fertilization, but further increases in GHG emissions that lead to even higher GHG concentrations and higher surface temperatures may eventually lead to plant die offs that could ultimately reduce carbon uptake. Climate change also serves as a useful illustration of the potential of both global stock externalities (increased probability of climate change due to increased atmospheric concentrations of GHGs) and local flow externalities (reduced air quality from simultaneous emission of joint pollutants, such as volatile organic compounds) to arise from the same pollution source. To further explore these issues we present an example of the "more is less" result using a stylistic model of climate change, but first we present a dynamic model that includes both stock and flow externalities and derive theoretical results.

2 A Model of Production with Stock and Flow Externalities

We present a dynamic model that incorporates both stock and flow externalities in production. We assume that the objective is to maximize welfare (W), defined as the discounted present value of social utility, and which is a function of the stock of pollution (a) and the production of a good (x). Utility is defined as the sum of consumer and producer surplus, adjusted for flow externalities, less the affect of the stock externality defined by $D(a)$, where D is increasing in a, i.e.,

$$U(a,x) = \int_0^x [P(z) - C^s(z)]\, dz - D(a) \equiv \Pi(x) - D(a)$$

where x is quantity produced, P is the inverse demand, and C^s is the social marginal cost of production.[2]

A dynamic constraint governs the change in the stock of pollution, $\dot{a}$, and is determined by the instantaneous increase in pollution γx, which is proportional to production x by a factor γ, and the decay of the stock of pollution, $d(a)$, which might be increasing or decreasing in a, depending upon the level of a.[3] The flow externality, τ_f, is the instantaneous externality which arises at the time the pollution is emitted. By definition, the marginal cost associated with the flow externality plus the private marginal cost of production, $C^p(x)$, equals the social marginal cost of production $C^s(x)$, thus $\tau_f \equiv C^s - C^p$.

Our analysis examines the case without discounting because, if the counterintuitive "more is less" occurs at a zero discount rate, it will also hold true with a positive discount rate. Indeed, for the right profile we can always find a discount rate sufficiently high for the counterintuitive case to occur.

The dynamic problem is to maximize welfare defined as:

$$W = \int_0^\infty U(v(t))dt, \quad v(t) \equiv (a(t), x(t)) \in R \times X$$

where $X = [0, B]$ is a given bounded interval, and W is maximized subject to the following dynamic constraint and initial condition:

$$\dot{a}(t) = x(t) - d(a(t)), \quad a(0) = a_0, \tag{1}$$

where a_0 is the initial level of the stock pollutant.

We assume that a long-term steady state is desirable and thus solve for processes $v \in V$, where

$$V = V[R \times X], \quad \lim_{t \to \infty} a(t) = a^*. \tag{2}$$

In this case, V represents the set of $(a(t), x(t))$ such that a is continuously differentiable, and x is continuous and piecewise differentiable.

To develop the model further, we define the following set of admissible processes in Definitions 1 and 2 in the appendix. Under these definitions, the optimal control problem is to determine the feedback rule $x(a)$ that solves $v \in A\max W$. If W is not bounded from above, we provide an alternative definition of optimality called Catching Up (CU) or Overtaking (OT) optimality that is used to distinguish possible unbounded value functions. CU or OT optimality formalizes the idea that, without discounting, one dollar a day forever is less than two dollars a day forever although both sums are infinite.

Our model may represent either a single firm in a competitive world, or an entire competitive industry. In the absence of intervention, market equilibrium requires that $P = C^p$, where private marginal cost is strictly increasing in x, and an equilibrium price and quantity of x can be defined for any level a. If a is a constant, the solution collapses to the standard result of static models with flow externalities, namely that welfare is maximized when $P = C^s$.

To demonstrate the counterintuitive or "more is less" result we first specify key variables defined formally in Definitions 3 and 4 of the Appendix. In particular, we introduce a sustainable utility rate, S, defined as $S(a) = \Pi(d(a)) - D(a)$ and a total utility rate, K, defined as $K(a, x) = \Pi(x) - D(a) + \Pi'(x) \cdot [d(a) - x]$. The total utility rate, $K(a, x)$, has the same value as the Hamiltonian along an optimal path but the costate is replaced by $-\Pi'(x)$. These definitions are required to state and prove Theorem 1, given in the appendix, that shows that the long-term steady-state $(a^*, x^* = d(a^*))$ is a saddle point for $K(a, x)$ and is determined by $S'(a^*) = 0$.

Using the above result, we can formally state Proposition 1, and which is proved in the appendix, to characterize the feedback solution and optimal production path to our dynamic problem.[4]

Proposition 1 *The optimal steady state is to the left of* $\hat{a} = \max d(a)$. *The separatrix part of the feedback solution is strictly decreasing to the right of* $\hat{a}$. *If the total utility rate,* $K(a, x)$, *is quasi-concave on the set L defined by* $\{(a, x) : a > a^*, 0 < x < x^*, x < d(a)\}$ *then the separatrix part of the feedback solution is concave below* $d(a)$.

To derive the counterintuitive or "more is less" result using a system of taxes we must next write the Hamiltonian for the dynamic optimization problem

$$H(a, x, \lambda) = U(a, x) + \lambda[x - d(a)] \tag{3}$$

where λ is the costate variable for the stock of pollution, a. The variable λ also represents the shadow cost and is given by

$$-\lambda = U'(x).$$

The corrective tax, ignoring the flow externality, is thus

$$\sigma(x) = -\lambda = \Pi'(x)$$

on the optimal path $x(a)$. Alternatively, we can rewrite the σ tax as

$$\sigma = P - C^s = P - C^p - (C^s - C^p) = \tau - \tau_f.$$

In other words, and as previously shown by Wirl (1994) and Farzin (1996), the optimal corrective tax exceeds, in the presence of stock and flow externalities, the shadow cost of pollution. Thus, the tax which corrects for both flow and stock externalities, defined as τ, is the difference between the consumer price and producer price, $\tau(x) = P(x) - C^p(x)$, and can be calculated at both the firm and industry level. As a result,

$$\tau(x) = \sigma(x) + \tau_f(x), \tag{4}$$

and which holds true at both the optimal steady state and on the path to the steady state. We can now compare σ and τ corrective taxes and their affect on production, emissions and pollution to show the counterinuitive result of "more is less" if flow externalities are ignored in the determination of an optimal set of emissions taxes.

3 The Effects of Ignoring Flow Externalities

At any given pollution level, σ must be lower than τ because σ ignores the flow externality. As a result, we would expect that a σ-tax would be associated with more pollution, more production and a lower tax payment at all times. In fact, because emissions differ with the τ and σ taxes, the time path of pollution will also differ and, thus, the possibility exists that σ may lead to a steady state with more aggregate pollution but, surprisingly, less production and a higher tax level.

To derive the time path of a with the σ-tax, and which ignores the flow externality, we must first derive the feedback rule for production that corresponds to σ. If we define the stock-externality part of the optimal tax as $\sigma(x(a))$, for any given a-level, we can obtain a new market equilibrium, y, defined by

$$\tau(y) = \sigma(x(a)). \tag{5}$$

This relationship is illustrated in Figure 1. Expression (5) is in terms of only y and a, and can be used to solve for y as a function of a. Thus we obtain new feedback rule, $y(a) > x(a)$, and the time path of a can be given by

$$\dot{a} = y(a) - d(a).$$

This result is stated formally in the following proposition, and proved in the appendix.

Proposition 2 *The σ-tax yields a production that is always higher than the optimal, x, for a given level of the stock pollution, except for very high pollution levels where both taxes may choke the production.*

The feedback rule based on σ leads to a different steady state with a higher a. This steady state, $a^{\#}$, can be found by substituting $y = d(a)$ into (5):

$$\tau(d(a)) = \sigma(x(a))$$

which eventually yields $a^{\#} > a^*$. At the steady state, $y^{\#} = d(a^{\#})$, which can be compared with $x^* = d(a^*)$. The counterintuitive or "more is less" result, stated formally in definition

5 of the appendix, is a steady state where $y(a^{\#}) < x(a^*)$, and thus $\sigma > \tau$, In other words, the tax that ignores the flow externality ultimately results in lower production and higher tax than the optimal corrective tax. The counterintuitive or "more is less" result, however, can only occur if $d(a)$ is non-linear and non-monotone, as stated in Proposition 3.

Proposition 3 *If the decay-function, $d(a)$, is monotonically increasing, the σ tax will always lead to a steady state with higher production and lower tax than the τ tax.*

The intuition behind the counterintuitive result is that, starting from the same initial situation, a tax that ignores the flow externality results in higher production and higher emissions over time than if the flow externality were corrected for in an optimal tax. Thus, over time, the σ tax results in a higher level of cumulative pollution or emissions. At the steady state, however, current emissions must equal the rate of decay of the emissions. Thus if the rate of decay decreases with the level of pollution, as we might expect with many natural processes, then for a sufficiently high stock pollution level the lower rate of decay must be compensated by a higher tax at the steady state. Thus, the σ tax at the steady state that only corrects for the stock externality may be higher than the τ tax that corrects for both the flow and stock externality. Moreover, at the steady state the τ tax that corrects for both externalities will result in a higher output and lower cumulative emissions (less decay) than the σ tax that ignores the flow externality.

For completeness, we can specify the necessary and sufficient conditions for the counter-intuitive case to occur. First, we define the steady state corresponding to τ as (a^*, x^*) and a^{**} as the solution of $x^* = d(a)$ for $a > a^*$ if it exists, and $a^{**} = \infty$ otherwise. In this case, a^{**} is a knife-edge case because it corresponds to the level of accumulated pollution at which production (and taxes) are exactly the same, whether we are to the left or right of the maximum of the decay function. This point is shown in Figure 2.

We can now define the inequality at the steady state between the σ tax that ignores the flow externality and the τ tax that corrects for both the flow and stock externalities, given that the σ tax has a higher stock of pollution than the τ tax at the steady state.

Proposition 4 *If $d(a)$ is quasi-concave, (a^*, x^*) is the steady state corresponding to τ and $(a^{\#}, y^{\#})$ is the steady state corresponding to σ, then $\sigma(y^{\#}) > \tau(x^*)$ iff $a^{\#} > a^{**}$.*

Figure 2 illustrates the two cases with two different decay functions—a case where the counterintuitive or "more is less" result occurs (the left-hand panel) and a case in which it does not (the right-hand panel). When comparing the cases in Figure 2, note that in a-space the shadow price is the same whether we use x or y. By contrast, in time space the development of the shadow price will differ for the two policies.

From the left-hand panel in Figure 2 we can verify the effects of discounting. Discounting implies a less conservative policy and will shift the optimal paths to the right. Thus if the counterintuitive case holds for a given discount rate, such as zero, it must also hold true for a higher discount rate. In fact, if $a^{**} < \infty$, it will always be possible to find a discount rate which is sufficiently high for the counterintuitive case to occur.

We are now able to derive the necessary and sufficient conditions for the counterintuitive or "more is less" result to occur. First, we note that the values a^* and a^{**} can be found without solving the complete problem, or solving any differential equations. Thus, we can assume the following quantities are known:

$$\begin{aligned} b &= \max d(a),\ J = [0, b] \subset X,\ \ \Delta D^* = D(a^*) - D(a^{**}) \\ \underline{M} &= X \in J\min\left[-\Pi''(x)\right], \quad \overline{M} = X \in J\max\left[-\Pi''(x)\right] \end{aligned} \tag{6}$$

Given the above, the necessary condition for the "more is less" result is stated by Proposition 5, and proved in the appendix.

Proposition 5 $\Delta D^* \geq \tau_f(y^\#)\left[y^\# - x(a^\#)\right]$ *is a necessary condition for the counterintuitive case to occur for cases covered by Theorem 1 (see Appendix).*

The economic interpretation of Proposition 5 is that the left-hand side, ΔD^*, is interpreted as the difference in damage associated with being at a^{**} instead of a^*. The right-hand side of the inequality is interpreted as the tax payment associated with the flow externality when staying at $a^\#$. The intuition behind this proposition is that if the damage difference associated with the knife-edge case is less than the initial flow-tax when jumping from the steady state $a^\#$ to the optimal path $x(a^\#)$, then the counterintuitive case may occur.

The sufficient condition for the counterintuitive result to occur is provided by Proposition 6, and proved in the appendix.

Proposition 6 *If there exists a* z *such that* $0 < z < x^*$ *and if* $\underline{\tau_F} = x \in N\min\left[\tau_f(x)\right]$ *where* $N = [z, x^*] \subseteq J \subseteq X$, *then* $2\overline{M}^2\Delta D^* < \underline{M}\tau_f^2$ *is sufficient for the counterintuitive case to occur for cases covered by Theorem 1 (see Appendix).*

Proposition 6 states that if the damage difference associated with the knife-edge case is less than a given value, than the counterintuitive case must occur. To illustrate the potential implications of the result, and to show how to derive the optimal corrective tax in the presence of stock and flow externalities, we present an example from the climate change literature.

4 "More is Less": A Climate Change Example

The possibility that ignoring the flow externality may eventually reduce production and increase tax payments can be illustrated using a numerical simulation from the climate change literature. The example shows that the "more is less' result is not only a theoretical possibility, but may also occur in practice.

The parameters used in the model defined below are stylized, but are derived from the literature on climate change and are used to illustrate the theoretical results. The model specifies linear demand and linear marginal cost functions and a quadratic damage function. We assume current emissions of CO_2 are 22 giga tonnes (Gt-CO_2), which is the private market equilibrium in our model when marginal costs are normalized to one, and production is measured as emissions. The cumulative anthropogenic emissions of CO_2, less decay, are assumed to be 625 Gt-CO_2 above the pre-industrial level. In other words, a is normalized to the pre-industrial level such that $d(0) = D(a) = 0$. The pre-industrial level is a natural equilibrium without emissions, and at this level there is no damage associated with the cumulative carbon emissions. The climate change model is given below:

$$
\begin{aligned}
P(x) &= 15 - 0.64 \cdot x, \\
C^p(x) &= 1 + 0.05 \cdot x, \\
C^s(x) &= 1 + 0.12 \cdot x, \\
\gamma x &= x, \qquad D(a) = 0.000005 \cdot a^2, \\
d(a) &= \max(0, 21 \cdot \exp\left(-(a - 600)^2 \cdot 0.512 \times 10^{-5}\right) - 3.32)
\end{aligned}
$$

The aggregate decay function, $d(a)$, for atmospheric carbon is the subject of intense study and debate. We simply use a non-monotone function to illustrate the relative effects of σ and τ taxes. For illustrative purposes, we assume that today's decay is just above the maximum decay and which implies the effects of global warming already have started.

With the exception of Wirl (1994) and Farzin (1996), existing models (such as Sinclair 1992, 1994; Ulph and Ulph 1994) do not explicity consider flow externalities as a function of the current level of emissions. Of these models, only Farzin and Tahnoven (1996) examine the effect of different rates of uptake of carbon in the atmosphere on the time paths of corrective taxes, but they ignore the flow externalities associated with GHG emissions. A feature of their model, shared with ours, is that they explicitly consider the affect of the decay function on their results. In their case, they develop their results using a decay function that is analogous to non-autonomous linear decay. Assuming multiple carbon stocks, each with different but constant rates of decay, they find that for carbon levels in excess of pre-industrial levels, the corrective tax may be decreasing or U-shaped over time. They conclude that the optimal carbon tax is "$\cdots$ sensitive to the submodel describing the accumulation of atmospheric CO_2." (Farzin and Tahvonen 1996, p. 533).

The specification of demand and supply functions are based on some assumptions about elasticities. If current production is 22, this implies a demand elasticity of -0.16 and a supply elasticity of 1.9. This is in accordance with Jorgenson and Wilcoxen (1990) who state that short-term demand elasticities are about one tenth of the long-term, and probably lie between -0.1 and -0.2. As this model is dynamic and adaptive, only short-term elasticities are of interest. The supply elasticity is in accordance with Burniaux et al. (1992) who state that supply elasticities from countries within OPEC vary between one and three.

The parameters above imply $\Pi(x) = 14 \cdot x - 0.38 \cdot x^2$, and, hence, the corrective taxes are

$$
\begin{aligned}
\tau(x) &= 14 - 0.69 \cdot x, \\
\sigma(x) &= 14 - 0.76 \cdot x, \\
\tau_f &= 0.07 \cdot x.
\end{aligned}
$$

Under this numerical specification, we obtain the following quantities:

$$
\begin{array}{lll}
b = 17.68 & a^{**} = 638.5 & \Delta D^* = 0.462 \\
\underline{M} = 0.76 & \overline{M} = 0.76 &
\end{array}
$$

Thus, the steady states resulting from $\tau(x)$ versus $\sigma(x)$ are

$$\begin{array}{lll} a^* = 561.5 & x^* = 17.52 & \tau(x^*) = 1.91, \\ a^\# = 669.3 & y^\# = 17.17 & \sigma(y^\#) = 2.15. \end{array}$$

The counterintuitive result corresponds with the sufficiency criterion given in Proposition 6, namely, $2\overline{M}^2 \Delta D^* < \underline{M}\tau_f^2$ is fulfilled in the region $11.97 < x < x^*$. In other words, starting at the same initial condition, a carbon tax that ignores flow externalities will initially be lower than a tax that accounts for both the stock and flow externalities. However, at the steady state, the optimal carbon tax rate (τ) will be lower, the output will be higher and the cumulative carbon emissions (less decay) will be lower than the σ tax that ignores the flow externalities. Thus, by incorporating non-monotone decay in the pollution stock in a model of GHG emissions, which better represents the actual physical processes, we show the importance of accounting for both stock and flow externalities. Moreover, in contrast to the accepted view that flow externalities affect only transient consumption (Wirl 1994), we find that a failure to consider flow externalities in a model of GHG emissions may affect both the time paths and steady states of production, emissions, and taxes.

The example illustrates the large potential differences that may arise in steady-state cumulative emissions depending upon whether the corrective tax does, or does not, ignore the flow externalities associated GHG emissions. For instance, at the steady state, cumulative carbon emissions (less decay) are almost 20 percent higher when flow externalities are ignored than when both flow and stock externalities are corrected for in a tax. The "more is less" result potentially has significant policy implications for the setting of targets under the United Nations Framework Convention on Climate Change, and indeed for all regulators wishing to address stock externalities. In particular, if corrective taxes and emission targets are set at too low a level, or if decision-makers ignore the transitory environmental costs associated with GHG emissions, then such corrective policies could result in lower world GDP, higher concentrations of GHGs and higher tax payments than if optimal taxes that consider both the stock and flow externalities were applied.

5 Concluding Remarks

Using a dynamic model with both flow and stock externalities and non-monotone decay of the stock pollutant, we prove the possibility that, at the steady state, an optimal corrective tax may result in less total emissions, but also lower tax payments, than a corrective tax which ignores flow externalities. A similar result also applies in the absence of a flow externality. If a tax to correct a stock externality is set at too low a level then, at the steady state, such a tax may result in both higher cumulative emissions (less decay) and higher tax payments than if the optimal corrective tax were applied from the initial period.

The intuition for the counterintuitive or "more is less" result is that ignoring the flow externality, or setting too low corrective tax for a stock externality, results in initially higher production and higher emissions. Over time, this leads to a higher cumulative level of the stock pollutant and eventually, depending the nature of decay of the stock pollutant, the tax payments may become higher than if the optimal tax was applied from the beginning period.

In order to arrive at a steady state, emissions of the stock pollutant must equal the decay of the stock pollutant, but if the decay of the stock of pollution decreases with the level of the stock then, for sufficiently high pollution levels, the reduced decay associated with a higher level of the pollution stock must be compensated for by lower production. Thus, at the steady state, the corrective tax that ignores the flow externality, or is set at too low a level, may result in higher tax payments than the optimal corrective tax and will also have a higher steady-state level of the stock of pollution and a lower level of output.

The "more is less" result provides important insights for corrective tax policies where there exist both stock and flow externalities. The paper suggests that decision makers that fail to consider flow externalities, or set their emission target reductions or corrective taxes at too low a level in the presence of stock externalities, risk imposing significant transitory and permanent costs in terms of higher future tax payments, higher steady-state levels of the stock pollution and lower future output.

Appendix

Definition 1 *The set of admissible processes, A, is defined as all processes that satisfy (1) and (2).*

Definition 2 *The usual assumptions about the input functions are:*
1. Π, D and d are C^2-functions in their arguments whenever the arguments are positive.
2. $D : R \to R_+$ is strictly increasing and convex for positive arguments, and identically equal to zero for non-positive arguments. The state $a = 0$ is, by definition, a steady state without emissions and can be interpreted as the preindustrial level, that is $d(0) = 0$ and $D(0) = 0$. No damage is associated with the preindustrial level.
3. $\Pi : X \to R_+$ is strictly increasing and strictly concave.
4. $d : R \to R_+$ is strictly increasing for $0 < a < \hat{a}$ and strictly decreasing for $a > \hat{a}$. Further $a \to 0^+ \lim \left[\Pi'(d(a)) \cdot d'(a) - D'(a)\right] > 0$.

Definition 3 *Sustainable utility rate, S, is defined as the utility obtained when a is fixed at certain level and given by $S(a) = \Pi(d(a)) - D(a)$*

Definition 4 *Total utility rate, K, is defined as $K(a, x) = \Pi(x) - D(a) + \Pi'(x) \cdot [d(a) - x]$.*

Definition 5 *The counterintuitive case: The case where σ leads to a steady state where $y(a^{\#}) < x(a^*)$, and hence $\sigma > \tau$, that is a steady state with lower production and higher tax, is called the counterintuitive ("more is less") case.*

Statement and Proof of Theorem ??

Theorem 1 *The Catching–Up (CU) optimal production for the problem $v \in ACU - opt\, W$ where $\frac{\partial^2}{\partial a^2} K(a, x) < 0$ on $R \times X$, is given by*

$$x(a) = \max(0, z(a)) \text{ where } K(a, z(a)) = \max S(a) = S(a^*). \tag{7}$$

The long-term steady-state $(a^*, x^* = d(a^*))$ is a saddle point for $K(a, x)$ and is determined by $S'(a^*) = 0$.

The feedback solution given by (7) satisfies all the conditions in the Mangasarin sufficiency theorem for the Catching-Up optimality as stated in theorem 3.13 in Seierstad and Sydsæther (1987). The usual assumptions in Definition 2 ensure the regularity. The Hamiltonian for our problem, $H(a, x, \lambda) = \Pi(x) - D(a) + \lambda \cdot (x - d(a))$, is conserved since time does not enter the problem explicitly, i.e., $H(a, x, \lambda) = K_0$ (constant) In the equilibrium we have $K_0 = H(a, d(a), \lambda) = S(a)$ and K_0 must be chosen as the global maximum of the sustainable utility rate $S(a)$. The assumptions done are sufficient to ensure that $S(a)$ atains its global maximum in a critical point a^*. This can be seen from the usual assumption which states that $S'(0^+) > 0$ and $S'(a) < 0$ for $a \geq \hat{a} = \arg\max d(a)$ and the implied regularity of the sustainable utility rate. Hence a^* is to the left of $\hat{a}$. The strict concavity of Π and $\partial^2 K(a, x)/\partial a^2 < 0$ is sufficient to ensure that $H(\cdot)$ is strictly concave in (a, x). Hence the CU-optimal problem has the unique solution given by the feedback expression implicitly defined in (7). This solution is constructed by the separatrix solution going into the equilibrium point (having $\lambda = -\Pi'(x(a)) < 0$) and the corner solution $x = 0$ when the separatrix yields a negative production. The optimal Hamiltonian, $H(\cdot)$, is equal in value to the total utility rate $K(\cdot)$ on the separatrix part of the feedback solution.

The new condition imposed by the Catching-Up optimality is given by $t \to \infty \lim\inf \lambda(t) \cdot [A(t) - a(t)] \geq 0$. The optimal policy is implies $a = a(t)$ and any other admissible policy implies $a = A(t)$. This CU-condition is trivial and the limit is zero due to the fact that the all admissible solutions end up in a^* and $|\lambda| = |\Pi'(x)|$ is bounded.

In our case we can show that the feedback solution is stronger than CU optimal. Indeed we have OT optimality: The OT-criterion is fulfilled if $\exists\, t_0$ such that $\Lambda(t) \geq 0 \;\forall t \geq t_0$ where

$$\Lambda(t) \equiv \int_0^t [\Pi(x) - D(a)]\, ds - \int_0^t [\Pi(y) - D(A)]\, ds.$$

In this case, (a, x) represents the separatrix solution and (A, y) represents other admissible solutions. These must satisfy $K(A, y) = K_0 < S^* = \max S(a)$ in order to yield a steady state. We have $K_0 = U(A, y) + \Pi'(y) \cdot (d(A) - y) = U(A, y) - \Pi'(y(A)\, \dot{A}$:

$$\int_0^t [\Pi(y) - D(A)]\, ds = K_0 \cdot t + \int_{a_0}^{A} \Pi'(y(s)) ds.$$

Inserted, this yields

$$\begin{aligned} \Lambda(t) &= (S^* - K_0) \cdot t - \int_{a_0}^{A} \Pi'(y(s)) ds + \int_{a_0}^{a} \Pi'(x(s)) ds \\ &\geq (S^* - K_0) \cdot t - \Psi \geq 0, \quad t \geq t_0. \end{aligned}$$

where we can choose $\Psi = \int_{a_0}^{a^*} \{\Pi'(x(s)) + \Pi'(y(s))\} ds$ and t_0 sufficiently large.

Proof of Proposition 1

Theorem 1 implies $S'(a^*) = \Pi'(d(a^*)) d'(a^*) - D'(a^*) = 0$, which implies that $d'(a^*) > 0$. Therefore the feedback intersects with $d(a)$ to the left of its maximum. Differentiating $K(a, x) = S^*$ yields $\Pi'' \cdot (d - x) \cdot x' = D' - \Pi' \cdot d'$. To the right of $\hat{b} = \max d$ we have $d - x > 0$ and $d' < 0$ and the monotonicity of x' follows now from the concavity of Π.

Concavity of the separatrix solution can be shown by differentiating $K(a, x) = S^*$ twice. This yields

$$x'' \frac{\partial K}{\partial x} = -\left[\frac{\partial^2 K}{\partial a^2} + 2x' \frac{\partial^2 K}{\partial a \partial x} + (x')^2 \frac{\partial^2 K}{\partial x^2} \right]$$

Quasi-concavity implies that the right-hand side is non-negative and $\frac{\partial K}{\partial x} < 0$ on L.

Proof of Proposition 2

As $\tau(y) = \sigma(y) + \tau_f(y) = \sigma(x) > \sigma(y)$, and as $\sigma = U_x$ is strictly decreasing, the proposition is proved for all pollution levels associated with positive production, i.e. $a \leq \hat{a}$ and $K(\hat{a}, 0) = S^*$.

Proof of Proposition 3

Steady states are, by definition, intersections between the $d(a)$-curve and the production feedback-paths $x(a)$ and $y(a)$. Since $y(a)$ is not lower than $x(a)$, the intersection between $y(a)$ and an increasing $d(a)$-curve will always imply that $y(a)$ is higher than $x(a)$ in steady state.

Proof of Proposition 4

The proof of this proposition can be seen directly from Figure 2. This case occurs if and only if $x(a)$ intersects with d at a higher value than $y(a)$. Quasi-concavity of $d(a)$ implies that this occurs if and only if $a^\# > a^{**}$.

Proof of Proposition 5

The steady state resulting from a σ-tax is denoted $(a^\#, y^\#)$. As S is concave, $S(a^{**}) \geq S(a^\#)$ is a necessary and sufficient condition for the counterintuitive case to occur (see Proposition 4). The concavity of the sustainable utility rate is a direct consequence of the concavity of Π and $\frac{\partial^2}{\partial a^2} K(a, x) < 0$.

It is easily verified that $K(a, x) = S^*$ is equivalent to

$$S^* - S(a) = \int_x^{d(a)} [\Pi'(x) - \Pi'(s)]\, ds.$$

Thus,

$$S(a^{**}) - S(a^\#) = \int_{x(a^\#)}^{y^\#} \left[\Pi'(x(a^\#)) - \Pi'(s) \right] ds - \Delta D^*. \tag{8}$$

From this result, it follows that

$$\Delta D^* \geq \int_{x(a^\#)}^{y^\#} \left[\Pi'(x(a^\#)) - \Pi'(s) \right] ds$$

is a necessary and sufficient condition for the counterintuitive case not to occur. The lower limit of integration is less than the upper limit according to Proposition 3, and Π' is decreasing. Thus,

$$\Delta D^* \geq \int_{x(a^\#)}^{y^\#} \left[\Pi'(x(a^\#)) - \Pi'(y^\#) \right] ds = \tau_f(y^\#) \cdot \left[y^\# - x(a^\#) \right]$$

is necessary for the counterintuitive case to occur.

Proof of Proposition 6

Let δ denote the left-hand side of equation (8). It will be shown that this proposition implies $\delta > 0$. Recalling $\sigma(x) = \Pi'(x)$ equations (4) and (8) imply

$$\Delta D^* + \delta \;=\; \int_{x(a^\#)}^{y^\#} \left[\Pi'(x(a^\#)) - \Pi'(s) \right] ds$$

$$= \int_{x(a^\#)}^{y^\#} \left[\Pi'(x(a^\#)) - \Pi'(y^\#) \right] ds - \int_{x(a^\#)}^{y^\#} \left[\Pi'(s) - \Pi'(y^\#) \right] ds$$
$$= \tau_f(y^\#) \left[y^\# - x(a^\#) \right] - \int_{x(a^\#)}^{y^\#} \left[\Pi'(s) - \Pi'(y^\#) \right] ds.$$

The first integral, together with (6), yield

$$\frac{1}{2}\underline{M} \left[y^\# - x(a^\#) \right]^2 \leq \Delta D^* + \delta \leq \frac{1}{2}\overline{M} \left[y^\# - x(a^\#) \right]^2.$$

This, together with last inequality, yield

$$\frac{\tau_f(y^\#)}{\overline{M}} \leq y^\# - x(a^\#) \leq \frac{\tau_f(y^\#)}{\underline{M}}.$$

It follows immediately that

$$\Delta D^* + \delta \geq \frac{1}{2}\underline{M} \left[y^\# - x(a^\#) \right]^2 \geq \frac{1}{2}\underline{M} \left[\frac{\tau_f(y^\#)}{\overline{M}} \right]^2.$$

If ΔD^* is smaller than the right-hand side, then this is sufficient for $\delta > 0$ and hence for the counterintuitive case to occur. The right-hand side requires that $(a^\#, y^\#)$ has been solved and can be ensured by securing that ΔD^* is less than the smallest value that the right-hand side may take in the interval of interest.

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Endnotes:

1. Papers that do use a nonlinear decay function include Forster (1975) and Tahvonen and Salo (1996).
2. For conciseness, functional dependence of the variables is suppressed in the text.
3. We define $\gamma \equiv 1$ which is equivalent to measuring a and x in the same units.
4. This expression for the optimal tax as a feedback control law can also be found in Sandal and Steinshamn (1998).