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Macroeconomic Coordination and Inflation Targeting in a Two-Country Model

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Abstract

This paper deals with macroeconomic coordination and its stabilization within a new Keynesian framework. The dynamic treatment of a two-country model is made by simulation, using the linear quadratic algorithm. We compare the optimal monetary policy rule for three types of equilibria: macroeconomic coordination, Nash and Stackelberg, using parameters that reflect the relative size and degree of openness of the economies. Under the strict inflation target, we obtain higher output and inflation volatilities due to each economy's reaction to the other country's policy. The only exception is the case of optimal macroeconomic coordination rule. This dynamic model finds that macroeconomic coordination policy is better than non-coordination rules, supporting the traditional result found in static models.

Keywords: Macroeconomic Coordination, Monetary Policy Rules.
JEL Classification: E52, E61, F42.

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1. Introduction

Nowadays the formation of big economic blocks aiming at macroeconomic stabilization is becoming even more important, as one can note in Eurozone. Monetary policy is coordinated by just one Central Bank.

The literature on macro coordination is considerable and started with the paper of Hamada (1976) and followed by Canzoneri & Gray (1985), Rogoff (1985), Kehoe (1989), and Canzoneri & Henderson (1991). One of the first papers about economic coordination among countries was by Robinson (1937). It involves a trade game among countries and the strategies and retaliations among partners in response to adverse situations. The main policy instruments are: depreciation of the exchange rate, wage reduction, exports subsidies and tariffs retaliations. This started an extensive research agenda focusing on trade policy cooperation among nations.

Hamada (1976, 1985), using a box called the Hamada diagram, analyzed the monetary policy and exchange regimes, where the potential gains from macro coordination became more visible. Using the diagram, it was possible to show that Nash and Stackelberg equilibria were inferior solutions than coordination (which was located on the Pareto contract curve).

Along the same line of showing that cooperation was superior, there is the Canzoneri & Gray (1985) paper. They analyzed the result of the same exogenous shock (for example an oil shock) on two blocks (in the case, the US and the rest of the world (ROW)). The analysis involved three types of externalities of the macro policy decision: i) externality with a negative symmetry (begger-thy-neighbor effect), where an expansionary policy in one country exports unemployment to the other; ii) externality with a positive effect (locomotive effect), where an expansionary policy in one country raises the GDP in the other; iii) externality with asymmetry, where the expansion in the US increases the product in ROW, but the expansion in ROW decreases the product in the US. They clearly pointed out that for regimes with positive or negative externalities there is room for coordination, giving superior results than the Nash or Stackelberg equilibria. But for the case of the asymmetrical externality, we do not get a clear result.

Hamada (1976), Canzoneri & Grey (1985) and Walsh (1998) reach similar conclusions with different model. These models emphasize that coordination is desirable from an economic point of view. The major drawbacks of the mentioned models are: they are static and the policy instruments to control are not clearly defined. All the policy decisions are taken at the same time and do not consider the delay effect of policy transmission. Understanding the macro coordination becomes more difficult when the policy decisions are not synchronized and when they are gradual.

Rogoff (1985), using a monetary model, shows that a cooperative solution may be inferior to the non-cooperation, when the authorities do not take into account the reaction of the private sector. When the authorities for both countries try to boost the employment level, the private sector may become afraid of exchange rate depreciation and may adjust the wage and price level, increasing inflation. Rogoff claims that coordination involves credibility issues about the commitment of the authorities to fighting inflation.

Kehoe (1989) rejected Rogoff (1985)'s point of view presenting a counter example where a government can maximize the welfare of the economy bringing about better results with macro-coordination than with Nash equilibrium. When there is a common strategy by the private agents and the government, these models raise questions about credibility and intertemporal inconsistency.

All the quoted papers are two-country models. When more than two countries are involved the following cases are presented: i) all the countries work in coordination; ii) there is no coordination among them and iii) only a sub-set of those countries is willing to coordinate their policy. Partial coordination is only sustainable when no inside country (*insiders*) or nor outside country (*outsiders*) is willing to change the *status quo*. These questions of insiders and outsiders and others related to incentives (*free-riding)* are addressed by Espinosa-Vega & Yip (1994).

In the 90's, there are some papers considering how monetary policy should be conducted. Among them the inflation-targeting approach is the major theoretical and practical reference as a monetary rule and this framework has been adopted in many countries. The inflation-targeting framework allows us to treat the interaction among the major variables in a simple manner than the big econometric models.

The present paper uses the inflation-targeting framework in a two-country model allowing us to consider macroeconomic stabilization and coordination between two nations. The inflation-targeting model is an extension of Ball (1998). The parameters are set in a way to characterize the difference in size and degree of openness of the two representative countries.

Three optimal monetary rules used in the paper are: a) macroeconomic coordination equilibrium, b) Nash equilibrium and c) Stackelberg equilibrium. The reaction function depends on the output of the two economies, inflation, exchange rate shock, lagged exchange rate and a reaction rule that takes into account the other country inflation. The optimal rules were found using a linear quadratic model. Several simulations were performed in order to calculate the variances on the inflation, output and interest rates when economies are under demand, cost and/or exchange shocks.

The dynamic model shows us that the output and inflation stabilization is more efficient when the coordination rule is used. The greater is the welfare gains the more dependent and open the country is. If macro coordination is impossible, the country that has more relevant information assumes a leader position, getting welfare gains in terms of inflation and output volatilities.

Without coordination, monetary rules with more weight on inflation turned out to get less stabilization on inflation and output than other policies rules. Hence, the more dependent and open is the country, the less weight should be placed on inflation, to avoid an increase in the output and inflation volatilities. The relevance of this kind of model that allows the interaction of two economies is increasing lately, as we move to a more global and integrated world.

After this introduction, in section 2 we deal with the two-country model specification. In section 3 we compare the optimal monetary policy rule for the three types of already mentioned equilibria. Section 4 presents the simulation results and in section 5 a few paragraphs summarize the main conclusions.

2. Two-Country Model

The core of the two-country model used in this paper is based in Ball (1998), adding up the externalities of the other economy output. The model has five equations: the domestic and foreign country demand, the domestic and foreign supply and the fifth equation that connects both economies by the exchange rate. The model specification is:

$$y_t = a_1 \rho_{t-1} - a_2 r_{t-1} + a_3 y_{t-1} + a_4 y^*_{t-1} + u_t \tag{1}$$

$$y^*_t = -a^*_1 \rho_{t-1} - a^*_2 r^*_{t-1} + a^*_3 y^*_{t-1} + a^*_4 y_{t-1} + u^*_t \tag{2}$$

$$\pi_t = \pi_{t-1} + b_1 y_{t-1} + b_2 (\rho_{t-1} - \rho_{t-2}) + e_t \tag{3}$$

$$\pi^*_t = \pi^*_{t-1} + b^*_1 y^*_{t-1} - b^*_2 (\rho_{t-1} - \rho_{t-2}) + e^*_t \tag{4}$$

$$\rho_t = \theta (r^*_t - r_t) + v_t \tag{5}$$

where y_t is the log of the output gap (real output minus the potential one), r is the real interest rate, ρ is the real exchange rate – an increase means real depreciation in the domestic economy – π is the inflation rate, u is the demand shock, e is the cost-push shock and v is the exchange rate shock; a_i e b_i are the structural parameters of the economy. The asterisks mean external variables and parameters. All the shocks are white noise, meaning zero mean and constant variance.

The model is linear around the steady state values. Inflation, real interest rate, and exchange rate are considered zero in the steady state.

Phillips Curves

Equations (3) and (4) present the Phillips curves. Each one relates inflation with its lagged value, lagged output gap, changes in the exchange rate and the contemporaneous cost shock. A change in the exchange rate affects inflation due to imported prices. The equation merges the imported and domestic inflation.

The specification for the domestic inflation is similar to a Phillips curve for a closed economy as shown by equation 6.

$$\pi_t^d = \pi_{t-1} + by_{t-1} + e_t^{'} \tag{6}$$

Imported inflation is given by the total inflation of the previous period added to a proportion of the lagged output gap. Imported prices follow a purchase power parity, so this inflation is given by:

$$\pi_t^i = \pi_{t-1} + (\rho_{t-1} - \rho_{t-2}) \tag{7}$$

where imported inflation is a result of the total inflation of the last period plus any change in the exchange rate in the last period. On the other hand, inflation in the present period, given by equation (3), is a weighted average of domestic inflation and imported one, taking the share of imported goods as ϕ. The following identities hold: $b_1 = (1-\phi)b$, $b_2 = \phi$ e $e_t = (1-\phi)e_t^{'}$.

Real Exchange Rate

Equation (5) connects the two economies by the real exchange rate, which relates it to the interest rate differential. This relationship captures the financial market behavior: an increase in the real interest rate turns the domestic asset more attractive and so causes exchange rate appreciation. Other things that affect the exchange rate are the shocks in the exchange rate, which capture the expectations and the confidence of the private agents. Equation (5) can be obtained by a linear approximation of the balance of payments equilibrium equation.

The balance of payments equation has the current account expression (TC) and the capital equations (MCA). The current account is positively related to the real exchange rate and the capital equation is positively related to the real interest rate differential. Hence:

$$TC(\rho_t) + MCA(r_t - r_t^* + \rho_t - E_t\rho_{t+1}) = 0 \tag{8}$$

The linear approximation of the equation (8) brings us to the equation (5), less the exchange rate shock. In the absence of the bubbles and under rational expectations give us that $E_t\rho_{t+1} = 0$.

There are other theories about the exchange rate behavior. Some of them are focused on variables as wealth and debt. Others consider purchase power parity and uncovered interest parity. Our paper emphasizes the role of the trade balance and of the interest rate differential and no attention to the role of wealth and debt stocks.

Parameters of the model

The calibration was based on results found in the literature[1]. Some parameters are set to capture the difference in the degree of openness and in the relative size of the economies.

Table 1 presents the results of the calibration for an open economy. Ball (1998) shows the results for the American economy; Haldane & Batini (1999) for the UK; Bonomo & Brito (2001) and Freitas & Muinhos (2001) for Brazil. Walsh (1998, p. 472) uses data based on other author's papers and with the exception of this work and Ball (1998) all the others results are for quarterly models.

Table 1. Parameters of the Structural Model

	Ball (1998)	H&B (1998)	B&B (2001)	F&M (2001)	Walsh (1998)
a_3	0.8	0.8	0.91	0.73	0.8
a_2	0.6	0.5	0.51	0.39	0.35
a_1	0.2	0.2	0.08	-	0.04
b_1	0.4	0.4	0.32	0.31	-
b_2	0.2	0.4	0.1	0.2	0.2

where:

a_1: demand elasticity for the exchange rate

a_2: demand elasticity for the real interest rate

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[1] Our purpose here is not the estimation of the parameters of the structural model for a particular economy, but to calibrate them to find a stylized model in order to simulate different policy objectives.

a_3 : auto-regressive parameter

b_1 : inflation elasticity in relation to the demand

b_2 : inflation elasticity in relation to the exchange (passthrough)

Comparing all the parameters used in the small-scale structural model, Table 1 points out the consistency in the magnitude and the sign of the parameters used by those authors. The IS curve and the Phillips curve in the models by Ball (1998) and by Freitas & Muinhos (2001) are backward-looking. Haldane & Batini (1998) model has a backward looking IS curve and the Phillips curve is a weighted average of backward-looking and forward-looking terms with a small weight in the last term. The exchange rate parameter in the IS curve and in the Phillips curve from Bonomo & Brito (2001) is rather small, showing how closed is the Brazilian economy compared with the US and the UK.

Table 2 shows the parameters used in the simulations that are based on those presented in Table 1.

Table 2. Parameters for the Two Economies

Domestic	Foreign
$a_1 = 0.1$	$a_1^* = 0.2$
$a_2 = a_2^* = 0.45$	$a_2 = a_2^* = 0.45$
$a_3 = a_3^* = 0.8$	$a_3 = a_3^* = 0.8$
$a_4 = 0.1$	$a_4^* = 0.2$
$b_1 = 0.3$	$b_1^* = 0.4$
$b_2 = 0.2$	$b_2^* = 0.4$

Those parameters are meant to represent two stylized facts: that the domestic economy is more closed, with a smaller pass through from exchange rate to inflation, $b_2 < b_2^*$; and it is less dependent on the foreign country's output, meaning that the demand of the other economy will affect less the domestic economy than vice-versa, $a_4 < a_4^*$.

3. Optimal Equilibrium Rule

In this section we present some details about how to obtain the optimal equilibrium rule. Except for some particular dynamic equations, most of them do not have algebraic solutions and they need iterated computer algorithm to solve. Svensson (1997) presents a particular dynamic equation with an algebraic solution.

In the optimal dynamic solution of the two-country model we use the algorithm of linear quadratic method. This method is extensively used in *Real Business Cycle Theory (RBC),* where the return function is maximized. In our case, it is a loss function, which is minimized.

The linear quadratic algorithm is based on Díaz-Giménez (1999) and it is implemented on computer language Matlab 5.1. In subsection below, we give details on how we implemented the algorithm to obtain three types of solutions: macroeconomic coordination equilibrium, Nash equilibrium and Stackelberg equilibrium.

3.1 Macroeconomic Coordination Equilibrium

Two countries obtain macroeconomic coordination when they minimize a joint objective function with same weight on the output gap, under the control of their respective monetary instruments, r and r^*. That is:

$$\min_{r_i, r_i^*} \sum_{i=0}^{\infty} \beta^i E_t \{ \frac{1}{2}(\pi_{t+i+1}^2 + \lambda y_{t+i+1}^2) + \frac{1}{2}[(\pi_{t+i+1}^*)^2 + \lambda (y_{t+i+1}^*)^2]\} \tag{9}$$

subject to:

$$y_{t+1} = -(a_1\theta + a_2)r_t + a_1\theta r_t^* + a_1 v_t + a_3 y_t + a_4 y_t^* + u_{t+1} \tag{10}$$

$$y_{t+1}^* = -(a_1^*\theta + a_2^*)r_t^* + a_1^*\theta r_t - a_1^* v_t + a_3^* y_t^* + a_4^* y_t + u_{t+1}^* \tag{11}$$

$$\pi_{t+1} = \pi_t + b_1 y_t - b_2 \rho_{t-1} + b_2 v_t - b_2 \theta r_t + b_2 \theta r_t^* + e_{t+1} \tag{12}$$

$$\pi_{t+1}^* = \pi_t^* + b_1^* y_t^* + b_2^* \rho_{t-1} - b_2^* v_t + b_2^* \theta r_t - b_2^* \theta r_t^* + e_{t+1}^* \tag{13}$$

Each restriction equation can be separated in three parts: i) state variables at time t; ii) control variables at time t and iii) shocks at time t + 1. Defining the state variables by s_i, where i = 1, 2, 3 or 4. Then:

$$s_{1t} = a_1 v_t + a_3 y_t + a_4 y_t^*$$

$$s_{2t} = -a_1^* v_t + a_3^* y_t^* + a_4^* y_t$$

$$s_{3t} = \pi_t + b_1 y_t - b_2 \rho_{t-1} + b_2 v_t$$

$$s_{4t} = \pi_t^* + b_1^* y_t^* + b_2^* \rho_{t-1} - b_2^* v_t \tag{14}$$

The value function of macroeconomic coordination in dynamic programming format is:

$$V(s_1, s_2, s_3, s_4) = \min_{r, r^*} E\{\frac{1}{2}[s_3 - b_2\theta r + b_2\theta r^*]^2 + \frac{1}{2}\lambda[s_1 - cr + a_1\theta r^*]^2 + \frac{1}{2}[s_4 + b_2^*\theta r - b_2^*\theta r^*]^2 +$$

$$\frac{1}{2}\lambda[s_2 - c^* r^* + a_1^*\theta r]^2 + \beta V(s_1', s_2', s_3', s_4')\} \tag{15}$$

subject to:

$$s_1' = a_3 s_1 + a_4 s_2 + (a_4 a_1^* \theta - a_3 c) r + (a_1 a_3 \theta - a_4 c^*) r^* + a_1 v' + a_3 u' + a_4 u^{*'}$$

$$s_2' = a_4^* s_1 + a_3^* s_2 + (a_3^* a_1^* \theta - a_4^* c) r + (a_1 a_4^* \theta - a_3^* c^*) r^* - a_1^* v' + a_3^* (u^*)' + a_4^* u'$$

$$s_3' = b_1 s_1 + s_3 - b_1 c r + b_1 a_1 \theta r^* + b_1 u' + b_2 (v' - v) + e'$$

$$s_4' = b_1^* s_2 + s_4 - b_1^* c^* r^* + b_1^* a_1^* \theta r + b_1 (u^*)' - b_2^* (v' - v) + (e^*)' \tag{16}$$

where $c = a_1\theta + a_2$, $c^* = a_1^*\theta + a_2^*$ and s_i' is next period state variable for $i = 1$, 2, 3 or 4.

The return function is:

$$R(s_1,s_2,s_3,s_4,r,r^*) = E\{\frac{1}{2}[s_3 - b_2\theta r + b_2\theta r^*]^2 + \frac{1}{2}\lambda[s_1 - cr + a_1\theta r^*]^2 + \frac{1}{2}[s_4 + b_2^*\theta r - b_2^*\theta r^*]^2 +$$

$$\frac{1}{2}\lambda[s_2 - c^* r^* + a_1^*\theta r]^2\} \qquad (17)$$

The optimal monetary policy rules with coordination are obtained by simulation using expressions (15) to (17) with parameters given by Table 2. These rules are function of six arguments $f(y, y^*, \pi, \pi^*, \rho_{-1}, v)$ as shown by expressions below. The coefficients of these arguments are taken for the specific case with $\lambda = 1$, the weight attributed to the output gap in the loss function. Therefore:

$$r_t = 1.6005 y_t + 0.6806 y_t^* + 1.2027 \pi_t + 0.2063 \pi_t^* + 0.1843 v_t - 0.1580 \rho_{t-1} \qquad (18)$$

$$r_t^* = 1.1734 y_t + 1.2225 y_t^* + 0.7177 \pi_t + 0.8072 \pi_t^* - 0.2860 v_t + 0.1793 \rho_{t-1} \qquad (19)$$

The signals of the coefficients of above reactions functions are all coherent with the literature. The interest rate reacts positively to the output gap and inflation rate to both economies.

We generate many samples for output, inflation and interest rate under optimal rules and taking into account the demand, the supply and the exchange rate shocks. After that, the variances of inflation and output were calculated. The variances obtained with different values of weight (given to output gap in the loss function) are plotted on a single graph showing the trade-off between the inflation rate and output variances.

3.2 Nash Equilibrium

The Nash equilibrium is a non-coordinated policy. The authorities choose interest rate to minimize a loss function, taking as given the interest rate of the other country. Each country decides their policy, taking into account that the other nation has already decided and would not change it during this period. The Nash equilibrium treatment in this section is similar to that is taken by Walsh (1998, p.266). The home country loss function is:

$$\min_{r_i} \sum_{i=0}^{\infty} \beta^i E_t \frac{1}{2}(\pi_{t+i+1}^2 + \lambda y_{t+i+1}^2) \tag{20}$$

and foreign country loss function is:

$$\min_{r_i^*} \sum_{i=0}^{\infty} \beta^i E_t \frac{1}{2}[(\pi_{t+i+1}^*)^2 + \lambda (y_{t+i+1}^*)^2] \tag{21}$$

The equilibrium treatment is similar to both countries. Then, we take home country to focus. Taking the aggregate demand and Phillips curve expression at time $t+1$, we have:

$$y_{t+1} = -(a_1\theta + a_2)r_t + a_1\theta r_t^* + a_1 v_t + a_3 y_t + a_4 y_t^* + u_{t+1} \tag{22}$$

$$\pi_{t+1} = \pi_t + b_1 y_t - b_2 \rho_{t-1} + b_2 v_t - b_2 \theta r_t + b_2 \theta r_t^* + e_{t+1} \tag{23}$$

Following the same treatment given to coordinated solution, each restriction equation can be separated in three parts: i) state variables at time t; ii) control variables at time t and iii) shock at time t + 1. The two state variables s_1 and s_3 are defined as:

$$s_{1t} = a_1 v_t + a_3 y_t + a_4 y_t^* \tag{24}$$

$$s_{3t} = \pi_t + b_1 y_t - b_2 \rho_{t-1} + b_2 v_t \tag{25}$$

The value function of Nash equilibrium in dynamic programming format is:

$$V(s_1, s_3) = \min_r E\{\frac{1}{2}[s_3 - b_2\theta r + b_2\theta r^*]^2 + \frac{1}{2}\lambda[s_1 - cr + a_1\theta r^*]^2 + \beta V(s_1', s_3')$$

subject to:

$$\begin{aligned} s_1' &= a_3 s_1 - a_3 cr + a_1 a_3 \theta r^* + a_1 v' + a_3 u' + a_4 y^* \\ s_3' &= b_1 s_1 + s_3 - b_1 cr + b_1 a_1 \theta r^* + b_1 u' + b_2 (v' - v) + e' \end{aligned} \tag{26}$$

where $c = a_1\theta + a_2$ and s_i' is next period state variable for $i = 1$ or 3.

The optimal monetary policy rules to home country, *r*, is a function of five arguments $f(y, y^*, \pi, \rho_{-1}, v)$ while foreign country has $f(y, y^*, \pi^*, \rho_{-1}, v)$ as its rule, r^*. The coefficients of these arguments are taken for the specific case with $\lambda = 1$, the weight attributed to the output gap in the loss function. Therefore:

$$r_t = 1.28619 y_t + 0.11871 y_t^* + 1.1217 \pi_t + 0.34305 v_t - 0.22434 \rho_{t-1} \tag{27}$$

$$r_t^* = 0.6642 y_t^* + 0.09414 y_t + 0.7191 \pi_t^* - 0.38178 v_t + 0.28764 \rho_{t-1} \tag{28}$$

The graphic of trade-off between the inflation rate variance and output variance is obtained in similar way as coordinated equilibrium.

3.3 Stackelberg Equilibrium

Stackelberg equilibrium, also known as leader-follower equilibrium, is another example of uncoordinated policy. The authorities choose the interest rate to minimize the loss function, taking into account how the other policy authority will respond to the leader's choice of interest rate. We take home country as leader. The external reaction function is given by Nash equilibrium, $r_t^* = j_1 s_{2t} + j_2 s_{4t}$, where j_1 and j_2 are the coefficients that depend on weight attributed to output gap variance in loss function.

The value function of Stackelberg equilibrium in dynamic programming format is:

$$V(s_1, s_2, s_3, s_4) = \min_r E\{\frac{1}{2}[s_3 - b_2\theta r + b_2\theta r^*]^2 + \frac{1}{2}\lambda[s_1 - cr + a_1\theta r^*]^2 + \beta V(s_1', s_2', s_3', s_4')$$

subject to:

$$s_1' = a_3 s_1 + a_4 s_2 + (a_4 a_1^* \theta - a_3 c) r + (a_1 a_3 \theta - a_4 c^*) r^* + a_1 v' + a_3 u' + a_4 u^{*'}$$

$$s_2' = a_4^* s_1 + a_3^* s_2 + (a_3^* a_1^* \theta - a_4^* c) r + (a_1 a_4^* \theta - a_3^* c^*) r^* - a_1^* v' + a_3^* (u^*)' + a_4^* u'$$

$$s_3' = b_1 s_1 + s_3 - b_1 c r + b_1 a_1 \theta r^* + b_1 u' + b_2 (v' - v) + e'$$

$$s_4' = b_1^* s_2 + s_4 - b_1^* c^* r^* + b_1^* a_1^* \theta r + b_1 (u^*)' - b_2^* (v' - v) + (e^*)'$$

$$r^* = j_1 s_2 + j_2 s_4 \tag{29}$$

where $c = a_1\theta + a_2$, $c^* = a_1^*\theta + a_2^*$ and s_i' is next period state variable for $i = 1$, 2, 3 or 4.

The optimal monetary policy rules to home country, *r*, is a function of six arguments $f(y,y^*,\pi,\pi^*,\rho_{-1},\nu)$ while to foreign country has $f(y,y^*,\pi^*,\rho_{-1},\nu)$ as its rule, r^*. The coefficients of these arguments are taken for the specific case with $\lambda=1$, the weight attributed to the output gap variance in the loss function. Thus:

$$r_t = 1.3458y_t + 0.4272y_t^* + 1.1036\pi_t + 0.3029\pi_t^* + 0.1744\nu_t - 0.0996\rho_{t-1} \tag{30}$$

$$r_t^* = 0.0940y_t + 0.6638y_t^* + 0.7191\pi_t^* - 0.3817\nu_t + 0.2876\rho_{t-1} \tag{31}$$

The leader reaction function has the output gap and inflation rate of foreign country as its arguments while the follower takes only leader's output gap on its optimal reaction function.

The graphic of trade-off between the inflation rate and output gap variances is obtained in similar way as before.

4. Simulation Results

Volatilities of inflation and output are used to measure the performance of different monetary policy rules following Taylor (1999), Ball (1998), Svensson (1998). The policy rule that conducts to less inflation and output volatilities is considered the best one.

We ran some simulations to obtain the inflation rate, the output gap and the interest rate volatilities as shown in Table 3. Each optimal rule is obtained with parameter values given by Table 2 and taking the same weight attributed to the inflation rate and output gap in the loss function. To obtain the variances, we consider demand, cost-pushing and exchange rate shocks. All shocks are white noise.

Table 3. Volatilities of Inflation rate, Output Gap and Interest rate

	Home country			Foreign country		
Equilibrium type	Var(π)	Var(y)	Var(r)	Var(π^*)	Var(y^*)	Var(r^*)
Coordination	3.11	2.57	4.71	2.39	2.46	4.38
Nash	3.25	2.86	3.61	4.43	4.27	2.40
Leader (home)	2.98	2.76	3.98	4.46	4.90	2.61
Leader (foreign)	3.11	3.02	3.72	2.51	2.59	3.80

Table 3 shows that under macroeconomic coordination policy, both countries have less inflation and output gap volatilities than any other type of policy. Nash equilibrium has clearly worse inflation and output gap volatilities than other rules for both countries. But comparing Nash equilibrium with Stackelberg equilibrium one can see that the leader has better performance. We found 2.98 for inflation volatility and 2.76 for output volatility when the home country is the leader; clearly those values are lesser than 3.25 and 2.86 under Nash equilibrium.

Taylor (1999) brings us a remark about volatility of policy instrument in his robustness analyses of different monetary policy rules. Table 3 shows that the volatility of interest rate is higher in the case of coordination equilibrium. The less volatilities of output gap and inflation rate under coordination come from an aggressive policy response to the shocks. Briefly, the coordination equilibrium conducts to less inflation and output volatilities but higher interest rate and exchange rate volatilities.

In short, our two-country inflation target dynamic framework found that macroeconomic coordination is desirable. Figure 1 below confirms this fact. We can get the same result simulating different combinations of shocks. This result is in line with the static models of Canzoneri & Gray (1985), Fielding & Mizen (1996), Walsh (1998, p. 259).

The next two figures show the trade-off of home and foreign countries under optimal rules.

Figure 1. Trade-off of Efficient Frontier for Home Country

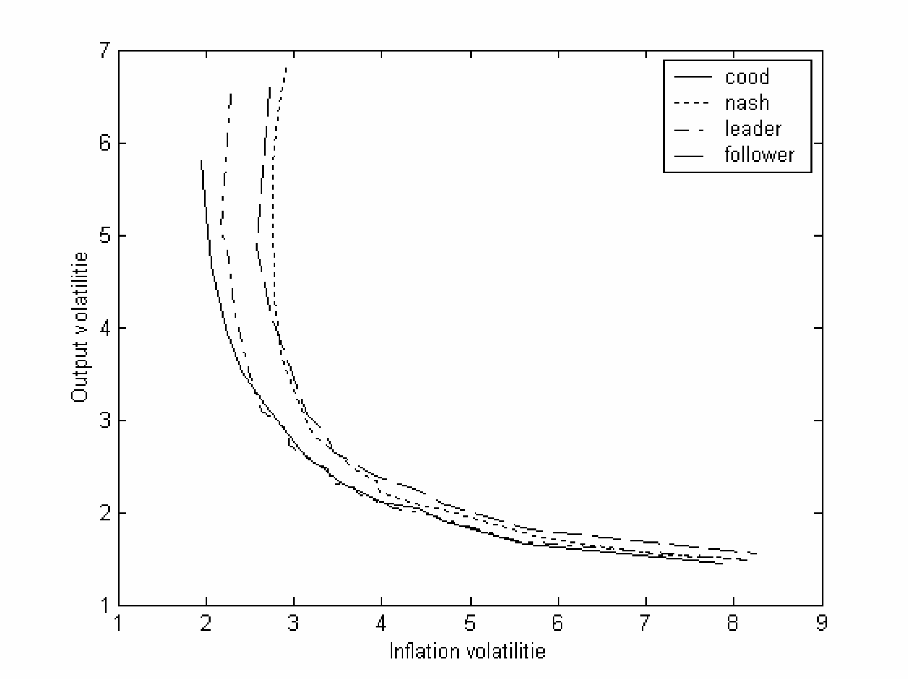
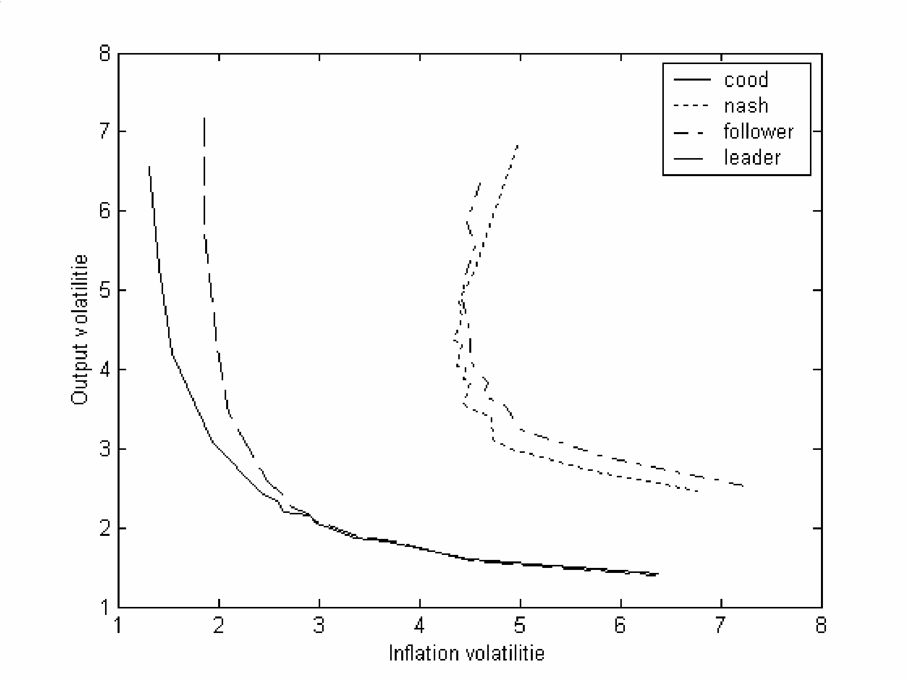


Figure 2. Trade-off of Efficient Frontier for Foreign Country



Figures 1 and 2 show the efficient frontier – lesser volatilities for inflation and output gap. Each simulation is divided in two steps: i) optimal rules of home and foreign countries are obtained from each type of equilibrium and; ii) using these rules and considering all types of shocks we generate the first and the second moments of the relevant variables.

The figures also show that when we assign more weight to inflation in the objective function we obtain an increase of the inflation and output volatilities[2] (except in the coordination case). Under macroeconomic coordination rule both countries have a common objective function and the trade-off between output and inflation volatilities is still working independent of the weight of the objective function.

Our two-country model's equations point out that the stabilization of output gap and inflation rate occurs through three channels: the interest rate, the exchange rate and foreign output. Except for the coordination case, the lower weight in output (meaning higher commitment with lower inflation) conducts to greater volatilities of inflation and output. On the other hand, higher output gap stabilization does not result in inflation rate destabilization.

Under Nash equilibrium rule, as we assign more weight on inflation stabilization in the loss function, we obtain higher volatility in inflation and output especially in the foreign country, which is parameterized with a greater degree of openness and as a more dependent economy. For this type of country it is not recommended a strict inflation goal but a flexible target.

The optimal rules coefficients depend on the structural parameters of the model and the weight given to output gap in the loss function. We point out two structural parameters: the degree of openness and relative size of the country. The simulation takes the parameters as given in Table 2. Needless to say, $a_4 < a_4^*$ characterizes that the home country is less dependent than the foreign country and $b_2 < b_2^*$ means that domestic country is less open compared to foreign country.

Another simulation is shown in Table 5 using a greater difference on the degree of openness and relative size of two countries[3]. Table 4 shows the coefficients using original parameters. The coefficients of new simulation are consistent with the signals and magnitudes of the previous one.

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[2] This figure is done changing the parameter value λ in [0, + ∞). If λ < 1 then we assign more weight on inflation. If λ > 1, it means more weight on output stabilization.

[3] This simulation is done increasing the difference on the parameters, ($a_4 < a_4^*$,). It means that the home country is bigger than before and the foreign country is smaller than before. If the difference of $b_2 < b_2^*$ increases, it means a "smaller" degree of openness of the home country and "greater" degree of openness of the foreign country.

Table 4. Nash Equilibrium with $a_4 = 0.1 < a_4^* = 0.2$ and $b_2 = 0.2 < b_2^* = 0.4$.

	y	y^*	π	π^*	ν	ρ_{-1}
r	1.28619	0.11871	1.1217	0	0.34305	-0.22434
r^*	0.09414	0.6642	0	0.7191	-0.38178	0.28764

Table 5. Nash Equilibrium with $a_4 = 0.05 < a_4^* = 0.3$ and $b_2 = 0.1 < b_2^* = 0.5$.

	y	y^*	π	π^*	ν	ρ_{-1}
r	1.51657	0.071425	1.2459	0	0.26744	-0.12459
r^*	0.11538	0.57524	0	0.6689	-0.41137	0.33445

The closer is the economy (home country in our example) less important is the other country's variables, the exchange rate shock, and lagged exchange rate variables on r. This means that the greater is the degree of openness and dependence of the other economy (foreign country in our case) the smaller is the monetary policy reaction through the interest rate (r*), in response to the inflation rate and the output gap. We obtain the same results with other types of optimal rules.

5. Concluding Remarks

Summing up, we can point to the following main conclusions:

- The macro coordination equilibrium brings about less volatile output and inflation than Nash and Stackelberg equilibrium;
- the stabilization of output and inflation are greater for more dependent and more open economy;
- the country which has more information and adopts a leader position presents a more stable economy;
- in the absence of coordination, a more strict anti-inflation policy results in a greater output volatility, being worse in a more dependent and open economy;

- a more dependent and open economy responds more aggressively to a exchange rate shock.

In the two-country model, it was possible to illustrate some stylized facts and derive some conclusions about different aspects of the monetary policy. However it is worth to stress that some important aspects such as fiscal policy and the structural features of the economy are not taken into account in this paper. So it is necessary to treat with caution our simulation results for guidance to monetary policy. We believe that our dynamic framework dealing with interacting economies is a starting point for a promising research agenda.

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