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Uncovered Interest Parity with Fundamentals: A Brazilian Exchange Rate Forecast Model[◊]

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Abstract

One of the most challenging elements of the inflation-targeting framework is the exchange rate forecast. Wadhwani (1999) proposed a UIP, where real variables like the unemployment differential, the current account differential, and the excess return of financial assets affect the expected exchange rate. The objectives of this paper are first to include, as in Wadhwani (1999), some real variables to anchor exchange rate expectations. In our case, the long-run value of the exchange rate is determined by balanced external accounts. Second, we use this approach to simulate the behavior of key macroeconomic variables in an inflation-targeting structural model for Brazil. Finally, we compare the results with those of a random walk specification. The impulse responses under the UIP-with-fundamentals model seemed to be more realistic than those obtained by using other specifications for exchange rate forecasts.

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1 – Introduction

Forecasting the nominal exchange rate path is one of the most challenging aspects of an inflation-targeting framework. According to our estimates, the pass-through from nominal exchange rate movements to inflation in Brazil is around 10% in each quarter[1]. Therefore, an accurate forecast of the nominal value of the currency is very important for the efficiency of an inflation-targeting regime. If the evaluation of the future exchange rate path can be made more precise, it may reduce the variance in output and inflation.

Uncovered Interest Parity (UIP), which relates the expected nominal depreciation to the nominal interest rate differential has been a popular condition used in exchange rate forecasting. But UIP has been questioned as an adequate tool to forecast future exchange rates because many empirical tests have found a negative correlation between exchange rate and interest differential, in contradiction to what is predicted by UIP[2]. This has led us to consider what can be gained and lost with other models for forecasting the exchange rate.

A simple alternative is to assume that the exchange rate follows a random walk and is not co-integrated with any observable series that can be modeled. Therefore, expectations of future exchange rate should be equal to the current value. This first approach, although simple and transparent, does not preclude the risk of, on occasion, large forecast errors in the exchange rate and hence inflation. And although exchange rates appear to have random walk-like properties, we cannot be sure that the econometric tests at our disposal are subtle enough to distinguish random walks from other processes with potentially very different forecasts over one and two year horizons.

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[1] The estimated coefficient a_{22} in equation 8 is approximately 0.10 and significant at conventional levels. In Muinhos (2001) many different specifications of the Phillips curve are estimated. In a shorter sample, which started in 1995, the pass-through coefficient was 0.10 (with a t-statistic of 3.25) when there was no forward-looking term for inflation and 0.09 (with t-statistic of 3.0) with the forward looking term. With a larger sample, starting in 1980, the pass-through was 0.11 and the t-statistic was 3.77.

[2] See Wadhawani (1999) and Taylor (1995)

Another simple alternative is to suppose that the real exchange rate will remain constant, according to purchasing power parity (PPP). To derive the nominal exchange rate path, we have to forecast the difference between the domestic and the foreign price level. According to a survey by Taylor (1995), PPP holds in the post-war period until the early 1970´s, when the Bretton-Woods system was abandoned. The validity of PPP was seriously questioned with the high variability of the major currencies that followed. For high frequency data, key findings were made by Meese and Rogoff (1983), (1988) , whose tests overwhelmingly rejected PPP in favor of the random walk hypothesis up to the one-year horizon[3].

Some recent tests of the co-integration between nominal exchange rate and relative prices that support the mean reversion property of the real exchange rate series, a finding that is consistent with PPP.[4] This is especially true when the authors use very long samples, covering several decades. Froot and Rogoff (1995) and Rogoff (1996) estimated that the convergence of PPP is very slow with a half-life of three or four years, using linear models[5].

The need to equalize the return of different nominal assets, avoiding arbitrage, yields the UIP relationship, which can be written as follows:

$$E_t e_{t+1} = e_t + i_t - (i_t^* + x_t) \quad . \tag{1}$$

where e_t is the nominal exchange rate at time t, defined as units of domestic currency needed to buy one unit of foreign currency (in such a way that increases in "*e*" means a

[3] MacDonald (1999) summarises the results of Meese and Rogoff (M+R) (op cit) as follows:

> *"M+R took the simple flexi-price monetary model (which relates an exchange rate to relative short term interest rates), the Dornbusch-Frankel model (which essentially adds a long term interest differential to the flexi-price model) and a Hooper-Morton model (which adds a wealth term and a risk premium to the Dornbusch-Frankel model). (...) Additionally, M+R considered a wide array of univariate models as well as a vector autoregression comprising exchange rates, relative short-term interest rate, relative inflation rates and current account. The currencies studies were the dollar-pound, dollar-mark, dollar-yen and the trade weighted dollar, and the sample period was March 1973 to November 1980, with the out-of sample forecasts conducted over the sub-period December 1976 to November 1980. "*

[4] Froot and Rogoff (1995) present three stages of PPP tests. The first uses the PPP as the null hypothesis, based on an idea of Cassel (1922) that PPP is a central tendency with temporary shocks. A second stage considers the real exchange rate as a random walk and the third tests for cointegration. The third test did not produce any further conclusion besides those already found in the second stage.

devaluation), "*i*" is the nominal interest rate of one-period maturity, x the risk premium, the superscript "*" relates to the foreign economy and E_t is the expectations taken at time t.

For example, Wadhwani (1999) discusses a simple test that is unfavorable to the UIP, based on the estimation of the following equation:

$$\Delta e_{t+k} = \alpha + \beta(i_t - i_t^*) + \upsilon_{t+k} \qquad (2)$$

Although UIP requires β=1, the literature has frequently estimated values of β smaller than one and even negative. Allowing for a risk premium in equation (2) may imply a β<1 but it is unlikely to imply that the true β is close to zero or negative (Taylor 1995). Meese and Rogoff (1988) failed to reject the null hypothesis of no cointegration between the real exchange rate and the real interest rate for dollar, yen and German mark for different periods. Meredith and Chinn (1998), however, found evidence for UIP using interest rate differentials embodied in bonds of longer maturity.

Each of these insights into the determinants of the exchange rate has its appeal. Rather than consider them as strict substitutes, it seems more natural (and not necessarily theoretically inconsistent) to combine them with the aim of retaining their information content.

In doing this, we are following quite closely the approach of Wadhwani's (1999). He suggested that the main reason for the failure of a standard UIP approach is that it is too restrictive: "*the UIP straitjacket, which requires variables like unemployment/growth to only affect exchange rate through interest rate.* [6]" Instead he adapted UIP to allow for other influences as follows:

$$\Delta e_{t+k} = \alpha + \beta(i_t - i_t^*) - \rho(q_t - \bar{q}, Z_t) \qquad (3)$$

where Z_t depends on other nominal assets as bond and stocks; and $q_t - \bar{q}$ is the estimated deviation of the real exchange rate, which depends on the difference in

[5] Many authors have tried to estimate non-linear models that attempt to allow for a non-convergence band. For example Taylor and Peel suggested that the speed of convergence might increase with the deviation from equilibrium when there are nonlinear factors governing the cost of arbitrage.

current account/GDP ratio, unemployment rate and net foreign asset/GDP ratio; and on the relative ratio of wholesale and consumer price indexes[7].

In this paper, we first add to the standard UIP the concept of a long run exchange rate equilibrium based on balanced external accounts. Second, we use this approach to simulate the behavior of key macroeconomic variables in an inflation-targeting structural model for Brazil. Finally, we compare the results with those of a random walk specification. In our adaptation of the standard UIP condition, we assume that, at some point in the future, real exchange rate will converge to equilibrium, anchoring expectations in a forward-looking model. This equilibrium exchange rate is determined within the model as the value that clears the balance of payments. The spot price of the long-run exchange rate equilibrium will depend of the interest differential corrected by the risk premium as predicted by the standard UIP condition.

The next section presents different specifications for the exchange rate equation. The third section describes the small-scale inflation-targeting model to be used in the simulations, whose results and interpretation are presented in the fourth section. The final section is left for the concluding remarks.

2 – The Central Bank of Brazil Exchange Rate Forecast Models

In order to forecast the nominal exchange rate path in our inflation-targeting structural models we are working with three alternatives. First we model a random walk with monetary surprises (RWMS) that relates movements of nominal exchange rate to movements in the interest differential adjusted by the risk premium. The second alternative is an UIP specification. Finally, the third procedure is a weighted average between the forecasts given by the UIP and the random walk hypothesis.

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[6] Wadhwani also suggested that the UIP failure may to some lesser extent have come about because of the noise introduced in the signal-extraction process by uninformed investors.

[7] J.P. Morgan's Emerging Markets Real Exchange Rate Model *(2000)* incorporates the deeper factors such as productivity, the terms of trade and trade openness that might affect the equilibrium real exchange rate in its determination of the current nominal exchange rate. In their model, higher productivity, better terms of trade and less openness should all cause the real exchange rate to appreciate over time.

The first approach, the so-called RWMS, is in fact a UIP in first difference. It can be easily derived in the following way:

$$E_t e_{t+1} - e_t = i_t - i_t^* - x_t \tag{4}$$

where x_t is the risk premium. Taking the first difference in equation (4) and assuming that the difference in exchange rate expectation is a white noise process:

$E_t e_{t+1} - E_{t-1} e_t = \eta_t$ [8],

will yield the RWMS model:

$$\Delta e_t = \Delta i_t^* + \Delta x_t - \Delta i_t + \eta_t = \Delta(i_t^* + x_t - i_t) + \eta_t \tag{5}$$

Therefore, unlike the traditional UIP, where exchange rates variations depend on the levels of interest rate differentials, in the RWMS approach only changes in interest rates differentials cause movements in exchange rate. Despite the strong assumptions embodied in the RWMS model, it presents two desirable features: i) in this specification there is no need to make hypothesis concerning future exchange rates; ii) it combines the random walk hypothesis with the desirable feature that exchange rates are sensitive to variations in the interest rate differential.

For simulation purposes, the foreign interest rate path is considered exogenous. The risk premium is modeled as either being exogenous or as being endogenously determined according to the Brazilian macroeconomic fundamentals, like fiscal variables or the behavior of the balance of payments. The latter model of the risk premium can be written as:

$$\Delta X_t = \gamma_1 \Delta X_{t-1} + \Delta PR_{t-3} + \sum \gamma_j \Delta z_{j,t-t_j} \tag{6}$$

where:

X is the risk premium, measured as the spread over treasury,

PR is the Public Sector Borrowing Requirement (PSBR) in primary concept, as percentage of GDP, and

z are other exogenous variables that affect the country risk.

[8] This assumption means that no major disturbance in the exchange rate expectations will occurs for the next period.

The second approach to forecast the nominal exchange rate path is using UIP with "model-consistent expectations". Given an exogenously equilibrium nominal exchange rate at some period *K* ahead, and then, using a model consistent UIP, the expected nominal exchange rate path is calculated from period 0 to *K*. From *K*+1 on, the future nominal exchange rate path follows a Purchase Power Parity (PPP). According to this model, an increase in the domestic interest rate leads to a contemporaneous fall in the nominal exchange rate, which begins to devaluate thereafter in order to offset the interest rate differential.

The third and final strategy to forecast the exchange rate is a variation of the previous method, and is called UIP with "adaptive expectations". In order to allow for persistence in the exchange rate, the exchange rate path is a linear combination of the model consistent UIP and the past value of the exchange rate.

3- The UIP with Fundamentals: The Five-Equation Model

In order to work with our new proposal of UIP[9], we have to build a complete set of equations that characterizes a small-scale inflation-targeting model. We present an aggregate demand equation, a Phillips equation, an interest rate rule (Taylor rule), the UIP and an equation of the balance of payments. The hypothesis in our UIP with fundamentals is that the expected real exchange rate equalizes the current account balance with the capital account *K* periods ahead.

The IS equation is very simple. The output gap depends on itself with a lag, on the lagged real interest rate and real exchange rate.

$$h_{t+1} = a_{10} + a_{11}h_t + a_{12}(i_t - \pi_t) + a_{13}\theta_t + u_t \qquad (7)$$

Where h is the log of the output gap, θ is real exchange rate, i is the nominal interest rate, π is consumer inflation, and u is the error term[10].

[9] This approach is only a new exercise proposed by the authors and it has not been used on the exchange rate forecast models of the Central Bank of Brazil.
[10] We are adopting the simplifying assumption that potential output is non-stochastic. Therefore, shocks in output gap reflect only aggregate demand disturbances.

The Phillips equation is compatible with any open economy Keynesian model with the restriction of long-term nominal neutrality, which means a vertical Phillips equation in the long run. This restriction implies the coefficients associated with the nominal variables should sum up to 1.

$$\pi_t = a_{21}\pi_{t-1} + a_{22}\pi_{t-2} + (1 - a_{21} - a_{22})(e_t - e_{t-1}) + a_{24}h_{t-1} + \varepsilon_t \qquad (8)$$

Where ε is the cost-push disturbance and (e_t – e_{t-1}) is the nominal exchange rate variation.

The interest rate is an exogenous variable, treated as the instrument of the monetary policy, but we are considering that the policy maker set this variable following a simple rule, like a Taylor rule as stated below:

$$i_t = a_{30} + a_{31}(\pi_{t-1} - \pi^*_{t-1}) + a_{32}h_{t-1} \qquad (9)$$

Exchange rate determination is based on the UIP, as stated in equation (5). In order to estimate the exchange rate path, however, it is necessary to anchor the exchange rate in some point in the future. The way we achieve this result is by assuming that at period $t+K$ nominal exchange rate will be consistent with the clearance of balance of payments. For each period between t and $t+K$, the nominal exchange rate will evolve according to the interest rate differential corrected by the risk premium, as predicted by the UIP hypothesis. Therefore, the following 2 equations determine the path of exchange rate:

$$E_t e_{t+n} = -\sum_{j=n}^{K-1} E_t(i_{t+j} - i^*_{t+j} - x_{t+j}) + E_t e_{t+K}\text{ , for } n < \text{k} \qquad (10)$$

$$E_t e_{t+K} = \theta_{t+K} - p^f_{t+K} + p_{t+K} \qquad (11)$$

where θ is the expected real exchange rate that clear the balance of payment K periods ahead, and x_t is an exogenous risk premium that follows an AR(1) process.

The fifth equation is the balance of payment clearance:

$$CA + BS = BC$$

where CA is the capital account, BS is the balance of services and BC is the trade balance. Both CA and BS are treated as exogenous and BC is determined by[11]:

$$BC = \sum_{j=1}^{7} \alpha_j . Q_j(y, \theta).) P_j \qquad (12)$$

Where Pj, are the price index for agricultural, semi-industrialized and industrialized export and are the price index for capital, durables, non-durables and raw material imported goods goods. Qj are the quantitative index for the same export and import goods, which depend on the output gap and the real exchange rate. αs are the weights to transform the indexes in US$ terms.

3.1 - The System Solution

Assuming that the balance of payment will clear K periods ahead, the economic system specified in equations 7 to 12 can be described by a quasi-linear system of equations. Taking expectation with respect to the information set available at time t-1, the model can be written as following:[12]

$$h_{t+1} = a_{10} + a_{11} h_t + a_{12}(i_t - \pi_t) + a_{13}\theta_t$$
$$\pi_t = a_{21}\pi_{t-1} + a_{22}\pi_{t-2} + (1 - a_{21} - a_{22})(e_t - e_{t-1}) + a_{24} h_{t-1}$$
$$i_t = a_{30} + a_{31}(\pi_{t-1} - \pi^*_{t-1}) + a_{32} h_{t-1}$$

[11] where $e^v = [e^{v_1} \quad e^{v_2} \quad \ldots \quad e^{v_N}]$ with $v = [v_1 \quad v_2 \quad \ldots \quad v_N]$

[12] In order to simplify the notation, we will refer to variables expectation without using $E_{t-1}(.)$.

$$qxb_t = exoqxb_t + a_{71}.y_t + a_{72}.qxb_{t-1} + a_{73}.qxb_{t-1}$$

$$qxs_t = exoqxs_t + a_{81}.\theta_t + a_{82}.y_t + a_{83}.qxs_{t-1}$$

$$qxm_t = exoqxm_t + a_{91}.qxm_{t-1} + a_{92}.qxm_{t-2} + a_{93}.\theta_t + a_{94}.y_{t-1} + a_{95}.y_{t-2}$$

$$qkap_t = exoqkap_t + a_{101}.qkap_{t-1} + a_{102}.y_{t-2} + a_{103}.\theta_t$$

$$qmbc_t = exoqmbc_t + a_{111}.qmbc_{t-1} + a_{112}.y_t + a_{113}.\theta_t$$

$$qmnd_t = exoqmnd_t + a_{121}.qmnd_{t-1} + a_{122}.y_t + a_{123}.\theta_t$$

$$qint_t = exoqqint_t + a_{131}.qint_{t-1} + a_{132}.y_t + a_{133}.\theta_t$$

$$e_{t+n} = -\sum_{j=n}^{K-1}(i_{t+j} - i^*_{t+j} - x_{t+j}) + e_{t+K} \qquad n < K$$

$$e_{t+K} = \theta_{t+K} - p^f_{t+K} + p_{t+K}$$

where *exoqmmm* denotes the exogenous component in the respective quantum equation.

The θ_{t+K} is the solution of the following non-linear equation:

$$CA + BC = \sum_{j=1}^{7}\alpha_j.Q_j(y,\theta).)P_{j\,j}$$

Remaining that K is the number of quarters necessary for achieve the balance of payment equilibrium, assuming exogenous paths for the balance of services and capital account. Using this hypothesis, we represent each variable in a different equation for the K periods, so the resulting system will have $[11(K+1) - 1]$[13] linear equations, as described in the table A.1 and the non-linear equation that solves for θ.[14]

The system is solved for time t, generating expected paths for all endogenous variables from period t up to $t+K$. At time $t+1$, the system takes the solution for t as given, and solves again, yielding the solution for $t+1$, which is used to generate the solution for $t+2$, etc.

[13] There are 10 endogenous variables – i, π, e, qxb, qxs, qxm, qkab, qmbc, qmnd and qint – that should be solved from t to t+K, and 1 endogenous variable – h – that is solved from (t+1) to (t+K). Therefore, there are [11(t+K) – 1] linear equations.

[14] Appendix 1 explains how we found the solution for this non-linear system.

In addition, we assumed that the system is in the steady state and the variables are defined as deviation from their equilibrium values. Hence, in the absence of shocks the system will stay in a trivial equilibrium. In order to evaluate the dynamic properties of the system, we assumed shocks in some key variables and the resulting impulse responses are shown in the following section[15].

4 – Simulations

All the coefficients of the system are calibrated based on previous estimations. The system is solved 70 periods ahead subject to demand, supply, interest rate, and risk premium shocks. The purpose of the simulations is to compare the impulse response using different hypotheses of the nominal exchange rate path, using the random walk with monetary surprises and the UIP with fundamentals. We run the simulation using different periods in which the balance of payment is expected to clear. The results are robust across different hypothesis. Hence, as the hypothesis $K = 12$ is more credible, we will keep it on all the simulations from now on

Graph 1 shows the impulse responses of a 1% shock in inflation. We consider it as a supply shock[16]. One clear result is that there is no great difference between the two hypotheses in the output gap response to the inflation shock. The inflation and the interest rate response are similar in the very short run (the 4 first periods) and after that, both responses with random walk hypothesis converge very rapidly. The main difference is concerned to the exchange rate. Under the RWMS model, the real exchange rate keeps appreciated for a long time while the nominal exchange returns to the equilibrium almost immediately. That slow convergence occurred because the only force that drives this model back to the equilibrium is the real exchange rate term in the IS curve. The appreciation of the real exchange rate reduces aggregate demand and

[15] Since the model is non-linear, we needed to try shocks of different magnitudes to evaluate the impulse response functions and they did not significantly differ from the one we present below, with 1% deviations from equilibrium.

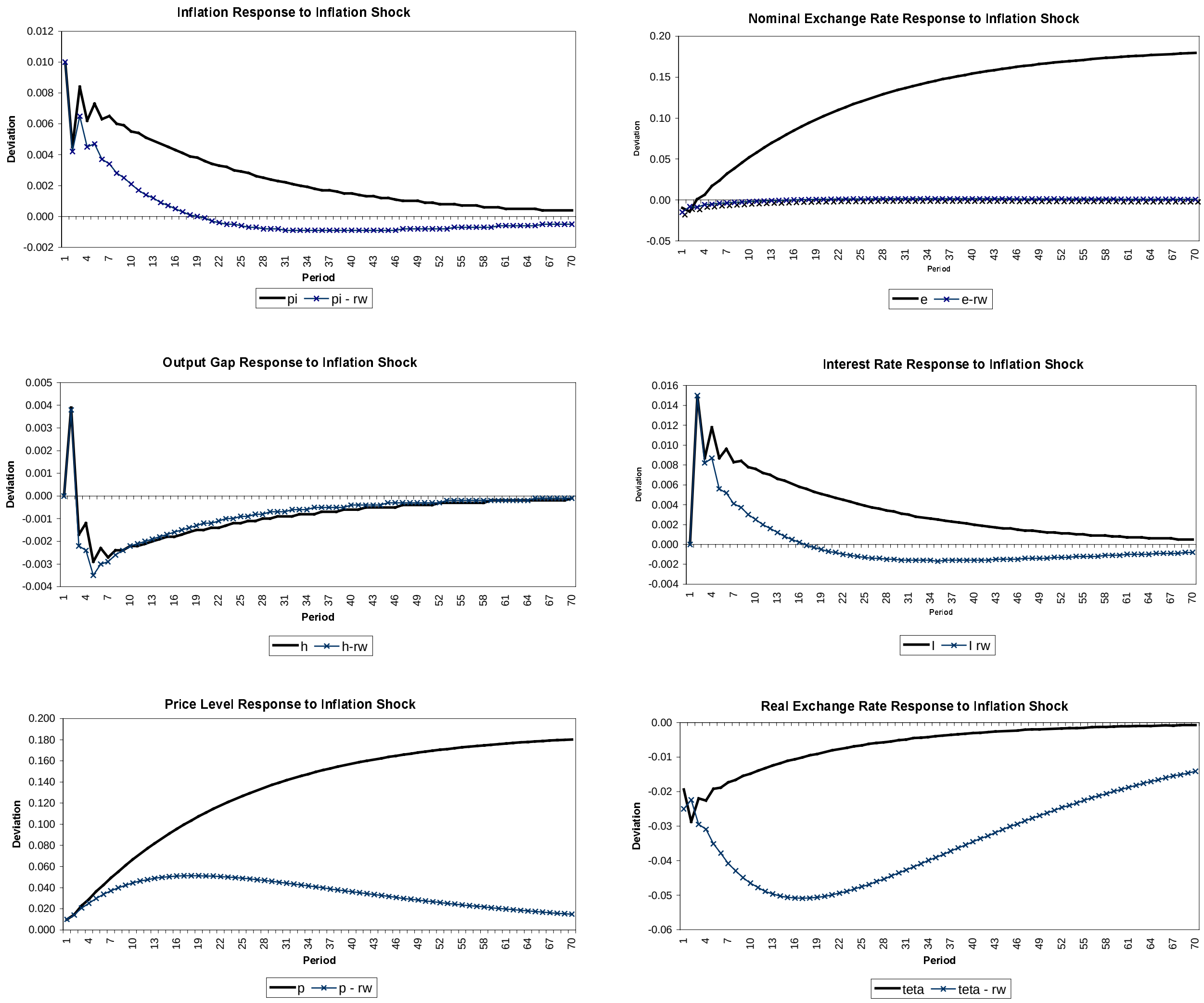
[16] This supply shock should be interpreted as cost-push shocks and not as shocks in potential output, like productivity shocks. As mentioned in footnote 7, we made the assumption that potential output is non-stochastic.

then (to the extent that it affects GDP) lowers inflation through the output gap. That corrects the imbalance in the exchange rate by decreasing the price level. On the other hand, in the UIP model, forward-looking expectations anchor the real exchange rate, reducing the likelihood of long-term deviation from the equilibrium.

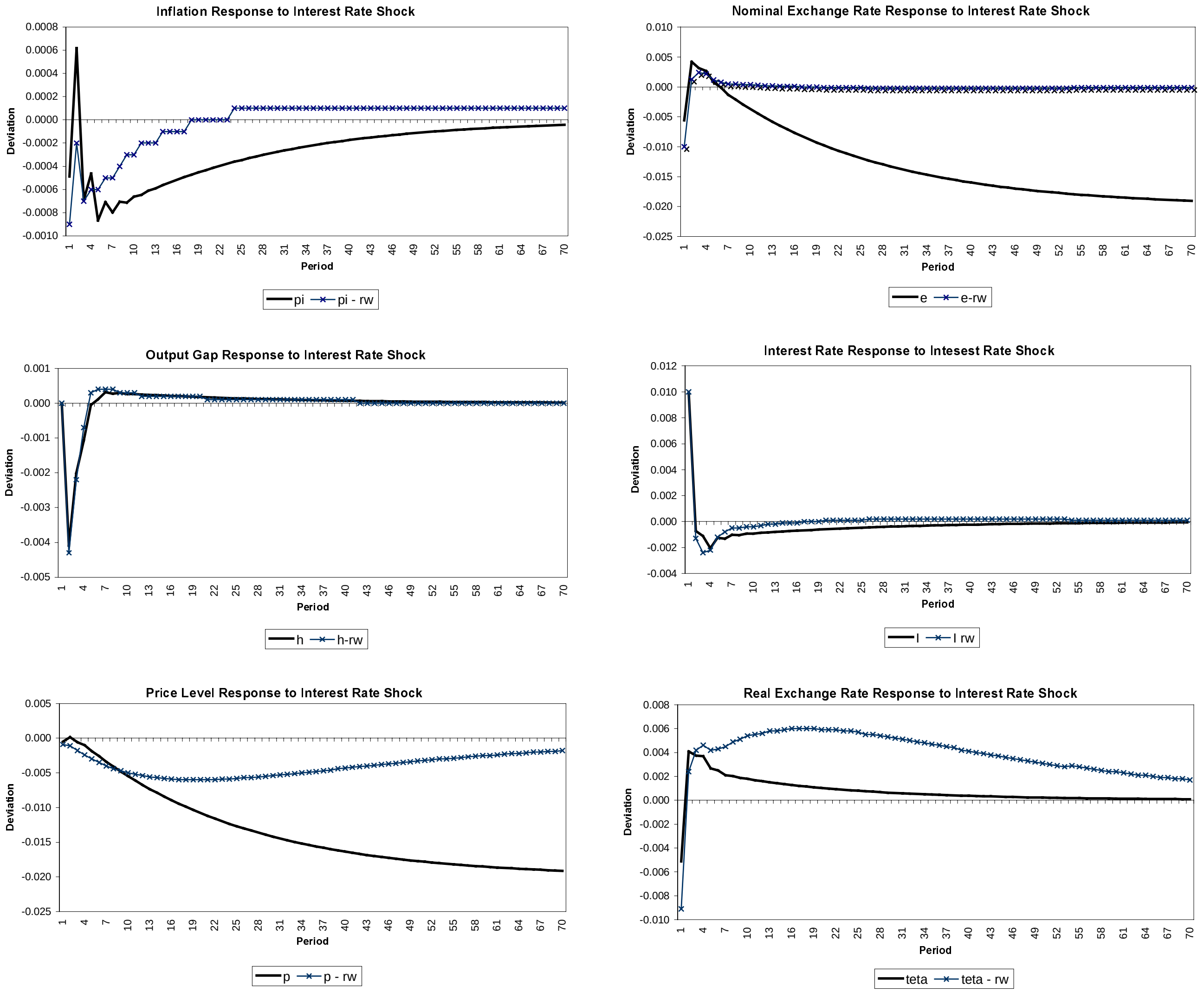
Graph 2 shows the impulse response to an interest rate shock. It is worth noting that interest rate and output gap responses are very similar and converge very quickly back to the equilibrium. The inflation responses, at first sight, look different, but when we observe the price level responses, they are similar in the short run. The real exchange rate with RWMS keeps undervalued during a longer period, when compared to the other hypothesis.

The impulse responses to an output gap shock are presented in Graph 3. The conclusions for both hypotheses are very similar to the inflation shock. However, the impulse response functions to a risk shock (Graph 4) diverge a lot to the impulse responses due to the others shocks and between both hypotheses. The RWMS responses converge much faster to the equilibrium, even for the real exchange rate, and the amplitude of the responses are also smaller than the UIP hypothesis. Hence, in the occurrence of this kind of shock, a monetary policy action will be different, depending upon the adopted hypothesis.

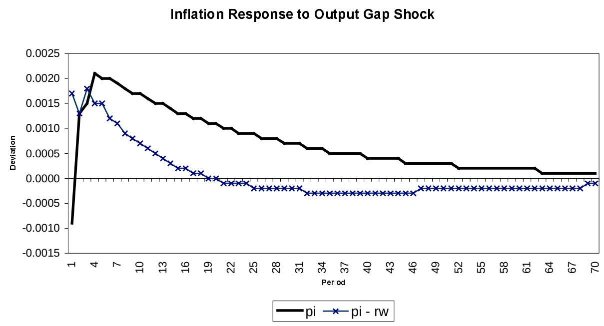
Graph 1: Inflation shock

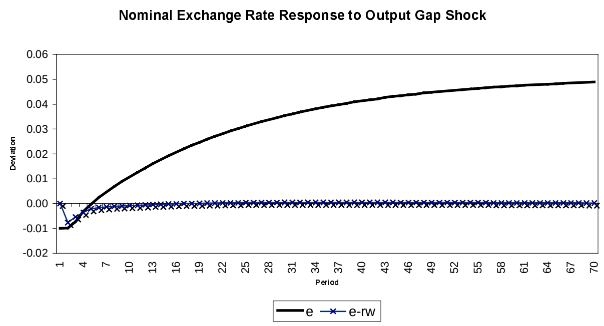


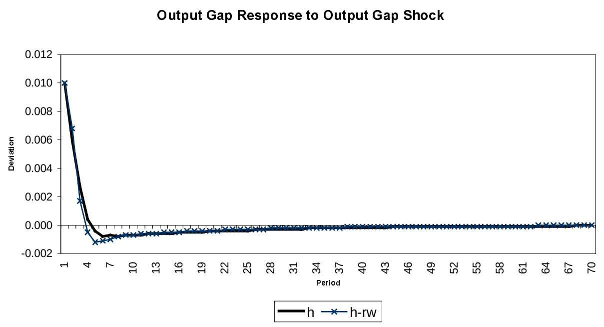
Graph 2: Interest rate shock

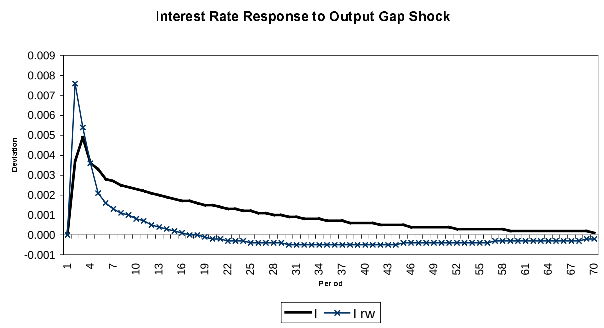


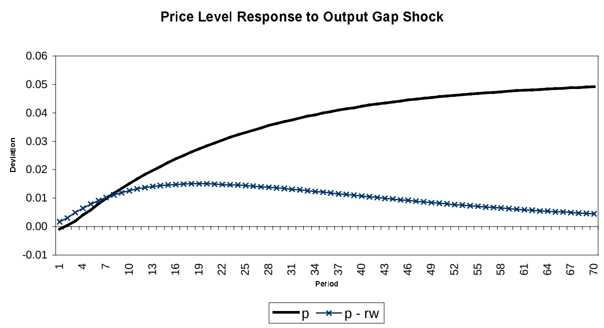
Graph 3: Output gap shock

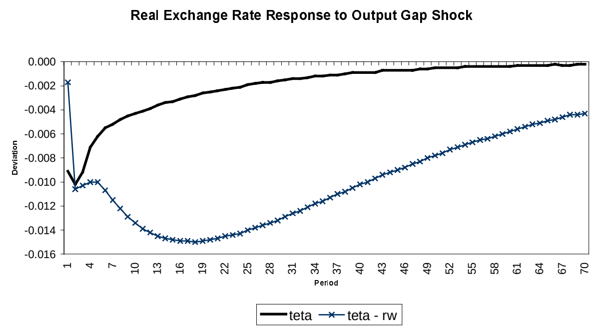




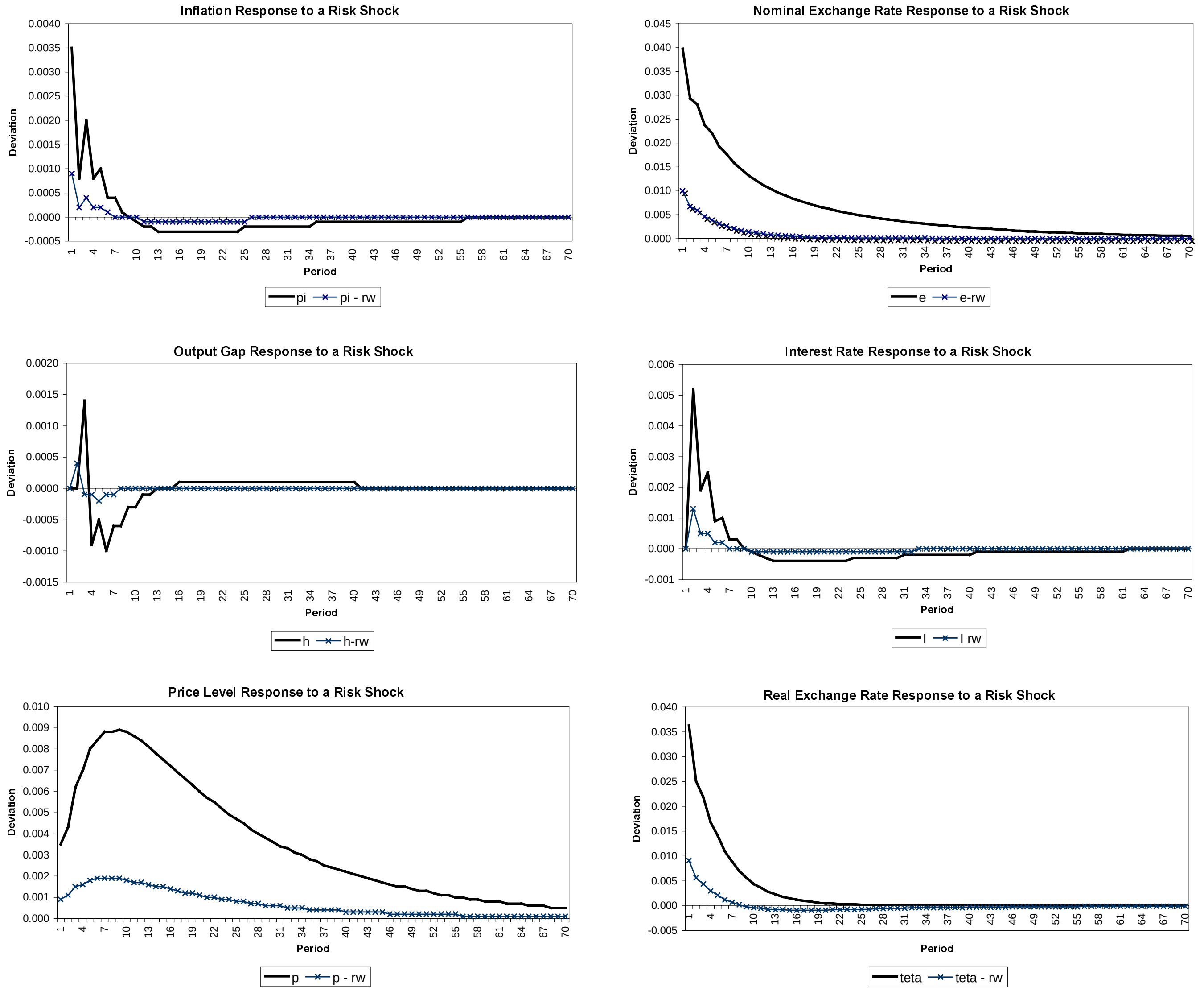








Graph 4: Risk shock



5 – Conclusions and Final Remarks

Our UIP-plus-fundamentals model allows for the components of balance of payments to affect the equilibrium real exchange rate. The role of the equilibrium exchange rate is to provide a terminal condition for UIP with rational expectations.

When the new approach is used for Brazilian data, the impulse responses of the UIP-with-fundamentals model appeared to be more realistic than those obtained from the RWMS model. For example, the response to a supply shock implied a much quicker return to equilibrium of the real exchange rate under the UIP with fundamentals model than under the RWMS one.

The simulation results change slightly when the expected time horizon to clear the balance of payments is altered. Furthermore, all the impulse responses have the same expected shape and the real variables return to the steady state value after a plausible lag.

From the impulse response functions we could see that inflation shocks result in much slower convergence than do output gap or risk premium shocks. Interest rate and inflation take approximately 40 quarters to converge to a 0,2%-deviation from equilibrium, for both hypotheses. This is related to the fact that interest rate has a direct impact on output gap and exchange rate but an indirect effect on inflation in our model of the transmission mechanism. Shocks in risk premium are followed by the quickest convergence to equilibrium: it takes up to 6 periods for interest rate, inflation and output gap converge back to equilibrium.

There are several interesting extensions to the model developed in this paper that we hope to make the subject of future work. The equilibrium condition might be reformulated as a constant current account to GDP ratio, or even a steady net external debt to GDP ratio. Another extension could be to endogenize the equilibrium criterion so that it would imply solving for the real exchange rate together with other system variables at the terminal date. A richer model of the transmission mechanism, including a forward-looking Phillips curve, more endogenous variables as the risk premium, other rules for the interest rate, etc., could also be explored. Another interesting step would be to try out the exchange rate models on countries with a longer period of exchange rate floating and inflation targeting, such as Australia, Canada, UK, and New Zealand.

References

Bogdanski, Joel , Alexandre A. Tombini and Sergio R. C. Werlang (2000)- “Implemeting Inflation Targeting in Brazil” – **Banco Central do Brasil Working Paper Series** nº1

Emerging Markets Research Team (2000) **“An Introduction to J. P. Morgan’s Emerging Markets Real Exchange Rate Model: Theory and Econometrics”,** mimeo.

Enders, Walter (1995) **Applied Econometric Time Series,** John Wiley & Sons, Inc

Freitas, Paulo S. and Marcelo K. Muinhos (1999) “A Simple Model for Inflation Targeting in Brazil” **- Banco Central do Brasil Working Paper Series** nº 18

Froot, Kenneth and Kenneth Rogoff (1995) “Perspectives on PPP and Long-Run Real Exchange Rates” in Grossman G. and K. Rogoff (eds) in **Handbook of International Economics Vol. 3** Elsevier

Meredith, G. & Chinn, M (1998): Long-Horizon Uncovered Interest Rate Parity, **NBER Working Paper** 6797

MacDonald, Ronald (1999) “Exchange Rate Behaviour: Are Fundamentals important?” **The Economic Journal** 109 (November) F672-F691

Meese, Richard and Kenneth Rogoff (1983) “Empirical exchange rate models of the seventies: do they fit out of sample” **Journal of International Economics** vol**.** 14 pp. 3-24

Meese, Richard and Kenneth Rogoff (1988) "Was it real? The exchange rate-interest differential relation over the modern floating-rate period" **Journal of Finance** 43

Muinhos, Marcelo (2001) **"Inflation Targeting in an open financially integrated emerging economy: the case of Brazil" –** Banco Central do Brasil -mimeo

Obstfeld, Maurice and Kenneth Rogoff (1996) – **Foundations of International Macroeconomics** The MIT Press, Cambridge MA

Rogoff, Keneth (1996) " The Purchasing power parity puzzle" in **Journal of Economic Literature** vol. 44, pg 1-19

Taylor, Mark P (1995) "The Economics of Exchange Rate" – **Journal of Economic Literature** Vol. XXXIII March 1995 pp. 13-47

Taylor, John B. (1998) - **Monetary Policy Rules** – University of Chicago Press, Chicago

Taylor, M. P. and D. A. Peel (2000) " Non Linear adjustment, long-run equilibrium and exchange rate fundamentals" in **Journal of International Money and Finance** vol. 19 n.1 p. 33-54

Wadhwani, Sushil B. (1999) - **Currency Puzzles** LSE Lecture on 16 September 1999

Walsh, Carl (1998)- **Monetary Theory and Policy** The MIT Press, Cambridge MA

Appendix 1

We used the following procedure to solve a system with 10(K+1)-1 linear and 1 non-linear (the Balance of Payments clearance) equations. The first step was to separate the linear from the non-linear part of the model. The linear part can be written in matrix notation as the following system

(Eq. A.1) AX=B + E,

where:

- A is a [11(K+1)-1]x[11(K+1)-1] matrix with the coefficients of the endogenous variables of the model;

- X is a [11(K+1)-1] column vector of the endogenous variables (π, h, i, e, qxb, qxm, qkab, qmbc, qmnd, qint);

- B is a [11(K+1)-1] column vector summarizing the exogenous variables, ie, each element of this vector is the product of the exogenous variables times their respective coefficients;

- E is a [11(K+1)-1] error term column vector.

Table A.1 below describes the variables pertaining to this linear system:

Table A.1

Left Hand Side Variables	Period	Right Hand Side Variables: Exogenous and Pre-Determined	Right Hand Side Variables: Endogenous
π	T	π(t-1), π(t-2), h(t-1), e(t-1), pf(t-1), pf(t)	E(t)
	t+1	π(t-1), h(t), pf(t), pf(t+1)	π(t), e(t), e(t+1)
	t+j, j = 2 ... K	pf(t+j-1), pf(t+j)	π(t+j-2), π(t+j-1), e(t+j-1), e(t+j)
h	t+1	h(t)	i(t), p(t)
	t+j, j = 2 ... K	None	i(t+j-1), π(t+j-1), h(t+j-1)
i	T	π*(t), h(t), i(t-1)	π(t)
	t+j, j = 1 ... K	π*(t+j)	π(t+j), h(t+j), i(t+j-1)
e	t+j, j = 0 ... K-1	if(t+j)...if(t+K-1), x(t+j)...x(t+K-1)	i(t+j)...i(t+K-1), e(t+K)
	t+K	p(t-1), pf(t+K)	π(t)...π(t+K), θ(t+K)
qxb	T	pxb(t-1), wy(t), qxb(t-1), qxb(t-2)	Y(t)
	t+1	pxb(t), wy(t+1), qxb(t-1)	Y(t+1), qxb(t)
	t+j, j = 2 ... K	pxb(t+j-1), wy(t+j)	y(t+j), qxb(t+j-2), qxb(t+j-1)
qxs	T	pxs(t-1), wy(t), qxs(t-1)	y(t), θ(t)
	t+j, j = 1 ... K	pxb(t+j-1), wy(t+j)	y(t+j), qxs(t+j-1), θ(t)
qxm	T	wy(t), wy(t-1), y(t-1), y(t-2), qxm(t-1), qxm(t-2), qxm(t-3)	θ(t)
	t+1	wy(t+1), wy(t), y(t-1), qxm(t-1), qxm(t-2)	y(t), qxm(t), θ(t+1)
	t+2	wy(t+2), wy(t+1), qxm(t-1)	y(t), y(t+1), qxm(t), qxm(t+1), θ(t+2)
	t+j, j = 3 ... K	wy(t+j), wy(t+j-1)	y(t+j-1), y(t+j), qxm(t+j-2), qxm(t+j-1), θ(t+j)
qkp	T	pkp(t), tkp(t), qkp(t-1), y(t-2)	θ(t)
	t+1	pkp(t+1), tkp(t+1), y(t-1)	θ(t+1)
	t+j, j = 2 ... K	pkp(t+j), tkp(t+j)	y(t+j-2), θ(t+j)
qmbc	T	pmbc(t), tmbc(t-1), qmbc(t-1)	y(t), θ(t)
	t+j, j = 2 ... K	pmbc(t+j), tmbc(t+j-1)	y(t+j), qmbc(t+j-1), θ(t+j)
qmnd	T	qmnd(t-1)	y(t), θ(t)
	t+j, j = 2 ... K	None	y(t+j), qmnd(t+j-1), θ(t+j)
qint	T	pint(t), tint(t-1), qint(t-1)	y(t), θ(t)
	t+j, j = 2 ... K	pint(t+j), tint(t+j-1)	y(t+j), qint(t+j-1), θ(t+j)

where:

endogenous	**Exogenous**
π - inflation	π* - inflation target
p - price index	**if** - foreing interest rate
h - output gap	**wy** - world GDP
i - interest rate	**pxb** - basic goods price

e - exchange rate
θ - real exchange rate
qxb - basic goods export quantum
qxs - semi-manufaturated goods export quantum
qxm - manufaturated goods export quantum
qkp - kapital goods import quantum

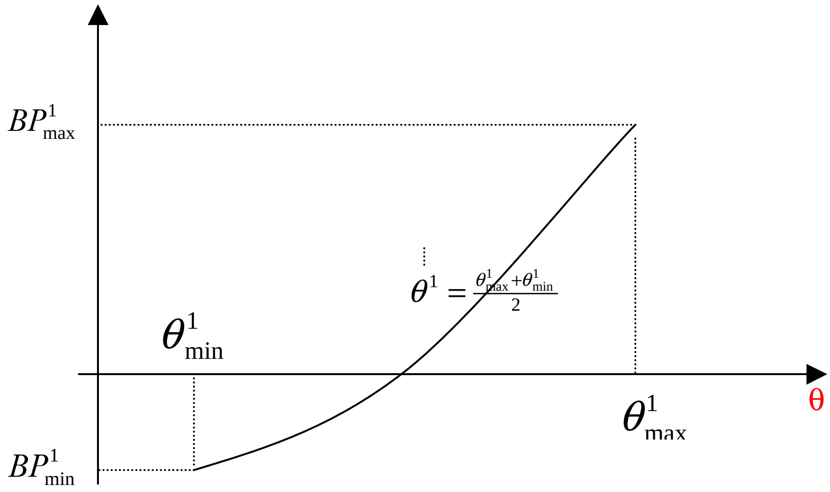
qmbc - durable goods import quantum
qmnd - non-durable goods import quantum
qint - raw material import quantum

pxs - semi-manufaturated goods quantum
pkp, tkp - kapital goods price and tax
pmbc, tmbc - durable goods price and tax
pint, tint - raw material price and tax

Given the exogenous and pre-determined variables, this system has a unique solution for each θ_{t+K}, so that X = X(θ_{t+k}). Furthermore, according to (13), the balance of payments is a function of θ_{t+k} and X, and we can rewrite it as in Equation A.2 below, where, for simplicity, we will refer to θ_{t+k} as θ.

(Eq. A.2) $\quad BP = BP(X, \theta_{t+k}) = BP(X(\theta_{t+k}), \theta_{t+k}) = BP(\theta)$

Therefore, the following step is to interact the linear part of the model, Equation A.1, with the non-linear equation, Equation A.2, to determine the real exchange rate that will clear the balance of payments. This is done by using the bipartition numerical method. It consists of choosing two values for θ ($\theta^1_{\min}$ e $\theta^1_{\max}$) in such a way that $BP^1_{\min}$ = BP($\theta^1_{\min}$) < 0 e $BP^1_{\max}$ = BP($\theta^1_{\max}$) > 0. This is illustrated in Figure A.1 below.



i) In the n^th^ iteration, for n ≥ 0, set $\theta^{n+1} = \frac{\theta^{n+1}_{max}+\theta^{n+1}_{min}}{2}$. Solve the linear part of the system to get the path for the output gap and recalculate the Balance of Payment, BP^{n+2} = BP(θ^{n+1}) . If $\left|BP^{n+2}\right| < \varepsilon$, where ε >0 is a (small) pre-determined value, we consider the system reached the final solution, with $\theta_{t+k} = \theta^{n+1}$ and X = X(θ^{n+1}). Otherwise:

i-a) if $BP^{n+2} < 0$, then $\theta^{n+2}_{min} = \theta^{n+1}$ and $\theta^{n+2}_{max} = \theta^{n+1}_{max}$.

i-b) if $BP^{n+2} > 0$, then $\theta^{n+2}_{max} = \theta^{n+1}$ and $\theta^{n+2}_{min} = \theta^{n+1}_{min}$.

ii) Add 1 to n and return to step (i).

At time t, this procedure will yield a trajectory for each of the endogenous variables from t to t+K. At time t+1, the system takes the solution for t as given and repeats the procedure again, yielding trajectories for the endogenous variables from t+1 to t+K+1. At time t+2 we take the solution for t+1 as given and repeat again the algorithm. We keep doing this exercise until we get the response functions for 70 periods.

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