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Generalized Hyperbolic Distributions and Brazilian Data*

José Fajardo [a] and Aquiles Farias [b]

Abstract

The aim of this paper is to discuss the use of the Generalized Hyperbolic Distributions to fit Brazilian assets returns. Selected subclasses are compared regarding goodness of fit statistics and distances. Empirical results show that these distributions fit data well. Then we show how to use these distributions in value at risk estimation and derivative price computation.

Key words:
Generalized Hyperbolic Distributions, Derivatives Pricing, Fat Tails, Fast Fourier Transformation
JEL classification: C52, G10

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1 Introduction

Since Mandelbrot (1963), the behavior of assets returns have been extensively studied. Using low frequency data, he shows that log returns present heavier tails than the Gaussian's, so he suggested the use of Pareto stable distributions. Unfortunately these distributions present too fat tails, fact that is refused by empirical evidence. Using high frequency data others "stylized facts" of real-life returns have been studied namely: volatility clustering, long range dependence and aggregational Gaussianity. Many econometric models have been suggested to explain part of these asset return behavior, among then we can mention the Generalized autoregressive conditionally heteroscedastic model(GARCH). Unfortunately, GARCH can not explain long range dependence. Other models have been suggested to capture this behavior, we refer the reader to Rydberg (1997) for a survey of this models.

An usual classification of the models developed in the literature is: discrete time models and continuous time models. In this paper we will work upon the later class. An important class called diffusion models has been largely used by the authors, but the use of a Brownian Motion implies the Gaussian distributions of log-returns, fact that is very well-known as not satisfied by the majority of the asset returns. Recently a class of distributions called Generalized Hyperbolic Distributions (GHD) have been suggested to fit financial data. The development of this distributions is due to Barndorff-Nielsen (1977). He applied the Hyperbolic subclass to fit grain size of sand subjected to continuous wind blow. Further, in Barndorff-Nielsen (1978), the concepts were generalized to the GHD. Since its development, GHD were used in different fields of knowledge like physics, biology [1] and agronomy, but Eberlein and Keller (1995) were the first to apply these distributions to finance. In their work they use Hyperbolic subclasses to fit German data. In Keller (1997), expressions for derivative pricing are developed and Prause (1999) applies GHD to fit financial data, using German stocks and American indexes, extending Eberlein and Keller (1995) work. He also prices derivatives, measures Value at Risk and extends to

[1] To an application to other fields of knowledge we suggest Blæsild and Sørensen (1992)

the multivariate case of these distributions. In early 90's Blæsild and Sørensen (1992) developed a computer program called Hyp which was used to estimate the parameters of Hyperbolic subclass distributions up to three dimensions. Prause (1999) develops a program to estimate the GHD parameters, but the structure of these programs are not freely available.

In the Brazilian Market some works have been carried on to study these stylized facts. Using the Hyp software Fajardo et al. (2001) analyses the goodness of fit of Hyperbolic distributions (subclass of the GHD) and Duarte and Mendez (1999), Issler (1999),Mazuchelli and Migon (1999) and Pereira et al. (1999) use the GARCH model to study Brazilian data.

In this paper we generalize Fajardo et al. (2001) using GHD to fit Brazilian data, moreover we show how to price derivatives and estimate Value at Risk which do not appear in Fajardo et al. (2001). The main difficulty of the paper is to create the parameter estimation algorithm and the Fast Fourier Transformation (FFT) to obtain the t-fold convolution of the GHD, since in most cases this family is not closed under convolution.

The paper is organized as follows: in section 2 we present the Generalized Hyperbolic Distributions and their subclasses, section 3 describes the GHD estimation procedures, and section 4 presents the data used for this estimation. In section 5 we show the results obtained in GHD estimation, in section 6 we apply some statistical tests and distances to evaluate the goodness of fit. In section 7 we apply GHD to price derivatives. And in section 8 we test the feasibility of VaR measures using GHD. In the last section we have the conclusions.

2 Generalized Hyperbolic Distributions

The density probability function of the one dimensional GHD is defined by the following equation:

$$D_{GH}(x;\alpha,\beta,\delta,\mu,\lambda) = a(\lambda,\alpha,\beta,\delta)(\delta^2+(x-\mu)^2)^{\frac{(\lambda-\frac{1}{2})}{2}} K(\lambda,\alpha,\delta,\mu,\beta) \tag{1}$$

with,

$$K(\lambda,\alpha,\delta,\mu,\beta) = K_{\lambda-\frac{1}{2}}(\alpha\sqrt{\delta^2+(x-\mu)^2})\exp(\beta(x-\mu)) \tag{2}$$

where,

$$a(\lambda,\alpha,\beta,\delta) = \frac{(\alpha^2-\beta^2)^{\frac{\lambda}{2}}}{\sqrt{2\pi}\alpha^{(\lambda-\frac{1}{2})}\delta^\lambda K_\lambda(\delta\sqrt{\alpha^2-\beta^2})} \tag{3}$$

is a norming factor to make the curve area total 1 and

$$K_\lambda(x) = \frac{1}{2}\int_0^\infty y^{\lambda-1} exp\left(-\frac{1}{2}x\left(y+y^{-1}\right)\right) dy$$

is the modified Bessel function [2] of third kind with index λ.

The parameters domain are:

$$\mu,\lambda \in \mathbb{R}$$
$$-\alpha < \beta < \alpha$$
$$\delta,\alpha > 0.$$

where μ is a location parameter, δ is a scale factor, compared to Gaussians σ in Eberlein (2000), α and β determine the distribution shape and λ defines the subclasses of GHD and is directly related to tail fatness (Barndorff-Nielsen and Blæsild, 1981)). In fig. 1 we have that the log-density is hyperbolic while Gaussian distribution log-density is a parabola, for this reason it is called Generalized Hyperbolic.

We can do a reparametrization of the distribution so that the new parameters are scale invariant. The new parameters are defined in equations 4.

$$\begin{gathered} \zeta = \delta\sqrt{\alpha^2-\beta^2} \quad \varrho = \frac{\beta}{\alpha} \\ \xi = (1+\zeta)^{-\frac{1}{2}} \quad \chi = \xi\varrho \\ \bar{\alpha} = \alpha\delta \qquad \bar{\beta} = \beta\delta \end{gathered} \tag{4}$$

[2] For more details about Bessel functions, see Abramowitz and Stegun (1968).

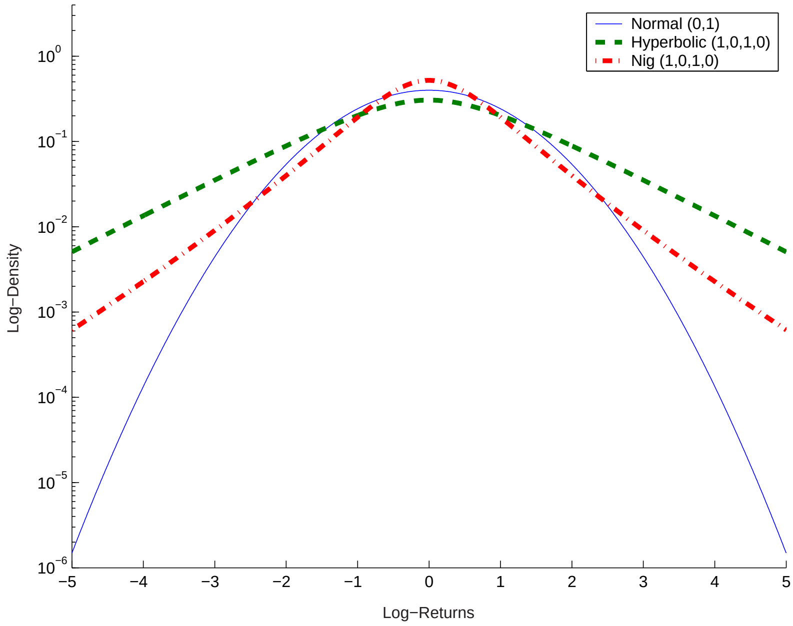

Fig. 1. Comparison among Normal, Hyperbolic subclass and NIG centered and symmetric log-densities

The GHD have semi-heavy tails, this name due to the fact that their tails are heavier than Gaussian's, but they have finite variance, which is clearly observed in (5):

$$gh(x;\lambda,\alpha,\beta,\delta) \sim \mid x \mid^{\lambda-1} \exp\left((\mp\alpha+\beta)x\right) \text{ as } x \to \pm\infty \tag{5}$$

Many distributions can be obtained as subclasses or limiting distributions of GHD. We cite as examples the Gaussian distribution, Student's T and Normal Inverse Gaussian. We refer to Barndorff-Nielsen (1978) and Prause (1999) for a detailed description. A negative aspect of these distributions is that in most cases they are not closed under convolution, which makes derivative pricing more difficult.

Using Bessel functions simplifications when its index is $\mathbb{N}+\frac{1}{2}$ we can get simpler densities to some subclasses. When $\lambda = 1$ we have the Hyperbolic Distribution subclass. As showed in (6) the Bessel function appears only in the norming factor, which makes maximum likelihood estimation easier. The simplified density is given by:

$$hyp(x;\alpha,\beta,\delta,\mu) = \frac{\sqrt{\alpha^2-\beta^2}}{2\delta\alpha K_1(\delta\sqrt{\alpha^2-\beta^2})} exp\left(-\alpha\sqrt{\delta^2+(x-\mu)^2}+\beta(x-\mu)\right) \quad (6)$$

These distributions are not closed under convolution.

When we make $\lambda = -0.5$, and using Bessel functions properties, we get a distribution called Normal Inverse Gaussian distribution whose density is given by:

$$nig(x;\alpha,\beta,\delta,\mu) = \frac{\alpha\delta}{\pi}\exp\left(\delta\sqrt{\alpha^2-\beta^2}+\beta(x-\mu)\right)\frac{K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\sqrt{\delta^2+(x-\mu)^2}} \quad (7)$$

This name is due to the fact that it can be represented as a mixture of a Generalized Inverse Gaussian with a Normal distribution. More details on these distribution can be found in Rydberg (1997), Keller (1997), Barndorff-Nielsen (1997) and Barndorff-Nielsen (1998). This subclass has the desired closed under convolution property (see (8)). This fact turns this subclass more adequate to price derivatives.

$$nig^{*t}(x;\alpha,\beta,\delta,\mu) = nig(x;\alpha,\beta,t\delta,t\mu); \quad (8)$$

3 Estimation Algorithm

For the estimation of GHD parameter we use maximum log-likelihood estimators, assuming log-returns independence, because it is the only non biased method (see Prause (1999)). This method was also used by Blæsild and Sørensen (1992) in the development of Hyp software, used to estimate multivariate Hyperbolic subclass ($\lambda = 1$) parameters.

Finding the maximum log-likelihood parameters consist in searching the parameters that maximize the following function:

$$L = \sum_{i=1}^{n} log\left(GH(x_i;\alpha,\beta,\delta,\mu,\lambda)\right) \quad (9)$$

This estimation consists in a numerical optimization procedure. We use the Downhill Simplex Method which makes no use of derivatives, developed by Nelder and Mead (1965), with some modifications (due to parameter restrictions). It is worth noting that Prause (1999) used a Bracketing Method, but our Downhill Simplex Method showed to be more consistent.

This method requires starting values to begin optimization, and in this case we followed Prause (1999) who used a symmetric distribution ($\beta = 0$) with a reasonable kurtosis ($\xi \approx 0.7$) to equalize the mean and variance of the GHD to those of the empirical distribution. This is done because when we use a symmetric distribution and fix the kurtosis, we have easy solvable equations, reducing computational efforts.

In all numerical optimization we have to define the tolerance of the search, and we decided to use 1×10^{-10}. This tolerance was applied in absolute ways to the function evaluation and to the parameters sum variation.

The numerical maximum likelihood estimation does not have a convergence analytical proof, but even using different starting values it has showed empirical convergence (Prause, 1999).

4 Data

The empirical evaluation use Brazilian assets that have the minimum liquidity requirement. Our sample consists of 14 assets and the Ibovespa index. The assets also represent different sectors of economy and public, private and privatized institutions. The data consisted of the daily log-returns which were calculated using:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

The price of the assets were adjusted according to their rights like dividends, splits, groupings, etc.

The samples with their respective periods are in table 4, point out that when the sample starting date is not 07/01/1994 it is because the asset started to be traded only after that date, which is the case of the assets that resulted of Telebras privatization. The starting date was chosen due to the Real plan (brazilian currency stabilization plan), that brought some stability to the prices avoiding daily correction of asset prices.

Table 1. Samples

Asset	***Ticker***	***Start***	***End***
Banco Itaú - PN	Itau4	07/01/1994	12/13/2001
Banco do Brasil - PN	Bbas4	07/01/1994	12/13/2001
Bradesco - PN	Bbdc4	07/01/1994	12/13/2001
Cemig - PN	Cmig4	07/01/1994	12/13/2001
Cia Siderúrgica Nacional - ON	Csna3	07/01/1994	12/13/2001
Eletrobrás - PNB	Elet6	07/01/1994	12/13/2001
Embratel Participações - PN	Ebtp4	09/21/1998	12/13/2001
Ibovespa	Ibvsp	07/01/1994	12/13/2001
Petrobrás - PN	Petr4	07/01/1994	12/13/2001
Petrobrás Distribuidora - PN	Brdt4	07/04/1994	12/13/2001
Tele Celular Sul - PN	Tcsl4	09/21/1998	12/13/2001
Tele Nordeste Celular - PN	Tnep4	09/21/1998	12/13/2001
Telemar - PN	Tnlp4	09/22/1998	12/13/2001
Telesp - PN	Tlpp4	07/01/1994	12/13/2001
Vale do Rio Doce - PNA	Vale5	07/01/1994	12/13/2001

5 Empirical Results

In this section we present the empirical estimation results.

5.1 Hyperbolic subclass

In table 2 we have the estimated parameters and the log-likelihood value. All samples but Cemig have asymmetric distributions estimations since β is different from 0. The same samples were submitted to Hyp software Blæsild and Sørensen (1992) and the results were equivalent.

Table 2. Estimated parameters of Hyperbolic subclass ($\lambda = 1$) and log-likelihood values.

Sample	α	β	δ	μ	***Log-Likelihood***
Bbas4	41.5931	3.896030	0.0130788	-0.005505	3512.08
Bbdc4	47.5455	-0.000629	2.11E-08	-1.45E-09	3984.49
Brdt4	51.7172	4.103200	0.011870	-0.003185	3925.06
Cmig4	43.3673	5.07E-07	0.0103856	0.000362	3677.76
Csna3	47.4118	0.008238	2.11E-08	3.84E-11	3976.50
Ebtp4	36.7618	3.808810	0.0196585	-0.00749485	1409.48
Elet6	41.1231	1.172670	0.0145371	-0.00120626	3522.58
Ibvsp	57.6958	-0.006950	0.00957707	0.00118714	4165.69
Itau4	49.9390	1.749500	2.02E-08	2.28E-09	4084.89
Petr4	45.7651	0.797027	0.010191	2.75E-05	3755.75
Tcsl4	35.5804	0.000417	0.032395	0.001370	1325.28
Tlpp4	41.7147	-0.005093	4.65E-07	1.34E-07	3753.45
Tnep4	34.9981	3.461250	0.0310437	-0.00540191	1314.64
Tnlp4	42.7018	0.002519	0.020710	0.000345821	1501.83
Vale5	48.7391	2.988860	0.00560929	-0.00170568	3955.42

In fig. 3, at the end of the paper, we have the Vale do Rio Doce (Vale5) Hyperbolic subclass estimation compared to the Gaussian estimation and Empirical distribution. The figure leads us to visually evaluate the better fit of Hyperbolic subclass.

The Hyperbolic subclass seems to better fit the leptokurtic behavior of the empirical curve. To see the fitness of the tails of the distribution we refer to log-density graphic 4. We can see again that, visually, the Hyperbolic distribution is closer to the empirical distribution.

5.2 *Normal Inverse Gaussian subclass*

The Normal Inverse Gaussian Distribution (NIG) ($\lambda = -0.5$) has been very used and for German data (Prause, 1999) it presented better fit than Hyperbolic. The estimated parameters and the log-likelihood values are in table 3.

Table 3. Estimated NIG parameters.

Sample	α	β	δ	μ	***Log-Likelihood***
Bbas4	26.1863	3.3516300	0.0356299	-0.00485126	3512.65
Bbdc4	25.2340	0.0026819	0.0234468	7.6154E-05	3978.71
Brdt4	36.7793	3.7894200	0.0324629	-0.00289173	3922.97
Cmig4	27.0195	0.0008486	0.0326901	0.00031385	3681.63
Csna3	25.3325	2.4752800	0.0235746	-0.00149070	3949.86
Ebtp4	22.8770	3.7054400	0.0424283	-0.00746229	1412.84
Elet6	23.6626	0.0033938	0.0343010	2.6479E-05	3532.29
Ibvsp	31.9096	-0.0034818	0.0232961	0.00122217	4178.17
Itau4	30.7352	0.0014784	0.0248846	0.00081258	4065.40
Petr4	25.3411	0.0008998	0.0284943	0.00067786	3764.37
Tcsl4	24.5055	-0.0002506	0.0555594	0.00123335	1327.02
Tlpp4	20.3812	0.0037757	0.0249410	0.00032556	3763.65
Tnep4	21.8374	0.0013490	0.0519268	0.00118522	1317.41
Tnlp4	26.2133	0.0009838	0.0384229	0.00038867	1505.29
Vale5	26.6233	0.0047853	0.0249197	9.7056E-05	3956.39

In fig. 5 we have the density graphics, while in fig. 6 we show the log-density graphics of Vale do Rio Doce asset. Graphically we can not see much difference between Hyperbolic and NIG distributions, but both seem better than Normal. The χ^2- test of NIG are in table 9.

5.3 *Generalized Hyperbolic*

A GHD is obtained through the λ freedom. Who first tested it empirically to financial data was Prause (1999). Following him Raible (2000) published his work using the same distributions.A big difficulty appeared when the parameters δ and μ tended simultaneously to zero (Raible, 2000). The numerical solution to this problem was to use specific treatments to the case following Hanselman and Littlefield (2001) and Abramowitz and Stegun (1968).

In Brazil they have never been used since Fajardo et al. (2001) only fit the Hyperbolic subclass. Table 4 contains the estimations parameters for all samples studied.

Table 4. Estimated GHD Parameters.

Sample	α	β	δ	μ	λ	***L-Likelihood***
Bbas4	30.7740	3.52665	0.02946	-0.00507	-0.0492	3512.73
Bbdc4	47.5455	-0.00063	2.1E-08	-1.4E-09	1	3984.49
Brdt4	56.4667	3.44169	0.00259	-0.00259	1.4012	3926.68
Cmig4	1.4142	0.74908	0.05150	-0.00038	-2.0600	3685.43
Csna3	46.1510	0.00941	2.2E-08	4.7E-11	0.6910	3987.52
Ebtp4	3.4315	3.43159	0.06704	-0.00708	-2.1773	1415.64
Elet6	1.4142	0.01203	0.05244	8.7E-05	-1.8987	3539.06
Ibvsp	1.7102	-1.66835	0.03574	0.00199	-1.8280	4186.31
Itau4	49.9390	1.74950	2.0E-08	2.3E-09	1	4084.89
Petr4	7.0668	0.48481	0.04163	0.00032	-1.6241	3767.41
Tcsl4	1.4142	-3.3E-06	0.08609	0.00114	-2.6210	1329.64
Tlpp4	6.8768	0.49049	0.03588	2.3E-05	-1.3333	3766.28
Tnep4	2.2126	2.21267	0.07857	-0.00280	-2.2980	1323.66
Tnlp4	1.4142	0.00208	0.05897	0.00045	-2.1536	1508.22
Vale5	25.2540	2.61339	0.02645	-0.00146	-0.6274	3958.47

As desired, the GHD estimations had higher log-likelihood values than its subclass, but in Bradesco and Itaú Samples where it is equal. The major samples had λ between -0.62 e -2.62 that is similar to the results obtained by Prause (1999). In figs. 2 and 7 we have the density and log-density graphics.

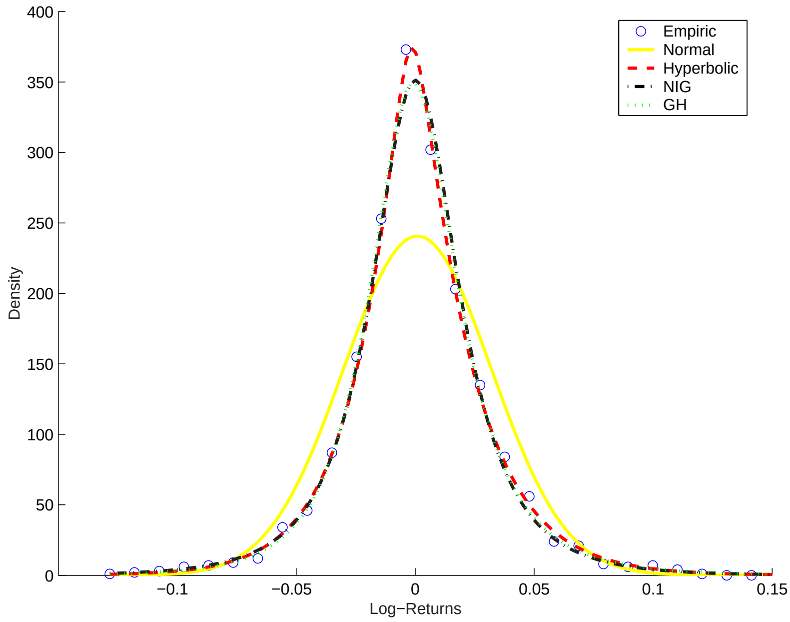

Fig. 2. Vale do Rio Doce Densities: Empiric x Hyperbolic x Normal x NIG x GH

6 Testing Goodness of Fit

In this section we test the goodness of fit, to this end we use the following tests and distances:

- χ^2 test: this test was used by Eberlein and Keller (1995) and Fajardo et al. (2001). This test is not recommendable for evaluating continuous distributions (see Press et al. (1992)), on the other hand Blæsild and Sørensen (1992) report that although the chi-square test tends to reject statistical test for large samples, our tests do not report that fact (table 8). This fact is due to the particular behavior of Brazilian market.
- Kolmogorov distance: this test is more suitable than chi-square test for continuous distributions. Its expression is given by:

$$KS = \max_{x \in \mathbb{R}} |F_{emp}(x) - F_{est}(x)| \tag{10}$$

- Kuiper distance: this is another distance evaluation used to test goodness of fit of continuous distributions. The main difference between Kuiper and Kolmogorov distance is that the first consider upper differences different from lower differences and in the late all distances are considered equally. Its expression is given by:

$$KP = \max_{x\in\mathbb{R}}\{F_{emp}(x) - F_{est}(x)\} + \max_{x\in\mathbb{R}}\{F_{est}(x) - F_{emp}(x)\} \tag{11}$$

- Anderson & Darling distance: a third distance evaluation used was the Anderson & Darling distance (12). The main difference between it and Kolmogorov's distance is that the first pays more attention to tail distances (Hurst et al., 1995).

$$AD = \max_{x\in\mathbb{R}} \frac{|F_{emp}(x) - F_{est}(x)|}{\sqrt{F_{est}(x)(1 - F_{est}(x))}} \tag{12}$$

Following we present the results obtained with each test.

6.1 Chi-Square Test

We present the Chi-Square test for GHD and the test for the Hyperbolic and NIG subclasses are presented in tables 8 and 9 at the end of the paper.

Table 5. χ^2-test for the GHD

Sample	***Statistic***	***P-Value***	***Degrees of Freedom***
Bbas4	23.6516	0.0876783	15
Bbdc4	34.1268	0.000902191	14
Brdt4	66.2152	2.01218E-08	21
Cmig4	21.2875	0.165019	15
Csna3	141.597	0	19
Ebtp4	13.6279	0.341022	11
Elet6	21.383	0.268148	17
Ibvsp	13.5203	0.511037	13
Itau4	32.5035	0.0819379	22
Petr4	15.3088	0.718927	18
Tcsl4	18.9641	0.162971	13
Tlpp4	22.5389	0.0840841	14
Tnep4	13.3699	0.522905	13
Tnlp4	16.5175	0.225828	12
Vale5	16.1775	0.462554	15

From table 5 we observe that with 5% of significance level we can not reject the null hypothesis of GHD behavior for 12 assets, in the NIG case we can not reject the null hypothesis in 11 assets and in the Hyperbolic subclass case we can not reject the null hypothesis in 9 assets.

6.2 Kolmogorov Distance

We present in table 6 the Kolmogorov distances of the NIG, Hyperbolic and GH distributions. In the Gh case all samples but CSNA3 can not be rejected using 1% of significance using Kolmogorov test and Ibovespa index got a P-value of 99.69%.

Table 6. Kolmogorov distances.

Sample	***Normal***	***Hyperbolic***		***NIG***		***GH***	
	KS	***KS***	P-Value	***KS***	P-Value	***KS***	P-Value
Bbas4	0.0585	0.0202	0.4446	0.0252	0.1938	0.0236	0.2611
Bbdc4	0.0682	0.0279	0.1112	0.0282	0.1052	0.0279	0.1112
Brdt4	0.0505	0.0240	0.2380	0.0303	0.0664	0.0252	0.1914
Cmig4	0.0559	0.0238	0.2440	0.0256	0.1779	0.0270	0.1354
Csna3	0.0744	0.0355	0.0192	0.0382	0.0092	0.0501	0.0002
Ebtp4	0.0699	0.0253	0.6818	0.0259	0.6537	0.0234	0.7694
Elet6	0.0598	0.0150	0.7968	0.0123	0.9415	0.0103	0.9897
Ibvsp	0.0661	0.0208	0.3967	0.0166	0.6833	0.0093	0.9970
Itau4	0.0681	0.0347	0.0233	0.0340	0.0276	0.0347	0.0233
Petr4	0.0640	0.0142	0.8526	0.0133	0.8993	0.0126	0.9294
Tcsl4	0.0458	0.0220	0.8307	0.0236	0.7594	0.0253	0.6823
Tlpp4	0.0784	0.0193	0.4924	0.0225	0.3062	0.0233	0.2691
Tnep4	0.0584	0.0187	0.9405	0.0239	0.7456	0.0219	0.8342
Tnlp4	0.0597	0.0178	0.9615	0.0188	0.9387	0.0178	0.9616
Vale5	0.0751	0.0099	0.9931	0.0121	0.9497	0.0108	0.9813

6.3 *Kuiper Distance*

In table 10, at the end of the paper, we have the Kuiper distances of Hyperbolic subclass, NIG and GH distributions. In Hyperbolic case we verify that 13 samples can not be rejected using 1% of significance. In NIG case we can not reject the null hypothesis for 12 samples (1% of significance). The Kuiper test rejects with 1% only two samples, in GH case, but even in the rejected samples the distance evaluated in the above estimates are smaller then Normal distances.

6.4 *Anderson & Darling Distance*

We present the results in table 11. We observe that this distance clearly shows the difference of fitness in the distributions tails. Analyzing the distances in comparison with the Hyperbolic we can deduce that the NIG is better as far as tail fitness is concerned. The Anderson and Darling test shows that GHD fit better in tails than Hyperbolic and are similar to NIG distances.

7 Derivative Pricing

Since Black and Scholes (1973), closed formula for European calls have been analyzed, but these models assume that the underlying distribution of the log-returns is Normal. More recently Prause (1999) and Raible (2000) presented the Lévy Generalized Hyperbolic process, where they assume that the log-returns of assets follow a GHD or one of its subclasses. Now we price European calls with Brazilian assets.

7.1 *Generalized Hyperbolic Distributions Convolution*

The first step on derivative pricing is calculating the GHD convolution, except for NIG subclass. Such subclass has the closed formula in (13).

$$NIG^{*t}(x;\alpha,\beta,\delta,\mu) = NIG(x;\alpha,\beta,t\delta,t\mu) \tag{13}$$

To solve the convolution problem using other subclasses we use Fourier transforms. The characteristic function is obtained using a Fourier transform and a transformed function multiplication is similar to the original function convolution, so we follow these steps:

1 - Apply Fourier transform in estimated GHD density.

2 - Multiply this transform by as many convolutions as we need.

3 - Apply the Inverse Fourier transform to obtain the GHD with t-fold convolution.

To easy calculation we use symmetric and centered distribution ($\beta, \mu = 0$) to guarantee that the functions are real (Press et al., 1992). So, we follow Prause (1999) and find a GHD as a function of a centered and symmetric GHD. This function is in (14).

$$GH^{*t}(x;\alpha,\beta,\delta,\mu,\lambda) = \frac{e^{\beta x}}{M_0^t(\beta)} gh^{*t}(x-\mu t;\lambda,\alpha,0,\delta,0) \tag{14}$$

Where $M_0^t(\beta)$ represents the moment generating function with parameter $\beta = 0$, powered to t and evaluated in β as an argument. Then we apply the fourier transform in centered and symmetric GHD, obtaining (15). Then we should apply the inverse Fourier transform, but it doesn't have an analytical solution.

$$GH^{*t}(x;\alpha,0,\delta,0,\lambda) = \frac{1}{\pi}\int_0^{\infty} cos(ux)\varphi(u;\alpha,\delta\lambda)^t du \tag{15}$$

To solve this kind of problem we use the Cooley and Tukey (1965) algorithm called Fast Fourier Transformation (FFT). We refer to Brigham (1988) and Press et al. (1992) for details on this algorithm applications.

The FFT calculates the Fourier transform and the inverse Fourier transform in an efficient way. The main concern here is related to variable transformations from frequency to time domain [3].

[3] To details about this variable transformation and a Matlab example we refer to Hanselman

After FFT application we have the density of symmetric and centered with t-fold convolution. To get the desired density we use (14).

7.2 Option Pricing Using Esscher Transforms

To price options with underlying assets following diffusions driven by Lévy processes we have to find an Equivalent martingale measure. Esscher (1932) presented a transform that was used by Gerber and Shiu (1994) for derivative pricing.

In GHD case this transformation to risk-neutral world is in (16).

$$GH^{*t,\vartheta}(x;\alpha,\beta,\delta,\mu,\lambda) = \frac{e^{\vartheta x}}{M^t(\vartheta)} GH^{*t}(x;\alpha,\beta,\delta,\mu,\lambda) \tag{16}$$

To find the ϑ parameter we have to solve (17).

$$r = log \frac{M(\vartheta+1)}{M(\vartheta)} \tag{17}$$

Where r is the risk free interest rate in the same period of estimated data and M is the moment generating function. The solution of this equation is obtained through numerical optimization.

The last step is to obtain the European Call prices. In this step we follow Keller (1997).

$$C_{GH} = S_0 \int_{\log \frac{K}{S_0}}^{\infty} GH^{*t,\vartheta+1}(x)dx - e^{-rt} K \int_{\log \frac{K}{S_0}}^{\infty} GH^{*t,\vartheta}(x)dx \tag{18}$$

where K is the strike price and S_0 is the stock price. In this case the Put-Call parity is valid, in order to calculate a Put price we use (19).

$$P_{GH} = C_{GH} - S_0 + e^{-rt} K \tag{19}$$

and Littlefield (2001).

7.3 Empirical Evidence

In figs. 8, 9 e 10 we have graphics with the Vale do Rio Doce Call behavior when changing certain parameters and, as expected, the major sensibility of Call prices are when the Option is at the money. Then we do comparative analysis of GHD call prices and Black and Scholes (1973) call prices of this asset. We obtain figs. 11, 12 e 13 that contain the difference between the prices. We can see clearly the desired W-Shape.

8 Value At Risk

The Value At Risk represents the worst loss, given a time period and a probability in market normal conditions (Jorion, 1997). In this section we briefly explore the parametric VaR using Normal and GHD as asset log-returns distributions. In fig. 14 we have the VaR graphics for different probability levels, and we can see that the GHD get closer to empirical probability.

Another way to test the efficiency of VaR models is Back Testing (Jorion, 1997) and we considered a portfolio with one asset only (Vale do Rio Doce) with an initial portfolio value of R$ 1,00. The initial sample used consisted of 252 observations, starting in 07/01/1994, reaching 1590 out of sample tests. Each day the VaR for 1 trading day holding period with 1% of probability was calculated. If the real loss were bigger then the predicted we consider this one exception. Then we aggregated this observation and repeated the steps to another day.

The results of the test are in table 7, that brings the number of exceptions and the Kupiec (1995) test P-Value whose null hypothesis is “The two probabilities are equal”.

This method of evaluation has as a major criticism the fact that it measures exceptions but do not measures the size of error, but we can see, only by using it, that the GHD represents better risk measures.

Table 7. Exceptions and Kupiec-test p-value.

Distribution	***Exceptions***	***Probability***	***P- Value***
Normal	21	0.013208	0.22
Hyperbolic	17	0.010692	0.78
N.I.G.	16	0.010063	0.98
G.H.	16	0.010063	0.98

9 Conclusions

In this paper we evaluated the goodness of fit of Generalized Hyperbolic Distributions to Brazilian log-return assets and showed that they are better to model asset log-returns than Gaussian distribution. Then we used Fast Fourier Transformation and Esscher transforms to option pricing and we compensate a part of Black and Scholes (1973) mispricing. In the last section we calculated VaR measures and showed that GHD improve risk measures.

The main limitations of the model are the non-market parameters, as volatility is in Normal distributions, the computational effort to parameter estimation and derivative pricing and finally the utilization of numerical calculus that require attention in precision determination. It is important to observe the trade-off between the use of a subclass or the Generalized class. The use of Hyperbolic subclass provides faster parameter estimation and the NIG easies the derivative pricing (since it is closed under convolution).

Last, we have shown many empirical evidence in favor of the use of this GHD distributions to fit Brazilian data, indeed the same analysis can be carried on to analyze other Latin American markets.

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Table 8. Hyperbolic χ^2 tests.

Sample	***Statistic***	***P-Value***	***Degrees of Freedom***
Bbas4	23.4443	0.0930294	15
Bbdc4	34.1268	0.0009022	14
Brdt4	70.7405	1.308E-09	21
Cmig4	21.3939	0.1607190	15
Csna3	55.3857	1.690E-05	22
Ebtp4	13.9941	0.3941480	12
Elet6	34.1289	0.0062942	17
Ibvsp	34.4265	0.0007819	14
Itau4	32.5035	0.0819379	22
Petr4	25.8397	0.1694790	19
Tcsl4	20.6518	0.1431640	14
Tlpp4	34.9870	0.0005964	14
Tnep4	16.6171	0.2871920	13
Tnlp4	19.5150	0.1404500	13
Vale5	12.2598	0.6807120	14

Table 9. NIG χ^2 tests.

Sample	***Statistic***	***P-Value***	***Degrees of Freedom***
Bbas4	24.3353	0.071753	15
Bbdc4	30.8658	0.003959	14
Brdt4	80.2756	2.64E-12	21
Cmig4	21.2789	0.165371	15
Csna3	92.3705	2.44E-15	22
Ebtp4	14.3936	0.364241	12
Elet6	26.8696	0.071477	17
Ibvsp	23.5482	0.061650	14
Itau4	56.8740	3.51E-06	21
Petr4	18.5013	0.575688	19
Tcsl4	20.2143	0.160487	14
Tlpp4	21.0947	0.127084	14
Tnep4	19.8779	0.126934	13
Tnlp4	18.0578	0.205481	13
Vale5	16.9344	0.337203	14

Table 10. Kuiper distances.

Sample	***Normal***	***Hyperbolic***		***NIG***		***GH***	
	KP	***KP***	P-Value	***KP***	P-Value	***KP***	P-Value
Bbas4	0.1133	0.0332	0.2495	0.0391	0.0742	0.0370	0.1187
Bbdc4	0.1299	0.0462	0.0109	0.0496	0.0038	0.0462	0.0109
Brdt4	0.0969	0.0413	0.0419	0.0450	0.0152	0.0414	0.0406
Cmig4	0.1022	0.0352	0.1662	0.0392	0.0694	0.0419	0.0356
Csna3	0.1299	0.0677	0.0000	0.0754	0.0000	0.1000	1 E-14
Ebtp4	0.1259	0.0442	0.4531	0.0452	0.4124	0.0393	0.6615
Elet6	0.1190	0.0290	0.4651	0.0228	0.8434	0.0188	0.9754
Ibvsp	0.1306	0.0280	0.5278	0.0253	0.6975	0.0172	0.9924
Itau4	0.1164	0.0470	0.0086	0.0554	0.0005	0.0470	0.0086
Petr4	0.1225	0.0254	0.6948	0.0259	0.6648	0.0226	0.8574
Tcsl4	0.0839	0.0418	0.5533	0.0431	0.4980	0.0424	0.5260
Tlpp4	0.1549	0.0351	0.1668	0.0363	0.1309	0.0349	0.1748
Tnep4	0.1101	0.0364	0.7757	0.0448	0.4274	0.0412	0.5768
Tnlp4	0.1177	0.0349	0.8357	0.0375	0.7370	0.0336	0.8761
Vale5	0.1332	0.0191	0.9701	0.0236	0.8020	0.0186	0.9782

Table 11. Anderson & Darling Distance

Sample	***Normal***	***Hyperbolic***	***NIG***	***GH***
Bbas4	137028000	3.14961	0.809321	1.19094
Bbdc4	51579.5	0.128786	0.168094	0.12879
Brdt4	485.583	0.21863	0.147332	0.24283
Cmig4	10296	0.451259	0.221778	0.07197
Csna3	7.14072	0.071032	0.0764042	0.1546
Ebtp4	118781	2.27445	0.509757	0.0762
Elet6	51495.5	0.473038	0.183155	0.08368
Ibvsp	72825.7	2.68092	0.371791	0.0831
Itau4	4.6648	0.075514	0.0680073	0.07551
Petr4	67.0476	0.167398	0.0720301	0.04054
Tcsl4	1849990	2.86085	1.08921	0.22114
Tlpp4	51523.5	0.517512	0.304372	0.05785
Tnep4	305.017	0.769552	0.205116	0.17429
Tnlp4	119529	5.94561	1.21541	0.17534
Vale5	51523.5	4.7077	0.74826	0.39979

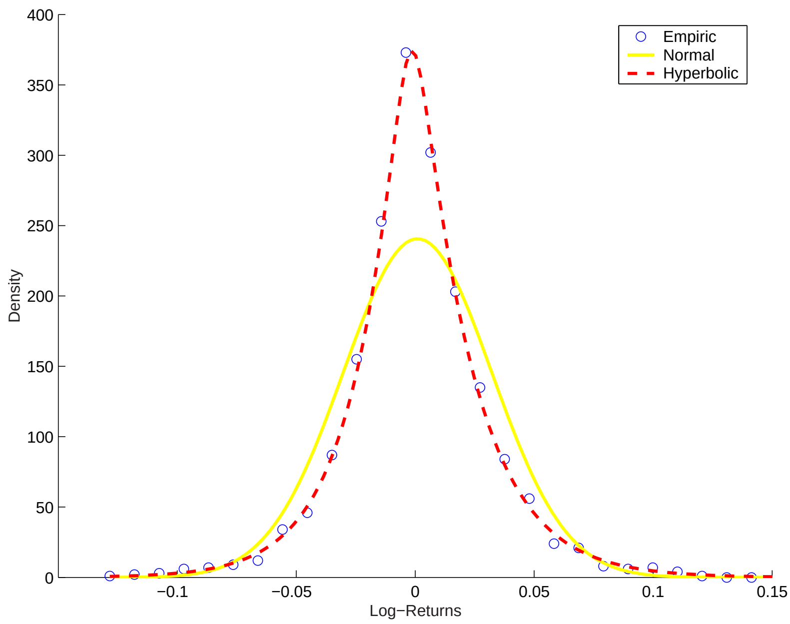

Fig. 3. Vale densities: Empiric x Hyperbolic x Normal

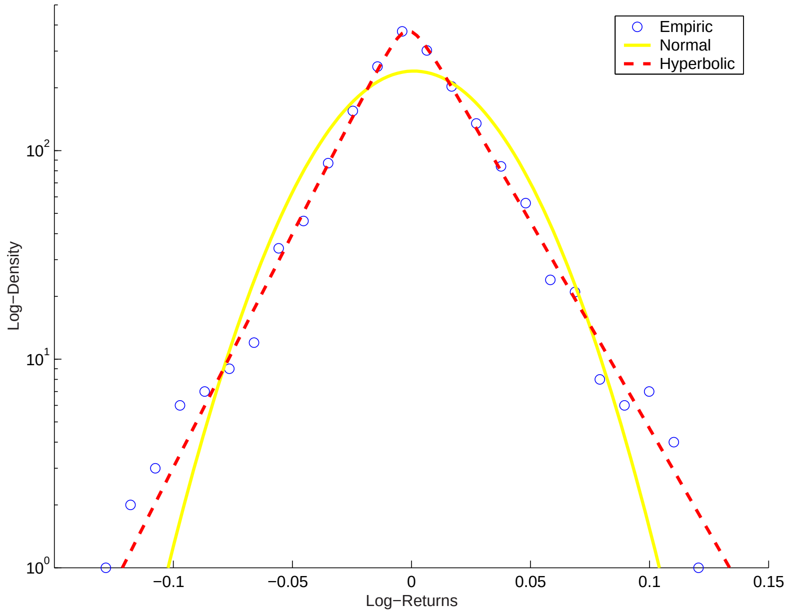

Fig. 4. Vale Log-densities: Empiric x Hyperbolic x Normal

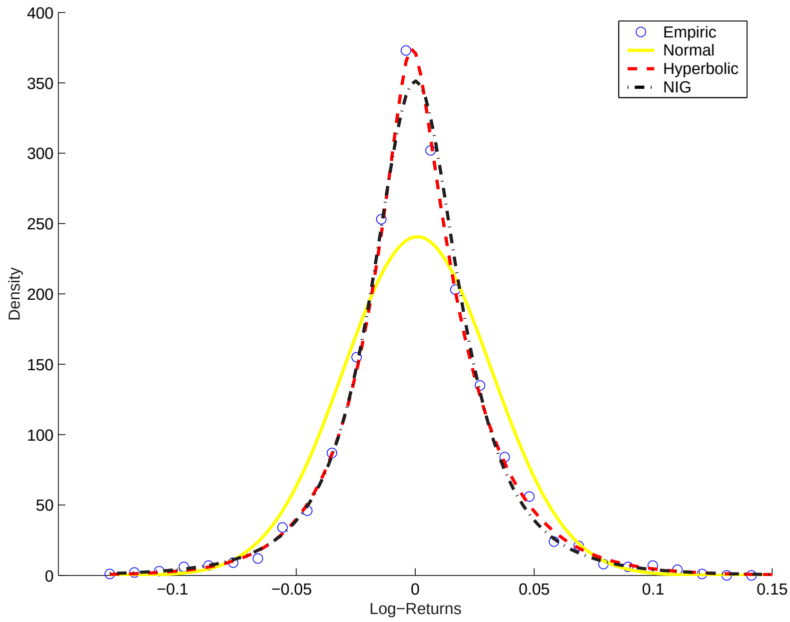

Fig. 5. Vale do Rio Doce Densities: Empiric x Hyperbolic x Normal x NIG

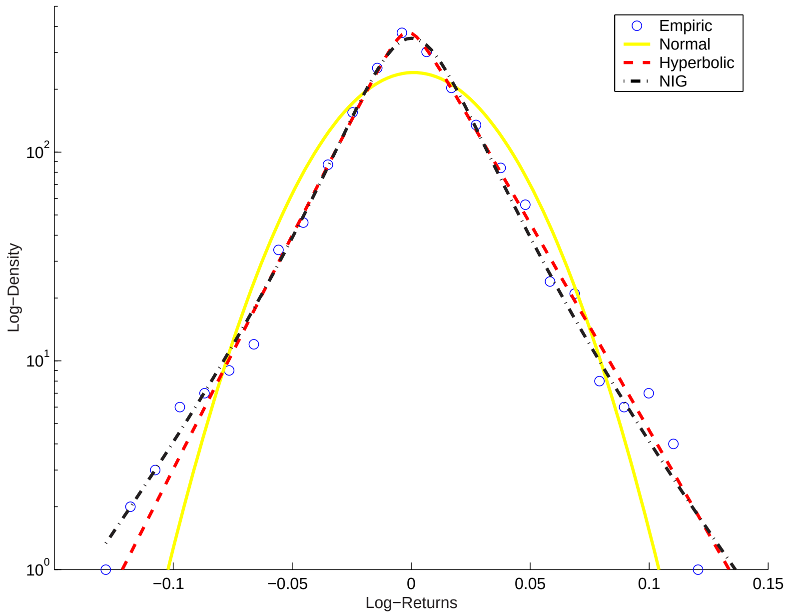

Fig. 6. Vale do Rio Doce Log-Densities: Empiric x Hyperbolic x Normal x NIG

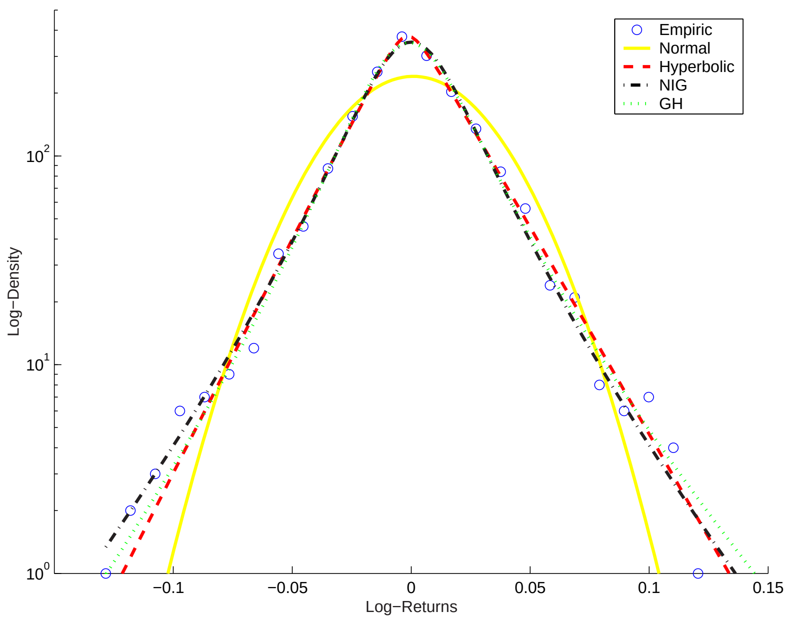

Fig. 7. Vale do Rio Doce Log-Densities: Empiric x Hyperbolic x Normal x NIG x GH

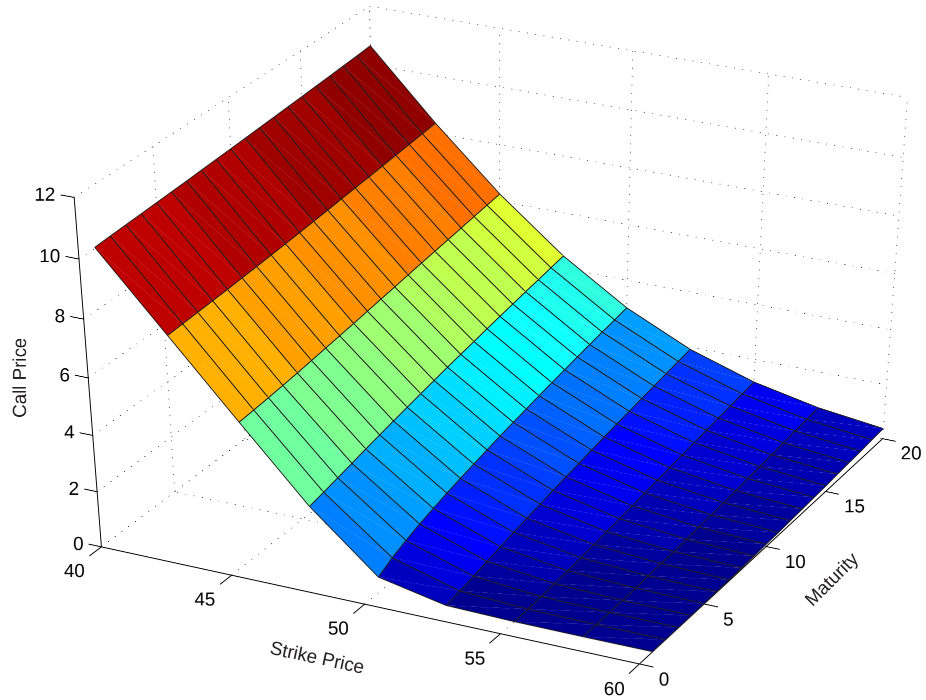

Fig. 8. Vale do Rio Doce Call price with $S_0 = 50$ and risk free interest rate 19% using Hyperbolic subclass.

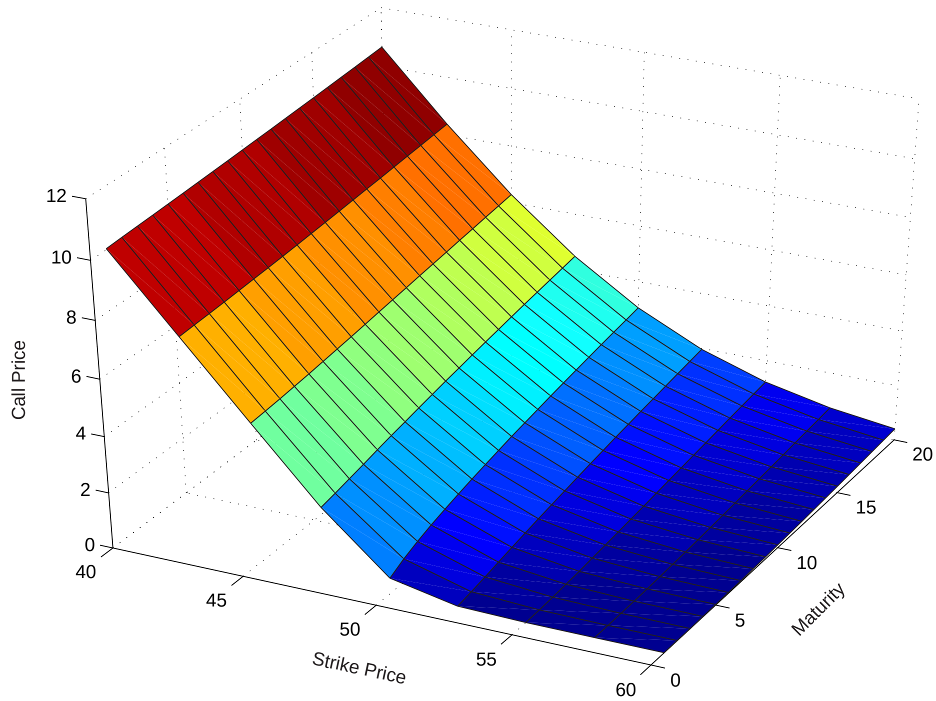

Fig. 9. Vale do Rio Doce Call price with $S_0 = 50$ and risk free interest rate 19% using Generalized Hyperbolic distribution.

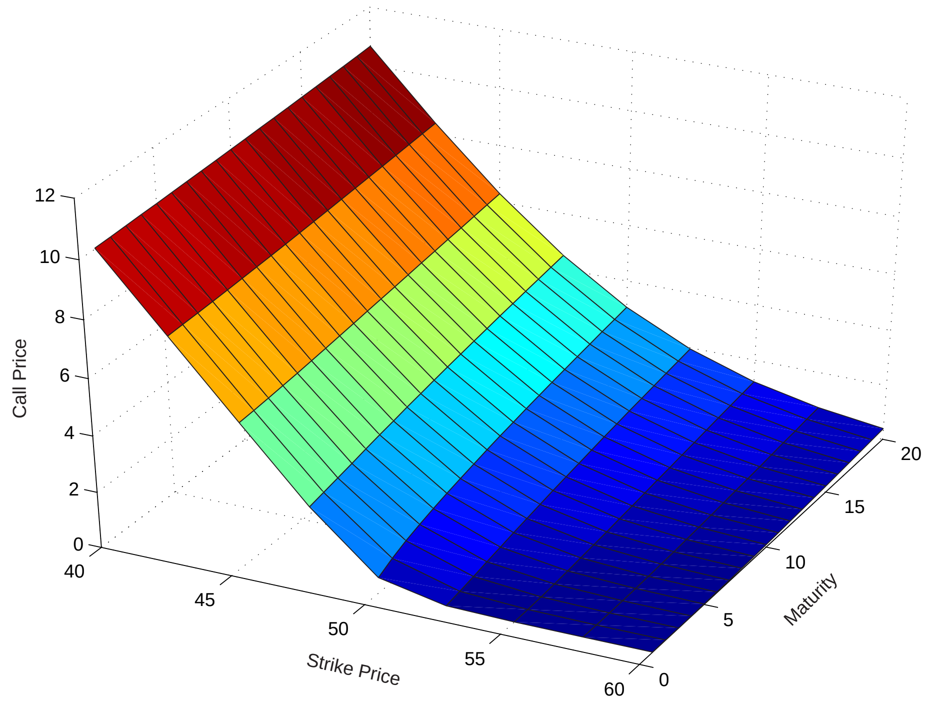

Fig. 10. Vale do Rio Doce Call price with $S_0 = 50$ and risk free interest rate 19% using Normal Inverse Gaussian Distribution.

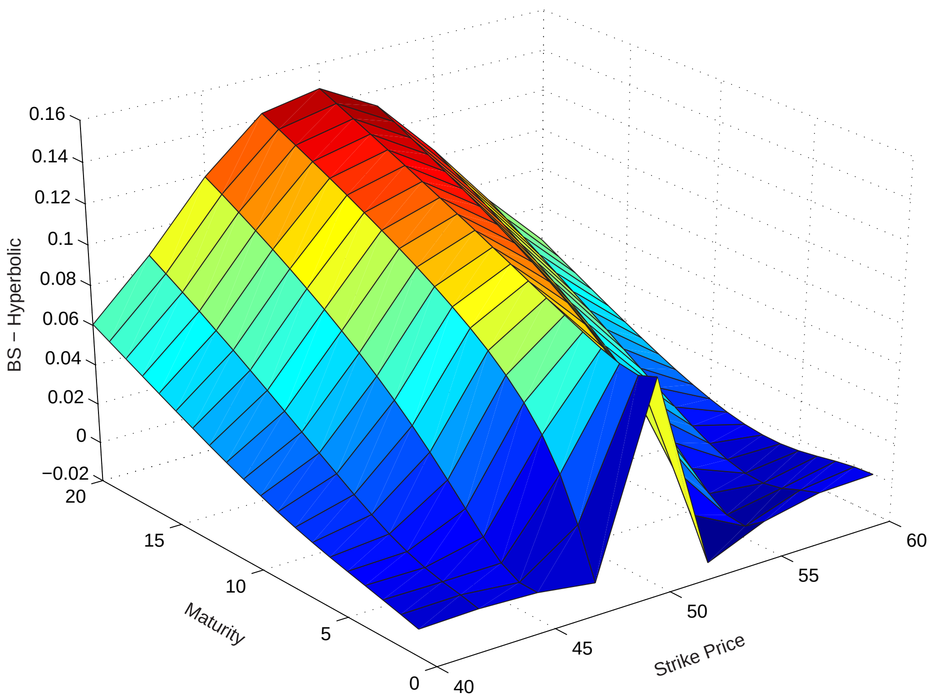

Fig. 11. Black and Scholes minus Hyperbolic Vale do Rio Doce Call prices for various maturities and strike prices .

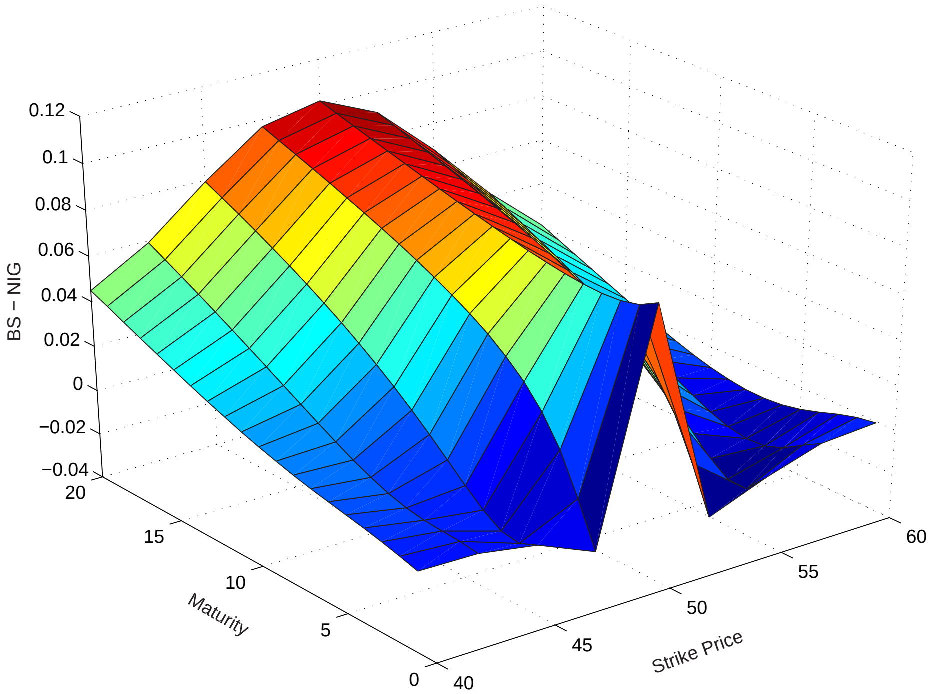

Fig. 12. Black and Scholes minus NIG Vale do Rio Doce Call prices for various maturities and strike prices.

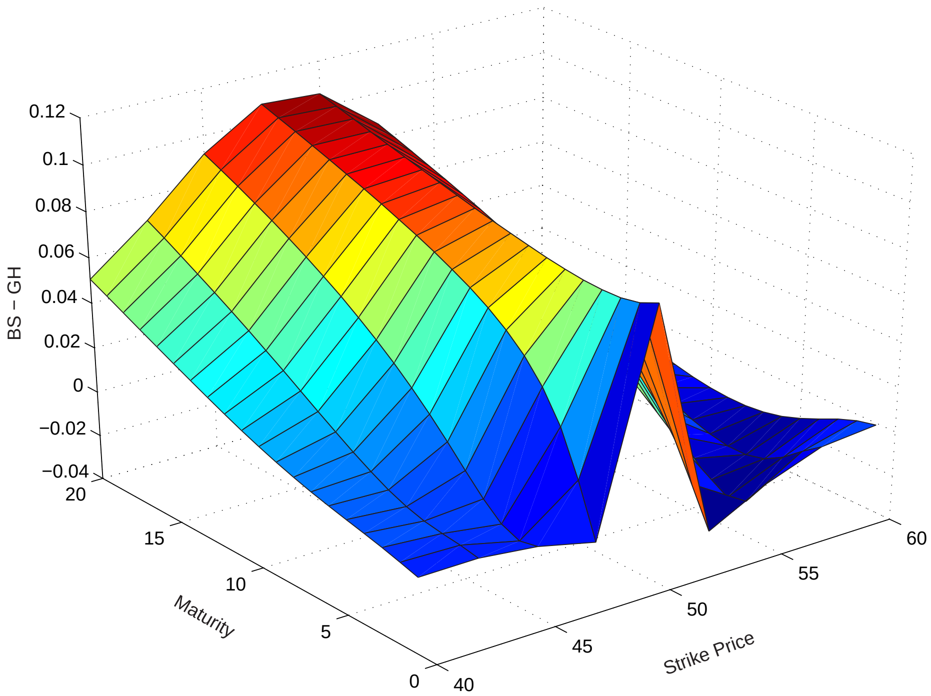

Fig. 13. Black and Scholes minus GHD Vale do Rio Doce Call prices for various maturities and strike prices.

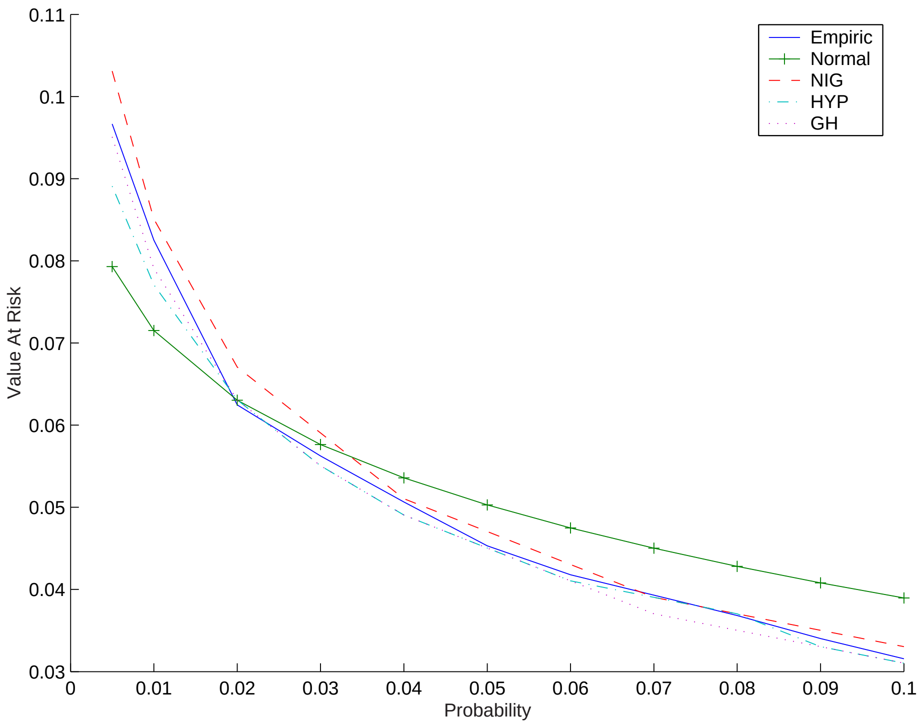

Fig. 14. Value At Risk of portfolio consisting of Vale do Rio Doce assets for different probabilities with 1 trading day holding period and the portfolio value of R$ 1,00.

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