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When half the truth is better than the truth: A Theory of aggregate information cascades*

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Abstract

We introduce a new model of *aggregate information cascades* where only one of two possible actions is observable to others. When called upon, agents (who decide in some random order that they do not know) are only informed about the total number of others who have chosen the observable action before them. This informational structure arises naturally in many applications. Our most important result is that only one type of cascade arises in equilibrium, the aggregate cascade on the observable action. A cascade on the unobservable action never arises. Our results may have important policy consequences. Central agencies, for example in the health sector, may optimally decide to *withhold* information from the public.

1 Introduction

A hiring committee must make a decision on a job candidate who has just been interviewed. The candidate mentions that three other companies have already made him an offer, information that the committee can verify. On the other hand, the committee can only speculate on how many rival companies have already rejected the candidate's job application.

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A manager of a venture capital firm discusses a project with an inventor who needs capital to develop a new product. The inventor has already secured funds from two other venture capital firms, information that the present manager can verify. The manager will also have some private information about the viability of the project but he can only speculate about how often the inventor was turned down by other rival firms who thought that the project was bad.[1]

A restaurant goer must decide whether or not he wants to dine at a particular restaurant he stands in front of. He has some private information on how good the restaurant is, and he is able to peer through the window to see how many others have already decided to dine there. But he can only speculate about how many others stood before the same door and decided to pass.

What these examples have in common is that agents who have to decide between two options have only aggregate information about one of the two options (offering a job, financing a project, dining in a restaurant), simply because the choice of the other option is not observable. In this paper we study the properties of social learning in this type of environment.

This informational environment appears to arise rather naturally in many social interactions. Like in the case of the restaurant goer or the venture capital firm, in many circumstances, a decision maker can gather some aggregate information (how many firms have already adopted a new technology, invested in a specific project, etc.), but he can rarely observe all the individual decisions. Clearly, if the decision is binary, knowing the number of agents who have made a certain decision also helps to update on the number of agents who have made the opposite decision. But this is not equivalent to knowing it. And as we will show below this makes an important difference for social learning.

The social learning literature so far has focussed on situations where, in principle, *all* available actions are observable. The standard model of informational cascades (Banerjee, 1992, and Bikhchandani et al., 1992), for

[1] This example could also be extended to the market for syndicated loans where several banks jointly offer funds to a borrowing firm. See for example Sufi (2007) for an empirical analysis of the effect of information provision between several lenders and the borrower on the syndicate structure of the contract.

instance, contemplates a sequence of binary decisions which are all observable. Agent n knows whether each predecessor in the sequence, from agent 1 to agent $n-1$, decided in favor of one option or the other. Several studies since have relaxed these stringent assumptions, some of which we discuss briefly below. However, the question of what happens if some actions are not observable at all has not been addressed yet.

In the standard sequential model of Banerjee (1992) and Bikhchandani et al. (1992) informational cascades arise: at a certain point in the sequence, agents rationally neglect their own private information, i.e., they choose the same action independently of the information they receive (for instance because they follow the decisions of the predecessors). In particular, different types of cascades can arise. If the decision is binary, say, between investing and not, there can be cascades where, from a certain point onwards, all decision makers decide to invest, as well as cascades where, from a certain point onwards, all decisions makers decide not to invest. At a first glance, one could think that this may be the case in our set up, too. If a restaurant goer sees many people in a restaurant, he could disregard his information and just join the crowd; and if he sees the restaurant empty, he could decide to go somewhere else independently of his private signal. However, we can prove that, on the contrary, only the first cascade is possible. In equilibrium cascades on the unobservable action cannot arise and a restaurant about which some people have read good reviews will not remain empty for ever.

In many scenarios with binary actions, one of the available actions arises naturally as the observable action. There are, however, also some important cases where *third parties* may have the power to decide what kind of information is provided to agents. An example is the disclosure policy of a health agency. A central agency in health policy must decide how to disclose information on the adoption of a new treatment. One possibility is to inform the doctors on how many others have already decided to adopt the new treatment. Another is to inform them on how many have considered doing it but have judged that it is preferable to stick to the old practice. A third possibility is to reveal both, the number of doctors in favor of the new practice and the number of physicians in favor of the old one. Since in equilibrium there cannot be a cascade on the unobservable action and since the central agency can choose which action to make observable, it can essentially *rule out* *one of the two cascades*—by withholding information. In summary, we like to argue that the aggregate information set up that we introduce here has not only several intriguing properties—some of which are in stark contrast to the predictions of the standard model—it also has potentially important policy implications.

Other papers in the social learning literature have studied what happens when we remove the strong assumption that agents can observe the entire history of individual decisions.[2] Smith and Sørensen (1998) study a sequential decision model in which agents can only observe unordered random samples from predecessors' actions (e.g., because of word of mouth communication). With unbounded private signals complete learning eventually obtains in their model. Similarly, Banerjee and Fudenberg (2004) present a model in which, at every time, a continuum of agents choose a binary action after observing a sample of previous decisions (and, possibly, of signals on the outcomes). This can be interpreted as a model of word of mouth communication in large populations. The authors find sufficient conditions (on the sampling rule, etc.) for herding to arise, and conditions for all agents to settle on the correct choice. Çelen and Kariv (2004) extend the standard model of sequential social learning by allowing each agent to observe the decision of his immediate predecessor only. The prediction of these authors is that behavior does not settle on a single action. Long periods of herding can be observed, but switches to the other action occur. As time passes, the periods of herding become longer and longer, and the switches increasingly rare. Finally, Larson (2006) is close to our paper in that he analyzes a situation in which agents observe the pooled average action of a population of their predecessors (before making a choice in a continuous action space). In contrast to our work, the focus of the study is not on whether a cascade occurs or not, but on the speed of learning (since the continuous action space guarantees that complete learning eventually occurs).

The remainder of the paper is organized as follows. In Section 2 we introduce the formal model. We present its equilibrium analysis in Section 3. Section 4 contains an example. Section 5 concludes with a discussion.

[2]For comprehensive surveys of the literature see, among others, Gale (1996), Hirshleifer and Theo (2003), Chamley (2004) and Vives (2007).

2 The Model

In our economy there are n agents who have to decide in sequence whether or not to take up a certain option. For convenience, we shall refer to this choice as the decision about whether or not to *invest*. Time is discrete and indexed by $t = 1, 2, ..., n$. Each agent makes his choice only once in the sequence. Agent i's ($i = 1, 2, ..., n$) action space is given by $\{0, 1\}$, where 1 is interpreted as investment. Player i's action is denoted by $I_i \in \{0, 1\}$. An agent's payoff π_i depends on his choice and on the true state of the world $\omega \in \{0, 1\}$. The prior probability of $\omega = 1$ is $r \in (0, 1)$. If $\omega = 1$ agent i receives a payoff of 1 if he chooses to invest, and a payoff of zero otherwise; vice versa if $\omega = 0$. That is,

$$\pi_i = \omega I_i + (1 - \omega)(1 - I_i).$$

The sequence in which agents make their choices is randomly determined before the first agent makes a decision, and agents are, w.l.o.g., (re-)numbered according to their positions: agent i chooses at time i only. All sequences are equally likely. The agents are, however, *not* informed about which sequence has been chosen. Furthermore, they do not know their own position in the sequence. When called upon, agent i is only informed about the total number of agents before him who have decided to invest. In other words, the decision to invest is assumed to be the only *observable* action. This means that, while the aggregate number of investments is observable, each individual decision to invest or not is not publicly known. We denote the total number of agents who have invested before agent i by T_i, i.e., agent i is informed about $T_i = \sum_{j=1}^{i-1} I_j$. In addition to observing T_i, each agent i receives a private signal $\sigma_i \in \{0, 1\}$ that is correlated with the true state ω. In particular, we assume that each agent receives a symmetric binary signal distributed as follows:

$$\Pr(\sigma_i = 1 \mid \omega = 1) = \Pr(\sigma_i = 0 \mid \omega = 0) \equiv q.$$

Note that, conditional on the state of the world, the signals are i.i.d.. We shall refer to $\omega = 1$ as the "good state" and to $\omega = 0$ as the "bad state." A signal pointing in the direction of the good state ($\sigma_i = 1$) shall be called a "good signal" and a signal pointing in the opposite direction ($\sigma_i = 0$) a "bad signal." We assume that $1 > q > r$ and that $r + q > 1$. These conditions ensure that, in the one-agent case, an agent would invest after a good signal but not after a bad signal, which renders the problem interesting. Note that these two conditions also imply that $q > \frac{1}{2}$, i.e., that the signal respects the monotone likelihood ratio property. Finally, the signal is not perfectly informative, which makes social learning possible and relevant.

Agent i's information set is, therefore, represented by the couple (T_i, σ_i). An agent's strategy $\mathfrak{I}_i$ maps (T_i, σ_i) into an action, i.e.,

$$\mathfrak{I}_i : \{0, 1, 2, ..., n-1\} \times \{0, 1\} \to \{0, 1\}.$$

An agent's mixed strategy induces, for each (T_i, σ_i), a probability with which the agent invests. We denote the probability with which agent i invests after observing (T_i, σ_i) by $\mathcal{I}_i(T_i, \sigma_i)$.

To conclude the description of our model, it is useful to introduce the notion of an *aggregate information cascade*. The definition is virtually identical to the standard definition of information cascade, with the characteristic that histories are summarized by the *aggregate* statistic T_i.

Definition 1 *An aggregate information cascade (AIC) occurs when, along the equilibrium path, there is a critical value of T_i after which all agents choose an action independently of their signal. In particular:*

In an aggregate up cascade (AUC) there is a critical value T^{UP} such that if $T_k = T^{UP}$ all agents from k onwards choose to invest regardless of their private signals. Consequently, there is some k such that $T_{k+j} = T_k + j$ for all $j = 1, ..., n-k$.

In an aggregate down cascade (ADC) there is a critical value T^{DOWN} such that if $T_k = T^{DOWN}$ all agents from k onwards choose not to invest regardless of their private signals. Consequently, in an ADC there is some k such that $T_{k+j} = T_k$ for all $j = 1, ..., n-k$.

There is one small curiosity that can arise in our model. In some cases there are multiple equilibria such that an equilibrium that triggers a cascade can coexist with one that does not. In these cases, players who coordinate at T^{UP} on the AUC equilibrium could revert to the other equilibrium at $T^{UP}+1$ (as no more new information is revealed). We shall rule this out, i.e., we shall assume that once agents have coordinated on an aggregate information cascade they will stay coordinated on that cascade.[3]

[3] This is intuitive as coordination on complicated switching patterns is perhaps less salient

We are now ready to start analyzing the equilibrium decisions in our economy.

3 Equilibrium Analysis

The ultimate goal of our analysis is to understand the social learning process that occurs in our economy. Each agent can learn about the true state of the world from the aggregate information that he receives about other agents' choices. This can lead to better decisions. On the other hand, it may be that also in our economy, as in the canonical model of social learning of Banerjee (1992) and Bikhchandani et al. (1992), there is room for information cascades. In such a case, the process of information aggregation will not be efficient. We will show that, indeed, "up cascades" of investments are possible even in our set up, as they are in the canonical model. In contrast, "down cascades" of non-investments never occur in equilibrium.

We shall restrict the entire analysis to symmetric Perfect Bayesian Nash equilibria (PBNEs). For convenience, we shall sometimes drop the qualification and simply speak of an "equilibrium."[4]

To start our analysis, it is convenient to focus first on the case of $T_i = 0$, in which an agent observes that no one has invested before him. At a first glance, the decision problem in such a situation appears to be fairly complicated. If the agent knew that $T_i = 0$ simply because he is the first decision maker, then he should certainly follow his private signal, since that is the only information available. If, instead, he knew that he is not the first decision maker, then he could decide not to invest independently of the signal, as other agents have already chosen the non-investment option. Intuitively, one might think that $T_i = 0$ is pretty bad information if there are many players. Suppose that n is very large and you observe that nobody has invested before you. But at the same time your own private signal is good. Would you trust your own signal? Of course, the answer to this question would depend on the other

than coordination on a cascade. Moreover, for AUCs one can use a refinement argument to get the same result. If there is the slightest uncertainty about which equilibrium players coordinated on at T^{UP} when observing $T^{UP} + 1$ the indifference will be broken as the increase in T might now actually be due to an additional good signal.

[4] Our economy is represented by a symmetric game and there is nothing in the environment that could help agents to coordinate on an asymmetric outcome. Therefore, the restriction to symmetric equilibria is very natural.

agents' strategies. While the problem is made hard due to the fact that the agent does not know his position in the sequence, it is made easier due to the fact that the only thing that matters about other agents' strategies is what these specify for the very same case of $T_i = 0$.

To attack the problem, let us start with the following definition:

Definition 2 *An initially-pure equilibrium (IPE) is an equilibrium that prescribes pure actions for $T_i = 0$ and both possible signal realizations $\sigma_i = 0$ and $\sigma_i = 1$.*

Note that there can be mixing in an IPE after observing $T_i > 0$. The definition of an IPE just excludes the cases in which an agent mixes after observing $T_i = 0$. We are able to establish some results that focus on $T_i = 0$. First, we prove that in any IPE agents must follow their signal after observing $T_i = 0$: there cannot exist IPEs in which an agent plays independently of his signal or plays against it.

Lemma 1 *In any IPE, an agent follows his own signal if he observes that nobody has invested so far, i.e., $\mathcal{I}_i(0, \sigma_i) = \sigma_i$ for all i.*

Proof We prove this by contradiction. Suppose that for $T_i = 0$ agents choose either to invest always or never (independently of their private signals). Consider the latter possibility first, i.e., consider a pure-strategy equilibrium with $\mathcal{I}_i(0, 0) = \mathcal{I}_i(0, 1) = 0$. Then, along the equilibrium path, nobody ever invests and, for any agent $i = 1, ..., n$, $T_i = 0$. Hence, $T_i = 0$ does not reveal any information on the true state of the world. Since the posterior probability that $\omega = 1$ is still r, agent i is better off by following his informative signal σ_i. Next, consider the case of investment after $T_i = 0$, i.e., an equilibrium with $\mathcal{I}_i(0, 0) = \mathcal{I}_i(0, 1) = 1$. In this case, along the equilibrium path, only the first agent in the sequence observes that nobody else has invested before. That is, $T_i = 0$ if and only if $i = 1$. Hence, after observing $T_i = 0$ agent i knows that he is the first agent in the sequence and, thus, should follow his signal. Finally, suppose that for $T_i = 0$ agents choose to play against their private information, i.e., consider a pure-strategy equilibrium with $\mathcal{I}_i(0, \sigma_i) = 1 - \sigma_i$. Then, along the equilibrium path, after observing $T_i = 0$, agent i knows that he is either the first in the sequence or all other agents before him have received good signals. In both cases, he should follow his signal.■

While we have shown that in any IPE an agent who observes zero investments should follow his signal, it remains unclear whether such equilibria exist. The next lemma identifies a necessary and sufficient condition under which an IPE does indeed exist.

Lemma 2 *An IPE exists if and only if* $r \geq \frac{1-q^n}{2-(1-q)^n-q^n}$.

Proof We first prove that it is indeed optimal for an agent i to follow his own good signal after $T_i = 0$ provided that everybody else follows his signal after $T_i = 0$, and that the condition stated in the lemma holds. (Notice that what another agent j does for $T_j > 0$ is irrelevant for agent i's optimal choice of $\mathcal{I}_i(0, \sigma_i)$). Assuming such behavior of others, an agent i who observes $T_i = 0$ and $\sigma_i = 1$ attaches to the good state a posterior of

$$\Pr(\omega = 1 \mid T_i = 0, \sigma_i = 1) = \frac{rq \sum_{j=1}^{n}(1-q)^{j-1}}{rq \sum_{j=1}^{n}(1-q)^{j-1} + (1-r)(1-q)\sum_{j=1}^{n} q^{j-1}}.$$

He will follow his good signal if this posterior is at least 1/2, i.e., if

$$rq \sum_{j=1}^{n}(1-q)^{j-1} \geq (1-r)(1-q)\sum_{j=1}^{n} q^{j-1}.$$

Solving for the sums and rearranging the terms, we get the condition in the lemma. To complete the proof we have to show that an agent i who assumes that the others play according to the rules stated in the lemma and who observes $T_i = 0$ and $\sigma_i = 0$ does *not* invest, i.e., we need that

$$\Pr(\omega = 1 \mid T_i = 0, \sigma_i = 0) = \frac{r(1-q)\sum_{j=1}^{n}(1-q)^{j-1}}{r(1-q)\sum_{j=1}^{n}(1-q)^{j-1} + (1-r)q\sum_{j=1}^{n} q^{j-1}} < \frac{1}{2},$$

or

$$r(1-q)\sum_{j=1}^{n}(1-q)^{j-1} < (1-r)q\sum_{j=1}^{n} q^{j-1},$$

which can be written as

$$\frac{r}{(1-r)} < \frac{q^2}{(1-q)^2}\frac{1-q^n}{1-(1-q)^n}. \tag{1}$$

Since $r < q$ we also have $\frac{r}{1-r} < \frac{q}{1-q}$. Hence, inequality (1) holds if $\frac{q}{1-q}\frac{1-q^n}{1-(1-q)^n} > 1$. This can be rewritten as $2q > 1 + q^{n+1} - (1-q)^{n+1}$ which is obviously true for $q > 1/2$.■

Notice that the condition imposed in the lemma is always fulfilled if $r \geq 1/2$, i.e., when the good state is initially at least as likely as the bad state, an IPE always exists.

We now turn our attention to Perfect Bayesian Nash equilibria that are not initially pure. The next lemma trivially follows from Bayesian updating. We state it formally because we shall need it later on. The lemma after that shows that, in an equilibrium that is not an IPE, agents who observe $T_i = 0$ never invest if their signal is bad, but will invest with some positive probability if their signal is good.

Lemma 3 *(i) In any equilibrium,* $\mathcal{I}_i(T_i, 1) \geq \mathcal{I}_i(T_i, 0)$ *for all* T_i.

(ii) In any equilibrium, if $0 < \mathcal{I}_i(T_i, 0) < 1$ *then* $\mathcal{I}_i(T_i, 1) = 1$*, and if* $0 < \mathcal{I}_i(T_i, 1) < 1$ *then* $\mathcal{I}_i(T_i, 0) = 0$ *for all* T_i.

Proof In equilibrium, each agent will infer the same information from observing a particular value of T_i. Whatever the posterior induced by just observing T_i, it follows immediately from Bayes' rule that an agent who has an additional good signal will be more optimistic about the good state than an agent with a bad signal. The first part of the lemma results from this consideration and from expected payoff maximization. The second part follows from the same argument and the additional observation that mixing requires indifference.■

Lemma 4 *In any equilibrium that is not an IPE,* $\mathcal{I}_i(0, 0) = 0$ *and* $0 < \mathcal{I}_i(0, 1) < 1$ *for all* i.

Proof Given Lemma 3 we just need to rule out an equilibrium with $0 < \mathcal{I}_i(0, 0) < 1$ and $\mathcal{I}_i(0, 1) = 1$. For an agent to be indifferent between investing and not after observing $T_i = 0$ and $\sigma_i = 0$ we need $\Pr(\omega = 1 \mid T_i = 0, \sigma_i = 0) = 1/2$. Using Bayes' rule, this can be re-written as

$$r \Pr(T_i = 0, \sigma_i = 0 \mid \omega = 1) = (1-r) \Pr(T_i = 0, \sigma_i = 0 \mid \omega = 0),$$

or

$$r \sum_{j=1}^{n} (1-q)^j (1-p)^{j-1} = (1-r) \sum_{j=1}^{n} q^j (1-p)^{j-1},$$

where p denotes the probability with which all other agents who see $T_i = 0$ and $\sigma_i = 0$ invest. Rewriting this as

$$\sum_{j=1}^{n} \left[\left(r(1-q)^j - (1-r)q^j \right) (1-p)^{j-1} \right] = 0$$

makes it obvious that there is no $p > 0$ that solves the equation: since $q > \max\left\{\frac{1}{2}, r\right\}$the left-hand side is strictly negative for any positive p.■

Having characterized equilibria that are not initially pure, we must discuss whether they exist. The next lemma introduces a necessary and sufficient condition for such mixed-strategy equilibria to exist.

Lemma 5 *(i) Mixed-strategy equilibria with $\mathcal{I}_i(0,0) = 0$ and $0 < \mathcal{I}_i(0,1) < 1$ for all i exist if and only if there is a $p \in (0,1)$ that solves*

$$r[1 - (1-pq)^n] = (1-r)[1 - (1 - p(1-q))^n].$$

(ii) If such a p exists, it is unique.

(iii) A mixed strategy equilibrium does not exist for $r \geq \frac{1}{2}$.

Proof The first part of the lemma follows from observing that agent i's indifference between investing and not investing after observing $T_i = 0$ and $\sigma_i = 1$ requires $\Pr(\omega = 1 \mid T_i = 0, \sigma_i = 1) = 1/2$. If all other agents $j \neq i$ use $I_i(0,0) = 0$ and $I_i(0,1) = p$, after applying Bayes' rule and some algebraic manipulation, this equality becomes

$$rq \sum_{j=1}^{n} (1-pq)^{j-1} = (1-r)(1-q) \sum_{j=1}^{n} (1 - p(1-q))^{j-1}, \qquad (2)$$

which is equivalent to the equation in the lemma.

For the second part, observe that $\Pr(\omega = 1 \mid T_i = 0, \sigma_i = 1)$ is strictly decreasing in p.[5]

Finally, note that for $r \geq \frac{1}{2}$ the left-hand side of (2) is strictly greater than the right-hand side for any value of p, which proves the last part of the lemma.■

This lemma completes our characterization of equilibrium decisions after observing $T_i = 0$. In the following proposition we summarize what we have learned so far.

Proposition 1 *(i) If $r \geq 1/2$ agents who observe $T_i = 0$ follow their signal in all equilibria.*

(ii) If $\frac{1-q^n}{2-(1-q)^n-q^n} \leq r < 1/2$ there is an equilibrium where agents who observe $T_i = 0$ follow their signal but there may also be other (mixed-strategy) equilibria where agents who observe $T_i = 0$ follow their signal if it is bad and mix if it is good.

(iii) If $r < \frac{1-q^n}{2-(1-q)^n-q^n}$ there can only be equilibria where agents who observe $T_i = 0$ follow their signal if it is bad and mix if it is good.[6]

Proof The proposition follows immediately from the four previous lemmas and the observation that $\frac{1-q^n}{2-(1-q)^n-q^n} < 1/2$.■

Our analysis essentially shows that, when facing a situation with no previous investments, an agent should either follow his signal or use a mixed strategy (only if the signal is good). An agent should never decide independently of his signal, neither should he decide against it. This clearly indicates that we should not observe a "down cascade" where all agent choose not to invest. In other words, to go back to one of our examples, a restaurant will

[5] While this is very intuitive (the higher p the more likely it is that an agent i's potential predecessors had bad signals if T_i is still zero) the easiest procedure to show this formally is as follows. Let $A = \Pr(T_i = 0, \sigma_i = 1 \mid \omega = 1)$ and define B accordingly for the bad state. Note that $A = q \sum_{j=1}^{n} (1 - pq)^{j-1}$ and $B = (1 - q\_ \sum_{j=1}^{n} (1 - p(1 - q))^{j-1}$. It is easy to see that the claim follows if and only if $AB' > A'B$ (where A' is short for the derivative of A with respect to p). It is also easy to establish that $A > B$. Finally, it remains to be shown that $B' > A'$. For that simply compare the summands in both expressions one by one.

[6] Notice that the third part of the proposition touches on an existence problem. For obvious reasons we have restricted our analysis to symmetric equilibria—in case of bad priors these may fail to exist.

not stay empty forever only because it is empty when it opens. While this puts already a lot of structure on the equilibrium solution of our game, we still need to investigate what happens for different values of the aggregate investment T_i.

To this purpose, we establish in the next step an intuitive monotonicity result, according to which a higher value of T_i is always good news: when an agent observes a higher number of investments made before him, he cannot be less willing to invest himself. Once this monotonicity lemma is established, we will be able to prove two fundamental results about aggregate cascades.

Lemma 6 *In any equilibrium, if* $T_i' < T_i''$ *then* $\mathcal{I}_i(T_i', \sigma_i) \leq \mathcal{I}_i(T_i'', \sigma_i)$ *for both* $\sigma_i = 0$ *and* $\sigma_i = 1$*. In particular, if* $0 < \mathcal{I}_i(T_i', \sigma_i) < 1$*, then* $\mathcal{I}_i(T_i'', \sigma_i) = 1$*.*

Proof *See Appendix.*■

While this lemma seems very intuitive (how could a fuller restaurant be worse news than an emptier?) it is actually not trivial to prove it. At the very core of the proof there is, however, some very basic logic operating. Essentially, it is the earlier monotonicity result (in Lemma 3) which is driving this one. Agents with good signals are more likely to invest than agents with bad signals. Good signals are more likely to be generated in the good state than in the bad state. Hence, T_i grows, on average, "faster" in the good state than in the bad state. Hence, the higher T_i the more confident can we be about being in the good state.

Equipped with Lemma 6 we are now ready to state our two main propositions that characterize which forms of cascades will or will not arise. In particular, we will see that aggregate down cascades *never* arise, while aggregate up cascades are *always* part of an equilibrium.

Proposition 2 *(i) In any equilibrium,* $\mathcal{I}_i(0, 1) > 0$*, and* $\mathcal{I}_i(T_i, 1) = 1$ *for all* $T_i > 0$*, i.e., an agent with a good signal always invests with positive probability (and invests with probability one after observing at least another investment) and an ADC never occurs in equilibrium.*

(ii) In any equilibrium, there can be at most one T^{MIX} *for which* $0 < \mathcal{I}_i(T^{MIX}, 0) < 1$*. For all* $T_i < T^{MIX}$ *agents with bad signals follow their signal and do not invest. For all* $T_i > T^{MIX}$*, an AUC occurs in which agents invest independently of their signal.*

Proof The first part of the proposition follows from Proposition 1 and Lemma 6. The second part follows again from Lemma 6.■

The first part of the proposition clearly implies that there are no cascades on the unobservable action. In particular, after observing at least one investment, agents with a good signal always invest. Incidentally, we note that such a result just comes from an equilibrium argument. One could imagine that, when facing a "low" value of T_i, in order to make his decision, agent i should consider all possible sequences and attach a probability to the event that he is the first in the sequence, or the second, etc. After all, a low number of investments may merely come from the fact that only few agents had the opportunity to invest so far, in which case the low value of T_i should be considered good news. Or it could arise from many agents having the option of investing but few only using it, in which case the low T_i should be viewed as bad news. All this inference process could be quite complicated. Our analysis solves the problems by just invoking some equilibrium arguments.

The second part of the proposition hints at the possible role of aggregate up cascades. But from all we have established so far it could be that $T^{MIX} \geq n$, i.e., that agents always follow their signal (or mix) such that an AUC never arises. The next proposition, however, shows that AUCs do arise—and are, in fact, part of *any* equilibrium.

Proposition 3 *AUCs are part of any equilibrium. In particular, in any equilibrium* $\mathcal{I}_i(T_i, \sigma_i) = 1$ *for all* $T_i > \frac{n}{2}$.

Proof Consider an agent i who observes $T_i > \frac{n}{2}$ and suppose he *knew* that he were the last agent in the sequence. Further suppose there were no AUC. If $T_i = T^{MIX}$, then for $T_i = T^{MIX} + 1$, an AUC would occur by Proposition 2. If, instead, $T_i \neq T^{MIX}$, then, due to Lemma 6, this agent knows that there were at least T_i good signals and no more than $n - T_i - 1$ bad signals. Hence, even if this agent's own signal is bad, he knows that there were altogether more good signals than bad signals and he will decide to invest. Of course, agent i can't be sure that he really is the last agent. But if he isn't, this means that there were *fewer* bad signals so far, while he can still be sure that there were T_i good signals. Hence, an agent who observes $T_i > n/2$ will always invest and, thus, trigger an AUC.■

	$\sigma_i = 0$	$\sigma_i = 1$
$T_i = 0$	0	$mixed, 1$
$0 < T_i < T^{MIX}$	0	1
$T_i = T^{MIX}$	$mixed$	1
$T_i > T^{MIX}$	1	1
$T_i \geq T^{UP}$	1	1

Table 1: Structure of all equilibria. Entries indicate whether the agent mixes or invests with probability 0 or 1.

The value $\frac{n}{2}$ is just an upper bound for the critical mass of observable choices that triggers an AUC. Depending on the parameters' values, AUCs may well be triggered earlier. But AUCs are indeed part of *all* equilibria. Of course, this does not necessarily imply that AUCs will actually be triggered, since there is always the possibility of sufficiently many bad signals occurring such that the critical T_i that triggers an AUC may not be reached.

We summarize the structure of equilibria in Table 1. The rows in the table indicate possible values of T_i, while the columns indicate the two possible signal realizations. Note that not necessarily all the values of T_i in the table exist. In particular, T^{MIX} might not exist. For this reason, the last two rows of the table have the same entries. Notice also that if T^{MIX} exists, then $T^{UP} = T^{MIX} + 1$. In any case, the basic structure of all equilibria is captured in the table and is nicely monotonic.

4 An example

Let us now illustrate our theory through a simple example. The example shows how constructive the results that we have illustrated above are. Consider the case in which $n = 3$ and $r \geq 1/2$. From Proposition 1 we know that $I_i(0, \sigma_i) = \sigma_i$ and from Proposition 2 we know that $I_i(2, \sigma_i) = 1$ and that $I_i(1, 1) = 1$. Thus, the two propositions alone immediately give us the equilibrium actions for five out of the six possible contingencies agents can face. The only remaining question is now what agents do after observing $T_i = 1$ and $\sigma_i = 0$. As is clear from the results illustrated in Table 1, this depends on further conditions on r and q. Let us first check under which conditions agent i rationally follows his bad signal. Recall that we are analyzing a symmetric equilibrium, therefore suppose each other agent j chooses $I_j(1,0) = 0$. Then it is optimal for agent i to do the same if his posterior for the good state is not bigger than 1/2, i.e., if

$$\frac{r[q(1-q) + 2q(1-q)^2]}{r[q(1-q) + 2q(1-q)^2] + (1-r)[q(1-q) + 2q^2(1-q)]} \leq \frac{1}{2}$$

which is equivalent to

$$q \geq 2r - \frac{1}{2} \equiv \underline{q}.$$

Similarly, $I_i(1,0) = 1$ is optimal if

$$r[q(1-q) + q(1-q)^2] \geq (1-r)[q(1-q) + q^2(1-q)].$$

which is equivalent to

$$q \leq 3r - 1 \equiv \overline{q}.$$

Note that $\underline{q} \leq \overline{q}$. Hence, in the case in which $\underline{q} > 0$ and $\overline{q} < 1$, we obtain three equilibrium regions. For $q < \underline{q}$ there is a unique pure-strategy equilibrium in which $I_i(1,0) = 1$ and an AUC starts with $T_i = 1$. For $q > \overline{q}$ there is a unique pure-strategy equilibrium in which $I_i(1,0) = 0$ and an AUC starts only with $T_i = 2$. Finally, for $\underline{q} \leq q \leq \overline{q}$ both the two pure-strategy equilibria exist and there is a mixed-strategy equilibrium as well—with $I_i(1,0) = \frac{1+2q-4r}{q-r}$.

5 Discussion

We have introduced a new model of information cascades. The crucial difference between our model and those already in the literature is that only one action taken by agents is observable by others. When it is their turn to make the binary decision, agents simply receive aggregate information about how many others before them took the observable action. We argue that this setup arises naturally in many scenarios: for example, when entrepreneurs seek investors they will typically not inform them about how many others have turned them down before, but, surely, they will mention who else decided previously to invest in their project. This asymmetry in observability dramatically affects all equilibria in such games. Most importantly, there can be no "down cascades:" if an action is unobservable, there can never be an information cascade where agents take this action.

Our result has important implications. In particular, it implies that a new, good project (e.g., a technological innovation, a new product or service, a new medical treatment) will not be neglected for ever simply because there is lack of interest at the beginning. Sooner or later (i.e., as soon as people start receiving good information on it) the new project will start diffusing. A lack of initial interest will not represent a barrier to future adoption because of informational considerations.

Our study has also an important consequence for applications where a third party can decide which information it is to release. In the introduction we mentioned the case of an agency in health policy. Such an agency must decide how to disclose information on the adoption of a new treatment: to inform the doctors on how many others have already decided to adopt the new treatment; inform them on how many have considered doing it but have decided to stick to the old practice; or, finally, to reveal both, the number of doctors in favor of the new practice and the number of physicians in favor of the old one. Can the way the information is disclosed make a difference for the diffusion of the new treatment? Suppose the agency is uncertain about the effects of the new treatment and considers as the worst case scenario the situation in which the new treatment is widely adopted while ultimately resulting in worse health outcomes than the old treatment, for instance because of side effects. Which disclosure policy should the agency employ? Intuitively, one would think that the disclosure of *all* available information should maximize social welfare. This is, however, only true if the welfare analysis focuses on doctors and excludes patients. The standard intuition does hold: With more information the doctors will always be better off. But as soon as the doctors' interests are not perfectly aligned with that of patients' (perhaps because patients care more about their own lives than their doctors do) the picture changes. Since the agency may want to maximize patients' welfare rather than doctors it may, in the light of our results, optimally *withhold* some information. By withholding information on a particular decision (e.g., the number of doctors who decided to adopt the new treatment), the agency can, in fact, guarantee that an informational cascade on that decision does not occur.[7] This may rule out the worst case scenario where many patients

[7] In their seminal paper on information cascades, Bikhchandani et al. (1992) have argued that the adoption of medical procedures is often based on fairly weak information and that

die because of severe side effects of the new treatment. A full-fledged welfare analysis is beyond the scope of our paper, but these considerations show that it can be an important topic for future research.

Models of social learning have been extensively tested in the laboratory, with results sometimes supportive and sometimes less encouraging for the theoretical analysis.[8] We have studied the behavior of human subjects in a laboratory setting that reproduces the model of this paper (Guarino et al., 2008). Our preliminary results are encouraging. In just two simple treatments we find that the main comparative statics go all in the right directions. In particular, while we observe cascades on the observable action, cascades on the unobservable actions either do not occur (in one treatment) or occur only rarely (in the other). While this evidence is not yet conclusive, and although some interesting anomalies emerge, our experimental results show that our theory is able to capture some of the behavior we observe in the laboratory.

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in many cases doctors tend to imitate others. As an example they cite the widespread use of tonsillectomies in the sixties and seventies and argue that it was essentially an information cascade. In the sixties and seventies, according to Bikhchandani et al. (1992), the sheer fact that the majority of physicians employed the procedure overrode any private information individual doctors might have had against tonsillectomies. And this was a "wrong cascade"—a cascade that generated the worst outcome, since it eventually turned out that tonsillectomies did more harm than good.

[8] See, e.g., Anderson and Holt (1997), Huck and Oechssler (2000), Goeree et al. (2007), Kübler and Weizsacker (2004, 2005) and Weizsacker (2007).

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6 Appendix

A Proof of Lemma 6

The proposition is equivalent to saying that in any equilibrium $\mathcal{I}(t_i, \sigma_i) \leq \mathcal{I}(t_i + 1, \sigma_i)$ for any $t_i = 0, 1, 2...$and both $\sigma_i = 0$ and $\sigma_i = 1$. Because of expected payoff maximization, this inequality holds if, whenever $\Pr(\omega = 1 \mid$

$T_i = t, \sigma_i) \geq \frac{1}{2}$, we have $\Pr(\omega = 1 \mid T_i = t+1, \sigma_i) > \frac{1}{2}$.

There are four relevant possibilities:

1. $\Pr(\omega = 1 \mid T_i = t, \sigma_i = 0) > \frac{1}{2}$ and $\Pr(\omega = 1 \mid T_i = t, \sigma_i = 1) > \frac{1}{2}$
2. $\Pr(\omega = 1 \mid T_i = t, \sigma_i = 0) < \frac{1}{2}$ and $\Pr(\omega = 1 \mid T_i = t, \sigma_i = 1) > \frac{1}{2}$
3. $\Pr(\omega = 1 \mid T_i = t, \sigma_i = 0) = \frac{1}{2}$ and $\Pr(\omega = 1 \mid T_i = t, \sigma_i = 1) > \frac{1}{2}$
4. $\Pr(\omega = 1 \mid T_i = t, \sigma_i = 0) < \frac{1}{2}$ and $\Pr(\omega = 1 \mid T_i = t, \sigma_i = 1) = \frac{1}{2}$

Case 1 is the case of an informational cascade. In such a case,

$$\Pr(\omega = 1 \mid T_i = t, \sigma_i) = \Pr(\omega = 1 \mid T_i = t+1, \sigma_i)$$

for both σ_i, and therefore the proposition obviously holds.

Now let us consider Case 2. In this case we want to show that $\Pr(\omega = 1 \mid T_i = t+1, \sigma_i = 1) > \frac{1}{2}$ (while nothing must be shown for the case of a bad signal). Suppose not, i.e., suppose $\Pr(\omega = 1 \mid T_i = t+1, \sigma_i = 1) \leq \frac{1}{2}$. Let us consider, first, the case of the strict inequality.

By Bayes' rule,

$$\Pr(\omega = 1 \mid T_i = t+1, \sigma_i = 1) =$$
$$= \frac{\Pr(T_i{=}t{+}1|\omega{=}1,\sigma_i{=}1)\Pr(\omega{=}1|\sigma_i{=}1)}{\Pr(T_i{=}t{+}1|\omega{=}1,\sigma_i{=}1)\Pr(\omega{=}1|\sigma_i{=}1)+\Pr(T_i{=}t{+}1|\omega{=}0,\sigma_i{=}1)\Pr(\omega{=}0|\sigma_i{=}1)}.$$

As we suppose that this is strictly smaller than $\frac{1}{2}$ we know that

$$\frac{\Pr(T_i = t+1 \mid \omega = 1, \sigma_i = 1)}{\Pr(T_i = t+1 \mid \omega = 0, \sigma_i = 1)} < \frac{\Pr(\omega = 0|\sigma_i = 1)}{\Pr(\omega = 1|\sigma_i = 1)}.$$

which is equivalent to

$$\frac{\Pr(T_i = t+1 \mid \omega = 1)}{\Pr(T_i = t+1 \mid \omega = 0)} < \frac{\Pr(\omega = 0|\sigma_i = 1)}{\Pr(\omega = 1|\sigma_i = 1)}.$$

By the law of total probabilities,

$$\Pr(T_i = t+1 \mid \omega = 1) \tag{3}$$
$$= \Pr(T_i = t+1 \mid \omega = 1, T_{i-1} = t)\Pr(T_{i-1} = t|\omega = 1) \tag{4}$$
$$+ \Pr(T_i = t+1 \mid \omega = 1, T_{i-1} = t+1)\Pr(T_{i-1} = t+1|\omega = 1) \tag{5}$$
$$= q\Pr(T_{i-1} = t|\omega = 1) + \Pr(T_{i-1} = t+1|\omega = 1). \tag{6}$$

Notice that the last equality comes from the fact that we are analyzing Case 2 and that we are assuming (by contradiction) no investment after observing $t+1$.

Now the decision problem of agent $i-1$ is identical to the one of agent i. So, by applying recursively the same law, we obtain:

$$\begin{aligned}
&\Pr(T_i = t+1 \mid \omega = 1) \\
&= q\Pr(T_{i-1} = t|\omega = 1) + \Pr(T_{i-1} = t+1|\omega = 1, \sigma_i = 1) \\
&= q\Pr(T_{i-1} = t|\omega = 1) + [q\Pr(T_{i-2} = t|\omega = 1) + \Pr(T_{i-2} = t+1|\omega = 1)] \\
&+ q\Pr(T_{i-1} = t|\omega = 1) + q\Pr(T_{i-2} = t|\omega = 1) + [q\Pr(T_{i-3} = t|\omega = 1) \\
&+ \Pr(T_{i-3} = t+1|\omega = 1)] + ... \\
&= q\Pr(T_{i-1} = t|\omega = 1) + q\Pr(T_{i-2} = t|\omega = 1) + q\Pr(T_{i-3} = t|\omega = 1) \\
&+ ... + q\Pr(T_{i-m} = t|\omega = 1)]
\end{aligned}$$

for some m (note that m depends on the value of i: indeed, for any value of i there is an m such that $\Pr(T_{i-m} = t+1|\omega = 1) = 0$). Similarly, conditioning on $\omega = 0$,

$$\begin{aligned}
&\Pr(T_i = t+1 \mid \omega = 0) \\
&= (1-q)\Pr(T_{i-1} = t|\omega = 0) + (1-q)\Pr(T_{i-2} = t|\omega = 0) \\
&+ ... + (1-q)\Pr(T_{i-m} = t|\omega = 0).
\end{aligned}$$

Some algebraic computations show that for any pair of terms in the two expressions above, the following inequality holds:

$$\frac{\Pr(T_{i-j} = t|\omega = 1)}{\Pr(T_{i-j} = t|\omega = 0)} \geq \frac{\Pr(T_i = t|\omega = 1)}{\Pr(T_i = t|\omega = 0)}.$$

Since we know that $\Pr(\omega = 1 \mid T_i = t, \sigma_i = 1) > \frac{1}{2}$ and, therefore,

$$\frac{\Pr(T_i = t \mid \omega = 1)}{\Pr(T_i = t \mid \omega = 0)} > \frac{\Pr(\omega = 0|\sigma_i = 1)}{\Pr(\omega = 1|\sigma_i = 1)}$$

simple algebra shows that

$$\frac{\Pr(T_i = t+1 \mid \omega = 1)}{\Pr(T_i = t+1 \mid \omega = 0)} > \frac{\Pr(\omega = 0|\sigma_i = 1)}{\Pr(\omega = 1|\sigma_i = 1)},$$

a contradiction.

Note that the same proof holds true when, by contradiction, we assume that

$$\Pr(\omega = 1 \mid T_i = t+1, \sigma_i = 1) = \frac{1}{2}.$$

The only difference is that in such a case

$$\begin{aligned}&\Pr(T_i = t+1 \mid \omega = 1)\\ &= q\Pr(T_{i-1} = t|\omega = 1) + s\Pr(T_{i-1} = t+1|\omega = 1),\end{aligned}$$

where s represents the probability by which an agent receiving the good signal decided not to invest. This change does not affect the above inequalities.

Finally, note that the proofs for Case 3 (for both the good and the bad signal) and Case 4 are identical to Case 2 just described, with the exception that in Case 3,

$$\Pr(T_i = t+1 \mid \omega = 1, T_{i-1} = t) = q + (1-q)u,$$

and

$$\Pr(T_i = t+1 \mid \omega = 0, T_{i-1} = t) = qu + (1-q),$$

where u is the probability of investment by an agent receiving a bad signal; similarly, in Case 4,

$$\Pr(T_i = t+1 \mid \omega = 1, T_{i-1} = t) = qu$$

and

$$\Pr(T_i = t+1 \mid \omega = 0, T_{i-1} = t) = (1-q)u.$$

■