The Dual Economy in Long-run Development†

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Abstract

This paper provides a dynamic model of the dual economy in which differences in productivity across sectors arise endogenously. Rather than relying on exogenous price distortions, duality arises because of differences between sectors in the separability of their fertility and labor decisions. The model demonstrates how a dual economy will originate, persist, and eventually disappear within a unified growth framework. It is also shown that agricultural productivity growth will exacerbate the inefficiencies of a dual economy and slow down long-run growth.

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1 Introduction

While neo-classical growth theory has concerned itself primarily with one-sector models, most developing countries contain within them multiple economies operating in distinctly different manners and typically with distinctly different levels of productivity. Lewis (1954) brought this concept of dual economies into focus, but the idea is as old as Ricardo and Malthus. More recently Banerjee and Duflo (2005) and Temple (2005) have suggested that a better understanding of growth and development requires the explicit adoption of models that incorporate heterogeneity within economies.

The relevance of the dual economy for understanding income differences across countries has been established by several recent empirical papers. Chanda and Dalgaard (2008), Cordoba and Ripoll (2008), Restuccia et al (2008) and Vollrath (2009) all document that the inefficiencies inherent in a dual economy can explain over half of the observed variation in total factor productivity (TFP) at the country level. Temple and Woessmann (2004) document that labor reallocation has a significant influence on country-level growth rates. An older literature on structural transformation (Chenery and Syrquin, 1975; Chenery, Robinson, and Syrquin, 1986; Kuznets, 1966) focused on the movement of labor from agriculture to industry as an important source of growth in output per capita.

While informative, this literature does not provide any insight on *why* the dual economy exists, nor how it changes over time. Development theory focused primarily on the static problems of how wage differentials could persist. The Harris and Todaro (1970) model of migration and Stiglitz's (1974) efficiency wage theory, among many, provided explanations of wage differences but do not explore the dynamic consequences. More recently, dynamic treatments of two-sector economies provided by Hayashi and Prescott (2008), Gollin, Parente and Rogerson (2004), Caselli and Coleman (2001) and Mourmouras and Rangazas (2007) have incorporated dual economy effects, but they all rely on exogenously given distortions to prices or preferences to generate their results, similar to the earlier efforts of Jorgenson (1961) and Ranis and Fei (1961).[1]

As Temple (2005) noted in his overview of the dual economy and growth, what is missing is an analysis of how dualism arises and evolves within the process of development. This paper provides a dynamic model of the dual economy in which the inefficient allocation of labor across sectors is the result of optimizing behavior by individuals. In contrast to previous work, here the extent of duality (as captured by differences in average and marginal products of workers between sectors) evolves along with the economy, and does not appeal to exogenous institutional or market structure differences to explain why some countries display dual

[1]The literature on economic growth has numerous examples of multi-sector models. Matsuyama (1992), Laitner (2000), Gollin et al (2002) and Kongasmut et al (2001) examine labor movements between sectors, but no duality is allowed to exist. Unified growth models such as Galor and Mountford (2008), Galor et al (in press), Goodfriend and McDermott (1995), Tamura (2002), and Hansen and Prescott (2002) involve the transition between sectors, but do so without exploring the dual nature of the economy. Kogel and Prskawetz (2001) present a growth model in which agricultural workers earn their average product while industrial workers earn their marginal product, but the ramifications of this are not explored.

economies while others do not. Similar to unified growth models (see Galor, 2005 for a complete review) that explain the coincident changes in fertility, education, and income, the model presented here shows how the dual economy is embedded within the process of long-run development.

To achieve this the idea of non-separability is borrowed from the development literature and incorporated within a model of optimal fertility choice.[2] As documented below, the dual economy manifests itself not only in labor productivity levels but in fertility rates as well. Fertility, as has become standard, is presumed to take time. For individuals in the traditional sector, their fertility decision is not separable from their production decision. They internalize the effect of their time allocation on the marginal product of their labor, realizing that as they work more they are lowering the value of their time. This induces higher fertility as the time cost of children is lower. Modern sector individuals work for a constant wage, and so the effect is not present for them. In equilibrium, traditional sector individuals will work less time and have higher fertility. Their marginal and average products will be lower than the modern sector, and a dual economy will exist.

This result is not driven by any labor market distortion, and individuals are freely mobile, ensuring that utility is equalized between sectors. What this implies is that while the dual economy is not output maximizing, it is welfare-maximizing.

The dynamics of the dual economy are examined by looking at productivity changes (exogenous at first, endogenous in an extension) in the two sectors. What is shown is that productivity changes in the traditional and modern sectors are not created equal, so to speak. Modern sector productivity changes raise the time cost of children in both sectors, inducing all individuals to spend more time on work, less time on children and shrinking the gap in worker productivity between sectors. Thus modern sector productivity growth is capable of eliminating the dual economy in the course of development.

In contrast, traditional sector productivity growth, due to a low income elasticity of demand for its output, can have a perverse effect on development. An increase in traditional productivity will actually lower the time spent working, raising fertility, and reducing output per capita in the future by decreasing resources per person. The somewhat counter-intuitive result is that productivity improvements in the 'backwards' sector can actually exacerbate the dual economy effect and, similar to the work of Matsuyama (1992) and Galor and Mountford (2008), slow down the aggregate growth rate.[3]

Aside from the implications regarding productivity, an important item to note is what the model tells

[2]Typically, the separability question in the development literature involves the distinction between labor demand on a family farm versus the labor supply of the family. Benjamin (1992) cannot reject the hypothesis that family labor supply is independent of farm labor demand. However, this result is based on household size being constant. In the model presented here, there is non-separability in the labor supply and fertility decision, and hence household size is in fact endogenous.

[3]The implications of this model are also related to several papers on endogenous fertility and income distribution, as in Dahan and Tsiddon (1998) and De la Croix and Doepke (2003). Here, duality creates the possibility of higher fertility and lower income levels, while in these other models differential fertility due to human capital differences may limit development.

us about evaluating dual economies. There is nothing fundamentally "wrong" with dual economies in this context. There are no distortions or institutional failures in the dual economy conceived of here, so there is nothing to fix. We can establish that the dual economy has consequences for aggregate output per capita, but we cannot judge the welfare effects of this.

To proceed, section 2 discusses the evidence regarding the dual economy and its manifestation in productivity and fertility differences between sectors of economies. Section 3 then presents the general model of duality, showing how it arises endogenously from a model of optimal fertility choice. The dynamic implications of the dual economy are examined in section 4, including an extension of the model to allow for endogenous productivity growth in a unified growth framework. Section 5 concludes.

2 Dualism in Production and Fertility

It will be useful to pursue a digression on semantics before reviewing the evidence on dualism. The phrase "dual economy" has several connotations. 'Modern sector dualism' typically deals with the underemployment of labor in the urban sector, while 'traditional sector dualism' focuses more on the coincidence of a low productivity traditional sector alongside a modern, commercially oriented sector.[4] In this paper, I focus exclusively on the latter type of dualism, laying aside urban labor market issues completely.

What is meant by the 'traditional sector'? Numerous definitions are available, but typically this refers to a sector of production that is predominantly rural, is limited by some fixed resource (e.g. land), is engaged in subsistence rather than commercial production, and is demonstrably poorer than the 'modern' sector (see Lewis, 1954; Ranis, 1988; Basu, 1997).

In the work that follows I will use the term 'agricultural' to refer to a sector that is generally rural, non-commercial, and that relies heavily on certain fixed factors of production such as land. I will use the terms 'modern' or 'manufacturing' to refer to the sector that is generally urban and commercially oriented. These labels are for convenience only, and are not meant to imply any strict divisions.

2.1 Differential Productivity

The most obvious feature of the dual economy is the pronounced difference in productivity between the agricultural and modern sectors. The (presumed) relatively low marginal product of agricultural laborers suggests that some improvement to living standards could be achieved by the transfer of labor into the modern sector.

[4]See Bertrand and Squire (1980) for a more careful definition of these terms.

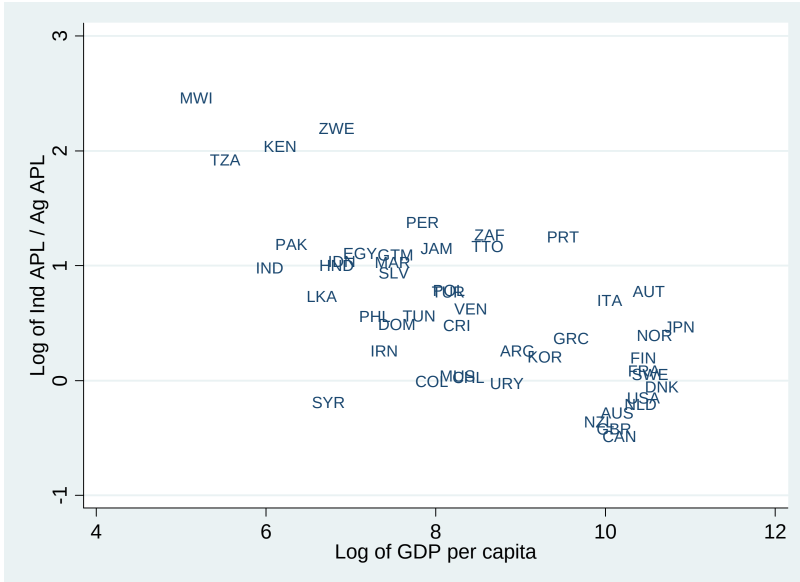

Figure 1: Relative Labor Productivity and Income per Capita

The *prima facie* evidence of these productivity differences comes from the observation that output per worker in agriculture is typically much lower than that of the manufacturing sector. Figure 1 plots the log of the relative labor productivity between the manufacturing and agricultural sectors against the log of income per capita for a cross-section of countries. As can be seen, this ratio declines markedly as income increases.

Similar evidence is presented in Temple (2005, Table 1), who documents that the relative labor productivity in manufacturing is nearly six to one for Sub-Saharan African countries in 1996, and even 1.7 to one in the OECD. As he points out, though, differences in average products do not necessarily mean that marginal products differ.

Assume that both agriculture and manufacturing have Cobb-Douglas production functions, and let labor's share in agriculture equal θ_A, while it's share in manufacturing is θ_M. The ratio of marginal products is therefore

$$\frac{MPL_M}{MPL_A} = \frac{\theta_M}{\theta_A}\frac{APL_M}{APL_A}. \tag{1}$$

If it is the case that $\theta_M < \theta_A$, then it is theoretically possible for $APL_M > APL_A$ to be consistent with equality in marginal products. Duality, as traditionally understood, would not hold because there was no scope for increasing output by transferring labor between sectors.

However, it is hard to actually reconcile the observed gaps in average products with the equalization of marginal products. If we assume, as we do for aggregate production functions, that the labor share in output is essentially equal across countries, then this suggests that θ_M/θ_A is constant across countries. If this is true, then it is impossible for *all* countries to have $MPL_M = MPL_A$ given that their ratios of average product differ.

Perhaps this just indicates that it is not true that θ_M/θ_A is the same in every country. If we still want to assert that marginal products are equalized across sectors, then it must be that θ_M/θ_A is the reciprocal of the ratio of average products. For several developing nations, the ratio of average products is on the order of 8 to one. Therefore θ_M must be only one-eighth of θ_A. If labor's share in agriculture is 0.6 (see Jorgenson and Gollop, 1992), then this implies that labor's share in manufacturing is only 0.075. This seems absurdly low. If we want to assert that labor's share in manufacturing is equal to only 0.2, we have the nonsensical result that the labor share in agriculture must be equal to 1.6, or that labor earns over 100% of total output.

So while the observations on average product cannot directly show that marginal products differ between sectors, it is hard to escape the conclusion that they do. Previous models of the dual economy have used exogenous institutional distortions to explain why wages may persistently differ between sectors, but they are not capable of explaining the relationship between income and duality in figure 1.

One possibility is that the dual economy and overall poverty are both outcomes of some deeper structural flaw or institutional failure across countries. However, the inverse relationship of relative productivity and income holds over time even for those countries that are the richest today.

David (2005), in his re-examination of the data on real income in the early years of the United States, finds evidence of a dual economy. In 1840, the relative labor productivity of agricultural workers (narrowly defined) was only 40% of non-agricultural workers. Even on a per man-hour basis, the ratio is still 77%. Over the whole period of 1790-1860, David concludes that the average product of labor was relatively low in the agricultural sector, but that this was primarily accounted for through fewer hours of work rather than lower productivity per hour. This difference between the agricultural and modern sectors will arise endogenously within the model presented here, and just as David documents for the U.S., a portion of the increase in output per capita will result from an increase in hours worked by all individuals as well as a shift of workers from agriculture to manufacturing.

2.2 Differential Fertility

The differential in time allocations that accounts for the gap in labor productivity occurs alongside similar differentials in fertility. For the United States, Grabill, Kiser and Whelpton (1958) report fertility data which covers the whole period of industrialization in the United States, 1800-1950.[5]

The overall decline in fertility in this period can be broken into three components: a decline in urban fertility, a decline in rural fertility, and the shift of population from rural to urban areas. Grabill et al (1958)

[5]Due to data limitations in the U.S. Census their fertility measures are child/woman ratios. This ratio is not ideal because it does not control for age structure, but it is available and does provide a decent portrayal of the experience of American fertility over time.

calculate the contribution of each factor to the overall decline in this period. They find that about 56 percent of the decline is attributable to the fall in rural fertility, with 24 percent accounted for by urban fertility decline and 20 percent to the shift from rural to urban areas.[6] So while there is a differential in fertility between sectors, over time this differential shrinks and fertility converges to a uniformly low level.

This pattern is repeated in the European experience, as summarized by Sharlin (1986) using the data from the Princeton European Fertility Project. Sharlin confirms is that rural marital fertility was indeed higher than urban marital fertility at virtually every date that data were collected. When the demographic transition began, it began in the urban areas before spreading to the rural sector. This trend is confirmed in the more detailed studies of Germany by Knodel (1974) and Belgium by Lesthaeghe (1977).

As the demographic transition continues, fertility remains always higher in rural areas but the differential decreases. The pattern is thus similar to what we identified above for the United States - a convergence of rural fertility to urban fertility. Sharlin also finds that the declines in both rural and urban fertility play the predominant role in lowering overall fertility, confirming the finding in the U.S. that the shift of population from rural to urban areas was of secondary importance.

One interesting aspect of fertility behavior that is highlighted by Sharlin is the effect of city size. Knodel (1974), Lesthaeghe (1977) and Livi-Bacci (1977) all present evidence that as the population of the urban area increases, fertility rates fall. This effect appears to be tied to the nature of work in these areas, as those places engaged in more industrial work - urban or not - are more likely to have lower fertility. Thus Sharlin suggests that if we could look at the fertility of agricultural and non-agricultural workers we would find higher differentials than those found between urban and rural areas. The model presented in this paper fits neatly into this framework, as the distinction is truly between agricultural and non-manufacturing workers, and actual residence is not important.

These long-run experiences of the currently developed world show that duality in fertility can exist and then disappear over time, but do currently developing countries fit within this framework? One of the earliest looks at the rural-urban differential comes from Kuznets (1974). He considers a cross-section of nations from the late 1950's and early 1960's. Using child/woman ratios he calculates the rural-urban differential and finds that rural areas are consistently more fertile, but that the gaps are not terribly large. [7]

Similar findings come from another cross-sectional study by Findley and Orr (1978) which looks at fertility behavior around 1970 in thirty-eight developing countries. They find again that differentials in rural-urban

[6] These percentages are tied to the time period in question. Using a later beginning date would lower the initial rural population share and decrease rural fertility's role in the overall decline, while raising urban fertility's role. Grabill et al (1958) do not find changing the time frame alters significantly the share of overall decline caused by rural to urban shifts.

[7] Kuznets does find several countries in Africa in which the urban child/woman ratio is actually higher in urban areas than in rural ones, but he points out this could be caused by differences in age structure which the child/woman ratio does not account for.

fertility rates are closely associated with a countries overall position in the demographic transition. In their seven highest fertility countries, there is essentially no differential. As they examine countries with lower overall fertility the differential widens substantially. For the lowest fertility countries they examine - with overall total fertility rates (TFR) of around 4.5 - urban TFR is only 60% of rural TFR.

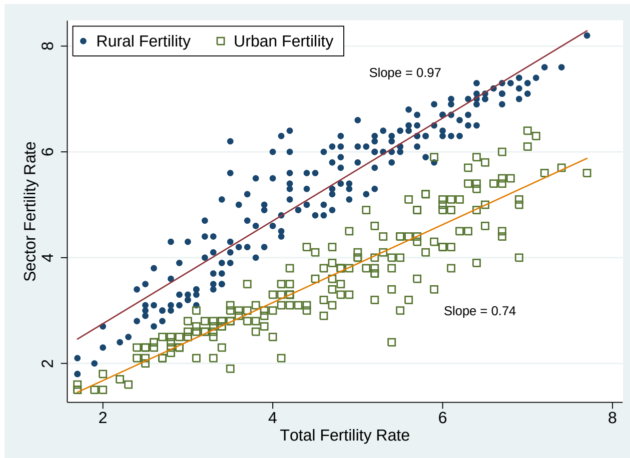

Figure 2: Sector Fertility Rates and Total Fertility

Note: Data is from the Demographic and Health Survey (DHS), showing urban and rural fertility rates for each of 172 different surveys from 74 different developing countries in various years. The slopes are estimated by OLS.

More recent data confirms the dual nature of fertility, and the convergence of fertility rates as economies develop. Figure 2 plots urban and rural fertility rates for all of the surveys undertaken by the Demographic and Health Survey project. As can be seen, rural fertility is consistently higher than urban fertility in this set of developing countries. Additionally, as the total fertility rate falls, rural fertility falls faster than urban fertility, indicated by the higher slope for the rural observations.

3 The Dual Economy

The evidence reviewed provides several distinct stylized facts that must be accounted for when trying to model the dual economy. First, differences in labor productivity (and likely in marginal productivity) exist between the agricultural and the modern sector. Secondly, these differences in productivity evolve along with the economy, shrinking in size at higher levels of aggregate output per capita.

Similarly, fertility differences exist within the economy, and these differences also evolve. As the economy develops both rural and urban fertility decline, but rural fertility appears to fall faster so that the sectoral rates converge.

To explain these facts, the model presented here will combine a standard model of fertility choice with an assumption regarding the non-separability of fertility and production in the agricultural sector.[8] Non-separability implies that individuals in the agricultural sector appreciate the effect of their fertility choice on the marginal product of their time. They will see a rising marginal product of labor as they spend more time raising children, and in equilibrium this will lead to agricultural individuals having more children than manufacturing sector workers. No distortions in wages or prices are required, and free mobility between sectors will be allowed.

This section presents general results regarding the dual economy in equilibrium. Subsequent sections explore the dynamics of the dual economy and the effect of productivity changes in both sectors.

Consider a two-sector economy consisting of L_t people at time t. Of these people, a share $a \in (0,1)$ work in the agricultural sector while the remaining $1-a$ work in manufacturing. The labels agricultural and manufacturing are for convenience, and could alternately be labeled the "traditional" and "modern" sectors.

Each individual has a unit of time, with the share $s \in (0,1)$ allocated to productive work, and the remaining $1-s$ time spent raising children. Individuals must consume a fixed amount of agricultural goods, $\overline{a}$, at a relative price p^A. They receive utility from the amount of manufacturing goods they consume as well as from the number of children they have. Their decision problem is thus one of allocating time between productive work and raising children, conditional on working enough to acquire the subsistence amount of agricultural goods.

3.1 Production

Agricultural production is a constant returns to scale function of labor effort and land. Capital is ignored throughout to simplify the analysis, but this does not materially alter the results. Total agricultural production is denoted as

$$Y_t^A = A_t^A F(R, E_t^A) \tag{2}$$

where A_t^A is total factor productivity in the agricultural sector, R is the total amount of land (resources) in the sector, and $E_t^A = s_t a_t L_t$ is the total labor effort expended. F is constant returns to scale, and has typical concave properties

$$F_R > 0, F_{RR} < 0, F_E > 0, F_{EE} < 0, F_{RE} > 0 \tag{3}$$

Each of the $a_t L_t$ individuals working in agricultural has property rights over an equal amount of land, $R/a_t L_t$. Therefore the total income of an agricultural worker is, given the constant returns to scale production

[8] A simpler form of this type of model can be found in Weisdorf (2006), who does not consider fertility levels.

function, equal to

$$I_t^A = p_t^A A_t^A F\left(\frac{R}{a_t L_t}, s_t\right) \tag{4}$$

where p_t^A is the relative price of agricultural goods, in terms of manufacturing goods. Notice that the income is increasing in s_t, but given the shape of the production function there are diminishing returns to the time of the agricultural worker. This will factor into the decision problem of the individual as they decide how to allocate time between fertility and work. Higher fertility, in the agricultural sector, will not "cost" as much because increasing fertility will actually raise the marginal return to work.

For the initial analysis, it is not necessary to specify completely the manufacturing sector production function. What is important is not the nature of production, but rather the nature of the labor market. From the perspective of a manufacturing worker, the wage rate per unit of effort is constant. Income for a manufacturing worker is simply

$$I_t^M = w_t^M s_t \tag{5}$$

where w_t^M is the wage rate paid in manufacturing. To explore the full dynamics of the model, details regarding this wage will be required, but for the current purposes it is sufficient to work with w_t^M alone.

3.2 Individuals

Individuals receive utility from their consumption of the manufactured good (c_i^M) and from the number of children they have (n_i). Similar to the work of Strulik and Weisdorf (2008), the utility function is presumed to be quasi-linear. This is a convenient way of capturing a "hierarchy of needs" such that fertility is a higher priority than consumption of manufacturing goods. In addition, there is no direct income effect on fertility. Changes in fertility will arise only from changes in relative prices, and this will depend upon the sector (agriculture or manufacturing) that generates the higher income levels.

Utility is written as

$$U_i = c_i^M + g(n_i) \tag{6}$$

while the function g has the properties that $g' > 0$ and $g'' < 0$. Fertility is a linear function of the time spent on child-rearing

$$n_i = \overline{n}(1 - s_i) \tag{7}$$

where $\overline{n}$ is simply a parameter scaling time into numbers of children.

In terms of consumption, each individual must fulfil a basic subsistence constraint first, eating $\overline{a}$ units of the agricultural output.[9] Once they have met this requirement, their manufacturing consumption is given

[9]Note that the food consumption of children is presumed to be zero, but including their food requirements into the budget

by

$$c_t^M = I_t^i - p_t^A \overline{a} \tag{8}$$

where i refers to the sector in which an individual works.

Putting (8) and (7) into the utility function from (6), along with the definitions of income from (4) and (5) yields the following utility functions for individuals in the two sectors

$$\begin{aligned} U_t^A &= p_t^A A_t^A F\left(\frac{R}{a_t L_t}, s_t^A\right) - p_t^A \overline{a} + g(\overline{n}(1 - s_t^A)) && (9) \\ U_t^M &= w_t^M s_t^M - p_t^A \overline{a} + g(\overline{n}(1 - s_t^M)) && (10) \end{aligned}$$

3.3 Equilibrium

Recall that all individuals must consume a quantity $\overline{a}$ of the agricultural good. Therefore, it must be the case that

$$\overline{a} L_t = A_t^A F\left(R, s_t^A a_t L_t\right) \tag{11}$$

which states that total agricultural production must be sufficient to provide the subsistence consumption to all individuals. Given the choice of time allocation in agriculture the share of individuals employed in agriculture, a_t, is fixed by this equation.

One last assumption is necessary before proceeding. Individuals are assumed to be freely mobile between sectors, and this will ensure that the utility of individuals in the two sectors is equalized, or $U_t^A = U_t^M$. With this, a definition of the equilibrium can be provided.

Definition 1 *At time t, define an* ***equilibrium*** *as a set of prices and allocations $(p_t^{*A}, a_t^*, s_t^{*A}, s_t^{*M})$ that, given the state of the economy (A_t^A, w_t^M, R, L_t), fulfills the following four conditions*

- s_t^{*A} *maximizes* U_t^A
- s_t^{*M} *maximizes* U_t^M
- $U_t^A = U_t^M$
- $\overline{a} L_t = A_t^A F\left(R, s_t^{*A} a_t^* L_t\right)$

At this point it is possible to demonstrate that a dual economy situation exists, which is spelled out in the following proposition.

constraint would not change the essential results of the paper. This would increase the marginal cost of children, lowering fertility for each individual.

Proposition 1 *In equilibrium, a dual economy exists. Specifically this means that the following hold:*

(A) Manufacturing workers allocate more time to productive work than agricultural workers, $s_t^{*M} > s_t^{*A}$

(B) Manufacturing workers have lower fertility than agricultural workers, $n_t^M < n_t^A$

(C) Output ***per worker*** *is higher in the manufacturing sector,* $w_t^M s_t^{*M} > p_t^{*A} A_t^A F\left(\frac{R}{a_t^* L_t}, s_t^{*A}\right)$

(D) The marginal product of labor effort is higher in the manufacturing sector, $w_t^M > p_t^{*A} A_t^A F_E\left(\frac{R}{a_t^* L_t}, s_t^{*A}\right)$

Proof. See Appendix ■

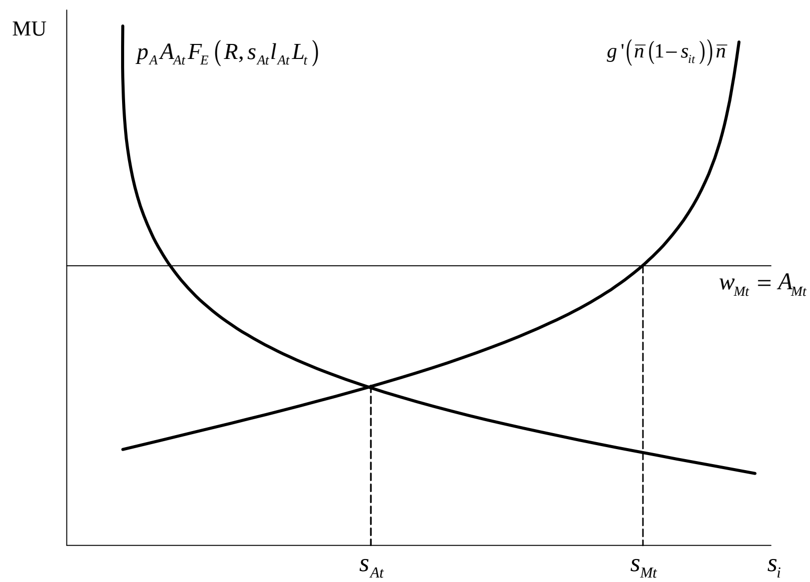

Figure 3: Determination of Optimal Time Allocations

The essential intuition of the proposition is captured in figure 3. Here the marginal utility of fertility is plotted, as well as the marginal utility derived from productive work in the two sectors. In the manufacturing sector, individuals take the wage as a given, and so the marginal utility is flat. In contrast, the marginal utility of productive work declines with s_t^A because agricultural workers are internalizing the effect of their own time. To achieve an equality of utility between the two types of workers, it must be the case that the optimal solution for agricultural workers involves lower marginal utility from both activities. The equilibrium for agricultural workers shows that their marginal productivity (which in this case is identical to their marginal utility) must be lower than the marginal productivity of manufacturing workers. This discrepancy in marginal products is the subject of the following corollary.

Corollary 1 *Given the results of Proposition 1, output per worker can be increased by a transfer of labor from agriculture to manufacturing.*

Proof. (A) follows from part (D) of Proposition 1, which states that the marginal product of agricultural workers is lower than workers in manufacturing. Therefore, transferring labor from agriculture to manufacturing will increase output per capita. ■

It is crucial to note that while Corollary 1 says that a transfer of labor can increase *output*, it does not increase *welfare*. This explains why a dual economy situation may persist despite the *prima facie* evidence that the economy is inefficient. Corollary 1 thus provides a potential explanation for the cross-country results highlighted in the introduction.

Why does this effect arise? A common explanation of dual economy effects is that while agricultural workers are paid their average product (because of poor property rights over land, for example), manufacturing workers are paid their marginal product. Marginal products are not equalized, and therefore there is an inefficiency in the economy.

This effect is present here, as I have assumed that agricultural workers are paid their average produce. But note that there is an additional inefficiency at work in this model. From part (C) of Proposition 1, the average product of agricultural labor is actually *lower* than the marginal product of manufacturing labor. Thus the economy exhibits even lower efficiency than the typical dual economy model. This difference arises because the agricultural workers internalize the effect of their choice on their own productivity, and this is necessary to generate not just the inefficiency in production but the differential fertility levels observed in the data.

3.4 Productivity and Population Changes

Given this basic model, how do changes in productivity (in either sector) alter the structure of the economy and affect the nature of the dual economy?

To proceed, let manufacturing wages be described by the following

$$w_t^M = A_t^M w(a_t) \tag{12}$$

where A_t^M is a measure of total factor productivity in manufacturing and the function $w(a_t)$ describes the relationship of the wage to the share of people engaged in manufacturing. It is presumed that $w' > 0$, meaning that manufacturing wages fall when the share of workers in manufacturing $(1 - a_t)$ increases.

Now we can state the following

Proposition 2 *Given manufacturing wages as in (12),*

(A) An increase in A_t^A has the following effects:

- *The share of labor in agriculture, a_t^*, falls*
- *The time allocations to productive work, s_t^{*A} and s_t^{*M}, both fall*
- *There is an indeterminate effect on aggregate fertility*

(B) An increase in A_t^M has the following effects:

- *The share of labor in agriculture, a_t^*, falls*
- *The time allocations to productive work, s_t^{*A} and s_t^{*M}, both rise*
- *Aggregate fertility falls*

Proof. See Appendix ■

From Proposition 2, it becomes clear that the kind of technological progress experienced by an economy matters for long-run output per capita. If agricultural productivity increases, labor is "pushed" into the manufacturing sector, the typical result in two-sector models. However, by lowering wages in the manufacturing sector this reduces the marginal utility of work and raises fertility in the manufacturing sector. The agricultural sector responds by increasing fertility as well, as the marginal productivity of their work must fall to ensure that labor is willing to flow into the manufacturing sector. The increases in sector-level fertility act to dampen the fertility-lowering effect of the shift into manufacturing.

In contrast, an increase in manufacturing productivity induces an increase in work effort, and lower fertility, in that sector. In response to the higher utility in manufacturing, labor is "pulled" out of agriculture. To maintain agricultural output, time spent working by agricultural individuals has to increase, lowering their fertility as well. Overall, there is a clear shift towards lower fertility.

The long-run impacts of productivity growth depend crucially on the sector in which it arises. In typical models, increasing agricultural productivity causes industrialization. However, these models normally take the population size as fixed. Here, we see that agricultural productivity growth, relative to manufacturing productivity growth, will cause fertility to be higher. In the future there will be a relatively larger population and this will act like a *decrease* in agricultural productivity as resources per person decline. So over time, agricultural productivity increases will limit development relative to manufacturing productivity increases. To see these dynamic effects more clearly will require a more detailed specification.

4 Dynamics of Development

The economy described in the previous section shows that dualism can result even with optimizing agents once we allow for endogenous fertility choice. The implications of Proposition 2 are that the *type* of productivity growth can have important consequences. That is, the long-run effects of growth in agricultural productivity are not the same as the effects of manufacturing productivity growth.

In this section I provide several simulations of a dual economy to demonstrate how it responds to different patterns of productivity growth (both exogenous and endogenous) over time. These experiments highlight the importance of the differential fertility responses to productivity changes by sector.

To do this, specific functional forms are assumed for production and utility that result in a tractable model that can be analyzed analytically and numerically. The algebra is tedious, and has been relegated to the Appendix. The most important point is that manufacturing output, for simplicity, is presumed to be linear in labor effort (and capital is excluded from the analysis). This makes the problem tractable, but removes the possibility that agricultural productivity growth will lower the optimal time allocation in manufacturing by lowering the marginal product in that sector. If this effect were present, then the potentially detrimental effects of agricultural productivity growth explored in this section would only be strengthened.

Several types of simulations are considered. In the first, two initially identical economies experience exogenous productivity "revolutions", one in the agricultural sector and one in the manufacturing sector. For the second, sustained differences in exogenously given productivity growth rates are examined for their affect on development.

Finally, productivity growth in both sectors is made endogenous, and the simulation shows how the dual economy evolves over time from the initial Malthusian era to the period of sustained growth. This simulation highlights how the dual economy can be seen as simply another feature of the process of long-run development, alongside fertility change and the structural transformation.

4.1 Productivity Revolutions

In this simulation, two economies are compared. Both have some latent exogenous productivity growth of 0.2% per year in each sector. The economies vary only in a productivity shock that strikes them at period 20 (a total of 100 periods are simulated):

- *Agricultural Revolution:* a 15% increase in A_t^A
- *Industrial Revolution:* a 15% increase in A_t^M

At the time of the shock, both simulations have an agricultural share of labor of approximately 70%, or they are relatively under-developed.

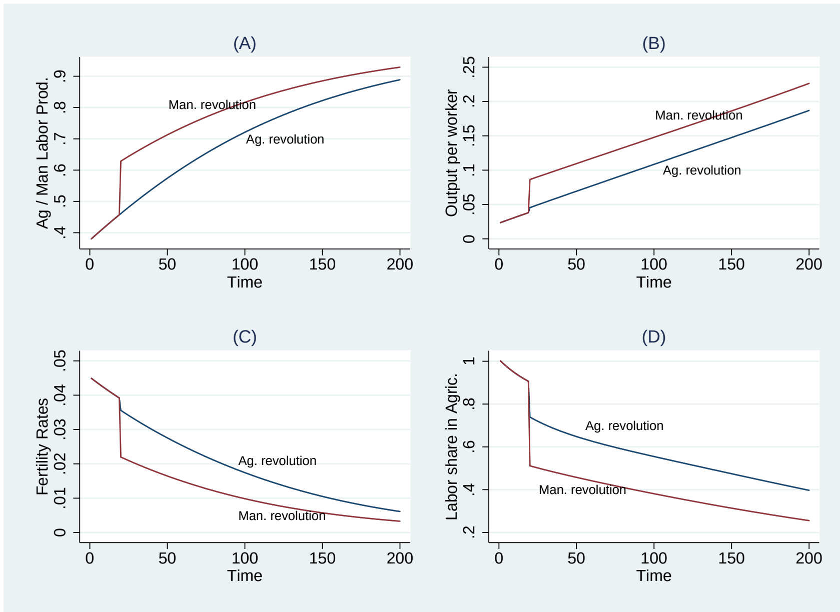

Figure 4: Productivity Revolutions

Note: The panels track the evolution of various characteristics under two different regimes of productivity revolutions. "Ag. revolution" refers to an agricultural total factor productivity growth (TFP) of 15% in period 20. "Man. revolution" refers to manufacturing TFP growth of 15% in period 20. Productivity growth in both sectors in both economies is equal to 0.2% in all other periods.

Figure 4 plots the results of the simulations for several important variables. Panel (A) shows the relative labor productivity of agriculture to manufacturing. As can be seen, both sectors begin with a distinct dual economy, with agricultural labor productivity only about 40% of that in manufacturing. With an Industrial Revolution, though, this ratio jumps distinctly to nearly 65% and is always higher than when an Agricultural Revolution occurs. The Agricultural Revolution has no effect on the dual economy at first because agricultural productivity changes do not induce higher work efforts.

Panel (B) shows how the two revolutions influence output per capita. As can be seen, the Industrial Revolution puts the economy on a permanently higher level of output per capita, while the growth rates remain identical. The source of this advantage is the increase in work effort that the Industrial Revolution initiates relative to the Agricultural. The higher work efforts also lead to lower fertility, as seen in figure (C). This lower fertility allows the economy with the Industrial Revolution to retain its higher output per capita by reducing the size of the population relative to the resource endowment.

Finally, panel (D) shows a somewhat counter-intuitive result. The Agricultural Revolution reduces the share of labor employed in that sector; this is the typical "push" idea that normally informs multi-sector models of industrialization. However, the Industrial Revolution produces an even larger drop in the agricultural labor share, even though they do not experience any significant increase in productivity in that sector. The difference comes from the increased work effort induced by the Industrial Revolution.

When A_t^M jumps by 15%, this increases the marginal value of time for manufacturing workers, and because of the increase in the relative price of agricultural goods, for agricultural workers as well. Thus the optimal allocation of time shifts towards work and away from fertility in both sectors. Given the concave nature of utility from fertility, agricultural workers have to increase their time allocation to work by more than manufacturing workers to achieve the same increase in marginal utility. This narrows the gap in their work effort, and narrows the dual economy effect. In addition, the increased work effort in agriculture acts like a productivity increase, lowering the share of people necessary to provide agricultural goods. Thus an Industrial Revolution can induce a greater structural transformation in the dual economy than a similar Agricultural Revolution is capable of.

4.2 Differential Growth Rates

Rather than examining singular "revolutions" in productivity, in this section the development of economies is tracked under sustained differences in productivity growth across sectors. The initial conditions of the simulations are identical to those of the previous section. The only difference is that productivity growth is concentrated in one sector versus the other in the two parallel simulations.[10]

- *Agricultural-led growth:* A_t^A grows at the rate of 1.5% per period, while manufacturing productivity A_t^M grows only at 0.1% per period.

- *Manufacturing-led growth:* A_t^A grows only at 0.1% per period, while A_t^M grows at 1.5% per period.

From this common starting point, each simulation is run forward for 200 periods. Figure 5 compares the path of several variables across the different productivity regimes. Panel (A) shows the ratio of agriculture output per worker to manufacturing output per worker, the *prima facie* evidence of a dual economy. As can be seen, in period zero under both regimes this ratio is only 0.38. Very quickly, though, manufacturing-led growth increases this to nearly one. In contrast, agricultural-led growth, by limiting the changes made to time allocations, shows only a very slow increase in the relative productivity of agricultural workers. Thus the dual economy persists much longer under agricultural growth.

The consequences of this persistence are apparent in panel (B). Here output per worker is tracked, and under both regimes it begins at a value of 0.023. By the end of the simulation output per capita is nearly

[10]The exact growth rates used in these simulations are not crucial, and were chosen solely to highlight the distinction in performance.

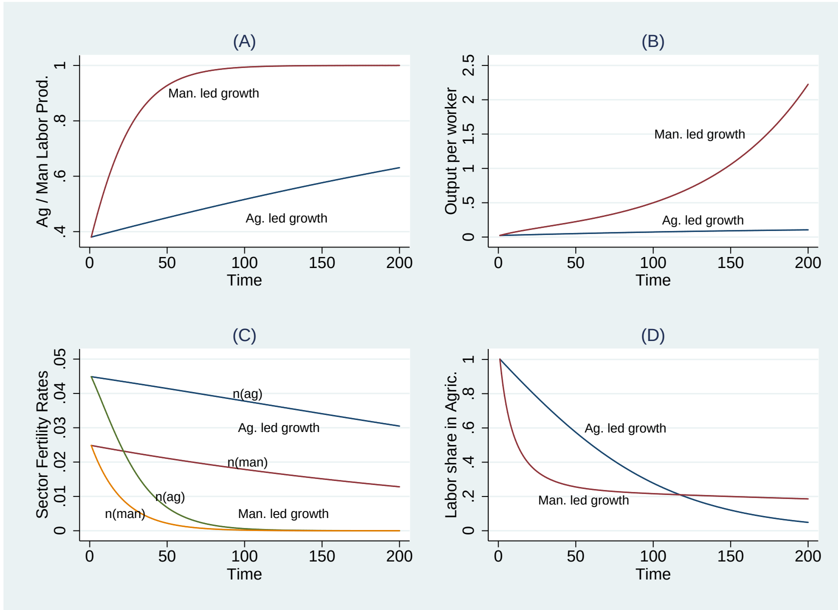

Figure 5: Development under Different Productivity Regimes

Note: The panels track the evolution of various characteristics under two different regimes of productivity growth. "Ag. led growth" refers to agricultural total factor productivity growth (TFP) of 1.5% per year, while manufacturing TFP grows at 0.1% per year. "Man. led growth" refers to agricultural TFP growth of 0.1% and manufacturing TFP growth of 1.5% per year.

5 times larger under agricultural growth, but this is overwhelmed by the nearly 100 fold increase in output per capita under manufacturing-led growth.

One of the main reasons for this disparity is the relative fertility levels in the two regimes. Panel (C) shows how both agricultural and manufacturing fertility change over time. With agricultural-led growth, fertility in both sectors declines slowly over time, leading to large population increases that literally eat up much of the productivity benefits of agricultural productivity change. In contrast, manufacturing-led growth shows rapid decreases in fertility in both sectors as people allocate more time to work when A_t^M goes up quickly. As a result, the manufacturing-led growth regime is trying to support fewer individuals, and this offsets their lower agricultural productivity level. Note as well that manufacturing-led growth shows a convergence of fertility rates between sectors while agricultural-led growth does not demonstrate this.

Finally, panel (D) plots the share of labor engaged in agriculture, which begins at essentially 100%. With manufacturing-led growth, this share drops very quickly, showing earlier industrialization than the agricultural-led regime. However, the manufacturing-led regime only slowly declines below 20% in the long run as the relatively low level of A_t^A in period 200 means a larger fraction of individuals must remain in that sector. In the agricultural-led regime, industrialization occurs more slowly, but ultimately is nearly complete.

Overall, the simulations show how divergent development can be depending on which sector experiences productivity increases. The agricultural-led regime does industrialize as people are "pushed" out of agriculture. However, this type of development retains large gaps in output per worker across sectors, as well as relatively high fertility. Ultimately, industrialization occurs within the framework of a dual economy and output per person increases only slowly.

The manufacturing-led regime industrializes as well due to the "pull" of higher wages, but this process is not necessarily as complete as in the agricultural-led regime. In contrast, though, the dual economy disappears relatively quickly and fertility falls as well. Ultimately output per capita is significantly higher due to the lower fertility and higher fraction of time spent working.

4.3 Unified Growth and the Dual Economy

Both of the previous sets of simulations took productivity growth to be exogenous. This section incorporates endogenous change in productivity to demonstrate how the dual economy fits within a unified growth framework. The economy will start out in a Malthusian era, with stagnant growth in population and income per capita. Despite the stagnation, agricultural productivity is improving over time and once the economy exits the Malthusian era the dual economy appears. Coincident with the arrival of the dual economy is a surge in fertility and the beginning of the structural transformation. As productivity increases continue the economy enters a modern growth era with low population growth, high income per capita growth, and the disappearance of the dual economy.

To facilitate this, some method of incorporating endogenous productivity growth must be included. Rather than complicate the exposition with a complete micro-economic model of innovation, I adopt a "reduced form" version of endogenous productivity growth similar to Kremer (1993) and Jones (1995a,b). Growth in productivity in the two sectors is described by

$$A_{t+1}^A - A_t^A = \delta_A a_t L_t (A_t^A)^\phi \tag{13}$$

$$A_{t+1}^M - A_t^M = \delta_M (1-a_t) L_t (A_t^M)^\phi \tag{14}$$

where δ_i is a parameter controlling the speed of innovation, and ϕ measures the returns to scale in innovation. If $\phi = 0$, then the arrival of new innovations is independent of the stock of knowledge, and the growth rate of productivity declines with productivity. If $\phi = 1$ then there are increasing returns to knowledge, and the growth rate depends only on the scale of the sector. To obtain realistic results in the simulations, a value of ϕ is chosen that is less than one, which accords with Jones (1995a,b) evidence on the long-run growth rate of innovation.

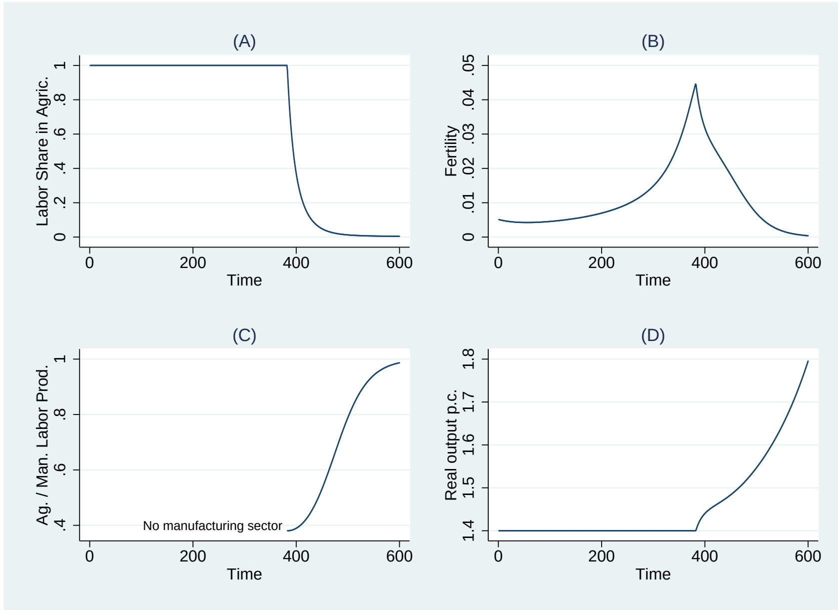

Figure 6: The Dual Economy with Endogenous Growth

Note: The panels track the evolution of the economy from the Malthusian era of low fertility, stagnant output per capita, and a large share of workers in the agricultural sector to the modern era of sustained growth, low fertility, and a primarily manufacturing workforce.

Figure 6 shows the time path of several variables over the 600 periods the simulation was run. Panel (A) shows that for nearly the first 400 periods, the economy works exclusively in the agricultural sector. This is due to the fact that initial productivity in that sector is set low initially, and the only way to generate the subsistence requirement is for all individuals to work in agriculture. Productivity is increasing in the agricultural sector during this period, while because $a_t = 1$, there is no productivity growth in the manufacturing sector.

Initially, as panel (B) shows, fertility is low. All individuals work in agriculture, and all individuals spend nearly 100% of their time working to feed themselves. Fertility is non-zero, though, and this begins to generate larger gains in A_t^A over time. Gains in A_t^A free up more time for fertility, and prior to industrialization around period 400 fertility is already increasing. Fertility peaks as agricultural productivity increases to the point where some individuals are freed to work in the manufacturing sector.

Panel (C) shows how the dual economy is evolving during this process. Prior to the release of the first laborers to manufacturing, there is no dual economy because all individuals work in the agricultural sector. Without a manufacturing sector, there can be no difference in relative labor productivity. However, once some individuals enter the manufacturing sector, they immediately begin to spend more time working than their agricultural peers. At the low levels of A_t^M present around period 400 of the simulation, the duality is severe, and agricultural workers produce only 40% of what a manufacturing worker does.

Once manufacturing workers are present, A_t^M begins to increase endogenously, and this generates a flow of workers into manufacturing, while also raising the time allocated to work in both sectors. This causes fertility to decline from its peak, and narrows the gap in output per worker between the sectors. Thus the dual economy endogenously disappears in this unified treatment, and this is driven by the increasing productivity growth of the manufacturing sector.

Ultimately, panel (D) shows how these factors all operate together to generate a Malthusian era of stagnant output per capita that gives way to an era of both high output per capita growth and high fertility around period 400, and finally to the era of modern growth where output per capita growth is high, but fertility has fallen to nearly zero.

The dual economy can be seen as a natural outgrowth of the process of development. Along with the Demographic Transition and the structural transformation, the appearance and gradual dissolution of the dual economy is an integral part of the long-run growth of economies.

5 Conclusion

This papers attempts to show that the dual economy is an integral part of the development of economies. Unlike previous attempts, the model presented here shows how a dual economy can arise endogenously in a model of optimal fertility.

Non-separability in the traditional, agricultural sector means that they internalize the effect of their own effort on their marginal product. Understanding that further work lowers the value of their time, they optimally shift towards having more children and working less. This generates a difference with modern sector individuals who face a constant wage rate for their time.

The model implies that agricultural productivity growth has negative consequences that offset some or all of its initial benefits. Work efforts fall, and fertility rises following agricultural productivity changes; the dual economy persists. In contrast, manufacturing productivity increases work efforts in both sectors and lowers fertility, resulting in higher long-run output per capita.

Ultimately, the persistence of the dual economy in developing countries is not a reflection of embedded institutional or market failures, but rather is a consequence of slow modern sector development. This paper's characterization of the dual economy is in many ways more hopeful than earlier explanations. First, it suggests that one should not interpret differences in output per worker as indicative of differences in welfare between sectors. Second, unlike models relying on exogenous distortions, this model shows that duality does not imply that anything is fundamentally "wrong" with an economy.

Appendix A: Proofs

Proposition 1

Proof. To see (A), consider what would happen if $s_t^{*M} = s_t^{*A} = s^*$. If that were true, then fertility would be identical in both sectors, and the utility, $g(\overline{n}(1-s^*))$ would be identical as well. From the manufacturing optimization problem, it must be that $g'(\overline{n}(1-s^*)) = w_t^M$. From the agricultural problem, it must be that $g'(\overline{n}(1-s^*)) = p_t^A A_t^A F_E\left(\frac{R}{a_t L_t}, s_t^*\right)$, so therefore it would have to be that $p_t^A A_t^A F_E\left(\frac{R}{a_t L_t}, s_t^*\right) = w_t^M$. If this is true, then the difference in utility between the agricultural and manufacturing sectors would have to be $U_t^A - U_t^M = p_t^A A_t^A \left(F\left(\frac{R}{a_t L_t}, s_t^*\right) - F_E\left(\frac{R}{a_t L_t}, s_t^*\right) s_t^*\right) > 0$ where the last inequality follows from the fact that F is concave. So if $s_t^{*M} = s_t^{*A} = s^*$ then agricultural utility is larger than manufacturing utility, which violates the free mobility condition. For the free mobility condition to hold, the value of s_t^{*A} must fall relative to s_t^{*M}, as this will lower the average product of an agricultural worker to the point where it equals the marginal product of a manufacturing worker. The proof of (B) follows directly from the definition of fertility in equation (7) and the fact that $s_t^{*M} > s_t^{*A}$. (C) follows from the proof of part (A). (D) comes from noting that (C) implies that the average product of an agricultural worker is less than $w_t^M s_t^M$. Because of the concave shape of the agricultural production function, the marginal product of labor in agriculture is less than its average product. Therefore $w_t^M s_t^M > p_t^A A_t^A F_E\left(\frac{R}{a_t L_t}, s_t^{*A}\right)$ and (D) follows from the fact that $s_t^M < 1$. ■

Proposition 2

Proof. (A) An increase in A_t^A, given the subsistence requirement in (11), requires that $s_t^A a_t$ falls so that total agricultural supply is equal to demand. Consider what would happen if a_t went up. From (12), this would cause manufacturing wages to rise. In addition, this increase in wages will increase the share of time spent working, s_t^M, which is immediately apparent from figure (3), and thus utility in the manufacturing sector. Free mobility requires that agricultural utility increases as well, which can only be achieved with an increase in the marginal product of labor, which increases s_t^A. However, if s_t^A goes up, and we have assumed that a_t has increased, then $s_t^A a_t$ goes up, and this contradicts the subsistence requirement that $s_t^A a_t$ goes down. For this subsistence requirement to hold, it must be that a_t falls, which lowers the manufacturing wage, lowering s_t^M, and requiring that s_t^A falls as well. Both time allocations thus fall, and by definition fertility in each sector rises. With fertility in both sectors increasing, but labor transferring from the high-fertility agricultural sector to the low-fertility manufacturing sector, the aggregate effect is indeterminate.

(B) When A_t^M goes up, this increases w_t^M at every level of a_t. As noted in for part (A), an increase in w_t^M will induce an increase in s_t^M, lowering fertility in the manufacturing sector and raising U_t^M. Free mobility requires U_t^A to increase, which is achieved by an increase in the marginal product of labor in agriculture, resulting in s_t^A increasing, lowering fertility in the agricultural sector. With s_t^A higher, it must be that a_t falls so that the subsistence requirement in (11) holds. With both s_t^A and s_t^M rising, fertility in both sectors is lower by definition. In addition, individuals are transferred from the high-fertility agricultural sector to the low-fertility manufacturing sector. Therefore, aggregate fertility must fall. ■

Appendix B: Simulation

Agricultural production is presumed to be Cobb-Douglas in land and labor effort, as in

$$Y_t^A = A_t^A R^\alpha (s_t^A a_t L_t)^{1-\alpha} \tag{15}$$

where α represents the elasticity of output with respect to land. Given the assumptions regarding property rights, output per person in the agricultural sector can be written as

$$I_t^A = A_t^A \left(\frac{R}{a_t L_t}\right)^\alpha (s_t^A)^{1-\alpha}. \tag{16}$$

Manufacturing output is, for simplicity, assumed to be completely linear in labor effort,

$$Y_t^M = A_t^M s_t^M (1 - a_t) L_t. \tag{17}$$

The wage rate per unit of labor effort is simply $w_t^M = A_t^M$ in this case, and there is no effect of $(1 - a_t)$ on the wage rate.

Individual utility for someone in sector i is assumed to have the following form

$$U_t^i = c_t^{Mi} + (n_t^i)^{1-\alpha} \tag{18}$$

where fertility shows diminishing marginal utility, and the assumption that the exponent is equal to $1-\alpha$ is made so that a clear analytical solution can be obtained. Fertility can be written as $n_t^i = \overline{n}(1 - s_t^i)$. Using the budget constraint in (8) we can write utility in the two sectors as

$$\begin{aligned} U_t^A &= p_t^A A_t^A \left(\frac{R}{a_t L_t}\right)^\alpha (s_t^A)^{1-\alpha} - p_t^A \overline{a} + \hat{n}(1 - s_t^A)^{1-\alpha} && (19) \\ U_t^M &= A_t^M s_t^M - p_t^A \overline{a} + \hat{n}(1 - s_t^M)^{1-\alpha} && (20) \end{aligned}$$

where $\hat{n} = \overline{n}^{1-\alpha}$.

The final condition is the subsistence constraint, as is (11), which given the agricultural production function in (15) yields

$$\overline{a} L_t = A_t^A R^\alpha (s_t^A a_t L_t)^{1-\alpha}. \tag{21}$$

Now applying the concept of equilibrium from Definition 1, we must have individuals maximizing the utility functions in (20), while free mobility ensures that $U_t^A = U_t^M$, and subsistence requirements are met as in (21). First, solving for the optimal time allocation in the manufacturing sector yields

$$s_t^{*M} = 1 - \left(\frac{\hat{n}(1-\alpha)}{A_t^M}\right)^{1/\alpha} \tag{22}$$

and knowing this, define the following term

$$\hat{U}_t \equiv A_t^M s_t^{*M} + \hat{n}(1 - s_t^{*M})^{1-\alpha} \tag{23}$$

which reflects the utility of manufacturing individuals if their subsistence requirement were zero.

Knowing this, the rest of the equilibrium can be found. First, maximizing U_t^A over s_t^A yields the following result

$$\frac{s_t^{*A}}{1 - s_t^{*A}} = \left(\frac{p_t^A A_t^A}{\hat{n}} \left(\frac{R}{a_t L_t}\right)^\alpha\right)^{1/\alpha} \tag{24}$$

which when inserted back into U_t^A and used with the free mobility condition and the definition of $\hat{U}$ gives

$$s_t^{*A} = 1 - \left(\frac{\hat{n}}{\hat{U}}\right)^{1/\alpha}. \tag{25}$$

Given s_t^{*A}, the optimal share of labor in agriculture is obtained from the subsistence constraint in (21) which yields

$$a_t^* = \left(\frac{\overline{a} L^\alpha}{A_t^A R^\alpha (s_t^{*A})^{1-\alpha}}\right)^{1/(1-\alpha)} \tag{26}$$

Table 1: Simulation Parameter Values

Parameter	(1) Base Model	(2) Unified Model
Land (R)	10	1
Land share (α)	0.3	0.3
Subsistence req. ($\overline{a}$)	1.4	1.4
Fertility constant ($\overline{n}$)	0.054	0.54
Initial population (L_0)	1	1
Initial agric. productivity (A_0^A)	2.4	1.5
Initial manuf. productivity (A_0^M)	0.115	0.115

and finally the price of agricultural goods can be solved from (24) giving

$$p_t^{*A} = \frac{\hat{n}}{A_t^A}\left(\frac{a_t^* L_t}{R}\frac{s_t^{*A}}{1-s_t^{*A}}.\right)^{\alpha} \tag{27}$$

Given s_t^{*A}, s_t^{*M}, a_t^*, and p_t^{*A}, we have all the information available to solve for other variables of interest within a given period. Specifically, output in each sector follows directly from (15) and (17). Output per capita is found simply by dividing their total by L_t. The relative output of agricultural workers to manufacturing workers can be directly backed out from the production functions.

The dynamic effects of changes in productivity depend entirely in this set-up on the fertility consequences. Population evolves as follows

$$L_{t+1} = (1+n_t)L_t \tag{28}$$

where

$$n_t = a_t\overline{n}(1-s_t^{*A}) + (1-a_t)\overline{n}(1-s_t^{*M}). \tag{29}$$

As population increases, this acts much like a *decrease* in agricultural productivity, raising agricultural prices and inducing more people to stay in agriculture.

The parameters in column (1) of table 1 yield initial values of $s_t^{*M} = 0.54$ and $s_t^{*A} = 0.17$ for the baseline model, which results in output per worker in the agricultural sector being only 38% of that in the manufacturing. To meet subsistence, $a_0 = 0.999$.

Once endogenous productivity growth is introduced, the parameters of the innovation equations in (14) must be specified as well. For the simulations presented in this paper, the value of $\phi = 0.99$, while $\delta_A = 0.001$ and $\delta_M = 0.00001$. These values were chosen by trial and error so as to yield maximum growth in productivity in any given period below 5%.

In addition to the endogenous productivity parameters, the baseline model is modified in the manner indicated in column (2) of table 1. These act mainly to reduce the agricultural output of the economy, meaning that subsistence cannot be achieved with fewer than 100% of individuals in the agricultural sector. With $a_t = 1$, the necessary work effort in agriculture is

$$s_t^A = \left(\frac{\overline{a}L_t^{\alpha}}{A_t^A R_t^{\alpha}}\right)^{1/(1-\alpha)}. \tag{30}$$

Once A_t^A is sufficiently large so that the subsistence requirement can be met with less than 100% of the individuals in agriculture, then the equilibrium is calculated as before.

References

[1] Banerjee, Abhijit and Esther Duflo. 2005. "Growth Theory through the Lens of Development Economics." in *Handbook of Economic Growth*, ed. Aghion, Phillipe and Steven Durlauf. North Holland: New York.

[2] Basu, Kaushik. 1997. *Analytical Development Economics: The Less Developed Economy Revisited.* MIT Press: Cambridge, MA.

[3] Benjamin, Dwayne. 1992. "Household Composition, Labor Markets, and Labor Demand: Testing for Separation in Agricultural Household Models." *Econometrica* 60(2):287-322.

[4] Bertrand, T. and L. Squire. 1980. "The Relevance of the Dual Economy: A Case Study of Thailand." *Oxford Economic Papers* 40:55-73.

[5] Caselli, Francesco and John W. Coleman. 2001. "The U.S. Structural Transformation and Regional Convergence: A Re-interpretation." *Journal of Political Economy* 109(31):584-616.

[6] Chackiel, Juan and Susana Schkolnik. 1996. "Latin America: Overview of the Fertility Transition, 1950-1990." in *The Fertility Transition in Latin America*, ed. Jose Miguel Guzman, Susheela Singh, German Rodriguez and Edith A. Pantelides. Oxford, UK: Oxford University Press.

[7] Chanda, Areendam and Carl-Johan Dalgaard. 2008. "Dual Economies and International Total Factor Productivity Differences: Channelling the Impact from Institutions, Trade, and Geography." *Economica* 75:629-661.

[8] Chenery, Hollis and Moshe Syrquin. 1975. *Patterns of Development, 1950-1970.* Oxford University Press: London.

[9] Chenery, Hollis and Sherman Robinson and Moshe Syrquin. 1986. *Industrialization and Growth: A Comparative Study.* Oxford University Press: New York.

[10] Cordoba, Juan Carlos and Marla Ripoll. 2008. "Agriculture, Aggregation, and Development Accounting." Working paper.

[11] Dahan, M. and D. Tsiddon. 1998. "Demographic Transition, Income Distribution and Economic Growth." *Journal of Economic Growth* 3:29-52.

[12] David, Paul A. 2005. "Real Income and Economic Welfare Growth in the Early Republic." Stanford University Working Paper.

[13] de la Croix, David and Matthias Doepke. 2003. "Inequality and Growth: Why Differential Fertility Matters." *American Economic Review* 93:1091-1113.

[14] Findley, Sally and Ann Orr. 1978. *Patterns of Urban-Rural Fertility Differentials in Developing Countries: A Suggested Framework*. Washington, DC: Center for Advanced Studies.

[15] Galor, Oded. 2005. "From Stagnation to Growth: Unified Growth Theory." in *Handbook of Economic Growth*, ed. Aghion, Phillipe and Steven Durlauf. North Holland: New York.

[16] Galor, Oded and David N. Weil. 2000. "Population, Technology, and Growth: From Stagnation to the Demographic Transition and Beyond." *American Economic Review* 90:806-828.

[17] Galor, Oded and Omer Moav. 2002. "Natural Selection and the Origins of Economic Growth." *Quarterly Journal of Economics* 117(4):1133-1192.

[18] Galor, Oded and Omer Moav and Dietrich Vollrath. (forthcoming). "Inequality in Land Ownership, the Emergence of Human Capital Promoting Institutions and the Great Divergence." *Review of Economic Studies.*

[19] Galor, Oded and Andrew Mountford. 2008. "Trading Population for Productivity: Theory and Evidence." *Review of Economic Studies* 75:1143-1179.

[20] Gollin, Douglas, Steven Parente and Richard Rogerson. 2002. "The Role of Agriculture in Development." *American Economic Reivew Papers and Proceedings.*

[21] Gollin, Douglas, Steven Parente and Richard Rogerson. 2004. "Farm Work, Home Work and International Productivity Differences." *Review of Economic Dynamics* 7:827-850.

[22] Goodfriend, Marvin and John McDermott. 1995. "Early Development." *American Economic Review* 85(1):116-133.

[23] Grabill, Wilson H., Clyde V. Kiser and Pascal K. Whelpton. 1958. *The Fertility of American Women.* New York: John Wiley and Sons, Inc.

[24] Graham, Bryan and Jonathan Temple. 2003. "Rich Nations, Poor Nations: How Much can Multiple Equilibrium Explain?" *Journal of Economic Growth* 11(1):5-41.

[25] Hansen, Gary and Edward C. Prescott. 2002. "Malthus to Solow." *American Economic Review* 1205-1217.

[26] Harris, J. R. and Michael P. Todaro. 1970. “Migration, Unemployment and Development: A two-sector Analysis.” *American Economic Review* 60:126-42.

[27] Jones, Charles I. 1995a. “R&D-Based Models of Economic Growth.” *Journal of Political Economy* 103(4):759-784.

[28] Jones, Charles I. 1995b. “Time Series Tests of Endogenous Growth Models.” *Quarterly Journal of Economics* 110(2):495-525.

[29] Jorgenson, Dale W. 1961. “The Development of a Dual Economy.” *Economic Journal* 71(282):309-334.

[30] Jorgenson, Dale W. and Frank M. Gollop. 1992. “Productivity Growth in U.S. Agriculture: A Post-war Perspective.” *American Journal of Agricultural Economics* 74(3):745-750.

[31] Knodel, John E. 1974. *The Decline of Fertility in Germany, 1871-1939*. Princeton, NJ: Princeton University Press.

[32] Kogel, Tomas and Alexia Prskawetz. 2001. “Agricultural Productivity Growth and Escape from the Malthusian Trap.” *Journal of Economic Growth* 6:337-357.

[33] Kongasmut, Piyabha, Sergio Rebelo and Danyang Xie. 2001. “Beyond Balanced Growth.” *Review of Economic Studies* 68(4):869-882.

[34] Kremer, Michael. 1993. “Population Growth and Technological Change: One Million B.C. to 1990.” *Quarterly Journal of Economics* 108:681-716.

[35] Kuznets, Simon. 1966. *Modern Economic Growth*. Yale University Press: New Haven.

[36] Kuznets, Simon. 1974. “Rural-Urban Differences in Fertility: An International Comparison.” *Proceedings of the American Philosophical Society* 118(1):1-29.

[37] Laitner, John. 2000. “Structural Change and Economic Growth.” *Review of Economic Studies* 67:545-561.

[38] Lesthaeghe, Ron J. 1977. *The Decline of Belgian Fertility, 1800-1970*. Princeton, NJ: Princeton University Press.

[39] Lewis, W. Arthur. 1954. “Economic Development with Unlimited Supplies of Labor.” *The Manchester School* 22:139-191.

[40] Livi-Bacci, Massimo. 1977. *A History of Italian Fertility during the Last Two Centuries*. Princeton, NJ: Princeton University Press.

[41] Matsuyama, Kiminori. 1992. "Agricultural Productivity, Comparative Advantage, and Economic Growth." *Journal of Economic Theory* 58:317-334.

[42] Mourmouras, Alex and Peter Rangazas. 2007. "Reconciling Kuznets and Habbakuk in a Unified Growth Theory." IUPUI Working Paper 2007-04.

[43] Ranis, Gustav. 1988. "Analytics of Development: Dualism." in *Handbook of Development Economics, Vol I*, ed. Chenery, H. and T. N. Srinivasan. North Holland: New York.

[44] Ranis, Gustav and John C. H. Fei. 1961. "A Theory of Economic Development." *American Economic Review* 51(4):533-565.

[45] Restuccia, Diego and Dennis Tao Yang and Xiaodong Zhu. 2008. "Agriculture and Aggregate Productivity: A Quantitative Cross-Country Analysis." *Journal of Monetary Economics* 55:234-50.

[46] Sharlin, Allen. 1986. "Urban-Rural Differences in Fertility in Europe during the Demographic Transition," in *The Decline of Fertility in Europe*, ed. Ansley J. Coale and Susan Cott Watkins. Princeton, NJ: Princeton University Press.

[47] Stiglitz, Joseph E. 1974. "Alternative Theories of Wage Determination and Unemployment in LDCs: The Labour Turnover Model." *Quarterly Journal of Economics* 88:194-227.

[48] Strulik, Holger and Jacob Weisdorf. 2008. "Population, Food, and Knowledge: A Simple Unified Growth Theory." *Journal of Economic Growth* 13(3):195-216.

[49] Tamura, Robert. 2002. "Human Capital and the Switch from Agriculture and Industry." *Journal of Economic Dynamics and Control* 27:207-242.

[50] Temple, Jonathan. 2005. "Dual Economy Models: A Primer for Growth Economists." *The Manchester School* 73(4):435-478.

[51] Temple, Jonathan and Ludger Woessmann. 2004. "Dualism and Cross-country Growth Regressions." *Journal of Economic Growth* 11(3):187-228.

[52] Vollrath, Dietrich. 2009. "How Important are Dual Economy Effects for Aggregate Productivity?" *Journal of Development Economics* 88:325-334.

[53] Weisdorf, Jacob. 2006. "From Domestic Manufacture to Industrial Revolution: Long-Run Growth and Agricultural Development." *Oxford Economic Papers* 58(2): 264-287.