Z

# DISCUSSION PAPER

EVALUATION OF THE EMPIRICAL PERFORMANCE OF TWO-STAGE BUDGETING AIDS, QUAIDS AND ROTTERDAM MODELS BASED ON WEAK SEPARABILITY

by

André DECOSTER Frederic VERMEULEN

**Public Economics** 

Center for Economic Studies Discussion Paper Series DPS 98.07



## Katholieke Universiteit Leuven Departement Economie

Naamsestraat 69 B-3000 Leuven

### Evaluation of the empirical performance of two-stage budgeting AIDS, QUAIDS and Rotterdam models based on weak separability

André Decoster\* and Frederic Vermeulen\*\*

Centrum voor Economische Studiën Naamsestraat 69 B-3000 Leuven Belgium Tel. : +32-(0)16326806 Fax : +32-(0)16326796 E-mail : frederic.vermeulen@econ.kuleuven.ac.be

#### Abstract :

Microsimulation models for indirect taxation require detailed underlying demand systems, in order to be policy relevant. A possible solution for the econometric problem (lack of necessary degrees of freedom) is the separability concept and the closely related notion of two-stage budgeting. In this paper, weak separability is applied on the Almost Ideal Demand System (AIDS), its quadratic extension QUAIDS and the Rotterdam model. These two-stage budgeting demand systems were estimated on Belgian time series data and were evaluated by means of a comparison of their elasticities (both partial and total), goodness-of-fit measures and their forecasting accuracy. Though the rank three QUAIDS model does not dominate the others in every respect (at least for time series data), it has nice theoretical properties which can on their own be a justification for the use of the system.

<sup>&</sup>lt;sup>\*</sup> Katholieke Universiteit Leuven, Campus Kortrijk and Centrum voor Economische Studiën, Katholieke Universiteit Leuven.

<sup>&</sup>lt;sup>\*\*</sup> Centrum voor Economische Studiën, Katholieke Universiteit Leuven.

This research has been financially supported by the Fund for Scientific Research - Flanders (contract FWO G.0327.97) and the DWTC (contracts PE/VA/07 and DB/01/032). We would like to thank Erik Schokkaert and participants of an internal workshop for their useful suggestions. All remaining errors are ours of course.

#### **1** Introduction

The idea of separability allows to model optimising behaviour of the economic agents as separate parts of a larger whole, without taking account of all possible interactions between economic variables. Within consumer theory (especially in agricultural applications), separability is often assumed or tested for a specific group of commodities (e.g. food), where conditional demand is modelled with the budget spent on these commodities and their prices as the only explanatory variables (see, e.g., Eales and Unnevehr, 1988, Hayes, Wahl and Williams, 1990 and Sellen and Goddard, 1997). The implicit assumption of this approach is that there is multi-stage budgeting, which means that the consumer takes her decisions in sequential steps. In its simplest form, two-stage budgeting, the consumer first allocates her total budget to broad commodity groups or aggregates (food, clothing, shelter,...), while in a second step group expenditures are allocated to the different items within that group (e.g. meat, fruit,...). Other applications of demand analysis model these consecutive steps and take the concept of two-stage budgeting explicitly into account (see, e.g., Baker, Blundell and Micklewright, 1989 and Edgerton, 1997). An advantage of this approach is that total instead of partial elasticities can be derived. Perhaps more important, with regard to practical applications, is that the number of commodities to model is almost unlimited, which allows for fairly disaggregated demand systems. These can, e.g., be used in microsimulation models for indirect taxation, where it can be important that these are able to simulate changes in indirect taxes on particular commodities rather than on broad commodity groups.

The present study intends to evaluate the performance of three two-stage demand systems for 32 commodities, which are to be used in ASTER, a static microsimulation model for indirect taxes (see Decoster, 1995). Due to the fact that we do not dispose of a long time series of Belgian individual household data (like the UK Family Expenditure Survey data) to capture precise price effects, estimation was done on aggregated time series data. (Though it might be possible to estimate price effects on a single household budget survey, see Deaton, 1987 and 1990). Therefore, before the systems will be used in ASTER, income effects will be re-estimated in the future on budget survey data and linked to the price effects estimated on time series. Another approach would be to find an optimal combination of micro (on budget survey data) and macro (on time series data) estimations using a minimum-distance estimator (see, e.g., Nichèle and Robin, 1995). Although the demand systems will be used in ASTER in an adapted form, it may be worth while to evaluate them on the basis of time series data (which is the usual approach, see, e.g., Parks, 1969, Klevmarken, 1979 and Barten, 1993). Evaluation of the three two-stage demand systems will be done by means of a comparative study of goodness-of-fit measures, the elasticities and the forecasting

performance of the models. As a benchmark, the empirical performance of a naive model was also evaluated.

The question of which separability concept is most appropriate to model two-stage demand systems is not easy to solve, because separability is a flag which covers many cargo's (for an overview see, e.g., Blackorby, Primont and Russell, 1978 and Pudney, 1981). Although appealing concepts like quasi-homothetic separability (e.g., Blackorby, Boyce and Russell, 1978) and quasi separability (e.g., Rossi, 1987) proved to be useful in empirical applications, we have chosen for the well-known weak separability. The reason for this is that this concept is easily imposed on one of the systems we wish to evaluate, namely the Rotterdam demand model which was first proposed by Theil and Barten (see, e.g., Barten, 1969). A slightly different approach will be followed to apply weak separability on Deaton and Muellbauer's (1980a) Almost Ideal Demand System (AIDS) and its extension the Quadratic Almost Ideal Demand System (QUAIDS) of Banks, Blundell and Lewbel (1997). In these cases weak separability will be assumed, rather than explicitly imposed as in the case of Rotterdam.

The structure of the paper is as follows. In the second section, the approach to model a two-stage demand system by means of AIDS, QUAIDS and Rotterdam is described. The data and some general estimation results are discussed in the third section. Section 4 discusses the elasticities of the three systems, while some goodness-of-fit measures are presented in the fifth section. Finally, the sixth section presents the results of the evaluation of the out-of-sample forecasting performance of the three systems and a naive model. Section 7 concludes.

#### 2 Specification of two-stage budgeting AIDS, QUAIDS and Rotterdam systems

#### 2.1 Utility maximisation under two-stage budgeting

Weak separability implies that the direct utility function can be written in the following form :

(1) 
$$u = v(q) = f[v_1(q_1), \dots, v_G(q_G), \dots, v_N(q_N)]$$

where *v* is a strictly quasi concave, increasing and differentiable function, *q* is the commodity vector, *f* is some increasing function and  $v_1, v_2, ..., v_N$  are well-behaved subutility functions with non-overlapping subvectors  $q_1, q_2, ..., q_N$ . A utility function of the form of equation (1) gives birth to second stage Marshallian demands for all goods *i* of group *G* of the form :

$$(2) q_i = g_{Gi}(x_G, p_G)$$

where  $x_G$  equals expenditures on group G and  $p_G$  is the vector of within-group prices. These second stage demands are the result of the maximisation of  $v_G$  subject to  $\sum_{i \in G} p_i q_i = x_G$  and have all the usual properties of demand functions, since they are derived from a standard utility maximisation procedure. Thus far the second stage of the two-stage budgeting model.

Contrary to the second stage budgeting, the allocation of total expenditures x to group expenditures poses more problems. Consider equation (1), where the subutility functions are replaced by their respective values :

(3) 
$$u = f(u_1, \dots, u_G, \dots, u_N)$$

where  $u_G$  is the utility level of group G assigned by the group utility function  $v_G(q_G)$ . Equation (3) is to be maximised subject to  $\sum_G c_G(u_G, p_G) = x$ , where  $c_G(u_G, p_G)$  is the group cost function which minimises the cost to reach the group utility level  $u_G$  with a given within-group price vector  $p_G$ , that is  $c_G(u_G, p_G) = \min_{q_G} \left( \sum_{k \in G} p_k q_k; v_G(q_G) = u_G \right)$  and which

itself is a representation of the within-group preferences. It is easily seen that in general this maximisation problem requires all individual prices in order to be solved. To be empirically useful however, it should be possible to derive a maximisation problem which uses a single price and quantity index for each commodity group. The necessary and sufficient conditions for consistent commodity aggregation were derived by Gorman (1959) and are rather restrictive. A first possibility makes use of homothetic preferences at the second stage, which implies the independence of the within-group budget shares of the group expenditures. Another solution needs group indirect utility functions of the Gorman generalised polar form and invokes strong or additive separability<sup>1</sup>. Given the strong Gorman conditions, which are empirically implausible, an approximate solution will be needed to solve the first stage problem.

This approximate solution is described in Deaton and Muellbauer (1980b). As can be seen from the above maximisation problem, there are in general no quantity indices  $Q_G$  and exogenous price indices  $P_G$  such that  $P_G Q_G = x_G = c_G (u_G, p_G)$ . A first step to obtain these price and quantity indices is unravelling the group cost functions as follows :

(4) 
$$c_G(u_G, p_G) = c_G(u_G, p_G^0) \cdot \frac{c_G(u_G, p_G)}{c_G(u_G, p_G^0)}$$

<sup>&</sup>lt;sup>1</sup> Strong or additive separability is much less general than weak separability, in that the utility function must have the following additive form under some monotone transformation :  $u = f \left[ v_1(q_1) + v_2(q_2) + ... + v_N(q_N) \right]$ 

where  $p_G^0$  is a base period price vector. The second term of the right-hand side is the true cost-of-living price index for group *G* and is denoted by  $P_G(p_G, p_G^0; u_G)$ . The first term of the right-hand side can be interpreted as the money cost of reaching utility level  $u_G$  with the base period price vector  $p_G^0$ . Consequently, this term can be considered as a quantity index and will be denoted by  $Q_G$ . The subgroup utility level  $u_G$  is given by the indirect utility function  $\mathbf{y}_G(Q_G, p_G^0)$  which is the inverse of  $c_G(u_G, p_G^0)$ . Now we have the following maximisation problem for the first stage :

(5) 
$$\max_{Q_G} u = f \Big[ \mathbf{y}_1 \Big( Q_1, p_1^0 \Big), \dots, \mathbf{y}_G \Big( Q_G, p_G^0 \Big), \dots, \mathbf{y}_N \Big( Q_N, p_N^0 \Big) \Big]$$

subject to  $\sum_{G} P_G(p_G, p_G^0; u_G) Q_G = x$ . The endogeneity problem still exists of course, due to the presence of the group utility level in the true cost-of-living price indices. However, under certain conditions the latter can be approximated by, e.g., Laspeyres or Paasche price indices which are independent of the group utility level. These are first-order approximations of the true cost-of-living indices which are weighted by respectively base period and current period group utility. For the Laspeyres or Paasche price indices to be a good approximation to the true index, one of the following conditions has to be satisfied :  $p_G$  has to be close to  $p_G^0$ ,  $p_G$  is relatively proportional to  $p_G^0$  or finally, substitution effects between commodities are small.

After solving the maximisation problem of equation (5), we get the following general form for the first stage Marshallian demand functions :

(6) 
$$Q_G = g_G(P_1, ..., P_G, ..., P_N, x)$$
 for  $G = 1, ..., N$ 

where  $P_G$  is a Paasche or Laspeyres price index and  $Q_G$  is a quantity index which is implicitly defined by  $x_G/P_G$ . Combining equation (6) with equation (2), we finally get an easily implementable, though approximate, solution for the two-stage budgeting problem.

The estimation of both the first and second stage demand functions will produce parameter estimates, which can be used to derive partial expenditure and price elasticities. Edgerton (1997) showed that, given the above approach, the total expenditure elasticities of commodities  $i \in G$  equal :

(7) 
$$\boldsymbol{e}_i = \boldsymbol{e}_G \cdot \boldsymbol{e}_i^G$$

where  $\mathbf{e}_i = \frac{\P q_i}{\P x} \frac{x}{q_i}$ ,  $\mathbf{e}_G = \frac{\P Q_G}{\P x} \frac{x}{Q_G}$  and  $\mathbf{e}_i^G = \frac{\P q_i}{\P x_G} \frac{x_G}{q_i}$  (i.e., respectively the total, the first

stage and the second stage expenditure elasticity).

The total uncompensated price elasticities of commodities  $i \in G$  have the form :

(8) 
$$\boldsymbol{e}_{ij}^{u} = \boldsymbol{d}_{GH} \cdot \boldsymbol{e}_{ij}^{uG} + \boldsymbol{e}_{i}^{G} \cdot \boldsymbol{w}_{j}^{H} \cdot \left(\boldsymbol{d}_{GH} + \boldsymbol{e}_{GH}^{u}\right)$$

where 
$$\mathbf{e}_{ij}^{u} = \frac{\P q_i}{\P p_j} \frac{p_j}{q_i}$$
 (*i*  $\hat{\mathbf{I}}$  *G*,  $j \in H$ ),  $\mathbf{e}_{GH}^{u} = \frac{\P Q_G}{\P P_H} \frac{P_H}{Q_G}$  and  $\mathbf{e}_{ij}^{uG} = \frac{\P q_i}{\P p_j} \frac{p_j}{q_i}$  (*i*  $\in$  *G*,  $j \in G$ ) are

respectively the total uncompensated price elasticity, the first stage uncompensated price elasticity and the within-group uncompensated price elasticity,  $w_j^H$  is the within-group budget share and  $d_{GH}$  is the Kronecker delta which equals 1 if G = H and 0 otherwise.

Compensated price elasticities can be calculated in the usual way, using the Slutsky equation in terms of elasticities. Partial first and second stage compensated price elasticities (with respectively *total* utility *u* and *group* utility  $u_G$  held constant) are obtained as follows :

(9) 
$$e_{GH}^{c} = e_{GH}^{u} + w_{H} \cdot e_{G}$$
  $(G, H = 1, ..., N)$ 

(10) 
$$\boldsymbol{e}_{ij}^{cG} = \boldsymbol{e}_{ij}^{uG} + \boldsymbol{w}_j^G \cdot \boldsymbol{e}_i^G \qquad (i \, \widehat{\boldsymbol{I}} \ G, j \, \widehat{\boldsymbol{I}} \ G)$$

where  $w_H$  is the group budget share. Total compensated price elasticities (with *total* utility *u* held constant) can be calculated as follows :

(11) 
$$\boldsymbol{e}_{ij}^{c} = \boldsymbol{e}_{ij}^{u} + \boldsymbol{w}_{j} \cdot \boldsymbol{e}_{i} \qquad (i \, \hat{\boldsymbol{I}} \, \boldsymbol{G}, j \, \hat{\boldsymbol{I}} \, \boldsymbol{H})$$

where  $w_j = w_H$ .  $w_j^H$  is the total budget share of commodity  $j \hat{I} H$ .

#### 2.2 Two-stage budgeting AIDS and QUAIDS models

The above approach is now applied on Deaton and Muellbauer's (1980a) well-known AIDS and its quadratic extension QUAIDS (Banks, Blundell and Lewbel, 1997). The former is a socalled rank two demand system (see Lewbel, 1987, 1989 and 1990). Recent empirical analysis on micro data, however, suggested that demand systems should be rank three, which implies that they would be able to display a greater variety of shapes of the Engel curves than rank two models (see, e.g., Lewbel, 1991, Blundell, Pashardes and Weber, 1993 and Banks, Blundell and Lewbel, 1997). Following this result Banks et alii (1997), derived a complete class of integrable, rank three, quadratic logarithmic expenditure share systems and proposed the appealing model QUAIDS which belongs to that class and which nests AIDS.

Demand systems of the above class have indirect utility functions of the form :

(12) 
$$y(x, p) = \left[ \left( \frac{\log x - \log a(p)}{b(p)} \right)^{-1} + I(p) \right]^{-1}$$

where  $\frac{\log x - \log a(p)}{b(p)}$  is the indirect utility function of a PIGLOG demand system and I(p)

is a differentiable, homogeneous of degree zero function of *p*. One particular member of this class of demand systems is QUAIDS and is specified as follows :

(13) 
$$\log a(p) = a_0 + \sum_k a_k \log p_k + \frac{1}{2} \sum_k \sum_j g_{kj}^* \log p_k \log p_j$$

$$(14) \qquad b(p) = \prod_i p_i^{b_i}$$

(15) 
$$I(p) = \sum_{i} I_{i} \log p_{i}$$

where equations (13) and (14) are the AIDS specification of the PIGLOG cost function. Filling in the above three equations in equation (12) results in the QUAIDS indirect utility function :

(16) 
$$\mathbf{y}(x,p) = \left[ \left( \frac{\log x - a_0 - \sum_k a_k \log p_k - \frac{1}{2} \sum_k \sum_j g_{kj}^* \log p_k \log p_j}{\prod_i p_i^{b_i}} \right)^{-1} + \sum_i I_i \log p_i \right]^{-1}$$

which corresponds to the following cost function :

(17) 
$$\log c(u, p) = a_0 + \sum_k a_k \log p_k + \frac{1}{2} \sum_k \sum_j g_{kj}^* \log p_k \log p_j + \frac{u \prod_i p_i^{b_i}}{1 - u \sum_i I_i \log p_i}$$

If all  $I_i$  coefficients in equation (17) are set equal to 0, then the QUAIDS cost function reduces to that of AIDS. Applying Roy's identity on equation (16) (or alternatively applying Shephard's lemma on equation (17) and substituting *u* for the indirect utility function), we get the QUAIDS budget share equations :

(18) 
$$W_i = \boldsymbol{a}_i + \boldsymbol{b}_i \log\left(\frac{x}{a(p)}\right) + \frac{l_i}{\prod_k p_k^{\boldsymbol{b}_k}} \left(\log\left(\frac{x}{a(p)}\right)\right)^2 + \sum_j \boldsymbol{g}_{ij} \log p_j$$

for i = 1,...,n and where log a(p) can be approximated by the Stone price index  $\sum_{k} w_k \log p_k$ (see Deaton and Muellbauer, 1980a). The QUAIDS budget shares reduce to those of AIDS if  $I_i = 0$  for all *i*. In that case the rank three Engel curves of QUAIDS reduce to rank two Working-Leser Engel curves.

Adding-up requires the following restrictions to be satisfied :  

$$\sum_{i} a_{i} = 1, \sum_{i} b_{i} = 0, \sum_{i} l_{i} = 0 \text{ and } \sum_{i} g_{ij} = 0 \text{ for all } j. \text{ Homogeneity is satisfied if } \sum_{j} g_{ij} = 0$$

for all *i*. The conditions to satisfy symmetry and negativity are most easily shown by using the matrix *K*, which consists of the coefficients  $k_{ij}$ :

(19) 
$$k_{ij} = \frac{p_i p_j s_{ij}}{x} = g_{ij} + b_i b_j \log\left(\frac{x}{a(p)}\right) + \frac{b_i l_j + b_j l_i}{b(p)} \left[\log\left(\frac{x}{a(p)}\right)\right]^2 + \frac{2l_i l_j}{\left[b(p)\right]^2} \left[\log\left(\frac{x}{a(p)}\right)\right]^3 - d_{ij} w_i + w_i w_j$$

where  $s_{ij}$  is the compensated price effect or Slutsky effect. Slutsky symmetry is satisfied if for all  $i,j \ g_{ij} = g_{ji}$ , while the negativity restriction is satisfied if the matrix *K* is negative semidefinite.

As already mentioned above, because QUAIDS is a rank three model, its Engel curves have the possibility to display a greater variety of shapes than the rank two AIDS. This is easily seen by making use of the expenditure elasticity:

(20) 
$$\boldsymbol{e}_{i} = 1 + \frac{\boldsymbol{b}_{i}}{W_{i}} + \frac{2\boldsymbol{I}_{i}}{W_{i}\boldsymbol{b}(p)} \log\left(\frac{\boldsymbol{x}}{\boldsymbol{a}(p)}\right)$$

First, remark the difference between the QUAIDS expenditure elasticity and that of AIDS :

$$(21) \qquad \boldsymbol{e}_i = 1 + \frac{\boldsymbol{b}_i}{W_i}$$

Commodities are luxury goods or necessities throughout the whole expenditure range ( $b_i > 0$  respectively  $b_i < 0$ ). Contrary to this, in the QUAIDS case the character of the commodities depends on the level of total expenditures. With a positive  $b_i$  and a negative  $I_i$ , e.g., the elasticity will be greater than unity at low levels of expenditure. If total expenditures increase, and the second term in the right-hand side of equation (20) becomes more important, the expenditure elasticity eventually becomes less than unity. Equation (20) allows thus for certain goods being luxuries at some income levels and necessities at others. Uncompensated price elasticities under AIDS and QUAIDS are respectively given by :

(22) 
$$\boldsymbol{e}_{ij}^{u} = \frac{-\boldsymbol{b}_{i} \cdot \boldsymbol{W}_{j}}{\boldsymbol{W}_{i}} + \frac{\boldsymbol{g}_{ij}}{\boldsymbol{W}_{i}} - \boldsymbol{d}_{ij}$$

(23) 
$$\boldsymbol{e}_{ij}^{u} = \frac{-\boldsymbol{b}_{i} \cdot \boldsymbol{w}_{j}}{\boldsymbol{w}_{i}} + \frac{\boldsymbol{g}_{ij}}{\boldsymbol{w}_{i}} - \boldsymbol{d}_{ij} - \left(\frac{2\boldsymbol{I}_{i}}{\boldsymbol{b}(p)}\log\left(\frac{\boldsymbol{x}}{\boldsymbol{a}(p)}\right)\right) \cdot \frac{\boldsymbol{w}_{j}}{\boldsymbol{w}_{i}} - \frac{\boldsymbol{I}_{i}\boldsymbol{b}_{j}\left(\log\left(\frac{\boldsymbol{x}}{\boldsymbol{a}(p)}\right)\right)^{2}}{\boldsymbol{w}_{i}\boldsymbol{b}(p)}$$

It is now easy to translate equation (18) into a two-stage demand system. The first stage cost function of QUAIDS (which is the dual representation of equation (5) and which consists of total utility and Paasche or Laspeyres price indices) can be written as follows :

(24) 
$$\log C(u, P) = a_0 + \sum_G a_G \log P_G + \frac{1}{2} \sum_G \sum_H g_{GH}^* \log P_G \log P_H + \frac{u \prod_G P_G^{b_G}}{1 - u \sum_G I_G \log P_G}$$

which gives the AIDS cost function if all  $I_G$  coefficients are set equal to 0. Applying Shephard's lemma and after substituting *u* for the indirect utility function, we get the QUAIDS (and of course AIDS under the above condition) first stage budget share equations :

(25) 
$$w_G = a_G + b_G \left( \log x - \sum_H w_H \log P_H \right) + \frac{I_G}{\prod_H P_H^{b_H}} \left( \log x - \sum_H w_H \log P_H \right)^2 + \sum_H g_{GH} \log P_H$$

for G = 1,...,N and where total expenditures are deflated by the Stone price index. With the appropriate index changes, the same theoretical restrictions as above apply to (25).

The second stage of the two-stage allocation problem amounts to applying Shephard's lemma and substituting the group utility by the group indirect utility function on the following second stage QUAIDS cost function :

- LG

(26) 
$$\log c_G(u_G, p_G) = \mathbf{a}_0^G + \sum_{i \in G} \mathbf{a}_i^G \log p_i + \frac{1}{2} \sum_{k \in G} \sum_{j \in G} \mathbf{g}_{kj}^{G^*} \log p_k \log p_j + \frac{u_G \prod_{k \in G} p_k^{\mathbf{b}_k}}{1 - u_G \sum_{k \in G} I_k^G \log p_k}$$

This results in the within-group QUAIDS budget shares :

$$(27) \ w_i^G = a_i^G + b_i^G \left( \log x_G - \sum_{j \in G} w_j^G \log p_j \right) + \frac{I_i^G}{\prod_{j \in G} p_j^{b_j^G}} \left( \log x_G - \sum_{j \in G} w_j^G \log p_j \right)^2 + \sum_{j \in G} g_{ij}^G \log p_j$$

for  $i \in G$  and G = 1,...,N and where the same theoretical restrictions have to be satisfied as in the first stage. Equation (27) reduces to the AIDS budget shares if all  $I_i^G$  are set equal to 0.

#### 2.3 Weak separability imposed on the Rotterdam demand system

Up to now, weak separability of preferences was assumed which made the above two-stage modelling possible. This assumption, which implies a certain structure of the Slutsky matrix, can be easily tested for smaller commodity breakdowns (see, e.g., Goldman and Uzawa, 1964 and Moschini, Moro and Green, 1994). However, by lack of enough degrees of freedom, this formal testing is almost impossible with a demand system consisting of 32 commodities. Another approach consists of the explicit imposition of separability on utility functions, production functions or demand systems (see, e.g., Barten and Turnovsky, 1966, Byron, 1970 and Berndt and Christensen, 1973). Due to the specific functional form of Rotterdam, where the Slutsky effects are directly captured by the price coefficients, weak separability or blockwise dependence is very easily imposed on it (see Theil, 1976).

Consider the Rotterdam demand equations :

(28) 
$$w_i d\log q_i = b_i \left( d\log x - \sum_k w_k d\log p_k \right) + \sum_j c_{ij} d\log p_j \qquad i = 1,...,n$$

where  $b_i = p_i \frac{\P q_i}{\P x}$  and  $c_{ij} = \frac{p_i p_j s_{ij}}{x}$ . Adding-up is satisfied if the real expenditure parameters  $b_i$  sum to one, i.e.  $\sum_i b_i = 1$ , and the price parameters  $c_{ij}$  satisfy the condition  $\sum_i c_{ij} = 0$  for all j. The homogeneity restriction requires that for all  $i \sum_j c_{ij} = 0$ . Symmetry is satisfied if for all i and  $j \ c_{ij} = c_{ji}$ . Finally, negativity requires the matrix Cconsisting of the elements  $c_{ij}$  being negative semidefinite.

Weak separability implies the following structure of the Slutsky effects for all  $i \in G$ and  $j \in H$ :

(29) 
$$s_{ij} = \boldsymbol{d}_{GH} s_{ij}^G + \boldsymbol{I}_{GH} \frac{\boldsymbol{\Re} q_i}{\boldsymbol{\Re} \boldsymbol{x}_G} \frac{\boldsymbol{\Re} q_j}{\boldsymbol{\Re} \boldsymbol{x}_G}$$

where  $d_{GH}$  is the Kronecker delta,  $s_{ij}^{G}$  is the within-group Slutsky effect and  $I_{GH}$  is the intergroup substitution effect. Substituting the Slutsky effects of equation (29) into the price coefficients of equation (28) and summing over all goods belonging to commodity group *G* results in the following first stage demand equations :

(30) 
$$\sum_{i \in G} w_i d \log q_i = b_G \left( d \log x - \sum_H w_H d \log P_H^1 \right) + \sum_H c_{GH} d \log P_H^2 \quad \text{for } G = 1, ..., N$$

where  $b_G = \sum_{i \in G} b_i$ ,  $c_{GH} = \frac{I_{GH}}{x}$ ,  $d \log P_H^1 = \sum_{k \in H} w_k^H d \log p_k$  (the Divisia price index) and

 $d\log P_H^2 = \sum_{k \in H} \frac{\P(p_k q_k)}{\P x_H} d\log p_k$  (the Frisch price index). The same theoretical restrictions as in

equation (28) apply to the first stage Rotterdam equations.

Remark that in this first stage allocation model, the restrictive Gorman conditions or the approximate solution as in the former section, are evaded by the use of two price indices per commodity group. This approach assumes that the correct first stage allocation is known at a certain period's prices and total expenditures. As long as there are only small changes in these explanatory variables (so that the associated coefficients can be treated as constants), the consumer is able to continuously update her group expenditures by means of the above two sets of price indices (see Gorman, 1970).

The allocation of group expenditures to within-group commodities can be written as follows :

(31) 
$$w_i^G d\log q_i = b_i^G \left( d\log x_G - \sum_{j \in G} w_j^G d\log p_j \right) + \sum_{j \in G} c_{ij}^G d\log p_j$$

where  $b_i^G = p_i \frac{\P q_i}{\P x_G}$  and  $c_{ij}^G = \frac{p_i p_j s_{ij}^G}{x_G}$ . Note that the same restrictions apply as in the first stage demand model.

The first and second stage parameter estimates can now be linked to obtain total parameter estimates, as if the system was estimated in one shot rather than in two stages. Note that this approach differs from the case where weak separability was assumed and where first and second stage elasticities are directly linked to each other. It can be shown that the Rotterdam total parameter estimates can be derived as follows:

$$(32) b_i = b_G \cdot b_i^G$$

(33)  $c_{ij} = \boldsymbol{d}_{GH}.c_{ij}^G.w_G + c_{GH}.b_i^G.b_j^H$ 

These can then be used to calculate the total expenditure and uncompensated price elasticities which are given by :

(34) 
$$\boldsymbol{e}_{i} = \frac{\boldsymbol{b}_{i}}{W_{i}}$$
  
(35)  $\boldsymbol{e}_{ij}^{u} = \frac{\left(\boldsymbol{c}_{ij} - \boldsymbol{b}_{i}W_{j}\right)}{W_{i}}$ 

In the next section, we focus on the estimation of the above two-stage budgeting AIDS, QUAIDS and Rotterdam demand models.

#### 3 Data and first stage estimation results

The two-stage demand models AIDS, QUAIDS and Rotterdam were estimated on aggregated data of the Belgian National Accounts from 1953-1989<sup>2</sup>. The first stage consists of a thirteen commodity breakdown : (1) food, (2) beverages, (3) tobacco, (4) clothing, (5) rent, (6) heating, (7) lighting, (8) durables, (9) housing, (10) personal care, (11) transportation, (12) leisure goods and (13) services. Four commodity groups were further disaggregated : food, beverages, heating and transportation<sup>3</sup>. This resulted in the joint modelling of 32 commodities. Both AIDS and QUAIDS were estimated in first differences by making use of Zellner's Seemingly Unrelated Regressions (SUR). The two-stage Rotterdam model was estimated by maximum likelihood estimation within the DEMMOD estimation package, developed by A.P. Barten. Intercept terms have been added to all models at both stages, in order to capture possible time trends (e.g., as a result of taste changes). To deal with the population increase, expenditure per capita appears at the right-hand side. With regard to the perfect nonlinear aggregation properties of AIDS and QUAIDS, this can be done under the assumption that the expenditure distribution and the demographic composition remained the same during the sample period (Deaton and Muellbauer, 1980b). Due to the fact that concavity of the cost function cannot be maintained over the whole priceexpenditure space under AIDS and QUAIDS, only the adding-up, homogeneity and symmetry conditions were explicitly imposed on these systems. On the contrary, the Rotterdam equations were estimated with all the theoretical restrictions imposed.

Due to limitations of space, not all estimations (five complete systems per two-stage demand model) can be discussed thoroughly. Therefore attention is restricted to some general results of the first stage estimations.

<sup>&</sup>lt;sup>2</sup> Data of the commodities within the group heating were only available from 1973-1989.

<sup>&</sup>lt;sup>3</sup> Food consists of (1) bread, (2) meat, (3) fish, (4) dairy, (5) oils and fats, (6) potatoes, vegetables and fruit, (7) coffee, tea and chicory, (8) sugar and jam and (9) other food. Beverages is broken down in (1) water and soft drinks, (2) beer, (3) alcohol and (4) wine and others. The commodity group heating is divided in (1) coal, (2) gas, (3) electrical heating and (4) oil fuel. Finally, transportation consists of (1) costs for own transportation, (2) diesel oil, (3) gasoline, (4) LPG, (5) public transportation and (6) other means of transportation.

Table 1 presents the expenditure and own-price parameter estimates and the accompanying standard errors of the first stage AIDS, QUAIDS and Rotterdam systems. As can be seen from the results, most of the parameter estimates are significantly different from zero at a significance level of 0.05. Important for the QUAIDS case is that nine parameters associated with the quadratic real expenditure term ( $I_G$ ) are significantly different from zero. The parameter estimates on their own are not so illuminating to compare the different demand systems. Moreover, opposite to the AIDS and Rotterdam cases, it is impossible to determine with a glimpse which goods are luxuries and which are necessities, respectively inferior and normal for QUAIDS from table 1. Therefore, it makes sense to write the parameter estimates into elasticities and to check whether they are not conflicting with a priori expectations. This question will be taken up in the next section.

(Standard error	0	•	parameter sig	• •				
,	AI			QUAIDS	0	Rotterdam		
	$\boldsymbol{b}_G$	$g_{GG}$	$\boldsymbol{b}_G$	$I_{G}$	<b>g</b> GG	$b_G$	$c_{GG}$	
Food	-0.10233	0.09184	0.14194	0.08115	0.12234	0.16669	-0.13170	
	(0.0370)*	(0.0251)*	(0.0589)*	(0.0228)*	(0.0321)*	(0.0420)*	(0.0282)*	
Beverages	0.00276 (0.0196)	0.02829 (0.0122)*	-0.04721 (0.0406)	-0.01454 (0.0142)	0.02180 (0.0126)	0.04964 (0.0180)*	-0.01912 (0.0073)*	
Tobacco	-0.01536	0.00787	-0.03604	-0.00737	0.01130	0.01002	-0.01292	
	(0.0068)*	(0.0003)*	(0.0180)*	(0.0063)	(0.0038)*	(0.0056)	(0.0022)*	
Clothing	0.04540	0.03688	0.10108	0.02040	0.04095	0.12913	-0.05050	
	(0.0221)*	(0.0078)*	(0.0324)*	(0.0129)	(0.0090)*	(0.0241)*	(0.0096)*	
Rent	-0.09949	0.09589	-0.02041	0.02734	0.09833	-0.01026	-0.00567	
	(0.0084)*	(0.0040)*	(0.0164)	(0.0057)*	(0.0041)*	(0.0087)	(0.0025)*	
Heating	0.01812	0.03367	-0.08521	-0.03860	0.03977	0.08599	-0.01505	
	(0.0232)	(0.0037)*	(0.0345)*	(0.0134)*	(0.0052)*	(0.0328)*	(0.0063)*	
Lighting	-0.00135	0.01505	-0.03807	-0.01263	0.01552	0.01136	-0.00243	
	(0.0070)	(0.0022)*	(0.0130)*	(0.0046)*	(0.0027)*	(0.0055)*	(0.0015)	
Durables	0.16355	0.09547	-0.05388	-0.07039	0.10300	0.26140	-0.06293	
	(0.0418)*	(0.0267)*	(0.0600)	(0.0225)*	(0.0301)*	(0.0397)*	(0.0245)*	
Housing	-0.00485	0.01170	-0.06370	-0.02052	0.02479	0.04746	-0.03513	
	(0.0111)	(0.0081)	(0.0255)*	(0.0088)*	(0.0082)*	(0.0106)*	(0.0085)*	
Personal care	-0.00757	0.06058	-0.00905	-0.00037	0.06304	0.06670	-0.01014	
	(0.0229)	(0.0114)*	(0.0386)	(0.0142)	(0.0115)*	(0.0247)*	(0.0131)	
Transportation	-0.01465	0.06774	-0.09538	-0.03043	0.07661	0.04919	-0.01543	
	(0.0173)	(0.0107)*	(0.0343)*	(0.0121)*	(0.0121)*	(0.0148)*	(0.0084)	
Leisure goods	0.00275	0.07730	0.11010	0.03718	0.08620	0.08421	-0.02458	
	(0.0205)	(0.0107)*	(0.0337)*	(0.0124)*	(0.0113)*	(0.0204)*	(0.0107)*	
Services	0.01302	0.04048	0.09583	0.02879	0.03479	0.04847	-0.01319	
	(0.0221)	(0.0135)*	(0.0373)*	(0.0129)*	(0.0109)*	(0.0241)*	(0.0131)	

 Table 1

 First stage restricted expenditure and own-price parameter estimates

 Standard errors between brackets \* parameter significant at 0.05 significance lev

The results of the statistical testing of the theoretical restrictions are shown in table 2. In the case of Rotterdam, which has been estimated by means of maximum likelihood, the likelihood ratio test was used. To test homogeneity and symmetry for AIDS and QUAIDS,

the T° test statistic of Gallant and Jorgenson (1979), which is analogous to the likelihood ratio test, was retained<sup>4</sup>. As can be seen from the results homogeneity is not rejected at the 0.05 significance level for AIDS and QUAIDS. On the contrary, homogeneity is rejected for the Rotterdam case. However, the likelihood ratio test is strongly biased towards rejection of the homogeneity restriction for systems with a large number of equations. A test which is better fit for large systems is the Laitinen test statistic (Laitinen, 1978). On the basis of the latter, homogeneity cannot be rejected for Rotterdam at the 0.05 significance level (1.44 < F(12,11) = 2.79). The much stronger symmetry restriction is rejected for all three demand systems. As can be seen from table 2, also the negativity condition is rejected at the 0.05 significance level<sup>5</sup>.

Table 2										
Testing the theoretical restrictions on the first stage AIDS, QUAIDS and Rotterdam systems										
	AIDS	QUAIDS	Rotterda							
			m							
	$T^{\circ}$	$T^{\circ}$	2LL	$c^{2}(0.05)$						
Homogeneity	5.0314	9.7136	38.5124	21.0261						
Symmetry	160.9655	121.4562	156.1276	85.9515						
Negativity			36.6022	9.4877						

The rejection of the theoretical restrictions is not at all a new result (see, e.g., Barten, 1969, Christensen, Jorgenson and Lau, 1975 and Deaton and Muellbauer, 1980a). The question arises in how far one should be worried by the violation of the theoretical restrictions. From an empirical point of view, one can say that one should not lay too much weight on the non-satisfaction of the theoretical restrictions. If according to the data the concavity of the cost function is rejected and the theory says that this is a necessary condition, who cares? Moreover, given that most of the parameter estimates are significantly different from zero, the demand model is able to predict fairly well (which is a primary aim for a demand system that is a possible basis for a good microsimulation model). However, the violation of the theoretical restrictions by means of a cost function, it is necessary that the latter is concave. Also the calculation of true cost-of-living indices and optimal taxation results are only possible with well-behaved cost functions.

To conclude this section, we will test whether AIDS is a restriction on QUAIDS. This is done for both the first and second stage estimations. Therefore, the Gallant and

<sup>&</sup>lt;sup>4</sup> The change in the least-squares criterion function which is minimised under SUR, multiplied by the number of observations can be seen as an asymptotically valid chi-square test with degrees of freedom equal to the difference in the number of free parameters in the unrestricted and the restricted models.

<sup>&</sup>lt;sup>5</sup> It is not clear how many degrees of freedom one should take into account to test the negativity condition using a likelihood ratio test, because this condition is an inequality restriction. Following Barten and Geyskens (1975), the number of negative Cholesky values (which are a by-product of the Cholesky decomposition of the Rotterdam matrix of price coefficients) under the symmetry condition is taken as the number of degrees of freedom.

Jorgenson  $T^{\circ}$  test can be used again. Except for the commodity group heating, AIDS is a restriction at the 0.05 significance level on the basis of table 3. As to the first stage estimation, this strengthens the results of table 1, where nine  $I_G$ 's were significantly different from zero. This seems to suggest that the extension of AIDS with a quadratic term in deflated expenditure is justified.

Tuble 0									
Is AIDS a restriction on QUAIDS ?									
		$T^{\circ}$	$c^{2}(0.05)$	Conclusion					
First stage	$I_G = 0 \ G = 1,, N$	58.3829	21.0261	Restriction					
Food	$I_i^G = 0 \ i \in G$	17.3791	15.5073	Restriction					
Beverages	$I_i^G = 0 \ i \in G$	10.9677	7.8147	Restriction					
Heating	$I_i^G = 0 \ i \in G$	4.4699	7.8147	No restriction					
Transportation	$I_i^G = 0 \ i \in G$	57.4435	11.0705	Restriction					

Table 3

After the more general discussion of the (partial) first stage estimation results, the following sections will focus on the empirical performance of the total two-stage demand systems.

#### 4 Comparison of the partial and total elasticities

Table 4 presents the expenditure, the uncompensated and the compensated own-price elasticities of the first stage estimations of the three systems evaluated at budget shares of 1987. As can be seen from the results, the elasticities differ largely in magnitude across the different demand systems. Only in six cases the goods have the same character with regard to the expenditure elasticities (i.e. food, lighting, personal care and transportation are evaluated as necessities, while clothing and durables can be seen as luxury goods). All commodities are evaluated as price inelastic. Remark that two goods (heating and transportation under both AIDS and QUAIDS) have positive compensated own-price elasticities, which is the most clear indication of the rejection of the negative semidefiniteness of the Slutsky matrix.

Table 4           First stage expenditure, uncompensated and compensated own-price elasticities										
	•	QUAIDS	-		QUAID	Rotter.	AIDS		Rotter.	
		·			S			·		
	$\boldsymbol{e}_{G}$	$\boldsymbol{\theta}_{G}$	$oldsymbol{e}_{G}$	$\boldsymbol{e}_{GG}^{u}$	$oldsymbol{e}_{GG}^{u}$	$\boldsymbol{e}_{GG}^{u}$	$\boldsymbol{e}_{GG}^{c}$	$oldsymbol{e}_{GG}^{~c}$	$oldsymbol{e}_{GG}^{c}$	
Food	0.421	0.863	0.929	-0.378	-0.356	-0.901	-0.304	-0.203	-0.736	
Beverages	1.069	0.563	1.246	-0.294	-0.455	-0.530	-0.251	-0.433	-0.480	
Tobacco	0.024	-0.331	0.618	-0.484	-0.280	-0.807	-0.484	-0.285	-0.797	
Clothing	1.602	1.786	1.714	-0.556	-0.547	-0.799	-0.436	-0.412	-0.670	
Rent	0.159	0.354	-0.088	-0.090	-0.087	-0.038	-0.071	-0.045	-0.049	
Heating	1.570	0.807	2.448	0.040	0.143	-0.514	0.090	0.168	-0.437	
Lighting	0.923	0.303	0.635	-0.140	-0.132	-0.147	-0.124	-0.127	-0.136	
Durables	2.225	1.676	1.986	-0.449	-0.350	-0.739	-0.151	-0.126	-0.474	
Housing	0.876	0.446	1.204	-0.696	-0.381	-0.939	-0.662	-0.364	-0.892	
Pers. care	0.932	0.926	0.601	-0.449	-0.426	-0.158	-0.345	-0.323	-0.091	
Transport	0.790	0.527	0.702	-0.016	0.083	-0.269	0.039	0.120	-0.220	
Leisure	1.029	1.363	0.910	-0.176	-0.161	-0.350	-0.080	-0.033	-0.265	
Services	1.169	1.479	0.650	-0.487	-0.624	-0.225	-0.397	-0.510	-0.175	

Table 5 shows the partial expenditure, uncompensated and compensated own-price elasticities of the second stage demand systems, evaluated at 1987 within-group budget shares.

Table 5									
Second stage expenditure, uncompensated and compensated own-price elasticities									
Food									
	AIDS	QUAIDS	Rotter.	AIDS	QUAIDS	Rotter.	AIDS	QUAIDS	Rotter.
	$\boldsymbol{e}_i^G$	$\boldsymbol{e}_i^G$	$\boldsymbol{e}_i^G$	$\boldsymbol{e}_{ii}^{uG}$	$\boldsymbol{e}_{ii}^{uG}$	$e_{ii}^{uG}$	$e_{ii}^{cG}$	$\boldsymbol{e}_{ii}^{cG}$	$\boldsymbol{e}_{ii}^{cG}$
Bread	-0.063	0.414	-0.043	-0.466	-0.546	-0.447	-0.474	-0.492	-0.453
Meat	1.733	1.423	1.684	-0.920	-0.889	-0.942	-0.297	-0.377	-0.336
Fish	0.593	0.893	0.405	-0.710	-0.681	-0.444	-0.673	-0.625	-0.419
Dairy	0.143	0.229	0.022	-0.144	-0.028	-0.115	-0.126	0.001	-0.112
Oils	0.310	0.697	0.824	0.470	0.414	-0.211	0.485	0.448	-0.170
Vegetables.	1.457	1.328	1.520	-0.510	-0.474	-0.670	-0.312	-0.294	-0.464
Coffee	0.588	0.078	1.075	0.083	0.211	-0.215	0.100	0.213	-0.184
Sugar	1.245	1.440	1.167	-0.645	-0.695	-0.645	-0.569	-0.607	-0.574
Other food	0.520	0.983	0.396	-0.938	-1.277	-0.737	-0.915	-1.233	-0.719
Beverages									
	AIDS	QUAIDS	Rotter.	AIDS	QUAIDS	Rotter.	AIDS	QUAIDS	Rotter.
	$\boldsymbol{e}_i^{~G}$	$oldsymbol{e}_i^{~G}$	$\boldsymbol{e}_i^{G}$	$e_{ii}^{uG}$	$e_{ii}^{uG}$	$e_{ii}^{uG}$	$e_{ii}^{cG}$	$oldsymbol{e}_{ii}^{\ cG}$	$\boldsymbol{e}_{ii}^{cG}$
Soft drinks	0.922	0.574	0.691	-0.406	-0.045	-0.254	-0.177	0.097	-0.083
Beer	0.980	0.885	1.323	-0.899	-0.857	-1.018	-0.580	-0.569	-0.587
Alcohol	0.885	1.374	1.371	-0.786	-0.765	-1.055	-0.678	-0.596	-0.887
Wine	1.132	1.322	0.744	-0.940	-0.858	-0.691	-0.597	-0.457	-0.465
Heating									
	AIDS	QUAIDS	Rotter.	AIDS	QUAIDS	Rotter.	AIDS	QUAIDS	Rotter.
	$\boldsymbol{e}_i^{G}$	$\boldsymbol{e}_i^{~G}$	$\boldsymbol{e}_i^{~G}$	$e_{ii}^{uG}$	$e_{ii}^{uG}$	$e_{ii}^{uG}$	${oldsymbol{e}}_{ii}^{\ cG}$	$oldsymbol{e}_{ii}^{cG}$	$e_{ii}^{cG}$
Coal	0.185	0.976	0.855	-0.185	-0.231	-0.172	-0.166	-0.131	-0.085
Gas	0.787	0.868	0.512	-0.480	-0.477	-0.275	-0.204	-0.172	-0.095
Electrical	1.055	1.037	0.649	-0.461	-0.553	-0.098	-0.324	-0.418	-0.014
Oil fuel	1.362	1.106	1.575	-0.786	-0.548	-0.858	-0.219	-0.087	-0.201
Transport									
	AIDS	QUAIDS	Rotter.	AIDS	QUAIDS			QUAIDS	Rotter.
	$\boldsymbol{e}_i^{G}$	$\boldsymbol{e}_i^{~G}$	$\boldsymbol{e}_i^{~G}$	$e_{ii}^{uG}$	$e_{ii}^{uG}$	$e_{ii}^{uG}$	$e_{ii}^{cG}$	$oldsymbol{e}_{ii}^{\ cG}$	$e_{ii}^{\ cG}$
Own trans.	2.037	1.329	1.990	-1.163	-0.939	-1.123	-0.306	-0.380	-0.286
Gasoline	0.907	1.702	0.994	-0.606	-1.269	-0.876	-0.301	-0.698	-0.542
Diesel oil	-0.975	-1.520	-1.757	-0.493	-1.286	-0.541	-0.592	-1.439	-0.718
LPG	0.034	0.314	-0.673	1.133	1.312	-0.544	1.133	1.313	-0.547
Public	-0.682	0.948	-0.060	-0.243	-0.172	-0.426	-0.306	-0.085	-0.432
trans. Other	-0.012	-1.416	0.489	-1.072	-2.089	-0.746	-1.073	-2.155	-0.723
Other	-0.012	-1.410	0.409	-1.072	-2.009	-0.740	-1.073	-2.100	-0.123

In general the same conclusions as in the first stage estimation can be drawn. Elasticities are rather different across the different demand systems and most of the goods are evaluated as price inelastic. The elasticities not only differ largely in magnitude, also the commodity character, with regard to the expenditure elasticities, differs from one system to another. The law of demand (negative compensated own-price elasticities) is violated in a couple of cases, which points to the fact that group cost functions are not concave as they should be.

	1		1		I	1			
	AIDS	QUAIDS	Rotter.	AIDS	QUAIDS	Rotter.	AIDS	QUAIDS	Rotter.
	$\boldsymbol{e}_i$	$\boldsymbol{e}_i$	$\boldsymbol{e}_i$	$\boldsymbol{e}_{ii}^{u}$	$\boldsymbol{e}_{ii}^{u}$	$\boldsymbol{e}_{ii}^{u}$	$\boldsymbol{e}_{ii}^{c}$	$\boldsymbol{e}_{ii}^{c}$	$\boldsymbol{e}_{ii}^{c}$
Bread	-0.027	0.357	-0.040	-0.471	-0.511	-0.452	-0.471	-0.503	-0.453
Meat	0.730	1.227	1.565	-0.533	-0.559	-1.186	-0.487	-0.481	-1.087
Fish	0.250	0.771	0.376	-0.687	-0.645	-0.431	-0.684	-0.636	-0.427
Dairy	0.060	0.198	0.020	-0.133	-0.009	-0.113	-0.131	-0.005	-0.112
Oils	0.131	0.601	0.765	0.480	0.436	-0.201	0.481	0.441	-0.194
Vegetables	0.614	1.146	1.413	-0.387	-0.358	-0.728	-0.372	-0.330	-0.694
Coffee	0.248	0.067	0.999	0.094	0.212	-0.214	0.095	0.212	-0.209
Sugar	0.524	1.242	1.085	-0.598	-0.638	-0.647	-0.592	-0.625	-0.635
Other food	0.219	0.848	0.368	-0.924	-1.249	-0.727	-0.922	-1.242	-0.724
Soft drinks	0.986	0.323	0.860	-0.244	0.033	-0.148	-0.234	0.036	-0.140
Beer	1.047	0.498	1.649	-0.673	-0.700	-0.880	-0.660	-0.694	-0.859
Alcohol	0.946	0.774	1.708	-0.710	-0.673	-1.006	-0.705	-0.669	-0.997
Wine	1.211	0.745	0.927	-0.698	-0.639	-0.560	-0.683	-0.630	-0.548
Tobacco	0.024	-0.331	0.618	-0.484	-0.280	-0.807	-0.484	-0.285	-0.797
Clothing	1.602	1.786	1.714	-0.556	-0.547	-0.799	-0.436	-0.412	-0.670
Rent	0.159	0.354	-0.088	-0.090	-0.087	-0.038	-0.071	-0.045	-0.049
Coal	0.291	0.788	2.094	-0.165	-0.117	-0.122	-0.164	-0.114	-0.115
Gas	1.236	0.701	1.253	-0.193	-0.129	-0.146	-0.179	-0.121	-0.132
Elec.heat.	1.657	0.837	1.588	-0.318	-0.399	-0.048	-0.311	-0.396	-0.042
Oil fuel	2.137	0.892	3.856	-0.196	-0.021	-0.704	-0.167	-0.010	-0.653
Lighting	0.923	0.303	0.635	-0.140	-0.132	-0.147	-0.124	-0.127	-0.136
Durables	2.225	1.676	1.986	-0.449	-0.350	-0.740	-0.151	-0.126	-0.474
Housing	0.876	0.446	1.204	-0.696	-0.381	-0.939	-0.662	-0.364	-0.892
Pers. care	0.932	0.926	0.601	-0.449	-0.426	-0.158	-0.345	-0.323	-0.091
Own	1.610	0.701	1.397	-0.320	-0.334	-0.699	-0.273	-0.313	-0.658
transp.									
Gasoline	0.717	0.897	0.698	-0.306	-0.650	-0.779	-0.289	-0.629	-0.762
Diesel oil	-0.770	-0.802	-1.233	-0.590	-1.452	-0.629	-0.596	-1.458	-0.638
LPG	0.027	0.166	-0.472	1.133	1.313	-0.547	1.133	1.313	-0.548
Public	-0.539	0.500	-0.042	-0.305	-0.078	-0.432	-0.309	-0.075	-0.432
trans.									
Other trans.	-0.010	-0.747	0.343	-1.073	-2.160	-0.727	-1.073	-2.162	-0.725
Leisure	1.029	1.363	0.910	-0.176	-0.161	-0.350	-0.080	-0.033	-0.265
Services	1.169	1.479	0.650	-0.487	-0.624	-0.225	-0.397	-0.510	-0.175

 Table 6

 Total expenditure, uncompensated and compensated own-price elasticities

Tables 4 and 5 presented partial elasticities which were obtained by respectively the first and second stage estimates. Table 6 concentrates on the total elasticities, which are obtained by linking first and second stage estimates (in the Rotterdam case) or by linking first and second stage elasticities (AIDS and QUAIDS). Given the rather different elasticities across the first and second stage demand systems, it should not be striking that this conclusion also applies to the total elasticities. The systems rather agree with each other with regard to the

uncompensated own-price elasticities, which are in most of the cases price inelastic. Less agreement between the systems can be found with regard to the expenditure elasticities. Only 12 of the 32 commodities have the same character across the models (most of these goods are necessary). The most striking differences are given by coal (which is a strong luxury good in Rotterdam !) and oil fuel. On the other hand, the expenditure elasticity of diesel oil is similar across the demand systems. Though the inferior character of that commodity remains somewhat counterintuitive. The total price elasticities are mainly influenced by the second stage price elasticities (see equation (8)). An obvious consequence of this is that the violation of the law of demand within the second stage is carried over to the total compensated own-price elasticities. This is the case for soft drinks in the QUAIDS case and oils, coffee, LPG in both the AIDS and the QUAIDS cases.

Following former studies (see, e.g., Parks, 1969, Klevmarken, 1979, Decoster and Schokkaert, 1990 and Barten, 1993), we can conclude that the obtained elasticities differ largely across demand systems and that on the basis of the latter no system outperforms the others in a convincing way. Moreover, on the average most of the above elasticities seem more or less reasonable. However, it should be borne in mind that ex post almost every elasticity can be defended. To be able to discriminate against a system, we will in the next sections pay attention to the empirical performance with regard to goodness-of-fit and forecasting accuracy of the three two-stage demand systems.

#### 5 Goodness-of-fit measures

In this section, we restrict our attention to the sample period performance, which is examined by some goodness-of-fit measures.

	Coefficients of determination								
	AIDS		QUA	AIDS	Rotterdam				
	$R^2$	R²(adj.)	$R^2$	R²(adj.)	$R^2$	R²(adj.)			
Food	0.4729	0.3564	0.4962	0.3626	0.6588	0.5834			
Beverages	0.3079	0.1549	0.2631	0.0678	0.3840	0.2479			
Tobacco	0.5277	0.4233	0.5631	0.4474	0.7167	0.6541			
Clothing	0.3950	0.2614	0.4079	0.2510	0.6813	0.6109			
Rent	0.9437	0.9312	0.9709	0.9632	0.1549	-0.0319			
Heating	0.7820	0.7338	0.8128	0.7632	0.5725	0.4780			
Lighting	0.5724	0.4779	0.6779	0.5925	0.3962	0.2627			
Durables	0.6620	0.5873	0.6719	0.5850	0.7047	0.6394			
Housing	0.2625	0.0995	0.3823	0.2186	0.5286	0.4244			
Personal care	0.5029	0.3931	0.4983	0.3653	0.3103	0.1579			
Transportation	0.7118	0.6481	0.7638	0.7012	0.3133	0.1615			
Leisure goods	0.5119	0.4040	0.5857	0.4759	0.5189	0.4126			
Services	0.3294	0.1813	0.4754	0.3339	0.3483	0.2043			

## Table 7Coefficients of determination

Table 7 shows the coefficients and adjusted coefficients of determination of the first stage of the three demand systems. Remark that the coefficient of determination is only an approximation of the goodness-of-fit of an individual equation. The reason for this is that in general, system estimation does not minimise the residual sum of squares of a single equation and consequently does not maximise the explained part of the regression (see, e.g., Berndt, 1991). Moreover, the coefficients of determination are not comparable between Rotterdam and the other two systems because of the fact that the dependent variables are not the same. On the contrary, AIDS and QUAIDS are comparable to each other. On the basis of the adjusted coefficients of determination, QUAIDS seems to provide the best fit. Only for beverages, clothing, durable goods and personal care, a higher adjusted coefficient of determination is obtained in the AIDS case.

A better goodness-of-fit measure is Theil's information inaccuracy (Theil and Mnookin, 1966). Opposite to the coefficient of determination, this measure takes the whole (two-stage) demand system into account in that it gives each commodity an appropriate weight in the measure. This measure, which is based on information theory, is for a single year defined as follows :

(36) 
$$I_t = \sum_i w_{it} \log \frac{w_{it}}{\hat{w}_{it}}$$

where  $w_{it}$  and  $\hat{w}_{it}$  are respectively the observed and the estimated budget shares of commodity *i* in year  $t^6$ . A measure for the whole prediction set (or parts of it) is provided by the average information inaccuracy :

$$(37) \qquad \overline{I} = \frac{1}{T} \sum_{t} I_{t}$$

where *T* is the number of periods. The procedure to obtain predicted budget shares of the three demand systems was as follows. Things are most simple in the AIDS and QUAIDS cases, where changes in first and second stage budget shares were sequentially estimated by means of the first difference form of equations (25) and (27) and linked to each other by :

(38) 
$$\hat{w}_{it} = \hat{w}_{Gt} \cdot \hat{w}_{it}^G = \left(w_{G,t-1} + \Delta \hat{w}_{Gt}\right) \cdot \left(w_{i,t-1}^G + \Delta \hat{w}_{it}^G\right)$$

Use is made of the budget share change decomposition to obtain the budget shares in the Rotterdam case :

(39) 
$$dw_i = w_i d\log q_i + w_i d\log p_i - w_i d\log x$$

$$I_t \approx \frac{1}{2} \sum_{i} \frac{\left(\hat{w}_{it} - w_{it}\right)^2}{w_{it}}$$

<sup>&</sup>lt;sup>6</sup> It can be shown that the information inaccuracy is positive as soon as there are pairwise differences in observed and estimated budget shares. Moreover, it also takes into account the relative forecasting errors. This is easily seen when the observed and estimated budget shares are not too far from each other, where in that case the information inaccuracy can be approximated as follows :

To get the changes in the first level budget shares  $w_G$ , equation (39) is summed over all commodities  $i \in G$ :

(40) 
$$dw_G = \sum_{i \in G} dw_i = \sum_{i \in G} w_i d\log q_i + \sum_{i \in G} w_i d\log p_i - \sum_{i \in G} w_i d\log x$$

where the first part of the right-hand side is the dependent variable of the first stage Rotterdam demand system (see equation (30)) and the other two parts are changes in prices and expenditures which are taken as given. Remark that in practice the finite change version of equation (40) is applied<sup>7</sup>. Predicted changes of the second stage budget shares are obtained by the finite version of equation (39) where the indices are appropriately changed (see section 2.3).

Table 8 shows the results of the application of the above concept on the two-stage AIDS, QUAIDS, Rotterdam and a naive model for each year separately, for four subperiods and for the entire sample period<sup>8</sup>. This naive model predicts no change at all in the budget shares, i.e. :

 $(41) \qquad \hat{W}_{it} = W_{i,t-1}$ 

It corresponds with the assumption that all expenditure elasticities are equal to unity, while own- and cross-price elasticities are respectively minus one and zero.

		Table 8									
Average infor	Average information inaccuracies of the in-sample budget share predictions										
0	AIDS	QUAIDS		Naive							
				model							
$I_{1974}$	5.606E-04	6.699E-04	5.963E-04	1.320E-03							
$I_{1975}$	7.018E-04	7.608E-04	1.008E-03	2.157E-03							
$I_{1976}$	6.005E-04	4.487E-04	5.753E-04	1.426E-03							
$I_{1977}$	4.515E-04	3.766E-04	5.429E-04	1.951E-03							
$I_{1974-77}$	5.786E-04	5.640E-04	6.806E-04	1.713E-03							
$I_{1978}$	4.444E-04	4.163E-04	5.980E-04	9.589E-04							
$I_{1979}$	3.476E-04	3.717E-04	4.005E-04	1.109E-03							
$I_{1980}$	4.156E-04	4.535E-04	3.781E-04	1.094E-03							
$I_{1981}$	5.916E-04	4.871E-04	5.347E-04	2.210E-03							
$I_{1978-81}$	4.498E-04	4.322E-04	4.778E-04	1.343E-03							
$I_{1982}$	6.697E-04	4.909E-04	7.006E-04	7.640E-04							
$I_{1983}$	3.849E-04	3.340E-04	5.318E-04	9.906E-04							
$I_{1984}$	4.558E-04	4.213E-04	5.024E-04	6.371E-04							
$I_{1985}$	8.126E-04	8.117E-04	7.547E-04	6.069E-04							
$I_{1982-85}$	5.808E-04	5.145E-04	6.224E-04	7.496E-04							
$I_{1986}$	5.272E-04	4.113E-04	7.563E-04	4.492E-03							
$I_{1987}$	2.442E-04	1.496E-04	6.004E-04	1.482E-03							
$I_{1988}$	4.982E-04	3.630E-04	9.700E-04	1.276E-03							
$I_{1989}$	3.603E-04	3.067E-04	4.546E-04	5.646E-04							
$I_{1986-89}$	4.075E-04	3.077E-04	6.953E-04	1.954E-03							
$I_{1974-89}$	5.042E-04	4.546E-04	6.190E-04	1.440E-03							

<sup>&</sup>lt;sup>7</sup> Due to the fact that the variation in  $\log p_i$  and  $\log x$  is larger than the change in the budget share,  $w_{it}$  can be replaced by  $w_{i,t-i}$  in the finite change version of equation (40).

<sup>&</sup>lt;sup>8</sup> Remark that average information inaccuracies could only be calculated from 1974-89, due to the fact that there were only data available for the commodity group heating from 1973 on.

As can be seen from the results, QUAIDS has the smallest average information inaccuracy for the entire sample period and for each of the four subperiods. AIDS seems to occupy the second place in fitting the data and has a fit that is not much worse than that of QUAIDS. This is in line with the conclusions that possibly could be drawn after the discussion of the estimation results. There it was shown that according to the Gallant and Jorgenson  $T^{\circ}$  test AIDS was a restriction on QUAIDS for four of the five systems within the two-stage allocation process. Moreover, at the first stage, nine of the thirteen coefficients associated with the quadratic expenditure term were significantly different from zero. The three theoretical demand systems tend to be superior to the naive model, which is the last comer in the ranking. This clearly shows that the investigated demand models highly gain in explanatory power in comparison with the constant share model. These conclusions are not contradicted by the results for each year separately, where the same ranking as above is obtained in nine of the sixteen cases. This stresses the dominance of QUAIDS, though closely followed by AIDS and Rotterdam, over the naive model.

A good fit is only one evaluation criterion for a demand system. The crucial acid test for the latter is its ability to forecast budget shares, given observed explanatory variables which were not included in the sample period. This is done in the next section.

#### 6 Forecasting accuracy

In this section the out-of-sample forecasting performance of the three two-stage demand systems is examined. A first useful measure to discriminate between models is again Theil's average information inaccuracy. Table 9 shows this measure applied on the three demand systems and the naive model for the entire prediction sample (1990-95), two subsamples and each year apart<sup>9</sup>.

Table 9										
Average information inaccuracies of the out-of-sample predictions										
	AIDS	QUAIDS	Rotterdam	Naive						
				model						
$I_{1990}$	1.181E-03	1.710E-03	8.701E-04	8.993E-04						
$I_{1991}$	1.393E-04	1.458E-04	1.195E-04	1.946E-04						
$I_{1992}$	2.553E-04	2.683E-04	3.022E-04	4.726E-04						
$I_{1990-92}$	5.252E-04	7.082E-04	4.306E-04	5.222E-04						
$I_{1993}$	1.155E-03	1.910E-03	1.345E-03	6.335E-03						
$I_{1994}$	3.580E-04	3.662E-04	3.325E-04	5.074E-04						
$I_{1995}$	3.178E-04	3.319E-04	3.812E-04	3.573E-04						
$I_{1993-95}$	6.102E-04	8.693E-04	6.862E-04	2.400E-03						
$I_{1990-95}$	5.676E-04	7.887E-04	5.584E-04	1.461E-03						

<sup>&</sup>lt;sup>9</sup> The predicted budget share was put equal to the observed budget share for 9 of the 32 commodities and that due to the fact that out-of-sample data for these goods were not available.

Things are a bit reversed in this (small sample) forecasting exercise. On average the Rotterdam model seems to predict best, though it is very closely followed by AIDS. Moreover, both systems occupy three times the first place if the information inaccuracy for each separate year is considered. QUAIDS, which had the best goodness-of-fit, occupies the third place. The three demand systems clearly forecast better than the naive model which predicts no change in the budget shares.

The robustness of the above forecasting results can in some sense be examined by the pairwise comparison of the three systems and the naive model by means of some nonparametric tests. In this case the null hypothesis of no difference in prediction accuracy of two models is tested against the alternative that one of the models produces better forecasts. Consider the following specification of the loss function for system A at time t for share i, which is in general a function of the forecast and the observed budget shares :

(42) 
$$g(\hat{w}_{Ait}, w_{it}) = \left| \frac{\hat{w}_{Ait} - w_{it}}{w_{it}} \right|$$

The choice of the specific functional form of the loss function is arbitrary and depends on the application considered (see, e.g., Diebold and Mariano, 1995). In our application the absolute percentage forecast error was chosen, in order to make the loss function independent of the weights of the equations within a system. A first nonparametric test that will be applied is the sign test (see Lehmann, 1975). The null hypothesis of equal forecast accuracy of two systems *A* and *B* is in this case a zero median loss differential  $d_{it}$ :

(43) 
$$med\left[\left|\frac{\hat{w}_{Ait} - w_{it}}{w_{it}}\right| - \left|\frac{\hat{w}_{Bit} - w_{it}}{w_{it}}\right|\right] = med(d_{it}) = 0$$

The sign test can be specified as follows<sup>10</sup>:

$$(44) \qquad S = \sum_{t}^{T} \sum_{i}^{N} I_{+} \left( d_{it} \right)$$

where  $I_+(d_{it}) = 1$  if  $d_{it} > 0$  and  $I_+(d_{it}) = 0$  otherwise. The sign test statistic *S* follows the binomial distribution, but in large samples the distribution

(45) 
$$S_N = \frac{S - \frac{1}{2}TN}{\frac{1}{2}\sqrt{TN}}$$

tends to the normal distribution. The results of the pairwise application of the sign test on AIDS, QUAIDS, Rotterdam and the naive model are shown in table 10. The null hypothesis of equal forecast accuracy is tested against the alternative hypothesis that the first

<sup>&</sup>lt;sup>10</sup> Note that we preferred to use two summation signs in order to draw attention to the fact that each system consists of N budget shares and that the forecasting sample contains T years (where T and N equal respectively 6 and 23 in our exercise).

mentioned system produces more accurate forecasts<sup>11</sup>. Only the AIDS - Rotterdam comparison gives a decisive answer, in favour of the former, at a significance level of 0.05. If the level of significance is raised to 0.10, AIDS predicts better than the other demand systems and the naive model.

Table 10									
Sign test statistics and probability that null hypothesis is not rejected									
	S	$S_N$	$P_{Ho}$						
AIDS-QUAIDS	60	-1.53226	0.063						
AIDS-Rotterdam	58	-1.87276	0.031						
AIDS-Naive model	61	-1.36201	0.087						
Rotterdam-QUAIDS	68	-0.17025	0.433						
QUAIDS-Naive model	68	-0.17025	0.433						
Rotterdam-Naive	68	-0.17025	0.433						

A test which is in many cases more powerful than the sign test, is the Wilcoxon signed-rank test which is illustrated below (see also Lehmann, 1975). This test uses both the signs of the loss differences and the magnitude of these differences. The null hypothesis in this case is not only a zero median loss differential, but also an equal distribution of the forecast errors. The test is specified as follows:

(46) 
$$W = \sum_{t}^{T} \sum_{i}^{N} I_{+}(d_{it}) \operatorname{rank}(|d_{it}|)$$

which sums the ranks of the absolute values of the positive loss differentials. In large samples the test statistic  $W_N$  is asymptotically standard normal :

(47) 
$$W_N = \frac{W - \frac{TN(TN+1)}{4}}{\sqrt{\frac{TN(TN+1)(2TN+1)}{24}}}$$

Table 11, where the Wilcoxon signed-rank test results are shown, shades the picture that arose in the case of the sign test results.

#### Table 11

Wilcoxon signed-rank test statistic and probability that null hypothesis is not rejected

	W	$W_N$	$P_{Ho}$
AIDS-QUAIDS	4392	-0.8576	0.1949
AIDS-Rotterdam	4342	-0.9638	0.1685
AIDS-Naive model	4490	-0.6493	0.2578
Rotterdam-QUAIDS	4414	-0.8108	0.2090
QUAIDS-Naive model	4596	-0.4240	0.3372
Rotterdam-Naive model	4792	-0.0074	0.4960

 $<sup>^{11}</sup>$  There are as many positive loss differentials as negative when S=69 (median).

None of the models is able to beat one of the others at a significance level of, say, 0.05 or 0.10<sup>12</sup>. The results of the pairwise comparison of Rotterdam and QUAIDS and QUAIDS and the naive model are tightened up as compared with the sign test results. Contrary to this, taking into account the distribution of the forecast errors weakens the results of the pairwise comparisons of AIDS and the other models and of the comparison of Rotterdam and the naive model.

To conclude this section, we can state that on the basis of Theil's average information inaccuracies the theoretically derived demand systems seem to predict better than the naive model which predicts no change in budget shares. A conclusion which does not conflict with the results of the nonparametric tests. More care is needed to discriminate against the three demand systems when they are compared with each other. Although Rotterdam and AIDS had the smallest full sample average information inaccuracies, no system is able to beat another at fairly low levels of significance on the basis of the nonparametric tests.

#### 7 Conclusion

In this paper, the empirical performance was evaluated of three two-stage budgeting demand systems based on weak separability : the Almost Ideal Demand System, its quadratic extension QUAIDS and the Rotterdam demand system. Weak separability was explicitly imposed on the latter, while for the others we had to resort to an approximate solution. The three systems were estimated on Belgian time series data and were evaluated by means of a comparison of the elasticities (both partial and total), goodness-of-fit measures and out-of-sample forecasting accuracy.

On the basis of a comparison of the elasticities, a ranking of the models is neigh on impossible. Though the elasticities differ largely in magnitude across the demand systems, most of them seem quite reasonable. An exception of this are some positive compensated own-price elasticities under AIDS and QUAIDS, which points to the violation of the negativity restriction. In the light of this, the ability of the Rotterdam system to impose the negativity restriction seems to be an advantage over the other two systems.

More illuminating are the goodness-of-fit measures and the forecasting accuracy of the two-stage budgeting demand systems. On the basis of Theil's average information inaccuracies, QUAIDS seems to have the best fit. All three theoretically derived demand systems tend to be superior to a naive model, which predicts no change in budget shares. This shows that the investigated demand systems are able to explain the data more

 $<sup>^{12}</sup>$  The median, taking into account the distribution of the prediction errors, equals 9591 in the Wilcoxon signed-rank test case.

accurately than the naive model. This conclusion also arises with respect to the (out-ofsample) forecasting accuracy of the systems. Things are less clear if one compares the three demand systems with each other. The out-of-sample average information inaccuracies seem to suggest that Rotterdam predicts best (though very closely followed by AIDS). The forecasting results on the basis of the average information inaccuracies are not so robust, however. On the basis of some nonparametric tests, no system is able to discriminate against another at fairly low levels of significance.

By the ambiguity of the above results (and following earlier results on single stage demand systems), it is rather difficult to proclaim one of the two-stage demand systems winner of the contest. If one takes into account not only the above results, but also the nice theoretical implications of the rank three QUAIDS model, which is able to capture more variety in Engel curves than rank two systems as AIDS and Rotterdam, one can be inclined to favour QUAIDS. The full exploitation of this feature, however, will only take place when the system is applied on budget survey data, which show much more heterogeneity than aggregate data. A topic for further research, therefore, will be the re-estimation of the expenditure parameters of the systems on micro data, which will be linked to the price parameters obtained by time series data. These adapted demand systems can then form a basis for a new comparison of their empirical performance.

#### References

Baker, P., R. Blundell and J. Micklewright (1989), "Modelling household energy expenditures using micro-data", *The Economic Journal*, 99, pp. 720-738.

Banks, J., R. Blundell and A. Lewbel (1997), "Quadratic Engel curves and consumer demand", *The Review of Economics and Statistics*, 74, pp. 527-539.

Barten, A.P. (1969), "Maximum likelihood estimation of a complete system of demand equations", *European Economic Review*, 1, pp. 7-73.

Barten, A.P. (1993), "Consumer allocation models : choice of functional form", *Empirical Economics*, 18, pp. 129-158.

Barten, A.P. and E. Geyskens (1975), "The negativity condition in consumer demand", *European Economic Review*, 6, pp. 227-260.

Barten, A.P. and S. Turnovsky (1966), "Some aspects of the aggregation problem for composite demand equations", *International Economic Review*, 7, pp. 231-259.

Berndt, E. (1991), *The practice of econometrics : classic and contemporary*, Addison-Wesley Publishing Company, Reading.

Berndt, E. and L.R. Christensen (1973), "The translog function and the substitution of equipment, structures and labor in U.S. manufacturing, 1929-68", *Journal of Econometrics*, 1, pp. 81-114.

Blackorby, C., R. Boyce and R. Russell (1978), "Estimation of demand systems generated by the Gorman polar form : a generalization of the S-branch utility tree", *Econometrica*, 46, pp. 345-363.

Blackorby, C., D. Primont and R. Russell (1978), *Duality, separability and functional structure : theory and economic applications*, North-Holland, New York.

Blundell, R., P. Pashardes and G. Weber (1993), "What do we learn about consumer demand patterns from micro data ?", *American Economic Review*, 83, pp. 570-597.

Byron, R.P. (1970), "A simple method for estimating demand systems under separable utility assumptions", *Review of Economic Studies*, 37, pp. 261-274.

Christensen, L.R., D.W. Jorgenson and L.J. Lau (1975), "Transcendental logarithmic utility functions", *American Economic Review*, 65, pp. 367-383.

Deaton, A. (1987), "Estimation of own- and cross-price elasticities from household survey data", *Journal of Econometrics*, 36, pp. 7-30.

Deaton, A. (1990), "Price elasticities from survey data. Extensions and Indonesian results", *Journal of Econometrics*, 44, pp. 281-309.

Deaton, A. and J. Muellbauer (1980a), "An almost ideal demand system", *American Economic Review*, 70, pp. 312-326.

Deaton, A. and J. Muellbauer (1980b), *Economics and consumer behavior*, Cambridge University Press, Cambridge.

Decoster, A. (1995), "A microsimulation model for Belgian indirect taxes with a carbon / energy tax illustration for Belgium", *Tijdschrift voor Economie en Management*, 40, pp. 133-156.

Decoster, A. and E. Schokkaert (1990), "Tax reform results with different demand systems", *Journal of Public Economics*, 41, pp. 277-296.

Diebold, F.X. and R.S. Mariano (1995), "Comparing predictive accuracy", *Journal of Business & Economic Statistics*, 13, pp. 253-263.

Eales, J. and L. Unnevehr (1988), "Demand for beef and chicken products : separability and structural change", *American Journal of Agricultural Economics*, 70, pp. 521-532.

Edgerton, D. (1997), "Weak separability and the estimation of elasticities in multistage demand systems", *American Journal of Agricultural Economics*, 79, pp. 62-79.

Gallant, A. and D. Jorgenson (1979), "Statistical inference for a system of simultaneous, nonlinear, implicit equations in the context of instrumental variable estimation", *Journal of Econometrics*, 11, pp. 275-302.

Goldman, S.M. and H. Uzawa (1964), "A note on separability in demand analysis", *Econometrica*, 32, pp. 387-398.

Gorman, W.M. (1959), "Separable utility and aggregation", Econometrica, 27, pp. 469-481.

Gorman, W.M. (1970), "Two-stage budgeting" in C. Blackorby and A.F. Shorrocks (eds.), *Separability and aggregation. Collected works of W.M. Gorman. Volume I*, Clarendon Press, Oxford, pp. 22-29.

Hayes, D., T. Wahl and G. Williams (1990), "Testing restrictions on a model of Japanese meat demand", *American Journal of Agricultural Economics*, 72, pp. 556-566.

Klevmarken, N.A. (1979), "A comparative study of complete systems of demand equations", *Journal of Econometrics*, 10, pp. 165-191.

Laitinen, K. (1978), "Why is demand homogeneity so often rejected ?", *Economics Letters*, 1, pp. 187-191.

Lehmann, E.L. (1975), Nonparametrics. Statistical methods based on ranks, Holden-Day, San Francisco.

Lewbel, A. (1987), "Characterizing some Gorman Engel curves", *Econometrica*, 55, pp. 1451-1459.

Lewbel, A. (1989), "A demand system rank theorem", Econometrica, 57, pp. 701-705.

Lewbel, A. (1990), "Full rank demand systems", *International Economic Review*, 31, pp. 289-300.

Lewbel, A. (1991), "The rank of demand systems : theory and nonparametric estimation", *Econometrica*, 59, pp. 711-730.

Moschini, G., D. Moro and R. Green (1994), "Maintaining and testing separability in demand systems", *American Journal of Agricultural Economics*, 76, pp. 61-73.

Nichèle, V. and J.-M. Robin (1995), "Simulation of indirect tax reforms using pooled micro and macro French data", *Journal of Public Economics*, 56, pp. 225-244.

Parks, R.W. (1969), "Systems of demand equations : an empirical comparison of alternative functional forms", *Econometrica*, 37, pp. 629-650.

Pudney, S. (1981), "An empirical method of approximating the separable structure of consumer preferences", *Review of Economic Studies*, 48, pp. 561-577.

Rossi, N. (1987), "An intertemporally quasi separable demand system", *The Review of Economics and Statistics*, 69, pp. 449-457.

Sellen, D. and E. Goddard (1997), "Weak separability in coffee demand systems", *European Review of Agricultural Economics*", 24, pp. 133-144.

Theil, H. (1976), *Theory and measurement of consumer demand*, vol. 2, North-Holland, Amsterdam.

Theil, H. and R. Mnookin (1966), "The information value of demand equations and predictions", *Journal of Political Economy*, 74, pp. 34-45.

#### **DISCUSSION PAPERS 1997**

- DPS 97.01 Hans Dewachter, Geert Gielens and Dirk Veestraeten, An Assessment of Central Banks.' Stand on Exchange Rate Stabilization Policies, February. (International Economics).
- DPS 97.02 Anneleen Peeters, *How Hiring and Firing Costs Affect Labour Demand in a Model of Uncertainty*, February. (International Economics).
- DPS 97.03 Hans Eyssen, Are West-African Immigrants Discriminated in Cote DIvoire?, March. (Development Economics).
- DPS 97.04 Wim Lagae, The Absorption of the Effects of Debt Relief Operations by European Export Credit Agencies: An Institutional Analysis, March. (Development Economics).
- DPS 97.05 Jenke ter Horst and Marno Verbeek, *Estimating Short-Run Persistence in Mutual Fund Performance*, March. (Econometrics).
- DPS 97.06 Hans Dewachter and Hanno Lustig, A Cross-Country Comparison of CPI as a Measure of Inflation, April. (Financial Economics).
- DPS 97.07 Yunus Aksoy and Yohanes E. Riyanto, *Exchange Rate Pass-Through in Vertically Related Markets*, May. (International Economics).
- DPS 97.08 Paul De Grauwe, *The Indeterminacy of the Euro Conversion Rates*. *Why it Matters and how it can be solved*, June. (International Economics).
- DPS 97.09 Jozef Konings, Hylke Vandenbussche and Reinhilde Veugelers, *Union Wage* Bargaining and European Antidumping Policy, June (Financial Economics).
- DPS 97.10 Francis Vella and Marno Verbeek, Using Rank Order as an Instrumental Variable: An Application to the Return to Schooling, May. (Econometrics).
- DPS 97.11 Jozef Konings and Patrick P. Walsh, *The Effect of Real Exchange Rate Movements and Ownership on the Life Cycle of Manufacturing Plants in Ireland (1973-1994)*, June. (Financial Economics).
- DPS 97.12 Johan Eyckmans, *Balancedness of games with multilateral environmental externalities*, August. (Public Economics).
- DPS 97.13 Patrick Van Cayseele and Dave FURTH, *Price Leadership and Buyouts*, September. (Financial Economics).
- DPS 97.14 Mark De Broeck, Dominique Guillaume and Emmanuel Van der Stichele, *Small and Big Bangs in Bond Markets*, September (Financial Economics).
- DPS 97.15 Hanno Lustig, *Re-examining the Synchronization of European Business Cycles*, September (Financial Economics).
- DPS 97.16 Joã P. Cocco, Francisco J. Gomes and Pascal J. Maenhout, A Two-Period Model of Consumption and Portfolio Choice With Incomplete Markets, June (Financial Economics).

- DPS 97.17 Stefan Dercon and Daniel Ayalew, *Demobilisation and Reintegration of Ex-Soldiers in Ehtiopia*, October (Development Economics).
- DPS 97.18 Stefan Dercon and Pramila Krishnan, In Sickness and in HealthRisk-Sharing within Households in Rural Ethiopia, October (Development Economics).
- DPS 97.19 Paul Collier, Stefan Dercon and John Mackinnon, *Density versus Quality in Health Care Provision: The Use of Household Data for Budgetary Choices in Ehtiopia* October (Development Economics).
- DPS 97.20 van der Lijn Nick and Marno Verbeek, *Excess demand, repressed inflation, and forced saving in the Soviet Union*, October (Econometrics).
- DPS 97.21 Lorelei Crisologo-Mendoza and Dirk Van de gaer, *Population Growth and Customary Law on Land: The Case of Cordillera Villages in the Philippines*, October (Development Economics).
- DPS 97.22 Tom Van Puyenbroeck, *Two-stage optimal control problems subject to an isoperimetric constraint*, October (Public Economics).
- DPS 97.23 Erik Schokkaert, Geert Dhaene and Carine Van de Voorde, *Risk Adjustment* and the Trade-off Between Efficiency and Risk Selection, November (Public Economics).
- DPS 97.24 Paul De Grauwe, Hans Dewachter and Dirk Veestraeten, *Stochastic Process Switching and Stage III of EMU*, November (International Economics).
- DPS 97.25 Dirk Heremans, *Regulation of Banking and Financial Intermediation*, November (Financial Economics).
- DPS 97.26 Knud J. Munk, Agricultural Policy a Public Economic Explanation, November (Public Economics).
- DPS 97.27 Hans Dewachter, Can Markov Switching Models Replicate Chartist Profits in the Foreign Exchange Market? November (International Economics).
- DPS 97.28 Paul De Grauwe and Frauke Skudelny, *The Impact of EMU on Trade Flows*, December (International Economics).

#### **DISCUSSION PAPERS 1998**

- DPS 98.01 Louis Baeck, *Thematisation and Canon Building in Post-War Development Studies*, January (Development Economics).
- DPS 98.02 Hans Dewachter and Hanno Lustig, *Sticky Prices and the Nominal Effects of Real Shocks*, January (International Economics).
- DPS 98.03 Ilse Frederickx, *Health in Rural Tanzania: The Determinants of Health Status, Health Care Demand and Health Care Choice, January (Development Economics).*
- DPS 98.04 Paul De Grauwe, *European Unemployment. A Tale of Demand and Supply*, February (International Economics).
- DPS 98.05 Joã Cocco, Francisco Gomes and Pascal Maenhout, *Consumption and Portfolio Choice over the Life-Cycle*, March (Financial Economics).
- DPS 98.06 Yunus Aksoy and Hanno Lustig, *Circular Aspects of Exchange Rates and Market Structure*, January (International Economics).
- DPS 98.07 André Decoster and Frederic Vermeulen, *Evaluation of the Empirical Performance of Two-Stage Budgeting AIDS, QUAIDS and Rotterdam Models Based on Weak Separability*, April (Public Economics).
- DPS 98.08 Erik Schokkaert and Luc Van Ootegem, *Preference Variation and Private Donations*, April (Public Economics).
- DPS 98.09 Erik Schokkaert, Mr. Fairmind is Post-Welfarist: Opinions on Distributive Justice, April (Public Economics).
- DPS 98.10 Dirk Van de gaer, Michel Martinez and Erik Schokkaert, *Measuring Intergenerational Mobility and Equality of Opportunity*, April (Public Economics).
- DPS 98.11 Paulo Augusto Nunes, *Testing the Validity of WTP values from a Contingent Valuation Survey in Portugal*, April (Public Economics).
- DPS 98.12 Paulo Augusto Nunes, *Measuring the WTP for Recreation and Biodiversity Protection Programs*, April (Public Economics).
- DPS 98.13 Laurens Cherchye and Tom Van Puyenbroeck, Learning from Input-Output Mixes in DEA: A Proportional Measure for Slack-Based Efficient Projections, February (Public Economics).
- DPS 98.14 Jerzy Mycielski and Yohanes Riyanto, *On the Interaction between Taste and Distance and Its Implications on the Vertical Distribution Arrangement*, May (Financial Economics).
- DPS 98.15 Jerzy Mycielski, Yohanes Riyanto and Filip Wuyts, *Product Differentiation* and the Equilibrium Structure of the Manufacturer-Retailer Relationship, May (Financial Economics).

- DPS 98.16 Hans Degryse and Patrick Van Cayseele, *Relationship Lending within a Bankbased System: Evidence from European Small Business Data*, April (Financial Economics).
- DPS 98.17 Pramila Krishnan, Tesfaye Gebre Selassie and Stefan Dercon, *The Urban Labour Market During Structural Adjustment: Ethiopia 1990-1997*, April (Development Economics).
- DPS 98.18 Bart Capáu and Stefan Dercon, Prices, Local Measurement Units and Subsistence Consumption in Rural Surveys: An Econometric Approach with an Application to Ethiopia, March (Development Economics).
- DPS 98.19 Stefan Dercon and Pramila Krishnan, Changes in Poverty in Rural Ethiopia 1989-1995: Measurement, Robustness Tests and Decomposition, March (Development Economics).
- DPS 98.20 Jenke R. ter Horst, Theo E. Nijman and Marno Verbeek, *Eliminating Biases in Evaluating Mutual Fund Performance from a Survivorship Free Sample*, June (Econometrics).
- DPS 98.21 Hilke Vandenbussche and Jozef Konings, *Globalization and the effects of national versus international competition on the labour market. Theory and evidence from Belgian firm level data*, August (Financial Economics).
- DPS 98.22 Wim Moesen and Laurens Cherchye, *The Macroeconomic Performance of Nations Measurement and Perception*, August (Public Economics).
- DPS 98.23 Anneleen Peeters, Interim Employment and a Leading Indicator for the Belgian Labour Market, September (International Economics, IERP 137).
- DPS 98.24 Wessel Marquering and Marno Verbeek, An Empirical Analysis of Intertemporal Asset Pricing Models with Transaction Costs and Habit Persistence, September (Econometrics).
- DPS 98.25 Filip Abraham and Joeri Van Rompuy, *Is Belgium ready for EMU? A look at national, sectoral and regional developments,* September (International Economics, IERP 138).
- DPS 98.26 Sara Ochelen, Stef Proost and Kurt Van Dender, *Optimal Pricing for Urban Road Transport Externalities*, September (Public Economics).
- DPS 98.27 Knud Munk, Optimal Support to Low-skilled Households, July (Public Economics).
- DPS 98.28 Wim Moesen and Philippe Van Cauwenberge, *The Status of the Budget Constraint, Federalism and the Relative Size of Government: A Bureaucracy Approach*, September (Public Economics).
- DPS 98.29 Laurens Cherchye, *The Measurement of Macroeconomic Performance: Comparison of DEA-Based Alternatives*, August (Public Economics).
- DPS 98.30 Jügen Janssens, Volatility and Risk Premia on Belgian Secondary Long Term Government Bond Markets, October (Financial Economics).

- DPS 98.31 Stef Proost and Kurt Van Dender, *Effectiveness and Welfare Impacts of Alternative Policies to Address Atmospheric Pollution in Urban Road Transport* (Public Economics).
- DPS 98.32 Inge Mayeres and Stef Proost, Marginal Tax Reform, Externalities and Income Distribution (Public Economics).