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## Inflation, Price Competition and Consumer Search Technology

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### Abstract

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This paper studies an  $(S, s)$  pricing model from the perspective of inflation and price competition in search markets. I present a model in which consumers' search technologies can influence firms' price setting, price dispersion, and the market structure. The result shows that although price competition among firms is more intensified in markets where consumers' search technologies are more efficient, price inflation is counter-intuitively, more likely to increase monopoly power of firms and to stimulate entry in these markets. The model also provides new empirical implications for firms' price setting behaviors.

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# 1 Introduction

Recent emergence of electronic commerce and Internet market places has improved consumers' search-purchase technologies relative to visiting physical stores. Consumers go online and comparison shop between hundreds of vendors with much less effort than using Yellow Pages or newspaper advertisements. Online consumers are price sensitive.<sup>1</sup> Evidence shows these information technologies have also altered firms' price setting behaviors and the market structure: Bailey [1998] finds Internet retailers change prices more frequently than traditional retailers; Brynjolfsson and Smith [2000] observe that e-retailers make price changes that were up to 100 times smaller than those made by bricks-and-mortar sellers; Latcovich and Smith [2001] find the online book market is more concentrated, at 93% in the US, than traditional book retail industry, at only 45%. The objective of this paper is to explore the macroeconomic implications of consumers' search technologies.

I study the consequence of inflation on firms' price setting and search market behaviors. Bénabou [1988] establishes an important channel through which expected inflation can impact on the market structure and price setting behaviors of firms who face menu costs of changing nominal prices: an increased price dispersion, arising from inflation, increases consumers' willingness to search for a low price, which in turn reduces monopoly power of firms and induces exit from the market. In the present paper, I generalize Bénabou's model to study the role of consumer search technology play in the effects of inflation.

For the formulation of consumers' search technologies, I follow from Burdett and Judd's [1983] noisy sequential search model, where the probability  $q_1$  that one search elicits exactly one price quotation determines the equilibrium outcome.<sup>2</sup> Assuming  $0 < q_1 < 1$ , I study a situation in which more (or less) efficient consumer search technology implies a larger (or smaller) number of price quotations a consumer observes on average during one search.

In this setup, I show that there exist two different types of equilibrium: in the first type of equilibrium firms' pricing strategies are bounded by price competition among firms, whereas in the second type of equilibrium firms' pricing strategies are bounded by consumers' search. If consumers' search technologies are more efficient, it is likely that consumers observe a larger number of other firms' price. Thus, a firm expects more price-sensitive consumers and more elastic demand. In this case, price competition binds firms' price setting and the first type of equilibrium emerges. Conversely, if consumers' search technologies are less efficient, a firm expects demand to be less elastic. In this case, consumers' search binds firms' price setting and the second type of equilibrium emerges.

Within this framework, the effect of inflation depends on the type of equilibrium and thereby on consumer search technology. A greater price dispersion, arising from an increase in the rate of

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<sup>1</sup>See Ellison and Ellison [2001], Goolsbee [2000] and Goolsbee and Chevalier [2002].

<sup>2</sup>They show without inflation and price adjustment costs that the unique equilibrium outcome is: (1) monopoly outcome if  $q_1 = 1$ ; (2) competitive outcome if  $q_1 = 0$ ; (3) price dispersion if  $0 < q_1 < 1$ .

inflation, makes consumers less price sensitive and mitigates competition among firms in the first type of equilibrium. On the contrary, a greater price dispersion makes consumers more willing to search for a low price and imposes a greater pressure from consumer search on firms' pricing in the second type of equilibrium. Therefore, an inflation is more likely to increase monopoly power of firms and stimulate entry when consumers' search technologies are more efficient, but to decrease monopoly power and induce exit when consumers' search technologies are less efficient.

This result fills an important gap in the existing literature: Assuming  $q_1 = 0$ , Fishman [1992] shows that Bertrand undercutting implies a positive relationship between inflation and monopoly power of firms; Assuming  $q_1 = 1$ , Bénabou [1988] shows that consumer search implies a negative relationship between inflation and monopoly power of firms. Assuming  $0 < q_1 < 1$ , my result shows that the former (or latter) case is more likely to emerge when consumers observe a larger (or smaller) number of price quotations on average during search.<sup>3</sup>

Further implication of the result is that although an improved search technology increases the amount of competition among firms, it might not benefit consumers under long lasting inflation. Intensified competition, driven by an improved search technology, generates a prolonged inflationary period of gradual increases in real prices. This further implies a long-lasting duplication of resource costs to extract monopoly rent. Hence, the result suggests that a tension might exist for policy prescriptions between controlling price inflation and monopoly power of firms.

This paper is organized as follows. Section 2 setups of the model. Section 3 shows existence and characterization of equilibrium. Section 4 presents numerical results on comparative statics of the model. The model delivers empirical implications that are consistent with the existing evidence. Section 5 discusses these issues and concludes. All the proofs are collected in the Appendix.

## 2 The Model

### 2.1 Overview

The model is built on the framework constructed by Bénabou [1988], so it is useful to review his model briefly. Bénabou considers a monopolistically competitive market with consumer search. There are three main ingredients. First, the economy is characterized by constant inflation. Firms incur real (menu) costs in adjusting nominal prices. The optimal price adjustment strategy of individual firms is a staggered  $(S, s)$  policy: a firm waits until its real price reaches some lower bound,  $s$ , whereupon it instantaneously resets its nominal price so that its real price returns to the upper bound,  $S > s$ .

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<sup>3</sup>Other related works are Bénabou [1992] that generalizes the baseline model by introducing heterogeneous search costs, and Diamond [1993] that provides an alternative model, a sticker price model, where a firm cannot change the price of goods already in inventory. In my model, search costs are homogeneous and there is no difference in price-adjustment costs between the goods newly produced and the goods in inventory.

These individual price policies determine in aggregate the market price distribution. Second, facing the market price distribution, consumers incur real (search) costs to become informed of the individual prices. The optimal strategy of consumers is the reservation rule: they accept any price no greater than a reservation price. Finally, entry and exit of firms occur till the net profits of the operating firms are driven down to zero. Thus, firms' free-entry determines the density of firms per consumers.

In the present paper, I allow for consumers' search technology to be stochastic which is taken as deterministic in Bénabou's model. In particular, I allow for consumers to observe more than one price. The consumers' search-purchase behaviors in aggregate constitute a stochastic demand curve. Because consumers might observe the other firms' prices, individual firms face a competitive pressure even when the consumers' search reservation price does not bind their pricing policy. This mechanism, which is absent in Bénabou's model, allows me to study the influence of consumers' search technology on firms' pricing policies and price competition under inflation.

## 2.2 Basic setup

Time is continuous, denoted by  $t$  and lasts forever. The market consists of a continuum of identical firms and consumers. Denote by  $\mu \in (0, \infty)$  the ratio of consumers to firms.  $\mu$  will be determined endogenously later by free-entry of firms. Firms are long-lived and produce a homogeneous good which requires a fixed cost  $h > 0$ , and a constant marginal cost  $c > 0$ . These are real costs. Firms post nominal prices for the good. Nominal prices are eroded at the constant inflation rate  $g > 0$ . Each nominal price-adjustment entails a fixed real cost  $\beta > 0$ . There is no capacity constraint, so that firms can satisfy all the forthcoming demand at the price they post.

At each moment in time, a new set of short-lived consumers enters the market to purchase one unit of the good. Consumers search for the best price. Search is costly and requires real costs  $\gamma > 0$ . I will be more specific about the consumers' search technology shortly.

Following Bénabou [1988, 1992], I focus my attention on a stationary and symmetric equilibrium which has the following properties: All firms follow the identical staggered  $(S, s)$  price policy, that is, a firm keeps its nominal price unchanged for an interval  $T > 0$  until its real price reaches a trigger value  $s > 0$ , whereupon it instantaneously resets its nominal price so that its real price returns to a target value  $S > s$ , which is optimal given consumers' search rule; Consumers search rule, which is represented by a reservation price  $r > 0$ , is optimal given the firms'  $(S, s)$  price policy; the ratio of consumers to firms  $\mu > 0$  is determined by free entry of firms. Further, it is important to note that when all firms follow the identical staggered  $(S, s)$  price rule with a constant inflation, the only time-invariant distribution of real prices is log-uniform. Hence in a stationary-symmetric equilibrium, the market price distribution  $F(p)$  satisfies:

$$dF(p) = \frac{d \ln p}{\ln(S/s)} \quad (1)$$

for all  $p$  in  $(s, S]$ . Under this distribution, aggregate nominal prices grow at the rate  $g > 0$ , which keeps the aggregate consistency and the stationarity of individual firms'  $(S, s)$  price strategies.<sup>4</sup>

### 2.3 Consumers' Search

Given the market price-distribution  $F(p)$ , I shall begin with consumers' problem. Consumers know the distribution of prices,  $F(p)$ , but need to become informed of which firm is charging which price. For how consumers obtain information about the individual prices, I will follow from a model of noisy sequential search developed by Burdett and Judd [1983].<sup>5</sup> In particular, I allow for the number of price quotations per search, denoted as  $n$ , to be a random variable. Following their setup, I assume a sample contains at least one price offer, i.e.,  $n \geq 1$ . I also assume that  $n$  follows a Poisson distribution  $\Pr[n = k] = \frac{e^{-\lambda} \lambda^k}{k!}$ . Consumers do not know how many price quotations they observe ex ante, but do know that they arrive at the rate  $\lambda$  on average.  $\lambda$  is an exogenous parameter. For a higher value of  $\lambda$ , one search elicits a larger number of price quotations on average, thus a higher value of  $\lambda$  represents a more efficient search technology of consumers.

The objective of consumers is to minimize the expected cost of purchasing one unit of the good. Notice that because a sample might contain more than one price quotations, consumers will buy at the lowest price they observe. Hence, consumers face the distribution of the lowest price that they expect to observe during one search conditional on  $n \geq 1$ . Let  $J(p)$  denote this conditional distribution.

**Lemma 1** *Suppose consumers observe  $n \geq 1$  price quotations during one search that follows a Poisson distribution with mean  $\lambda$ . Given  $F(p)$ , the distribution of the lowest price consumers observe is given by the closed form expression:*

$$J(p) = \frac{1 - e^{-\lambda F(p)}}{1 - e^{-\lambda}}.$$

$J(p)$  is a non-decreasing function of  $p$  and satisfies  $\lim J(p) = 0$  as  $p \rightarrow s$  and  $J(S) = 1$ , where  $S$  (or  $s$ ) is the upper (or lower) bound of the market price distribution  $F(p)$ .

The distribution  $J(p)$  on which consumers base for search-purchasing decision depends on  $\lambda$  and the market price distribution  $F(p)$ . Note  $J(p)$  is constant over time and known to consumers. Given  $J(p)$ , the optimal strategy for consumers is determined by a reservation price,  $r$ : a consumer continues searching until she/he finds a price no greater than  $r$ .<sup>6</sup> If  $r$  denotes the lowest price observed to date,

<sup>4</sup>See Caplin and Spulber [1987] for a proof. Bénabou [1989] shows if firms follow a randomized  $(S, s)$  strategy to limit storage by speculators, then any initial distribution of real prices converges to the log uniform steady-state distribution. Bhaskar [1992] shows staggered pricing equilibria are stable when multidimensional strategic interactions among firms are taken into consideration.

<sup>5</sup>For a dynamic version of noisy sequential search model, see Burdett and Coles [1997].

<sup>6</sup>Implicit here is that the first sample is free so that all consumers carry out the searching activity whenever they have willingness to purchase. Recall problems are irrelevant once resources spent on search are assumed to be negligible relative to consumers' income. Further, I make the standard assumption that the search is instantaneous and consumption cannot be postponed. See Bénabou [1988, 1992] for more detailed discussion.

the consumer is indifferent between accepting the price  $r$  and searching for another sample. Formally,  $r$  satisfies:

$$\gamma = \int_s^{\min[S,r]} (r - p) dJ(p). \quad (2)$$

The L.H.S. of this equation represents the marginal cost of search, whereas the R.H.S. represents the marginal expected gain. Any consumer's search-purchasing decision is thus characterized by his/her reservation price  $r$ .  $\gamma$  is common to all consumers, hence they have the identical reservation price  $r$ . I assume a high consumption value of the good relative to search costs. From this it follows that it is the search reservation price, not the consumption value, that matters to firms' pricing strategies.

## 2.4 Profits

Given the consumers' search rule described above, the next step is to derive firms' expected profits. From the standpoint of individual firms, consumers' stochastic sampling implies that a random number of consumers observe their price. Specifically, each firm receives  $\mu\lambda$  consumers on average at each moment in time. If a given firm has a real price  $p$  in  $F(p)$  at some point in time, then let  $D(p)$  define the expected number of sales the firm makes at that time. Note  $D(p)$  is time-invariant. The firm makes no sales when its price is greater than  $r$ , hence  $D(p)=0$  for  $p > r$ . For  $p \leq r$ , because any given consumer might observe more than one price, the firm can sell to the consumer only when its price is the lowest among prices that she/he observes. The following lemma computes  $D(p)$ .<sup>7</sup>

**Lemma 2** *Given  $F(p)$  and the consumers' search reservation price  $r$ , the instantaneous expected number of sales faced by a firm that has a price  $p$  is given by*

$$D(p) = \begin{cases} 0 & \text{if } p > r \\ \mu\lambda e^{-\lambda F(p)} & \text{if } p \leq r. \end{cases}$$

The expected number of sales of a firm is decreasing in  $p \leq r$ . This is because there is a possibility that a consumer who observes the firm's price may also observe the other firms' price and thereby it is more likely that the firm sells to the consumer if its price is lower. Given  $D(p)$ , the corresponding instantaneous expected-profits of a firm that has a real price  $p$  in  $F(p)$  is given by  $\Pi(p) = (p - c)D(p)$ . Note  $\Pi(p)$  is stationary.  $\Pi(p)$  has the following important property.

**Lemma 3** *The instantaneous expected-profit function  $\Pi(p)$  is strictly quasi-concave in  $p$ .*

As shown in the next subsection the strict quasi-concavity of  $\Pi(p)$  ensures the uniqueness of the  $(S, s)$  price policy.

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<sup>7</sup>Similar demand functions are in Wilde [1977], Butters [1977], and Stahl [1989].

## 2.5 $(S, s)$ price policy

The objective of a firm is to maximize its total discounted expected profits by choosing its nominal price. Given a continuum of firms, an individual firm's price has no influence on the entire price distribution in the market. With a constant inflation  $g > 0$  and a positive price-adjustment cost  $\beta > 0$ , individual firms face a tradeoff between the cost of adjusting its nominal price and the cost of letting inflation erode its real price. Sheshinski and Weiss [1977] show that the optimal price strategy of a firm who faces such a tradeoff is the  $(S, s)$  type: a firm keeps its nominal price unchanged until its real price  $p$  depreciates to a trigger value  $s$  whereupon it increases the nominal price so that  $p$  reaches a target value  $S > s$ .

Denote by  $S', s'$  a real price band of a firm. Then, the total discounted expected profits of a firm pursuing a stationary  $(S', s')$  price rule, just prior to the first price change, defines:

$$V(S', s') = -\beta + \int_0^{T'} \Pi(S' e^{-gt}) e^{-\rho t} dt + V(S', s') e^{-\rho T'},$$

where  $T' \equiv \ln(S'/s')/g$  denotes the duration of time that nominal price remains fixed, and  $\rho > 0$  denotes the discount rate. The firm's problem is to select the level of real prices  $S'$  and  $s'$  so as to maximize  $V(S', s')$ . The optimal solution is unique and satisfies the following conditions.

**Lemma 4** *Given  $F(p)$  and the consumers' search rule, the optimal  $(S', s')$  price policy of a firm satisfies*

$$S' \leq r \tag{3}$$

and the first-order conditions,

$$\Pi(S') - \Pi(s') - \rho\beta = 0 \tag{4}$$

$$\Pi(s') - \rho V(S', s') = 0. \tag{5}$$

Further, the solution to the first-order conditions satisfies the second-order conditions and is unique.

Firm expects no sales during the time that its real price is greater than the consumers' reservation price  $p > r$ . Thus, the optimality of  $(S', s')$  policy requires  $S' \leq r$ . Equations (4), (5) are the standard first order conditions. The expected gains from delaying a price adjustment are the expected profits just prior to the adjustment, which is equal to  $\Pi(s')$ , plus the interest on adjustment costs,  $\rho\beta$ , whereas the expected loss from such a delay is the expected profits just after the price adjustment  $\Pi(S')$ . These expected gains and losses are equated at the optimum as stated in condition (4). The change in real price drives the marginal expected profits during the price cycle  $T'$  to zero on average as stated in condition (5).<sup>8</sup> Lemma 3 guarantees that the optimal solution to the first order-conditions is unique.

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<sup>8</sup>Integrating  $V(S', s')$  by parts in equation (5) leads to  $\int_0^{T'} \Pi'(S' e^{-gt}) e^{-(\rho+g)t} dt = 0$ .

In symmetric equilibrium, an individual firm's optimal policy  $(S', s')$  must coincide with the other firms' price rule  $(S, s)$ . Hence, the equilibrium value of real price band  $(S, s)$  satisfies the first-order conditions (4), (5).

## 2.6 Entry

Following Bénabou [1988, 1992], I allow for the density of firms per consumer  $1/\mu$  to be determined endogenously by free entry. This is the final requirement for a monopolistically competitive equilibrium. Entry and exit occur until the firms operating in the market earn zero expected net profits, just to cover fixed costs in equilibrium.

$$\rho V(S, s) = h \quad (6)$$

Since firms cover all costs, they are willing to operate and satisfy demand. It would be useful to mention here the role played by entry and exit of firms. Consider for example a situation in which fixed costs  $h$  gets higher. Then, some firms exit from the market and the average market share  $\mu\lambda$  of the remaining firms increases given values of  $\lambda$ . This implies that for any given price  $p$ , the instantaneous expected profit for the remaining firms  $\Pi(p)$  increases in response to exit of firms. This process continues until the corresponding value  $\rho V(S, s)$  covers the fixed costs  $h$  as expressed in equation (6).

## 3 Equilibrium

### 3.1 Market equilibrium

**Definition 1** *A symmetric steady-state equilibrium in this economy is consumer search reservation price  $r > 0$ , a pair of real prices  $S, s > 0$ , the market distribution of real prices  $F(p)$ , and the density of firms operating in the market  $1/\mu > 0$  such that:*

- *Given  $F(p)$  and firms'  $(S, s)$  price rule,  $r$  satisfies equation (2);*
- *Given  $F(p)$  and  $r$ , firms' price rule is a staggered  $(S, s)$  policy that satisfies inequality (3) and the first-order conditions (4), (5);*
- *The market real-price distribution is stationary and is consistent with the individual firms' staggered  $(S, s)$  policy, that is,  $F(p)$  satisfies equation (1);*
- *$1/\mu$  is determined by the free-entry condition (6).*

Noting  $S \leq r$ , the equilibrium solution  $\mu, r, S, s$ , can be described by the following system of equations:

$$r = \gamma + E_J(p) \quad (7)$$

$$S = \min \left[ \left( s + \frac{\rho\beta}{\mu\lambda} \right) e^\lambda - c(e^\lambda - 1), r \right] \quad (8)$$

$$s = c + \frac{h}{\mu\lambda} \quad (9)$$



and the free-entry condition (6). Condition for consumer search (7) is obtained by applying  $S \leq r$  to equation (2), where

$$E_J[p] = S \frac{\lambda(\frac{s}{S} - e^{-\lambda})}{(1 - e^{-\lambda})(\lambda - \ln(\frac{S}{s}))} \in (s, S]$$

represents the lowest price consumers expect during search. Equation (8) is obtained by applying  $S \leq r$  to equation (4), and equation (9) is obtained by applying the free-entry condition (6) to equation (5).

In what follows, I use  $\frac{1}{\sigma} - 1 \equiv (S - s)/s$  as a measure of price dispersion. A higher (or lower) value of  $\frac{1}{\sigma}$  implies a larger (or smaller) dispersion of real prices.<sup>9</sup> Observe from equation (8) that there are two cases: either  $S$  can be determined independent of the search reservation price  $r$  or not. In the following two subsections, I show the existence and characterization of equilibrium for each case in separation. Either type of equilibrium can emerge.

Finally, equation (9) shows that the lower bound of real-prices  $s$  is increasing in the number of firms  $1/\mu$ . This means, individual firms compensate a smaller expected profit, driven by entry of firms, by setting the trigger value  $s$  at a higher level. This positive relationship holds irrelevant of how the upper bound  $S$  is determined.

### 3.2 Equilibrium without binding reservation price

I shall first consider the case with  $S < r$ . In this case, the real-price band  $(S, s)$  is determined regardless of the consumer reservation price  $r$ . As will be discussed in detail later, this equilibrium has essentially a similar property to Fishman's [1992], so I will refer to it as F-type equilibrium.

**Definition 2 (F-type equilibrium)** *Define:*

$$\Omega_F \equiv \{(\gamma, \beta, h, c, g, \rho) \in (0, \infty)^6, \lambda \in [1, \infty) \text{ s.t. } S < r\}.$$

*If  $\{\gamma, \beta, h, c, \lambda, g, \rho\} \in \Omega_F$ , then define an F-type equilibrium to be the steady-state equilibrium in Definition 1.*

The equilibrium value of  $\mu$ ,  $S$ ,  $s$  for an F-type is determined by equations (6), (8), (9) that satisfy  $S < r$ . The standard fixed-point argument implies the existence of equilibrium.

**Theorem 1 (Existence of F-type equilibrium)** *Suppose  $\{\gamma, \beta, h, c, \lambda, g, \rho\} \in \Omega_F$ . Then, there exists an F-type equilibrium that satisfies:  $\mu \in (0, \infty)$  and  $r > S > s > c$ .*

Given  $\{\gamma, \beta, h, c, \lambda, g, \rho\} \in \Omega_F$ , Theorem 1 establishes the existence of an F-type equilibrium. Notice  $\Omega_F$  is non-empty. In an F-type equilibrium, price-competition binds effectively the target value

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<sup>9</sup>Lemma 2 in Bénabou [1988] proves that the coefficient of variation is monotone in  $1/\sigma$  for the log-uniform distribution.

$S$  of individual firms. The role played by price competition can be captured by the price elasticity of expected number of sales, which is given by  $\alpha \equiv \frac{dD(p)/dp}{D(p)/p} = \frac{\lambda}{\ln(1/\sigma)}$ . Note  $\alpha$  is increasing in  $\lambda$  and decreasing in  $1/\sigma$ . This means, the more price quotations consumers observe on average, the more price sensitive consumers are and the more elastic demand a firm faces. Conversely, the more dispersed market prices are, the less price sensitive consumers are and the less elastic demand a firm faces.

There are two important properties of the F-type equilibrium concerning the relationship between price dispersion, the number of firms, and the average level of market prices. First, equations (9), (11) yield:

$$\frac{1}{\sigma} = \frac{c + e^\lambda(h + \rho\beta)/\mu\lambda}{c + h/\mu\lambda} \quad (10)$$

which shows the positive relationship between the price dispersion  $1/\sigma$  and the number of firms  $1/\mu$ . This reflects the fact that when price dispersion is larger, firms face a less elastic expected demand. Given firms' pricing is free from the search-reservation price (i.e.,  $r > S$ ), a less elastic expected demand lets individual firms set the target value  $S$  at a higher level, and the resulting higher expected-profits stimulate entry of firms.

Second, applying the free-entry condition (6) to the first order conditions (4) yields:

$$S = c + \frac{e^\lambda(h + \rho\beta)}{\mu\lambda}. \quad (11)$$

$S$  is increasing in  $1/\mu$ . Since  $s$  is also increasing in  $1/\mu$ , the entire real-price band is at a higher level when there are a larger number of firms.

Finally, it is instructive to compare the role played by price-competition in the F-type equilibrium with that considered in Fishman's [1992]. Both share a common mechanism through which price competition binds firms' price-adjustment strategies. In Fishman's model, however, consumers observe more than one price quotations with probability one. In this case, firms know that any consumers observe the rival's price without fail, making the Bertrand undercutting dictate the equilibrium. In his model, the price-competition ensures equal profits in equilibrium only by itself, while the equilibrium price dispersion arises solely from real price erosions (due to inflation) and adjustment costs. In contrast, my model retains a monopolistic element in pricing, because consumers might observe only one price. There is still room for high-price firms to survive due to the presence of some consumers who find only their price and not others. This opens up a room for an equilibrium interaction between the price dispersion and the number of firms.

### 3.3 Equilibrium with binding reservation price

Consider next the case with  $S = r$ . In this case, the search reservation price effectively binds the upper bound  $S$  of market prices. This equilibrium has essentially a similar property to the equilibrium with binding reservation price studied in Bénabou's [1988], so I will refer to it as B-type equilibrium.

**Definition 3 (B-type equilibrium)** *Define:*

$$\Omega_B \equiv \{(\gamma, \beta, h, c, g, \rho) \in (0, \infty)^6, \lambda \in [1, \infty) \text{ s.t. } S = r\}.$$

If  $\{\gamma, \beta, h, c, \lambda, g, \rho\} \in \Omega_B$ , then define a B-type equilibrium to be the steady-state equilibrium in Definition 1.

Given that equation (8) holds with  $S = r$ , the equilibrium value of  $S$ ,  $s$ ,  $\mu$ , in a B-type equilibrium is determined by equations (6), (7), (9). The standard fixed-point argument implies the existence of equilibrium.

**Theorem 2 (Existence of B-type equilibrium)** *Suppose  $\{\gamma, \beta, h, c, \lambda, g, \rho\} \in \Omega_B$ . Then, there exists a B-type equilibrium that satisfies:  $\mu \in (0, \infty)$  and  $S = r > s > c$ .*

Given  $\{\gamma, \beta, h, c, \lambda, g, \rho\} \in \Omega_B$ , Theorem 2 establishes the existence of a B-type equilibrium. Notice  $\Omega_B$  is non-empty. Equations (7), (9) yield:

$$\frac{1}{\sigma} = \frac{\gamma}{(c + h/\mu\lambda)(1 - L(1/\sigma))} \quad (12)$$

where  $L(1/\sigma) \equiv \frac{E_J[p]}{S} = \frac{\lambda(\sigma - e^{-\lambda})}{(1 - e^{-\lambda})(\lambda - \ln(\frac{1}{\sigma}))}$  is strictly decreasing in  $1/\sigma$ . The R.H.S. of the above equation is strictly decreasing in both  $1/\sigma$  and  $1/\mu$ . Hence, opposite to F-type equilibrium, the B-type equilibrium implies the negative relationship between price dispersion and the number of firms. This reflects the fact that as price dispersion becomes larger, consumers expect a lower price to observe during search. Given  $S = r$ , this implies individual firms must set the target value  $S$  at a lower level. The resulting lower expected-profits lead to exit of some firms, which continues till the remaining firms' expected-profits cover the fixed costs, to satisfy the free-entry condition (6).

As in F-type equilibrium, the level of entire price-band is strictly increasing in the number of firms in a B-type equilibrium. Remembering that the lower bound  $s$  is increases in  $1/\mu$ , this can be seen by noting from equation (7) or  $S = \frac{\gamma}{1 - L(1/\sigma)}$  that the upper bound  $S$  is increasing in  $1/\mu$ .

## 4 Comparative statics

The analysis so far has established the existence of the steady-state equilibrium, and an important difference in the equilibrium properties: in an F-type equilibrium, the upper bound of real prices  $S$  is bounded by price-competition among firms, and a change in price dispersion has a *positive* effect on the average real price and the number of firms; in a B-type equilibrium,  $S$  is bounded by the consumers' search reservation price, and a change in price dispersion has a *negative* effect on the average real price and the number of firms.

In this section, I use numerical simulations of the model for comparative statics on two important parameters,  $\lambda$  and  $g$ . For simulation exercises, the baseline parameters I have chosen are:  $\gamma = 0.01$ ,  $\beta = 1.5$ ,  $h = 15.0$ ,  $c = 1.0$ ,  $\lambda = 1.4$ ,  $\rho = 0.05$ ,  $g = 0.03$ . Although the purpose is not to calibrate any specific evidence, implicit here is to generate a small menu cost relative to expected profits over the price-cycle. I chose the set of parameters to generate  $\beta / \int_s^S (p - c)D(p)dp = 0.027$ .<sup>10</sup> Under this set of parameters, the equilibrium is in the F-type. The results presented here are robust to the choice of parameters.

To illustrate how a switch of the type of equilibrium can occur, suppose that equilibrium is initially in the F-type, that is,  $\{\gamma, \beta, h, c, \lambda, g, \rho\} \in \Omega_F$ . Differentiation of equation (7) yields:

$$\frac{dr}{d(1/\sigma)} = \frac{dS}{d(1/\sigma)}L(1/\sigma) + S\frac{dL(1/\sigma)}{d(1/\sigma)}. \quad (13)$$

In the RHS of this equation, the first term is positive and the second term is negative. This shows, there exist two opposing effects of an increase in price dispersion on the reservation price  $r$ . On the one hand, a larger price dispersion implies a less elastic expected demand to firms and hence leads to a higher average market price (i.e.,  $dS/d(1/\sigma) > 0$ ). This effect increases the consumers reservation price  $r$ . On the other hand, a larger price dispersion implies a higher marginal expected gain to consumers from one search (i.e.,  $dL(1/\sigma)/d(1/\sigma) < 0$ ). This effect decreases  $r$ . Thus, if  $r$  increases with  $1/\sigma$  in an F-type equilibrium, its increment must be less than that of  $S$ . Hence, if the equilibrium switches to the B-type equilibrium (i.e.,  $S = r$ ), then there must be a sufficiently large price dispersion.

#### 4.1 Consumers' search technology

Taking  $g$  as given, I first examine comparative statics with respect to  $\lambda$ . The following result provides a useful benchmark.

**Proposition 1** *The steady-state equilibrium is in the F-type in the limit as  $\lambda \rightarrow \infty$  and satisfies:  $1/\mu \rightarrow 0$ ,  $S \rightarrow c$ ,  $s \rightarrow c$ ,  $r \rightarrow \gamma + c$ ,  $1/\sigma \rightarrow 1$  as  $\lambda \rightarrow \infty$ .*

The result shows an equilibrium is in the F-type when  $\lambda$  takes high values.

Figure 1(a) - 1(b) plot simulation results for the responses of real prices  $r, S, E_F(p), s$  and the number of firms  $1/\mu$  to changes in  $\lambda$ .  $E_F(p)$  denotes the average market price. The figures show that all real prices, price dispersion and the number of firms decrease with  $\lambda$ . The underlying mechanism is that a larger  $\lambda$  implies a more elastic expected demand and intensifies price competition among firms.

<sup>10</sup>Levy, Bergen, Dutta, and Venable [1997] estimate the size of menu costs measured in terms of labor units, on the basis of direct observation of the price changing process in several large US grocery chains during 1991-1992. They report that the total menu cost is 0.7% of the entire revenue and 2.8% of the gross margin. The annual inflation rate during this time period was 3%. In a related firm-level study, Zbaracki et. al. [2004] find that managerial costs (information gathering, decision making, and communication costs) and customers costs (communication and negotiation costs) of price adjustment are significantly larger than physical menu costs.

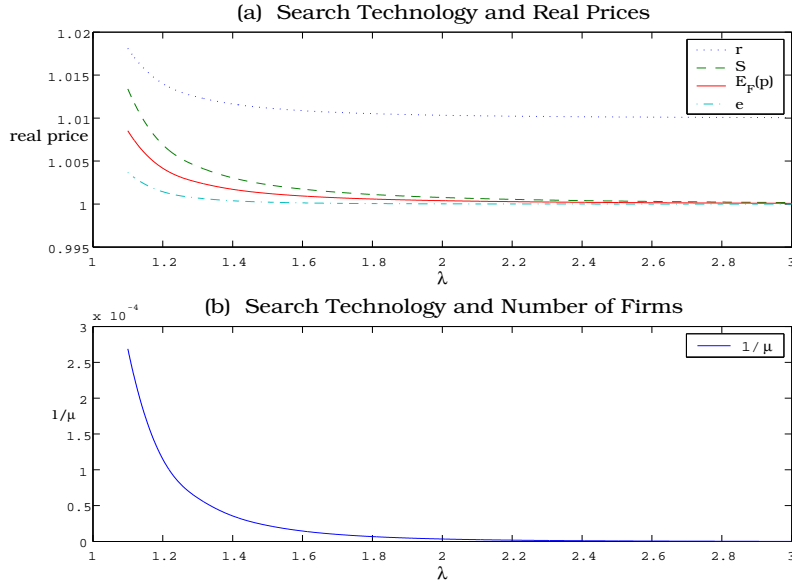


Figure 1: Search Technology, Real Prices, and Number of Firms

This makes individual firms' price adjustments smaller and more frequent, and induces exit of firms. Hence, both the average market price and price dispersion decrease with  $\lambda$ .<sup>11</sup>

**Result 1** *As consumers observe a larger number of price quotations on average, the average real price, price dispersion and the number of firms all decrease, while price adjustments by individual firms become smaller and more frequent, and vice versa.*

Result 1 is consistent with a set of empirical findings on individual markets mentioned in the introductory section. Internet commerce, in which consumers have improved search technologies, leads to more frequent and smaller price adjustments by individual firms, more elastic demand, a higher concentration ratio, and lower prices.<sup>12</sup> Interpreting  $\lambda$  as an average search intensity of consumers, which varies across markets, the negative relationship between search intensity and price is consistent with Sorensen's [2000] findings that a larger search intensity reduces price dispersion in retail markets for prescription drugs.

## 4.2 Inflation

To present comparative statics results on  $g$ , the following proposition provides a useful benchmark.

<sup>11</sup>The results remain the same even when the equilibrium is in a B-type region, which emerges for lower value of  $\lambda$  with assuming another parameter values, for example, a lower value of  $\gamma, h$  and/or a higher value of  $g, \beta$ . Numerical tables for these cases are available upon request from the author.

<sup>12</sup>Brown and Goolsbee [2000] and Scott, Morton and Zettlemeyer [2002] find Internet leads to lower prices for life insurance and for cars, respectively.

**Proposition 2** *The F-type equilibrium satisfies:  $1/\mu \rightarrow 0$ ,  $S \rightarrow c$ ,  $s \rightarrow c$ ,  $r \rightarrow \gamma + c$ ,  $1/\sigma \rightarrow 1$  as  $g \rightarrow 0$ . Further, the steady-state equilibrium is in the B-type in the limit as  $g \rightarrow \infty$  and satisfies:  $1/\mu \in (0, \infty)$ ,  $S = r > s > c$ ,  $1/\sigma \rightarrow 1/\sigma_+ \in (0, \infty)$  as  $g \rightarrow \infty$ .*

Supposing that equilibrium is initially in the F-type, Proposition 2 shows that a switch of the type of equilibrium from the F-type to the B-type occurs at some  $g \in (0, \infty)$ . It will become clear shortly that the results below are essentially the same even if I suppose equilibrium is initially in the B-type, given that an F-type equilibrium emerges for large values of  $\lambda$  as shown above.

**Inflation, real prices, and the number of firms:** Figure 2(a) - 2(b) plot numerical results for the responses of real prices  $r, S, E_F(p), s$ , and the number of firms  $1/\mu$  to changes in the rate of inflation  $g$ . The figures show a clear occurrence of the equilibrium switching: an equilibrium is the F-type for low values of  $g$  but is the B-type for high values of  $g$ . An increase in  $g$  leads to an increase in price dispersion.<sup>13</sup> A larger price dispersion implies a less elastic expected demand to firms, and a larger expected return of search to consumers. As already mentioned, these two effects work oppositely on the consumer reservation price  $r$ , hence the upper bound  $S$  increases more than  $r$  (see equation (13)). In the benchmark case with  $\lambda = 1.4$ ,  $S = r$  for  $g \geq 20.9\%$ . As  $g$  increases, all real prices and the number of firms increase in the F-type region but decrease in the B-type region. Hence, an equilibrium switching displays non-monotonic responses of equilibrium to an increase in  $g$ .

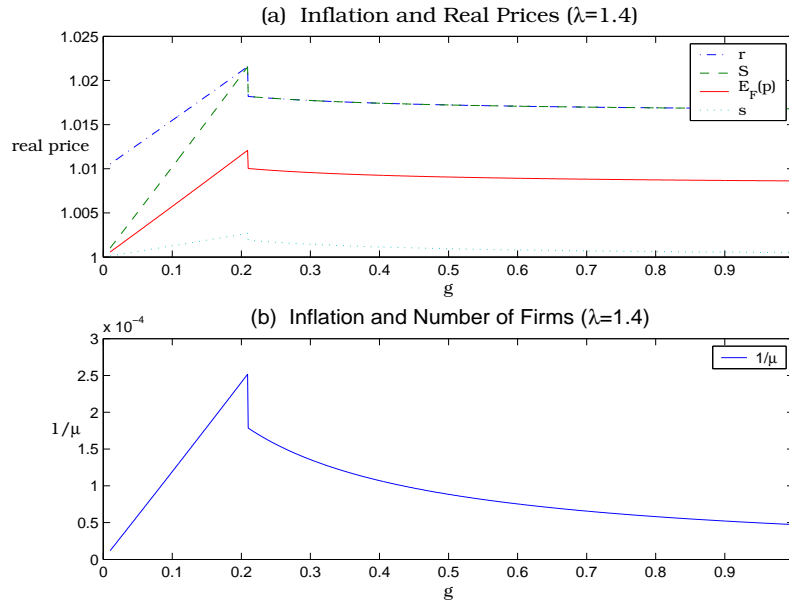


Figure 2: Inflation, Real Prices, and Number of Firms ( $\lambda = 1.4$ )

<sup>13</sup>For empirical evidence on the positive relationship between inflation and price dispersion, see Vining and Elwertowski [1976], Lach and Tsiddon [1992], and Eden [2001] for example.

Figure 3(a) - 3(d) plot simulation results on real prices with  $\lambda = 1.1$ ,  $\lambda = 1.5$ ,  $\lambda = 2.0$ ,  $\lambda = 2.5$  respectively. These figures show that an equilibrium switches at a lower (or higher) level of  $g$  when  $\lambda$  takes a lower (or higher) value. For  $\lambda = 1.1$ , after large increases in real prices in the F-type region, a switch occurs at  $g = 5.6\%$  above which all real prices decrease with  $g$ ; for  $\lambda = 1.5$ , 2.0, a switch occurs at  $g = 27.1\%$ , 68.2% respectively; while for  $\lambda = 2.5$  all real prices increase with  $g \leq 100\%$  within which a switch does not occur. The dependence of the non-monotonicity on  $\lambda$  is intuitive. When  $\lambda$  is low (or high), the amount of price competition among firms is small (or large) so that individual firms make a larger (or smaller) price-increase in response to an increase in  $g$ . As a result, when  $\lambda$  is low the upper bound  $S$  increases readily to reach  $r$  as  $g$  increases, whereas when  $\lambda$  is high the equilibrium remains in the F-type region for a wide range of  $g$ .

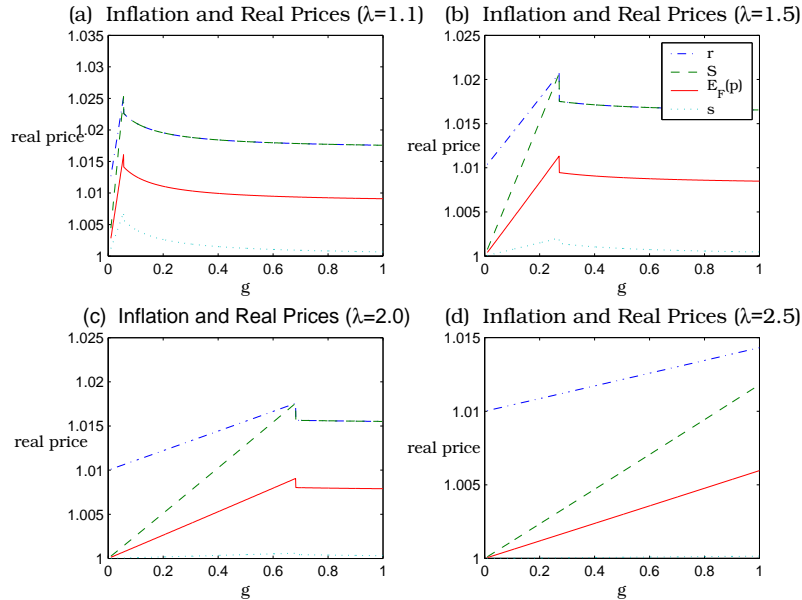


Figure 3: Inflation, Real Prices, and Number of Firms ( $\lambda = 1.1, 1.5, 2.0, 2.5$ )

Finally, the non-monotonicity result is significantly sensitive to values of  $\lambda$ . To confirm this point, I have calculated the elasticity of the turning point inflation rate with respect to a change in parameters. Denote this elasticity by  $g_i^*$  for parameter  $i \in \{\lambda, \gamma, \beta, h\}$ . Lower search costs  $\gamma$ , higher price-adjustment costs  $\beta$ , or lower entry costs  $h$  all contribute to make an equilibrium switching arise at a lower inflation rate. However, their effects are quantitatively small relative to  $\lambda$ :  $g_\gamma^* = 0.29$ ,  $g_\beta^* = -0.05$ , and  $g_h^* = 0.02$ , while  $g_\lambda^* = 0.74$ .<sup>14</sup> The overall results can be summarized as follows.

<sup>14</sup>The elasticity of the turning point inflation rate is defined as  $g_i^* \equiv \frac{(\bar{g}_{i,j+1} - \bar{g}_{i,j})/\bar{g}_{i,j}}{(i_{j+1} - i_j)/i_j}$  where  $\bar{g}_{i,j}$  represents the turning point inflation rate for the  $j$ th value of a parameter  $i$ . The number reported is averaged over a parameter interval that satisfies  $\bar{g}_{i,j} \in (0, 1.000]$ , which is found to be  $\gamma = (0, 0.06]$ ,  $\beta = (0, 5.5]$ ,  $h = (0, 20.1]$ , and  $\lambda = (1, 2.3]$  respectively. The result is robust to any choice of this parameter interval.

**Result 2** *An increase in the rate of inflation increases price dispersion, and is more likely to increase the average real price and the number of firms when consumers observe a larger number of price quotations on average, and vice versa. Further, the impact of inflation on real prices and firms' entry-exit behaviors is significantly sensitive to parameter for consumers' search technologies  $\lambda$ .*

**Inflation and resource costs:** In this economy, the overall resource costs are the total of menu costs and fixed costs, given by  $\frac{1}{\mu} \left( h + \frac{\beta}{T} \right)$ . Entry of firms, i.e., a higher  $1/\mu$ , implies larger total fixed costs, and a more frequent price-adjustment by individual firms, i.e., a higher  $1/T$ , implies larger total menu costs. Figure 4(a) plots numerical results on the responses of total resource costs to changes in  $g$ . An increase in  $g$  induces entry of firms and increases total resource costs in the F-type region, whereas it induces exit of firms and decreases total resource costs in the B-type region. This result reflects the fact that the contributions of menu costs are minor. Noting the equilibrium is the F-type for a wider range of  $g$  when  $\lambda$  takes a higher value, the result can be summarized as follows.

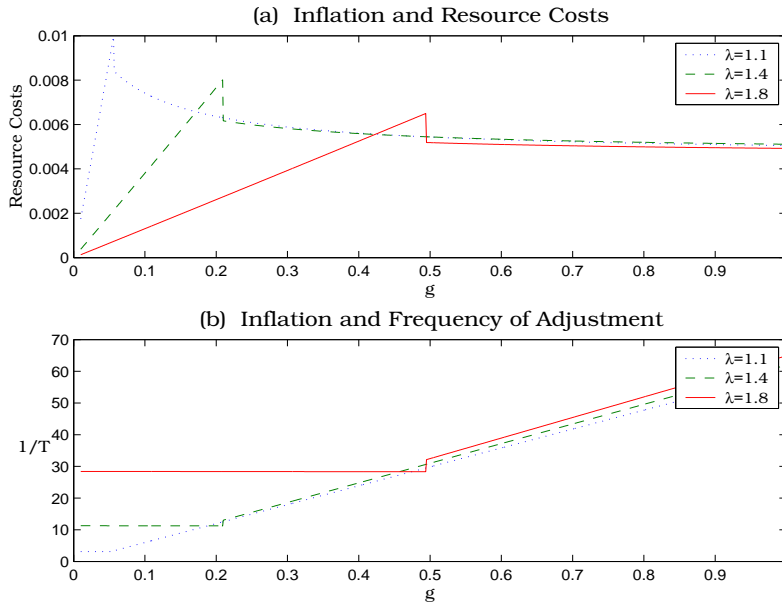


Figure 4: Inflation, Resource Costs, and Frequency of Price Adjustments

**Result 3** *An increase in the inflation rate is more likely to increase the total resource costs spent on entry and price-adjustments when consumers observe a larger number of price quotations on average, and vice versa.*

**Inflation and the frequency of price adjustments:** Figure 4(b) plots numerical results for the responses of the frequency of price-adjustments  $1/T$  to changes in  $g$ . In the present framework, inflation



affects  $1/T$  through the following two channels: directly, an increase in  $g$  increases the speed at which real prices hits the lower bound  $s$ , which shortens the length of time that nominal price remains fixed,  $T \equiv \ln(1/\sigma)/g$ ; indirectly, an increase in  $g$  leads to a larger size of price adjustments by individual firms, which decreases the frequency of price adjustments,  $1/T$ .<sup>15</sup> The result shows that the latter effect is relatively stronger than the former effect in the F-type region, whereas the former effect is relatively stronger than the latter effect in the B-type region. This reflects an aggregate feedback effect on  $1/T$  present in the B-type equilibrium: exit of some firms, if induced by an increased willingness to search, partially restores the average market shares of the remaining firms and motivates them to make smaller and more frequent price-adjustments. Consequently, an increase in  $g$  leads to less frequent price adjustments in the F-type region, but to more frequent price adjustments in the B-type region. For  $\lambda = 1.1$ , prices are adjusted every 16.7, 8.8, 1.8, 0.03 weeks for the annual inflation rate  $g = 1\%$ , 10%, 50%, 100%, while for  $\lambda = 1.4$  and 1.8, the corresponding durations are 4.6, 4.6, 1.7, 0.8 weeks and 1.8, 1.8, 1.6, 0.8 weeks, respectively. Because a B-type equilibrium emerges at a higher inflation rate and lower values of  $\lambda$ , the result can be summarized as follows.<sup>16</sup>

**Result 4** *An increase in the inflation rate is likely to yield more frequent price-adjustments by individual firms when consumers observe a smaller number of price quotations on average, and vice versa.*

## 5 Conclusion

This paper has studied the relationship between search market behaviors and price competition in an inflationary economy. I extended Bénabou's [1988] seminal work by using a noisy sequential search model developed by Burdett and Judd [1983]. A novel insight into the consequence of expected inflation has been obtained: an inflation is more likely to increase monopoly power of firms and to stimulate entry of firms in markets where consumers' search technologies to observe firms' prices are more efficient, and vice versa.

The model has also delivered a couple of new empirical implications on firms' price setting behaviors. Result 1 provides a testable hypothesis: given the rate of inflation, demand is more elastic in markets where consumers search technologies are more efficient, and hence firms' price adjustments are smaller and more frequent. This is consistent with the existing evidence on digital economy, and

<sup>15</sup>In partial equilibrium  $(S, s)$  models in which the demand function is exogenously given, the relationship between inflation, real prices, and the frequency of price adjustments depends on the curvature of the profit function. See Koenieckzny [1990, 1993] and Bénabou and Koenieckzny [1994].

<sup>16</sup>The empirical evidence concerning the frequency in individual markets is not convergent, ranging from a week on average (Ariga, Matsui, Watanabe [2001] and Dutta, Bergen, Levy, and Venable [1999]) to 11 months on average (Kashyap [1995]) to 3-7 years (Cecchetti [1986]). Wolman [2000] provides an excellent survey of this literature. Recently, Bills and Klenow [2004] find the average frequency is about three months in the US, and Dhyne et al [2005] find it more than six months in Europe.

observations by Carlton [1986] and Caucutt, Gosh and Kelton [1999] that firms' price changes are more frequent in more competitive markets where demand is more elastic. It is important to note that Bils and Klenow [2004] find this relationship is not robust. The result obtained in my paper (Result 4) also suggests that this relationship can be altered once inflation and the macroeconomic effects are taken into consideration.

A recent work by Konieczny and Skrzypacz [2006] finds an interesting evidence that shows firms' price adjustments are more frequent and smaller when consumers' search is more intensive. Using several commodity characteristics as a proxy, they show that consumers' search intensity that varies across the units of purchases, can explain the relationship between inflation and the frequency of price adjustments observed in high inflation economies. In contrast to their formulation, varying search intensities introduced in my model reflect market characteristics rather than commodity characteristics. It will be interesting to test if the difference in the notion of search intensity yields similar or different empirical outcomes. I will leave a closer look at these empirical issues for future research.

## Appendix

### Proof of Lemma 1

Suppose that a sample contains  $n$  price quotations drawn from the distribution  $F(p)$ , where  $n \geq 1$  is a positive integer and  $F(p)$  is well-defined and has no mass point. Given that any price observed is an independent random variable, the probability that the lowest price in the  $n$  sampled prices is greater than a price  $p$  is given by  $(1 - F(p))^n$ . Then  $1 - (1 - F(p))^n$  represents the probability that the lowest price in the  $n$  sampled prices is no greater than  $p$ . From the assumption that  $n$  follows a Poisson distribution with  $\lambda$ , it follows that:

$$\begin{aligned}
 & \Pr[\text{the lowest price in } n \text{ prices} \leq p] \\
 &= \sum_{n=1}^{\infty} \Pr[n \text{ prices are sampled}] \times \Pr[\text{the lowest price in } n \text{ prices} \leq p] \\
 &= \sum_{n=1}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} [1 - (1 - F(p))^n] \\
 &= 1 - \sum_{n=0}^{\infty} \frac{e^{-\lambda(1-F(p))} [\lambda(1 - F(p))]^n}{n!} e^{-\lambda F(p)} \\
 &= 1 - e^{-\lambda F(p)}
 \end{aligned}$$

Noting that the Poisson probability  $\frac{e^{-\lambda} \lambda^n}{n!}$  is defined over  $n = 0, 1, 2, \dots$ , the distribution of the lowest price conditional on  $n \geq 1$  that is faced by consumers can be derived as follows:

$$J(p) = \frac{\Pr[\text{the lowest price in } n \text{ prices} \leq p]}{\Pr[n \geq 1]} = \frac{1 - e^{-\lambda F(p)}}{1 - e^{-\lambda}}$$

where I used  $\Pr[n \geq 1] = 1 - \Pr[n = 0] = 1 - e^{-\lambda}$ . This completes the proof of Lemma 1.

### Proof of Lemma 2

If  $p > r$ , then  $D(p) = 0$  due to the reservation-price nature of demand. If  $p \leq r$ , then noting that the density of consumers who observe a firm's price is given by  $\mu n$  leads to:

$$\begin{aligned}
 D(p) &= \sum_{n=1}^{\infty} \mu n \times \Pr[n \text{ prices are observed}] \times \Pr[p \text{ is the lowest price in } n \text{ prices}] \\
 &= \mu \sum_{n=1}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} n (1 - F(p))^{n-1} \\
 &= \mu \lambda \sum_{n=0}^{\infty} \frac{e^{-\lambda(1-F(p))} (\lambda(1 - F(p)))^n}{n!} e^{-\lambda F(p)} \\
 &= \mu \lambda e^{-\lambda F(p)}.
 \end{aligned}$$

This completes the proof of Lemma 2.

### Proof of Lemma 3

If  $p > r$ , then  $\Pi(p) = 0$ . If  $p \leq r$ , then define  $\hat{p}$  to be the price that satisfies  $\frac{d\Pi(p)}{dp} \Big|_{p=\hat{p}} = 0$ . Applying  $dF(p)/dp = 1/p \ln(1/\sigma)$  for all  $p \in [s, S]$  (see equation (1)), it is immediate that  $\hat{p} = \frac{\lambda c}{\lambda - \ln(1/\sigma)}$ . A straightforward algebra implies:

$$\frac{d^2\Pi(p)}{dp^2} \Big|_{p=\hat{p}} = -\mu \lambda^2 e^{-\lambda F(\hat{p})} \left[ \frac{dF(p)}{dp} \Big|_{p=\hat{p}} + (\hat{p} - c) \frac{d^2F(p)}{dp^2} \Big|_{p=\hat{p}} \right] = -\mu \lambda^2 e^{-\lambda F(\hat{p})} \frac{c/\hat{p}^2}{\ln(1/\sigma)} < 0.$$

Hence,  $\Pi(p)$  is strictly quasi-concave in  $p \in [0, r]$ . This completes the proof of Lemma 3.

## Proof of Lemma 4

Begin with the optimality of  $S' \leq r$ . Suppose  $S' > r$ . Then the corresponding value function, denoted as  $W = W(S', s')$ , can be written as

$$W = -\beta + \int_{\omega}^{T'} \Pi(re^{-g(t-\omega)})e^{-\rho t} dt + We^{-\rho T'},$$

where  $\Pi(re^{-g(t-\omega)}) = \mu\lambda e^{-\lambda F(re^{-g(t-\omega)})}(re^{-g(t-\omega)} - c)$  and  $\omega \equiv \ln(S'/r)/g$  represents a time interval for no sales. Notice any sales will be made only when  $\omega < T' = \ln(S'/s')/g$ . Now, consider a deviation by the firm who adjusts its price to  $r$  instead of  $S'$  at some point in time and then resumes the original  $(S', s')$  policy (described above) after a time  $T' - \omega$ . This deviation yields a payoff denoted as  $X = X(r, s')$  given by:

$$X = -\beta + \int_0^{T'-\omega} \Pi(re^{-gt})e^{-\rho t} dt + We^{-\rho(T'-\omega)} = -\beta + \left[ \int_{\omega}^{T'} \Pi(re^{-g(t-\omega)})e^{-\rho t} dt + We^{-\rho T'} \right] e^{\rho\omega}.$$

The latter expression implies  $X + \beta = (W + \beta)e^{\rho\omega}$  and hence  $X > W$ , which shows that  $S' > r$  is never optimal.

Given  $S' \leq r$  and Lemma 3, the optimality and uniqueness of the  $(S', s')$  price policy can be shown by using the procedure developed by Sheshinski and Weiss [1977], that is, by rewriting  $V = V(S', s')$  as follows:

$$V = \frac{1}{g(S' \frac{\rho}{g} - s' \frac{\rho}{g})} \left[ -\beta g S' \frac{\rho}{g} + \int_{s'}^{S'} \Pi(z) z^{-1 + \frac{\rho}{g}} dz \right],$$

where  $z \equiv S'e^{-gt}$  and  $\Pi(z) = \mu\lambda e^{-\lambda F(z)}(z - c)$ . This expression allows one to obtain the first order conditions (4), (5). Further, the second-order conditions are satisfied due to the strictly quasi-concavity of  $\Pi(p)$  as shown in Lemma 3. Hence, the optimal solution  $(S', s')$  is unique. This completes the proof of Lemma 4.

## Proof of Theorem 1

The analysis so far has established that equations (6), (7), (8), (9) describe the necessary and sufficient conditions for an F-type equilibrium given  $\{\gamma, \beta, h, c, \lambda, g, \rho\} \in \Omega_F$ . All that remains here is to find a solution  $\mu, r, S, s$  to these conditions. The proof takes two steps. Assuming  $r > S$ , Step 1 establishes a solution  $\mu \geq 0$  to equations (8), (9) for  $\sigma \equiv \frac{s}{S} \in [\sigma_*, 1]$ , where  $\sigma_* \in (0, 1)$ . Let  $\mu = \mu(\sigma)$  denote this solution. Using  $\mu = \mu(\sigma)$  obtained and equation (8), Step 2 solves the value function  $V = V(S, s)$  for  $\sigma \in [0, 1]$ . With a slight abuse of notation, let  $V(\mu(\sigma), \sigma)$  denote this solution. An equilibrium is then identified by noting that equation (6) requires that  $\sigma$  must satisfy the fixed point condition,

$$V(\mu(\sigma), \sigma) = \frac{h}{\rho}, \tag{14}$$

where  $h/\rho$  is a positive constant. Step 2 then shows there exists a solution  $\sigma \in (\sigma_*, 1)$  to this condition (14). Applying  $\sigma$  obtained to  $\mu = \mu(\sigma)$  identifies an equilibrium value of  $\mu \in (0, \infty)$ , which further identifies an equilibrium value of  $S > s > c$ . Applying these values to equation (7), an equilibrium value for  $r > S$  can be identified by choosing a set of parameters that satisfies  $\gamma > S(1 - L(1/\sigma))$ , where  $L(1/\sigma) \equiv \frac{E_J[p]}{S} = \frac{\lambda(\sigma - e^{-\lambda})}{(1 - e^{-\lambda})(\lambda - \ln(1/\sigma))}$ . Note  $\Omega_F$  is non-empty since the RHS of this inequality is bounded. By construction, this solution satisfies equations (6), (7), (8), (9) and  $r > S$ , and hence describes an F-type equilibrium.

**Step 1** Equations (8), (9) solve for  $\mu = \mu(\sigma) \geq 0$  that is continuous and strictly increasing in  $\sigma \in (0, 1)$  and satisfies:  $\mu(\sigma) = 0$  when  $\sigma = \sigma_* \equiv \frac{e^{-\lambda}h}{h + \rho\beta}$  and  $\mu(\sigma) \rightarrow \infty$  as  $\sigma \rightarrow 1$ .

**Proof of Step 1.** The claim is immediately shown from equation (10), which is constructed by equations (8), (9). This completes the proof of Step 1.

**Step 2** There exists a solution  $\sigma \in (\sigma_*, 1)$  to the fixed point condition (14).

**Proof of Step 2.** The value function  $V = V(S, s)$  given in the text can be written as follows:

$$V = \frac{-\beta + \mu\lambda \ln(1/\sigma) (S\Theta_s(\sigma) - c\Theta_c(\sigma))}{1 - \sigma^{\frac{\rho}{g}}} \quad (15)$$

where

$$\Theta_s(\sigma) \equiv \frac{\sigma^{\frac{\rho+g}{g}} - e^{-\lambda}}{\lambda g - (\rho + g) \ln(1/\sigma)}; \quad \Theta_c(\sigma) \equiv \frac{\sigma^{\frac{\rho}{g}} - e^{-\lambda}}{\lambda g - \rho \ln(1/\sigma)}.$$

Notice  $\Theta_s(\sigma), \Theta_c(\sigma) > 0$  for any  $\sigma \in (0, 1)$  and  $\lim_{\sigma \rightarrow 1} \Theta_s(\sigma) = \lim_{\sigma \rightarrow 1} \Theta_c(\sigma)$ . Substituting out  $S$  in  $V$  using equation (8) and  $\sigma \equiv s/S$ , and applying the solution  $\mu = \mu(\sigma)$  established in Step 1, the fixed point condition (14) can be written as follows:

$$\Lambda_F(\sigma) \equiv \frac{\beta \left(1 + \frac{\rho \ln(1/\sigma) \Theta_s(\sigma)}{\sigma - e^{-\lambda}}\right) + \left(1 - \sigma^{\frac{\rho}{g}}\right) h/\rho}{\frac{1 - e^{-\lambda}}{\sigma - e^{-\lambda}} \Theta_s(\sigma) - \Theta_c(\sigma)} = c\mu(\sigma)\lambda \ln(1/\sigma) \quad (16)$$

$\Lambda_F(\sigma)$  is continuous in  $\sigma \in (0, 1)$ . When  $\sigma = \sigma_*$ , note the numerator of  $\Lambda_F(\sigma_*)$  becomes:

$$\frac{h}{\rho} \left( \frac{1}{\eta} - e^{-\lambda \frac{\rho}{g} \eta^{\frac{\rho}{g}}} \right) \left( 1 - \frac{\lambda + \ln(1/\eta)}{\lambda + \frac{\rho+g}{\rho} \ln(1/\eta)} \right) > 0$$

where  $\eta \equiv \frac{h}{h + \rho\beta} < 1$ . Since the denominator of  $\Lambda_F(\sigma_*)$  is negative, it follows that:  $\Lambda_F(\sigma_*) < 0 = c\mu(\sigma_*)\lambda \ln(1/\sigma_*)$ . Further, applying the l' Hospital's rule once yields:

$$\lim_{\sigma \rightarrow 1} c\mu(\sigma)\lambda \ln(1/\sigma) = c\lambda \lim_{\sigma \rightarrow 1} \frac{d\mu}{d\sigma} \sigma (\ln(1/\sigma))^2 = e^\lambda (h + \rho\beta) - h \quad \text{as } \sigma \rightarrow 1$$

where the last equality can be obtained by using equation (10). This leads to:  $c\mu(\sigma)\lambda \ln(1/\sigma) < \Lambda_F(\sigma) \rightarrow +\infty$  as  $\sigma \rightarrow 1$ . Therefore, there exists  $\sigma \in (\sigma_*, 1)$  that satisfies equation (16). This completes the proof of Step 2.

## Proof of Theorem 2

First of all, Theorem 1 implies that there exists a set of parameters such that  $\gamma = S(1 - L(1/\sigma))$  holds, so  $\Omega_B$  is non empty. Hence, the proof is to find a solution  $\mu, S, s$  to equations (6), (7),(9) that satisfy  $S = r$ . The proof follows from a similar two-step approach as before. Given  $S = r$ , Step 1 establishes a solution  $\mu \geq 0$  to equations (7), (9) for  $\sigma \equiv \frac{s}{S} \in [\sigma_+, 1]$ , where  $\sigma_+ \in (0, 1)$ . Viewing equation (6) as a fixed point requirement, Step 2 finds a fixed point  $\sigma \in (\sigma_+, 1)$  by using the solution obtained in Step 1 and equation (7). This solution further identifies an equilibrium solution of  $\mu \in (0, \infty)$  and  $S = r > s > c$ . Hence, Step 1 and 2 establish that there exists an equilibrium solution  $S = r, s, \mu$  that satisfies equations (6), (7), (9) and by construction, this solution describes a B-type equilibrium.

**Step 1** Equations (7), (9) solve for  $\mu = \mu(\sigma) \geq 0$  that is continuous and strictly decreasing in  $\sigma \in (0, 1)$  and satisfies:  $\mu(\sigma) \rightarrow \infty$  as  $\sigma \rightarrow \sigma_+ \in (0, 1)$  and  $\mu(\sigma) \rightarrow 0$  as  $\sigma \rightarrow 1$ , where  $\sigma_+$  is a unique solution to  $\gamma\sigma_+ = c(1 - L(1/\sigma_+))$ .

**Proof of Step 1.** The claim is immediately shown from equation (12), which is constructed by equations (7), (9). This completes the proof of Step 1.

**Step 2** Given  $\mu = \mu(\sigma)$  established in Step 1, there exists a solution  $\sigma \in (\sigma_+, 1)$  that satisfies equations (6),(7),(9).

**Proof of Step 2.** Substituting out  $S$  in  $V$  from equation (15) by using equation (7), and applying the solution  $\mu = \mu(\sigma)$  established in Step 1, the fixed point requirement  $V = h/\rho$  can be written as follows:

$$\Lambda_B(\sigma) \equiv \frac{\beta + \left(1 - \sigma^{\frac{\rho}{g}}\right) h/\rho}{\ln(1/\sigma) \left( \frac{\gamma}{1-L(1/\sigma)} \Theta_s(\sigma) - c\Theta_c(\sigma) \right)} = \mu(\sigma)\lambda. \quad (17)$$

$\Lambda_B(\sigma)$  is continuous in  $\sigma \in (0, 1)$ . It is immediate that:  $\Lambda_B(\sigma) < \mu(\sigma)\lambda$  as  $\sigma \rightarrow \sigma_+$ . Further, applying the l' Hospital's rule once yields

$$\lim_{\sigma \rightarrow 1} \frac{\gamma \ln(1/\sigma)}{1 - L(1/\sigma)} = \frac{\lambda\gamma(1 - e^{-\lambda})}{\lambda - 1 + e^{-\lambda}} \quad \text{as } \sigma \rightarrow 1$$

which leads to  $\lim_{\sigma \rightarrow 1} \Lambda_B(\sigma) = \frac{\beta(\lambda - 1 + e^{-\lambda})}{\lambda\gamma(1 - e^{-\lambda})} > 0 = \lim_{\sigma \rightarrow 1} \mu(\sigma)\lambda$  as  $\sigma \rightarrow 1$ . Therefore, there exists  $\sigma \in (\sigma_+, 1)$  that satisfies equation (17). This completes the proof of Step 2.

## Proof of Proposition 1

Suppose that the equilibrium in the B-type as  $\lambda \rightarrow \infty$ . There are two cases. Suppose first  $\sigma < 1$  as  $\lambda \rightarrow \infty$ . Then, noting  $\lim_{\lambda \rightarrow \infty} \Omega_s(\sigma) = \lim_{\lambda \rightarrow \infty} \Omega_c(\sigma) = 0$  and  $\lim_{\lambda \rightarrow \infty} L(1/\sigma) = \sigma$  as  $\lambda \rightarrow \infty$  for any  $\sigma \in [0, 1]$  yields:  $\sigma < 1$  as  $\lambda \rightarrow \infty$  implies  $\Lambda_B(\sigma) \rightarrow \infty$  as  $\lambda \rightarrow \infty$ . However, equations (7), (9) imply  $\mu(\sigma)\lambda < \infty$  as  $\lambda \rightarrow \infty$ , hence  $\sigma < 1$  as  $\lambda \rightarrow \infty$  contradicts to the fixed point condition (17). Suppose next  $\lim_{\lambda \rightarrow \infty} \sigma = 1$ . In this case again, noting  $\Lambda_B(\sigma) \rightarrow \infty > \mu(\sigma)\lambda$  as  $\lambda \rightarrow \infty$  yields a contradiction to (17). Hence, the equilibrium is not the B-type as  $\lambda \rightarrow \infty$ .

Suppose next that the equilibrium in the F-type as  $\lambda \rightarrow \infty$ . Then, observe that  $\lim_{\lambda \rightarrow \infty} \sigma = 1$  as  $\lambda \rightarrow \infty$  implies:

$$e^{-\lambda}(1 - \sigma) \left( \frac{1 - \sigma}{\sigma - e^{-\lambda}} \Omega_s(\sigma) - \Omega_c(\sigma) \right) \Lambda_F(\sigma) = e^{-\lambda}(1 - \sigma) \left\{ \beta \left( 1 + \frac{\rho \ln(1/\sigma) \Omega_s(\sigma)}{\sigma - e^{-\lambda}} \right) + \left( 1 - \sigma^{\frac{\rho}{g}} \right) \frac{h}{\rho} \right\} \rightarrow 0$$

as  $\lambda \rightarrow \infty$  and

$$\begin{aligned} & e^{-\lambda}(1 - \sigma) \left( \frac{1 - \sigma}{\sigma - e^{-\lambda}} \Omega_s(\sigma) - \Omega_c(\sigma) \right) c\mu(\sigma)\lambda \ln(1/\sigma) \\ &= e^{-\lambda}(1 - \sigma) \left( \frac{1 - \sigma}{\sigma - e^{-\lambda}} \Omega_s(\sigma) - \Omega_c(\sigma) \right) c(\sigma(h + \rho\beta) - he^{-\lambda}) \ln(1/\sigma) \rightarrow 0 \end{aligned}$$

as  $\lambda \rightarrow \infty$ , and hence satisfies the fixed point condition (16). The rest of the statements in proposition are immediate from applying  $\lim_{\lambda \rightarrow \infty} \sigma = 1$  as  $\lambda \rightarrow \infty$  to equations (8),(9). This completes the proof of Proposition 1.

## Proof of Proposition 2

Noting  $\Omega_s(\sigma) = \Omega_c(\sigma)$  as  $g \rightarrow 0$  for any  $\sigma \in [\sigma_*, 1]$  implies that  $\lim_{g \rightarrow 0} \sigma = 1$  as  $g \rightarrow 0$  satisfies the fixed point condition (16). Applying  $\sigma \rightarrow 1$  as  $g \rightarrow 0$  yields the rest of the statements in the first claim in proposition. To prove the second claim, suppose first that the equilibrium is the F-type as  $g \rightarrow \infty$ . Note  $\lim_{g \rightarrow \infty} \Omega_s(\sigma) = \lim_{g \rightarrow \infty} \Omega_c(\sigma) = 0$  as  $g \rightarrow \infty$  for any  $\sigma \in [\sigma_*, 1]$ , and hence  $\Lambda_F(\sigma) \rightarrow \infty$  as  $g \rightarrow \infty$ . As shown in Step 2 in the proof of Theorem 1, remember that the R.H.S. of the fixed point condition (16) does not depend on  $g$  and satisfies  $c\mu(\sigma)\lambda \ln(1/\sigma) < \infty$  for any  $\sigma \in [\sigma_*, 1]$ . This leads to the desired contradiction, hence the equilibrium is not the F-type as  $g \rightarrow \infty$ .

Suppose next the equilibrium is the B-type as  $g \rightarrow \infty$ . Then, it is immediate that  $\lim_{g \rightarrow \infty} \sigma = \sigma_+$  as  $g \rightarrow \infty$  satisfies the fixed point condition (17). Applying  $\sigma \rightarrow \sigma_+$  as  $g \rightarrow \infty$  to equations (7),(9) yields the rest of the statements in the second claim. This completes the proof of Proposition 2.

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