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# TRADE AND GROWTH: A SIMPLE MODEL WITH NOT-SO-SIMPLE IMPLICATIONS* 

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#### Abstract

We present a simple dynamic model of international trade and growth. Our equations linking exogenous and endogenous variables do not resemble those estimated by the empirical literature: Ours are not linear, despite the fact that our model is linear, they do not include variables used in this literature and include variables that have never been used in this literature.


Keywords: Trade, Growth, Liberalization
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## 1. INTRODUCTION

The question of the link between International Trade (IT) and growth is one of the most popular issues outside our profession. What do we have to say about it? On the one hand we have general theoretical results on the performance of open economies (Bardham (1965), Oniki and Uzawa (1965), Stiglitz (1970), Johnson, (1971), Deardorff (1973), Smith, (1977), Baxter (1992) and Chen (1992)). These papers provide conditions for the convergence to a steady state and study its properties. ${ }^{1}$ On the other hand, the empirical literature presents conflicting views on this relationship: Some authors (e.g. Sala-i-Martin (1997) and Frankel and Romer (1999)) maintain that a positive relationship between these two variables shows up in the data. Others (e.g. Levine and Renelt (1992) and Rodriguez and Rodrik (2000)) are skeptical about it. ${ }^{2}$ A characteristic of the empirical literature is that the equations to be estimated are not based on a formal model.

This paper presents a simple model of trade and growth that allows direct computation of equilibrium in order to establish a link between theory and empirical studies (see Ventura, (1997) for a similar attempt). Our model is a blend

[^1]of Ricardian ideas where trade is driven by differences in technology and there is only one factor of production, and the Harrod/Domar/Rebelo (1991) model of endogenous growth, where all relationships between variables are linear. In order to concentrate on the fundamentals the model leaves aside important topics like oligopoly, asymmetric information, externalities and any possible "friction". Innovation, a topic of immense empirical relevance, is also disregarded, see Grossman and Helpman (1991).

Our model yields predictions that look very different from those made by the empirical literature. Firstly, despite the fact that our model is linear, relationships among exogenous and endogenous variables are not linear: They are piecewise multiplicative with each specialization pattern yielding a different equation. Secondly, some variables used by the empirical literature do not play a role in our case, like the share of IT in GDP. Others play a role that is different from the one assumed by this literature like the investment share (in some cases), the initial level of income and the number of years that the economy has been open. Trade distortions -that in our case take the extreme form of autarky- affect growth in a very complicated way. Thirdly, variables that are important in our model have never been used by the applied literature, such as comparative advantage, specialization patterns and saving habits and technology of partner
countries. Moreover, once the mechanism that operates behind these variables is understood, it is difficult to think of a model where they do not play a role.

What do we learn from here? On the negative side the main lesson from our paper is that the simplest model of IT and growth produces results that are far from the "common sense" equations that have been estimated. On the positive side, our model provides a deeper understanding of the role of variables in the mechanism of trade and growth, some cautions about those already used, and a fresh set of variables to be used. In any case, more work is necessary to produce a workable theoretical model and to test the implications.

The rest of the paper goes as follows: Section 2 presents the model, Section 3 discusses the empirical implications and Section 4 concludes. An Appendix extends the model to international capital mobility and two factors.

## 2. THE MODEL

We assume two countries, $A$ and $B$, and two goods: a consumption good and a capital good. Capital does not depreciate. The aggregate stock of capital in country $i=A, B$ is denoted by $K^{i}$. The quantity of capital used in the production of the consumption (resp. capital) good is denoted by $K_{C}^{i}\left(\right.$ resp. $\left.K_{I}^{i}\right) i=A, B$.

We assume that capital is mobile between sectors, thus

$$
K^{i}=K_{C}^{i}+K_{I}^{i}, i=A, B
$$

Capital is assumed to be immobile between countries. In an Appendix we prove that under some additional assumptions the main conclusions of this paper hold under capital mobility. We assume that output is produced by means of capital alone. We may assume a second factor, labor, whose supply (including emigrants) is arbitrarily large. Then, either wages are at subsistence level (normalized to zero), or, in each country, capital-labor ratios are identical in each sector. The later interpretation is developed in an Appendix.

We assume constant returns to scale. The production function of consumption (resp. capital) goods in country $i=A, B$ is

$$
C_{P}^{i}=\beta_{C}^{i} K_{C}^{i} \quad\left(\text { resp. } \quad I_{P}^{i}=\beta_{I}^{i} K_{I}^{i}\right)
$$

where $C_{P}^{i}$ (resp. $I_{P}^{i}$ ) stands for the output of consumption (resp. capital) good and $\beta_{I}^{i}$ and $\beta_{C}^{i}$ are the average productivity of capital in country $i$ in the production of capital $\left(\beta_{I}^{i}\right)$ and consumption $\left(\beta_{C}^{i}\right)$ goods.

For international trade to be mutually advantageous we assume that

$$
\frac{\beta_{I}^{B}}{\beta_{C}^{B}}<\frac{\beta_{I}^{A}}{\beta_{C}^{A}},
$$

i.e., country $A$ (resp. $B$ ) has a comparative advantage in the production of the capital (resp. consumption) good.

Let $P^{i}$ be price of the consumption good in country $i$. The price of the output of the capital good is the numeraire. Let $r^{i}$ be the rental price of capital in country $i$. Since production takes place under constant returns to scale, the supply side is characterized by the following equations:

$$
\begin{gathered}
P^{i} C_{P}^{i} \leq r^{i} K_{C}^{i} \quad \text { or } \quad P^{i} \leq r^{i} \beta_{C}^{i} \quad \text { and if strict inequality holds } C_{P}^{i}=0 \\
I_{P}^{i} \leq r^{i} K_{I}^{i} \quad \text { or } \quad 1 \leq r^{i} \beta_{I}^{i} \quad \text { and if strict inequality holds } I_{P}^{i}=0 .
\end{gathered}
$$

Let $Y^{i}$ be national income in country $i$. Clearly, $Y^{i}=r^{i} K^{i}$.
The demand for consumption goods in country $i$, denoted by $C_{D}^{i}$, is assumed to be of Keynesian type, i.e., consumption is linear on real income,

$$
C_{D}^{i}=\frac{c^{i} Y^{i}}{P^{i}} \quad 0<c^{i}<1 \quad i=A, B
$$

In the conclusions we discuss how to obtain this function from utility maximization. Demand for investment in each country $\left(I_{D}^{i}\right)$ is assumed to be such that capital stock is fully utilized. Finally let $g^{i}$ be the rate of growth of capital in country $i$. As we will see all the relevant variables depend on the capital stock. Thus the rate of growth of any variable can be easily calculated from $g^{i}$.

For the time being let us consider that capital is given. In this case the model is a standard Ricardian model so we omit the calculations. We first solve the model for the case of autarky and then for all the possibilities opened up by free trade. The dynamic paths of the economy when investment accrues the capital stock will be considered later on.

### 2.1. Autarky.

In this case we do not have to distinguish between production and demand so we drop the corresponding subscript for $C$ and $I$. Equilibrium values of the variables under autarky are denoted with the superscript $*$. For $i=A, B$ we obtain,

$$
\begin{gather*}
I^{i *}=\beta_{I}^{i}\left(1-c^{i}\right) K^{i} . \quad C^{i *}=c^{i} \beta_{C}^{i} K^{i} . \quad r^{i *}=\beta_{I}^{i} \\
g^{i *}=\beta_{I}^{i}\left(1-c^{i}\right) . \quad P^{i *}=\frac{\beta_{I}^{i}}{\beta_{C}^{i}} \tag{2.1}
\end{gather*}
$$

Notice that under autarky the rate of growth equals the propensity to save $\left(1-c^{i}\right)$ divided by the capital-output ratio $1 / \beta_{I}^{i}$ of the capital goods sector. This is akin to the (warranted) rate of growth in the Harrod-Domar model.

Suppose now that both countries open up to international trade. Now we delete the superscript of $P$, since now domestic prices are international prices. We also calculate a new variable, $i^{i} \equiv \frac{\text { Value of Trade }}{\text { Income of } i}$ that measures the relative importance of trade in national income of country $i$. Given our assumption about relative efficiency of both countries, we have three possible specialization patterns:

### 2.2. Country $A$ produces the capital good and country $B$ produces the consumption good.

In this case, we denote equilibrium variables by an upper bar.

$$
\begin{gather*}
\bar{I}_{D}^{A}=\beta_{I}^{A}\left(1-c^{A}\right) K^{A} . \quad \bar{C}_{D}^{A}=\left(1-c^{B}\right) \beta_{C}^{B} K^{B} . \quad \bar{r}^{A}=\beta_{I}^{A} . \quad \bar{\imath}^{A}=c^{A} . \\
\bar{I}_{D}^{B}=\beta_{I}^{A} c^{A} K^{A} . \quad \bar{C}_{D}^{B}=c^{B} \beta_{C}^{B} K^{B} . \quad \bar{r}^{B}=\frac{\beta_{I}^{A} c^{A} K^{A}}{\left(1-c^{B}\right) K^{B}} . \quad \bar{\imath}^{B}=1-c^{B} . \\
\bar{g}^{A}=\beta_{I}^{A}\left(1-c^{A}\right) . \quad \bar{g}^{B}=\frac{\beta_{I}^{A} c^{A} K^{A}}{K^{B}} . \quad \bar{P}=\frac{\beta_{I}^{A} c^{A} K^{A}}{\left(1-c^{B}\right) K^{B} \beta_{C}^{B}} . \tag{2.2}
\end{gather*}
$$

Supply equations boil down to

$$
\frac{\left(1-c^{B}\right) \beta_{I}^{B}}{c^{A} \beta_{I}^{A}} \leq \frac{K^{A}}{K^{B}} \leq \frac{\left(1-c^{B}\right) \beta_{C}^{B}}{c^{A} \beta_{C}^{A}} .
$$

We will call this case Complete Specialization.

### 2.3. Country $A$ produces both goods and country $B$ specializes in the

 consumption good.In this case, we denote equilibrium variables by a hat.

$$
\begin{gather*}
\hat{I}_{D}^{A}=\beta_{I}^{A}\left(1-c^{A}\right) K^{A} . \quad \hat{C}_{D}^{A}=\frac{c^{A} K^{A}}{\beta_{C}^{A}} . \quad \hat{r}^{A}=\beta_{I}^{A} . \quad \hat{\imath}^{A}=\frac{\beta_{C}^{B}\left(1-c^{B}\right) K^{B}}{\beta_{C}^{A} K^{A}} . \\
\hat{I}_{D}^{B}=\frac{\beta_{C}^{B}\left(1-c^{B}\right) K^{B} \beta_{I}^{A}}{\beta_{C}^{A}} . \quad \hat{C}_{D}^{B}=c^{B} K^{B} \beta_{C}^{B} . \quad \hat{r}^{B}=\frac{\beta_{I}^{A} \beta_{C}^{B}}{\beta_{C}^{A}} . \quad \hat{\imath}^{B}=1-c^{B} . \\
\hat{g}^{A}=\beta_{I}^{A}\left(1-c^{A}\right) . \quad \hat{g}^{B}=\frac{\beta_{C}^{B} \beta_{I}^{A}\left(1-c^{B}\right)}{\beta_{C}^{A}} . \quad \hat{P}=\frac{\beta_{I}^{A}}{\beta_{C}^{A}} . \tag{2.3}
\end{gather*}
$$

Supply equations boil down to

$$
\frac{K^{A}}{K^{B}}>\frac{\left(1-c^{B}\right) \beta_{C}^{B}}{c^{A} \beta_{C}^{A}} \quad\left(\text { or } c^{A}>\hat{\imath}^{A}\right)
$$

We will call this case Country $B$ Specializes. It occurs for large values of $\frac{K^{A}}{K^{B}}$.

### 2.4. Country $A$ specializes in the capital good and country $B$ produces

 both goods.In this case we denote equilibrium variables by a tilde.

$$
\begin{gather*}
\tilde{I}_{D}^{A}=\beta_{I}^{A} K^{A}\left(1-c^{A}\right) . \quad \tilde{C}_{D}^{A}=\frac{c^{A} \beta_{I}^{A} K^{A} \beta_{C}^{B}}{\beta_{I}^{B}} . \quad \tilde{r}^{A}=\beta_{I}^{A} . \quad \tilde{\imath}^{A}=c^{A} . \\
\tilde{I}_{D}^{B}=\beta_{I}^{B}\left(1-c^{B}\right) . \quad \tilde{C}_{D}^{B}=c^{B} \beta_{C}^{B} K^{B} . \quad \tilde{r}^{B}=\beta_{I}^{B} . \quad \tilde{\imath}^{B}=\frac{c^{A} \beta_{I}^{A} K^{A}}{\beta_{I}^{B} K^{B}} . \\
\tilde{g}^{A}=\beta_{I}^{A}\left(1-c^{A}\right) . \quad \tilde{g}^{B}=\beta_{I}^{B}\left(1-c^{B}\right) . \quad \tilde{P}=\frac{\beta_{I}^{B}}{\beta_{C}^{B}} . \tag{2.4}
\end{gather*}
$$

Supply equations boil down to:

$$
\frac{\left(1-c^{B}\right) \beta_{I}^{B}}{c^{A} \beta_{I}^{A}}>\frac{K^{A}}{K^{B}} \quad\left(\text { or } 1-c^{B}>\tilde{\imath}^{B}\right) .
$$

We will call this case Country A Specializes. It occurs for small values of $\frac{K^{A}}{K^{B}}$. It is the only case in which growth rates of both countries equal those under autarky.

The previous results can be summarized as follows:

Proposition 1. a) $g^{A}$ is constant. b) $g^{B}$ is linearly increasing on $\frac{K^{A}}{K^{B}}$ under complete specialization and constant otherwise. It equals the autarkic growth rate when Country A Specializes and is larger than the autarkic rate in the other cases.

We now turn our attention to the consequences of capital accumulation under free trade. A good intuition about the shape of dynamic paths can be obtained by plotting both growth rates against $\frac{K^{A}}{K^{B}}$. We have three possibilities: That $g^{A}$ and $g^{B}$ cross, that $g^{A}>g^{B}$ everywhere or that $g^{B}>g^{A}$ everywhere. For the first possibility to occur since $g^{A}$ is constant and $g^{B}$ is increasing on $\frac{K^{A}}{K^{B}}$, it must be that $\tilde{g}^{A}>\tilde{g}^{B}$ and $\hat{g}^{A}<\hat{g}^{B}$. The second possibility arises when $\tilde{g}^{A}>\tilde{g}^{B}$ and $\hat{g}^{A}>\hat{g}^{B}$. The last possibility requires that $\tilde{g}^{A}<\tilde{g}^{B}$. Working out these conditions we obtain the following inequalities: For $g^{A}$ and $g^{B}$ to cross:

$$
\begin{align*}
& \frac{1-c^{A}}{1-c^{B}}>\frac{\beta_{I}^{B}}{\beta_{I}^{A}}  \tag{2.5}\\
& \frac{1-c^{A}}{1-c^{B}}<\frac{\beta_{C}^{B}}{\beta_{C}^{A}} \tag{2.6}
\end{align*}
$$

For $g^{A}>g^{B}$ everywhere, (2.5) above must hold and in addition,

$$
\begin{equation*}
\frac{1-c^{A}}{1-c^{B}}>\frac{\beta_{C}^{B}}{\beta_{C}^{A}} \tag{2.7}
\end{equation*}
$$

And for $g^{B}>g^{A}$ everywhere:

$$
\begin{equation*}
\frac{1-c^{A}}{1-c^{B}}<\frac{\beta_{I}^{B}}{\beta_{I}^{A}} \tag{2.8}
\end{equation*}
$$

Inequality (2.5) (resp. (2.8)) is equivalent to assuming that under autarky, country $A$ grows faster (resp. slower) than country $B$. Inequality (2.6) arises if either, country $A$ does not save much relative to country $B$ or if country $B$ is very productive relative to country $A$ in the consumption sector, with the reverse interpretation for (2.7). The next proposition studies the dynamic trajectories and show that they always end in one specialization pattern and that this pattern does not depend on the initial condition. Thus our mode1 is as well-behaved as it can possibly be.

Proposition 2. Starting from any initial value of $\frac{K^{A}}{K^{B}}$ :
a) Under (2.5) and (2.6), $\frac{K^{A}}{K^{B}}$ converges to Complete Specialization. Rates of growth of both countries converge to $\beta_{I}^{A}\left(1-c^{A}\right)$.
b) Under (2.5) and (2.7), $\frac{K^{A}}{K^{B}}$ converges to Country B Specializes. The difference between growth rates does not increase but it does not vanish either.
c) Under (2.8) $\frac{K^{A}}{K^{B}}$ converges to Country A Specializes. The difference between the growth rates does increase but it does not vanish either.

Proof. a) Suppose that $\frac{K^{A}}{K^{B}}$ is such that the country $B$ specializes. But there,

$$
\hat{g}^{A}=\beta_{I}^{A}\left(1-c^{A}\right)<\hat{g}^{B}=\frac{\beta_{C}^{B} \beta_{I}^{A}\left(1-c^{B}\right)}{\beta_{C}^{A}} .
$$

Thus $\frac{K^{A}}{K^{B}}$ decreases and we end up in the case of complete specialization.
If $\frac{K^{A}}{K^{B}}$ is such that country $A$ specializes, $\tilde{g}^{A}=\beta_{I}^{A}\left(1-c^{A}\right)>\tilde{g}^{B}=\beta_{I}^{B}\left(1-c^{B}\right)$.
Thus, $\frac{K^{A}}{K^{B}}$ grows and we will end up in the case of complete specialization.
If $\frac{K^{A}}{K^{B}}$ is such both countries specialize and $\bar{g}^{A}<\bar{g}^{B}$ the rate of growth of country $B\left(=\frac{\beta_{I}^{A} c^{A} K^{A}}{K^{B}}\right)$ decreases up to the point in which it equals the rate of growth of country $A$. If $\bar{g}^{A}>\bar{g}^{B}$, the opposite occurs. Thus,

$$
\bar{g}^{A}=\beta_{I}^{A}\left(1-c^{A}\right)=\bar{g}^{B}=\frac{\beta_{I}^{A} c^{A} K^{A}}{K^{B}} \text { and } \frac{K^{A}}{K^{B}}=\frac{1-c^{A}}{c^{A}} .
$$

b) Notice that here, $g^{A}>g^{B}$. Thus $\frac{K^{A}}{K^{B}}$ increases with time and, no matter what the initial $\frac{K^{A}}{K^{B}}$ is, we end up with country $B$ specialized. In this case the rate of growth of capital of country $B$ is the largest among all equilibrium rates.
c) Virtually identical to b) above.

Notice that the following trajectories are possible:
Under (2.5)-(2.6): $A$ Specializes $\Rightarrow$ Complete Specialization.
Under (2.5)-(2.6): $B$ Specializes $\Rightarrow$ Complete Specialization.
Under (2.5)-(2.7): $A$ Specializes $\Rightarrow$ Complete Specialization $\Rightarrow B$ Specializes.
Under (2.8): $B$ Specializes. $\Rightarrow$ Complete Specialization $\Rightarrow A$ specializes.
The first and the third possibilities generate a growth rate for country $B$ that
is constant, then increasing and then constant and the second and the fourth possibilities generate a growth rate for $B$ that is constant, then decreasing and then constant. Any part of these trajectories is possible too, i.e. if we start under complete specialization and (2.8) holds the trajectory will be Complete Specialization $\Rightarrow B$ Specializes, etc.

## 3. EMPIRICAL IMPLICATIONS

In this section we compare the implications of our simple model with the set up employed by the empirical literature.

1: Functional forms. There is not a single functional form that relates exogenous variables to growth rates: Country $A$ always grows at the same rate, under autarky or under any specialization pattern. In country $B$, functional forms depend on specialization patterns and trading opportunities. And when functional forms are stable, switching from autarky to free trade has no effect on growth.

Equations are not linear despite the linearity assumed throughout the model.
They are multiplicative so it they are captured better by using logarithms.
2: Variables used. The following variables have been linked to growth by the empirical literature:
i): Share of IT in GDP.
ii): Investment share.
iii): Exchange Rate and/or Trade distortions.
iv): Initial level of Income.
v): Number of Years that the economy has been open.

We see immediately the problem with i): The share of IT in GDP is not an independent variable: In other words, both the share of IT and the growth rate depend on the fundamentals of the economy but there is no causation at all between them. If this variable is used as a proxy, notice that under complete specialization or if $A$ specializes $g^{A}=\beta_{I}^{A}\left(1-i^{A}\right)$-so we get a negative relationship!and if $B$ specializes $\hat{g}^{A}$ is independent of $i^{A}$ and $i^{B}$. In the case of $B, \bar{g}^{B}=\frac{\beta_{I}^{A} i^{A} K^{A}}{K^{B}}$, $\hat{g}^{B}=\frac{\beta_{C}^{B} \beta_{I}^{A} \hat{i}^{B}}{\beta_{C}^{A}}$ and $\tilde{g}^{B}$ is independent of $i^{A}$ and $i^{B}$. Thus, our model does not support the view that to introduce the share of IT in GDP in the equations to be estimated is a good idea.

The problem with ii), the investment share -which equals $1-c^{i}$ - is that under complete specialization $B^{\prime} s$ rate of growth does not depend on $1-c^{B}$, but positively on $c^{A}$ ! Thus if we have in our sample many countries completely specialized in consumption goods, the relationship between $g^{i}$ and $1-c^{i}$ will be blurred.
iii) measures how changes in trade barriers affect growth. In our model these changes take the simple form of switching from autarky to free trade. Despite
this simplification this effect can not be captured by an additive dummy variable: Growth in country $A$ is unaffected and, in country $B$, the following possibilities may arise after a Trade Liberalization (TL):

- No effect at all. This occurs if after TL $A$ specializes and (2.8) holds.
- Level effect only: This occurs if after a TL $A$ specializes and (2.5)-(2.7) hold. In this case $g^{B}$ increases and remains constant thereafter.
- No effect in the short run. Acceleration until a certain point later on. This occurs if after a TL $A$ specializes and (2.5) holds. Under (2.6) both growth rates converge and under (2.7) they just approach each other.
- Acceleration in the short run and constancy thereafter. This occurs if after a TL both countries specialize and (2.5) holds. Under (2.6) both growth rates converge and under (2.7) they just approach each other.
- Level effect, deceleration and growth like in autarky later on. This occurs under (2.8) if after a TL either $B$ specializes or both countries specialize. In the first case the new growth rate is maintained for a while.
- Level effect, deceleration and convergence. This occurs under (2.5) and (2.6) if after a TL either $B$ specializes or both countries specialize.
iv), the initial level of income is inversely related in neoclassical models to the growth rate. Here it is related with relative capital stocks that determine special-
ization patterns. However, the initial level of income is an imperfect measure of relative capital stocks..
v), the number of years that the economy has been open, determines in our model the switch between specialization patterns. But the relationship between the latter and the growth rate depends on the relative initial capital stock -that determines the initial position- and growth rates of past periods -that determines the amount of time that a country spends in a given specialization pattern.

3: Variables omitted. We see that rates of growth depend on variables that, to the best of my knowledge, do not appear in the empirical literature like:
I) Comparative advantage in the production of capital or consumption goods.
II) Relative stock of capital/specialization patterns.
III) Saving habits and technology of partner countries.

It is unlikely that such variables could be absent in more complicated models. In fact, it is surprising that such variables have been forgotten because their role is clear: I) Countries with comparative advantage in goods that make the economy grow, behave differently than countries with no such advantage. For instance, the former countries are less likely to be affected by opening to trade or by tariffs. II) Relative size matters, because this size shapes specialization patterns that, in turn, explains supply. III) When development is demand oriented and capital
goods are imported and paid with exports of consumption goods, characteristics of trade partners are important. And specialization patterns depend in the long run on relative growth rates.

## 4. CONCLUSIONS

In this paper we show that even in a model where IT is never harmful to growth and all relationships are linear, the exact relationship between trade and growth cannot be captured by a single equation. ${ }^{3}$ It is clear that a full assessment of the impact of trade on growth needs a complicated model with general assumptions on factors, technology, etc., and where imperfections of competition and information play an important role. But it is unlikely that in such models the relationship between growth and other variables would be simpler than in our model.

A possible criticism of our paper is that our assumption of a constant fraction of income devoted to savings is not based on utility maximization. When the economy is always completely specialized there is a utility function that provides microfoundations to this assumption (see Bhagwati, Panagariya and Srinivasan [1998], p. 531). But when the economy moves between specialization patterns, it is not clear which preferences yield such an assumption. In any case, most of our

[^2]remarks in Section 3 are still valid with a variable saving rate because they remain applicable for any given savings rate. Predictions of a TL still hold as long as there is no switch from one specialization pattern to another. In any case, if the linear relationship between consumption and real income is lost we should expect that the relationship between trade and growth becomes even more complicated.

Another criticism is that an important part of the theory of IT is based on the Heckscher-Ohlin-Samuelson model with two factors and a differentiable production function and not on the Ricardian model. But our model is capable of incorporating a second factor under additional assumptions on technology, see Appendix. Again, more general assumptions will produce even more complicated models. Moreover, Baxter (1992, p. 713) has shown that the long run equilibrium of the Heckscher-Ohlin-Samuelson model displays Ricardian features. In particular "countries specialize according to comparative advantage". Finally, such a model produces vey often only level effects (see Lucas (1988) p. 12). A model like ours that is capable of producing level and growth effects seems preferable.

What do we learn from our exercise other than the complexity of the issue?
On this count our model suggests, at least, three things.
1: The relevance of variables not considered so far like comparative advantage, relative capital stock and consumption habits and technology of partner countries.

2: Some variables used so far, like the trade share, the savings rate, etc do not play the role that they were assumed to play.

3: Logarithmic and piecewise functional forms may capture the intended relationships better.

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## 6. APPENDIX

### 6.1. The model with capital mobile between countries.

Assume that capital flows from the country with the lowest rental rate to the country with the highest rental rate. We will see that the dynamic analysis presented in Proposition 2 still valid if $c^{A}=c^{B}$. This condition corresponds to the assumption often made in International Trade that tastes are identical in both countries. Recall that:

Under complete specialization, $\bar{r}^{A}=\beta_{I}^{A}$ and $\bar{r}^{B}=\frac{\beta_{I}^{A} c^{A} K^{A}}{\left(1-c^{B}\right) K^{B}}$.

If country B specializes, $\hat{r}^{A}=\beta_{I}^{A}$ and $\hat{r}^{B}=\frac{\beta_{I}^{A} \beta_{C}^{B}}{\beta_{C}^{A}}$. If country A specializes, $\tilde{r}^{A}=\beta_{I}^{A}$ and $\tilde{r}^{B}=\beta_{I}^{B}$.

There are three possible cases:

- $r^{A}>r^{B}$ for all $\frac{K^{A}}{K^{B}}$. In this case, capital flows from $B$ to $A$. This case arises when $\beta_{C}^{A}>\beta_{C}^{B}$. Notice that if $c^{A}=c^{B}$ this is just inequality (2.7). There $g^{A}>g^{B}$ without capital mobility, so this tendency is reinforced by capital mobility.
- $r^{A}<r^{B}$ for all $\frac{K^{A}}{K^{B}}$. In this case capital flows from $A$ to $B$. This case arises
when $\beta_{I}^{A}>\beta_{I}^{B}$ and $\beta_{C}^{A}<\beta_{C}^{B}$. Notice that if $c^{A}=c^{B}$ this case is identical to inequality (2.8) in the main text. In this case $g^{B}>g^{A}$ without capital mobility, so this tendency is reinforced by capital mobility.
$-r^{A}>($ resp. $<) r^{B}$ for some values of $\frac{K^{A}}{K^{B}}$. This case arises when $\beta_{I}^{A}<\beta_{I}^{B}$. Notice that if $c^{A}=c^{B}$ this case is identical to (2.5) in the main text. For $\frac{K^{A}}{K^{B}}>$ $($ resp. $<) \frac{1-c^{A}}{c^{A}}, r^{B}>\left(\right.$ resp. <) $r^{A}$, so capital flows from $B($ resp. $A)$ to $A$ (resp. $B)$. These conditions match exactly those in the model without capital mobility.


### 6.2. The model with a second Factor

If output is produced by capital and labor (denoted by $L$ ), supply equations read $P^{i} C_{P}^{i} \leq \pi^{i} K_{C}^{i}+w^{i} L_{C}^{i}$ and $I_{P}^{i} \leq \pi^{i} K_{I}^{i}+w^{i} L_{I}^{i}$, where now $\pi^{i}$ (resp. $w^{i}$ ) stands for the rental price of capital (resp. wages) in country $i$. Let $l_{C}^{i}$ (resp. $l_{I}^{i}$ ) be the labor/capital ratio in country $i$ in sector $C$ (resp. $I$ ). Thus, $L_{C}^{i} \equiv l_{C}^{i} K_{C}^{i}$ and $L_{I}^{i} \equiv l_{I}^{i} K_{I}^{i}$. If we assume that $l_{I}^{i} \equiv l_{C}^{i} \equiv l^{i}$, supply equations read $P^{i} C_{P}^{i} \leq$ $\left(\pi^{i}+w^{i} l^{i}\right) K_{C}^{i}$ and $I_{P}^{i} \leq\left(\pi^{i}+w^{i} l^{i}\right) K_{I}^{i}$. By redefining $r^{i} \equiv\left(\pi^{i}+w^{i} l^{i}\right)$ we obtain the equations for the model without labor. The model yields a linear wage-profit frontier, familiar to the researchers of linear models. It is customary to close these models by assuming some kind of bargaining between capitalist and workers.


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[^1]:    ${ }^{1}$ For a survey of the results on comparative dynamics see Dixit and Norman (1980).
    ${ }^{2}$ One branch of this literature uses cross-country regressions to search for linkages between growth rates and other variables. Another branch concentrates on the dynamic paths followed by a country or group of countries, especially after a mayor trade liberalization (TL).

[^2]:    ${ }^{3}$ See Baldwin and Seghezza (1996) for a similar point based in the european experience .

