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DEBT LIMITS AND ENDOGENOUS GROWTH *

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Abstract

This paper studies the consequences on growth and welfare of imposing limits to public borrowing. In the model economy, government spending may play two different roles, either as input in the production function, or providing services directly in the utility function. In these setups I study the effects of different fiscal policies with and without debt limits both in the balanced growth path and during the transitional dynamics. In the long run, if there is no limit, the growth effects of raising labor income taxes are negative, regardless of the role of government spending. However, the role public spending is crucial for the growth effects of changes in the ratio of public expenditures to output. In the presence of a limit to debt, higher labor tax rates have a positive effect on growth if government spending is productive. The opposite is true when private capital drives growth. Regarding welfare, raising labor income taxes imply a lower welfare cost of reducing debt than does cutting government spending, when this is productive.

Keywords: endogenous growth, debt limits, fiscal policy, transitional dynamics.

JEL Classification: E62, H3, H5, H63, O41.

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1 Introduction

This paper analyzes the growth and welfare effects of imposing limits to public borrowing. Macroeconomists have long debated the effects of government spending on economic growth. In addition to the way government spending is employed in the economy, research has also focused on the instruments to finance this expenditure, such as taxes and debt issue.

The effects of public debt in growth models has usually been analyzed by imposing only a no-Ponzi game condition on the limiting behavior of debt. Little attention has been paid to tighter constraints on public borrowing. Recently, however, this topic has gained growing interest because of the criteria imposed on the EMU countries by the Maastricht Treaty and later reinforced by the Stability Pact. These criteria required, among other things, the ratios of public debt and deficits over GDP not to be above 60% and 3%, respectively. Furthermore, it is widely recognized that high ratios of debt to GDP are not desirable for the economy. This has led many countries to reduce government deficits and control the rate of growth of public debt.

In this paper I analyze the effects of fiscal policy on growth and welfare when there are limits to public debt. In the model economy, government spending may play two different roles, either acting as an input to the production function, or providing services directly in the utility function. In these setups I study the effects of fiscal policy (changes in taxes and the ratio of government spending to output) with and without debt limits both in the balanced growth path and during the transitional dynamics.

The literature on the imposition of limits on public borrowing can be structured in two main branches. The first one investigates the consequences of the *credit market discipline hypothesis*.[1] This line of research states that individuals' behavior in credit markets may constrain government borrowing. In particular, private agents may ask for risk premia that would be increasing in the amount of outstanding public debt. The government's ability to pay for these premia will determine its access to borrowing from the private sector. It is in this way that credit market conditions limit government borrowing.

The second branch focuses on the effects of exogenously imposed limits to debt, for example in the way it is done by the Maastricht Treaty. In this context, Uctum and Wickens (1997)

[1] See for example Bayoumi, Goldstein and Woglom (1995).

examine from the econometric viewpoint, the effects of imposing debt ceilings on the government intertemporal budget constraint. Their analysis is applied to US and EU data since 1970. They find that current fiscal policy is not sustainable for most industrialized countries over an infinite horizon, but it is sustainable in the medium term in the absence of ceilings. Chari and Kehoe (1998) analyze the need for fiscal constraints in the implementation of monetary unions, specially in the case of the European Monetary Union. In a standard economic model with benevolent policy makers, they find that it is desirable to impose fiscal constraints whenever the monetary authority cannot commit to future policies. Finally, Woodford (1996) analyzes the role of limits on the rate of growth of public debt in order to maintain price stability.

None of these papers focuses on the effects on growth. However, if government spending affects the equilibrium of the economy, and is partially financed by issuing debt, it is important to analyze the consequences of limiting this source of financing. There is a vast literature on the growth effects of fiscal policies in endogenous growth models. Most papers like Barro (1990), Glomm and Ravikumar (1994), and Baier and Glomm (2001) focus on the growth effects of distortionary taxes when government spending affects private returns of the agents. However, most of them abstract from public debt. In contrast, the present work introduces government debt in a framework in which growth issues can be easily addressed.

The model developed here nests Barro's (1990) and Romer's (1986) models of growth. In the first case, productive government spending is introduced in the production function enhancing both capital and labor productivity, and permitting endogenous growth. In the second case, public spending enters the household's utility function and endogenous growth is generated by an externality involving learning by doing.

The analysis focuses on both the balanced growth path and the transitional dynamics. Due to the introduction of labor-leisure choice, no closed form analytical solution is available, so I recur to numerical solutions for the competitive equilibrium. Several simulations are carried out to study the effects of changes in fiscal variables (taxes on labor income, and the ratio public expenditures over output). I study how the outcome differs, depending on the role given to government spending in each economy and whether there is a debt limit or not. The analysis of the dynamics explains not only how growth rates are affected, but also shed some light on individuals' welfare.

I find that in the long run raising tax rates on labor has positive effects on growth when there are limits to debt and government spending is productive. However, when learning by doing drives growth, rising taxes on labor only serves to reduce the incentives to work, with a negative effect on the growth rate. A reduction in government spending has negative effects on growth if public spending is productive, but has negligible effects if public spending only affects utility, in both cases regardless of the presence of a debt limit.

These results are supplemented by a study of the dynamic effects of tightening fiscal policy to reduce public debt in order to attain a lower debt to output ratio in the case of productive government spending. Compared with the initial balanced growth path, raising taxes to lower debt leads the economy to a new balanced growth path with higher growth and lower taxes because of the role of government spending in this model. By the same reason, a fiscal policy consisting of reducing government spending over output has the opposite effects, reducing growth and output. Regarding welfare, if the government must achieve a lower debt limit, higher labor income taxes imply a lower welfare cost than reducing government spending.

The rest of the paper is organized as follows. Section 2 describes the model economy. Sections 3 and 4 characterize the competitive equilibrium and the balanced growth path, respectively. Section 5 covers the parameterization of the model. In Section 6, I report the results for the long run analysis. Section 7 deals with the dynamics of the model in response to changes in taxes and or in the government spending to output ratio, and Section 8 contains the welfare analysis. Finally, conclusions and extensions close the paper.

2 The model

In this section, I present an endogenous growth model in a general equilibrium framework. I consider an economy composed by three types of agents: households, competitive firms and a government. The population size is normalized to one, so that variables are in per capita terms. In this economy private agents take as given fiscal policies when making their decisions.

As mentioned above, the model extends to two different cases, each one displaying different externalities. First, externalities arise because of public productive spending in the production function *à-la-Barro* (1990); in the second case, externalities appear due to the existence of learning-by-doing and knowledge spillovers in the productive process *à-la-Romer* (1986). In this last setup, government spending only supplies public services and enters additively into the households' utility function.

2.1 Households

The economy consists of a large number of identical infinitely-lived individuals. Agents are endowed with one unit of time to be divided between leisure, $x(t)$, and labor, $l(t)$. Households consume a homogeneous good whose price is taken as numeraire and normalized to one. Individuals derive utility from leisure, and from consuming private goods. When government spending enters the utility function, individuals will also get some utility from public services. In general, the utility function $U[c(t), x(t), g(t)]$, takes the appropriate functional form according to the following CES utility function

$$U[c(t), x(t), g(t)] = \begin{cases} \dfrac{[c(t)^{\theta} x(t)^{1-\theta}]^{1-\sigma} + \eta [g(t)^{\psi}]^{1-\sigma}}{1-\sigma} & if \quad \sigma \neq 1 \\ \\ \theta \ln c(t) + (1-\theta) \ln x(t) + \eta \psi \ln g(t) & if \quad \sigma = 1, \end{cases} \tag{1}$$

where $c(t)$ is consumption per capita; $x(t)$ is the proportion of time devoted to leisure; $g(t)$ is government spending; $\sigma > 0$ refers to the intertemporal elasticity of substitution, which is constant; $\theta \in [0, 1]$ reflects the household's preference between consumption and leisure, and $\psi > 0$ is a parameter measuring the impact of $g(t)$ on the welfare of the household. The parameter ψ is assumed to be positive (so that public consumption yields a positive marginal utility) and the following expressions must hold $-\infty < 1 - \sigma < \dfrac{1}{1+\psi}$, and $\psi(1-\sigma) < 1$, to have a bounded utility.[2] This Cobb-Douglas specification of the utility function together with the constant returns to scale of the production function will allow for the existence of

[2] For the isoelastic utility function, ψ can also be interpreted as the marginal rate of substitution between public and private goods and leisure. For the learning-by-doing model if preferences for government spending are separable (or if the agent obtains no utility from government spending) then the wealth and substitution effects cancel and leisure remains unchanged, a condition required for the balanced growth in this model.

endogenous growth.[3] Finally, the parameter η has been introduced in order to study the effects of government spending entering or not the utility function, thus $\eta = \{0, 1\}$.

Households hold assets, $d(t)$, which return some interest payments. This interest plus labor income minus the amount spent in consumption, is devoted to the acquisition of new assets, as reflected in the following budget constraint:

$$\dot{d}(t) = r(t)d(t) + \omega(t)l(t) - c(t), \tag{2}$$

where $d(t)$ denotes the household's wealth, composed of the stock of capital and government bonds; and $r(t)$ and $\omega(t)$ refer to the interest rate and the after tax wage in terms of time t consumption.

The representative discounts at a rate $\rho > 0$. His decision problem is given by

$$\underset{\{c(t),x(t),\dot{d}(t)\}}{Max} \int_0^{+\infty} U[c(t), x(t), g(t)]e^{-\rho t}dt$$

$$subject \;\; to \;\; \dot{d}(t) = r(t)d(t) + \omega(t)l(t) - c(t)$$

$$x(t) + l(t) = 1,$$

$$c(t) \geq 0 \;\; for \;\; all \;\; t,$$

$$d(0) = d_0 \;\; taken \;\; as \;\; given,$$

and the no-Ponzi game condition on assets

$$\lim_{t \rightsquigarrow \infty} d(t) \exp \left\{ - \int_0^t r(\nu) d\nu \right\} \geq 0, \tag{3}$$

The Hamiltonian for the household's problem is

$$H[c(t), l(t), d(t), \lambda(t)] = e^{-\rho t}\{U[c(t), l(t)] + \lambda(t)[r(t)d(t) + \omega(t)l(t) - c(t)]\}, \tag{4}$$

where $\lambda(t) = \mu(t)e^{\rho t}$ is the shadow price associated to the household's budget constraint.

The first order conditions (FOC) for an interior solution to this problem are given by

$$\theta c(t)^{\theta(1-\sigma)-1} x(t)^{(1-\theta)(1-\sigma)} = \lambda(t), \tag{5}$$

[3] For a more detailed discussion, see King, Plosser and Rebelo (1988).

$$(1-\theta)c(t)^{\theta(1-\sigma)}x(t)^{(1-\theta)(1-\sigma)-1} = \lambda(t)\omega(t), \tag{6}$$

$$\dot{\lambda}(t) = \lambda(t)[\rho - r(t)], \tag{7}$$

together with the transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)d(t) = 0. \tag{8}$$

Equations (5)-(6) embody the two basic margins in this problem. First, the choice between $c(0)$ and $c(t)$, given by equation (5), evaluated at times 0 and t; and second, the choice between $c(t)$ and $x(t)$ that equating the marginal rate of substitution to the real wage.

2.2 Firms and technology

There is a large number of identical firms. Markets are competitive. The inputs are capital stock, labor and government expenditure. The representative firm produces a final good according to a Cobb-Douglas constant returns to scale production function. The production function is given by

$$y(t) = Ak(t)^{\alpha}[l(t)\bar{k}(t)^{\phi}g(t)^{1-\phi}]^{1-\alpha}, \tag{9}$$

where $\alpha \in [0,1]$, $y(t)$ is output, $A > 0$ is the scale parameter, $k(t)$ is private capital, $l(t)$ is labor, $\bar{k}(t)$ denotes the aggregate level of capital, and $g(t)$ is government expenditure. The parameter $\phi = \{0,1\}$ measures the relative weight of $\bar{k}(t)$ and $g(t)$ in the production function, giving two possible sources of endogenous growth.

Under the assumptions of competitive input markets and constant returns to scale in production technology, factors are paid their marginal products. For capital this means

$$R_k(t) = \alpha Ak(t)^{\alpha-1}[l(t)\bar{k}(t)^{\phi}g(t)^{1-\phi}]^{1-\alpha}, \tag{10}$$

and for labor

$$W(t) = (1-\alpha)Ak(t)^{\alpha}[\bar{k}(t)^{\phi}g(t)^{1-\phi}]^{1-\alpha}l(t)^{-\alpha}. \tag{11}$$

As a result of this, the interest rate equals the marginal productivity of capital after depreciation

$$r(t) = R_k(t) - \delta, \tag{12}$$

while for the after-tax wage rate it is

$$\omega(t) = (1 - \tau_w)W(t), \tag{13}$$

where τ_w denotes the tax rate on labor income.

2.3 Government

In this model, the government has a path for public expenditure, $g(t)$, that is financed through taxes and debt, the government needs not run a balanced budget at every moment of time. Thus, the path for government spending is financed by taxation but also by debt. Tax revenues come from flat-tax rates on labor income, and debt is issued as government bonds held by the households. The flow of government consumption is an exogenous constant fraction of total production denoted by ζ, that is,

$$\frac{g(t)}{y(t)} = \zeta \quad \text{and} \quad \zeta \in [0, 1]. \tag{14}$$

With these assumptions the government budget constraint is the following:

$$\dot{b}(t) = R_b(t)b(t) + g(t) - \tau_w W(t)l(t), \tag{15}$$

where $R_b(t)b(t)$ denotes public debt expenses, $g(t)$ is the flow of public expenditure, and the remaining term in the equation refers to the revenues from flat-tax rates on labor income, τ_w, that are constant. To completely describe the government's setup, there is the no-Ponzi game condition on public debt

$$\lim_{t \longrightarrow \infty} b(t) \exp\left\{-\int_0^t R_b(v)dv\right\} \leq 0. \tag{16}$$

Definition 1 *In the absence of a debt limit, a fiscal policy is a pair $\{\zeta, \tau_w\}$ constant over time which implies a path for government debt that satisfies the no-Ponzi game condition (16).*

2.3.1 The debt limit

Two possible scenarios are considered. In one case, the government will *never* be constrained in issuing debt except for the no-Ponzi game condition (the standard setup in the literature), whereas in the other case, there will be a limit imposed *at some time* T to the amount of debt over output in the economy. Let $\chi(t)$ denote the debt-to-output ratio, that is, $\frac{b(t)}{y(t)}$. Using this notation, the government budget constraint (15) can be expressed as follows

$$\dot{\chi}(t) = [R_b(t) - \gamma_y(t)]\chi(t) + \zeta - \tau_w(1-\alpha). \tag{17}$$

where $\gamma_y(t)$ is the growth rate of output, that is, $\gamma_y(t) = \frac{\dot{y}(t)}{y(y)}$. This second case is captured by the following chart:

$$\begin{array}{l} \text{for } t \leq T \longrightarrow \chi(t) \text{ evolves as (17)} \qquad \text{for } t' \geq T \longrightarrow \chi(t') \leq \bar{\chi} \\ \hline \quad \uparrow \qquad\qquad\qquad\qquad\qquad \uparrow \qquad\qquad\quad \uparrow \\ \text{time } t \qquad\qquad\qquad\qquad \text{time } T \qquad\quad \text{time } t' \end{array} \tag{18}$$

From $t \leq T$ the path for $\chi(t)$ is given by equation (17). At a certain time, T, the debt ceiling is enforced, and the government debt-to-output ratio cannot exceed the limit $\bar{\chi}$. For simplicity in the analysis, I will assume that once the limit is imposed, the government fixes the ratio debt over output at the debt limit. Therefore, $\chi(t) = \bar{\chi}$, and $\dot{\chi}(t) = 0$. This means that from $t' \geq T$ on, the government budget constraint (17) becomes

$$[r(t') - \gamma_y(t')]\bar{\chi} + [1 - \tau_w(t)](1-\alpha) + \zeta - 1 = 0. \tag{19}$$

Intuitively, constraining the issue of public debt will have important effects on the way government spending is financed. In the absence of limits, the public sector has two instruments available to pay back its expenditure. These instruments are debt and revenues from taxes. When one of these tools is restricted (for example debt), the other (in this case taxes) will have to adjust to keep the government budget constraint holding. Different models will react in a different way to changes in taxes, and consequently will display different paths for growth.

Therefore, fiscal policy in this scenario differs.

Definition 2 *If there is a limit to debt, a fiscal policy consists initially of a pair* $\{\zeta, \tau_w\}$ *constant over time with public debt determined by equation (15). Then when the limit is imposed, fiscal policy is a constant* ζ*, and a path for* $\tau_w(t)$ *that satisfy equation (19).*

3 Competitive equilibrium

As usual, given fiscal policy, conditions from utility maximization are combined with those of profit maximization, together with the balanced budget for the government and market clearing conditions to characterize the competitive equilibrium of this economy.

Notice that when $\eta = 0$, and $\phi = 0$ the model collapses to a setup *à-la-Barro*, in which government spending enters the production function enhancing both capital and labor productivity. However, if $\eta = 1$, and $\phi = 1$ it becomes a model in which government spending enters additively the utility function, and the production side exhibits learning-by-doing and knowledge spillovers *à-la-Romer*. More concretely, I will refer to the first case ($\eta = 0$, and $\phi = 0$) as the *Government in the Production Function* (GPF) model, and to the second case ($\eta = 1$, and $\phi = 1$) as the *Government in the Utility Function* (GUF) model.

In equilibrium, assuming symmetry among firms, aggregate and individual stocks of capital are the same, $\bar{k}(t) = k(t)$. Then using equation (14), output becomes

$$y(t) = [Ak(t)^{\alpha+\phi(1-\alpha)} l(t)^{1-\alpha} \zeta^{(1-\alpha)(1-\phi)}]^{\varphi},$$

and the marginal products for capital and labor are, respectively,

$$R_k(t) = \alpha k(t)^{\alpha\varphi+\phi\varphi(1-\alpha)-1} [Al(t)^{1-\alpha} \zeta^{(1-\alpha)(1-\phi)}]^{\varphi}, \tag{20}$$

and

$$W(t) = (1-\alpha) l(t)^{(1-\alpha)\varphi-1} [Ak(t)^{\alpha+\phi(1-\alpha)} \zeta^{(1-\alpha)(1-\phi)}]^{\varphi}, \tag{21}$$

where

$$\varphi = \frac{1}{1-(1-\phi)(1-\alpha)}.$$

In a competitive equilibrium, markets clear. Financial markets clearing implies

$$d(t) = k(t) + b(t), \tag{22}$$

that is, assets demanded by the household, $d(t)$, must equal total supply: private assets, $k(t)$, and public assets, $b(t)$.

It remains to state the clearing condition in the goods market

$$\dot{k}(t) = y(t) - c(t) - g(t) - \delta k(t). \tag{23}$$

Additionally, due to arbitrage conditions the following must hold:

$$r(t) = R_b(t) = R_k(t) - \delta.$$

Definition 3 *Taking as given the initial state,* $k(0)$ *and* $b(0)$*, and a fiscal policy, a competitive equilibrium path for the economy described above consists of sequences for quantities* $\{c(t), l(t), k(t), b(t)\}_{t=0}^{\infty}$*, and prices* $\{r(t), \omega(t)\}_{t=0}^{\infty}$*, such that:*

(i) the triplet $\{c(t), x(t), \dot{k}(t)\}_{t=0}^{\infty}$ *solves the representative household's problem;*

(ii) the pair $\{l(t), k(t)\}_{t=0}^{\infty}$ *solves the representative firm's problem;*

(iii) the labor market clears,

$$x(t) + l(t) = 1;$$

the market for goods clears,

$$\dot{k}(t) = y(t) - c(t) - g(t) - \delta k(t);$$

and capital markets clear,

$$d(t) = k(t) + b(t);$$

(iv) the government's budget constraint (15) holds,

$$\dot{b}(t) = R_b(t)b(t) + g(t) - \tau_w(t)W(t)l(t);$$

(v) and by no arbitrage, capital and public debt earn the same interest rate,

$$r(t) = R_b(t) = R_k(t) - \delta.$$

The first order conditions characterizing the competitive equilibrium are reported in the Appendix.

4 Balanced growth path

In this section the analysis concentrates on the balanced growth path,[4] to account for the long run effects of fiscal policies. Time between parenthesis is removed to denote steady-state variables.

Definition 4 *A balanced growth path is defined as a competitive equilibrium path in which consumption, government spending, output, debt and capital grow at the same rate, γ; and in which the time allocation (leisure, labor), interest and wage rates and the fiscal variables τ_w, and ζ are constant over time.*

On the balanced growth path all positive growth rates are the same rate, γ, which satisfies

$$\gamma = \frac{1}{1-\theta(1-\sigma)}(R_k - \delta - \rho),$$

where the following needs to hold

$$R_k > \rho + \delta > \theta(1-\sigma)\gamma + \delta,$$

to ensure both endogenous growth and bounded utility, respectively. I will analyze all growing variables in ratios of capital, $k(t)$.

The balanced growth path (hereafter, BGP) in this economy is described by the set of values of the variables $\{\gamma, l, \frac{c}{k}, \frac{y}{k}, \frac{b}{k}\}$ if there is no limit. If there is a limit to debt, the BGP is described either by $\{\gamma, l, \frac{c}{k}, \frac{y}{k}, \tau_w\}$ or by $\{\gamma, l, \frac{c}{k}, \frac{y}{k}, \zeta\}$, depending on which variable adjusts to satisfy the debt limit. These variables must solve the following system of equations:

$$\theta(1-l)(1-\tau_w)(1-\alpha)\frac{y}{kl} = (1-\theta)\frac{c}{k}, \tag{24}$$

$$\gamma = \frac{1}{1-\theta(1-\sigma)}\left[\alpha\frac{y}{k} - \delta - \rho\right], \tag{25}$$

$$\frac{y}{k} = A\left[l\left(\zeta\frac{y}{k}\right)^{(1-\phi)}\right]^{1-\alpha}, \tag{26}$$

$$\gamma = \frac{y}{k} - \zeta\frac{y}{k} - \frac{c}{k} - \delta, \tag{27}$$

[4] To ensure that the balanced growth path exists for this model, it is necessary to assume that the utility function has the CES form, as it is the case here, where $\sigma > 0$. See Lucas (1990) and Rebelo (1991).

and if there is no limit to debt

$$\zeta\frac{y}{k} + \left(\alpha\frac{y}{k} - \delta - \gamma\right)\frac{b}{k} - \tau_w(1-\alpha)\frac{y}{k} = 0, \tag{28}$$

or if there is a limit $\bar{\chi}$,

$$\zeta + \left(\alpha\frac{y}{k} - \delta - \gamma\right)\bar{\chi} - \tau_w(1-\alpha) = 0. \tag{29}$$

Equation (24) represents the labor supply decision by households that depends on the after tax wage rate and on consumption. Equation (25) is the growth rate of consumption that results from the individual's optimization problem. Equation (26) is the production function in terms of the output to capital ratio and labor. Equation (27) is the resource constraint. Finally, the next two equations, (28) and (29), represent the government budget constraint without and with limits, respectively.

In the presence of a debt limit, $\bar{\chi}$, then $\frac{b}{k}$ is determined by $\frac{y}{k}$, since the imposition of a limit implies that $\frac{b}{k} = \bar{\chi}\frac{y}{k}$, and $\bar{\chi}$ is fixed. This means that any change in fiscal policy engineered through taxes, τ_w, make ζ endogenous whereas changes in the ratio of government spending over output, ζ, will make labor tax rates endogenous.

5 Parameter values

In general, it is not possible to solve this model analytically. Actually, a closed form analytic solution can be obtained for certain versions of model, but not when the labor-leisure choice is made endogenous, as is the case here. To learn about the consequences of imposing limits to public debt with respect to the standard case, I perform dynamic simulations using parameter values which are conventional in public finance and macroeconomics literature.

The parameters of the model are σ, θ, ψ, α, δ, τ_w, ρ, A, and ζ. I assign values for σ, α, δ, and ζ according to standard literature on endogenous growth. The rest of parameters, θ, ρ, A, τ_w, are calibrated. Tables 1, and 2 summarize the results.

I set the intertemporal elasticity of substitution, σ, equal to 2. The elasticity of substitution between consumption and leisure, θ, is calibrated to match a proportion of leisure to labor around 0.4, as US data suggest. The discount parameter, ρ, is calibrated to get an annual real interest rate of 4%. The elasticity of substitution between public and private goods in the utility function, ψ, has no effect on the balanced growth path since $g(t)$ is not a choice variable for the household. Therefore, it need not be assigned any value.

As in Stokey and Rebelo (1995), I compare economies that are observationally equivalent: they are compared around an identical balanced growth path, but respond differently to any parameter change. To have the two models in the same steady-state, the adjustment is made through the technological parameter, A. The annual depreciation rate, δ, equals 10%, and has been taken from previous estimates in the literature for US data. Finally, the capital share of output, α, is assigned a value of 1/3.

Regarding fiscal variables, I need to determine the tax rate on labor income, τ_w, and the weight of government spending on output, ζ. The tax rate on labor has been chosen to be $\tau_w = 36.47\%$, which corresponds to a government spending to output ratio, ζ, of 24%. All these values imply a debt to output ratio, χ, equal to 65%. Table 2 reports the values for the main variables on the steady state.

6 Long run effects of fiscal policy

In principle, if a government wants to control its budget has three possible instruments, debt, taxes and government spending. Having one of them constrained (in this case debt) affects the allocation of the others (taxes and government spending). In order to control public debt (either to reduce the amount of outstanding debt or just to prevent it from increasing without control) the government can increase taxes or reduce government spending.

In this section, I analyze the long run effects of fiscal policy (changes in the labor tax rate, τ_w, in the government spending to output ratio, ζ) in the two models considered (GPF and GUF), and highlight the differences induced by the imposition of debt limits. This will be done abstracting from transitional dynamics. To understand the characteristics of the steady state in the presence of limits, I compare balanced growth paths for different labor income taxes and government spending over output ratios around a point at which the debt limit is just binding.

6.1 An increase in the labor tax rate (τ_w)

The first experiment consists of increasing labor tax rates from 36.47% to 41.47%, keeping all the rest of parameters unchanged. Figures 1 and 2 report the results for the *GPF* and *GUF* models, respectively. In the figures, the solid lines refer to the economy without debt limits, and the dashed lines denote the economy with the debt limit. Figure 3 shows the effects on the growth rate and the debt to capital ratio under debt limits for the two models considered, the GPF (solid line) and the GUF (dashed line).

As expected, the long run effects of rising taxes differ depending on the role of government spending in the model. In the absence of debt limits for the GPF model a rise in the labor tax rate has two opposite effects on labor supply. On one hand, it diminishes the wages effectively earned by households. This reduces the incentives to work, affecting negatively output, revenues from taxes, and therefore growth. On the other hand, it has a positive direct effect on government spending, and affects positively the productivity of labor, which raises labor supply. In the figures the first effect dominates, inducing a reduction of labor. Figure 1 shows that in the economy without debt limits the fall in labor reduces output and therefore the growth rate of the economy. Given that government spending is a constant fraction ζ of output (recall equation (14)), public consumption is also reduced, what enhances the fall in the growth rate. Private consumption is diminished too.

With limits to debt the two opposite effects of the rise in taxes on labor are still at work. However, government finances behave differently. Given that the ratio of public debt to output cannot change, the rising revenues from labor income are completely devoted to higher government spending. The mechanism can be derived from equation (29). In the GPF model, higher public expenditure increases the growth rate of the economy and this positive effect is transmitted to the rest of variables. Therefore, unlike in the model without limits, the final outcome is an increase in output, public spending, and growth.

The same results hold for the GUF model in the absence of limits. It is worth noticing that the effects on the growth rate are larger in the GPF than in the GUF model due to the externalities induced by productive public spending. The reason is that in the latter higher public spending does not affect labor productivity, whereas labor taxes do. As a result, in the GUF model the rise in tax rates reduces both public and private consumption. When there is a limit to debt issue, the rise in taxes allows for higher government spending, which weakens the negative effects of fiscal policy.

After analyzing the effects in each model, what is the main difference between models of introducing debt limits? In the presence of limits to debt, raising tax rates on labor has positive effects on growth when the economy's growth is propelled by public spending and there are limits on the debt-to-output ratio. When private investment drives growth, rising taxes on labor only serves to increase government spending and to reduce the incentives to work, with a negative effect on growth. This shows that the role of government spending has in the economy is crucial in determining the long run growth effects of changes in taxes when there is a limit to debt.

6.2 A fall in the government spending to output ratio (ζ)

Next, I consider the long run effects of changes in the share of government spending on output. I will assume that if there is a limit constraining public debt, the government has to change taxes, to maintain the budget constraint holding. The change in ζ is from 24% to 22%.

Figures 4, 5 and 6 show the results. Figure 4 refers to the *GPF model,* Figure 5 shows the *GUF model*. As before, the solid lines refer to the economy without debt limits, whereas the dashed lines denote the economy with a limit to debt. Figure 6 compares both models in terms of the effects on the growth rate and the debt to capital ratio, when there is a limit imposed.

Figure 4 shows that in the GPF model, reducing ζ affects negatively all variables. Notice that these reductions are less pronounced (or even positive as in the case of labor) if there is a limit to debt. Recall that now with the debt ceiling, a change in ζ implies a change in taxes to keep the government budget constraint (29) balanced. Having debt issue controlled by the limit, the tax rate implied by lower ζ need not be so high as before. This has a positive effect on labor supply, and prevents it from falling.

However, in the GUF model the same fall in ζ only affects individuals' welfare, with no direct effect on growth. Figure 5 shows a lower level of public consumption to output ratio induces lower output, and labor. The final effect on growth is negative. Keeping taxes constant, the resources from reducing ζ go to increase debt issue. Labor falls and so does output, reducing the growth rate. Notice, however, that in the economy with a debt ceiling the fall in ζ has the opposite effects as a rise in taxes, that is, increases the growth rate.

Summarizing, in the GPF model, reducing ζ affects negatively growth with stronger effects in the absence of limits to debt. The fall in ζ reduces growth both in the GPF and GUF models in the absence of limits, with stronger effects when government spending is productive.

7 Transitional dynamics

Although the analysis above has concentrated on the balanced growth path, the two models considered in this paper display transitional dynamics. The analysis of the dynamics focuses only on the GPF model.

To recover the equilibrium path of the variables, the following procedure is employed.

1. The set of optimal conditions for the competitive equilibrium (equations A1-A10 in the Appendix) has to be expressed in terms of the normalized variables. Therefore, growing variables are expressed in ratios to capital, $k(t)$.

2. The system is reduced to the least number of variables. I denote the vector of unknowns as $z(t) = \{\beta(t), \eta(t), \xi(t), l(t)\}$, where $\beta(t) = \dfrac{b(t)}{k(t)}$, $\eta(t) = \dfrac{c(t)}{k(t)}$, and $\xi(t) = \dfrac{y(t)}{k(t)}$. Notice that when the debt limit is imposed, there is an additional equation (the one imposed by the debt limit), and an additional unknown $\tau_w(t)$. Thus, $z(t)$ becomes $z'(t) = \{\beta(t), \eta(t), \xi(t), l(t), \tau_w(t)\}$.

3. To recover the path of the original series I need to characterize the balanced growth path to which the new variables would converge. Given the nonlinearity of the resulting model, I linearize it around the new balanced growth path in order to solve it. The linearized systems have the following structure

$$A\dot{\widetilde{z}}(t) + B\widetilde{z}(t) = 0, \text{ that is, } \dot{\widetilde{z}}(t) = P\widetilde{z}(t),$$

where $P = -A^{-1}B$, and $\widetilde{z}(t) = z(t) - \overline{z}$, where $\overline{z}$ denotes variables on the new balanced growth path. Once this system is solved, I obtain the path for the vector $\widetilde{z}(t)$ in terms of

Λ, the matrix of stable eigenvalues of matrix P. Stability requires the resulting series not to be explosive, that is, in continuous time the elements in Λ must be negative.

In what follows, I investigate the dynamics of the economy when fiscal policy is tightened in order to reduce the debt to output ratio to a new limit. Fiscal policy in this analysis will take two different forms. Recall that in the absence of debt limits, fiscal policy is defined as a pair $\{\zeta, \tau_w\}$ constant over time that imply a path for government debt consistent with the no-Ponzi game condition (16). In the presence of debt limits, fiscal policy consists of a constant ζ and a path for $\tau_w(t)$ that make the ratio of debt over output constant and equal to the limit imposed, $\bar{\chi}$. For simplicity, I will consider the case in which there is only one period of transition between regimes, that is, $T = 1$ in chart (18). This is the simplest way to study the dynamic effects of imposing the limit, since I avoid calculating the branch of the dynamics between the time of the announcement and the moment when the limit becomes active, T.

7.1 An increase in the labor tax rate (τ_w)

In this section, I will study the transitional dynamics of an economy that raises taxes in order to achieve a lower ratio of debt to output. As mentioned above, for simplicity the moment in which the debt limit is enforced is $T = 1$. The dynamics off balanced growth paths for an economy that raises taxes to attain a lower debt level are compared with the initial balanced growth path, that is, an economy growing at a constant growth rate without the imposition of any debt limit or any other change in fiscal policy, that will be taken as benchmark.

Figure 7 displays the results of a temporary rise in the labor tax rate from 36.47% to 40%, implying a drop in the debt to output ratio from 65% to 60%. In the figures, the solid lines refer to the model without limits to debt, and the dashed lines draw the results for the model with limits. The panels of the figure depict the paths for consumption, output, labor, government spending, the growth rate of capital, the debt to output ratio, and the labor income tax. All variables are expressed as fractions of their initial balanced growth path values.

After the initial exogenous change in taxes, the debt is reduced to hit the ceiling as imposed. What are the effects for the rest of variables? Since $\frac{b}{y}$ is constrained by the limit, taxes become endogenous, and converge to a new balanced growth path with lower labor income taxes. This affects positively labor, which increases. Although it may seem counterintuitive, lower taxes result in high government spending. Given the assumption of a fixed spending ratio, $\zeta = \frac{g}{y}$, and given the productive role of government spending in this economy, lower tax distortions result in more output and more government spending as well as increased consumption and growth.

That is, if the economy raises labor income taxes to reduce debt and maintain it at a fix ratio over output, the economy will converge to a new balanced growth path in which consumption, output, labor, and growth all will be higher, labor income taxes lower and government spending will increase with respect to an economy that stays at its initial balanced growth path.

7.2 A reduction in the government spending to output ratio (ζ)

Following with the analysis parallel to the balanced growth path, this subsection analyzes a reduction in the ratio of government spending to output, ζ, from 24% to 22%, once and for all at time $T = 1$. In this case, the economy uses changes in ζ to reduce its debt to output ratio and attain another balanced growth path with the debt limit. As before, two cases are compared, without limits or any other change in fiscal policy (the benchmark), and with limits. Recall that in the case with debt limits, the change in ζ makes labor tax rates, τ_w, endogenous.

As in the previous case, the solid lines refer to the model without limits to debt, and the dashed lines draw the results for the model with limits. The analysis will focus on the paths for consumption, output, labor, government spending, the growth rate of capital, the debt to output ratio, and labor income tax rates. As before, all variables are expressed as fractions of their initial steady-state values. Figure 8 reports the results.

When government spending is diminished to reduce the amount of debt over output, it affects negatively consumption, output, and the growth rate. Notice that reducing government consumption and debt allows the economy to enjoy lower labor income taxes. The immediate effect is a rise in labor supply. Thus, the reduction of ζ to attain a level of debt over output below the initial one, and stick to it, leads the economy to a new balanced growth path with lower consumption, output, growth rate, government spending, and taxes, and higher labor.

The main difference between this fiscal policy and the former relies in the sign of the effects. When taxes are raised to reduce $\frac{b}{y}$, the effects on consumption, output and the growth rate go in the opposite direction than when government spending is reduced. Although both policies are conducted to reduce the amount of outstanding debt, the dynamics off steady states confirm the results previously obtained in the long run analysis: increasing taxes in the presence of limits to debt affects positively the growth rate. Now, the dynamics adds the notion of what happens with consumption and labor. With initially higher taxes on labor income, the representative household enjoys higher consumption and lower taxes in the following periods. When government spending is reduced, consumption is lower and labor higher. What are the final effects on welfare is the focus of the next section.

8 Welfare analysis

In the two former sections, I have analyzed the effects on growth of different fiscal policies in economies with limits to debt. Raising labor income taxes had positive growth effects in contrast with reductions in government spending. However, what are the consequences for individuals' welfare? In this section, I study the welfare effects of the changes in fiscal policy analyzed before in the economy with debt limits and for the case in which government spending is a productive input (the GPF model).

The welfare cost of implementing a given fiscal policy comes from the comparison of the levels of welfare at the starting balanced growth path and during the transition off the balanced growth paths. In this economy, welfare on the balanced growth path, W_{BGP}, is given by

$$W_{BGP} = \int_0^{+\infty} U[c_{BGP}, x_{BGP}]e^{-\rho t}dt = \int_0^{+\infty} \left[\frac{(c_{BGP}^{\theta} x_{BGP}^{1-\theta})^{1-\sigma}}{1-\sigma} \right] e^{-\rho t}dt =$$

$$= \int_0^{+\infty} \left[\frac{(c_0^{\theta} e^{\theta \gamma_0 t} x_0^{1-\theta})^{1-\sigma}}{1-\sigma} \right] e^{-\rho t}dt,$$

where zero subscripts denote the initial balanced growth path, and where the coefficient of relative risk aversion, σ, has been set equal to 2. Note that at the initial balanced growth path all variables grow at the same rate, γ_0. Recall that $\eta = 0$ because the analysis focuses on the GPF model.

The level of welfare attained during the transitional dynamics, W_{TD}, is given by the following expression:

$$W_{TD} = \int_0^{+\infty} U[c(t), x(t)] e^{-\rho t} dt = \int_0^{+\infty} \left\{ \frac{[c(t)^{\theta} x(t)^{1-\theta}]^{1-\sigma}}{1-\sigma} \right\} e^{-\rho t} dt.$$

Recall from the discussion in previous sections that $c(t) = k(t)\eta(t)$, and $k(t) = k_0 e^{\gamma_k(t)t}$, where $\gamma_k(t)$ denotes the growth rate of capital at time t. Given the endogenous character of labor, I cannot study welfare implications of fiscal policy explicitly. Therefore I simulate the economy.

I follow Lucas (1987) and measure the welfare cost of fiscal policies as the proportion of consumption in the initial balanced growth path the agent would be willing to lose in order not to experience the change in consumption after the fiscal policy experiment. This cost will be denoted by ς, and can be computed as follows:

$$W_{BGP} = \int_0^{+\infty} U[c_{BGP}(1-\varsigma), x_{BGP}] e^{-\rho t} dt = W_{TD},$$

that is,

$$\varsigma = 1 - \left[\frac{W_{TD}}{W_{BGP}} \right]^{\frac{1}{\theta(1-\sigma)}},$$

where W_{TD} depends on τ_w and ζ.

Table 3 reports the welfare cost, ς, of the two fiscal policies analyzed as percentage of initial BGP consumption in the presence of limits to debt. The welfare cost associated with an increase in labor tax rates is lower than when government spending over output is reduced. In the former case, this is due to the increase in labor and the growth rate, that drive the economy to a new balanced growth path with higher levels of consumption. When government spending is reduced the welfare cost is much higher. The reason is the reduction in consumption and the increase in labor that can be seen in Figure 8.

Summarizing, a fiscal policy consisting on raising taxes to attain a lower debt to output ratio results in higher growth and less welfare cost than other fiscal policy that has government spending over output as its instruments.

9 Conclusions and extensions

The aim of this paper is to investigate the growth and welfare consequences of imposing debt limits on the government's budget constraint. The long run effects of increases in taxes on labor, and reductions in the government spending to output ratio are analyzed in two different endogenous growth models with labor-leisure choice, in an environment with and without limits to debt. The two models considered differ in the weight and role government spending is given, either as productive spending (entering in the production function), or as providing public services (in the utility function) being private capital what drives growth in the latter case.

The existence of debt limits is crucial for the growth effects of different fiscal policies. In the long run, if there is no debt limit, the growth effects of raising labor income taxes are negative regardless of the role of government spending, and vice versa. However, which role public spending plays in the economy is determinant for the growth effects of changes in the ratio of public expenditures to output. Interestingly, in the presence of a limit to debt, higher labor tax rates have a positive effect on growth if government spending is productive.

I also investigate the dynamic effects of using fiscal policy to reduce public debt in order to attain a debt limit with a lower debt to output ratio, and compare them with an economy without limits which stays at its balanced growth path. This analysis is done for the case in which government spending is a productive input. I find that raising taxes to lower debt leads the economy to a new balanced growth path with higher growth and lower taxes. This is due to the role of government spending in this model. By the same reason, a fiscal policy consisting of reducing government spending over output has the opposite effects, reducing growth and output. Regarding welfare, in the presence of limits to debt, higher labor income taxes imply a lower welfare cost than reducing government spending. The reason is the higher levels of consumption that the representative household enjoys if taxes are used as the instrument of fiscal policy.

The introduction of public debt and the imposition of limits to this borrowing in the way it is done in this paper is novel in the framework of endogenous growth models. Moreover, in contrast with traditional models of growth that focus on the growth effects of distortionary taxes disregarding debt issues, the setup presented here offers a lot of new possibilities to analyze the effects of different fiscal policies.

One interesting experiment would be to study the dynamics of the economy with a longer transitional period. This economy would receive at some time t the announcement of a debt limit becoming enforced at a given time $T > t$. This economy would undertake the appropriate fiscal policy measures in order to reduce $\chi(t)$ from t to T, and converge smoothly to the debt limit at time T. In this experiment fixing the time T will give us the exact change in fiscal policy needed at time t, and vice versa. This experiment will be useful to analyze, for example, the preliminary effects of the criteria imposed by the Maastricht Treaty, and the consequences of the possible fiscal policies implemented afterwards.

Furthermore, Barro (1990) finds that the tax rate that maximizes growth is the same that maximizes individuals' welfare. It would be interesting to investigate whether it is also the case here. In this sense, setting up the second best problem would allow the government to optimally design fiscal policy taking into account first order conditions from individuals' optimization. Here, the Ramsey problem may allow the government to choose just the optimal tax structure, taking as given $g(t)$; or deciding on both fiscal variables, when there are limits to public debt and therefore its financial options are constrained.

In conclusion, the introduction of limits on public debt in endogenous growth models inaugurates a new step in understanding the performance of fiscal policy in this environment, both in the long run and during the transition.

Appendix: First order conditions for the competitive equilibrium

The conditions for competitive equilibrium in the general setup are given by the following set of equations:

$$\frac{\theta(1-\tau_w)W(t)}{(1-\theta)} = \frac{c(t)}{[1-l(t)]}, \tag{A1}$$

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = [\rho - r(t)], \tag{A2}$$

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta(1-\sigma)-1}\left[(1-\theta)(1-\sigma)\frac{\dot{l}(t)}{1-l(t)} + \frac{\dot{\lambda}(t)}{\lambda(t)}\right], \tag{A3}$$

$$r(t) = \alpha\frac{y(t)}{k(t)} - \delta, \tag{A4}$$

$$W(t) = (1-\alpha)\frac{y(t)}{l(t)}, \tag{A5}$$

$$\dot{k}(t) = y(t) - g(t) - c(t) - \delta k(t), \tag{A6}$$

$$\dot{b}(t) = r(t)b(t) + g(t) - \tau_w W(t)l(t), \tag{A7}$$

$$g(t) = \zeta y(t), \tag{A8}$$

$$y(t) \begin{array}{l} \nearrow \quad Ak(t)^{\alpha}[l(t)g(t)]^{1-\alpha}, \\ \\ \searrow \quad Ak(t)l(t)^{1-\alpha}, \end{array} \tag{A9}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)d(t) = 0, \tag{A10}$$

where $\lambda(t)$ is the shadow price associated to the household's budget constraint. Equations (A1), (A2) and (A3) describe optimal choices of the household. Conditions (A4), and (A5) are the optimal input demands by firms. Equations (A6) and (A7) report the laws of motion of the two state variables of the system. Finally, equation (A8) describes fiscal policy, equation (A9) specifies the production function depending on the model considered, and equation (A10) states the transversality condition.

The system defined above fully describes the competitive equilibrium in the economy together with the constraint on $l(t) \in [0,1]$.

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Table 1: Parameter Values

Technology parameter GUF model	$\phi = 1, \quad A = 0.1799$
Technology parameter GPF model	$\phi = 0, \quad A = 2.1494$
Capital share of output	$\alpha = 1/3$
Depreciation rate	$\delta = 0.0238$
Government spending-to-output ratio	$\zeta = 0.24$
Labor tax rate	$\tau_w = 0.3647$
Inverse elasticity of intertemporal substitution	$\sigma = 2$
Discount parameter	$\rho = 0.0026$
Elasticity of substitution between consumption and leisure	$\theta = 0.4481$

Table 2: Balanced Growth Path Values*

Growth rate (γ)	0.0050
Nominal interest rate (r)	0.0098
Consumption-to-capital ratio ($\frac{c}{k}$)	0.0479
Government spending-to-capital ratio ($\frac{g}{k}$)	0.0242
Output-to-capital ratio ($\frac{y}{k}$)	0.1009
Public debt-to-capital ratio ($\frac{b}{k}$)	0.0656
Labor (l)	0.4198

*For the sake of comparison, steady state values are common to the two models (GUF and GPF).

Table 3: Welfare effects of fiscal policies

	Welfare cost (ς)
An increase in labor tax rates	
with limits	14.62%
A decrease in government spending over output	
with limits	25.71%

*The welfare cost of fiscal policies, ς, is expressed as percentage of initial BGP consumption.

Figure 1: Changes in the GPF model for different taxes on labor income. The solid line reports the model without debt limits, and the dashed line stands for the model with limits.

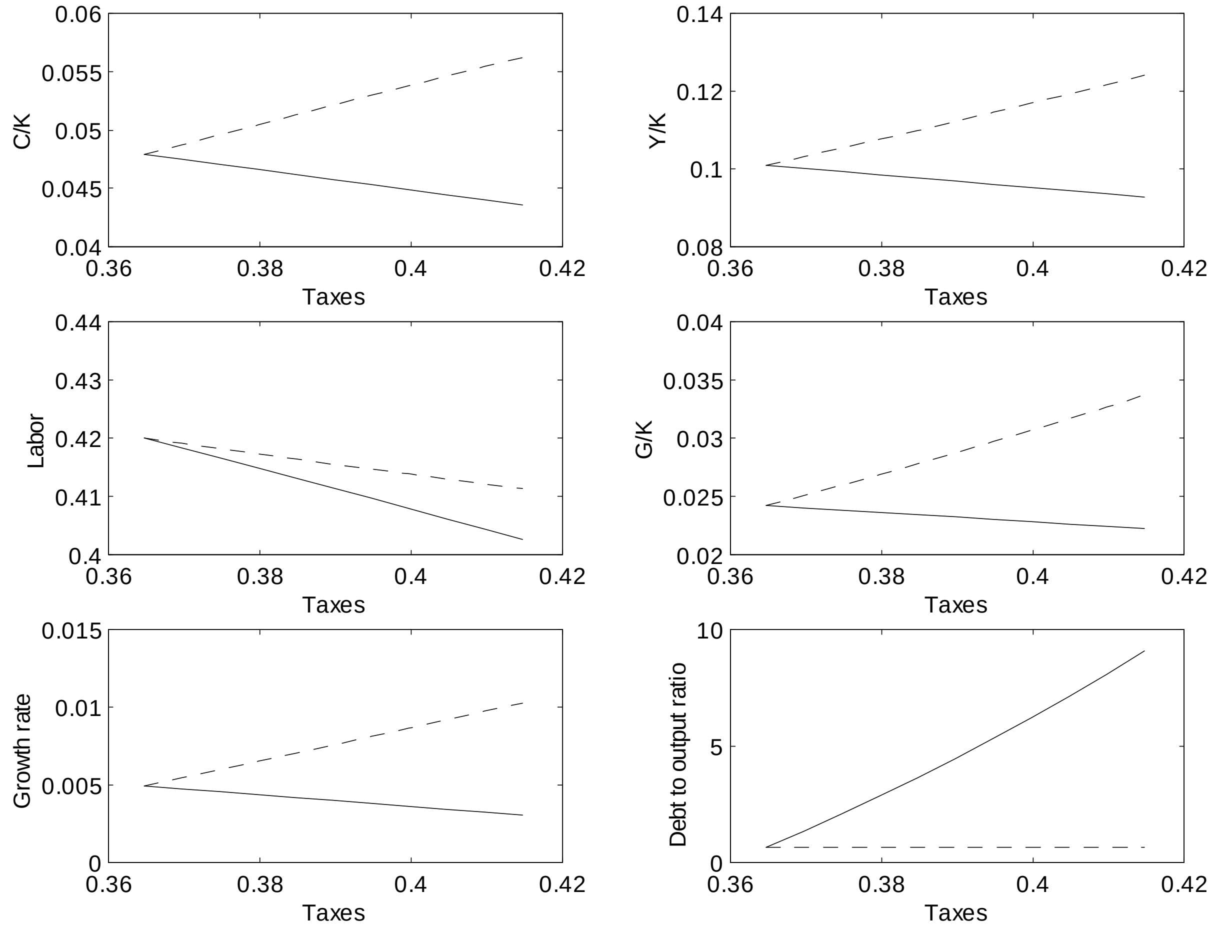

Figure 2: Changes in the GUF model for different taxes on labor income. The solid line reports the model without debt limits, and the dashed line stands for the model with limits.

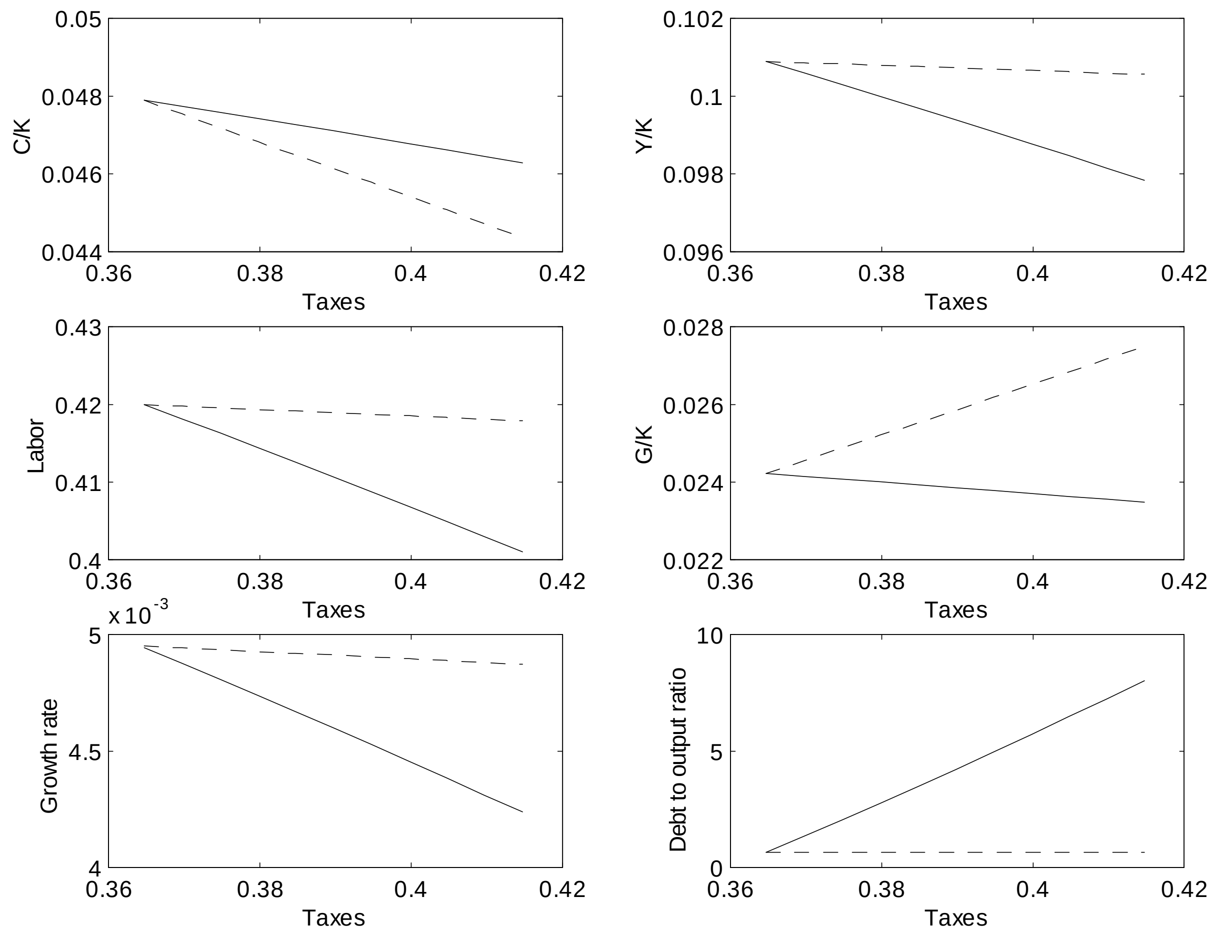

Figure 3: Changes in the GPF and GUF models for different tax rates on labor income. The solid line reports the GPF model, and the dashed line the GUF model, both cases in the presence of debt limits.

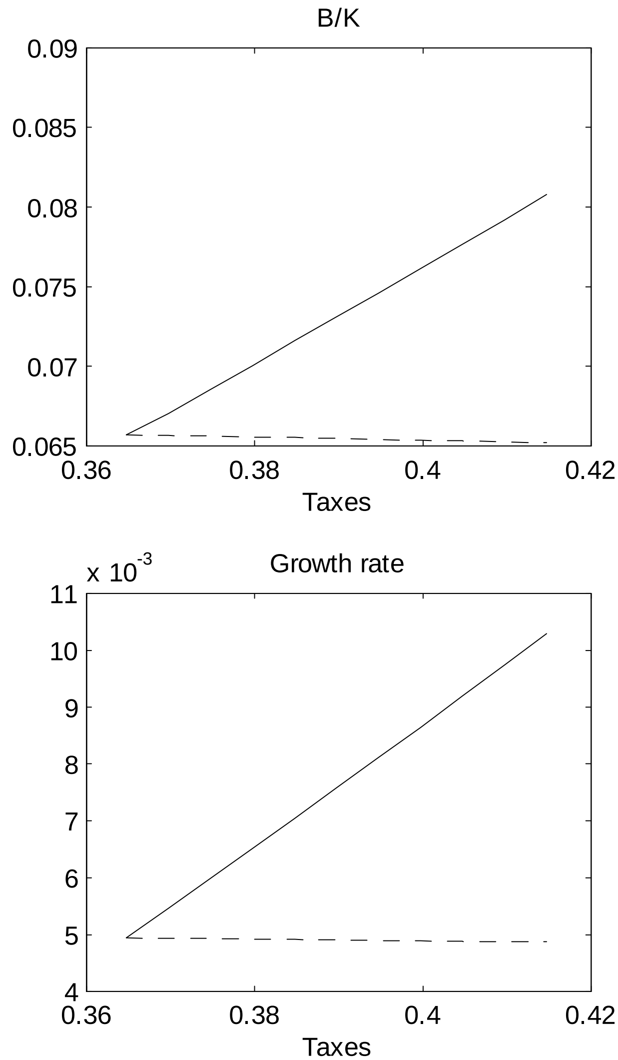

Figure 4: Changes in the GPF model for different ζ. The solid line reports the model without debt limits, and the dashed line stands for the model with limits.

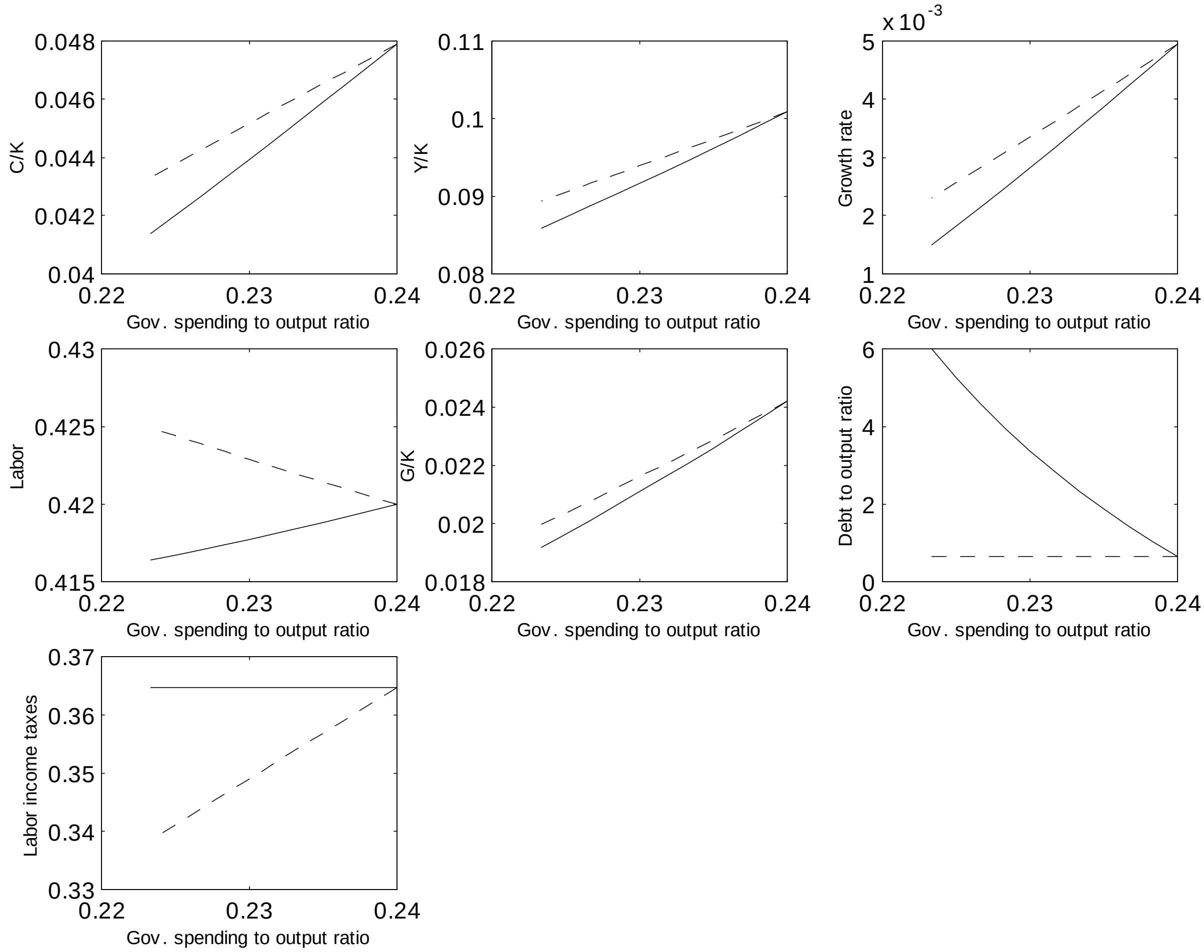

Figure 5: Changes in the GUF model for different ζ. The solid line reports the model without debt limits, and the dashed line stands for the model with limits.

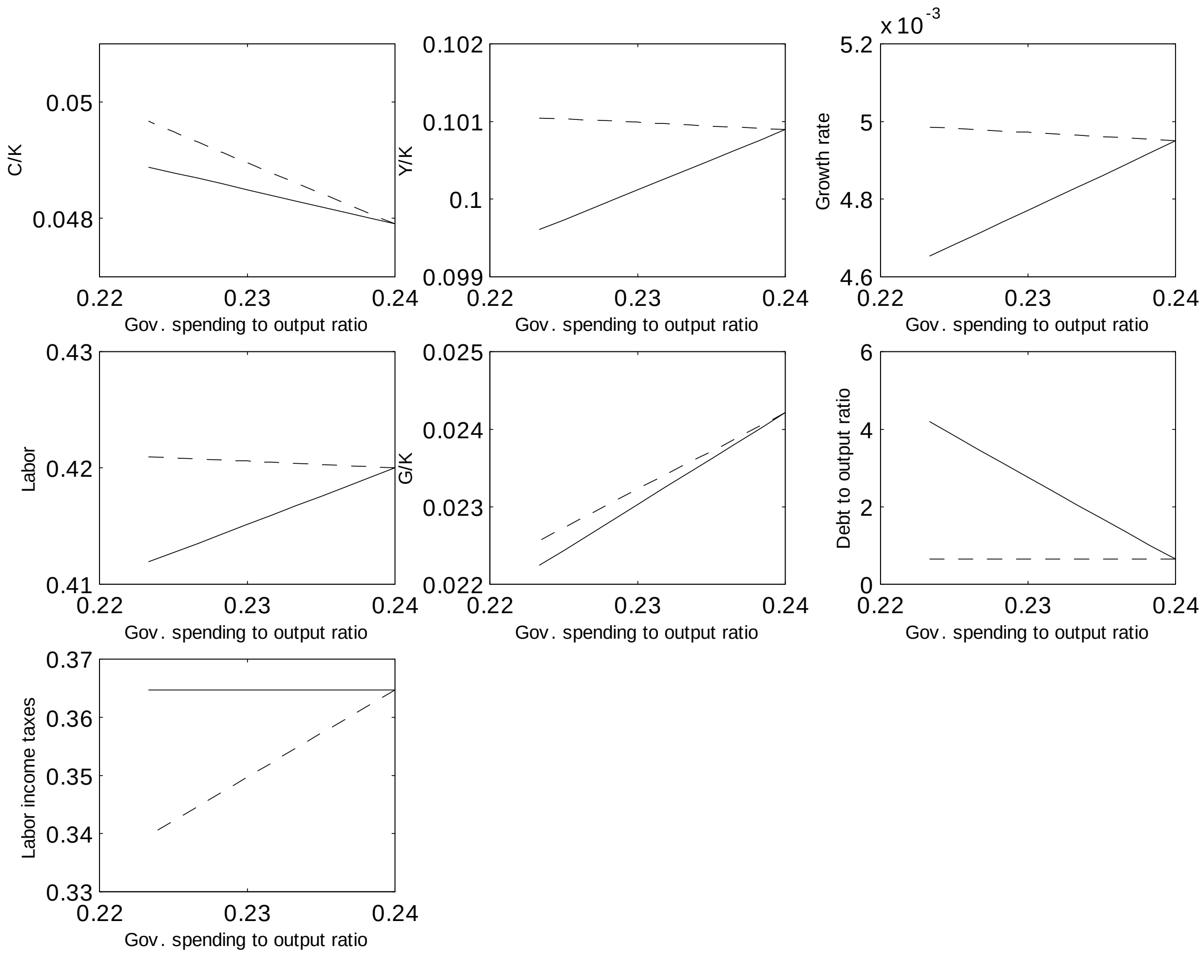

Figure 6: Changes in the GPF and GUF models for different ζ. The solid line reports the GPF model, and the dashed line stands for the GUF model, both cases in the presence of debt limits.

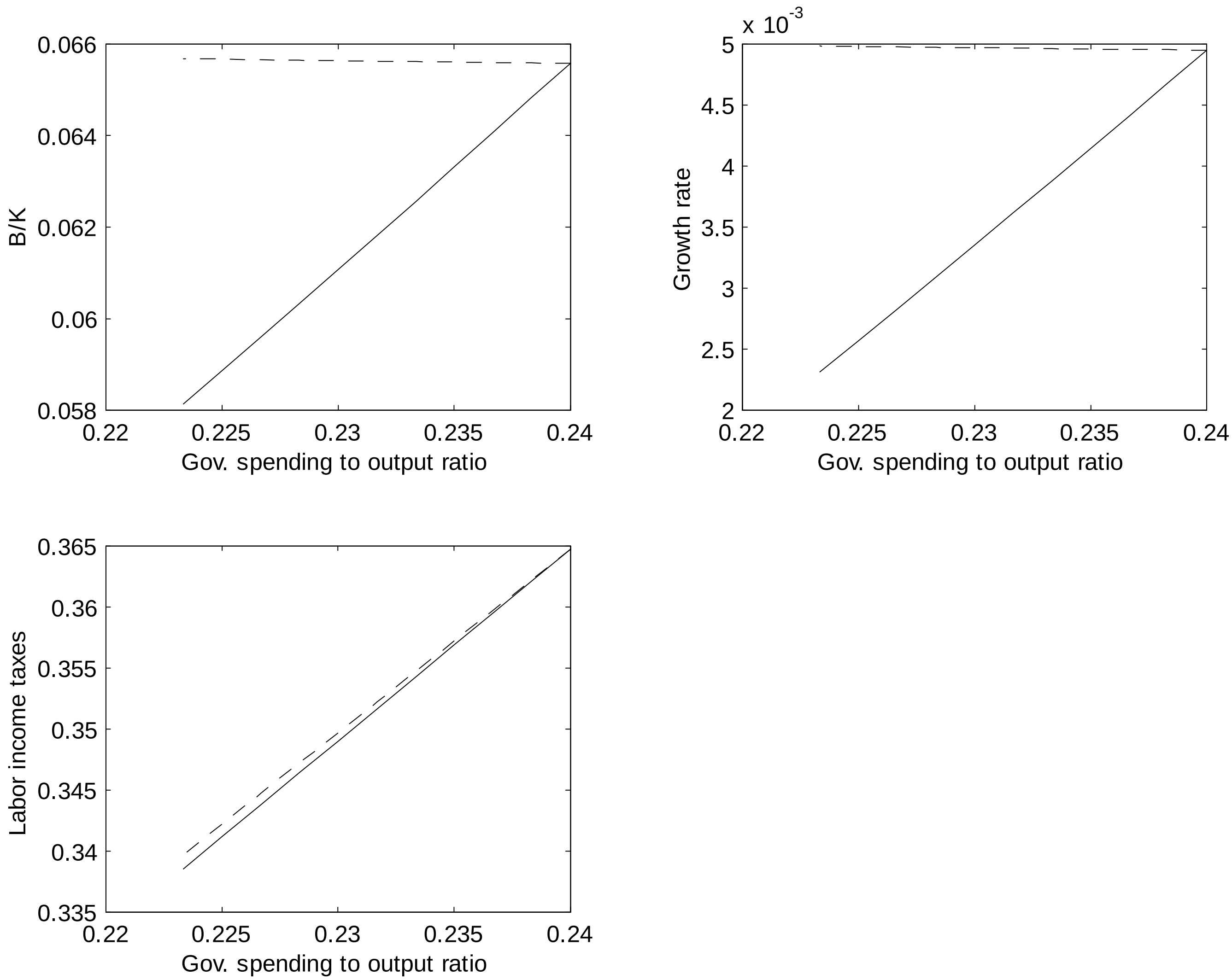

Figure 7: The GPF model after a rise in the labor tax rate. The solid line reports the model without limits to debt, and the dashed line stands for the model with debt limits. All variables are expressed as fractions of their initial BGP values.

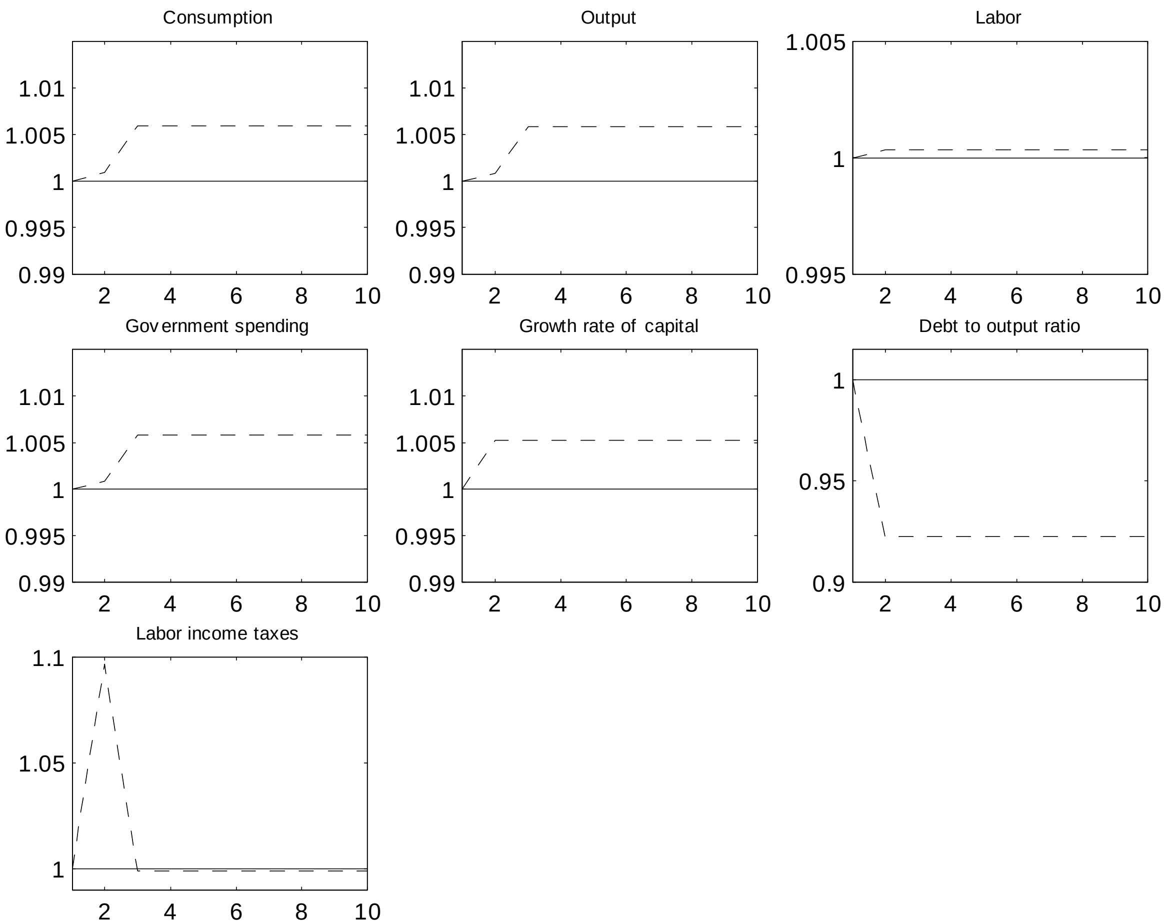

Figure 8: The GPF model after a fall in ζ. The solid line reports the model without limits to debt, and the dashed line stands for the model with debt limits. All variables are expressed as fractions of their initial BGP values.