MEASURING FISCAL DECENTRALISATION: AN ENTROPIC APPROACH[1]

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Fiscal decentralisation has attracted attention from government, academic studies, and international institutions with the aims of enhancing economic growth in recent years. One of the difficult issues is to measure satisfactorily the degree of fiscal decentralisation across countries. This study helps resolve the problem by developing the fiscal decentralisation index which accounts for both fiscal autonomy and fiscal importance of subnational governments. While the index is an advance on current practice, it is still not perfect as it assumes there is no dispersion of revenue and expenditure across regions. In response to this weakness, fiscal entropy and fiscal inequality measures are developed using information theory (Theil, 1967). It is shown how fiscal inequality can be decomposed regionally and hierarchically. These ideas are illustrated with Australia data pertaining to federal, state and local levels of governments.

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PREFACE

Title of thesis: *Fiscal decentralisation and economic growth*
Supervisors: Professor Ken Clements, Dr Michael McLure

Fiscal decentralisation has recently emerged as a fundamental issue in the literature on economic growth in developing countries. The issue has attracted the attention of both academics and international institutions such as the World Bank. The aim of my thesis is to enhance understanding of fundamental principles of fiscal decentralisation in public finance by: (1) developing the fiscal decentralisation index ("FDI") which takes the fiscal autonomy and the economic dispersion of subnational governments into consideration; and (2) investigating the possible relationship between fiscal decentralisation and economic growth based on the FDI.

The thesis will examine the following four main topics:

- The development of the fiscal decentralisation index. This index is basic in that it accounts for the fundamental influences in the fiscal federalism literature concerning the fiscal autonomy and importance of subnational governments.
- The extension of the basic index to derive the second and third approximations of fiscal decentralisation. These two indexes take into account the dispersion of subnational government revenue and expenditure between jurisdiction (in the second approximation) and within jurisdiction (in the third approximation).
- Empirical work on investigating the relationship between fiscal decentralisation and economic growth, employing an extension of the model of neoclassical economic growth due to Mankiw, Romer, and Weil (1992). The FDI enters into the production function to reflect differing levels of efficiency of the public sector across countries.
- The relationship between fiscal decentralisation and economic growth is examined for Australia and China and lessons from these countries for the fiscal constitution of Vietnam are considered.

The structure of the thesis is as follows:

Chapter 1: Introduction
Chapter 2: Aspects and explorations of fiscal decentralisation
Chapter 3: The development of the fiscal decentralisation index
Chapter 4: Entropy and fiscal inequalities: Extensions of the FDI
Chapter 5: Fiscal decentralisation and economic growth: A Cross-Country Analysis
Chapter 6: Applications of the FDI: Australia versus China
Chapter 7: Vietnam's fiscal changes and economic growth since 1975
Chapter 8: Conclusions

The following paper is mostly based on Chapters 3 and 4.

1. Introduction

To date, measurement of fiscal decentralisation in studies of public finances has been very crude. Typically, either revenue or expenditure from subnational governments ("SNGs") has been employed without taking into account the fiscal autonomy of lower level governments. For example, in his pioneering study, Oates (1972) used the national government share in total public revenue as the degree of fiscal centralisation. More recently, Woller and Phillips (1998) measured fiscal decentralisation in one of four ways: (1) the ratio of local government revenues to total government revenues; (2) the ratio of local government revenues less grants-in-aid to total government revenues; (3) the ratio of local government expenditures to total government expenditures and (4) the ratio of local government expenditures to total government expenditures less defence and social security expenditures. Similarly, Davoodi and Zou (1998) measured the level of fiscal decentralisation as the spending by SNGs as a fraction of total government spending. It is widely accepted that measurement of fiscal decentralisation in previous works has been undertaken on a superficial basis. There has been no recognition of the important distinction between subnational "revenue" and own sourced revenue over which subnational jurisdiction have policy autonomy.

The literature on fiscal federalism is extended in this study by developing a fiscal decentralisation index ("FDI") that is sensitive to fiscal autonomy of subnational governments and differences in expenditure and revenue between SNGs. Three indexes of fiscal decentralisation are introduced, with each one a more refined and extended version of its predecessor: (i) the "first approximation" index accounts for the fundamental influences of the fiscal autonomy and fiscal importance of subnational governments by focusing on the aggregate revenue and expenditure of SNGs as a whole; (ii) the "second approximation" index accounts for dispersion of revenue and expenditure across SNGs; and (iii) the third approximation accounts for differences in spending and revenue by all governments within each state, including local governments.

As discussed above, many previous attempts to measure the degree of fiscal decentralisation involve the use of some form of share of revenue/expenditure at lower-level jurisdictions in the national total. It is the claim of this paper that such an approach is inadequate as it completely ignores important distributional aspects of fiscal

arrangements. Consider two hypothetical economies, A and B. In both economies, government spending and revenue at the national level accounts for 50 percent of the total, so that the remaining 50 percent is the responsibility of subnational government. The difference is that in A there are only two large subnational institutions that have an equal share of the total 50 percent; while in B there are 100 subnational units, each accounting for 1 percent of the 50 percent total. It is clear that there is substantially more fiscal decentralisation in B as compared to A. However, an exclusive focus of the split of the total between the national and subnational levels would lead one to erroneously conclude that both economies exhibit the same degree of fiscal decentralisation. In other words, both the first and second moments of the distribution of revenue/expenditure are important for understanding the workings of fiscal arrangements. In this paper we develop measures of the dispersion of revenue and expenditure using ideas from information theory.

The paper is organised into seven sections. The distinct notions of fiscal autonomy and fiscal importance of subnational governments – two cornerstones of fiscal federalism literature - are discussed in Section 2, culminating in the development of the first approximation index to fiscal decentralisation. Elements of information theory as developed by Theil (1967) are presented in Section 3. This provides the analytical basis for subsequent approximate indexes to fiscal decentralisation. An extensive discussion of "entropy" and fiscal decentralisation is included in Section 4. Entropy is used to measure revenue equality, and then inequality, among subnational governments. In the context of fiscal decentralisation, total revenue inequality across regions can be divided into the between-state and within-state inequalities in terms of revenue and expenditure shares. The influence of between-state fiscal inequalities and within-state fiscal inequalities on the degree of fiscal decentralisation is discussed in Section 5; this material provides the foundations for the development of the second and third approximations to the FDI in subsequent work. A preliminary discussion of the nature of these indexes is presented in Section 6. Concluding remarks are given in Section 7.

2. Fiscal autonomy and importance: the development of the FDI

Tiebout's classic article "A pure theory of local expenditures" was published in 1956. In the next half century, the field of fiscal federalism developed substantially and contributed to a large body of literature on fiscal decentralisation. Seminal studies by Tiebout (1956), Musgrave (1959) and Oates (1972) laid the foundation for the significant discussions of fiscal decentralisation. Tiebout (1956) introduced the notion of local public expenditures to demonstrate that, in a fiscally decentralised country, perfect mobility of citizens between localities will result in competition among localities in providing goods and services and that the sequent migration between jurisdictions would serve to increase economic efficiency. Three years later, Musgrave (1959) laid the general foundations for modern public finance theory, stressing that the best allocation of scarce resources will be achieved whenever preferences and tastes of local citizens have been met. Subsequently, Oates (1972) argued that there should be variations in the provisions of public goods and services between governments since inhabitants in different regions have different tastes in their consumption patterns. From this perspective, subnational governments will better understand their local citizens in comparison to the national government which always provides the same bundles of goods across regions without regard to regional variations in tastes and preferences. In addition, if the national government is the only provider of public goods and services for the community, there will be no incentive for it to improve efficiency due to non existence of competition, whereas subnational governments have to face the fierce competitions from neighbourhoods. Oates formalised treatment of the issue by developing the first decentralisation theorem:

> "For a public good – the consumption of which is defined over geographical subsets of the total population, and for which the costs of providing each level of output of the good in each jurisdiction are the same for the central or the respective local government – it will always be more efficient (or at least as efficient) for local governments to provide the Pareto-efficient levels of output for their respective jurisdictions than for the central government to provide any specified and uniform level of output across all jurisdictions" (Oates, 1972, p. 35).

Discussion on fiscal decentralisation has generally centred on four main areas: the assignment of expenditure responsibility; revenue assignment (taxing powers);

intergovernmental fiscal transfers; and responsibility for subnational borrowing. Conceptually, these topics can be considered with respect to two broad categories: (i) fiscal autonomy of subnational governments; and (ii) relative fiscal importance of subnational governments (Vo, 2005). The fiscal autonomy of SNGs is primarily influenced by the assignment of taxing powers and supplementary tools such as intergovernmental fiscal transfers and fiscal equalisation, discretion over subnational borrowing and the assignment of responsibility for public provision of goods and services. In contrast, relative fiscal importance is most directly connected to the share of public sector expenditure responsibilities met by SNGs.

2.1 Fiscal autonomy of SNGs

Agreement on the distribution of taxing powers is difficult since the public sector players (national government and SNGs) approach their respective powers from two different perspectives. While the national government continues keeping important tax sources for economic stabilisation and income redistribution, SNGs typically focus on taxing powers to generate revenue to fund their provision of services which are fundamental to community welfare such as healthcare, education and public order. When the assignment of tax bases across levels of governments is extensive, the gap between spending responsibility and taxing power of SNGs will be minimal, leading to a high degree of fiscal autonomy of SNGs. Fiscal autonomy of SNGs implies that, to some extent, SNGs can arrange their own sourced revenue by exercising their taxing powers to cover costs occurring in the provision of public goods and services. In such circumstances, intergovernmental fiscal transfers will not represent a significant source of revenue for SNGs. It should be noted that, however, even in the absence of fiscal transfers ("grants"), SNGs will not enjoy full fiscal autonomy if they receive taxes or shares from revenue bases directly controlled and defined by the national government (McLure, 2001). The necessary condition for a significant level of fiscal autonomy is that SNGs themselves have the discretion to set the tax rates and/ or bases (so that they can adjust their revenue by varying the rates and/ or the bases) in response to fiscal demand for publicly provided services. If this is not the case, flexibility and the potential for creativity by SNGs for the efficient provision of public goods and services are limited.

In the event of a long-period mismatch between SNGs' spending responsibility and revenue capacity, vertical fiscal imbalance will inevitably emerge and must be managed by the national government through intergovernmental fiscal grants and advances. If SNGs are given adequately fiscal autonomy, ex-post vertical fiscal imbalance is expected to be minimised before any fiscal transfer takes place. However, it is also argued that if the national government focuses exclusively on filling the gap of vertical fiscal issues, this decision may reduce the incentive for the SNGs to increase their respective taxing powers and to manage public spending efficiently (Ahmad and Craig, 1997). One option for reducing the vertical fiscal imbalance without reform of tax assignment is to re-assign some spending responsibility for goods and services provision from SNGs to the national government. However, experience suggests that mismatch between spending and taxing will also provide some balancing role for the national government in fiscal transfers (Bird and Smart, 2002).

In essence, the greater the share of SNG expenditure funded from subnational own sourced revenue ("OSR"), the more fiscally decentralised a nation is. However, this is adjusted by the adjustment factor ("AF") from two major influences: (i) total proportion of intergovernmental grants received that are "untied" (i.e. unconditional); and (ii) the extent of SNG fiscal autonomy in borrowing decision. As a consequence, the relative level of autonomy, which can be called "fiscal autonomy", for SNGs can be defined as follows:

$$FA = \frac{\sum_{i=1}^{N} OSR_i}{\sum_{i=1}^{N} E_i} \times AF \,, \tag{2.1}$$

where OSR_i represents for the own sourced revenue for subnational region i; E_i represents for the expenditure made by subnational region i; AF represents the adjustment factor; and N is the number of subnational regions.

Fiscal autonomy of SNGs is fundamental and important feature of fiscal decentralisation. However, fiscal autonomy is only one aspect of fiscal decentralisation, which also depends on the proportion of national fiscal activity undertaken by SNGs, or their "fiscal importance".

2.2 Relative fiscal importance of SNGs

The principle of subsidiarity suggests that economic performances of the governments will be more responsive to consumer demands and to cost cutting pressures (i.e. more efficient) if services are provided by the lowest level of government possible. While foreign policy, defence, immigration, and international trade can be best formulated and implemented by the national government, SNGs are able to carry out some important tasks for regional and local communities such as law, order and public safety, education, health policy, as well as very local issues such as street lighting system, local sewerage, garbage collection, and local paper deliveries, etc. Services provided by the national government are consistent with the law of subsidiarity when demand is at a constant level across various subnational localities. However, when demand varies from location to location, national provision to a common standard leads to inefficient under-provision, in some areas, and inefficient over-provision, in other areas. In short, services provided by the national government assume tastes and preferences to be homogeneous across locations and for citizens within locations.

SNGs operate closely to local inhabitants so that they are the sole agents, who are in the best position to understand preferences, tastes and amount demanded. It is clear that levels of goods and services provided should not be exceeded the amount demanded by the community. This can avoid both under or overprovision of public goods and services. Moreover, a system of fees, users' charges can be considered useful and effective for the purpose of cost recovered (McLure and Martinez-Vazquez, 2004). The larger the portion of the total public spending cake attributable to SNGs, the higher the degree of fiscal importance and the more likely it is that the benefits from the law of subsidiarity will be realised. Consequently, the relative level that represents the fiscal importance of SNGs is defined as:

$$FI = \frac{\sum_{i=1}^{N} E_i}{TE} \quad , \tag{2.2}$$

where *TE* represents total public sector expenditures of the whole economy (including both expenditures from the national government and all SNGs).

2.3 The development of the fiscal decentralisation index

The notions of fiscal autonomy [equation (2.1)] and fiscal importance [equation (2.2)] of subnational governments may need to be used simultaneously to establish a reliable index of fiscal decentralisation. Such fiscal decentralisation index, as developed in Vo (2005), is:

$$FDI = \sqrt{\left(\frac{\sum_{i=1}^{N} OSR_i}{\sum_{i=1}^{N} E_i} \times AF\right) \times \left(\frac{\sum_{i=1}^{N} E_i}{TE}\right)} \tag{2.3}$$

(A) (B)

where OSR_i represents for the own sourced revenue for subnational region i; E_i represents for the expenditure made by subnational region i; AF represents the adjustment factor for the country and $0 \le AF \le 1$; TE represents total public sector expenditures of the whole economy (including expenditures from the national government and all SNGs); and N is the number of subnational regions.

As components (A) and (B) are to be both positive fractions, and $0 \le AF \le 1$, we can conclude that FDI will also be a positive fraction. Also, the higher the value of FDI, the more fiscally decentralised is the country.

2.4 Fiscal decentralisation index for selected countries

To illustrate the application of this index, it has been applied to a range of countries selected based on a different level of economic growth and different institutional structure of the governments. The results are reported in Table 1, which reports on countries from: the Organisation for Economic Cooperation and Development (OECD) (items 1 to 19 inclusive); the Association of South East Asian Nations (ASEAN) (items 20 to 22 inclusive); and other developing countries with middle level of income (items 23 to 26). In order to measure the dispersion of political institutions, both federal and unitary counties are considered. Table 1 also reveals that the degree of fiscal decentralisation in federal countries is generally higher than that of unitary countries since their SNGs' responsibilities and powers are often assured by their constitutions (this guarantee cannot

basically be found in the constitutions of unitary countries). Also, with developed countries, their subnational governments are more advanced in terms of managerial capability and experience in comparison with developing countries. As a result, fiscal decentralisation is expected to occur to a larger extent in developed countries.

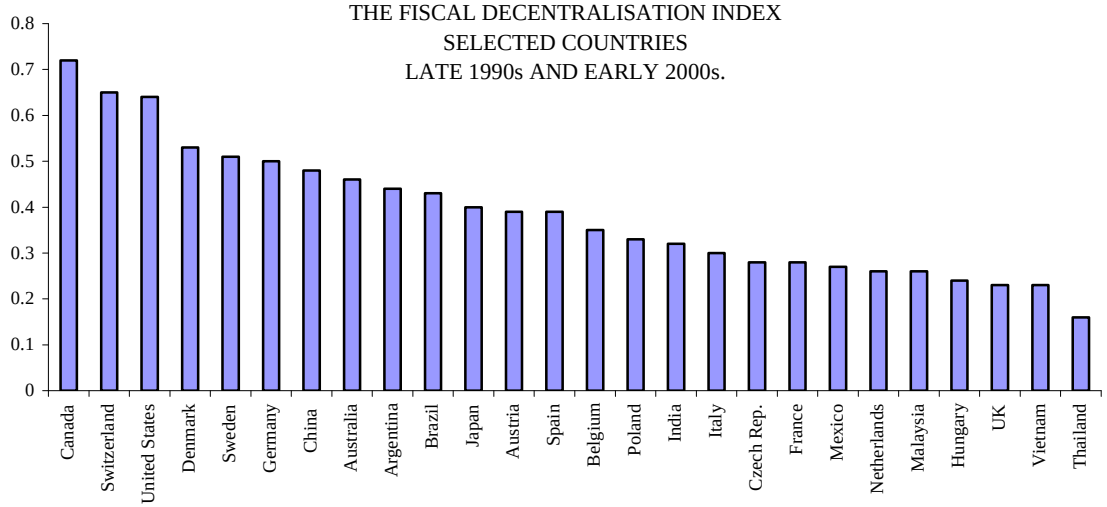
TABLE 1

FISCAL DECENTRALISATION INDEX OF SELECTED COUNTRIES

No.	Country	Year	Units	Total subnational owned source revenue $\sum_{i=1}^{n} OSR_i$	Total subnational expenditure $\sum_{i=1}^{n} E_i$	Total expenditure TE	Adjustment Factor AF	Fiscal Decentralisation Index FDI
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	Australia	2002	Bil. AUD	71	125	271	0.81	0.46
2	Austria	2002	Bil. Euro	29	40	115	0.63	0.39
3	Belgium	2001	Bil. Euro	23	50	134	0.69	0.35
4	Canada	2002	Bil. CAD	281	324	503	0.94	0.72
5	Germany	2002	Bil. Euro	351	444	1,066	0.75	0.50
6	Mexico	2000	Bil. Pesos	193	459	1,136	0.44	0.27
7	Switzerland	2001	Bil. Franc	86	105	166	0.81	0.65
8	United States	2001	Bil. USD	1,738	2,040	3,713	0.88	0.64
9	Czech Rep.	2002	Bil. Koruny	141	242	1,089	0.63	0.28
10	Denmark	2002	Bil. Kroner	294	462	788	0.75	0.53
11	France	2001	Bil. Euro	86	146	771	0.69	0.28
12	Hungary	2002	Bil. Forint	1,044	2,197	8,950	0.50	0.24
13	Italy	2000	Bil. Euro	96	163	544	0.50	0.30
14	Japan	2001	Bil JPY (000)	80	79	286	0.56	0.40
15	Netherlands	2002	Bil. Euro	22	72	211	0.63	0.26
16	Poland	2002	Bil. Zlotys	70	120	350	0.56	0.33
17	Spain	2000	Bil. Euro	74	90	276	0.56	0.39
18	Sweden	2001	Bil. Kroner	446	567	1,300	0.75	0.51
19	UK	2002	Bil. GBP	41	117	433	0.56	0.23
20	Malaysia	1997	Bil. Ringgit	9	13	68	0.50	0.26
21	Thailand	2002	Bil. BHT	76	138	1,379	0.44	0.16
22	Vietnam	2002	Bil. VND (000)	31	65	148	0.25	0.23
23	Argentina	2002	Bil. Pesos	25	36	91	0.69	0.44
24	Brazil	1998	Bil. Reais	146	182	400	0.50	0.43
25	China	1999	Bil. Yuan	759	1,155	1,637	0.50	0.48
26	India	1999	Bil. Rupees.(000)	1,188	2,676	5,870	0.50	0.32

Source: IMF, *Government Finance Statistic Yearbooks 1998, 2000, 2002 and 2004*.

FIGURE 1
THE FISCAL DECENTRALISATION INDEX
SELECTED COUNTRIES
LATE 1990s AND EARLY 2000s.



2.5 Potential weaknesses of the FDIs

Equation (2.3) has two potentially significant limitations. Firstly, revenue and expenditure in all SNGs is implicitly assumed to be equal. In effect, all regions are assumed to be a homogeneous fiscal mass. However, it is well known that SNGs typically involve large differences in revenue and spending, differences that could have significant implications for fiscal decentralisation. Secondly, the structure of the fiscal constitution is ignored. Subnational governments are not differentiated by type – the state government level is not distinguished from the local government level. These structural changes may also impact on fiscal decentralisation. For example, local councils have different distribution of revenue and spending within the same state. Furthermore, population, revenue, and expenditure across states are also different.

As equation (2.3) accounts only for the fundamental influences of the fiscal autonomy and fiscal importance of subnational governments while ignoring the impact of fiscal differences between them, it can only be considered as a “first approximation”. To redress these shortcomings, the fiscal decentralisation index will be extended by using information theory as developed by Theil (1967). The main goals of the extensions of the first approximation index are to account for the distributions of revenue and expenditure shares of all governments (including local governments) between the state jurisdictions

(in the second approximation) and the distribution of revenue and expenditure shares of all governments (including local governments) within a state jurisdiction. The concepts of "between-set entropy" and "within-set entropy" appear to have the potential to account for heterogeneity in fiscal shares across different levels of governments.

3. Entropy and information theory

Information theory provides a convenient way to summarise probability distributions and how they change with the receipt of new information. This section sets out the key principles of information theory, drawing on Theil (1967, Chapters 2 and 3). These principles are then applied into next section to measuring distributional aspects of fiscal arrangements

Any possibility occurs with the probability x with $0 \le x \le 1$. The message is considered to be a definite and reliable message if, with its presence, one possibility is confirmed to occur. Let $h(x)$ be information content of a definite and reliable message x, then $h(x)$ is the decreasing function of the probability x. Among all such decreasing functions, we choose:

$$h(x) = \log \frac{1}{x} = -\log x.$$

As will be shown subsequently, the reasons for choosing $-\log x$ are that it leads to (i) convenient decomposition and (ii) measures that have an axiomatic justification.

Until the message is released, no one can predict how large the "information content" will be since either $h(x_1), .., or\ h(x_N)$ with different probabilities $x_1 \neq ... \neq x_N$ can occur. However, the average or expected information content can be calculated before the message comes in since we know the probabilities:

$$H(x) = \sum_{i=1}^{N} x_i h(x_i) = \sum_{i=1}^{N} x_i \log \frac{1}{x_i} = -\sum_{i=1}^{N} x_i \log x_i.$$

As the product of $x_i \log x_i$ is always non-positive, $\sum_{i=1}^{N} x_i \log x_i \le 0$. Therefore, the negative of this sum, $H(x)$, cannot be negative. The measure $H(x)$ is the expected information of a distribution, which Theil calls "entropy". In addition, the value of the

entropy $H(x)$ has a lower limit of zero and the upper limit of $\log N$, where N represents a number of events or possibilities, so that $0 \le H(x) \le \log N$..

Unlike a direct and reliable message, the presence of an indirect message does not confirm anything, rather it only provides more information regarding the event which is likely to occur in the future. It is assumed we have N events as $E_1,..,E_N$ with the probabilities to occur are $x_1,..,x_N$, respectively. These probabilities are known as prior probabilities since they exist before the message comes in. When the message comes in, these probabilities become $y_1,..,y_n$, respectively. These are called posterior probabilities. The sum of these posterior probabilities is unity: $\sum_{i=1}^{N} y_i = 1,\ \ y_i \ge 0\ \ \forall\, i = 1,..,N.$

These posterior probabilities are also non-negative. If this turns out to be that one of these probabilities is one, all the others are zero, the message becomes direct message. The "probability ex post" is the probability of the event to occur after the message is released: y_i. In addition, "probability ex ante" is the probability of the event to be occurred before the message is released, still x_i in this case. Therefore, the information content in the case of "indirect message" is: $h(y_i, x_i) = \log\left(y_i / x_i\right)$. The expected information of the indirect message is as follows:

$$I(\boldsymbol{y} : \boldsymbol{x}) = \sum_{i=1}^{N} y_i \log \frac{y_i}{x_i}.$$

The expected information of an indirect message $I(\boldsymbol{y} : \boldsymbol{x})$ transforms the prior probabilities $\boldsymbol{x} = \left[x_1,..,x_N\right]'$ into posterior probabilities $\boldsymbol{y} = \left[y_1,..,y_N\right]'$. This can be shown that $I(\boldsymbol{y} : \boldsymbol{x})$ is non-negative.

4. Entropy and revenue inequality

In his influential study, Theil (1967) advocated the use of entropy-based measure for the analysis of income inequality. In this section, we apply Theil's notion of the entropy to public finance.

It is assumed that a country has Q states (the second level of governments) and P local councils (the third level of governments) and each local council belongs to one

state. Let $N = P + Q$ be a total number of local and state governments, the number of subnational governments (SNGs). It is further assumed that each SNG accounts for a non-negative fraction of total subnational revenue, to be denoted by r_i which for short we shall refer to as the "regional revenue share". The sum of these all revenue shares is equal to unity: $\sum_{i=1}^{N} r_i = \quad 1, \quad r_i \geq 0 \quad \forall\, i = 1, ..., N$. Let $\boldsymbol{r}$ denote the vector of the revenue shares $r_1, .., r_N$. The entropy of the revenue shares is defined as:

$$H(\boldsymbol{r}) = \sum_{i=1}^{N} r_i \log \frac{1}{r_i}. \tag{4.1}$$

The entropy $H(\boldsymbol{r})$ can be regarded as the measure of the equality with which revenue is distributed among the SNGs. When the revenue distribution is extremely equal in that each SNG has the same revenue share (i.e., $r_i = 1/N$) and the entropy is at its maximum: $H(\boldsymbol{r}) = \log N.$ At the other extreme, if only one SNG accounts for all revenue so that others have no revenue at all (i.e., $r_i = 1$ and $r_j = 0$ for $i \neq j$), the minimum value of the entropy is achieved: $H(\boldsymbol{r}) = 0.$ As a result, the range of the entropy is $0 \leq H(\boldsymbol{r}) \leq \log N.$

In the context of the distribution of revenue, it is more convenient to focus on revenue inequality, rather than revenue equality. Revenue inequality can be measured by deducting the entropy $H(\boldsymbol{r})$ from its maximum value, $\log N$:

$$\log N - H(\boldsymbol{r}) = \log N - \sum_{i=1}^{N} r_i \log \frac{1}{r_i} = \sum_{i=1}^{N} r_i \log N r_i. \tag{4.2}$$

Due to the constraints on the range of the entropy $H(\boldsymbol{r})$, it is clear that the range of this measure of revenue inequality is 0 -- perfect equality (when $H(\boldsymbol{r}) = \log N$) -- and $\log N$ -- maximum inequality (when $H(\boldsymbol{r}) = 0$). The entropy $H(\boldsymbol{r})$ is an attractive way to measure equality as it satisfies three axioms or tests described below.

4.1 Axiom 1: The proportionality test

The entropy (4.1) is expressed in terms of the revenue shares of SNGs. Thus, if all revenues changes proportionally, the shares do not change, and measure (4.2) remains

unchanged. This invariance of revenue inequality to a proportional change is the proportionality test.

4.2 Axiom 2: The "Haves and Have Nots" test

The upper limit of $H(\boldsymbol{r})$, increases with N, so that the maximum value of the inequality measure (4.2) rises with N. Consider two hypothetical countries. Firstly, in a two-subnational region country, there is perfect inequality when one SNG accounts for all revenue, and the other has no revenue. The entropy of the revenue shares is zero, and the value of (4.2) is $\log 2$. Secondly, in a society consisting of 10,000 SNGs, revenue inequality is at maximum when 9,999 SNGs have no revenue. The value of revenue inequality is now $\log 10,000$. It is obvious that revenue distribution in the latter is much more unequal than the first country. In the first country, one-half of the SNGs (one SNG) accounts for all revenue and the other half has no revenue. As a result, revenue inequality of the second country is as unequal as for the first country when one-half of the SNGs account for all revenue and when each of these has the same revenue. The concern is that whether revenue inequality, as expressed in equation (4.2), satisfies this condition. The following material reveals that this is true by showing that as a larger fraction of SNGs join the "revenue" group, revenue inequality falls. This establishes that revenue inequality will be uniquely determined by the size of the revenue group (which we call "the haves") relative to the "no-revenue" group ("the have nots").

Assume there is a set S which consists of M subnational governments where $0<M\leq N$. It is further assumed that SNGs in set S account for all revenue, so that SNGs outside set S have no revenue. Also, within set S, each SNG accounts for the same amount of revenue (i.e., for $i\in S, r_i = 1/M.$). The inequality measure (4.2) then becomes:

$$\sum_{i=1}^{N} r_i \log N r_i = \sum_{i\in \boldsymbol{S}_i} r_i \log N r_i = \frac{1}{M}\log N\frac{1}{M} + \frac{1}{M}\log N\frac{1}{M} + ...,$$

or:

(4.3) $$\sum_{i=1}^{N} r_i \log N r_i = \log\frac{N}{M} = \log\frac{1}{\theta},$$

where $\theta = M/N$ is the fraction of SNGs in the country who jointly account for all subnational revenue. The application of the last member of equation (4.3) to the second example above with $N = 10,000$ and $\theta = 5,000/10,000 = 1/2,$ reveals that revenue inequality is also $\log 2.$

From these two examples, we can conclude that when revenue is equally distributed among some groups of SNGs in the society, and the remaining SNGs outside these groups have no revenue, revenue inequality of the country is determined solely by the fraction θ -- the ratio of the number of SNGs in the group to the total number of SNGs. In both examples above, this ratio is $1/2,$ and the revenue inequality is $\log 2.$ This result is in consistence with intuition: when the number of SNGs receiving revenue, $M,$ increases, revenue distribution becomes more equal. The above discussion shows that as the inequality (4.3) decreases as the share of a number of SNGs which receive revenue rises, this measure satisfies the "Haves and Have Nots" axiom.

4.3 Axiom 3: The revenue transfer test

Consider an economy consisting two SNGs only A (rich) and B (poor) with the revenue shares r_A and r_B, where $r_A > r_B.$ Suppose that some revenue is transferred from A to $B,$ such that $dr_A + dr_B = 0.$ A reasonable measure of revenue inequality should indicate that such a transfer from the rich SNG to the poor SNG has the effect of decreasing inequality. Does equation (4.2) satisfy this property? The following material shows that it does have this property.

It is assumed that there are G sets of SNGs, to be denoted by $\boldsymbol{S}_1,..,\boldsymbol{S}_G,$ and each SNG belongs to one and only one set. Let N_g be a number of SNGs in set $\boldsymbol{S}_g$, with $\sum_{g=1}^{G} N_g = N.$ The entropy of revenue shares, equation (4.1), then can be expressed as:

$$H(\boldsymbol{r}) = \sum_{g=1}^{G} \left[\sum_{i \in \boldsymbol{S}_g} r_i \log \frac{1}{r_i} \right], \tag{4.4}$$

where the component inside the square brackets is the entropy of revenue shares within set $\boldsymbol{S}_g.$ Let R_g be the sum of revenue shares of all SNGs in set $\boldsymbol{S}_g$, $R_g = \sum_{i \in \boldsymbol{S}_g} r_i;$ this

R_g is the revenue share of group g with $\sum_{g=1}^{G} R_g = 1$. The entropy of revenue shares within set $\boldsymbol{S}_g$ can be expressed as:

$$\sum_{i \in \boldsymbol{S}_g} r_i \log \frac{1}{r_i} = R_g \left[\sum_{i \in \boldsymbol{S}_g} \frac{r_i}{R_g} \left(\log \frac{1}{r_i / R_g} \times \frac{1}{R_g} \right) \right]$$

$$= R_g \sum_{i \in \boldsymbol{S}_g} \frac{r_i}{R_g} \log \frac{1}{r_i / R_g} + R_g \log \frac{1}{R_g}.$$

Thus, if we define $H_g(\boldsymbol{r}_g) = \sum_{i \in \boldsymbol{S}_g} \frac{r_i}{R_g} \log \frac{1}{r_i / R_g}$, where $\boldsymbol{r}_g$ is the vector of r_i that fall under $\boldsymbol{S}_g$, as the within-set entropy, we have:

(4.5) $$\sum_{i \in \boldsymbol{S}_g} r_i \log \frac{1}{r_i} = R_g H_g(\boldsymbol{r}_g) + R_g \log \frac{1}{R_g}.$$

Combining equations (4.4) and (4.5), the total entropy becomes:

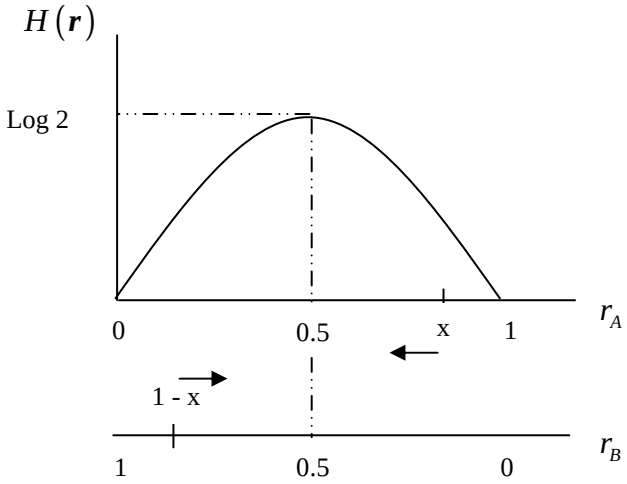
(4.6) $$H(\boldsymbol{r}) = \sum_{g=1}^{G} R_g H_g(\boldsymbol{r}_g) + \sum_{g=1}^{G} R_g \log \frac{1}{R_g}.$$

On the right-hand side of this equation, the first component is a weighted average of the within-set entropies $H_1(\boldsymbol{r}_1), ..., H_G(\boldsymbol{r}_G)$, with the group revenue shares $R_1, ..., R_G$ as the weights. The second term on the right of equation (4.6) is the between-set entropy, $\sum_{g=1}^{G} R_g \log(1/R_g)$.

We consider equation (4.6) in the context of SNGs A (rich) and B (poor) in two situations: (i) when they are the only SNGs of the country, so that $N = 2$; and (ii) when the nation is made up of A, B plus all other SNGs, so that $N > 2$. When $N = 2$, the country comprises two groups, $S_1 = A$, and $S_2 = B$, which we shall denote by S_A and S_B. Similarly, the revenue shares are $R_1 = r_A$ and $R_2 = r_B$, with $r_A + r_B = 1$. As there is only one SNG in each group, the within-group entropies are zero, $H_A(r_A) = H_B(r_B) = 0$, as is their weighted average. Accordingly, in this case, equation (4.6) simplifies to:

$$H(\boldsymbol{r}) = r_A \log \frac{1}{r_A} + r_B \log \frac{1}{r_B}.$$

This entropy is at its maximum when $r_A = r_B = 1/2.$ In that case, the entropy is $H(\boldsymbol{r}) = 1/2 \log 2 + 1/2 \log 2 = \log 2,$ as is illustrated below. From the graph, it is clear that any deviations from the equal shares of $r_A = r_B = 1/2$ will result in a lower value of the entropy, that is, higher revenue inequality. As A is richer than $B,$ the initial revenue distribution is represented in the graph by the shares $x > 1/2$ and $(1-x) < 1/2.$ When revenue is transferred from A to $B,$ both revenue shares move towards $1/2,$ the distribution becomes more equal and the entropy increases.



Next, consider the $N > 2$ case where there are three groups of SNGs: (i) Group A with only one SNG A; group B with SNG B; and (iii) group C with $(N-2)$ SNGs comprising every SNG in the economy except A and $B.$ These three groups are denoted by $\boldsymbol{S}_A$, $\boldsymbol{S}_B$, and $\boldsymbol{S}_C$. We assume that the joint revenue share of A and B is a constant, i.e. $r_A + r_B = R_{A+B} = \text{constant}.$ This implies that the revenue share of group $C,$ $R_C,$ is also constant at $1 - \mathrm{R}_{\mathrm{A+B}}.$ It is further assumed that there are no revenue transfers to or from the other SNGs of the society in $\boldsymbol{S}_C$. We now apply decomposition (4.6) to this economy. The weighted average of the within-group entropies, the first term on the right-hand side of equation (4.6), is:

$$(4.7) \qquad \sum_{g=1}^{G} R_g H_g(\boldsymbol{r}_g) = R_A H_A(r_A) + R_B H_B(r_B) + R_C H_C(\boldsymbol{r}_C) = R_C H_C(\boldsymbol{r}_C).$$

where $H_C(\boldsymbol{r}_C) = \sum_{i \in \boldsymbol{S}_C} \frac{r_i}{R_C} \log \frac{1}{r_i / R_C}$, with $\boldsymbol{r}_C$ is the vector of r_i that fall under group $\boldsymbol{S}_C$, is the within-group entropy of group C. The first and second components in the second step of equation (4.7), the within-group entropies for groups A and B, disappear because there is only one SNG in each group. In addition, the between-group entropy, the second term on the right-hand side of equation (4.6), now becomes:

$$\sum_{g=1}^{G} R_g \log \frac{1}{R_g} = R_A \log \frac{1}{R_A} + R_B \log \frac{1}{R_B} + R_C \log \frac{1}{R_C}. \tag{4.8}$$

Substituting equations (4.7) and (4.8) into equation (4.6), the total entropy for this three-group country becomes:

$$H(\boldsymbol{r}) = R_A \log \frac{1}{R_A} + R_B \log \frac{1}{R_B} + R_C \log \frac{1}{R_C} + R_C H_C(\boldsymbol{r}_C). \tag{4.9}$$

When we transfer revenue from A to B, with the distribution within $\boldsymbol{S}_C$ remaining unchanged, equation (4.9) can be expressed as:

$$H(\boldsymbol{r}) = R_A \log \frac{1}{R_A} + R_B \log \frac{1}{R_B} + \text{constant}. \tag{4.10}$$

The constant in (4.10) includes $R_C \log(1/R_C)$ and $R_C H_C(\boldsymbol{r}_C)$. In words, the total entropy of the three-group country is equal to the total entropy of two-group country plus a constant. Accordingly, the impact on inequality of a transfer from A to B is the same in the $N > 2$ case as it is in the $N = 2$ case.

To summarise this discussion, revenue inequality decreases if there is a transfer of revenue from the rich SNG to the poor SNG. This conclusion holds for a society with two-subnational regions $(N = 2)$, as well as in the higher-dimensional case $(N > 2)$. In short, it is clear that the measure of revenue inequality satisfies the revenue transfer test.

5. Decomposing revenue inequality

In the above, we decomposed revenue equality into within-set and between-set terms. We now show that revenue inequality can be similarly decomposed.

Recall from equation (4.6) that the entropy is decomposed into two distinct components: a weighted average of the within-set entropy and the between-set entropy.

Furthermore, as in (4.2), inequality is measured by the difference between the maximum value of the entropy, $\log N$ and the entropy $H(\boldsymbol{r})$. Thus, by combining equations (4.2) and (4.6), revenue inequality can be expressed as:

$$\log N - H(\boldsymbol{r}) = \log N - \sum_{g=1}^{G} R_g H_g(\boldsymbol{r}_g) - \sum_{g=1}^{G} R_g \log \frac{1}{R_g}. \tag{5.1}$$

The right-hand side of equation (5.1) remains unchanged if we subtract and add $\sum_{g=1}^{G} R_g \log N_g$, where R_g and N_g are the revenue share of and a number of SNGs in set $\boldsymbol{S}_g$, respectively:

$$\begin{aligned}\log N - H(\boldsymbol{r}) &= \sum_{g=1}^{G} R_g \left(\log N_g - H_g(\boldsymbol{r}_g)\right) + \log N - \sum_{g=1}^{G} R_g \log \frac{N_g}{R_g} \\ &= \sum_{g=1}^{G} R_g \left(\log N_g - \sum_{i \in \boldsymbol{S}_g} \frac{r_i}{R_g} \log \frac{1}{r_i / R_g} \right) + \sum_{g=1}^{G} R_g \log \frac{R_g}{N_g / N}.\end{aligned}$$

As the result, revenue inequality can be expressed as follows:

$$\log N - H(\boldsymbol{r}) = \sum_{g=1}^{G} R_g \left[\sum_{i \in \boldsymbol{S}_g} \frac{r_i}{R_g} \log \frac{r_i / R_g}{1 / N_g} \right] + \sum_{g=1}^{G} R_g \log \frac{R_g}{N_g / N}. \tag{5.2}$$

Result (5.2) reveals that revenue inequality consists of two distinct components: (i) a weighted average of within-set inequalities and (ii) a between-set inequality. The right-hand side of equation (5.2) parallels the decompositions given by equation (4.6). The meaning of the two components of equation (5.2) is discussed further in what follows.

5.1 The within-set inequalities

The first component of (5.2) is a weighted average of the within-set inequalities:

$$\sum_{g=1}^{G} R_g \left[\sum_{i \in \boldsymbol{S}_g} \frac{r_i}{R_g} \log \frac{r_i / R_g}{1 / N_g} \right]. \tag{5.3}$$

The term r_i / R_g is the conditional revenue share of SNG i within group $\boldsymbol{S}_g$, that is, SNG i's revenue share within the group. Also, N_g represents a number of SNGs in group $\boldsymbol{S}_g$.

Equation (5.3) comprises two weighted averages: (a) $Z_g = \sum_{i \in \boldsymbol{S}_g} \frac{r_i}{R_g} \log \frac{r_i / R_g}{1 / N_g}$, the

within-set revenue inequality for group $\boldsymbol{S}_g$, and (b) $\sum_{g=1}^{G} R_g Z_g$, the weighted average of the within-set revenue inequalities. We discuss each in turn.

If each SNG in set $\boldsymbol{S}_g$ receives an equal revenue share, then $r_i/R_g = k$ (say). However, as $\sum_{i\in \boldsymbol{S}_g}\left(r_i/R_g\right)=1$, it follows that $k = 1/N_g$. When each SNG has an equal share of the group's revenue, i.e., $r_i/R_g = 1/N_g$, $i \in \boldsymbol{S}_g$, then there is no dispersion of the revenue distribution within the group, the perfect equality. Accordingly, the extent to which the N_g ratios

$$\frac{r_i/R_g}{1/N_g}, \qquad i = 1,...,N_g \tag{5.4}$$

deviate from unity is a measure of revenue inequality within set $\boldsymbol{S}_g$. The within-set measure of revenue inequality, the term in square brackets of equation (5.3), is a weighted average of the logarithms of the ratios in equation (5.4), the weights being the conditional revenue shares.

5.2 The between-set inequality

The second term on the right-hand side of (5.2) is the between-set inequality:

$$\sum_{g=1}^{G} R_g \log \frac{R_g}{N_g/N}. \tag{5.5}$$

The basic ingredient of inequality (5.5) is the contrast between two sets of shares, the revenue shares of the G groups, $R_1,...,R_G$ and the corresponding population shares, $N_1/N,...,N_G/N$. If all groups receive their pro-rata shares of revenue based on population, i.e. $R_g = N_g/N$, $g = 1,...,G$, then there is no dispersion of revenue distribution and we have perfect between-set revenue equality.

In summary, total inequality consists of two components: the weighted average of the within-set inequality and the between-set inequality. Interestingly, it is clear that both components are of the form of the expected information content of an indirect message which was previously discussed in Section 3. For the within-set inequality, the prior and posterior probabilities are $1/N_g$ and r_i/R_g, respectively. Similarly, for a between-set

inequality, N_g/N and R_g are prior and posterior probabilities. Furthermore, from equation (5.2), the revenue inequality, can be written as:

$$\log N - H(\boldsymbol{r}) = \sum_{i=1}^{N} r_i \log N r_i = \sum_{i=1}^{N} r_i \log \frac{r_i}{1/N}. \tag{5.6}$$

The far right-hand side of equation (5.6) reveals that total revenue inequality can also be expressed in the form of the expected information content of an indirect message. In this case, the prior and posterior probabilities are $1/N$ and r_i, respectively. With this perspective, it is clear that the message that transforms the vector $[1/N,...,1/N]'$ into $[r_1,..., r_N]'$ is equivalent to two sub-messages. The first message transforms $[1/N_g,...,1/N_g]'$ into $[r_1/R_g,..., r_g/R_g]'$, $g = 1,..., G,$ which could be called "the within-set message", and the second message transforms $[N_1/N,..., N_G/N]'$ into $[R_1,..., R_G]'$, which is "the between-set message".

5.3 Why not per capita fiscal data?

The above fiscal inequalities are expressed in terms of the number of subnational governments, rather than a number of individuals. That is to say, according to our approach, per capita fiscal data are irrelevant for the measurement of fiscal decentralisation. Why?

Consider the two countries A and B discussed in Section 1. Country A has two local councils, each of the same size, whereas country B consists of 100 local councils (again, all of the same size). Revenue generated by country A is equal to the sum of revenue of the 100 councils in country B. As it has many more local councils, country B is more fiscally decentralised as compared to country A. This conclusion is perfectly reasonable and stands independently of the size of the population in the two countries, and how the population is distributed across the 100 local governments in country B.

Next, consider a real-world example from the fastest-growing state in Australia, Western Australia ("WA"). In terms of expenditure, the largest local government in WA is the City of Stirling, while the smallest is Shire of Three Springs. Columns 2 and 3 of

Table 2 show that expenditure in Stirling is almost 100 times greater than that in Three Springs. However, the population in Stirling is almost 250 times larger than that of Three Springs (columns 4 and 5). Spending per capita is therefore significantly higher in Three Springs than in Stirling, as presented in column 6.

TABLE 2
EXPENDITURE AND POPULATION IN TWO LOCAL COUNCILS
IN WESTERN AUSTRALIA, 2004

Local council	Expenditure		Population		Per capita expenditure	
	$' '000	Percent of WA total	Persons	Percent of WA total	$	Deflated
(1)	(2)	(3)	(4)	(5)	(6)	(7)
City of Stirling	100,405	5.97	182,047	9.06	552	0.66
Shire of Three Springs	1,338	0.08	722	0.04	1,853	2.00

Source: Unpublished ABS data.

Let s_i be the expenditure share of local council $i\,(i=1,2)$, and $S=\sum_{i=1}^{2}S_i$ be total expenditure, $s_i=S_i/S$ be the share, P_i be the population of i and $P=\sum_{i=1}^{2}P_i$ be total population, and $p_i=P_i/P$ be the corresponding share. Then the ratio of the expenditure share to the population share $\frac{s_i}{p_i}=\frac{S_i/S}{P_i/P}=\frac{S_i/P_i}{S/P}$ is "deflated" per capita expenditure of the i^{th} council. If all local councils receive their pro rata expenditure share based on population, then $s_i/p_i=1$ for each i. Column 7 of Table 2 shows that deflated per capita expenditure of Stirling and Three Springs is 0.66 and 2, respectively, so that Stirling receives much less its pro rata share and Three Springs much more. Accordingly, if per capita expenditure is considered, the smallest local government area, Three Springs, would, in effect, play a more important role in measuring the degree of fiscal decentralisation in WA. Such an approach clearly provides a misleading picture of the degree of fiscal decentralisation.

5.4 A note on notation

In the above discussion, the results are formulated in logarithmic terms. For future reference, it is convenient to take the antilogarithm of the inequality measure.

We start by expressing revenue inequality in terms of information theory as discussed in Section 3. Recall the second component on the right-hand side of equation (5.2), the between-set inequality, which is a weighted average of the logarithms of the ratios of the set revenue shares and the corresponding institutional shares, $\sum_{g=1}^{G} R_g \log \frac{R_g}{N_g/N}$. Let m_i and q_i be the revenue share and institutional share of the i^{th} region, that is, $m_i = M_i/M$, where M_i, M are the revenue of the i^{th} region and the total economy, and $q_i = Q_i/Q$, where Q_i, Q are the number of SNGs in the i^{th} region and the total number of SNGs in the economy. As a result, $\frac{m_i}{q_i} = \frac{M_i/M}{Q_i/Q} = \frac{M_i/Q_i}{M/Q}$. The numerator of this ratio is revenue per SNG of the i^{th} region, while the denominator is revenue per SNG. If $\boldsymbol{m} = [m_1, ..., m_N]'$ and $\boldsymbol{q} = [q_1, ..., q_N]'$, the between-region inequality can be expressed in terms of information theory as:

$$I(\boldsymbol{m}:\boldsymbol{q}) = \sum_{i=1}^{N} m_i \log \frac{m_i}{q_i}.$$

The ratio m_i/q_i is "deflated" per SNG revenue of the i^{th} set. The term "deflated" here means that revenue is expressed as relative to national revenue for SNG. The above $I(\boldsymbol{m}:\boldsymbol{q})$ is the logarithm of a weighted average of deflated revenue per SNG, so that the corresponding geometric mean is:

$$e^{I(\boldsymbol{m}:\boldsymbol{q})} = \prod_{i=1}^{N} \left(\frac{m_i}{q_i} \right)^{m_i}. \tag{5.7}$$

If all SNGs receive their pro rata share based on a number of SNGs, then $m_i/q_i = 1$ for each i, $\prod_{i=1}^{N} (m_i/q_i)^{m_i} = 1$ and there is no revenue dispersion. Accordingly, the further is the mean (5.7) away from unity, the greater is revenue inequality across sets. Similarly, on the expenditure side, the geometric mean is:

$$e^{I(\boldsymbol{s}:\boldsymbol{q})} = \prod_{i=1}^{N} \left(\frac{s_i}{q_i} \right)^{s_i}. \tag{5.8}$$

where $\boldsymbol{s} = [s_1, ..., s_N]'$ and $\boldsymbol{q} = [q_1, ..., q_N]'$ with s_i and q_i is the expenditure share and institutional share of the i^{th} region.

6. Australia's fiscal federalism

This section applies fiscal inequality measures to Australia. These inequalities are particularly relevant to the case of Australia because there is a great regional fiscal disparities. We start with a brief description of fiscal arrangements in Australia.

The Commonwealth of Australia was established in 1901 as a Federation in which six self-governing British colonies became the six states of Australia. The main purpose of this unification was to form a strong and open country by eliminating tariff on interstate trade. More than one century after its formation, modern Australia is still seen as a "young" country in comparison with many nations from the "old" world. The Commonwealth of Australia now consists of six states and two territories (hereafter referred to in aggregate as the "States") with a total number of local councils of 700. The eight "states" of Australia are New South Wales (NSW), Victoria (VIC), Queensland (QLD), South Australia (SA), Western Australia (WA), Tasmania (TAS) and two territories, Northern Territory (NT) and Australian Capital Territory (ACT). The first tier of government is occupied by the Commonwealth government. The second tier is represented by state governments. The third and lowest tier of governments is represented by local councils. In geographic terms, the three levels of governments are not mutually exclusive. The geographic region associated with each state includes both a state and many local governments. The geographic area associated with the Commonwealth government also includes state and local governments. As such, the fiscal authority of different levels of government overlaps – residents in each local government are influenced by fiscal activities of local, state, and Commonwealth governments.

However, ACT has no local governments because of its special nature of administration. As such, the ACT government performs two roles: one as the "state" government and another role as the "local" government. On this basis, the total number of local governments in Australia of 700 is allocated to seven "states", namely NSW (192 local governments), VIC (79), QLD (125), SA (68), WA (143), TAS (29), and NT (64).

6.1 Revenue and expenditure patterns

Table 3 reveals that the allocation of revenue and expenditure across local councils in Australia is significantly dispersed. Brisbane City Council in Queensland is the largest local council in Australia with revenue and expenditure shares of 7.7% and 6.9% of all local councils, respectively. The second and third biggest councils are the Gold Coast Council (QLD) and Melbourne City (VIC). On the other hand, Timber Creek (NT) is the smallest council with revenue and expenditure shares of around 0.0003% and 0.0019%, respectively. As can be seen from Figures 2 – 4, there is considerable dispersion of revenue and expenditure within and between states.

TABLE 3
REVENUE AND EXPENDITURE SHARES,
LOCAL COUNCILS, AUSTRALIA, 2000 - 2004

No.	Region	Number of local councils	Revenue shares (Percent of total)					Expenditure shares (Percent of total)				
			Mean	Median	Standard deviation	Min	Max	Mean	Median	Standard deviation	Min	Max
1	NSW	192	0.1744	0.0847	0.2082	0.0013	1.2487	0.1704	0.0913	0.1848	0.0015	0.8386
2	VIC	79	0.2559	0.1758	0.2410	0.0237	1.6629	0.2784	0.1946	0.2172	0.0273	1.3578
3	QLD	125	0.2191	0.0603	0.7495	0.0098	7.7079	0.2039	0.0609	0.6666	0.0144	6.9045
4	SA	68	0.0865	0.0459	0.1155	0.0056	0.7069	0.0899	0.0539	0.1034	0.0079	0.5400
5	WA	143	0.0617	0.0189	0.1016	0.0041	0.6308	0.0637	0.0257	0.0872	0.0069	0.5387
6	TAS	29	0.1004	0.0569	0.1156	0.0163	0.4592	0.1006	0.0602	0.1082	0.0210	0.4587
7	NT	64	0.0203	0.0096	0.0357	0.0003	0.2651	0.0259	0.0139	0.0364	0.0019	0.2665

Source: Unpublished data from ABS. Data are averages over the period 2000 – 2004.

FIGURE 2
DISTRIBUTION OF LOCAL COUNCIL SHARES
AUSTRALIA, 2000 – 2004
(Percent of total)

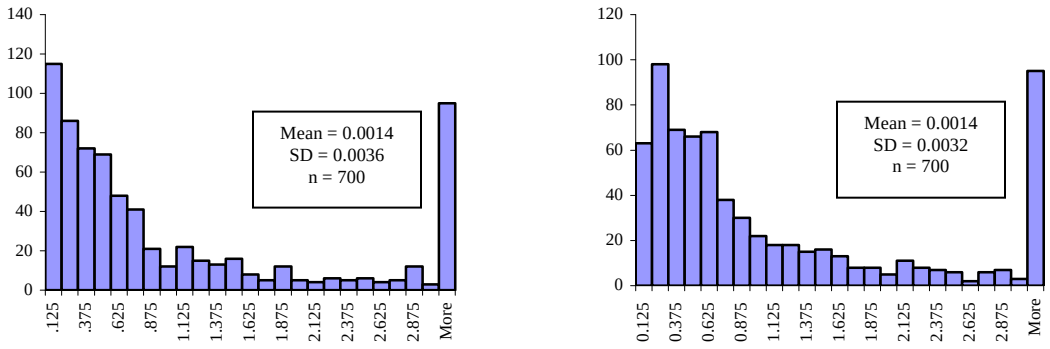


FIGURE 3
DISTRIBUTION OF REVENUE ACROSS LOCAL COUNCILS
AUSTRALIAN STATES, 2000 – 2004
(Percent of total)

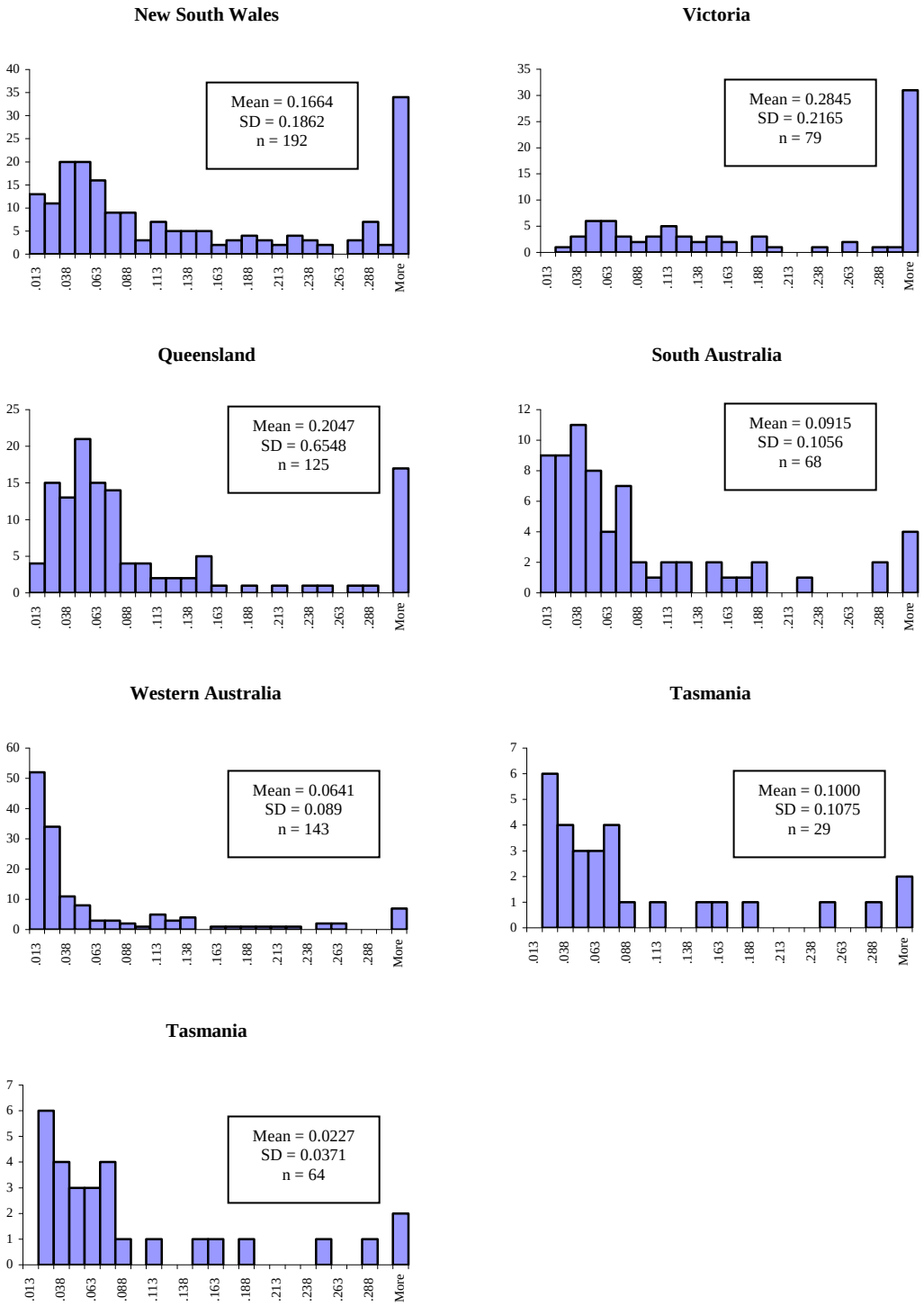
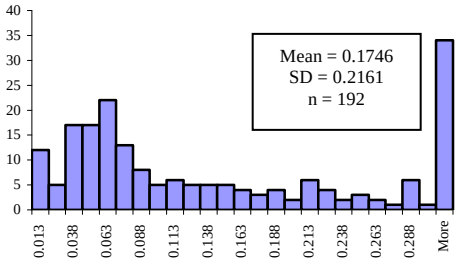
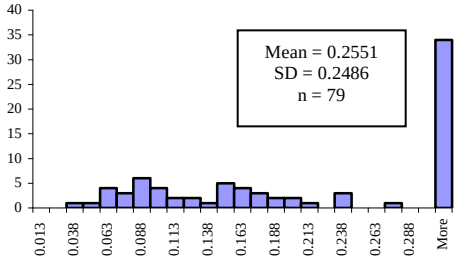


FIGURE 4
DISTRIBUTION OF EXPENDITURE ACROSS LOCAL COUNCILS
AUSTRALIAN STATES, 2000 – 2004
(Percent of total)

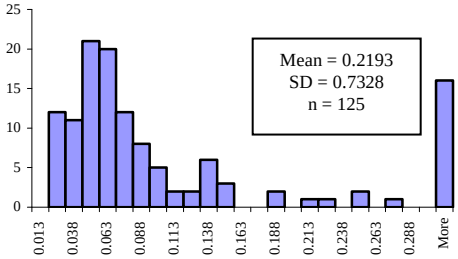
New South Wales



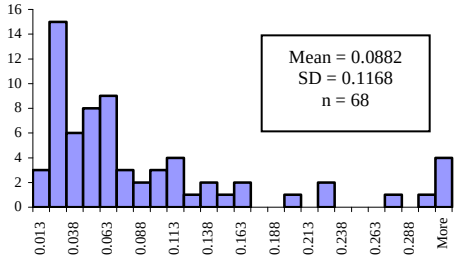
Victoria



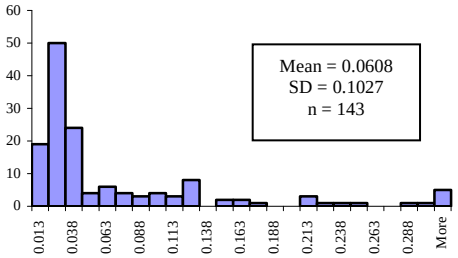
Queensland



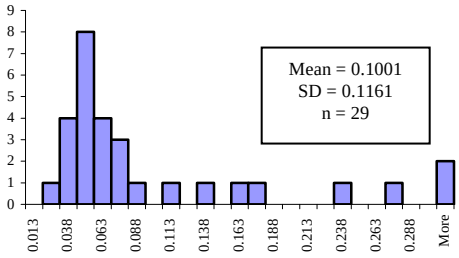
South Australia



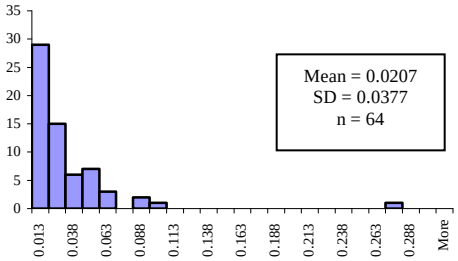
Western Australia



Tasmania



Northern Territory



6.2 Regional and hierarchical fiscal inequality

Table 4 provides a framework for the analysis of fiscal inequality when SNGs are identified geographically. Here, each subnational region consists of the state government and a number of local governments. Each row of the table represents one of the G regions in the country. Consider region g as an example. As indicated in column 2, there are n_g local councils in this region plus one state government, so there are $n_g + 1$ revenue shares, $r_{g1},..., r_{gn_g}, r_{g,n_g+1}$. Total revenue for g is $\sum_{k=1}^{n_g+1} r_{g,k} = R_g$, as presented in column 3. These total regional shares sum over the G regions to unity, that is, $\sum_{g=1}^{G} R_g = \sum_{g=1}^{G} \sum_{k \in \mathbf{S}_g} r_{gk} = 1$, as indicated by the last element of column 3. Column 4 of the table presents the number of all SNGs in each region, $N_1,..., N_G$, as well as the total number in the whole economy, N.

TABLE 4

THE ANALYTICS OF THE GEOGRAPHIC ALLOCATION OF REVENUE

Region	Revenue shares of subnational region		Number of subnational regions (state and local councils)
g	Individual shares	Total	
(1)	(2)	(3)	(4)
1	$r_{11},..., r_{1n_1}, r_{1,n_1+1}$	$R_1 = \sum_{k=1}^{n_1+1} r_{1k}$	$N_1 = n_1 + 1.$
⋮	⋮	⋮	⋮
g	$r_{g1},..., r_{gn_g}, r_{g,n_g+1}$	$R_g = \sum_{k=1}^{n_g+1} r_{gk}$	$N_g = n_g + 1.$
⋮	⋮	⋮	⋮
G	$r_{G1},..., r_{Gn_G}, r_{G,n_G+1}$	$R_G = \sum_{k=1}^{n_g+1} r_{Gk}$	$N_G = n_G + 1.$
Total		$\sum_{g=1}^{G} \sum_{k \in \mathbf{S}_g} r_{gk} = 1$	$N = \sum_{g=1}^{G} N_g$

To apply the above framework to the Australian case, we have $G = 7$, as there are six states and one territory that contain local councils (the ACT is excluded as it has no local councils). Each of the seven SNGs contains one state government and a number of

local councils. Take Western Australia as an example. As there are 143 councils in this state, there are 143 revenue shares $r_{g,1},...,r_{g,143}$, for $g = WA$, while the revenue share for the WA state government is $r_{g,144}$. The total of these 144 shares, $\sum_{k=1}^{144} r_{g,k} = R_g$, is the share of national revenue accounted for by WA. For Australia as a whole, there are 700 local councils and 7 states, so $N = 707$.

In accordance with the analysis in Section 5, total revenue inequality for Australia with $N = 707$ and $G = 7$ is:

$$\log 707 - H(\boldsymbol{r}) = \sum_{g=1}^{7} R_g \left[\sum_{i \in \boldsymbol{s}_g} \frac{r_i}{R_g} \log \frac{r_i / R_g}{1/N_g} \right] + \sum_{g=1}^{7} R_g \log \frac{R_g}{N_g / 707}.$$

The first component on the right-hand side is the within-state inequality for revenue shares of local and state governments:

$$\sum_{g=1}^{7} R_g \left[\sum_{i \in \boldsymbol{s}_g} \frac{r_i}{R_g} \log \frac{r_i / R_g}{1/N_g} \right],$$

The second component on the right-hand side is the between-set inequality:

$$\sum_{g=1}^{7} R_g \log \frac{R_g}{N_g / 707}.$$

Table 5 reveals that within-state fiscal inequality accounts for 96.3% and 96.9% total inequality in terms of revenue and expenditure, respectively. As a result, it is clear that the within-state fiscal inequality plays a more important role in total inequality of the distribution of revenue and expenditure across subnational regions in Australia. This is partly because each subnational region includes both state and local governments, and the state government is significantly larger than any local government within the same region. For example, for NSW, the total share $R_g = 31.7\%$ in 2004, the state government accounts for $r_{g,n_{g+1}} = 27.1\%$, leaving only 4.6% to be divided among the 192 local governments in NSW. Another reason for the dominance of the within-state component of fiscal inequality is the operation of the system of fiscal equalisation in Australia. Fiscal equalisation has a tendency to equal per capita revenue and expenditure among states, which causes the between-state to be low, or the within-state inequality to be high.

TABLE 5
GEOGRAPHIC ALLOCATION OF
FISCAL INEQUALITIES ACROSS SUBNATIONAL REGIONS
AUSTRALIA, 2004

Inequality measure	Revenue	Expenditure
Total inequality	1.727	1.763
Between-set inequality	0.063	0.054
Within-set inequality (WSI)	1.664	1.709
Inequality within:		
New South Wales	0.573	0.622
Victoria	0.348	0.343
Queensland	0.365	0.340
South Australia	0.106	0.123
Western Australia	0.228	0.215
Tasmania	0.029	0.034
Northern Territories	0.015	0.032
WSI as the percentage of total inequality	96.3	96.9

Total inequality can also be disaggregated in a hierarchical manner in which the two sets to be considered are: (i) the upper-level SNGs, the set consisting of the seven states and territories; and (ii) the lower-level SNGs, the 700 local councils. Table 6 below presents the results when fiscal inequality is decomposed in this way. The results show that when local councils and states are completely isolated in this way, the between-set inequality is much larger than the within-set inequality. The between-set inequality

TABLE 6
HIERARCHICAL ALLOCATION OF
FISCAL INEQUALITIES ACROSS LEVELS OF GOVERNMENTS
AUSTRALIA, 2004

Inequality measure	Revenue	Expenditure
Total inequality	1.727	1.763
Between-set inequality (BSE)	1.552	1.613
Within-set inequality	0.175	0.150
Inequality within:		
States governments	0.118	0.108
Governments of local councils	0.057	0.042
BSE as the percentage of total inequality	89.9	91.5

between the states and local councils accounts for about 89.9% of revenue inequality and 91.5% of expenditure inequality. This result also reflects the ideas discussed in the previous paragraph.

6.3 The applications to the second and third approximations

We now use the Australian data to illustrate the second and third approximations to the FDIs. We indicate how the previous development of the fiscal inequality can be used to extend the fiscal decentralisation indexes. These ideas are still preliminary and some detail still remains to be worked out.

FIGURE 5
REVENUE AND EXPENDITURE INEQUALITIES,
AUSTRALIAN STATES, 2004

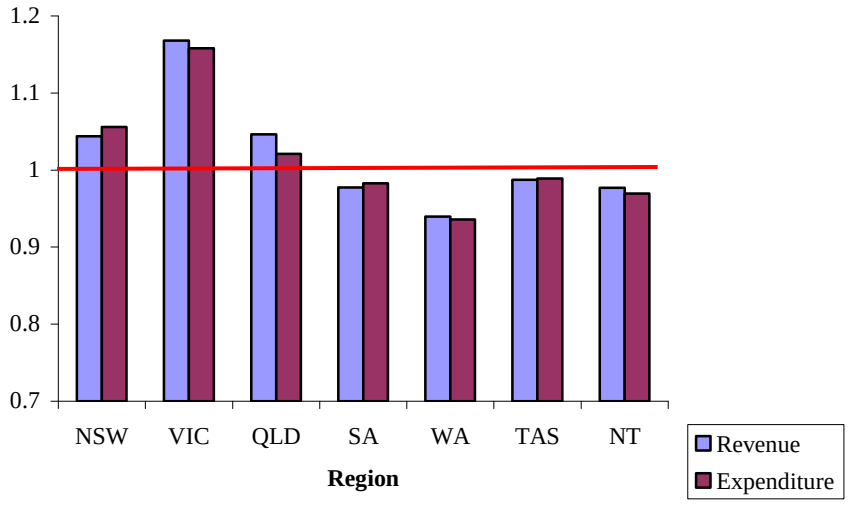


Figure 5 presents the results for the between-state fiscal inequalities for revenue and expenditure, using the geometric mean of revenue [equation (5.7)] and expenditure [equation (5.8)], for the $N = 7$ states. As previously discussed, if a region receives its pro rata revenue (or expenditure) share, then there is no revenue (or expenditure) dispersion across regions. Figure 5 reveals that, NSW, VIC and QLD receive on average more revenue than it would be justified based on pro rata share. On the other hand, SA, WA, TAS, and NT receive less revenue. These results confirm the view that there exists a dispersion of revenue and expenditure across states in Australia. As a consequence, the

first approximation index, equation (2.3), may not be a comprehensive measure of fiscal decentralisation in Australia.

The first approximation index (2.3) only shows the aggregate of total revenue and expenditure of subnational regions. When the dispersion of fiscal shares (for both revenue and expenditure) across states is insignificant, the first approximation index will be adequate. However, when there is a significant dispersion of revenue and expenditure across states, the first approximation may not be an accurate measure of the true degree of fiscal decentralisation because the impact of dispersion is not accounted for. In this case, a second approximation index is needed. The second approximation index modifies the first approximation index by incorporating the means (5.7) and (5.8).

The differences in terms of revenue and expenditure across subnational regions are considered by employing the between-set inequality of the distribution of revenue and expenditure across subnational regions. However, fiscal differences among local councils within the states have still been ignored. From Figures 3 and 4, it is clear that for all states there is a significant dispersion of revenue shares and expenditure shares across local councils within the same state. Consider again the local councils in WA. In terms of revenue and expenditure, the City of Stirling is the biggest local council in WA with its shares of 7.46% and 5.97% total region's revenue and expenditure, respectively. By contrast, the revenue share and expenditure share for the smallest local council in this state is 0.04% (the Shire of Nungarin) and 0.08% (the Shire of Three Springs). As the consequence, the differences among councils within the same state should also be considered. As such, total fiscal inequality among subnational regions, as discussed in Section 5, is incorporated in the third approximation of the FDI.

7. Concluding remarks

Economic aspect of fiscal decentralisation has recently attracted a noticeable increase in attention from academics and international institutions such as the World Bank. The question has been raised how fiscal decentralisation across countries can be measured. The main contribution of this paper is that it develops the fiscal decentralisation index ("FDI"), known as the first approximation index, which takes into

account both fiscal autonomy and fiscal importance of subnational governments – two corner-stones in the literature of fiscal federalism. Fiscal autonomy of subnational governments is defined by the ratio of own sourced revenue and expenditure made by subnational governments. To facilitate comparisons across countries, this ratio is subject to an adjustment factor that considers differences in fiscal arrangements such as the share of unconditional grants in total grants received, the extent to which subnational governments can access financial markets, and so on. While the first approximation is fundamental in nature, it ignores the role of dispersion of revenue and expenditure across subnational regions.

One of the main ideas of the paper can be illustrated with a simple example. Consider two hypothetical nations M and N which consist of four subnational regions: A, B, C and D, each with different level of revenue. It is assumed that government spending and revenue at the national level accounts for 50 percent of the total, so that the remaining 50 percent is the responsibility of subnational government. For simplicity, it is further assumed that expenditure of each SNG is equal to its revenue. Table 7 provides data for this example.

TABLE 7
ILLUSTRATING FISCAL INEQUALITY

	Region	Country M				Country N			
		Revenue	Share in total			Revenue	Share in total		
		($)	Actual	Average	Difference	($)	Actual	Average	Difference
	(1)	(2)	(3)	(4)	(5) = (4) – (3)	(6)	(7)	(8)	(9) = (8) – (7)
1	A	3,000	0.010	0.250	0.240	3,300	0.011	0.250	0.239
2	B	125,000	0.427	0.250	-0.177	271,390	0.926	0.250	-0.676
3	C	97,000	0.331	0.250	-0.081	10,810	0.037	0.250	0.213
4	D	68,000	0.232	0.250	0.018	7,500	0.026	0.250	0.224
5	Total	293,000	1.000	1.000	0.000	293,000	1.000	1.000	0.000
6	FDI		0.500	0.500			0.500	0.500	
7	Standard deviation		0.178	0.000			0.451	0.000	
8	Entropy		0.484	0.602			0.146	0.602	
9	Fiscal Inequality		0.118	0.000			0.456	0.000	

Revenue raised by all SNGs is equal in two countries, as presented in row 5 of columns 2 and 6. Total government expenditure, the sum of spending made by the national government and all SNGs, is $293,000 \times 2 = 586,000$. Column 2 shows that there is one

small region in country M, region A. Revenue from region B is almost double that of D and forty times higher than that of region A. Columns 3 and 4 present the actual and average revenue shares for 4 regions in country M. By contrast, in country N, there is one large and three small regions. Region B accounts for more than 92% of the total revenue of all regions, and the remaining 8% is spread across the three small regions A, C, and D.

Table 7 contains the following important points on the relationship between the first approximation index, entropy and fiscal inequality:

- The Adjustment Factor ("AF") is assumed to be 0.5 for both countries M and N. Using the index developed in Section 2.3, the first approximation to the FDI for both countries is: $FDI = \sqrt{\left(\frac{293,000}{293,000} \times 0.5\right)\left(\frac{293,000}{586,000}\right)} = 0.5.$ The same value of FDI applies to these countries irrespective of the distribution of revenue, as indicated by row 6 and columns 3, 4, 7, and 8. This reflects the fact that the first approximation only considers the aggregate level of revenue and expenditure of SNGs.
- Row 7 presents the standard deviations of the revenue shares of the two, 0.178 and 0.451. This clearly reveals that the distribution of revenue of country N is more dispersed than in M.
- Row 8 gives the values of the fiscal entropy, defined as $-\sum r_i \log r_i,$ where r_i is the revenue share of SNG $i.$ The entropy value in country M is 0.484 and 0.146 in country N, as shown in columns 3 and 7 of row 8, respectively. If we were to assume alternatively that each region accounts for the same share of 25%, as shown by columns 4 and 8, there is no inequality, so that fiscal entropy for both countries is $\log 4 = 0.602,$ as in row 8, columns 4 and 8.
- Row 9 presents the fiscal inequality, the difference between the maximum level of the entropy, $\log 4,$ or 0.602, and the actual level. Fiscal inequality is 0.118 and 0.456 for countries M and N, respectively. Higher fiscal inequality in N means a greater degree of revenue dispersion, and as a result, a lower degree of fiscal decentralisation because revenue is allocated more disproportionately across regions.

To summarise this example, according to the first approximation index, both countries exhibit the same degree of fiscal decentralisation. But as there is much more fiscal inequality in country N, it can be reasonably concluded that the true situation is markedly different: there is less fiscal decentralised in country N. This shows that the first approximation provides a misleading picture of the degree of fiscal decentralisation because it ignores the dispersion of revenue (and expenditure) across regions. As a result, further development of the first approximation index to reflect dispersion is desirable.

Another key idea of the paper is the analysis on the composition of fiscal inequality. Fiscal inequality can be decomposed into between-set and within-set components. The set can be defined in one of two alternative ways:

- *Geographically* whereby the country is split into a number of regions, and then fiscal inequality across SNGs contained in each region is examined. For example, in Australia there are eight states and territories, each containing one state government and a number of local governments (the only exception is the ACT that contains no local government).
- *Hierarchically* whereby the country is divided into two groups: (i) all state governments; and (ii) all local governments. For Australia, seven state and territory governments (excluding the ACT) are in one group and the 700 local governments are in the other.

FIGURE 6
FISCAL INEQUALITY BY COMPONENT
AUSTRALIA, 2004

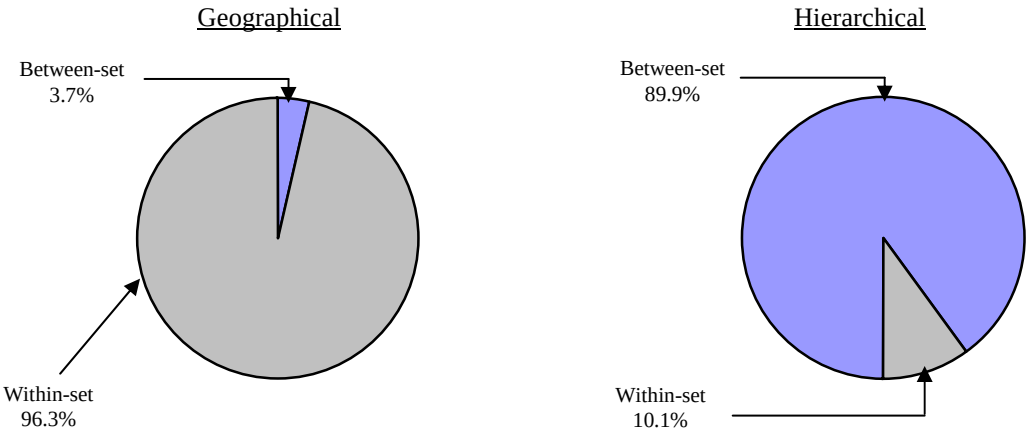


Figure 6 presents a summary of the results of fiscal inequality in Australia by component. The results reveal that within-set inequality plays a significant role when the set is defined on a geographic basis. The insignificance of the between-set inequality for regions can be partly explained by the application of fiscal equalisation in Australia whereby the Federal government allocates GST revenue among the states in a manner that gives the states equal capacity to provide a standard level of service provided their revenue raising (i.e. tax and royalty) is the same. Another reason for the dominance of the regional within-set component is that each region contains state and local governments; and in most cases, the state government is substantially larger than local governments. By contrast, when groups are defined hierarchically, the between-set inequality accounts for about 90% of the total inequality. This result reflects the fact that state governments account for a significant share in total revenue or expenditure of subnational governments.

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