UNDERSTANDING THE WORLD WOOL MARKET: TRADE, PRODUCTIVITY AND GROWER INCOMES

PART III: A MODEL OF THE WORLD WOOL MARKET*

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CHAPTER 3

A MODEL OF THE WORLD WOOL MARKET

3.1 Preamble

The purpose of this chapter is twofold: first, to present the theoretical structure of the model to be applied in Chapters 5 and 6; second, to derive alternative model closures. This and the next section provide a brief overview of the methodology underlying the theoretical structure of the model.

The model presented here is a comparative-static general-equilibrium model of the world economy, with a focus on the world wool market. We refer to the model as WOOLGEM: WOOL General Equilibrium Model. WOOLGEM represents the workings of the world wool market in very detailed form, as well as representing the nonwool global and regional economies in highly aggregated form. The principal purpose in constructing WOOLGEM is to provide projections of changes in endogenous variables, such as wool growers' incomes, exports and imports, due to changes in exogenous variables, such as factor productivity and import protection. When a nonzero shock is applied to an exogenous variable in WOOLGEM, the resulting projections of changes in endogenous variables indicate the variation in these variables from the values they would have had in the absence of the change in the exogenous variable, i.e., the model is comparative-static and provides no time path for changes in endogenous variables between initial and terminal equilibrium values.

The theoretical structure of WOOLGEM is flexible enough to allow the adoption of different assumptions with respect to the economic environment the user wishes to simulate. The different economic environments on which we will focus in this work are what are commonly termed the 'short run' and the 'long run'. The choice of economic environment affects the choice of both microeconomic, or industry, variables and macroeconomic variables, on the endogenous and exogenous lists, i.e., the model closure. These assumptions affect the response of the endogenous variables in each representative region and, as such, affect the projections that WOOLGEM generates when a shock is applied to an exogenous variable.

3.1.1 Model overview

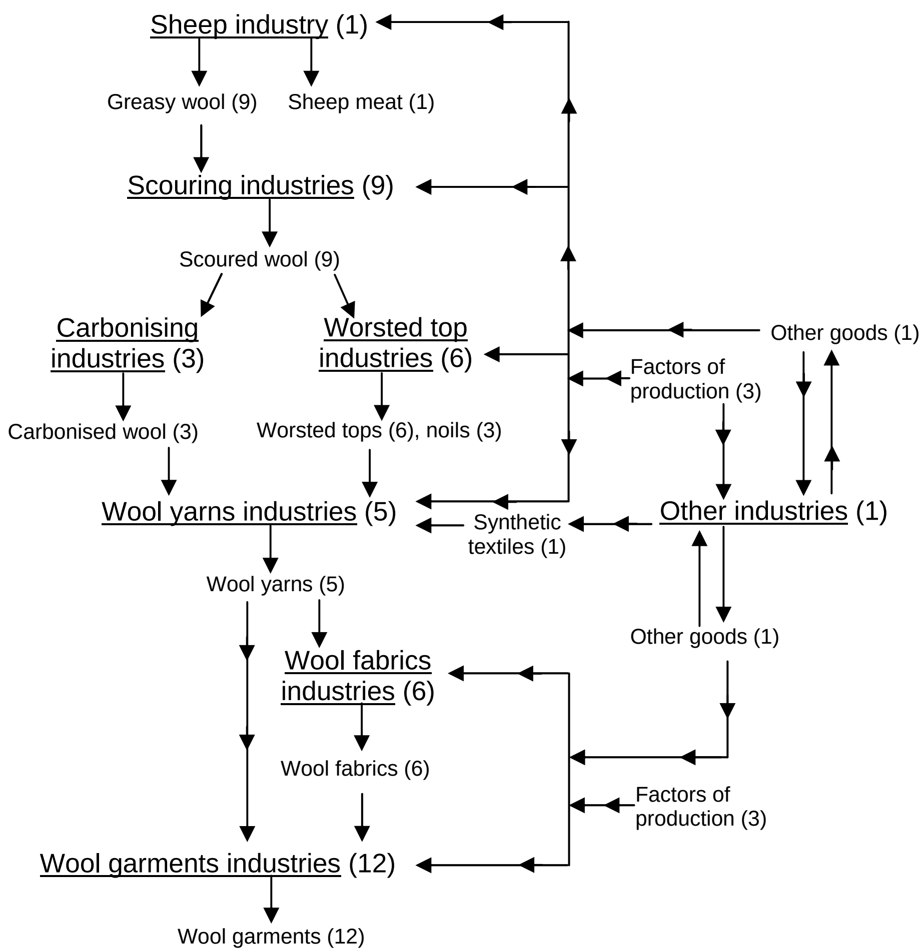
The WOOLGEM model represents a synthesis of two modelling traditions: (i) the partial-equilibrium commodity-specific approach and (ii) the computable-general-equilibrium approach.

Applying aspects of the first tradition to wool, the model represents the production of nine qualities of wool, distinguished by diameter and hauteur (or length). These nine qualities are then tracked through five successive processing stages, after which twelve different types of wool garments are consumed by a representative household. All of these activities are represented in different regions of the world. Production, processing and household demand for raw wool, wool textiles and wool garments vary significantly across regions of the world, so that significant trade occurs for all classes of products.

Applying aspects of the second tradition to wool, the model contains a comprehensive representation of the nonwool economy, i.e., a representation of the economy as a complete system of interdependent components – industries, households, investors, governments, importers and exporters (Dixon et al. 1992). As such, it completes and complements the commodity-specific aspects described above, by linking the wool economy in each region with the nonwool economy through domestic factor markets, domestic and international markets for intermediate inputs, and domestic and international markets for goods consumed by households. Further, it constrains the behaviour of the wool economy to assumptions about macroeconomic behaviour, such as a balance of trade constraint and household and government consumption constraints, in individual regions and globally. All of this is done at minimum cost, in terms of industry and commodity detail, by representing nonwool industries and commodities as a single composite industry and commodity.

Figure 3.1 summarises the industry and commodity structure of the model. The figure shows the dichotomous nature of the model: a detailed representation of the wool economy showing the processing stages through which greasy wool passes on its way to becoming wool garments; and a composite representation of the nonwool economy that is, nevertheless, fully linked to the wool economy through intermediate input and primary factor markets. The wool economy is represented as having a linear hierarchy where outputs from downstream processing industries are not used as inputs by upstream processing industries. This conforms to the 'Austrian' view of production. In contrast, the nonwool economy is represented as having 'whirlpools' of production and general interdependence between all the industries it represents via direct or indirect intermediate input usage, so that the other industries composite is a net supplier of the other goods composite. This conforms to a 'Leontief' view of production (Blaug 1978, p. 544; Dorfman et al. 1987, p. 205).

Figure 3.1 The industry and commodity structure of the WOOLGEM model



Note: Bracketed figures indicate the number of individual industries, commodities or factors of production in each region. Arrows indicate flows of inputs (commodities and factors of production) and outputs (commodities only) between industries.

3.2 A linear equation system

WOOLGEM can be represented as[1] Equation Section 3

$$\begin{aligned} F_1(X_t, K_t, L_t) &= 0 \\ &\vdots \\ F_m(X_t, K_t, L_t) &= 0, \end{aligned} \tag{3.1}$$

where K_t is the vector of industry capital stocks at time t and is assumed to be determined by investment decisions undertaken before time t. The vector of industry employment at

[1] This section draws on Dixon et al. (1982), Section 10.

time t, L_t, is determined by current hiring decisions. X_t is the vector of all other variables, both exogenous (e.g., factor productivity, tariff rates) and endogenous (e.g., prices, outputs). $F_i\left(i=1,\ldots,m\right)$ are m differentiable and continuous functions. Behavioural relationships (e.g., production and utility functions) and equilibrium conditions (e.g., market clearing and zero pure profits) are imposed on WOOLGEM via (3.1); thus there are *m* such relationships and conditions in WOOLGEM. As such, the values of all endogenous variables in (3.1) are equilibrium values, and any perturbation of the exogenous variables will lead to new *equilibrium* values for all endogenous variables.

We know from (3.1) that WOOLGEM contains *m* equations. There are also *e* exogenous variables that the user can shock to project changes in endogenous variables. Thus, *m* equations plus *e* exogenous variables gives the total number of components, *p*, in the model. The selection of the *e* (= *p* – *m*) exogenous variables will partly depend on the economic environment the user wishes to impose in any given simulation.

With the *e* exogenous variables and the setting of the values of behavioural parameters chosen, we can, in principle, solve (3.1). For instance, a short-run solution can be represented as[2]

$$\begin{bmatrix} X_t(N) \\ L_t \end{bmatrix} = G\begin{bmatrix} X_t(X) \\ K_t \end{bmatrix}; \qquad (3.2)$$

[2] When we refer to the short run here, and in the section on model closures, we are thinking of the length of time that "…must be long enough for local prices of imports to fully adjust to tariff increases, for major import users to decide whether or not to switch to domestic suppliers, for domestic suppliers to hire labour and to expand output with their existing plant, for new investment plans to be made but not completed, and for price increases to be passed onto wages and wage increases passed back to prices" (Dixon et al. 1982, pp. 65–6).

where $\begin{bmatrix} X_t(N) \\ L_t \end{bmatrix}$ are the endogenous subsets of (X_t, L_t), $\begin{bmatrix} X_t(X) \\ K_t \end{bmatrix}$ are the exogenous subsets of (X_t, K_t), and G is a vector function of length *m*. Thus, L_t is treated as endogenous whereas K_t is treated as exogenous in the short-run environment of WOOLGEM.

Using (3.2) we can calculate the short-run effects of shocks to any exogenous variables on any of the endogenous variables as

$$\begin{bmatrix} dX_t(N) \\ dL_t \end{bmatrix} = \left(\nabla G\right) \begin{bmatrix} dX_t(X) \\ \mathbf{0} \end{bmatrix}, \tag{3.3}$$

where ∇G is the matrix of first-order partial derivatives of G of dimension $m \times e$. Here, $\begin{bmatrix} dX_t(N) \\ dL_t \end{bmatrix}$ is the vector of changes in the equilibrium values of the endogenous variables due to the shocks to the vector of exogenous variables, $dX_t(X)$. $\begin{bmatrix} dX_t(N) \\ dL_t \end{bmatrix}$ represents changes in equilibrium values because equilibrium conditions are imposed on (3.3) via (3.1). Furthermore, $\begin{bmatrix} dX_t(N) \\ dL_t \end{bmatrix}$ only represents the impact of the changes in $dX_t(X)$; it does not represent actual changes in $\begin{bmatrix} dX_t(N) \\ dL_t \end{bmatrix}$ over any particular time period.

The underlying economic behaviour in WOOLGEM is highly nonlinear [see (3.1)] but is specified in linear form; thus WOOLGEM is a general-equilibrium model specified in derivative form. Linearisation of (3.1) is completed by totally differentiating each equation giving a system of linear homogeneous equations, i.e.,

$$\nabla F_i(dX_t, dK_t, dL_t) = \mathbf{0}, \tag{3.4}$$

where ∇F_i (i = 1,…,m) is a vector of first-order partial derivatives of F_i. (3.4) can be rewritten as

$$A\mathbf{v} = \mathbf{0}, \tag{3.5}$$

where A is an $m \times p$ matrix and $\mathbf{v}$ is the vector $(x_t, k_t, l_t)'$, with x_t, k_t and l_t being vectors of *percentage* changes in the elements of the vectors X_t, K_t and L_t.[3] Using (3.5) allows us to write (3.3) without having to use (3.2). Thus, we can avoid finding the explicit forms for the functions G in (3.2), and we can therefore write percentage changes (or changes) in the endogenous variables as linear functions of the percentage changes (or changes) in the exogenous variables, as in (3.3). To do this, we rearrange (3.5) as

$$A_n \boldsymbol{n} + A_x \mathbf{x} = \mathbf{0}, \tag{3.6}$$

where $\boldsymbol{n}$ and $\boldsymbol{x}$ are vectors of percentage changes in endogenous and exogenous variables. A_n are A_x are $m \times m$ and $m \times e$ matrices formed by selecting columns of A corresponding to $\boldsymbol{n}$ and $\boldsymbol{x}$. The percentage-change forms of (3.3) are then obtained by subtracting $A_x \boldsymbol{x}$ from both sides of (3.6) and premultiplying both sides by A_n^{-1} giving the reduced form, i.e.,[4]

$$\boldsymbol{n} = -A_n^{-1} A_x \mathbf{x}. \tag{3.7}$$

In this way, the many nonlinearities that underlie WOOLGEM and that are represented in (3.2) are avoided. Computing solutions to an economic model using (3.7) and assuming the coefficients of the A matrices are constant, is the method pioneered by Johansen (1960).

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[3] x_t, k_t and l_t can also be interpreted as vectors of natural-logarithmic changes in the elements of the vectors X_t, K_t and L_t, and this is true for all percentage changes presented in the remainder of this chapter.

[4] For A_n^{-1} to exist requires that A_n be square, which it is as it is of order $m \times m$, and that it have a nonzero determinant, which cannot be guaranteed. Nevertheless, we will proceed on the assumption that A_n^{-1} exists.

3.2.1 Linearisation errors

(3.7) only provides an approximate solution to the endogenous variables in (3.1) from shocks applied to the exogenous variables in (3.1). For a marginal change in $\boldsymbol{x}$ the approximation is accurate. But for a discrete change in $\boldsymbol{x}$ the approximation is inaccurate. The problem here is the standard one of numerical integration; that is,

$$dN = G\left[X^T\right] - G\left[X^I\right], \tag{3.8}$$

where dN is the change in the endogenous variables due to the change in the exogenous variables, X, from X^I (the initial values) to X^T (the terminal values).[5] Here we are assuming that the G functions are differentiable and continuous, so that a solution exists for the model underlying (3.8).

The problem of accurately calculating dN in (3.8), which is equivalent to allowing the coefficients of the A matrices in (3.7) to be nonconstant, is solved by breaking the change in the exogenous variables from X^I to X^T into i equal parts or, in the case of (3.7), breaking the percentage change in $\boldsymbol{x}$ into i equal percentage changes. The multistep solution procedure requires that there are many intermediate values of N in (3.8) between moving from X^I to X^T. The intermediate values of N are obtained by successively updating the values of N after the each of the i steps is applied. Once the value of N is updated for any given step, the coefficients of the A matrices in (3.7) are recomputed before (3.7) is solved again.

Updating of N occurs using formulae of the form

$$N^{new} = N^{old}\left(1 + \frac{n}{100}\right), \tag{3.9}$$

[5] The problem is discussed at length in Dixon et al. (1992), pp. 109–24.

where n is the percentage change in N from the current step. In WOOLGEM, N is typically a (dollar) value flow whose original value is taken from the initial database. Such flows are usually the product of prices and quantities. Thus n in (3.9) can be replaced by $(p+q)$, i.e., percentage change in price plus the percentage change in quantity. The specification of the update formulae ensures that there is a clearly defined system of nonlinear equations underlying the linearised representation in (3.7) (Hertel et al. 1992). WOOLGEM is implemented using the GEMPACK economic modelling software and the solved using the Gragg multistep algorithm available therein (Harrison and Pearson 1996).

3.2.2 Presentation and notation

In the following sections we describe the equation system of WOOLGEM, i.e., (3.1), in linearised form, i.e., (3.5). We do not present any derivations of the linearised equations but, instead, refer readers interested in the lineage of the linearised equations from their levels form to Chapter 2, which presents such derivations.

The presentation of the equations is done in thematic order. Note also that we will follow the notational convention of using upper case letters to denote variable levels and corresponding lower case letters for their percentage changes; lower case Greek letters are used to denote elasticity parameters. Further, only behavioural equations will be presented in linearised form; indices and accounting identities will be presented in levels form. We believe this aids in conveying the intuition of the model theory as behavioural equations are most easily interpreted in elasticity form, while indices and accounting identities are most easily interpreted in levels form.

WOOLGEM contains five classes of representative economic agents in each region: a firm for each industrial sector, a capital creator, a household, a government and an importer.[6] Due to the rich tax structure of the model, each representative agent has a unique purchaser's price; this is in addition to the basic or pre-tax price variable upon which all purchasers' prices are based. Thus we are forced to present the variables using an extensive notational convention, as outlined in Table 3.1.

Table 3.1 Notational convention for WOOLGEM variables

Prefixes		Suffixes		Superscripts	
P, p	price or price index	F, f	firms	F	primary factors
Q, q	quantity or quantity index	CRSH, crsh	CRESH	NF	non-primary factors
V, v	nominal value	CRTH, crth	CRETH	I	intermediate inputs
A, a	technical change	I, I	investors	T	trade
Y, y	income	H, h	households	B	broad composite
T, t	tax rate	X, x	exports	C	composite
TR, tr	tax revenue	G, g	governments	NM	nonmargin (exports)
Z, z	shift variable	S, s	stocks	M	margin (exports)
Θ	marginal share	M, m	importers		
W	budget share	D, d	domestic		
σ	substitution parameter				
θ	transformation parameter				
φ	income flexibility				

Note: Subscripts denote the range and order (dimension) of variables and parameters.

3.3 Primary factor demands

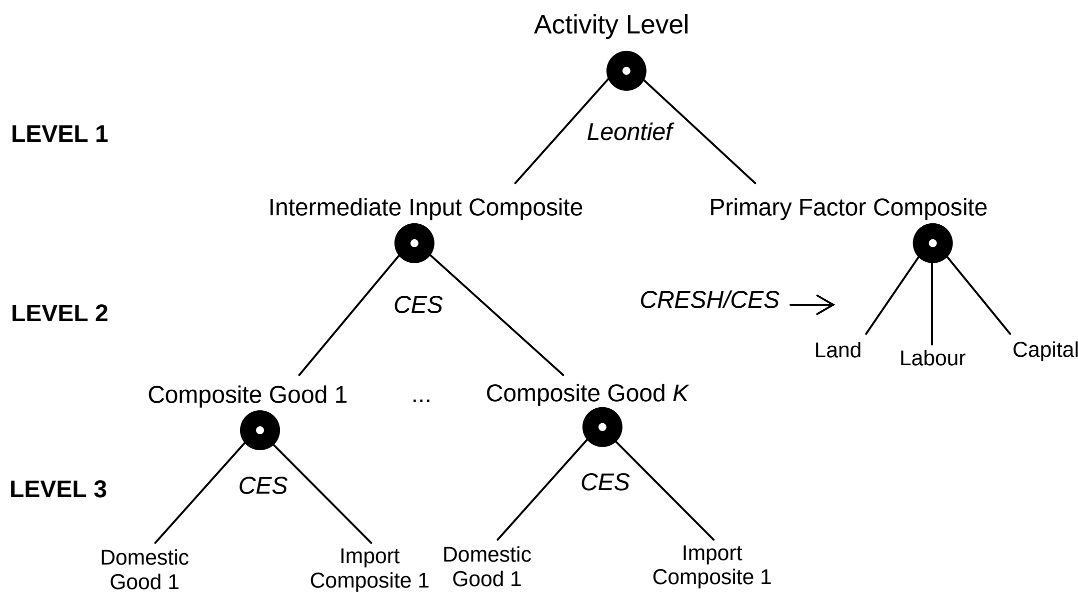
Firms in WOOLGEM are assumed to treat all factors of production (land, labour and physical capital) as variable, so that they rent their land and physical capital. Section 3.17 describes a short-run closure of WOOLGEM that includes the assumption that land and physical capital are fixed in each industry, and a long-run closure that includes the assumption of imperfect mobility of land and perfect mobility of capital between industries. So there exists a rental market for the use of land and capital by each industry and the rental prices of land and capital are taken as given by each industry as they attempt to minimise

[6] Note that the terms firm, industry and sector are used here interchangeably to refer to an industrial sector.

costs. The rental prices act to ensure market clearing for the land and capital used by each industry, such that demand and supply of land and capital by each industry are equated.[7]

Demands for primary factors are modelled using nested production functions consisting of two levels: at the top level, firms decide on their demand for the primary factor composite; at the second level, firms decide on their demand for individual factors of production, i.e., land, labour and (physical) capital – see the right-hand side (RHS) branch of figure 3.2 for a summary.

Figure 3.2 Input technology for industries



3.3.1 Level 1: demands for the primary factor composite

The underlying production technology applied by firms in demanding the primary factor composite (or value added) is Leontief. We are therefore assuming that firms' use of the primary factor composite is a fixed share of output, reflecting the idea that the share of

[7] An alternative assumption would be to treat land and capital as owned by firms; firms would then attempt to maximise profits subject to the availability of land and capital. Both assumptions yield identical results. The first approach yields the market-clearing rental per unit of fixed factor; the second approach yields the profit per unit of fixed factor (Dixon et al. 1982, p. 77).

output made up by value added is invariant to changes in relative prices and reflects characteristics intrinsic to the production of each good.

Adapting equation (2.4) (see Chapter 2, Section 2.3.1), the linearised form of the demand function for the primary factor composite is a function of the industry's activity level and primary factor technical change:

$$qf_{jr}^{F} = qf_{jr} + af_{jr}^{F} \; ; \; \forall j, r \; . \tag{3.10}$$

Equations (3.10) say that (the percentage change in) demand for the *effective* primary factor composite by the *j*-th industry $\left(j = 1, ..., J \right)$ in the *r*-th region $\left(r = 1, ..., R \right)$, qf_{jr}^{F}, is a positive (linear) function of (the percentage change in) the $\left(j, r \right)$-th industry's activity level, qf_{jr}, and Hicks-neutral technical change, af_{jr}^{F}. This is technical change of the form that is equally land-, labour- and capital-augmenting (Allen 1967, pp. 239–40).[8] Thus, (3.10) only consists of an expansion effect and a productivity term. Note that qf_{jr} is determined by the $\left(j, r \right)$-th industry's zero pure profit condition [see equation (3.34), Section 3.5.1].

3.3.2 Level 2: demands for individual primary factors

The underlying production technology applied in combining individual factors varies by type of industry; the sheep industry applies a CRESH (constant ratios of elasticities of substitution, homothetic) production function, whereas all other industries apply CES (constant elasticity of substitution) production functions. Both functional forms make demands for individual factors a function of the demand for the primary factor

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[8] In defining Hicks-neutral technical change, Allen only refers to labour and capital as his analysis confines itself to these two primary factors. But the concept can be extended to include land where it is part of the firm's primary factor bundle.

composite, technical change and prices. Adapting equations (2.32) (see Chapter 2, Section 2.3.3) and (2.18) (see Chapter 2, Section 2.3.2), we write

$$qf_{ijr}^{F} = qf_{jr}^{F} + af_{ijr}^{F} - \sigma crsh_{ir}^{F}\left(pf_{ijr}^{F} + af_{ijr}^{F} - pcrsh_{jr}^{F}\right),\ \forall i,r; j = Sheep\,, \tag{3.11}$$

$$qf_{ijr}^{F} = qf_{jr}^{F} + af_{ijr}^{F} - \sigma f_{jr}^{F}\left(pf_{ijr}^{F} + af_{ijr}^{F} - pf_{jr}^{F}\right),\ \forall i,r; j \neq Sheep\,, \tag{3.12}$$

where

$$PCRSH_{jr}^{F} = \sum\nolimits_{i=1}^{3} \frac{\left(VF_{ijr}^{F}/VF_{jr}^{F}\right)\sigma crsh_{ir}^{F}}{\sum_{k=1}^{3}\left(VF_{kjr}^{F}/VF_{jr}^{F}\right)\sigma crsh_{kr}^{F}} PF_{ijr}^{F} AF_{ijr}^{F}\ ,\ \forall r; j = Sheep\,, \tag{3.13}$$

$$PF_{jr}^{F} = \sum\nolimits_{i=1}^{3} \frac{VF_{ijr}^{F}}{VF_{jr}^{F}} PF_{ijr}^{F} AF_{ijr}^{F}\ ,\ \forall r, j\,. \tag{3.14}$$

Equations (3.11) and (3.12) are the demand functions for individual factors by the *Sheep* industry and nonsheep industries. The demand functions differ in two respects. The first difference is in the definition of the *effective* price of the primary factor composite. We can see from (3.12) and (3.14) that the relevant *effective* price of the primary factor composite in a CES demand function, pf_{jr}^{F}, is an arithmetic-weighted average of the *effective* prices of individual factors $\left(PF_{ijr}^{F} A_{ijr}^{F}\right)$ using factor payment shares $\left(\frac{VF_{ijr}^{F}}{VF_{jr}^{F}}\right)$ as weights; whereas the relevant *effective* price of the primary factor composite in a CRESH demand function, $pcrsh_{jr}^{F}$, is an arithmetic-weighted average of $\left(PF_{ijr}^{F} A_{ijr}^{F}\right)$ using $\left[\frac{\left(VF_{ijr}^{F}/VF_{jr}^{F}\right)\sigma crsh_{ir}^{F}}{\sum_{k=1}^{3}\left(VF_{kjr}^{F}/VF_{jr}^{F}\right)\sigma crsh_{kr}^{F}}\right]$ as weights, i.e, the ratio of the factor payment shares multiplied by the CRESH elasticity of substitution, $\sigma crsh_{ir}^{F}$, to the summation of factor payment shares multiplied by $\sigma crsh_{ir}^{F}$. (3.11) is a generalised form of (3.12) as it allows $\sigma crsh_{ir}^{F}$ to vary across individual factors. The second difference is the reason we assume CRESH production technology for the sheep industry; it allows us to take advantage of differences in the econometrically-estimated values of $\sigma crsh_{ir}^{F}$ across individual factors. Note that both price indices include factor-specific technical change $\left(AF_{ijr}^{F}\right)$, which is why they are referred to as *effective* price indices. Note also that VF_{ijr}^{F} and its aggregated form $\left(VF_{jr}^{F}\right)$ are taken from the model database.

(3.11) and (3.12) state that the *effective* demand for the *i*-th factor $\left(i = 1, 2, 3 = Land, Labour, Capital\right)$ is a function of an expansion effect and a substitution effect. If we set the percentage change in (effective) relative prices, $\left(pf_{ijr}^{F} - pcrsh_{jr}^{F}\right)$ or $\left(pf_{ijr}^{F} - pf_{jr}^{F}\right)$, and technology, af_{ijr}^{F}, to zero, then demand for factor *i* will move exactly with the firm's (percentage change in) demand for the *effective* primary factor composite, qf_{jr}^{F}; i.e., the expansion effect. This reflects constant returns to scale in the CRESH and CES production functions. Alternatively, if we set the (percentage) change in the firm's demand for the *effective* primary factor composite to zero, then demand for factor *i* will be a function of the change in the relative price of factor *i*, and the size of the elasticity of substitution between any pair of inputs, $\sigma crsh_{ir}^{F}$ or σf_{ir}^{F}, i.e., the substitution effect. The size of the substitution effect is determined by the value of the elasticity of substitution.

3.3.3 Taxes on factor usage

The prices of individual factors which appear in (3.11) and (3.12), pf_{ijr}^{F}, are purchasers' prices: that is, they are the prices actually paid by the purchaser, the firm in this case, and therefore they include factor- and industry-specific taxes on usage by firms;

$$PF_{ijr}^{F} = P_{ijr}^{F} TF_{ijr}^{F}, \tag{3.15}$$

where P_{ijr}^{F} is the supply (or basic) price of factor *i* used by the (j,r)-th industry, and TF_{ijr}^{F} is the power of the *ad valorem* tax on factor *i* used by the (j,r)-th industry, so that $TF_{ijr}^{F} = 1 + \overline{T}F_{ijr}^{F}$, where $\overline{T}F_{ijr}^{F}$ is the *ad valorem* tax on factor *i* used by the (j,r)-th industry.[9] Thus, if $\overline{T}F_{ijr}^{F}$ equals zero, then $TF_{ijr}^{F} = 1$.[10]

P_{ijr}^{F} can be used to define real value-added by industry, VA_{jr}^{F}, for use as a possible industry welfare measure, as follows:

$$VA_{jr}^{F} = \frac{\sum_{i=1}^{3} P_{ijr}^{F} QF_{ijr}^{F}}{PH_{r}}, \tag{3.16}$$

where PH_{r} is the consumer price index (CPI).

3.3.4 Supply (or basic) price of factors

P_{ijr}^{F} is determined differently across factors. For *i* = *Land, Capital*, the basic price paid by each industry is determined by market clearing, thus giving industry-specific rental prices of land and capital in each region. This is consistent with economic environments where land and capital are treated as either industry specific (i.e., the short run) or perfectly or imperfectly mobile (i.e., the long run). In these cases, the aggregate basic prices of land and capital are weighted averages of the industry prices of land and capital using factor payments at basic values $\left(V_{ijr}^{F}\right)$ as weights:

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[9] Using a transformed form of $\overline{T}F_{ijr}^{F}$ avoids a null column for $\overline{T}F_{ijr}^{F}$ in A_{n} [see equation (3.7)] when the initial tax rate is zero. This helps ensure that A_{n}^{-1} exists.

[10] Note that there is an *ad valorem* equivalent, like $\overline{T}F_{ijr}^{F}$, for all powers of *ad valorem* tax variables in the model. We shall not mention this for all remaining powers of *ad valorem* tax variables to be discussed, and so this point should be taken as implied by the reader.

$$P_{ir}^{F} = \sum_{j=1}^{J} \frac{V_{ijr}^{F}}{V_{ir}^{F}} P_{ijr}^{F}, \ i = Land, Capital. \tag{3.17}$$

In contrast, labour is treated as perfectly mobile in all economic environments; therefore each industry pays the same (basic) price for hiring labour. Here, the industry price of labour is indexed to the national price of labour,

$$P_{ijr}^{F} = P_{ir}^{F}, \ i = Labour, \tag{3.18}$$

and P_{ir}^{F} ($i = Labour$) is determined by market clearing at the national level.

We also define the real wage rate in each region as P_{ir}^{F} ($i = Labour$) deflated by the CPI $\left(PH_r\right)$:

$$W_r = \frac{P_{ir}^{F}}{PH_r}, \ i = Labour. \tag{3.19}$$

3.4 Intermediate input demands

Firms are assumed to able to vary their intermediate inputs that they use in production. In deciding their intermediate input usage, firms attempt to minimise costs but, analogous to the factor markets they face, they have no control over the prices of these inputs.

In combining intermediate inputs, all firms are assumed to use a three-level nested production structure. At level 1, firms decide on their use of the intermediate input composite; at level 2, firms decide on their use of individual intermediate input composites; and at level 3, firms decide on their use of individual intermediate inputs from different sources – see the left-hand side (LHS) branch of figure 3.2 for a summary.

3.4.1 Level 1: demands for the intermediate input composite

As is the case with the primary factor composite, Leontief production technology is applied by all firms in demanding the intermediate input composite. As before, we are assuming that firms' use of the intermediate input composite is a fixed share of output, reflecting the idea that the share of output made up by intermediate inputs is invariant to changes in relative prices and reflects characteristics intrinsic to the production of each good.

Similar to equation (3.10), the linearised form of the demand function for the intermediate input composite makes the composite a positive (linear) function of the industry's activity level, qf_{jr}, and intermediate input technical change:

$$qf^{I}_{jr} = qf_{jr} + af^{I}_{jr}\,,\ \forall j, r\,, \tag{3.20}$$

where qf^{I}_{jr} denotes demand for the intermediate input composite by the (j,r)-th industry, and af^{I}_{jr} denotes technical change in the use of the intermediate input composite.

3.4.2 Level 2: demands for individual intermediate input composites

The underlying production technology applied by all firms in combining individual intermediate input composites is CES. This functional form makes demands for inputs a function of demand for the higher level aggregate – the intermediate input composite – and relative prices;

$$qf^{I}_{ijr} = qf^{I}_{jr} - \sigma f^{I}_{ir}\left(pf^{I}_{ijr} - pf^{I}_{jr}\right),\ \forall i, j, r\,, \tag{3.21}$$

where

$$PF^{I}_{ijr} = \frac{VFD^{I}_{ijr}}{VF^{I}_{ijr}} PFD^{I}_{ijr} + \frac{VFM^{I}_{ijr}}{VF^{I}_{ijr}} PFM^{I}_{ijr}\,, \tag{3.22}$$

$$VF_{ijr}^{I} = VFD_{ijr}^{I} + VFM_{ijr}^{I} , \quad (3.23)$$

and

$$PF_{jr}^{I} = \sum_{i=1}^{K} \frac{VF_{ijr}^{I}}{VF_{jr}^{I}} PF_{ijr}^{I} . \quad (3.24)$$

(3.21) says that demand for the *i*-th intermediate input $(i = 1,...,K)$ by the (j,r)-th industry, qf_{ijr}^{I}, is a positive linear function of qf_{jr}^{I} – the expansion effect; and an inverse function of the price of the (i, j, r)-th intermediate input, pf_{ijr}^{I}, relative to the price of the intermediate input composite used by the (j,r)-th industry, pf_{jr}^{I}, adjusted by the CES between the *i*-th intermediate input and all other inputs in the *r*-th region, σf_{ir}^{I} – the substitution effect.

Note that PF_{ijr}^{I} is an average of the prices of domestic, PFD_{ijr}^{I}, and imported, PFM_{ijr}^{I}, intermediate inputs, weighted by the shares of domestic $\left(VFD_{ijr}^{I}\right)$ and imported $\left(VFM_{ijr}^{I}\right)$ intermediate inputs, in total imported intermediate inputs $\left(VF_{ijr}^{I}\right)$. Note also that PF_{jr}^{I} is an average of the prices of the *i* individual intermediate input composites, PF_{ijr}^{I}. The initial values of the *VF*s are taken from the model database.[11]

3.4.3 Level 3: demands for individual intermediate inputs by source

Similar to level 2, all firms combine individual intermediate inputs from different sources – domestic and foreign – using CES production technology:

$$qfd_{ijr}^{I} = qf_{ijr}^{I} - \sigma_{ir}^{T}\left(pfd_{ijr}^{I} - pf_{ijr}^{I}\right), \ \forall i, j, r , \quad (3.25)$$

[11] It is tedious to continually note that the initial values of nominal variables used to calculate shares, like those in (3.22)–(3.24), are taken from the model database. Thus, the reader should take this point as implied in the rest of this chapter, unless otherwise specified.

$$qfm_{ijr}^{I} = qf_{ijr}^{I} - \sigma_{ir}^{T}\left(pfm_{ijr}^{I} - pf_{ijr}^{I}\right),\ \forall i,j,r\,. \tag{3.26}$$

Thus, demand for domestic (imported) good *i* used as an intermediate input, by the (j,r)-th industry, qfd_{ijr}^{I} $\left(qfm_{ijr}^{I}\right)$, is a positive linear function of qf_{ijr}^{I}, and an inverse function of pfd_{ijr}^{I} $\left(pfm_{ijr}^{I}\right)$ relative to pf_{ijr}^{I}, and adjusted by the parameter σ_{ir}^{T}, i.e., the CES between any pair of inputs from different sources.

3.4.4 Taxes on intermediate input usage

All prices that appear in the intermediate input demand functions above are purchasers' prices, thus they include commodity- and industry-specific taxes on intermediate input usage:

$$PFD_{ijr}^{I} = PD_{ir}TFD_{ijr}^{I}\,, \tag{3.27}$$

$$PFM_{ijr}^{I} = PM_{ir}TFM_{ijr}^{I}\,. \tag{3.28}$$

Thus, PFD_{ijr}^{I} is the product of the basic (or supply) price of domestically-produced good *i* in region *r*, PD_{ir}, and the power of the tax on domestic good *i* used as an intermediate input by the (j,r)-th industry, TFD_{ijr}^{I}. Similarly, PFM_{ijr}^{I} is the product of the basic price of import composite *i* in region *r*, PM_{ir}, and the power of the tax on imported good *i* used as an intermediate input by the (j,r)-th industry, TFM_{ijr}^{I}. PD_{ir} is determined by the market-clearing condition for domestic commodities [see equations (3.90)–(3.91), Section 3.11.1] and PM_{ir} is defined in equation (3.72), Section 3.9.1.

3.5 Industry outputs and commodity supplies

All industries in WOOLGEM are modelled as multiproduct industries. In doing so, we assume input-output separability (see Chapter 2, Section 2.3.4). Therefore, industries never alter the set of commodities for which they are (net) suppliers, and so, the actual outputs producible by each industry are strictly limited by the initial data (see Chapter 4, table 4.13). Even though all industries are modelled as multiproduct industries there are only three classes of multiproduct industries: (i) the sheep industry in each region; (ii) the worsted tops industries in each region; and (iii) the other industries composite (see figure 3.1).

3.5.1 Industry supplies of individual commodities

Firms are assumed to be price takers in the market for their outputs. Given this condition, they attempt to maximise revenue in determining their mix of outputs using a production possibilities frontier (PPF). The PPF varies by type of industry: the sheep industry is assumed to determine its outputs using a CRETH PPF, whereas all other industries determine their outputs using a CET PPF. Thus, adapting equations (2.45) (see Chapter 2, Section 2.3.6) and (2.41) (see Chapter 2, Section 2.3.5), we write

$$qd_{ijr} = qf_{jr} - \theta crth_{ir}\left(pd_{ir} - pcrth_{jr}\right),\ \forall i,r; j = Sheep\,, \qquad (3.29)$$

$$qd_{ijr} = qf_{jr} - \theta f_{jr}\left(pd_{ir} - pf_{jr}\right),\ \forall i,r; j \neq Sheep\,, \qquad (3.30)$$

where

$$PCRTH_{jr} = \sum_{i=1}^{K} \frac{\left(VF_{ijr}/VF_{jr}\right)\theta crth_{ir}}{\sum_{k=1}^{K}\left(VF_{kjr}/VF_{jr}\right)\theta crth_{kr}} PD_{ir}\ ,\ \forall r; j = Sheep\,, \qquad (3.31)$$

$$PF_{jr} = \sum_{i=1}^{K} \frac{VF_{ijr}}{VF_{jr}} PD_{ir}\ ,\ \forall j,r\,. \qquad (3.32)$$

Equations (3.29)–(3.30) state that the supply of any domestically-produced output i (i = 1,...,K) by the (j,r)-th industry, qd_{ijr}, is a function of an expansion effect, determined by qf_{jr}, and a transformation effect, determined by the change in the relevant relative price, $(pd_{ir} - pcrth_{jr})$ or $(pd_{ir} - pf_{jr})$, adjusted by the relevant elasticity of transformation between any pair of outputs, $\theta crth_{ir}$ or θf_{jr}. Setting the change in the relative price to zero, supply for output i will move exactly with qf_{jr}; i.e., the expansion effect. This reflects constant returns to scale in the CRETH and CET PPFs. Alternatively, if we set the change in qf_{jr} to zero, then supply of output i will be a function of the price of output i relative to the relevant price of the industry's composite outputs and the size of $\theta crth_{ir}$ or θf_{jr}: so that if the price of output i rises relative to the relevant price of composite outputs, supply of output i will rise relative to qf_{jr}, i.e., the transformation effect.

The differences between the CRETH and CET PPFs are analogous to the differences between the CRESH and CES production functions. The first difference is in the definition of the average output price. The relevant average output price in a CET frontier, pf_{jr}, is an average of the individual output prices (PD_{ir}) using industry revenue shares $\left(\frac{VF_{ijr}}{VF_{jr}}\right)$ as weights [see (3.32)], whereas the relevant average output price in a CRETH frontier, $pcrth_{jr}$, is an average of PD_{ir} using $\left[\frac{(VF_{ijr}/VF_{jr})\theta crth_{ir}}{\sum_{k=1}^{K}(VF_{kjr}/VF_{jr})\theta crth_{kr}}\right]$ as weights, i.e., the ratio of the revenue shares multiplied by the CRETH elasticity of transformation, $\theta crth_{ir}$, to the sum of the revenue shares multiplied by $\theta crth_{ir}$ [see (3.31)]. Note that

(3.29) is a generalised form of (3.30) as it allows $\theta crth_{ir}$ to vary across individual factors. The second difference is the reason we assume a CRETH frontier for the sheep industry; it allows us to take advantage of differences in the econometrically-estimated values of $\theta crth_{ir}$ across individual outputs, namely, sheep meat and greasy wool (see figure 3.1).

Even though we write the output response functions like (3.30) for the j ($\neq Sheep$) industries in WOOLGEM, there is no choice to make for the single product industries. In the case of single product industries, the transformation effect is zero as the firm produces only one product, and output of this product will move strictly with the activity level of the firm.

For use in specifying the market-clearing condition for domestic goods (see Section 3.11.1), we define the supply of commodity outputs in each region, QD_{ir}, as

$$QD_{ir} = \sum_{j=1}^{J} QD_{ijr} \text{ , } \forall i,r \text{ .} \tag{3.33}$$

3.5.2 Zero pure profits and industry activity levels

All firms are assumed to operate in a perfectly competitive environment so that no firm earns pure profits. Thus, we impose a zero pure profits condition for all firms, i.e.,

$$PF_{jr}QF_{jr} = \left[PF^F_{jr}QF^F_{jr}AF^F_{jr} + PF^I_{jr}QF^I_{jr}AF^I_{jr} \right] TF_{jr} \text{ , } \forall j,r \text{ .} \tag{3.34}$$

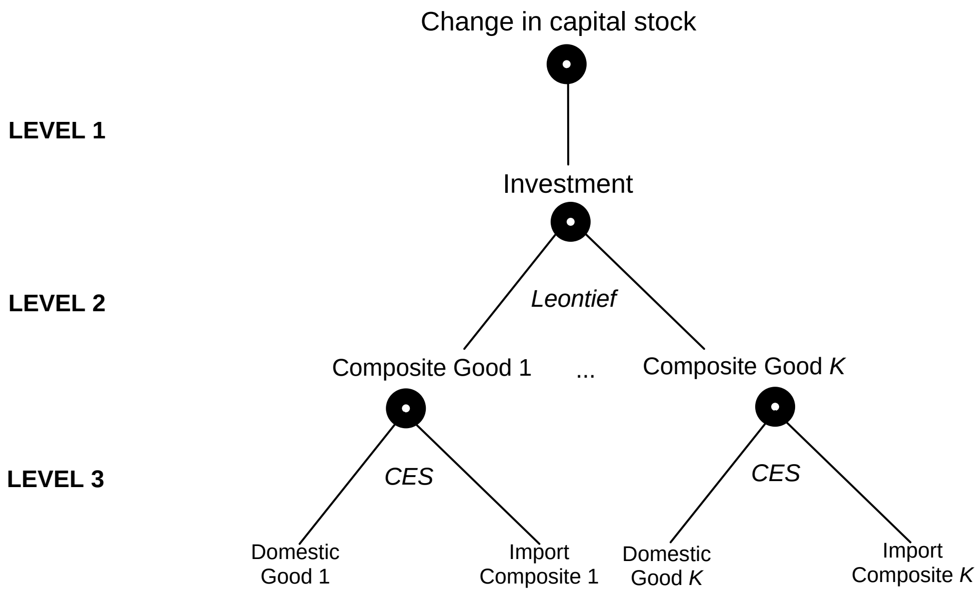
The LHS of (3.34) is total revenue from sales at supply (or basic) prices for industry j in region r. The bracketed term on the RHS of (3.34) is total payments for inputs at purchasers' prices by the (j,r)-th industry, to which is added the industry-specific (power of the) tax on output, TF_{jr}, so that the RHS of (3.34) is total costs for the (j,r)-th industry. Note that $\left(PF^F_{jr}QF^F_{jr}AF^F_{jr}\right)$ represents the effective value of the primary factor bundle as it includes Hicks-neutral technical change $\left(AF^F_{jr}\right)$, and $\left(PF^I_{jr}QF^I_{jr}AF^I_{jr}\right)$ represents the effective value of the intermediate input composite as it includes intermediate input technical change $\left(AF_{jr}^{I}\right)$.

With $af_{jr}^{F} = af_{jr}^{I} = 0$, industry revenue will move strictly with the change in industry costs. With no change in any of the prices in (3.34), any improvement in technology relating to the use of the (primary factor and intermediate) input bundle, i.e., $a_{jr}^{F} = a_{jr}^{I} < 0$, will initially raise the firm's activity level, QF_{jr}^{I}, from a given set of inputs. With no change in input or output prices, the ratio of revenue to costs will rise leading to nonzero pure profits, but this is prevented by (3.34) that will ensure that input and output prices adjust so as to restore zero pure profits. For example, as the ratio of revenue to costs rises, the firm's activity level will be forced to expand further that, in turn, will reduce output prices and raise input prices, *ceteris paribus*, thus causing industry revenue to fall and the effective cost of the input bundle to rise until zero pure profits are restored. In this way, (3.34) determines the firm's activity level.

3.6 Investment demands

WOOLGEM models investment within each region via a representative (physical) capital creator. This representative agent determines investment via a three-stage process. At the top level, the total demand for investment in a region is determined. At the second level, individual composite inputs to capital creation are determined. At the final level, individual inputs by source (domestic and foreign) are determined. Figure 3.3 summarises the three-stage procedure applied by the capital creator in determining inputs.

Figure 3.3 Input technology for capital creation



3.6.1 Level 1: demands for total investment

The percentage change in regional investment, qi_r, is driven by the equation

$$VI_r qi_r = V_{ir}^F qf_{ir}^F \text{ , } \forall r; i = Capital \text{ ,} \tag{3.35}$$

where

$$VI_r = \sum\nolimits_{i=1}^{K} VI_{ir} \text{ ,} \tag{3.36}$$

$$V_{ir}^F = \sum\nolimits_{j=1}^{J} V_{ijr}^F \text{ , } i = Capital \text{ ,} \tag{3.37}$$

$$QF_{ir}^F = \sum\nolimits_{j=1}^{J} QF_{ijr}^F \text{ , } i = Capital \text{ .} \tag{3.38}$$

The LHS of (3.35) is the change in real regional investment, i.e., the product of the initial value of regional investment, VI_r, and qi_r. The RHS of (3.35) is the change in the regional capital stock, i.e., the product of the initial value of regional capital at basic values, $V_{ir}^F\left(i = Capital\right)$, and $qf_{ir}^F\left(i = Capital\right)$. Thus, regional investment is purely determined by total industry demands for capital. Consequently, when industry capital stocks are assumed exogenous, as is usually the case in a short-run closure, qi_r will equal zero; when industry capital stocks are assumed endogenous, as in a long-run closure, qi_r will take on a nonzero value.

3.6.2 Level 2: demands for individual composite inputs to investment

At the next stage, a Leontief production function is used to determine the combination of individual composite inputs to capital creation. We are therefore assuming that the pattern of individual composite inputs in creating capital is unchanging, and reflects the underlying nature of capital goods:

$$qi_{ir} = qi_r \,, \ \forall i, r \,. \tag{3.39}$$

So demand for composite good *i* (*i* = 1,...,*K*) used as an input to investment in region *r*, qi_{ir}, moves strictly with qi_r.

3.6.3 Level 3: demands for individual inputs to investment by source

At stage 3, the capital creator combines individual inputs by source (domestic and foreign) using a CES production function;

$$qid_{ir} = qi_{ir} - \sigma_{ir}^{T}\left(pid_{ir} - pi_{ir}\right), \ \forall i, r \,, \tag{3.40}$$

$$qim_{ir} = qi_{ir} - \sigma_{ir}^{T}\left(pim_{ir} - pi_{ir}\right), \ \forall i, r \,, \tag{3.41}$$

where

$$PI_{ir} = \frac{VID_{ir}}{VI_{ir}} PID_{ir} + \frac{VIM_{ir}}{VI_{ir}} PIM_{ir} \tag{3.42}$$

and

$$VI_{ir} = VID_{ir} + VIM_{ir} \,. \tag{3.43}$$

Thus, with no change in relative prices of individual inputs to investment by source $\left(pid_{ir} = pim_{ir} = pi_{ir}\right)$, demand for individual inputs to investment from both sources $\left(qid_{ir}, qim_{ir}\right)$ will move with qi_{ir}, and the pattern of individual inputs to investment by source will remain unchanged, i.e., the expansion effect. With $qi_{ir} = 0$ and $\left(pid_{ir} \neq pim_{ir} \neq pi_{ir}\right)$, demand for individual inputs to investment by source will diverge $\left(qid_{ir} \neq qim_{ir}\right)$ and the pattern of individual inputs to investment by source will shift in favour of the cheaper source, i.e., the substitution effect.

Note that the price of individual composite inputs to investment, PI_{ir}, is an average of the domestic and imported prices of individual inputs to investment, PID_{ir} and PIM_{ir}, weighted by the shares of domestic and imported individual inputs to investment in individual composite inputs to investment, $\left(\frac{VID_{ir}}{VI_{ir}}\right)$ and $\left(\frac{VIM_{ir}}{VI_{ir}}\right)$.

For use in defining the rate of return on capital, we define the aggregate price of investment in each region as a weighted average of PI_{ir},

$$PI_{r} = \sum_{i=1}^{K} \frac{VI_{ir}}{VI_{r}} PI_{ir} . \tag{3.44}$$

3.6.4 Taxes on inputs to investment

PID_{ir} and PIM_{ir}, which appear above, are purchaser's prices, thus they include commodity-specific taxes on inputs used by the capital creator:

$$PID_{ir} = PD_{ir} TID_{ir} , \ \forall i, r , \tag{3.45}$$

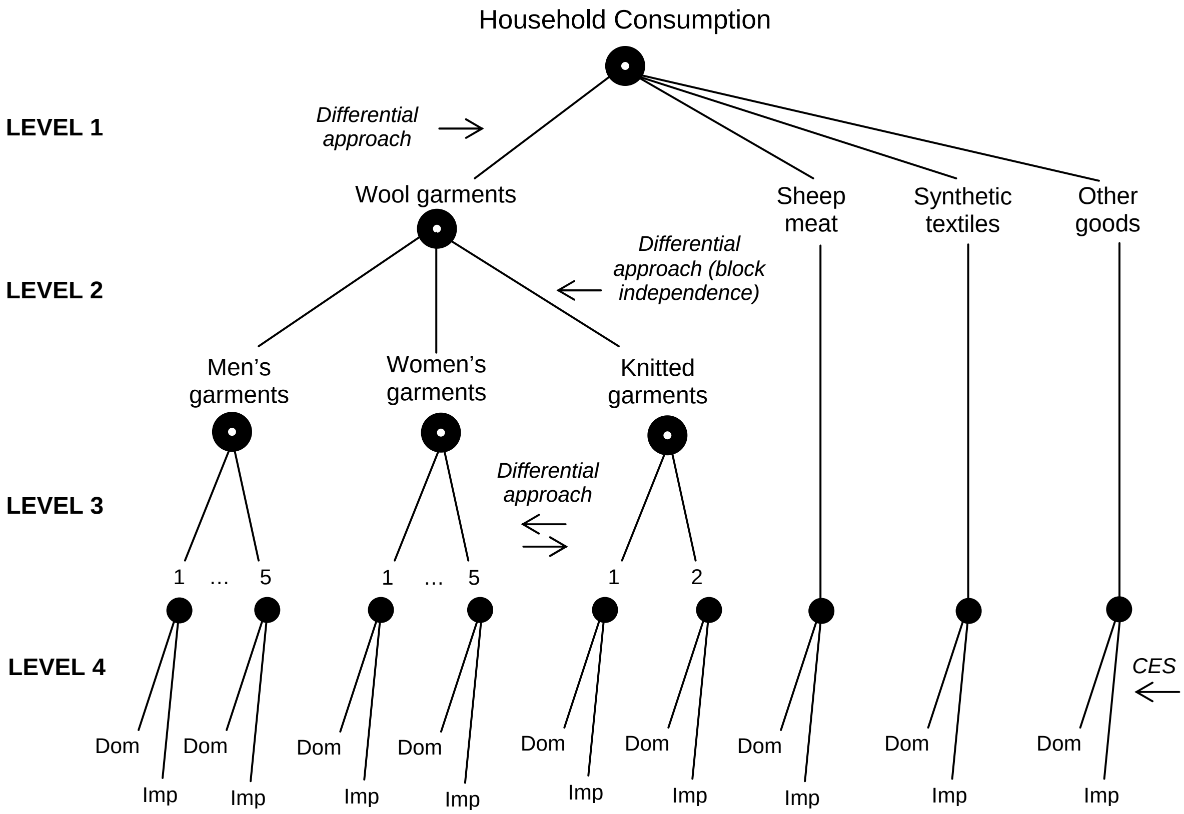
$$PIM_{ir} = PM_{ir} TIM_{ir} , \ \forall i, r . \tag{3.46}$$

Thus, PID_{ir} is the product of PD_{ir} and the power of the *ad valorem* tax on domestic good *i* used as an input to investment in region *r*, TID_{ir}. Similarly, PIM_{ir} is the product of PM_{ir} and the power of the *ad valorem* tax on imported good *i* used as an input to investment in region *r*, TIM_{ir}.

3.7 Household demands

Representative households in WOOLGEM determine demand for their inputs to utility maximisation via a four-stage procedure. At the top level, households determine demand for four broad composite commodities: sheep meat, wool garments, synthetic textiles, and other goods. At the second level, households determine demand for the three wool garments subgroups: men's wool garments, women's wool garments, and knitted wool garments. At level three, households determine demand for the 12 individual composite goods which make up each of the three wool garments subgroups, i.e., five men's and five women's wool garments goods, and two knitted wool garments goods. At the final level, households determine demand for all 15 individual goods from different sources: sheep meat (one good), wool garments (12 goods), synthetic textiles (one good), and other goods (one good). The four-stage procedure followed by households in determining their allocation of spending across inputs is summarised by figure 3.4.

Figure 3.4 Input technology for households



3.7.1 Level 1: household demands for broad composites

At level 1, households combine four broad composites – sheep meat, wool garments, synthetic textiles, and other goods – to maximise an implicit utility function (see Chapter 2, Section 2.3.7). This gives a differential demand system written in terms of income and uncompensated price elasticities [equation (2.55)] or compensated price elasticities [equation (2.62)]. Adapting equation (2.62), we write

$$qh_{ir}^{B} = \eta_{ir}^{B} qh_{r} + \sum_{j=1}^{4} \varepsilon_{ijr}^{B} ph_{jr}^{B}\ , \ i, j = 1,...,4 . \tag{3.47}$$

Equation (3.47) says that household demand for broad composite i in region r, qh_{ir}^{B}, is subject to an income effect and a substitution effect. The income effect for the (i,r)-th broad composite is the product of the (normalised) income elasticity of demand for the

(i,r)-th composite, η_{ir}^{B}, and demand for aggregate household consumption in the r-th region, qh_r. The substitution effect for the (i,r)-th broad composite is the sum of the compensated (own- and cross-) price elasticities of demand in region r, ε_{ijr}^{B}, multiplied by the price of the $\left(j,r\right)$-th broad composite, ph_{jr}^{B}.

Note that: (i) the values of η_{ir}^{B} used here satisfy Engel's aggregation; and (ii) the values of ε_{ijr}^{B} used here satisfy the homogeneity constraint (i.e., holding real income constant, an equiproportionate increase in all prices will lead to no change in the quantity demanded) and the symmetry constraint (i.e., the income-compensated price slopes of the demand equations are symmetric).[12]

Demand for aggregate household consumption, QH_r, is aggregate household expenditure, VH_r, deflated by the CPI:

$$QH_r = \frac{VH_r}{PH_r}. \tag{3.48}$$

VH_r is itself determined by applying a Keynesian consumption function, so that regional household expenditure is equal to the average propensity to consume in region r, Υ_r, multiplied by household income, YH_r;

$$VH_r = \Upsilon_r YH_r. \tag{3.49}$$

The regional and global CPIs are

$$PH_r = \sum_{i=1}^{K} \frac{VH_{ir}}{VH_r} PH_{ir}, \tag{3.50}$$

[12] For a formal definition of these restrictions, see Chapter 2, Section 2.3.7.

$$PH = \sum_{r=1}^{R} \frac{VH_r}{VH} PH_r \ . \tag{3.51}$$

Note that even though (3.50) is summed over the K goods, the budget shares, $\frac{VH_{ir}}{VH_r}$, are zero for most of these except for sheep meat (one good), wool garments (twelve goods), synthetic textiles (one good), and other goods (one good).

The prices of broad composites are defined as follows;

$$PH_{ir}^{B} = PH_{jr} ,\ i = 1,3,4; j = 1,...,K; i \in K , \tag{3.52}$$

$$PH_{ir}^{B} = \sum_{j \in WG} \frac{VH_{jr}}{VH_{ir}^{B}} PH_{jr} \ ,\ i = 2; j = 1,...,K; WG = 1,...,12; WG \subset K , \tag{3.53}$$

where

$$VH_{ir}^{B} = \sum_{j \in WG} VH_{jr} \ ,\ i = 2; j = 1,..., K; WG = 1,...,12; WG \subset K \ . \tag{3.54}$$

As sheep meat, synthetic textiles, and other goods (i.e., i = 1,3,4) are all elements of the set of K commodities, their broad composite price is the same as their individual composite price [see (3.52)]. In contrast, the wool garments broad composite price (i = 2) is an average of the prices of the WG (= 1,...,12) individual wool garments composites using their conditional budget shares as weights [see (3.53)–(3.54)].

3.7.2 Level 2: household demands for wool garments subgroups

At this level, households determine demand for the three wool garments subgroups (or blocks), S_1, S_2, S_3, that make up the wool garments broad composite – men's wool garments, women's wool garments, and knitted wool garments – applying Theil's (1980) differential approach to consumption theory. We assume *block independence* between these three subgroups so that utility derived from each block is assumed to be additive (see

Chapter 2, Section 2.3.9), giving the demand equations (2.90). Adapting these equations, we write

$$qh_{ir}^{C} = \frac{\Theta H_{ir}^{C}}{WH_{ir}^{C}} qh_{jr}^{B} + \phi_r \frac{\Theta H_{ir}^{C}}{WH_{ir}^{C}} \left(ph_{ir}^{*C} - ph_{jr}^{*B} \right),\ i = S_1, S_2, S_3;\ j = Wool\ garments. \quad (3.55)$$

Here, the demand for each of the $i \left(= S_1, S_2, S_3\right)$ wool garments subgroups, qh_{ir}^{C}, is a function of an expansion effect of the broad composite to which it belongs (the first term on the RHS) and a substitution effect (the second term on the RHS).

The expansion effect is equal to qh_{jr}^{B} multiplied by the ratio of the marginal share, ΘH_{ir}^{C}, to the budget share, WH_{ir}^{C}, of each subgroup; defined as

$$\Theta H_{ir}^{C} = \sum_{j \in S_i} \Theta H_{jr} \text{ and } WH_{ir}^{C} = \sum_{j \in S_i} \frac{VH_{jr}}{VH_r},\ i = S_1, S_2, S_3; j = 1, ..., K . \quad (3.56)$$

ΘH_{jr} in (3.56) are the marginal shares for the K goods, which are taken from the model database. Note that $\frac{\Theta H_{ir}^{C}}{WH_{ir}^{C}}$ is income elasticity for the (*i*,*r*)-th composite good.

The substitution effect consists of the price elasticity of demand for the *i*-th subgroup (i.e., the income flexibility, ϕ_r, multiplied by the income elasticity, $\frac{\Theta H_{ir}^{C}}{WH_{ir}^{C}}$) multiplied by the change in the relative price of the *i*-th subgroup, $\left(ph_{ir}^{*C} - ph_{jr}^{*B} \right)$, where

$$PH_{ir}^{*C} = \sum_{j \in S_i} \frac{\Theta H_{jr}}{\Theta H_{ir}^{C}} PH_{jr},\ i = S_1, S_2, S_3; j = 1, ..., K ; \quad (3.57)$$

$$PH_{ir}^{*B} = \sum_{j \in WG} \frac{\Theta H_{jr}}{\Theta H_{ir}^{B}} PH_{jr} \text{, and } \Theta H_{ir}^{B} = \sum_{j \in WG} \Theta H_{jr} ,$$

$$i = Wool\ garments;\ j = 1, ..., K; WG = 1, ..., 12; WG \subset K . \quad (3.58)$$

Thus $\left(ph_{ir}^{*C} - ph_{jr}^{*B}\right)$ in (3.55), for $i = S_1, S_2, S_3$ and j = *Wool garments*, is the price of subgroup i relative to the price of the wool garments broad composite. Note that both prices are Frisch indices as they use marginal shares, rather than budget shares, as weights.

3.7.3 Level 3: household demands for individual wool garments composites

At level 3, households determine demand for the *WG* (= 1,...,12) individual wool garments composites also using Theil's differential approach. Chapter 2, Section 2.3.9 shows how the differential demand system with block independence can be extended to commodities within groups, giving equations (2.96). Adapting equations (2.96), we write

$$qh_{ir} = \frac{\Theta H_{ir}}{WH_{ir}} \frac{WH_{jr}^{C}}{\Theta H_{jr}^{C}} qh_{jr}^{C} + \phi_r \frac{\Theta H_{ir}}{WH_{ir}} \left(ph_{ir} - ph_{jr}^{*C} \right),$$

$$i \in S_j; i = 1,...,12; j = S_1, S_2, S_3. \qquad (3.59)$$

(3.59) are the conditional demand equations. Demand for the i-th (i = 1,…,12) individual wool garment composite, qh_{ir}, is dependent upon two effects. The first of these is the change in demand for the subgroup to which it belongs, qh_{jr}^{C} $\left(j = S_1, S_2, S_3\right)$, adjusted by the income elasticity of the i-th individual wool garment composite, $\frac{\Theta H_{ir}}{WH_{ir}}$, multiplied by the inverse of the income elasticity of the subgroup to which it belongs, $\frac{WH_{jr}^{C}}{\Theta H_{jr}^{C}}$ $\left(j = S_1, S_2, S_3\right)$. The second effect is the change in the price of the i-th (i = 1,...,12) individual wool garments composite, ph_{ir}, relative to ph_{jr}^{*C}, $\left(j = S_1, S_2, S_3\right)$, adjusted by the price elasticity of demand for the i-th (i = 1,...,12) good, i.e., $\phi_r \left(\frac{\Theta H_{ir}}{WH_{ir}}\right)$.

For completeness, we set $qh_{ir} = qh^{B}_{jr}$, $i = j; j = 1,3,4$, that is, for sheep meat, synthetic textiles, and the other goods composite.

3.7.4 Level 4: households demands for individual goods by source

At the bottom level, households determine demand for the 15 individual goods from different sources using a CES utility function. Chapter 2, Section 2.3.10 derives the demand equations (2.100) by solving the utility maximising problem from a CES utility function. Adapting equations (2.100), we write

$$qhd_{ir} = qh_{ir} + \sigma^{T}_{ir}\left(phd_{ir} - ph_{ir}\right),\ \forall i,r, \tag{3.60}$$

$$qhm_{ir} = qh_{ir} + \sigma^{T}_{ir}\left(phm_{ir} - ph_{ir}\right),\ \forall i,r. \tag{3.61}$$

Thus, demand for the *i*-th domestic (imported) individual good is a positive linear function of qh_{ir} (the expansion effect) and the price of *i*-th domestic (imported) individual good $qhd_{ir}\left(qhm_{ir}\right)$ relative to ph_{ir}, adjusted by σ^{T}_{ir} (the substitution effect).

Note that

$$PH_{ir} = \frac{VHD_{ir}}{VH_{ir}} PHD_{ir} + \frac{VHM_{ir}}{VH_{ir}} PHM_{ir},\ \forall i,r, \tag{3.62}$$

i.e., PH_{ir} is an average of the price of the *i*-th domestic and imported good weighted by their respective budget shares.

3.7.5 Taxes on household inputs

PHD_{ir} and PHM_{ir}, which appear above, are purchaser's prices; thus they include commodity-specific taxes on consumption by households:

$$PHD_{ir} = PD_{ir} THD_{ir},\ \forall i,r, \tag{3.63}$$

$$PHM_{ir} = PM_{ir} THM_{ir}, \ \forall i, r. \tag{3.64}$$

That is, PHD_{ir} is the product of PD_{ir} and the power of the *ad valorem* tax on domestic good *i* used as an input by households in region *r*, THD_{ir}. Similarly, PHM_{ir} is the product of PM_{ir} and the power of the *ad valorem* tax on imported good *i* used as an input by households in region *r*, THM_{ir}.

3.8 Government demands

The representative government in each region is assumed to consume the same commodities as the representative household – sheep meat, wool garments, synthetic textiles, and other goods.

3.8.1 Government demands for individual goods by source

Demands by government for commodities by source are a fixed share of household consumption;

$$qgd_{ir} = qh_r, \ \forall i, r, \tag{3.65}$$

$$qgm_{ir} = qh_r, \ \forall i, r, \tag{3.66}$$

where qgd_{ir} $\left(qgm_{ir}\right)$ is government demand for domestic (imported) good *i* in region *r*. Thus, government consumption of individual goods by source is a fixed proportion of aggregate household demand $\left(QH_r\right)$. This assumes that governments rely on aggregate household consumption as an index of demand by households for the goods and services they provide.

For later use, we define the price of aggregate government consumption $\left(PG_r\right)$, as

$$PG_r = \sum_{i=1}^{K} \left(\frac{VGD_{ir}}{VG_r} PGD_{ir} + \frac{VGM_{ir}}{VG_r} PGM_{ir} \right), \ \forall r, \tag{3.67}$$

where PGD_{ir} $\left(PGM_{ir}\right)$ is the price of the *i*-th domestic (imported) good consumed by the government in the *r*-th region; and the value of government consumption $\left(VG_r\right)$ is defined as

$$VG_r = \sum_{i=1}^{K} \left(VGD_{ir} + VGM_{ir}\right), \ \forall r. \tag{3.68}$$

3.8.2 Taxes on government inputs

PGD_{ir} and PGM_{ir} are purchasers' prices that include commodity-specific taxes on domestic $\left(TGD_{ir}\right)$ and imported $\left(TGM_{ir}\right)$ consumption by governments:

$$PGD_{ir} = PD_{ir} TGD_{ir}, \ \forall i, r, \tag{3.69}$$

$$PGM_{ir} = PM_{ir} TGM_{ir}, \ \forall i, r. \tag{3.70}$$

Note that both TGD_{ir} and TGM_{ir} are powers of *ad valorem* taxes.

3.9 Trade demands

Exports in WOOLGEM are distinguished on a bilateral basis. Exports are further distinguished between types, that is, into either margin exports or nonmargin exports. Nonmargin exports are typical merchandise and services exports, whereas margin exports are exports of services which facilitate the export of merchandise and services, e.g., transport, communication, insurance and financial services.

3.9.1 Nonmargin export demands

Nonmargin traded commodities are demanded by firms, capital creators, households and governments. These demands relate to individual (nonmargin) import composites; that is, firms, households and governments do not choose between individual imports from different sources. The decision on goods from different sources – bilateral (nonmargin) export demands – is made by a representative importer using a CES production function (see Chapter 2, Section 2.3.2) giving demand functions of the form

$$qx_{isr}^{NM} = qm_{ir} - \sigma_{ir}^{T}\left(pm_{isr} - pm_{ir}\right),\ \forall i,s,r; \tag{3.71}$$

where

$$PM_{ir} = \sum_{s=1}^{R} \frac{VM_{isr}}{VM_{ir}} PM_{isr},\ \forall i,r; \tag{3.72}$$

and $\frac{VM_{isr}}{VM_{ir}}$ are the bilateral import shares at basic (or ex-duty) values.

Equation (3.71) states that the demand for any (nonmargin) import *i* from (source) region *s* (= 1,…,*R*) to (destination) region *r* (= 1,…,*R*) $\left(qx_{isr}^{NM}\right)$ is a function of an expansion effect and a substitution effect. If we set the change in relative prices $\left(pm_{isr} - pm_{ir}\right)$ to zero, then demand for imports of *i* from source *s* in destination *r* will move exactly with demand for composite imports of *i* $\left(qm_{ir}\right)$; i.e., the expansion effect. As previously observed, this reflects constant returns to scale in the CES production function. Alternatively, if we set $qm_{ir} = 0$, then qx_{isr}^{NM} will be a function of the change in the (basic or ex-duty) price of import *i* from source *s* in destination *r* $\left(pm_{isr}\right)$, relative to pm_{ir}, the average (basic) price of import *i* from source *s* in destination *r*, adjusted by σ_{ir}^{T}; i.e., the substitution effect.

Note that all variables relating to bilateral trade follow the notational convention that the first index refers to commodities, the second to source regions and the third to destination regions; so the index counters r and s can refer to either source regions or destination regions depending on the order in which they appear.

For use below, we define aggregate nonmargin exports in each region as

$$QX_s^{NM} = \sum_{i=1}^{K} \sum_{r=1}^{R} QX_{isr}^{NM} \text{ , } \forall s \text{ .} \tag{3.73}$$

3.9.2 Margin export demands

Demands for bilateral margin exports of good m used to ship good i from region s to region r $\left(QX_{misr}^{M}\right)$ are determined by the representative importer using a Leontief production function (see Chapter 2, Section 2.3.1) with bilateral nonmargin exports being the input. Thus, we write

$$qx_{misr}^{M} = qx_{isr}^{NM} \text{ , } m = \textit{Other goods}; \ \forall i,s,r \text{ .} \tag{3.74}$$

Equation (3.74) limits margin exports to the other goods composite for consistency with the model database.

For use below, we define aggregate margin exports in each (source) region as

$$QX_s^{M} = \sum_{i=1}^{K} \sum_{r=1}^{R} QX_{misr}^{M} \text{ , } m = \textit{Other goods}; \ \forall s \text{ .} \tag{3.75}$$

3.9.3 Taxes on exports

Export taxes are only applied on nonmargin exports. Thus, the f.o.b. prices of bilateral nonmargin and margin exports, PX_{isr}^{NM} and PX_{isr}^{M}, are defined as

$$PX_{isr}^{NM} = PD_{is} TX_{isr} \text{ , } \forall i,s,r \text{ ,} \tag{3.76}$$

$$PX_{isr}^{M} = PD_{ms} \text{ , } m = \textit{Other goods}; \ \forall i,s,r \text{ ,} \tag{3.77}$$

where TX_{isr} is the power of the destination-specific *ad valorem* bilateral export tax. Note that PX_{isr}^{M} is the price of the margin export required to ship good *i* from region *s* to region *r*.

3.9.4 Composite imports

We define composite imports of good *i* $\left(QM_{ir}\right)$ as the sum of all demands for imports (i.e., by firms, capital creators, households, and governments):

$$QM_{ir} = \sum_{j=1}^{J} QFM_{ijr}^{I} + QIM_{ir} + QHM_{ir} + QGM_{ir}, \ \forall i,r. \tag{3.78}$$

We define the c.i.f. price of bilateral imports, PM_{isr}^{CIF}, as a weighted average of the f.o.b. prices of margin and nonmargin exports,

$$PM_{isr}^{CIF} = \frac{VX_{isr}^{NM}}{VM_{isr}^{CIF}} PX_{isr}^{NM} + \frac{VX_{isr}^{M}}{VM_{isr}^{CIF}} PX_{isr}^{M}, \ \forall i,s,r; \tag{3.79}$$

where

$$VM_{isr}^{CIF} = VX_{isr}^{NM} + VX_{isr}^{M}, \ \forall i,s,r. \tag{3.80}$$

3.9.5 Taxes on imports

Source region-specific import taxes are applied to imports entering each destination region, thus the ex-duty (or basic) price of bilateral imports, PM_{isr}, is defined as

$$PM_{isr} = PM_{isr}^{CIF} TM_{isr}, \tag{3.81}$$

where TM_{isr} is the power of the *ad valorem* bilateral import tax.

3.9.6 Trade indices

We define the terms of trade in each region $\left(\mathrm{E}_r\right)$ in terms of the f.o.b. and c.i.f. prices of aggregate exports and imports;

$$\mathrm{E}_r = \frac{PX_r}{PM_r^{CIF}}, \ \forall r, \tag{3.82}$$

where

$$PX_r = \sum_{i=1}^{K} \sum_{s=1}^{R} \left[\frac{VX_{irs}^{M}}{VX_r} PX_{irs}^{M} + \frac{VX_{irs}^{NM}}{VX_r} PX_{irs}^{NM} \right], \ \forall r, \tag{3.83}$$

$$PM_r^{CIF} = \sum_{i=1}^{K} \sum_{s=1}^{R} \frac{VM_{isr}^{CIF}}{VM_r^{CIF}} PM_{isr}^{CIF}, \ \forall r, \tag{3.84}$$

and

$$VX_r = \sum_{i=1}^{K} \sum_{s=1}^{R} \left[VX_{irs}^{M} + VX_{irs}^{NM} \right], \tag{3.85}$$

$$VM_r^{CIF} = \sum_{i=1}^{K} \sum_{s=1}^{R} VM_{isr}^{CIF}. \tag{3.86}$$

Thus, PX_r and PM_r^{CIF} are defined using f.o.b. and c.i.f. values of exports and imports.

For possible use as an exogenous variable, we also define the ratio of the trade balance to GDP:

$$\mathrm{X}_r = \frac{\left(VX_r - VM_r^{CIF}\right)}{VGDP_r^{EXP}}. \tag{3.87}$$

Note that $VGDP_r^{EXP}$ is the value of GDP from the expenditure side [see (3.119) below].

3.10 Inventories demands

The change in the demand for stocks of good *i*, produced and held by industry *j* in region *r*, QS_{ijr}, represents a component of the *j*-th industry's sales. For use in defining total domestic sales, we define (the change in) stocks by commodity, QS_{ir}, as

$$QS_{ir} = \sum_{j=1}^{J} QS_{ijr} \text{ , } \forall i, r \text{ .} \tag{3.88}$$

For use in defining nominal GDP, we define the value of (the change in) industry stocks by region, VS_r, as the product of prices and quantities, where the basic price of the domestic good is the relevant price:

$$VS_r = \sum_{i=1}^{K} PD_{ir} QS_{ir} \text{ .} \tag{3.89}$$

3.11 Market-clearing conditions

The model assumes prices in all markets are determined by market clearing. Below we define the market-clearing conditions for traded commodities and factors of production.

3.11.1 Traded commodities

To determine the basic (or supply) price of each of the *K* domestic commodities $\left(PD_{ir}\right)$, we specify a market-clearing condition that relates the supplies and demands of domestic commodities to each other. Due to the existence of two types of exports (margin and nonmargin), we require two market-clearing conditions:

$$QD_{ir} = \sum_{j=1}^{J} QFD_{ijr}^{I} + QID_{ir} + QHD_{ir} + QGD_{ir} + QS_{ir} + \sum_{s=1}^{R} QX_{irs}^{NM} + QX_{r}^{M} \text{ ,}$$

$$i = \textit{Other goods}; \ \forall r \text{ ,} \tag{3.90}$$

$$QD_{ir} = \sum_{j=1}^{J} QFD_{ijr}^{I} + QID_{ir} + QHD_{ir} + QGD_{ir} + QS_{ir} + \sum_{s=1}^{R} QX_{irs}^{NM} ,$$

$$i \neq Other\ goods;\ \forall r . \tag{3.91}$$

(3.90) and (3.91) are the market-clearing conditions for the margin and nonmargin commodities, and they are identical except for the inclusion of aggregate margin exports $\left(QX_r^M\right)$ in (3.90). The LHSs of (3.90) and (3.91) represents the aggregate output of commodity *i* in region *r*. The RHSs of (3.90) and (3.91) represent total sales of commodity *i* produced in region *r*, consisting of:

(i) sales of domestic commodity *i* for intermediate input usage by industry *j* in region *r* summed across the *J* industries $\left(\sum_{j=1}^{J} QFD_{ijr}^{I}\right)$;

(ii) sales of domestic commodity *i* for intermediate input usage by the capital creator in region *r* $\left(QID_{ir}\right)$;

(iii) sales of domestic commodity *i* for household consumption in region *r* $\left(QHD_{ir}\right)$;

(iv) sales of domestic commodity *i* for government consumption in region *r* $\left(QGD_{ir}\right)$;

(v) (change in) aggregate sales of commodity *i* held as inventories in region *r* $\left(QS_{ir}\right)$;

(vi) sales of domestic commodity *i* produced and exported (as a nonmargin) by region *r* to destination region *s* summed across the *R* destination regions $\left(\sum_{s=1}^{R} QX_{irs}^{NM}\right)$; and

(vii) aggregate sales of domestic commodity *i* produced and exported (as a margin) by region *r* $\left(QX_r^M\right)$.

3.11.2 Factors of production

We define market-clearing conditions for the *i* (= 1,2,3) factors of production similarly to traded commodities,

$$Q_{ir}^{F} = \sum_{j=1}^{J} QF_{ijr}^{F} Z_{ir}^{F} \text{ , } i = 1,2,3; \forall r \text{ .} \tag{3.92}$$

(3.92) includes Z_{ir}^{F}, a factor-specific shift variable in each region, for possible use in choosing alternative treatments of factor allocation amongst industries. For the same reason, we add the following equation defining Γ_{jr}, the ratio of industry and regional land usage:

$$\Gamma_{jr} = \frac{QF_{ijr}^{F}}{Q_{ir}^{F}} \text{ , } i = Land; \forall j, r \text{ .} \tag{3.93}$$

3.12 Household income

Household income, YH_r, was introduced earlier in defining the household consumption function [see (3.49)]. As households are assumed to own all factors of production in WOOLGEM, YH_r is defined as

$$YH_r = \sum_{i=1}^{3} PH_{ir}^{F} Q_{ir}^{F} \text{ , } i = 1,2,3; \forall r \text{ ,} \tag{3.94}$$

i.e., YH_r consists of total factor income received by households in each region, which is itself defined as total demand for each of the three factors, Q_{ir}^{F}, multiplied by the factor price received by households, PH_{ir}^{F}, summed across the three factors.

3.12.1 Taxes on factor income

Factor-specific taxes on household income are applied to the basic price of each factor to give PH_{ir}^{F} as:

$$PH_{ir}^{F} = \frac{P_{ir}^{F}}{TH_{ir}^{F}} \text{ , } i = 1,2,3; \forall r \text{ ;} \tag{3.95}$$

where TH_{ir}^{F} is the power of the *ad valorem* tax on income from the *i*-th factor in the *r*-th region.

TH_{ir}^{F} is subject to the tax function,

$$TH_{ir}^{F} = \left(1 + \overline{T}H_{ir}^{F}\right) TH_{r}^{F}\ ,\ i = 1,2,3\ ; \tag{3.96}$$

where $\overline{T}H_{ir}^{F}$ is the *ad valorem* tax rate on income from the *i*-th factor in the *r*-th region, and TH_{r}^{F} is the factor-generic power of the tax on factor income in the *r*-th region and is initially set equal to one. (3.96) fixes the differences in TH_{ir}^{F} across factors in a region, even where TH_{r}^{F} is made endogenous.

For later use, we also define industry-specific rental prices received by households $\left(PH_{ijr}^{F}\right)$,

$$PH_{ijr}^{F} = \frac{P_{ijr}^{F}}{TH_{ir}^{F}}\ ,\ i = Land, Capital; \forall j, r\ . \tag{3.97}$$

3.13 Tax revenues

We have already shown the range of tax instruments, both direct and indirect, contained in the model. This section defines government revenue from (i) indirect taxes for use in defining GDP from the income side [see (3.117)], and (ii) all taxes for use in defining the government budget.

3.13.1 Indirect taxes

The model contains eight distinct indirect tax instruments. The bases for these instruments are industry output, factor usage by firms, intermediate input usage by firms, inputs to investment, household consumption, government consumption, imports and exports. Thus, we define tax revenue from all indirect taxes as

$$TR_r^{NF} = TRF_r + TRF_r^F + TRFD_r^I + TRFM_r^I + TRID_r + TRIM_r + TRHD_r + TRHM_r +$$

$$TRGD_r + TRGM_r + TRX_r + TRM_r \, , \; \forall r \, . \tag{3.98}$$

The aggregates on the RHS of (3.98) are defined below.

3.13.1.1 *Industry output*

Tax revenue is calculated as the difference between the tax-inclusive value and the tax-exclusive value of the relevant revenue base. Thus, tax revenue on industry output, TRF_r , is

$$TRF_r = \sum\nolimits_{j=1}^{J} \left(PF_{jr} QF_{jr} - \left[\frac{PF_{jr}}{TF_{jr}} \right] QF_{jr} \right) , \; \forall r \, , \tag{3.99}$$

where TF_{jr} is the power of the *ad valorem* tax on industry output, therefore $TF_{jr} \geq 1$. So TRF_r equals the tax-inclusive value of industry output $\left(PF_{jr} QF_{jr} \right)$ minus the tax-exclusive value of industry output $\left(\left[\frac{PF_{jr}}{TF_{jr}} \right] QF_{jr} \right)$, summed over the *J* industries.

3.13.1.2 *Factor usage*

Tax revenue from factor usage by firms, TRF_r^F , is

$$TRF_r^F = \sum\nolimits_{i=1}^{3} \sum\nolimits_{j=1}^{J} \left(PF_{ijr}^F QF_{ijr}^F - P_{ijr}^F QF_{ijr}^F \right) , \; \forall r \, , \tag{3.100}$$

that is, the tax-inclusive value of industry *j*'s expenditure on factor *i* $\left(PF_{ijr}^{F}QF_{jr}^{F}\right)$ minus the tax-exclusive value of industry *j*'s expenditure on factor *i* $\left(P_{ijr}^{F}QF_{ijr}^{F}\right)$, summed over the three factors and the *J* industries.

3.13.1.3 Intermediate input usage by firms

Tax revenue on domestic (imported) intermediate input usage by firms, $TRFD_{r}^{I}$ $\left(TRFM_{r}^{I}\right)$, is

$$TRFD_{r}^{I}=\sum_{i=1}^{K}\sum_{j=1}^{J}\left(PFD_{ijr}^{I}QFD_{ijr}^{I}-PD_{ir}QFD_{ijr}^{I}\right),\ \forall r, \tag{3.101}$$

$$TRFM_{r}^{I}=\sum_{i=1}^{K}\sum_{j=1}^{J}\left(PFM_{ijr}^{I}QFM_{ijr}^{I}-PM_{ir}QFM_{ijr}^{I}\right),\ \forall r. \tag{3.102}$$

3.13.1.4 Inputs to investment

Tax revenue on domestic (imported) intermediate input usage by the capital creator, $TRID_{r}$ $\left(TRIM_{r}\right)$, is

$$TRID_{r}=\sum_{i=1}^{K}\left(PID_{ir}QID_{ir}-PD_{ir}QID_{ir}\right),\ \forall r, \tag{3.103}$$

$$TRIM_{r}=\sum_{i=1}^{K}\left(PIM_{ir}QIM_{ir}-PM_{ir}QIM_{ir}\right),\ \forall r. \tag{3.104}$$

3.13.1.5 Household consumption

Tax revenue from domestic (imported) consumption by households, $TRHD_{r}$ $\left(TRHM_{r}\right)$, is

$$TRHD_{r}=\sum_{i=1}^{K}\left(PHD_{ir}QHD_{ir}-PD_{ir}QHD_{ir}\right),\ \forall r, \tag{3.105}$$

$$TRHM_{r}=\sum_{i=1}^{K}\left(PHM_{ir}QHM_{ir}-PM_{ir}QHM_{ir}\right),\ \forall r. \tag{3.106}$$

3.13.1.6 Government consumption

Tax revenue on domestic (imported) consumption by governments, $TRGD_r$ $\left(TRGM_r\right)$, is

$$TRGD_r = \sum_{i=1}^{K}\left(PGD_{ir}QGD_{ir} - PD_{ir}QGD_{ir}\right),\ \forall r, \tag{3.107}$$

$$TRGM_r = \sum_{i=1}^{K}\left(PGM_{ir}QGM_{ir} - PM_{ir}QGM_{ir}\right),\ \forall r. \tag{3.108}$$

3.13.1.7 Exports

Tax revenue from exports, TRX_r, is

$$TRX_r = \sum_{i=1}^{K}\sum_{s=1}^{R}\left(PX_{irs}^{NM}QX_{irs}^{NM} - PD_{ir}QX_{irs}^{NM}\right),\ \forall r. \tag{3.109}$$

3.13.1.8 Imports

Tax revenue from imports, TRM_r, is

$$TRM_s = \sum_{i=1}^{K}\sum_{r=1}^{R}\left(PM_{irs}QX_{irs}^{NM} - PM_{irs}^{CIF}QX_{irs}^{NM}\right),\ \forall s. \tag{3.110}$$

3.13.2 Direct taxes

Tax revenue from direct taxes is simply the difference between pre-tax and post-tax household income:

$$TRH_r^F = \sum_{i=1}^{3}\left(P_{ir}^F Q_{ir}^F - PH_{ir}^F Q_{ir}^F\right),\ \forall r. \tag{3.111}$$

3.13.3 Aggregate tax revenue

Total government revenue from taxes in a region is the sum of revenues from indirect and direct taxes:

$$TR_r = TR_r^{NF} + TRH_r^F \,,\ \forall r \,. \tag{3.112}$$

3.14 Government accounts

We define the government deficit as the difference between total government spending and government revenue $\left(VG_r - TR_r\right)$, and the ratio of the government deficit to GDP, Π_r, as

$$\Pi_r = \frac{\left(VG_r - TR_r\right)}{VGDP_r^{EXP}} \,. \tag{3.113}$$

Note that $VGDP_r^{EXP}$ is the value of GDP from the expenditure side [see (3.119) below].

3.15 Inter-industry mobility of rented factors

Section 3.3.4 has already specified perfect mobility of labour between industries in a region via equation (3.18). This assumption will apply regardless of what is assumed about the behaviour of total employment in a region. For the rented factors of production, land and capital, we wish to allow for inter-industry mobility within regions in a long-run environment. To accommodate this objective for land, we add the following equation that was first applied by Peter et al. (1996):

$$ph_{ijr}^F - ph_{ir}^F = \rho\left(qf_{ijr}^F - q_{ir}^F\right) + zph_{ijr}^F \,,\ i = Land; \forall j, r \,. \tag{3.114}$$

The LHS of (3.114) is the percentage-change form of the ratio of the rental price received by households for a unit of land in industry *j* to the average land rental price received by households. The term in parentheses on the RHS of (3.114) is the percentage-change form of the ratio of land used by industry *j* to total land usage. zph_{ijr}^F is a shift term. Letting (the parameter) ρ = 1 and setting zph_{ijr}^F as exogenous, (3.114) enforces a one-to-one relationship between the price and quantity ratios, where fast-growing (slow-growing) industries pay a premium (receive a discount) on the land they rent. Taking the view that land is a very immobile factor and specific to certain uses, we set $\rho = 10$; thus a small increase (decrease) in the use of land by an industry will lead to a significant increase (decrease) in the rental price paid by the industry, which, in turn, will discourage (encourage) a further increase (decrease) in the use of land by the industry.

To allow for inter-industry capital mobility within regions, we first define the post-tax (net of depreciation) rate of return on (a unit of) capital by industry, R_{jr};

$$R_{jr} = \frac{PH_{ijr}^{F}}{PI_{r}} - \Omega_{r}, \; i = Capital; \forall j, r. \tag{3.115}$$

So R_{jr} equals the ratio of the rental price of capital in industry j received by households over the (average) price of investment, minus the depreciation rate Ω_r. Using R_{jr}, we write the following allocation rule for inter-industry capital movements:

$$R_{jr} = R_r ZR_{jr}, \; \forall j, r. \tag{3.116}$$

That is, the post-tax (net of depreciation) rate of return on (a unit of) capital used by industry, R_{jr}, is indexed to the region-wide post-tax rate of return on capital, R_r, multiplied by the shift variable ZR_{jr}, which is initially set equal to one.

3.16 GDP indices

We define GDP from two perspectives: the income side and the expenditure side.

3.16.1 GDP from the income side

We define nominal GDP from the income side as the sum of total factor income plus indirect tax receipts:

$$VGDP_r^{INC} = \sum_{i=1}^{3} \sum_{j=1}^{J} V_{ijr}^F + TR_r^{NF} \text{ , } \forall r \text{ .} \tag{3.117}$$

Note that V_{ijr}^F is factor payments at basic (or supply) values defined as the product of prices and quantities:

$$V_{ijr}^F = P_{ijr}^F Q_{ijr}^F \text{ , } \forall i, j, r \text{ .} \tag{3.118}$$

Thus, V_{ijr}^F is inclusive of income taxes.

3.16.2 GDP from the expenditure side

We define nominal GDP from the expenditure side as the sum of household consumption, investment, government consumption, total exports net of imports, and the change in inventories:

$$VGDP_r^{EXP} = VH_r + VI_r + VG_r + VX_r - VM_r^{CIF} + VS_r \text{ , } \forall r \text{ .} \tag{3.119}$$

The attendant price index for (3.119) is

$$PGDP_r^{EXP} = \frac{VH_r}{VGDP_r^{EXP}} PH_r + \frac{VI_r}{VGDP_r^{EXP}} PI_r + \frac{VG_r}{VGDP_r^{EXP}} PG_r + \frac{VX_r}{VGDP_r^{EXP}} PX_r$$

$$- \frac{VM_r^{CIF}}{VGDP_r^{EXP}} PM_r^{CIF} + \frac{VS_r}{VGDP_r^{EXP}} PS_r \text{ , } \forall r \text{ .} \tag{3.120}$$

Dividing (3.119) by (3.120) gives real GDP:

$$QGDP_r^{EXP} = \frac{VGDP_r^{EXP}}{PGDP_r^{EXP}} \text{ , } \forall r \text{ .} \tag{3.121}$$

3.17 The complete model

As discussed in Section 3.2, we could represent the linear equations in WOOLGEM as in equation (3.5), which is reproduced below,

$$A\mathbf{v} = \mathbf{0}, \tag{3.122}$$

where A is an $m \times p$ matrix and $\mathbf{v}(\neq \mathbf{0})$ is the vector of the percentage change (or change) variables in WOOLGEM. The m rows of A represent the number of linearised equations in WOOLGEM and the p columns represent the number of linear variables in WOOLGEM. Thus A is rectangular as $p > m$, i.e., the number of variables exceeds the number of equations, and there exists an infinite number of solutions to (3.122). To generate a nontrivial solution to (3.122) we must set $(p-m)$ variables as exogenous, and most of these will have a value of zero. We specify two sets of exogenous variables: one for simulating a short-run environment and another for simulating a long-run environment.

3.17.1 A short-run closure

Table 3.2 contains the list of exogenous variables we choose for a short-run closure of WOOLGEM. The choice of exogenous variables is intended to represent an adjustment period, due to any perturbation, of between one to two years. The ratio of industry land usage and regional land usage, Γ_{jr}, is set as exogenous. Combined with fixed regional land usage, Q^F_{ir} ($i = Land$), setting Γ_{jr} as exogenous fixes the use of land in all industries, so that land is assumed to be industry specific in the short run. Next we set industry capital usage as exogenous, QF^F_{ijr} ($i = Capital$). Like land, we are assuming capital to be industry specific in the short run. With industry usage of land and capital set as exogenous, the industry factor demands equations, (3.11)–(3.12), determine the industry prices of land and capital, P^F_{ijr} ($i = Land, Capital$).

Table 3.2 Exogenous variables in short-run closure

Variable	Subscript range	Description	Identifier
Γ_{jr}	$j = 1,\ldots,J$; $r = 1,\ldots,R$	Ratio of industry land usage and regional land usage	(3.93)
Q^F_{ir}	$i = Land$; $r = 1,\ldots,R$	Regional land usage	(3.92)
QF^F_{ijr}	$i = Capital$; $j = 1,\ldots,J$; $r = 1,\ldots,R$	Industry capital usage	(3.11), (3.12)
A^F_{jr}	$j = 1,\ldots,J$; $r = 1,\ldots,R$	Hicks-neutral technical change by industry	(3.10)
A^F_{ijr}	$i = 1,\ldots,3$; $j = 1,\ldots,J$; $r = 1,\ldots,R$	Factor-specific technical change by industry	(3.11), (3.12)
A^I_{jr}	$j = 1,\ldots,J$; $r = 1,\ldots,R$	Technical change in the intermediate input composite, by industry	(3.20)
QS_{ijr}	$i = 1,\ldots,K$; $j = 1,\ldots,J$; $r = 1,\ldots,R$	(Change in) the demand for stocks by industry	(3.88)
T	various	All powers of *ad valorem* indirect tax rates	various
$\overline{TH}^F_{ir}$	$i = 1,\ldots,3$; $r = 1,\ldots,R$	Factor-specific *ad valorem* tax rate on household income	(3.96)
TH^F_r	$r = 1,\ldots,R$	Factor-generic power of the *ad valorem* tax rate on household income	(3.96)
W_r	$r = 1,\ldots,R$	Regional real wage	(3.19)
Ω_r	$r = 1,\ldots,R$	Regional depreciation rate	(3.115)
R_r	$r = 1,\ldots,R$	Region-wide post-tax (net of depreciation) rate of return on capital	(3.116)
Υ_r	$r = 1,\ldots,R$	Average propensity to consume in R–1 regions	(3.49)
PH	1	Global consumer price index (numeraire)	(3.51)

WOOLGEM can project the effects on endogenous variables, such as output, exports, imports, etc., from exogenous changes in production technology. But it cannot project or determine technical change itself; consequently we set all technical change variables, A^F_{jr}, A^F_{ijr} and A^I_{jr}, as exogenous. Neither can the model project changes in industry stocks QS_{ijr}. Thus, we set the change in the volume of inventories as an exogenous variable with zero change.

The many (powers of) indirect tax rates in WOOLGEM are also set as exogenous. By setting these variables to nonzero values, we can project the effects of changes in tax rates on the endogenous variables. For example, by setting percentage-change in import tariffs to nonzero values, we are able to observe the effects of historical changes or expected future changes in protection rates. We also set two direct tax instruments as exogenous, $\overline{T}H_{ir}^{F}$ and TH_{r}^{F}; the factor-specific *ad valorem* tax rate on income, and the factor-generic power of the *ad valorem* tax rate on household income. This fixes the income tax rates on all factors in each region. With government demands indexed to household demands [see (3.65)–(3.66)] and exogenous direct and indirect tax rates, the ratio of the government deficit to GDP Π_r [see (3.113)] is endogenous in the short run.

We also set the regional real wage rate as exogenous in all regions, W_r. This imposes the idea that total employment in each region can vary, implicitly through changes in regional unemployment rates. Thus, the model will project the change in employment necessary to maintain the existing real wage rate in each region. At the same time, industry employment is endogenous and labour moves between industries in a region so that industry prices of labour are equalised [see (3.18)].

Regional depreciation rates, Ω_r, are also set as exogenous with zero change. Although we also set the region-wide post-tax (net of depreciation) rate of return on capital, R_r in (3.116) as exogenous, this has no effect as the shift variable ZR_{jr}, which also appears in (3.116), is endogenous in the short-run closure. Thus R_{jr}, the industry post-tax rates of return on capital, can vary within a region, which is consistent with fixed industry capital usage in the short run.

Our choice of short-run exogenous variables is completed by placing two more variables on the exogenous list. To achieve macroeconomic closure in each region, we fix Υ_r, the average propensity to consume, in all regions except ROW (the Rest of World region) so that a household consumption function operates in all regions via Walras's law.[13]

[13] See Chapter 4, table 4.7 for a list of model regions.

This fixes savings rates in each region and allows the trade balance to be determined in the short run. The global CPI is also set as exogenous, thus serving as the numeraire.

3.17.2 A long-run closure

In altering the model closure for simulating the long run, we begin with our short-run closure in table 3.2 and move variables between the lists of exogenous and endogenous variables (i.e., we perform closure swaps); the closure swaps are listed in table 3.3. The choice of exogenous variables in the long-run closure is intended to represent an adjustment period, due to any perturbation, of between five to ten years. In the long run, we wish industry usage of all factors to be endogenous. Starting from our short-run closure, industry usage of labour is already endogenous. We then endogenise the ratio of industry and regional land usage Γ_{jr} and exogenise the shift term for land allocation across industries, ZPH^F_{ijr} (i = *Land*). This engages the land allocation rule in (3.114), which makes land only slightly mobile between industries in a region. Note that Q^F_{ir} (i = *Land*), regional land usage, is still exogenous.

Table 3.3 Closure swaps in moving from short-run to long-run closure

Exogenous to endogenous	Endogenous to exogenous	Subscript range	Description	Identifier
Γ_{jr}	ZPH^F_{ijr}	i = *Land*; $j = 1,\ldots,J$; $r = 1,\ldots,R$	Endogenise ratio of industry and regional land usage; exogenise shift term for land allocation across industries	(3.114)
QF^F_{ijr}	ZR_{jr}	i = *Capital*; $j = 1,\ldots,J$; $r = 1,\ldots,R$	Endogenise industry capital usage; exogenise shift term for capital allocation across industries	(3.116)
W_r	Q^F_{ir}	i = *Labour*; $r = 1,\ldots,R$	Endogenise regional real wage rate; exogenise regional labour usage	(3.92)
TH^F_r	Π_r	$r = 1,\ldots,R$	Endogenise all income tax rates; exogenise ratio of government deficit to GDP	(3.113)
Υ_r	X_r	$r = 1,\ldots,R$	Endogenise marginal propensity to consume in R–1 regions; exogenise ratio of the trade balance to GDP in R–1 regions	(3.87)

Next, we endogenise industry usage of capital, QF^F_{ijr} (i = *Capital*) and exogenise the shift term for capital allocation across industries, ZR_{jr}, in (3.116). With the region-wide post-tax rate of return on capital, R_r, already exogenous, this fixes the differences between industry post-tax rates of return on capital, R_{jr}, within a region and forces capital to move perfectly between industries within a region. This change also assumes that each region's capital stock can grow or depreciate without affecting the post-tax regional rate of return on capital, R_r, i.e., the regional capital supply function is close to horizontal and changes in regional capital stocks are almost purely demand driven.

We set the regional real wage rate, W_r, as endogenous and fix regional labour usage, Q^F_{ir} (i = *Labour*). This assumes that, in the long run, total employment in each region is a function of an imperfectly flexible national labour market where the real wage does not adjust to clear excess demands in the labour market. In this case, WOOLGEM would indicate the change in the real wage necessary to maintain initial employment levels from a given exogenous shock.

The last two closure swaps we make are to macroeconomic variables. We endogenise all income tax rates by removing TH^F_r from the exogenous list [see (3.96)], and we fix Π_r, the ratio of the government deficit to GDP. Thus, all income tax rates will adjust to ensure that the government savings position remains constant in the long run, reflecting the idea that the government savings position is a policy instrument which is exogenous to any given simulation. We also endogenise Υ_r, the marginal propensity to consume, in all regions except ROW; this turns off the household consumption function in (3.49) for all regions except ROW. At the same time, we fix the ratio of the trade balance to GDP, X_r, in all regions except ROW so that each region must return to its initial trading position with the rest of the world (via Walras's law) once the effects of any simulation have dissipated, i.e., once long-run equilibrium has been achieved in all markets. The last change makes household savings rates endogenous in each region (government savings have already been fixed through Π_r), and forces them to move in a way that achieves a fixed X_r. Also note that, given a fixed R_r (the regional post-tax rate of return on capital) and X_r, growth in regional capital stocks must be largely sourced from domestic savings, as a fixed X_r (ratio of the trade balance to GDP) implies a fixed ratio of net capital outflow (savings minus investment) to GDP.

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