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## **How Long is the Long Run? Evidence from the Foreign Exchange Market\***

by

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### Abstract

The aim of this paper is to estimate the length of the long run in the foreign exchange market. We do this by examining the link between exchange rates and relative prices, based on the implications of purchasing power parity (PPP) theory. Using a new approach, we test if the ratios of variances of exchange rates to prices are unity over all horizons, as implied by PPP. Through Monte Carlo simulations, we derive the variance ratios under the null of equal variances and examine the power and size of the test. We find evidence that PPP holds in the long run. While the long run based on the consumer prices appears to be “long”, about five years, the estimate of the long run based on the single good, Big Macs, is shorter (two years).

Keywords: foreign exchange market, purchasing power parity, long run, variance ratio.

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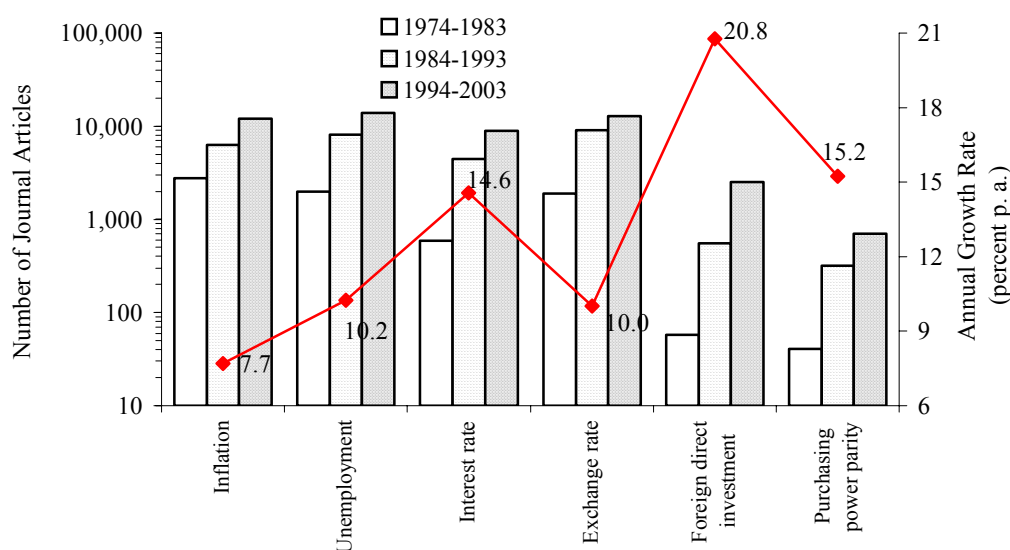
## 1. Introduction

The concept of purchasing power parity (PPP) is deeply rooted in the minds of many economists (Dornbusch and Krugman, 1976). PPP theory states that prices across countries should be equal when converted to a common currency (absolute PPP), or less strictly, the change in the exchange rate should be equal to the difference between inflation at home and abroad (relative PPP). Empirically, however, there is much heated debated regarding whether PPP holds. Professional confidence in PPP dipped after the publication of the paper “The Collapse of Purchasing Power Parities during the 1970s” by Frenkel (1981). Over the past three decades, the research devoted to PPP has been increasing dramatically and there is now general consensus that PPP is not a theory of short-term exchange rate determination, but it does offer a long-run equilibrium relationship between relative prices and exchange rates, at least for the major currencies.

To measure the extent of professional interest in PPP, we search for the total number of journal articles containing the phrase “purchasing power parity” in Econlit, a widely-used economic indexing database produced by the American Economic Association.<sup>1</sup> For comparison we also search for another five terms. The search results for each term are categorized into three sub-periods and are reported in Figure 1. The left-hand axis uses a logarithmic scale, so that the change in the height of the bars indicates the exponential rate of growth. The right-hand axis gives the growth rate, on an annual basis, for each topic. It can be seen that the number of articles on PPP has grown at an average rate of 15 percent p.a., second only to foreign direct investment. This growth rate clearly reflects the strong expansion of research interest in PPP over the past three decades, so that PPP research can now be described as “exploding” rather than “collapsing”.

Financial markets are also interested in PPP as a practical approach to valuing currencies and in making international price comparisons. However, PPP does not usually hold exactly and there are always prolonged deviations from PPP; for details, see surveys by Froot and Rogoff (1995),

FIGURE 1  
THE GROWTH OF ECONOMIC RESEARCH



<sup>1</sup> The material in this paragraph updates Lan (2004).

Rogoff (1996), Sarno and Taylor (2002), and Taylor and Taylor (2004). The focus of recent research on PPP is on the length of time taken to establish the PPP relationship between exchange rates and prices, that is, the length of the long run. This paper proposes a new test to examine this topic, based on the ratio of the variance of exchange rates to that of relative prices. Unlike previous tests of PPP, which involve traditional methods of regression or cointegration analysis for pairs of countries, our approach deals with all countries simultaneously.<sup>2</sup> The organisation of the paper is as follows. Section 2 provides theoretical background to the analysis. Section 3 presents the data and empirical variance ratios of exchange rates and relative prices. In Section 4, we simulate the confidence band of the variance ratios under the null hypothesis of equal variances under PPP. Section 5 examines the power and size of the new variance ratio test. We conduct a sensitivity analysis in Section 6 and compare the estimate of the length of the long run based on consumer price indexes with that based on the Big Mac Index from The Economist magazine. The final section concludes.

## 2. Theoretical Foundations

Let  $S_t$  be the nominal exchange rate in year  $t$ , defined as the domestic currency cost of a unit of foreign exchange, and  $P_t$  and  $P_t^*$  be the domestic and foreign price levels. Relative PPP then provides a strong link between the exchange rate and prices:

$$(1) \quad \Delta s_t = \Delta r_t,$$

where  $\Delta s_t = \log S_t - \log S_{t-1}$  is the log-change in  $S$ , and  $\Delta r = \log(P_t/P_{t-1}) - \log(P_t^*/P_{t-1}^*)$  is the difference between inflation at home and abroad. In words, equation (1) states that the change in the exchange rate equals the inflation differential. Clearly equation (1) implies that  $\text{var}(\Delta s_t) = \text{var}(\Delta r_t)$ , so that the volatility of exchange rates and prices coincide.

Consider equation (1) applied to the transition from year  $t-2$  to  $t-1$ , so that  $\Delta s_{t-1} = \Delta r_{t-1}$ . If we add both sides of this equation to equation (1) we obtain  $\log S_t - \log S_{t-2} = \log(P_t/P_{t-2}) - \log(P_t^*/P_{t-2}^*)$ , which we write as  $\Delta s_t^{(2)} = \Delta r_t^{(2)}$ , where  $\Delta s_t^{(2)} = \frac{1}{2}(\log S_t - \log S_{t-2})$  is the annualised log-change in  $S$  and  $\Delta r_t^{(2)} = \frac{1}{2}[\log(P_t/P_{t-2}) - \log(P_t^*/P_{t-2}^*)]$  is the corresponding annualised inflation differential. A similar argument establishes that for any horizon  $m$ ,

$$(2) \quad \Delta s_t^{(m)} = \Delta r_t^{(m)},$$

where, e.g.,  $\Delta s_t^{(m)} = \frac{1}{m}(\log S_t - \log S_{t-m})$ . If we define the variance ratio as  $\phi(m) = \text{var}[\Delta s^{(m)}] / \text{var}[\Delta r^{(m)}]$ , equation (2) then implies  $\phi(m) = 1$  for any value of  $m$ .<sup>3</sup> In

<sup>2</sup> One exception is Manzur (1990, 1993), which also avoid taking pairs of countries in isolation. But our approach is based on the ratio of two variances, while Manzur is based on Divisia index numbers.

<sup>3</sup> This variance ratio is to be distinguished from the variance ratio proposed by Cochrane (1988), which is used to detect a unit root in a time series. According to Cochrane, if the ratio of the variance of the  $k^{\text{th}}$  to the first differences is greater than  $k$ , there exists a unit root.

other words, PPP implies that over horizons, the variance of the exchange rates is the same as that of the inflation differential.<sup>4</sup>

If PPP does not hold, then the change in the inflation-adjusted, or, real, exchange rate,  $\Delta s_t - \Delta r_t$ , is non-zero and equations (1) and (2) have to be modified to

$$(3) \quad \Delta s_t = \Delta r_t + \varepsilon_t, \quad \Delta s_t^{(m)} = \Delta r_t^{(m)} + \varepsilon_t^{(m)},$$

where  $\varepsilon_t$  is the one-year change in the real exchange rate and

$$(4) \quad \varepsilon_t^{(m)} = \frac{1}{m} \sum_{j=0}^{m-1} \varepsilon_{t-j}$$

is the corresponding annualised change over horizon  $m$ . As an illustrative example, consider the case in which real exchange rate changes are orthogonal to inflation differentials, have a constant variance  $\sigma_\varepsilon^2$  and are independent over time.<sup>5</sup> Equation (4) then implies

$$(5) \quad \text{var}[\varepsilon^{(m)}] = \frac{\sigma_\varepsilon^2}{m},$$

and the variance ratio becomes

$$(6) \quad \phi(m) = 1 + \frac{\text{var}[\varepsilon^{(m)}]}{\text{var}[\Delta r^{(m)}]} = 1 + \frac{\sigma_\varepsilon^2}{m \times \text{var}[\Delta r^{(m)}]}.$$

If we make the reasonable assumption that  $\lim_{m \rightarrow \infty} \text{var}[\Delta r^{(m)}]$  is finite, then equation (6) reveals that when the horizon  $m \rightarrow \infty$ ,  $\phi(m) \rightarrow 1$ , so at long horizons we are back to the original case whereby the exchange rate and relative prices have the same volatility.

### 3. Exchange Rates and Prices Across Time and Countries

This section investigates the behaviour of exchange rates and prices over time and over a substantial number of countries. We denote countries by a  $c$  subscript and obtain the exchange rate and Consumer Price Index (CPI) data from the International Financial Statistics for

<sup>4</sup> We are not testing directly for PPP, which involves an examination of the comovement of logarithmic of variables  $S_t$ ,  $P_t$  and  $P_t^*$ . Rather we test it indirectly via its volatility implications. Note that while PPP implies equal variances and that  $\phi(m) = 1$  is not inconsistent with PPP, we cannot conclude that  $\phi(m) = 1$  always implies PPP. This is a qualification to our approach.

<sup>5</sup> The assumption that real exchange rate changes,  $\varepsilon_t$ , are independent over time requires some comment. This assumption implies that the real exchange rate has a unit root:  $\log(P_t/S_t P_t^*) = \log(P_{t-1}/S_{t-1} P_{t-1}^*) + \varepsilon_t$ . While such an assumption may appear to be strong, in fact it does not do too much violence to the truth (e.g., Meese, 1990, Mussa, 1979, 1990), and can be justified on the basis of the foreign exchange market being near efficient. However, it is to be emphasised that the above is just an example and in what follows, we make no *a priori* assumption about the behaviour of real exchange rates.

$c = 1, \dots, 50$  countries and  $t = 1974-2004$  with the US as the base country.<sup>6</sup> Thus the maximum value of the differencing span here is  $m_{\max} = 2004-1974=30$  years. For all countries and years, we plot  $\Delta s_{ct}$  against  $\Delta r_{ct}$  in panel A of Figure 2. It is obvious that many of the points are of a considerable distance from the  $45^\circ$  line and are thus not supportive of PPP. Panel B of Figure 2 contains a scatter plot of the annualized 30-year changes,  $\Delta s_{ct}^{(30)}$  and  $\Delta r_{ct}^{(30)}$ .<sup>7</sup> As the points are now much “closer” on average to the  $45^\circ$  line, visually there seems to be impressive support for PPP in Panel B. As we can interpret the 30-year changes as reflecting the long run, when all adjustments are complete, it can be seen why it is that many authors have emphasised that PPP only applies over the longer term, not on a year-to-year basis.

To measure the dispersion of the points around the  $45^\circ$  line in a given panel of Figure 2, we use the root-mean-squared error (RMSE), which has the property that if all points lie on the  $45^\circ$  line, the RMSE is zero. As can be seen from Panel A, which corresponds to annual changes, the RMSE is about 12 percent. The changes over the entire 30-year period (Panel B) give the much lower RMSE of about 2 percent. This reinforces the discussion in the previous paragraph that PPP holds better in the long run.

We have compared exchange rates and prices over one- and 30-year horizons. As can be seen from panel A of Figure 2, for  $m = 1$  the variance of exchange rates seems to be larger than that of relative prices with annual changes. That is, the scatter in Panel A is substantially “higher” than its “width” (the scales are the same on both axes). By contrast, in Panel B of the figure ( $m = 30$ ) the two variances seems to be more or less the same. This indicates that in the short run  $\phi(1) > 1$ , while for the long run  $\phi(30) \approx 1$ . This result seems to be not inconsistent with equation (6). The reason for  $\phi(1) > 1$  is the existence of  $\sigma_\varepsilon^2$ , which reflects the variability of real exchange rates, so that  $\phi(1) = 1 + \sigma_\varepsilon^2 / \text{var}[\Delta r] > 1$ .

As we know the two “end points” for the function  $\phi(m)$ ,  $\phi(1) > 1$  and  $\phi(30) \approx 1$ , it is natural to ask, what is the nature of the transition path in going from the short run to the long run? What period of time has to elapse until the ratio effectively hits unity? One way to investigate this issue is to analyse the variance ratio  $\phi(m)$  for  $1 < m < 30$ . To do this, for a given value of  $m$  we average over all countries and years to yield an estimate of  $\phi(m)$ .<sup>8</sup> The solid line in Figure 3

<sup>6</sup> The data we use are annual inflation rates for the 50 countries and the exchange rates in the first quarter of each year from 1974 to 2004. The countries are selected from the International Financial Statistics on the following basis: (i) CPIs are available for all years. The only exception to this rule is Cameroon whose CPI in 2004 is unavailable. We omit that country for that year. (ii) Exchange-rates data for all periods are available. The exceptions to this rule are EU member countries whose exchange rates after 1999 are unavailable and the Soloman Islands whose 2004 rate is not available. For these countries, in the computation of  $\text{var}[\Delta s]$  and  $\text{var}[\Delta r]$  we delete the pair of  $\Delta s_{ct}^{(m)}$  and  $\Delta r_{ct}^{(m)}$  whenever we cannot obtain either  $\Delta s_{ct}^{(m)}$  or  $\Delta r_{ct}^{(m)}$ . For example, there are only 38 pairs of observations when  $m=30$  mainly because of the missing exchange rates for EU countries. (iii) The one-year changes,  $\Delta s_t$  and  $\Delta r_t$ , are less than 80 percent in absolute value. In addition, it is to be noted that in 1993, the currencies of five African countries, Burkina Faso, Cameroon, Cote d’Ivoire, Niger, and Senegal jointly depreciated by about 65 percent. This is shown as the several points at the top end near the vertical axis in Panel A of Figure 2.

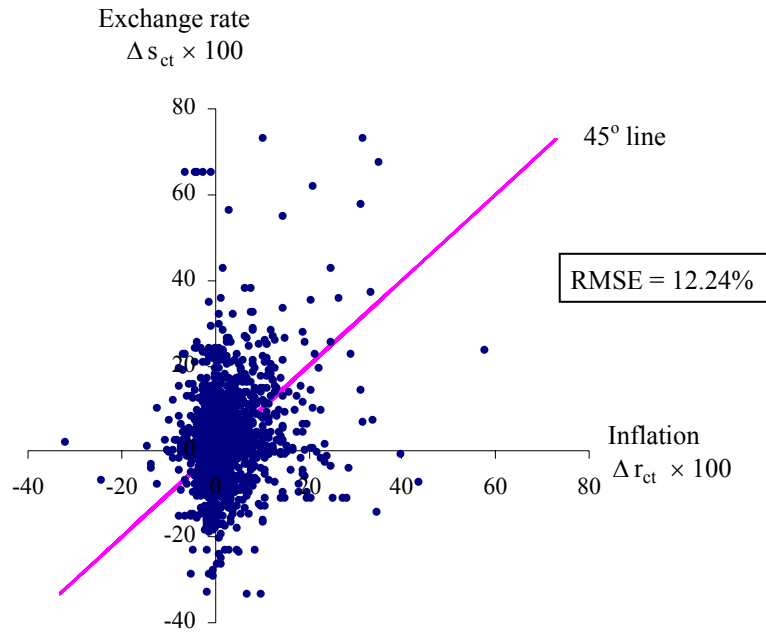
<sup>7</sup> Similar analyses are provided in Lothian (1985) and Obstfeld (1995).

<sup>8</sup> It is to be noted that all data used in this paper involve non-overlapping observations.

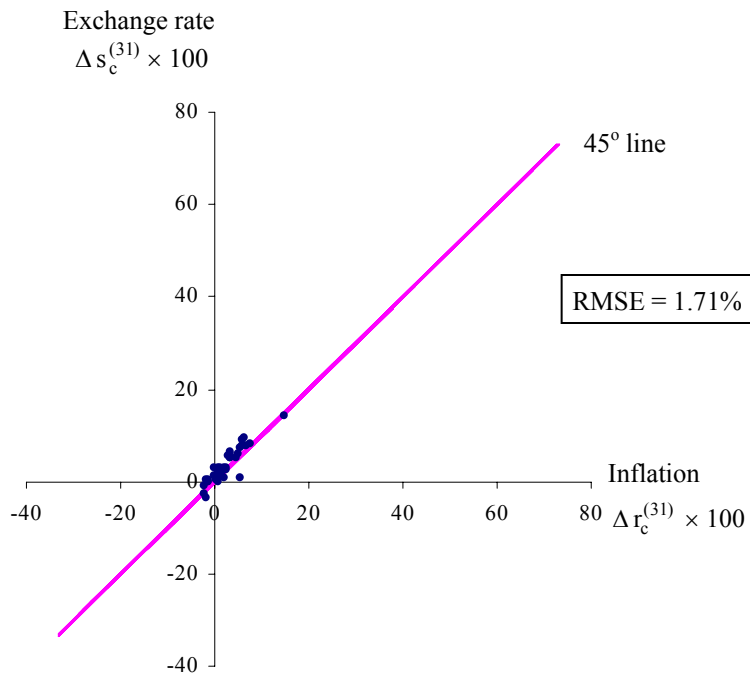
FIGURE 2

CHANGES IN EXCHANGE RATES AND RELATIVE PRICES FOR 50 COUNTRIES, 1974-2004  
(Annualised logarithmic changes)

A. Short run (all 50 countries, 30 years)

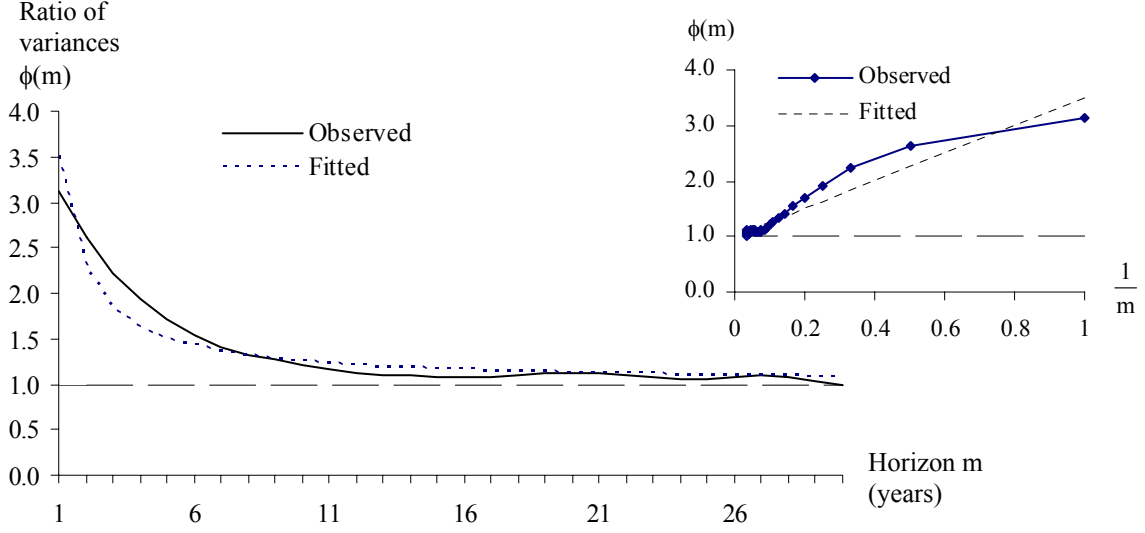


B. Long run (38 countries)



Note: There are only 38 observations in panel B due to missing observations, which are mainly associated with EU countries.

FIGURE 3  
VARIANCES OF EXCHANGE RATES AND RELATIVE PRICES



plots  $\phi(m)$  against  $m$ . As the value of  $\phi(m)$  is in the vicinity of 3 for  $m = 1$ , on average, exchange rates are about three times as volatile as relative prices on a year-to-year basis. However, the ratio decreases considerably as  $m$  increases, and gradually approaches one. This result is consistent with the idea that exchange rates overshoot relative prices in the short run, while the two variables have more or less the same variability at longer horizons (Dornbusch, 1976).

To analyse further the relationship between  $\phi(m)$  and  $m$ , we return to the special case in which the one-year real exchange rate changes  $\varepsilon_t$  are independently and identically distributed. We also assume that the variance of the annualised inflation differential over the  $m$ -year horizon  $\Delta r^{(m)}$  is a constant, denoted by  $\sigma_r^2$ . From equation (6), we then have

$$(7) \quad \phi(m) = 1 + \frac{\sigma_\varepsilon^2}{m\sigma_r^2}.$$

As  $\sigma_\varepsilon^2/\sigma_r^2$  is constant, the relationship between  $\phi(m)$  and  $1/m$  is linear and can be expressed as

$$(8) \quad \phi(m) = \alpha + \beta\left(\frac{1}{m}\right) + \zeta_m,$$

where  $\alpha$  and  $\beta$  are coefficients and  $\zeta_m$  is a disturbance term, interpreted as an approximation error. We estimate equation (8) by least-squares using the values of  $\phi(m)$  in Figure 3 and obtain  $\hat{\alpha} = 1.00$  (0.03) and  $\hat{\beta} = 2.51$  (0.15), where the figures in parentheses are standard errors. The fitted values of  $\phi(m)$ ,  $\hat{\phi}(m) = \hat{\alpha} + \hat{\beta}(1/m)$ , are presented as a dotted line in Figure 3. Figure 3 also contains a plot of the actual and fitted values of  $\phi(m)$  against  $1/m$  on the top-right corner. While the fit is not perfect, the actual and fitted values of  $\phi(m)$  are reasonably close, except for

the initial few years. Both estimated parameters are significant and the estimated value of  $\alpha$  is not significantly different from its theoretical value of unity. When the horizon is  $m=1$ , the predicted variance ratio is  $\hat{\phi}(1) = \hat{\alpha} + \hat{\beta} = 3.51$ , implying that on a one-year basis exchange rates are about three and a half times as volatile as the corresponding inflation differential, which is not inconsistent with the previous discussion. For horizons greater than five years, the predicted variance ratio is very close to its actual counterpart, providing evidence that the assumption of constant variances of exchange rates changes and inflation differentials are not grossly contradicted by the data.<sup>9</sup>

Consider the effects of a one-year shock to real exchange rate  $\varepsilon_t > 0$ . From equation (3), the nominal rate changes one-for-one in the same year. But as we proceed through time and if there are no subsequent shocks, the impact of this  $\varepsilon_t$  dies out. That is, from equation (4), for  $t' > t$ ,  $\varepsilon_{t'}^{(m)} = \varepsilon_t / m < \varepsilon_t$  for all horizons  $m > 1$ . Let  $m'$  be the horizon for which  $\varepsilon_{t'}^{(m')} \approx 0$ , so that the real exchange rate has effectively returned to its pre-shock value. We shall call  $m'$  “the long run”. As  $\text{var}[\varepsilon^{(m')}] \rightarrow 0$ ,  $\phi(m') \rightarrow 1$ . Figure 3 reveals that  $\phi(m)$  gradually declines to a value in the vicinity of one. The important question is, at what horizon  $m'$  can the variance ratio be considered to be sufficiently “close” to unity so that this  $m'$  can be regarded as the long run? We will analyse this question in the next section.

#### 4. Measuring the Long Run

As explained in Section 2, relative PPP gives the relationship between exchange-rate changes and relative-price changes. The variance ratio being one corresponds to the idea that there is one-for-one co-movement in these two variables. We thus suppose that the data-generating process is as follows:

$$(9) \quad H_0 : \Delta s_{ct}^{(m)} = \Delta r_{ct}^{(m)} + \varepsilon_{ct}^{(m)}.$$

To find out the length of the long run, we need to compare the data-based variance ratio plotted in Figure 3 with the distribution of the variance ratio under the null of equal variances of  $\Delta s_{ct}^{(m)}$  and  $\Delta r_{ct}^{(m)}$ . If for a particular value of  $m$  the empirical variance ratio falls into the 95 percent confidence band and stays there, we can conclude that the value of  $m$  constitutes the long run of the foreign exchange market.<sup>10</sup>

We derive the distributions of  $\phi(m)$  using the following Monte Carlo approach. The first equation of (3) for country  $c$  defines the one-year residuals  $\varepsilon_{ct} = \Delta s_{ct} - \Delta r_{ct}$ , and the corresponding  $m$ -year concept  $\varepsilon_{ct}^{(m)} = \Delta s_{ct}^{(m)} - \Delta r_{ct}^{(m)}$ . As the sample period is 1974-2004, there are 30 annual changes, i.e., for  $m=1$ . In general for horizon  $m$ , there are  $31-m$  observations. We apply the bootstrap approach to draw from these residuals. In simulation trial  $k$  ( $k = 1, \dots, 1000$ ), we draw  $31-m$  residuals for country  $c$  and denote them by  $\varepsilon_{ct,k}$ . Since the data-generating process is equation (9), the exchange rate change in trial  $k$  is given by adding the data-based

<sup>9</sup> In fact, the constancy of  $\beta$  in equation (8) is implied by the weak assumption that the ratio of the variances is constant.

<sup>10</sup> This value of  $m$  was referred to as  $m'$  in the previous section.



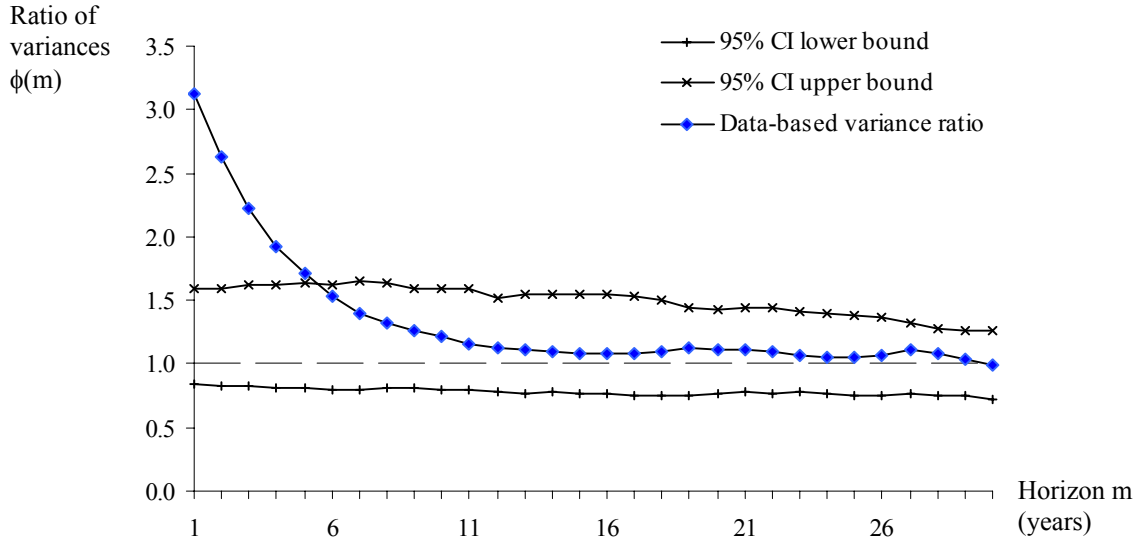
$\Delta r_{ct}^{(m)}$  to  $\varepsilon_{ct,k}^{(m)}$ , so that  $\Delta s_{ct,k}^{(m)} = \Delta r_{ct}^{(m)} + \varepsilon_{ct,k}^{(m)}$ . The variance ratio in trial  $k$  is thus  $\phi_k^*(m) = \text{var}[\Delta s_k^{(m)}] / \text{var}[\Delta r^{(m)}]$ . As  $\text{var}[\Delta s_k^{(m)}] = \text{var}[\Delta r^{(m)}] + \text{var}[\varepsilon_k^{(m)}] + \text{cov}[\Delta r^{(m)}, \varepsilon_k^{(m)}]$ , the simulated variance ratio can be written as

$$(10) \quad \phi_k^*(m) = 1 + \frac{\text{var}[\varepsilon_k^{(m)}] + \text{cov}[\Delta r^{(m)}, \varepsilon_k^{(m)}]}{\text{var}[(\Delta r)^{(m)}]}.$$

As the error terms in the above simulation,  $\varepsilon_{ct,k}^{(m)}$ , are bootstrapped from the data-based residuals, which are the exchange-rate changes from equation (9), the bootstrapped exchange rates,  $\Delta s_{ct,k}^{(m)}$ , are related to the observed distribution of  $\Delta s_{ct}^{(m)}$ . In addition, as in each trial  $k$  we fix the inflation differential at its data-based values,  $\Delta r_{ct}^{(m)}$ , the simulated variance ratios  $\phi_k^*(m)$  will be scattered around their observed counterparts  $\phi(m)$  and thus reflect the distribution of the data-based ratio  $\phi(m)$ . But as discussed previously, if PPP holds for all horizons, real exchange rate changes are negligible, i.e.,  $\varepsilon_t^{(m)} \approx 0$  for any  $m$ . To obtain the confidence band of the variance ratios under the null, we thus need to remove the effects of these real exchange rate changes. We do so by subtracting the second term on the right-hand side of equation (10) from  $\phi_k^*(m)$  to obtain the variance ratio under the null, denoted by  $\phi_k(m)$ . Finally, the 1,000 values of  $\phi_k(m)$  are sorted in the ascending order, and the 25<sup>th</sup> and 975<sup>th</sup> values, denoted by  $\phi_L(m)$  and  $\phi_U(m)$ , are the lower and upper bounds of the variance ratios under the null.

In Figure 4, we plot  $\phi_L(m)$  and  $\phi_U(m)$  for horizons  $m=1, \dots, 30$ , together with the data-based variance ratios. As can be seen, the data-based ratio falls into the confidence band under the null when  $m > 5$ . Thus after about five years, changes in exchange rates and relative prices have more or less the same variability, so that the long run of the foreign exchange market is about five years.

FIGURE 4  
VARIANCE RATIOS AND THE CONFIDENCE INTERVAL



## 5. The Power and Size of the Variance Ratio Test

In this section we examine the reliability of our test by investigating its power -- the percentage of cases the null is rejected when the null is in fact false. In addition, the size of the test is obtained by examining the power of the test under the null.

The null hypothesis of equal variances is described in equation (9). There are two kinds of alternative hypothesis corresponding to unequal variance when PPP does not hold. One is that the error terms  $\varepsilon_{ct}^{(m)}$  do not approach zero over longer horizons, so that the impact of the second term in equation (10) can not be ignored when  $m$  increases. The other kind of alternative is the slope coefficient in the data-generating process  $\beta \neq 1$ , i.e.,

$$(11) \quad H_1: \Delta s_{ct}^{(m)} = \beta \Delta r_{ct}^{(m)} + \varepsilon_{ct}^{(m)}, \quad \text{where } \beta \neq 1.$$

We will examine the test power in this case when the exchange rate under or over shoots inflation ( $\beta < 1$ ,  $\beta > 1$ ).

We proceed by specifying a value of  $\beta$  in equation (11) and in the  $k^{\text{th}}$  ( $k = 1, \dots, 1000$ ) trial, use bootstrapped residuals from the first equation in (3), as before, denoted by  $\varepsilon_{ct,k}$ . Equation (4) then defines  $\varepsilon_{ct,k}^{(m)}$ , as before. As discussed in Section 3, the variance of  $\varepsilon_{ct,k}^{(m)}$  is sufficiently small only after the long run and the long estimate is about five years as shown in Section 4. Accordingly, for horizons  $m \leq 5$  the shock terms will be big as compared to those in the long run if we bootstrap them from the large data-based error terms. Thus for  $m \leq 5$  we bootstrap the error terms from the residuals according to  $\varepsilon_{ct}^{(5)}$ , and for  $m > 5$  the error terms are bootstrapped from the data-based residuals,  $\varepsilon_{ct}^{(m)}$ . Then the exchange-rate changes are generated via equation (11) as  $\Delta s_{ct,k}^{(m)} = \beta \Delta r_{ct}^{(m)} + \varepsilon_{ct,k}^{(m)}$ . Finally, as before we calculate the variance ratio after the removal of the error effect to obtain  $\phi_k(m)$ . The percentage of cases that  $\phi_k(m)$  falls outside the confidence band of the variance ratio under the null,  $[\phi_L(m), \phi_U(m)]$ , gives the power of the test.

We analyse power for a range of values of  $\beta$  by specifying that it successively rises from 0 to 0.1 to 0.2, ..., until it hits 2.0. Figure 5 plots the power surface against  $\beta$  and  $m$ . As can be seen, for  $\beta \leq 0.8$  and  $\beta \geq 1.4$  the test power is very close to 100 percent. This means that the test has high power when the slope coefficient is far away from that under the null,  $\beta = 1$ . But when the slope is close to one, the power surface has a “valley” for horizons  $m \geq 2$ . This “valley” is consistent with lower power when the variability of exchange rates is more or less the same as that of relative prices. In summary, we can conclude that the test has high power to reject the null when the time horizon  $m \geq 2$  and when the volatility of exchange rates is substantially different to that of relative prices.

Figure 6 presents the size of the test (that is, power under the null). It can be seen that the size for all horizons except  $m = 1$  are less than 5 percent. This suggests that even when PPP holds, the null hypothesis of equal variances is rejected if the time horizon is one year. Only when  $m \geq 2$  does the variance ratio test have desirable size properties. In Section 4, we identified five years as

FIGURE 5  
TEST POWER FOR A RANGE OF VALUES OF THE SLOPE COEFFICIENT

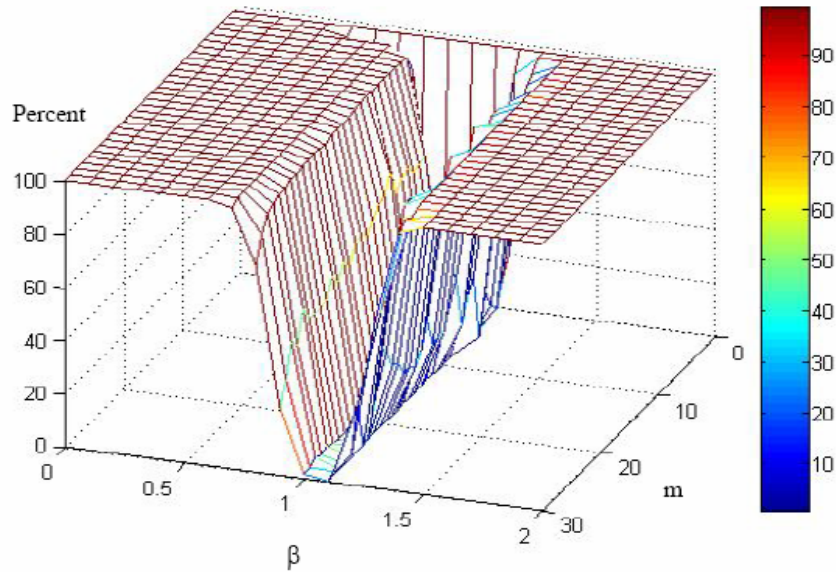
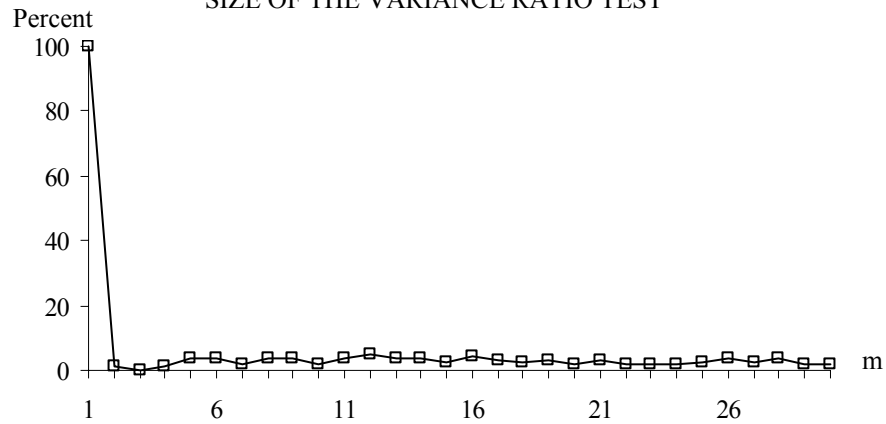


FIGURE 6  
SIZE OF THE VARIANCE RATIO TEST



the length of the long run. As this period is in the range of  $m \geq 2$  discussed in this and the previous paragraphs, we can be fairly confident that five years is a reliable estimate of the long run.

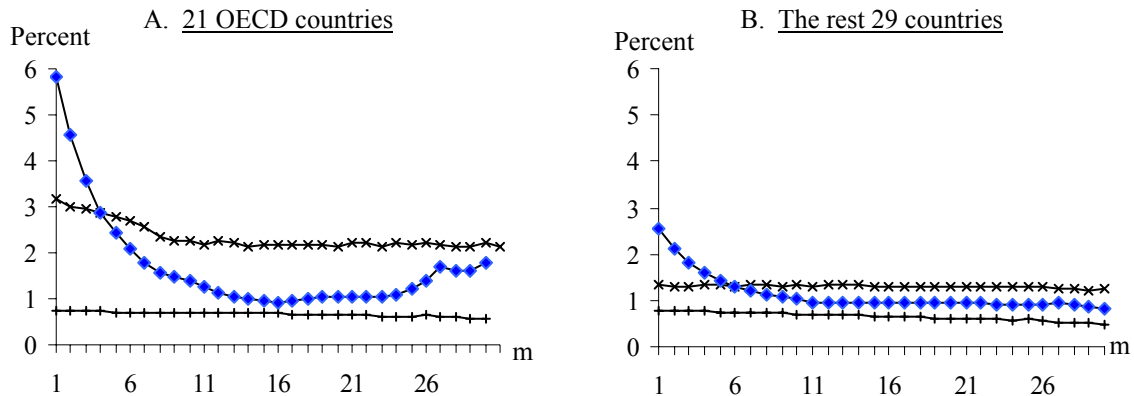
## 6. Sensitivity and Comparative Analyses

The long run of PPP reported in the literature is often in the range of four to five years (see, e.g., Froot and Rogoff, 1995). Our analysis, using the foreign exchange and consumer price data, shows that the long run is about five years, which is in line with the general consensus. To investigate whether our long-run estimate of five years is robust to subsets of sample periods and subsets of countries, in this section we conduct a sensitivity analysis. In addition, we compare our variance ratio results with those obtained using the prices of a single good, McDonald's Big Mac hamburgers.

We first examine the variance ratios for a sub-set of 21 OECD countries included in our sample. Panel A of Figure 7 shows that the long run for OECD is about four years. Panel B gives an estimate of six years for the remaining 29 countries. While these results are quite similar to the previous estimate of five years, OECD exchange rates are much more volatile than those of the remaining countries. An examination shows that this is mostly caused by the high variability of the OECD rates in the mid-1980s when the US dollar considerably appreciated and then fell.

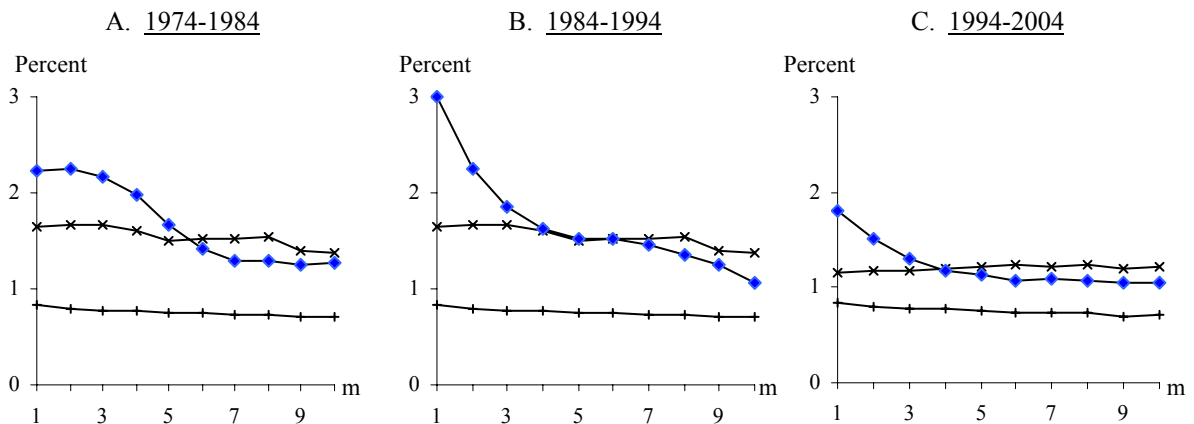
Next, we divide the whole sample period into three sub-periods, 1974-1984, 1984-1994, and 1994-2004. The estimates of long run corresponding to each of the three decades are 6, 4, and 4 years, as shown in panels A, B and C of Figure 8. A comparison of the three graphs reveals that in terms of the one-year changes (the first point in each graph), the exchange rates are more volatile in the sub-period of 1984-1994 than the other two sub-periods. It is also to be noted that exchange rate volatility was smallest in the most recent decade.

FIGURE 7  
VARIANCE RATIOS FOR SUBSETS OF COUNTRIES



Note: The legend is the same as in Figure 4.

FIGURE 8  
VARIANCE RATIOS FOR SUB-PERIODS



Note: The legend is the same as in Figure 4.

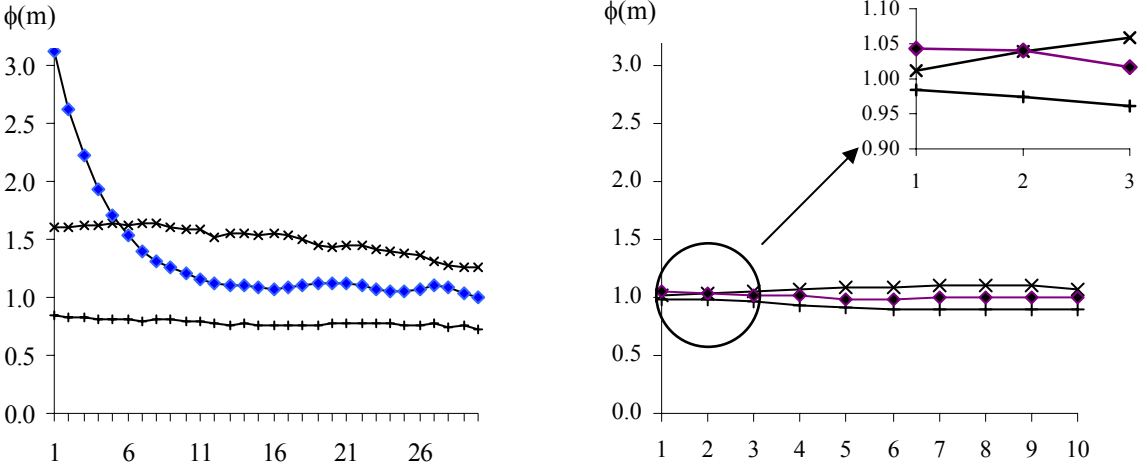
The CPIs used above refer to the cost of market basket in each country. As these baskets differs from one country to another, we are not really comparing like with like. To better control for differing market baskets, we redo the analysis with an identical basket by using the famous Big Mac Index (BMI) published by The Economist magazine. This index is based on the prices of a universal good -- a McDonalds' Big Mac hamburger, which is produced in about 180 countries in the world with almost identical ingredients. The Economist claims that the BMI is the “the world's most accurate financial indicator to be based on a fast-food item” (The Economist website). Several studies (Cumby, 1996, Lan, 2004, Parsley and Wei, 2003) find that the half-life estimates of the Big Mac PPP are shorter than those based on other price indices.

We use Big Mac prices in 24 countries and the US, and the corresponding 24 exchange rates versus the US dollar, for the period of 1994 to 2004 and Panel B of Figure 9 presents the variance ratio. <sup>11</sup> For comparison we also reproduce Figure 4 in Panel A of Figure 9. With the BMI, the variance ratio falls into the 95 percent confidence band after about two years. This result is in agreement with the previously-mentioned studies that find, relative to CPI, a faster adjustment to PPP when Big Macs are used.

7. Concluding Remarks

According to purchasing power parity, a country’s exchange rate equals the ratio of prices at home to those abroad, so that the proportionate change in the exchange rate equals the inflation differential. As such sharp hypotheses are rare in economics, it is not surprising that PPP has generated substantial controversy over a lengthy period. Among the contentious issues involving PPP, three can be highlighted. The first is the question of what causes what – does higher inflation at home lead to a depreciation of the exchange rate, or does causality go in the opposite direction? The second issue involves the period over which PPP can be expected to hold -- a day, a year, a decade, etc. Third, exactly what prices does the theory refer to -- consumer prices, GDP deflators, wages, or some individual commodities prices?

FIGURE 9  
VARIANCE RATIOS WITH THE CPI AND THE BIG MAC DATA  
A. The CPI B. The BMI



Note: The legend is the same as in Figure 4.

<sup>11</sup> See Lan (2004) for details of the data.

In this paper we have explored the meaning of the length of time required for PPP to hold. We used the idea that if exchange rates are proportional to relative prices, then the volatility of those two variables coincides. On a year-to-year basis, exchange rates are usually much more variable than prices; while as the time horizon increases to, say, a decade, the two variables tend to have a similar variance. In other words, PPP holds in the long run, but not the short run. We provided a new way of measuring exactly the length of the long run by examining the ratio of the variance of exchange rates to that of relative prices for various time horizons. This idea forms the basis of a new test of PPP, the variance ratio test. According to the test, the horizon for which the variance ratio falls close to unity and stays there, is the long run in so far as PPP is concerned.

We applied the variance ratio test to a large number of countries over the past three decades and estimated that the long run was about five years. An investigation of the power of the new test reveals that its performance was at least satisfactory, especially for large departures from the null hypothesis. The size properties of the test were also satisfactory.

A long run of five years means that after this period has elapsed, exchange rates fully adjust to relative prices. This result agrees with the general consensus of the long run estimate reported in the literature. We investigated the sensitivity of our result by (i) dividing countries into two groups -- OECD countries and the rest; and (ii) examining the data for three sub-periods. It is found that the results do not alter much. We also applied the methodology to the price of an individual good -- Big Mac hamburgers rather than Consumer Price Indexes -- and found that the Big Mac prices lead to a shorter long run of about two years.

Exchange rates are notoriously volatile and much research has been devoted to evaluating approaches to forecasting exchange rates. Until recently, the consensus view was that the random walk model was an unbeatable way to predict future exchange rates (Meese and Rogoff, 1983, Frankel and Rose, 1995, Rogoff, 1999). But of course even if the random walk is the best available approach, needless to say that does not necessarily implies forecasts that are particularly accurate. More recently, however, research has shown that for medium to long horizons PPP beats the random walk (Kong, 2000, Lan, 2004, Simpson and Grossmann, 2004). Our paper in providing further evidence in favour of PPP and the length of the long run may thus contribute to further improvements of exchange rate forecasts.

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