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TWO SHORT PAPERS ON MARIJUANA, LEGALISATION AND DRINKING

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EXOGENOUS SHOCKS AND RELATED GOODS: DRINKING AND THE LEGALISATION OF MARIJUANA*

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Abstract

The paper uses the substitutability between goods to model the transmission to other products of a consumption shock to one product. The framework is used to analyse the impact on drinking of legalisation of marijuana. For all types of consumers for example, the results indicate that legalisation would led to approximately a 4-percent increase in marijuana consumption, while beer, wine and spirits consumption would fall by 1 percent, 2 percent and almost 4 percent, respectively.

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1. Introduction

The interaction of goods in the consumer's utility function, as well as the operation of the budget constraint, means that a shock that affects the consumption of one product will have ramifications for the demand for related products. Thus while hot weather may well stimulate ice-cream sales, it would probably do so at the expense of other products; similarly, low-carb diets reduce the consumption of bread, pasta, etc., but have the effect of increasing other food items. This paper models such phenomena and applies the framework to analyse the possible impact on drinking of legalisation of marijuana consumption. For all types of consumers, the results indicate that legalisation would led to approximately a 4-percent increase in marijuana consumption, while beer, wine and spirits consumption would fall by 1 percent, 2 percent and almost 4 percent, respectively.[1]

2. The Model

In conventional consumption theory, the consumer chooses the quantity vector $\mathbf{q} = [q_1,...,q_n]'$ to maximise the utility function $u(\mathbf{q})$ subject to the budget constraint $\mathbf{p}'\mathbf{q} = M$, where $\mathbf{p}' = [p_1,...,p_n]$ is the price vector, and M is total expenditure ("income" for short). This leads to a system of Marshallian demand equations of the form $\mathbf{q} = \mathbf{q}(M,\mathbf{p})$. Consider now an extended version of this theory in which some scalar shift variable s affects tastes, so that the utility function now becomes $u(\mathbf{q},s)$. The associated demand equations now take the form $\mathbf{q} = \mathbf{q}(s,M,\mathbf{p})$, which we approximate for good i as

(1) $$Dq_i = \alpha_i Ds + \eta_i DM + \sum_{j=1}^{n} \eta'_{ij} Dp_j ,$$

[1] For related research pertaining to drug usage, see, e.g., Chaloupka and Laixuthai (1997), DeSimone (2002), Lee (1993), Model (1993), Pacula *et al.* (2003), Saffer and Chaloupka (1998, 1999a, b) and Williams (2004).

where D is the log-change operator; α_i is the elasticity of the consumption of good i with respect to the shift variable s; η_i is the i^{th} income elasticity; and η'_{ij} is the $(i,j)^{th}$ uncompensated price elasticity.

Let η_{ij} be the $(i,j)^{th}$ compensated price elasticity and $w_i = p_i q_i / M$ be the budget share of i, so that $\eta'_{ij} = \eta_{ij} - w_j \eta_i$, which is the Slutsky equation. Defining the change in real income as $DQ = DM - \sum_{i=1}^{n} w_i Dp_i$ and using the Slutsky equation, we can then express equation (1) as:

$$(2) \qquad Dq_i = \alpha_i Ds + \eta_i DQ + \sum_{j=1}^{n} \eta_{ij} Dp_j \ .$$

We interpret the shift variable s as a binary variable reflecting two regimes, such that Ds takes the value 0 (for regime 1) or 1 (regime 2). We can then write equation (2) under the regime 2 as

$$(3) \qquad Dq_i = \alpha_i + \eta_i DQ + \sum_{j=1}^{n} \eta_{ij} Dp_j \ .$$

To preserve the budget constraint, the coefficients of equation (3) satisfy $\sum_{i=1}^{n} w_i \alpha_i = 0$, $\sum_{i=1}^{n} w_i \eta_i = 1$, $\sum_{i=1}^{n} w_i \eta_{ij} = 0$, $j=1,...,n$. The coefficient α_i is interpreted as the log-change in consumption of good i resulting from the regime change when income and prices are held constant.

Let $\partial u / \partial q_k$ be the marginal utility of good k and suppose that the regime change causes this marginal utility to increase by $c \times \partial u / \partial q_k$, where $c > 0$, so that

$$(4) \qquad d\left(\log \frac{\partial u}{\partial q_k} \right) = c \, .$$

To interpret c, recall that for a budget-constrained utility maximum, each marginal utility is proportional to the corresponding price, $\partial u / \partial q_i = \lambda p_i$, where λ is the marginal utility of income. Accordingly for $i = k$, $d(\log \partial u / \partial q_k) = d(\log \lambda) + d(\log p_k)$, or in view of (4), $c = d(\log p_k)$, if λ is constant. This shows that c is an "equivalent price change", the fall in the price of k that would yield the same increase in consumption of the good as would the original shock. It can be shown (see Appendix) that equation (4) implies that:

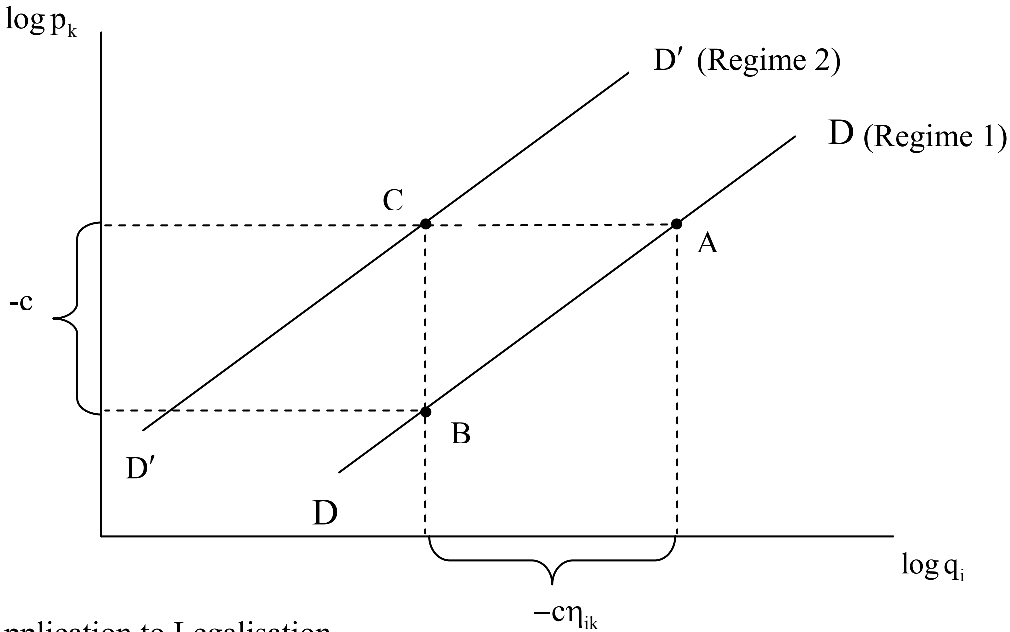
(5) $$\alpha_i = -c\,\eta_{ik}.$$

In words, the change in consumption of good i due to the regime change is proportional to that good's elasticity of demand with respect to the price of good k, with factor of proportionality (the negative of) c. This is an attractively-simple result linking the effect on consumption of i of the shock to k which involves the degree of substitutability between the two goods. Equation (5) also preserves the budget constraint as $\sum_{i=1}^{n} w_i \alpha_i = -c \sum_{i=1}^{n} w_i \eta_{ik} = 0$, where the last step follows from the aggregation constraint $\sum_{i=1}^{n} w_i \eta_{ij}$, given below equation (3). Accordingly, rule (5) serves to reallocate the fixed amount of income among the n goods following the change in regime.

Figure 1 illustrates the link between the substitution effect and the shift in the demand curve in the log-linear case when goods k and i are substitutes (so that $\eta_{ik} > 0$). With DD the cross relationship between consumption of i and the price of k under regime 1, a lowering of $\log p_k$ by c leads to the movement from the point A to B, and $\Delta \log q_i = -c\,\eta_{ik}$. According to equation (5), the regime change causes this demand curve to shift to the left by exactly the same amount, so that the point on the new demand curve, $D'D'$, corresponding to the original price must be C, directly above B.

FIGURE 1

SHOCKS AND SUBSTITUTES



3. Application to Legalisation

A survey of students at The University of Western Australia inquired about the effects on the consumption of marijuana of a possible decision to legalise it (Clements and Daryal, 2004). As the survey does not contain similar precise information on the impact of legalisation on alcohol consumption, we shall show how the effects on drinking can be estimated by employing the above framework.

For i = marijuana, the survey yields a value for α_i, which when combined with the own-price elasticity η_{ii} (from Table 1), we can obtain c from equation (5). Next, to estimate the change in consumption of alcoholic beverages following legalisation, we use in equation (5) for i = beer, wine and spirits and k = marijuana this c value, together with the cross-price elasticities given in the last column of Table 1. The results are given in Table 2 for various types of consumers identified by sex and the intensity of use of marijuana. The key results are: (i) For each user group, the consumption of spirits always falls the most with legalisation. Next is wine, and then comes beer. (ii) The largest fall in alcohol consumption is for the daily users. The effects for weekly, monthly and occasional users are not too dissimilar. (iii) For all types of consumers (panel H of the table), on average legalisation is estimated to cause beer consumption to fall by about 1 percent, wine by 2 percent and spirits by almost 4 percent, while marijuana usage increases by 4 percent. It should however be noted that as the standard errors are relatively large, the changes in alcohol consumption are not estimated too precisely.

TABLE 1

COMPENSATED PRICE ELASTICITIES

(Standard errors in parentheses)

Good	Beer	Wine	Spirits	Marijuana
Beer	-.17 (.09)	.03 (.02)	.06 (.03)	.08 (.04)
Wine	.09 (.05)	-.36 (.19)	.13 (.07)	.15 (.08)
Spirits	.17 (.09)	.13 (.07)	-.60 (.32)	.30 (.16)
Marijuana	.10 (.05)	.08 (.04)	.15 (.08)	-.33 (.17)

Source: Clements and Daryal (2004) and Clements and Lan (2005).

TABLE 2

EFFECTS OF LEGALISATION ON THE CONSUMPTION OF

ALCOHOLIC BEVERAGES AND MARIJUANA

(Standard errors in parentheses; percentage changes)

	Gender				Gender		
Good	Male	Female	All	Good	Male	Female	All
	A. Daily users				E. No longer a user		
Beer	-5.15 (5.15)	.00 (.00)	-4.58 (4.67)	Beer	-1.14 (1.40)	.00 (.00)	-.61 (.74)
Wine	-9.66 (9.83)	.00 (.00)	-8.59 (8.91)	Wine	-2.13 (2.65)	.00 (.00)	-1.14 (1.41)
Spirits	-19.32 (19.65)	.00 (.00)	-17.17 (17.81)	Spirits	-4.26 (5.29)	.00 (.00)	-2.27 (2.82)
Marijuana	21.25 (14.80)	.00 (.00)	18.89 (13.70)	Marijuana	4.69 (4.67)	.00 (.00)	2.50 (2.48)
	B. Weekly users				F. All users		
Beer	-1.98 (1.73)	-2.71 (2.30)	-2.26 (1.80)	Beer	-2.20 (1.68)	-1.50 (1.16)	-1.89 (1.40)
Wine	-3.71 (3.31)	-5.08 (4.42)	-4.24 (3.46)	Wine	-4.13 (3.23)	-2.81 (2.24)	-3.54 (2.71)
Spirits	-7.41 (6.62)	-10.16 (8.84)	-8.47 (6.92)	Spirits	-8.27 (6.47)	-5.63 (4.48)	-7.08 (5.42)
Marijuana	8.15 (4.07)	11.18 (5.08)	9.32 (3.19)	Marijuana	9.09 (2.28)	6.19 (1.78)	7.79 (1.49)
	C. Monthly users				G. Non-users		
Beer	-1.65 (1.44)	-2.21 (1.87)	-1.95 (1.56)	Beer	-.05 (.05)	-.09 (.09)	-.07 (.06)
Wine	-3.09 (2.76)	-4.15 (3.59)	-3.66 (3.00)	Wine	-.09 (.10)	-.17 (.17)	-.14 (.12)
Spirits	-6.17 (5.51)	-8.29 (7.18)	-7.33 (6.00)	Spirits	-.17 (.21)	-.35 (.34)	-.27 (.24)
Marijuana	6.79 (3.38)	9.12 (4.07)	8.06 (2.79)	Marijuana	.19 (.18)	.38 (.24)	.30 (.15)
	D. Occasional users				H. All types		
Beer	-2.64 (2.16)	-.94 (.83)	-1.77 (1.39)	Beer	-1.35 (1.03)	-.74 (.57)	-1.04 (.77)
Wine	-4.95 (4.15)	-1.77 (1.59)	-3.31 (2.68)	Wine	-2.52 (1.98)	-1.40 (1.11)	-1.94 (1.49)
Spirits	-9.89 (8.30)	-3.54 (3.17)	-6.63 (5.36)	Spirits	-5.05 (3.96)	-2.79 (2.21)	-3.88 (2.97)
Marijuana	10.88 (4.27)	3.89 (1.96)	7.29 (2.35)	Marijuana	5.55 (1.42)	3.07 (.86)	4.27 (.82)

Notes: 1. The zeros for the change in consumption of the four goods for two types of consumers arise because their consumption of marijuana was estimated by the survey not to change in response to legalisation.

2. For the derivation of the standard errors, see Clements and Lan (2005).

APPENDIX

To derive result (5) we use an extended version of Barten's (1964) fundamental matrix equation in consumption theory. The consumer chooses the quantity vector $\mathbf{q}$ to maximise utility $u(\mathbf{q},s)$, where s is the exogenous shock to preference, subject to the budget constraint $\mathbf{p}'\mathbf{q} = M$, $\mathbf{p}$ being the price vector and M income. The first-order conditions are the budget constraint and $\partial u / \partial \mathbf{q} = \lambda \mathbf{p}$, where λ is the marginal utility of income.

Differentiation of the first-order conditions with respect to M, $\mathbf{p}$ and s yields

$$\text{(A1)} \qquad \begin{bmatrix} \mathbf{U} & \mathbf{p} \\ \mathbf{p}' & 0 \end{bmatrix} \begin{bmatrix} \partial \mathbf{q} / \partial M & \partial \mathbf{q} / \partial \mathbf{p}' & \partial \mathbf{q} / \partial s \\ -\partial \lambda / \partial M & -\partial \lambda / \partial \mathbf{p}' & -\partial \lambda / \partial s \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \lambda \mathbf{I} & -\mathbf{u} \\ 1 & -\mathbf{q}' & 0 \end{bmatrix},$$

where $\mathbf{U} = \partial^2 u / \partial \mathbf{q} \partial \mathbf{q}'$, $\mathbf{I}$ is the identity matrix and $\mathbf{u} = \partial^2 u / \partial \mathbf{q} \partial s$. Solving (A1) for the second matrix on the left yields

$$\text{(A2)} \qquad \begin{bmatrix} \partial \mathbf{q} / \partial M & \partial \mathbf{q} / \partial \mathbf{p}' & \partial \mathbf{q} / \partial s \\ -\partial \lambda / \partial M & -\partial \lambda / \partial \mathbf{p}' & -\partial \lambda / \partial s \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} (\mathbf{p}'\mathbf{U}^{-1}\mathbf{p})\mathbf{U}^{-1} - \mathbf{U}^{-1}\mathbf{p}(\mathbf{U}^{-1}\mathbf{p})' & \mathbf{U}^{-1}\mathbf{p} \\ (\mathbf{U}^{-1}\mathbf{p})' & -1 \end{bmatrix} \begin{bmatrix} \mathbf{0} & \lambda \mathbf{I} & -\mathbf{u} \\ 1 & -\mathbf{q}' & 0 \end{bmatrix},$$

where $\Delta = \mathbf{p}'\mathbf{U}^{-1}\mathbf{p}$. Accordingly,

$$\text{(A3)} \qquad \frac{\partial \mathbf{q}}{\partial M} = \frac{1}{\Delta} \mathbf{U}^{-1}\mathbf{p},$$

$$\frac{\partial \mathbf{q}}{\partial \mathbf{p}'} = \frac{1}{\Delta} \left\{ \left[(\mathbf{p}'\mathbf{U}^{-1}\mathbf{p})\mathbf{U}^{-1} - \mathbf{U}^{-1}\mathbf{p}(\mathbf{U}^{-1}\mathbf{p})' \right] \lambda - \mathbf{U}^{-1}\mathbf{p}\mathbf{q}' \right\}$$

$$\text{(A4)} \qquad = \frac{1}{\Delta} \left[(\mathbf{p}'\mathbf{U}^{-1}\mathbf{p})\mathbf{U}^{-1} - \mathbf{U}^{-1}\mathbf{p}(\mathbf{U}^{-1}\mathbf{p})' \right] \lambda - \frac{\partial \mathbf{q}}{\partial M} \mathbf{q}',$$

in view of (A3). It also follows from (A2) that

$$\frac{\partial \mathbf{q}}{\partial s} = -\frac{1}{\Delta} \left[(\mathbf{p}'\mathbf{U}^{-1}\mathbf{p})\mathbf{U}^{-1} - \mathbf{U}^{-1}\mathbf{p}(\mathbf{U}^{-1}\mathbf{p})' \right] \mathbf{u},$$

which, when combined with (A4), becomes

(A5)
$$\frac{\partial \mathbf{q}}{\partial \mathrm{s}} = -\frac{1}{\lambda}\left[\frac{\partial \mathbf{q}}{\partial \mathbf{p}'} + \frac{\partial \mathbf{q}}{\partial \mathrm{M}}\mathbf{q}'\right]\mathbf{u}.$$

Equation (4) implies that the only non-zero element of **u** is the k^{th}, which equals $c\left(\partial u / \partial q_k\right) = c\lambda p_k$. As the term in square brackets in (A5) is the substitution matrix, it follows that

(A6)
$$\frac{\partial q_i / q_i}{\partial s} = -c\eta_{ik} \qquad i = 1, \ldots, n,$$

where $\eta_{ik} = \partial\left(\log q_i\right) / \partial\left(\log p_k\right)$, real income remaining constant. Interpreting the left-hand side of (A6) as the growth in q_i resulting from the regime change, α_i, this establishes result (5). For a further discussion, see Barten (1977) and Theil (1975, pp. 205-206).

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NOTES ON
PROJECTIONS OF
ALCOHOL CONSUMPTION
FOLLOWING MARIJUANA LEGALISATION[2]

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[2] Research support from the ARC is gratefully acknowledged.

Clements and Daryal (2005) develop a utility-maximising theory of how exogenous shocks to one market have implications for the consumption of related goods, and applied that theory to analyse the impacts on drinking of possible legalisation of marijuana. These notes set out the derivations of the standard errors of their projections.

1. A Brief Recapitulation

Let p_i and q_i be the price and quantity demanded of good i (i = 1,…,n) and let D be the log-change operator. We can then express the demand for good i as

$$(1) \qquad Dq_i = \alpha_i + \eta_i \, DQ + \sum_{j=1}^{n} \eta_{ij} \, Dp_j \, ,$$

where α_i is the change in consumption due to exogenous factors other than income and prices; η_i is the i^{th} income elasticity; DQ is the log-change in the consumer's real income; and η_{ij} is the $(i,j)^{th}$ compensated price elasticity. If we write w_i for the budget share of good i, then to preserve the budget constraint the coefficients of equation (1) for $i = 1,\ldots,n$ satisfy $\sum_{i=1}^{n} w_i \, \alpha_i = 0$, $\sum_{i=1}^{n} w_i \, \eta_i = 1$, $\sum_{i=1}^{n} w_i \, \eta_{ij} = 0$, $j = 1,\ldots,n$.

Let $\partial u / \partial q_k$ be the marginal utility of good k and suppose that an exogenous shock causes this marginal utility to increase by $c \times \partial u / \partial q_k$, where $c > 0$, so that

$$(2) \qquad d\left(\log \frac{\partial u}{\partial q_k} \right) = c \, .$$

The quantity c is an "equivalent price change", the fall in the price of k that would yield the same increase in consumption of the good as would the original shock. It can be shown that equation (2) implies that:

$$(3) \qquad \alpha_i = -c \, \eta_{ik} \, .$$

In words, the change in consumption of good i due to the exogenous shock is proportional to that good's elasticity of demand with respect to the price of good k, with factor of proportionality (the negative of) c. This is an attractively-simple result linking the effect on consumption of i of the shock to k, where the link involves the degree of substitutability between the two goods in question. Equation (3) also preserves the budget constraint as $\sum_{i=1}^{n} w_i \alpha_i = -c \sum_{i=1}^{n} w_i \eta_{ik} = 0$, where the last step follows from the aggregation constraint $\sum_{i=1}^{n} w_i \eta_{ij} = 0$, given below equation (1).

A survey of students at UWA yielded information regarding the effects on the consumption of marijuana of a possible decision to legalise it, but no precise information on the impact on alcohol consumption. These latter effects can be derived from the above framework as follows. For i = marijuana, the survey yields a value for α_i, which when combined with the own-price elasticity η_{ii} (from Table 1), we can obtain the value of c from equation (3). Next, to estimate the change in consumption of alcoholic beverages following legalisation, we use in equation (3) for i = beer, wine and spirits and k = marijuana this c value, together with the cross-price elasticities given in the last column of Table 1. The results are given in Table 2 for various types of consumers identified by sex and the intensity of use of marijuana.

TABLE 1

PRICE ELASTICITIES OF DEMAND

Good	Beer	Wine	Spirits	Marijuana
Beer	-.17	.03	.06	.08
Wine	.09	-.36	.13	.15
Spirits	.17	.13	-.60	.30
Marijuana	.10	.08	.15	-.33

Source: Clements and Daryal (2004).

TABLE 2

EFFECTS OF LEGALISATION ON THE CONSUMPTION OF

ALCOHOLIC BEVERAGES AND MARIJUANA

(Percentage changes)

Good	Gender			Good	Gender		
	Male	Female	All		Male	Female	All
	A. Daily users				E. No longer a user		
Beer	-5.15	.00	-4.58	Beer	-1.14	.00	-.61
Wine	-9.66	.00	-8.59	Wine	-2.13	.00	-1.14
Spirits	-19.32	.00	-17.17	Spirits	-4.26	.00	-2.27
Marijuana	21.25	.00	18.89	Marijuana	4.69	.00	2.50
	B. Weekly users				F. All users		
Beer	-1.98	-2.71	-2.26	Beer	-2.20	-1.50	-1.89
Wine	-3.70	-5.08	-4.24	Wine	-4.13	-2.81	-3.54
Spirits	-7.41	-10.16	-8.47	Spirits	-8.26	-5.63	-7.08
Marijuana	8.15	11.18	9.32	Marijuana	9.09	6.19	7.79
	C. Monthly users				G. Non-users		
Beer	-1.65	-2.21	-1.95	Beer	-.05	-.09	-.07
Wine	-3.09	-4.15	-3.66	Wine	-.09	-.17	-.14
Spirits	-6.17	-8.29	-7.33	Spirits	-.17	-.35	-.27
Marijuana	6.79	9.12	8.06	Marijuana	.19	.38	. 30
	D. Occasional users				H. All types		
Beer	-2.64	-.94	-1.77	Beer	-1.35	-.74	-1.04
Wine	-4.95	-1.77	-3.31	Wine	-2.52	-1.40	-1.94
Spirits	-9.89	-3.54	-6.63	Spirits	-5.05	-2.79	-3.88
Marijuana	10.88	3.89	7.29	Marijuana	5.55	3.07	4.27

As the price elasticities and the consumption projections are both random, we shall proceed to derive their standards errors.

2. The Price Elasticities

Clements and Daryal (2004) use Australian data to estimate the 4×4 matrix of price elasticities $\left[\eta_{ij}\right]$ under the simplifying assumption that tastes with respect to alcohol and marijuana can be characterised by a utility function of the preference independence form. This means that the $(i, j)^{th}$ price elasticity takes the form

$$\eta_{ij} = \phi\eta_i\left(\delta_{ij} - w_j\eta_j\right), \qquad (4)$$

where ϕ is the own-price elasticity of demand for the group of goods as a whole, and δ_{ij} is the Kronecker delta. Due to data limitations, Clements and Daryal (2004) also specified the values of the income elasticities as set out in the second column of Table 3. The other columns of that table contain the information on the budget shares. As the income elasticities and budget shares on the right-hand side of equation (4) are treated as known, the only remaining unknown parameter is ϕ, which Clements and Daryal estimate by GLS to be

$$\hat{\phi} = -0.429, \text{ with standard error } 0.227, \qquad (5)$$

so that $\operatorname{var}\phi = \left(0.227\right)^2$. The elasticities of Table 1 are derived from equations (4) and (5), and the information contained in Table 3.

Under the above assumptions, the only random variable on the right-hand side of equation (4) is ϕ. Accordingly, $\operatorname{var}\eta_{ij} = \left[\eta_i\left(\delta_{ij} - w_j\eta_j\right)\right]^2 \times \operatorname{var}\phi$, or in view of equation (4),

(6) $$\text{var}\,\eta_{ij} = \left(\frac{\eta_{ij}}{\phi}\right)^2 \text{var}\,\phi \ .$$

Note that this equation implies that the t-value of η_{ij}, $\eta_{ij}/\sqrt{\text{var}\,\eta_{ij}}$, equals that for ϕ, $\phi/\sqrt{\text{var}\,\phi}$. We use the values of the price elasticities of Table 1, together ϕ = -0.429 to evaluate the variances in equation (6) for i, j = 1,…,4 and the results are given in Table 4.

TABLE 3

INCOME ELASTICITIES AND BUDGET SHARES

Good i	Income elasticity η_i	Budget share w_i	Product of income elasticity and budget share $\eta_i w_i$
Beer	.5	.4	.2
Wine	1	.15	.15
Spirits	2	.15	.3
Marijuana	1.2	.3	.35
Sum		1.00	1.00

Source: Clements and Daryal (2004).

TABLE 4

PRICE ELASTICITIES AND THEIR STANDARD ERRORS

(Standard errors in parentheses)

Good	Beer	Wine	Spirits	Marijuana
Beer	-.17 (.09)	.03 (.02)	.06 (.03)	.08 (.04)
Wine	.09 (.05)	-.36 (.19)	.13 (.07)	.15 (.08)
Spirits	.17 (.09)	.13 (.07)	-.60 (.32)	.30 (.16)
Marijuana	.10 (.05)	.08 (.04)	.15 (.08)	-.33 (.17)

3. An Unattractive Procedure

The projected consumption of good i is defined by equation (3) for k = 4. As mentioned above, the value for the change in marijuana consumption α_4 is obtained form the survey, and we use equation (3) for i, k = 4 (marijuana) to obtain the equivalent price change as $c = -\alpha_4/\eta_{44}$. We then substitute this value back in equation (3) for i = 1, 2, 3, and k = 4 to yield

$$\alpha_i = \alpha_4\left(\frac{\eta_{i4}}{\eta_{44}}\right). \tag{7}$$

Using equation (4) to substitute for η_{ij} in the above, we obtain

$$\alpha_i = \alpha_4\left(\frac{-\phi\eta_i\left(w_4\eta_4\right)}{\phi\eta_4\left(1-w_4\eta_4\right)}\right) = \alpha_4\left(\frac{-\eta_i w_4}{1-w_4\eta_4}\right). \tag{8}$$

The third member of equation (8) contains three basic ingredients, (i) the change in marijuana consumption α_4, which is observed from the survey; (ii) the income elasticities of good i and of marijuana, η_i and η_4, which are both treated as known; and (iii) the budget share of marijuana w_4, also treated as known. As the only unknown demand parameter ϕ does not appear on the far right of equation (8), we conclude that in this formulation there can be no estimation uncertainty stemming from the consumption responses. We could, of course, use equation (8) in the form $\alpha_i = -\beta_i \times \alpha_4$, where β_i is the known quantity $\eta_i w_4/\left(1-w_4\eta_4\right)$, so that $SE\left(\alpha_i\right) = \beta_i \times SE\left(\alpha_4\right)$. But such a procedure would not be appealing as it treats all the demand responses as being known with certainty, which stretches imagination a bit.

4. The Consumption Projections

To avoid the above problems, we allow for uncertainty in the demand elasticities by ignoring equation (4) and adopt the following approach to shed light on the uncertainty of the consumption projections. While this is only an approximation, the approach strikes a balance between (i) retaining the benefits of the assumption of preference independence as a way to structure the numerical values of the elasticities, and (ii) avoiding the unpalatable implication of all the consumption responses being known with certainty.

In equation (7) for i =1,2,3, we treat all three terms on the right as random variables. The total differential of that equation is

$$\left(\frac{d\alpha_i}{\alpha_i}\right) = \left(\frac{d\alpha_4}{\alpha_4}\right) + \left(\frac{d\eta_{i4}}{\eta_{i4}}\right) - \left(\frac{d\eta_{44}}{\eta_{44}}\right),$$

so that if as an approximation we assume that the three variables are independent, we have

$$\text{(9)} \qquad \operatorname{var}\alpha_i = \alpha_i^2\left(\frac{\operatorname{var}\alpha_4}{\alpha_4^2} + \frac{\operatorname{var}\eta_{i4}}{\eta_{i4}^2} + \frac{\operatorname{var}\eta_{44}}{\eta_{44}^2}\right).$$

We use equation (9) to compute the variability of the consumption projections. We substitute in (9) the value from the survey for α_4, the consumption projections contained in Table 2 for α_i $(i = 1, 2, 3)$ and the price elasticities given in the last column of Table 4 for η_{i4} (i = 1,…,4). We also use the value of $\operatorname{var}\alpha_4$ from the survey, and $\operatorname{var}\eta_{i4}$ (i = 1,...,4) from the last column of Table 4. Table 5 gives the results for each of the eight types of consumer. As can be seen, compared to their point estimates, the standard errors of the projected changes in alcohol consumption are fairly large.

Equation (9) is an approximation, derived under some simplifying assumptions. Some insight into the impact of the assumptions is available from the "t-value" implied by equation (9),

TABLE 5

EFFECTS OF LEGALISATION ON THE CONSUMPTION OF

ALCOHOLIC BEVERAGES AND MARIJUANA: POINT ESTIMATES AND STANDARD ERRORS

(Standard errors in parentheses; percentage changes)

	Gender				Gender		
Good	Male	Female	All	Good	Male	Female	All
	A. Daily users				E. No longer a user		
Beer	-5.15 (5.15)	.00 (.00)	-4.58 (4.67)	Beer	-1.14 (1.40)	.00 (.00)	-.61 (.74)
Wine	-9.66 (9.83)	.00 (.00)	-8.59 (8.91)	Wine	-2.13 (2.65)	.00 (.00)	-1.14 (1.41)
Spirits	-19.32 (19.65)	.00 (.00)	-17.17 (17.81)	Spirits	-4.26 (5.29)	.00 (.00)	-2.27 (2.82)
Marijuana	21.25 (14.80)	.00 (.00)	18.89 (13.70)	Marijuana	4.69 (4.67)	.00 (.00)	2.50 (2.48)
	B. Weekly users				F. All users		
Beer	-1.98 (1.73)	-2.71 (2.30)	-2.26 (1.80)	Beer	-2.20 (1.68)	-1.50 (1.16)	-1.89 (1.40)
Wine	-3.71 (3.31)	-5.08 (4.42)	-4.24 (3.46)	Wine	-4.13 (3.23)	-2.81 (2.24)	-3.54 (2.71)
Spirits	-7.41 (6.62)	-10.16 (8.84)	-8.47 (6.92)	Spirits	-8.27 (6.47)	-5.63 (4.48)	-7.08 (5.42)
Marijuana	8.15 (4.07)	11.18 (5.08)	9.32 (3.19)	Marijuana	9.09 (2.28)	6.19 (1.78)	7.79 (1.49)
	C. Monthly users				G. Non-users		
Beer	-1.65 (1.44)	-2.21 (1.87)	-1.95 (1.56)	Beer	-.05 (.05)	-.09 (.09)	-.07 (.06)
Wine	-3.09 (2.76)	-4.15 (3.59)	-3.66 (3.00)	Wine	-.09 (.10)	-.17 (.17)	-.14 (.12)
Spirits	-6.17 (5.51)	-8.29 (7.18)	-7.33 (6.00)	Spirits	-.17 (.21)	-.35 (.34)	-.27 (.24)
Marijuana	6.79 (3.38)	9.12 (4.07)	8.06 (2.79)	Marijuana	.19 (.18)	.38 (.24)	.30 (.15)
	D. Occasional users				H. All types		
Beer	-2.64 (2.16)	-.94 (.83)	-1.77 (1.39)	Beer	-1.35 (1.03)	-.74 (.57)	-1.04 (.77)
Wine	-4.95 (4.15)	-1.77 (1.59)	-3.31 (2.68)	Wine	-2.52 (1.98)	-1.40 (1.11)	-1.94 (1.49)
Spirits	-9.89 (8.30)	-3.54 (3.17)	-6.63 (5.36)	Spirits	-5.05 (3.96)	-2.79 (2.21)	-3.88 (2.97)
Marijuana	10.88 (4.27)	3.89 (1.96)	7.29 (2.35)	Marijuana	5.55 (1.42)	3.07 (.86)	4.27 (.82)

$$\text{(10)} \qquad \frac{\alpha_i}{\sqrt{\operatorname{var}\alpha_i}} = \frac{-1}{\sqrt{\dfrac{\operatorname{var}\alpha_4}{\alpha_4^2} + \dfrac{\operatorname{var}\eta_{i4}}{\eta_{i4}^2} + \dfrac{\operatorname{var}\eta_{44}}{\eta_{44}^2}}} .$$

From the last column of Table 4, the t-values of the η_{ij} are of the order of 2, which means that each of the last two terms under the square-root sign in (10) is about $(1/2)^2 = 0.25$. If there were no uncertainty regarding the estimate of α_4, so that $\operatorname{var}\alpha_4 = 0$, then the right side of equation (10) becomes approximately $-1/\sqrt{0.25+0.25} \approx -1.4$. As a nonzero value of $\operatorname{var}\alpha_4$ has the effect of reducing the t-value absolutely, 1.4 plays the role of the maximum possible t-value. On the other hand, there is the ignored covariance between the elasticities. As the elasticitities are constrained by $\sum_{i=1}^{4} w_i \eta_{i4} = 0$, this covariance is likely to be negative; thus allowing for a negative covariance term in equation (10) would have the effect of increasing the (absolute) t-value of the α_i. These comments serve to highlight the qualifications that need to be kept in mind regarding the standard errors of Table 5.

References

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