UNIVERSIDAD CARLOS III DE MADRID working papers

Working Paper 06-64
Economics Series 26
November 2006

Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624 98 75

MONETARY PROPAGATION IN SEARCH-THEORETIC MONETARY MODELS*

Martin Menner[1]

Abstract

Shouyong Shi(1998) presents a general equilibrium model which shows a persistent monetary propagation mechanism. There the high persistence is obtained by a combination of search frictions in the goods and labor markets and the presence of final goods inventories. The present paper addresses the question of robustness of these results, especially, how sensitive are Shi's results to parameter changes and to different model specifications. Calibration of the parameters to intervals is used to perform a global sensitivity analysis. The calibration exercise reveals that the model is quite robust to changes in parameters. Comparing different model versions - including a CIA model which appears as a special case when buyers and sellers match always - we can disentangle and quantify the contributions of the various frictions in accounting for the persistent propagation. Search-frictions in the goods market and inventory holdings are necessary for persistent propagation of monetary shocks. Labor market frictions are not crucial but prolong the output responses and reduce their magnitude.

JEL Classification: D83, E19, E32, E49, J64

Keywords: Monetary Search-Theory, Labor Market Search, Inventory Holdings, CIA Model, Propagation Mechanism, DSGE Model

* **Acknowledgements:** This paper is based on the first part of my doctoral thesis. I would like to thank especially my thesis advisor Hugo Rodríguez Mendizábal, my tutor Fabio Canova, Xavier Cuadras Morato, Jordi Galí, Doerte Doemeland, Matthias Messner, Nicola Pavoni, Lutz Weinke and the participants of the CREI Macro Workshop at UPF, Barcelona for helpful discussions. I acknowledge the Ministry of Education of Spain for financial support by a FPU grant. All remaining errors are mine.

[1] Department of Economics. Universidad Carlos III de Madrid, Calle Madrid, 126 28903 Getafe (Madrid).
E-mail: mmenner@eco.uc3m.es

1 Introduction

The analysis of monetary economies is mostly undertaken in the framework of dynamic general equilibrium models with Walrasian goods markets. With Walrasian markets there is no role for money as a means of exchange, since all transactions take place in a centralized market place. Intrinsically valueless money can serve as a numeraire and a store of value, but is dominated in return by interest earning assets, so additional assumptions have to be made to let agents hold money in equilibrium. To introduce money in a Walrasian model mainly three assumptions are used. First, in the money-in-the-utility-function (MIUF) models agents derive direct utility from holding money.[1] Literally this could be the pleasure of counting one's money. But one can also interpret this utility as the transaction services derived from the durable good 'money balances'. Second, money-in-the-production-function (MIPF) models assume that the transaction services of money improve the production process.[2] Finally, a more direct way to model the role of money in transactions is to assume that agents have to hold cash-in-advance (CIA) to be able to buy at least some goods.[3] All these approaches are short-cuts that avoid the modelling of the role of money as means of exchange from first principles and share the feature that money is not essential.[4]

There exist other approaches that restrict the participation in otherwise Walrasian markets and generate thereby the need for a means of exchange. In the overlapping generations model of Samuelson[21] young agents can meet old agents to trade, but their potential trading partners when old are not born yet. Hence there arises the need of a store of value that can be used to be exchanged for goods when agents are old. However, if other tradable assets exist, money looses its role as means of exchange. Another drawback is that money is used in exchange only twice in a lifetime of an agent which is quite different from what we observe in reality. The turnpike model of

[1] Most models in the New Keynesian Macroeconomics literature use this approach. See, e.g., the Handbook article of Rotemberg and Woodford[18] for an overview.

[2] See e.g. Benhabib and Farmer[1] for an application. There money in the production function can lead to indeterminacy of equilibria even if the effect of money in production is assumed small. They argue that their flexible prices model with indeterminacy can mimic the behaviour of sticky prices models.

[3] The Limited-Participation models following Fuerst[7], [8] are commonly based on the assumption of a CIA constraint.

[4] Essentiality means that some of the allocations achievable in a monetary equilibrium cannot be achieved in an equilibrium without money.

Townsend[26] does not have this unpleasant feature. There, people have altering endowments over time and are spatially separated on a turnpike moving in opposite directions from one trading post to another. Money here also serves as medium of intertemporal exchange, although on a shorter time horizon. Both approaches set up an environment that gives money in principle an essential role[5]. However, they do not capture the role of money as a medium of exchange in overcoming the problems of an 'absence of a double coincidence of wants' or of 'indivisibilities'. The first problem raised by Jevons[10] is characterized by a situation where, say, agent A likes the good of agent B but not vice versa. The second problem arises if the two different goods to exchange are indivisible but differ in value, for example an exchange of 1 cow for a pair of shoes, so direct barter is not possible, either.

The search-theoretic monetary models of Kiyotaki and Wright[11],[12] and the following literature based on this approach set up an environment where money eases bilateral trade exactly in these dimensions.[6] Typically, a medium of exchange arises endogenously in these models. This can in principle be either 'commodity money'[7] or intrinsically useless 'fiat money', or both. Under certain parametrizations 'fiat money' gets valuable. In this literature it is mostly assumed that agents are matched randomly with each other.[8] In this case there cannot be commitment because agents will most probably not meet again. This rules out credit as a means of payment. If additionally agents have no memory or only partial memory about other agents' transaction histories society cannot enforce an equilibrium where agents always produce for others. Under these circumstances there exist only insufficient 'trigger strategies' to punish agents who refuse to produce for others, and hence money gets essential.[9]

[5] Alternative approaches where money is modelled in a way such that it becomes essential are among others: the trading-post models of Shubik[23],[24] and the asymmetric information model of Levine[14].

[6] See for example Rupert et al.[19], chapter 4, for an extensive overview over the literature based on the search-theoretic approach.

[7] If the farmer in the above example accepts 10 pairs of shoes for a cow although he needs only one pair - hoping to be able to exchange later the other pairs of shoes into other goods he likes - then shoes are used as a medium of exchange. If they are accepted widely as a medium of exchange they form a kind of 'commodity money'.

[8] Corbae, Temzelides and Wright[4] study models with endogeneous choice of trading partners instead of random matching. Also there a double-coincidence of wants problem creates a role for money, as long as agents are restricted to one bilateral trade per period.

[9] Kocherlakota[13] establishes that necessary conditions for the essentiality of money

Because this literature models the role of money as a means of exchange in an explicit way it provides a reasonable micro-foundation of money. Unfortunately, there is a high degree of heterogeneity of agents in these models: the pairwise exchange of goods in a random matching framework generates non-degenerate distributions of goods inventories and money holdings. The difficulty of keeping track of these distributions is usually overcome by making strong assumptions in order to limit the state space.[10] These, in turn, inhibit the examination of the effects of a constant rate of money creation and shocks to the money growth rate.

Shi[22] is the first successful attempt to study the monetary propagation mechanism of a monetary policy shock in a search-theoretic monetary model. He overcomes the problem of heterogeneity by using the assumption that the decision making unit - the household - is itself a continuum of agents. Hence, idiosyncratic risk is fully insured and a representative agent formulation can be used.[11] In addition, Shi's[22] model incorporates inventory holdings and labor-search.

In spite of the absence of a propagation mechanism due to capital accumulation Shi's[22] model features a highly persistent employment reaction to a monetary shock. The propagation of a positive monetary shock in this model can be summarized as follows: An increase in the money supply reduces the shadow value of money and hence the value of wages and sales. This alone would cause a reduction in labor supply and labor demand and therefore employment and output. This is the conventional 'expected inflation effect'. However, there is a 'search-enhancing effect' that works in the opposite direction: the falling shadow value of holding money makes search

are lack of complete memory and that agents cannot commit to future actions.

[10] The strongest assumption - present in the very first papers of this literature - is that agents can only hold one unit of goods or one unit of money. The following literature relaxed this assumption gradually, but still money holdings have to be either one unit or bounded above by a fixed number.

[11] Faig[6] uses a different interpretation for the same modelling device: the continuum of agents do not form a household but a village. Within a village people know each other and their respective trading histories, so credit and insurance contracts among villagers can be used to face idiosyncratic risk of the agents.

Recently Lagos and Wright [25] used a different modelling device to generate a degenerate distribution of money holdings. After decentralized trade for specialized goods a centralized market opens where agents trade for a general good in order to adjust their money balances. With quasi-linear preferences in one of the goods traded in the centralized market this implies that all agents end up with the same money holdings.

more profitable since a higher search intensity increases the probability of a suitable match where the now less valuable money can be exchanged for consumption goods. In other words, buyers spend more shoe-leather to get rid of the less valuable money. Hence, less unsold goods remain as inventories for next period. The reduction in goods supply, in turn, lets buyers search more intensively in the subsequent period, too. Thus, the search-enhancing effect induces an 'inventory effect' which in turn reinforces the search-enhancing effect. This feedback triggers a high demand for labor in subsequent periods and is stronger than the opposing inflation effect.

The first goal of this paper is to examine the model's overall sensitivity to parameter changes. To do this, a global sensitivity analysis is performed by following the calibration procedure in Canova[2],[3] to calibrate each parameter of the model to an interval. The results of this exercise indicate that the model is quite robust to parameter changes.

The second goal is to isolate the effects of the key elements of this complex model: goods-search (S), inventory holdings (I) and labor-search (L). Rather than looking at the effects of parametrizations where, for example, the model features almost no holdings of inventories because they depreciate at a high rate, I formulate variations of the model which explicitly abandon one or more of the elements each. A pairwise comparison of the model equations and the associated impulse responses then reveals the effects of the element in which the two models differ: **Goods search (S)** enables a monetary shock to affect sales through an enhanced search intensity. Without inventory holdings there is no further effect of goods search. The introduction of **inventories (I)** allows the above stated feedback mechanism between the search-enhancing effect and the inventory effect to come into play. Inventories provide the necessary link between today's increase in sales and tomorrows decrease in goods supply, which in turn enhances search intensity. Even in the absence of labor search this propagation mechanism generates a considerable persistence in the responses of the model variables. Costly **labor search (L)** adds three elements to the model: First, labor gets predetermined and hence there is no impact effect of a monetary shock to the level of employment. Second, the persistence of the reactions is increased by 4 quarters. Since employment cannot adjust immediately to a higher demand in period 2 inventories cannot be refilled as quickly and the excess demand is larger than in the model with a Walrasian labor market. This in turn implies that the search intensity in period 2 is higher than in the model without labor search. Hence, the feedback is stronger and the economy shows more persistent responses. Third, the reactions of employment are smoothed over a long period because of costly hiring and thus feature a hump shape. Hence, to get a hump-shaped employment response, the labor search friction is essential.

The remainder of paper is organized as follows: Section 2 contains a thorough description of the model of Shi[22] and its solution. In section 3 the calibration exercise is described in detail and the results of the general sensitivity analysis are documented. A pairwise comparison of model variations in order to isolate the respective roles of goods search, inventory holdings and labor search is performed in section 4. A discussion in section 5 summarizes the results and suggests possible extensions of the model.

2 The Complete Search-Theoretic Monetary Model

2.1 The Economy and its Matching Process

In the following I present Shi's[22] version of the search-theoretic monetary model. In this version there are two search frictions: costly labor search and costly search for consumption goods. The economy is populated by a continuum of households with measure one, denoted by H. Each household produces a distinct good with labor as the only input to production.[12] Each good $h \in H$ is storable only by its producer. The production function has the form $f(n) = n^{e_f}$, where n is the level of employment and $e_f < 1$, so that $f' > 0, f'' < 0$. Workers have to be hired through a costly search process. Each household $h \in H$ produces good h and wants to consume a subset of goods different from its own product. This induces a need for exchange before consumption is possible. In the absence of a centralized market with a Walrasian auctioneer households have to search for trading partners with the desired goods. Generally, there will be no double-coincidence of wants. The literature following Kiyotaki and Wright[11],[12] showed that in random search models under certain parametrizations fiat money gets valuable and is the only medium of exchange. To establish this in the present model would

[12] Shi [22] abstracts from the propagation through capital accumulation to focus on the role of search in the goods market for propagating monetary shocks. Menner[16] studies the additional effects of capital accumulation in the propagation of shocks.

require a more detailed consideration of the exchange patterns. Instead, Shi[22] assumes that fiat money is required in each transaction.

The matching in the goods market between sellers and buyers and in the labor market between producers and unemployed is assumed to be random. Hence, in a standard setup, where households consist of few individuals, individual agents would face idiosyncratic risks: a priori, buyers do not know whether they can find the desired good and exchange it for the money they carry with them or whether they have to carry their money home again without having found their consumption good; sellers do not know whether their product will be exchanged for money and whether they will be able to pay their workers; producers do not know whether their vacancies are filled, unemployed do not know whether they get hired or not, and so on. As a consequence, money holdings and inventories would not be equally distributed among households. In addition, the employment status as well as the number of people employed would be different among households. Hence, these variables would be individual state variables for each household. To avoid the need of tracking the distributions of money holdings, inventories, employment status and level of employment, it is assumed that the decision unit - the household - is itself a continuum of different agents. These members of the household share the bought consumption goods and regard the household's utility as the common objective. Wage payment regardless of whether the firms had a suitable match in the goods market is made possible by resource sharing of firms within a household. Inventory holdings as well as employees for the next period are shared among the firms of a household, too. Under these assumptions there is no idiosyncratic risk anymore due to the random matching process.

The household consists of five groups: one group of members enjoys leisure while the other four groups are active in markets: Entrepreneurs (set A_p with measure a_p), unemployed (A_u, measure u) workers (A_{nt}, measure $a_p n_t$), and buyers (A_b, measure a_b). The values of a_p, u and a_b are assumed to be constant, while the number of workers per firm n_t may vary over time. An entrepreneur consists of two agents: a producer and a seller. A producer in household h hires workers from other households to produce good h, which is sold by the seller. A worker inelastically supplies one unit of labor each period to other households' firms. A buyer searches with search intensity $s > 0$ to buy the household's desired good. The sellers' search intensity is set to 1. Thus, we focus only on the effect of monetary policy on buyers' search intensity. Let $B = a_b/a_p$ be the buyers/sellers ratio.

In the following a hat on a variable indicates that the household takes this variable and all its future values as given when making the decisions at t.

The total number of matches in the goods market is given by the matching function:

$$g(\hat{s}) \equiv z_1 (a_b \hat{s})^{\alpha} (a_p)^{1-\alpha}, \qquad \alpha \in (0,1).$$

By normalizing $z \equiv z_1 B^{\alpha-1}$ the matching rate per unit of search intensity is $g_b(\hat{s}) \equiv z\hat{s}^{\alpha-1}$, so that a buyer finds a desirable seller at a rate sg_b, and a seller meets a desirable buyer at a rate $g_s(\hat{s}) \equiv zB\hat{s}^{\alpha}$. Thus, the measure of the set of buyers with suitable matches, A_{b*}, is $sg_b a_b$ and that of sellers with suitable matches, A_{p*}, is $g_s a_{p.}$[13]

Each buyer j having found a seller $-j$ with his desired good exchanges $\hat{m}_t(j)$ units of money for $\hat{q}_t(-j)$ units of good $-j$, which implies a price of good $-j$ in this match of $\hat{P}_t(j) = \hat{m}_t(j)/\hat{q}_t(-j)$ and an average price of goods of $\hat{P}_t$.[14]

Each producer j can create vacancies $v_t(j)$ with a cost of $K(v_t(j))$. Unemployed workers have to search for a job and they do this by supplying one unit of search effort inelastically. A worker supplies inelastically one unit of labor each period and receives a wage $\hat{W}(j)$ in units of money. There is an exogenous constant job separation rate δ_n. The matching function in the labor market is linearly homogeneous. The number of matches between firms and unemployed workers is given by $(a_p\hat{v})^{A}(u)^{1-A}$ and the number of matches per vacancy is $\mu(\hat{v}) \equiv (a_p\hat{v}/u)^{A-1}$.

2.2 The Household's Decisions

At the beginning of period t the household distributes the available money M_t evenly among the buyers. Then the four active groups go to their respective markets and do not meet until the end of the period. While the agents are engaged in their transactions each household receives a lump sum monetary transfer τ_t from the central bank. At the end of the period the members of the household arrive at home carrying their trade receipts and residual balances and profits. They consume in equal parts the bought consumption goods. Then the money not spent by the buyers, the wages earned and the profits are added to the money balance of the household for next period's shopping.

[13] The notation $*$ stands for agents that are suitably matched in the current period.

[14] The notation $-j$ stands for an agent with whom agent j is matched.

Also, goods inventories and employees are shared among the household's firms.

Households decide at the beginning of each period about their consumption c_t and the amount of 'fiat' money they want to distribute to the buyers in the next period M_{t+1}. They choose the intensity of search for each buyer $s_t(j)$, the number of vacancies for each firm $v_t(j)$, as well as the inventory level and the amount of labor they have in each of their firms in period $t+1$, $i_{t+1}(j)$ and $n_{t+1}(j)$. Denote by s_t, v_t, i_{t+1}, and n_{t+1} the corresponding distributions over all j. In their decision households take the sequence of terms of trade and wages $\left\{\hat{q}_t, \hat{m}_t, \hat{W}_t\right\}_{t\geq 0}$ as given, as well as the initial distributions $\{M_0, i_0, n_0\}$. Since both buyers and sellers have a positive surplus from trade, it is optimal for households to chose M_{t+1}, n_{t+1} and i_{t+1} such that in period $t+1$ every buyer carries the required amount of money $\hat{m}_{t+1}$ and that every seller has $\hat{q}_{t+1}$ units of good h to be sold. The assumptions $M_0 \geq \hat{m}_0(j)\, a_b$ and $i_0(j) + f(n_0(j)) \geq \hat{q}_0(j)$ ensure that buyers and sellers carry the necessary amounts of money and goods also in period 0.

A household's utility depends on its utility from consuming c_t units of its preferred consumption good, the disutility of labor of the $a_p n_t$ workers and the disutility from searching for the a_b buyers, which depends on their search intensity. Finally the cost of maintaining vacancies for the a_p firms decreases the households utility. Each household maximizes its expected lifetime utility over an infinite time horizon choosing the series of variables $\Gamma_h \equiv \{c_t, s_t, v_t, M_{t+1}, i_{t+1}, n_{t+1}\}_{t\geq 0}$:

$$\max_{\Gamma_h} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(c_t) - \int_{A_{n_t}} \varphi dj - \int_{A_b} \Phi(s_t(j))\, dj - \int_{A_p} \Upsilon(v_t(j))\, dj \right] \right\} \tag{PH}$$

The maximization is subject to the following constraints that are explained below:

$$c_t \leq \int_{A_{bt^*}} \hat{q}_t(-j)\, dj, \tag{1}$$

$$\frac{M_{t+1}}{a_b} \geq \hat{m}_{t+1}(j), \qquad \forall j \in A_{bt+1^*}, \tag{2}$$

$$i_{t+1}(j) + f(n_{t+1}(j)) \geq \hat{q}_{t+1}(j), \qquad \forall j \in A_{pt+1^*}, \tag{3}$$

$$M_t+\tau_t-\int\limits_{A_{bt*}} \hat{m}_t(j)\,dj+\int\limits_{A_{n_t}} \hat{P}_t\hat{W}_t(-j)\,dj+\int\limits_{A_{pt*}} \hat{m}_t(-j)\,dj-\hat{P}_t\int\limits_{A_p} \hat{W}_t(j)\,n_t(j)\,dj \geq M_{t+1}, \tag{4}$$

$$\int\limits_{A_p} \left[(1-\delta_n)\,n_t(j)+v_t(j)\,\mu_t(j)-n_{t+1}(j)\right]dj \geq 0, \tag{5}$$

$$(1-\delta_i)\left[\int\limits_{A_p} \left[i_t(j)+f\left(n_t(j)\right)\right]dj-\int\limits_{A_{pt*}} \hat{q}_t(j)\,dj\right] \geq \int\limits_{A_p} i_{t+1}(j)\,dj. \tag{6}$$

Constraint (1) states that the household's consumption has to be bought by buyers which successfully meet a trading partner and have been endowed with sufficient money for the purchase of $\hat{q}_t$ each. Condition (2) ensures that next period all suitably matched buyers have a sufficient amount of money on hand, while (3) is a similar trading restriction for suitably matched sellers: each should have a sufficient stock of inventory and newly produced goods to satisfy the demand of the costumer. Inequality (4) restricts money holdings at the beginning of period $t+1$ not to exceed the money holdings at the beginning of period t plus the monetary injection minus the money spent plus the cash receipts from firms. Condition (5) states that a household can not allocate more workers of other households to its firms in period $t+1$ than those who worked in firms of the household in period t and have not quitted plus the newly hired workers. Finally, inequality (6) restricts inventories in period $t+1$ to be no larger than the fraction of the excess supply of goods in period t that has not depreciated. The quitting rate δ_n and the depreciation rate of inventories δ_i are assumed to be constant.

In order to express the first order conditions (FOC) conveniently denote the shadow price of money at the beginning of period $t+1$, measured in terms of period-t utility, by ω_{Mt}. Then ω_{Mt} is the multiplier of (4). Similarly let ω_{nt} be the shadow value of workers at the beginning of period $t+1$ and ω_{it} be the shadow price of inventory at the beginning of period $t+1$, again measured in terms of period-t utility. Thus, ω_{nt} and ω_{it} are the multipliers of (5) and (6). Also, let Λ_t, ω_{qt}, be the multipliers of equations (2) and (3), respectively, both measured in terms of period-t utility. Since utility is increasing in c_t, expression (1) takes the equality form and the right hand side (RHS) can be plugged into the utility function. The solution to the maximization problem is then given by the FOCs (with respect to M_{t+1}, i_{t+1}, n_{t+1}, s_t, and v_t):

$$\omega_{Mt} = \beta E \left\{ \omega_{Mt+1} + g_b(\hat{s}_{t+1}) s_{t+1}(j) \Lambda_{t+1}(j) \right\}, \tag{7}$$

$$\omega_{it} = \beta E \left\{ g_s(\hat{s}_{t+1}) \omega_{qt+1}(j) + (1 - \delta_i) \omega_{it+1} \right\}, \tag{8}$$

$$\omega_{nt} = \beta E \left\{ (1 - \delta_n) \omega_{nt+1} - \omega_{Mt+1} \hat{P}_{t+1} \hat{W}_{t+1}(j) \right\} + \omega_{it} f'(n_{t+1}(j)), \tag{9}$$

$$\Phi'(s_t(j)) = g_b(\hat{s}_t) \left[U'(c_t) \hat{q}_t(-j) - \omega_{Mt} \hat{m}_t(j) \right], \tag{10}$$

$$\mu(j) \omega_{nt} = \Upsilon'(v((j))), \tag{11}$$

with the slackness conditions associated with (2) and (3):

$$\Lambda_t(j) \left[\frac{M_t}{a_b} - \hat{m}_t(j) \right] = 0, \qquad \forall j \in A_{bt^*}, \tag{12}$$

$$\omega_{qt}(j) \left[i_t(j) + f(n_t(j)) - \hat{q}_t(j) \right] = 0, \qquad \forall j \in A_{pt^*}. \tag{13}$$

Equation (7) characterizes the dynamics of the shadow value of money held for period $t+1$. The value of holding one unit of money today must be equal to the discounted value of holding this unit tomorrow plus the discounted value of relaxing constraint (12) for $g_{bt+1} s_{t+1}$ desirably matched buyers. Expression (8) is the analogue for inventories held for period $t+1$: the value of holding one unit of inventory today is equal to the discounted value of the fraction $(1 - \delta_i)$ of this inventory which is still available tomorrow plus the discounted value of relaxing the trading constraint (13) for the g_{st+1} suitably matched sellers. The value of an additional worker is given in (9) by the discounted expected value of his value as a worker being further employed minus the discounted expected value of the wage to be paid to him plus the discounted expected value of his marginal contribution in production. The latter is given by the value of its marginal product when selling the good plus its value when storing it as inventory, as indicated by relation (8). Expression (10) is a static equation determining the search intensity of the buyers. The disutility of an additional unit of search effort is equalized to its benefit: the increase in the expected utility of trading $\hat{m}$ units of money for $\hat{q}$ units of consumption due to an increase of the probability of a successful match by g_{bt+1}. Equation (11) equates the expected benefit of an additional vacancy $\omega_{nt} \mu(j)$ to its marginal cost.

Hence, the solution to this decision problem is given by a system of 8 equations consisting of the above optimality conditions together with the laws of motion for money balances, employment and inventories (4) - (6). The system of equations has 8 unknowns once the terms of trade are specified and the equilibrium conditions are imposed. This is done in the following subsections.

2.3 Terms of Trade

There are no centralized markets in this model where prices could be determined. The early goods search literature assumed indivisible goods and indivisible money and that agents could only carry either 1 unit of a good or 1 unit of money. So the price was always 1 by definition. If one wants to allow for divisible goods and divisible money another device for price determination is needed. Here a Nash bargaining process is assumed as it is commonly done in the goods search literature.

2.3.1 Goods Market

To determine the terms of trade in each match and the associated price $P = m/q$, each agent is re-interpreted as an identity of a small measure Δ . After calculating the terms of trade contingent on Δ we take the limit $\Delta \to 0$. When a seller from household h meets a buyer of household $-h$, the trade at the terms $(q\Delta, \bar{m}\Delta)$ leads to the following surpluses in the two agents' households:[15]

$$\text{The seller's surplus:} \qquad \omega_{Mt}\bar{m}_t\Delta - \left[(1-\delta_i)\,\omega_{it} + \omega_{qt}\right] q_t\Delta \ .$$

$$\text{The buyer's surplus:} \qquad U\left(\bar{c}_t + q_t\Delta\right) - U\left(\bar{c}_t\right) - \left(\bar{\omega}_{Mt} + \bar{\Lambda}_t\right)\bar{m}_t\Delta \ .$$

Nash-bargaining with equal weights and taking the limit $\Delta \to 0$ leads to the expressions:

$$\begin{aligned} P_t\omega_{Mt} &= \omega_{qt} + (1-\delta_i)\,\omega_{it}, &(14)\\ U'\left(\bar{c}_t\right) &= P_t\left(\bar{\omega}_{Mt} + \bar{\Lambda}_t\right). &(15)\end{aligned}$$

[15] In what follows symbols with a bar refer to variables of household $-h$.

2.3.2 Wage Bargaining

The firm's surplus from hiring a new worker is given by:

$$\left[\omega_{nt} - \beta\left(1-\delta_n\right)\omega_{nt+1}\right]\Delta = \omega_{it}\left(f\left(n_{t+1}+\Delta\right) - f\left(n_{t+1}\right)\right) - \beta\omega_{t+1}W_{t+1}\Delta \ .$$

The increase in the household's utility from another member being working is:

$$\beta\left(\bar{\omega}_{t+1}W_{t+1} - \varphi\right)\Delta \ .$$

The bargaining outcome is the wage rate that maximizes the weighted Nash product of the two agent's surpluses, with weight $\sigma \in (0,1)$. After taking the limit $\Delta \to 0$, the wage reads:

$$W_{t+1} = \sigma\frac{\varphi}{\bar{\omega}_{t+1}} + (1-\sigma)\frac{\omega_{it}f'\left(n_{t+1}\right)}{\beta\omega_{t+1}}. \qquad (16)$$

2.4 Equilibrium

DEFINITION: A symmetric search equilibrium is a sequence of household's choices $\Gamma_h \equiv \{\Gamma_{ht}\}_{t\geq 0}$, $\Gamma_{ht} \equiv \left(c_t, s_t, v_t, M_{t+1,}i_{t+1}, n_{t+1}\right)$, expected quantities in trade $\hat{X} \equiv \left\{\hat{X}_t\right\}_{t\geq 0}$, $\hat{X}_t \equiv \left(\hat{m}_t, \hat{q}_t, \hat{W}_t\right)$, terms of trade $X \equiv \{X_t\}_{t\geq 0}$ and expected average variables $\hat{s} \equiv \{\hat{s}_t\}_{t\geq 0}$ and $\hat{v} \equiv \{\hat{v}_t\}_{t\geq 0}$ such that

(i) all variables are identical across households and relevant individuals;

(ii) given $\left\{\hat{X}_t\right\}_{t\geq 0}$ and (M_0, i_0, n_0), Γ_h solves (PH) with $(s,v) = (\hat{s}, \hat{v})$;

(iii) X_t satisfies (14), (15) and (16);

(iv) $\hat{X}_t = X_t \quad \forall\, t \geq 0$.

This definition implies that each household takes the sequence $\hat{X}$ as given when choosing Γ_h.[16]

Considering only symmetric equilibria the hats on aggregate variables and bars on household specific variables can be suppressed. For convenience define $\omega_t \equiv P_t\omega_{Mt}$, and $\lambda_t \equiv P_t\Lambda_t$. Hence (14) and (15) become:

$$\omega_{qt} = \omega_t - \left(1-\delta_i\right)\omega_{it}, \qquad (17)$$

$$\lambda_t = U'\left(c_t\right) - \omega_t. \qquad (18)$$

[16] A symmetric equilibrium is determined as follows: For any given $\hat{X}$ the household's choices are a correspondence $\Gamma_h = G\left(\hat{X}\right)$ (part (ii)). The bargaining problem determines X as a correspondence of the particular household's and other households' choices, $X = g(\Gamma_h, \Gamma_{-h})$ (part (iii)). Symmetry and (iv) imply that (Γ, X) solve $\Gamma = G\left(g(\Gamma,\Gamma)\right)$ and $X = g(G\left(X\right), G\left(X\right))$.

Shi[22] restricts attention to the equilibrium where $\lambda > 0$ and $\omega_q > 0$. In this economy with a storage technology the latter is valid whenever a seller prefers selling a good to hoarding it (i.e. as long as $\omega > (1-\delta_i)\,\omega_{it}$). A positive (but bounded) inventory is a sufficient condition for this. The former inequality requires that a buyer prefers spending to hoarding his money (i.e., $U' > \omega$). This is implied by a positive nominal interest rate, but is not given anymore under the so called 'Friedman rule', according to which the gross growth rate of money should be equal to the discount factor. Around the steady state these requirements for λ and ω_q being positive are fulfilled.

The equation defining this equilibrium can be further reduced as follows: Note that the price level is $P_t = m/q = M_t/\left(a_b q_t\right).$ Define the gross rate of money growth between periods t and $t+1$ as:

$$\gamma_t \equiv M_{t+1}/M_t.$$

Thus, the gross inflation rate between periods t and $t+1$ is given by:

$$P_{t+1}/P_t = \gamma_t q_t/q_{t+1}.$$

To simplify the calculations it is possible to express the equilibrium conditions in terms of the variables $(v, n, \omega_i, \omega, q)$ by elimination of other the remaining variables $(i, \omega_q, \lambda, c, \mu, \omega_n, W, m, s)$:

With $\omega_q > 0$, $i = q - f(n)$. The multiplier ω_q can be replaced by $\omega - (1-\delta_i)\,\omega_i$ through (17). When $\lambda > 0$, $c_t = a_p B z s_t^{\alpha} q_t$. With (18) λ can be eliminated. Under symmetry μ is a function of v and one can define $k(v) \equiv \omega_n = \Upsilon'(v)/\mu(v)$.

By definition $m = Pq$, so (10) takes the following form after substituting the expressions for g_b and c_t :

$$s_t^{1-\alpha}\Phi'(s_t) = z q_t\left[U'\left(a_p B z s_t^{\alpha} q_t\right) - \omega_t\right]. \tag{19}$$

The search intensity s is thus a decreasing function of ω and q:

$$s_t = s(\omega_t, q_t), \quad s_\omega < 0, \quad s_q < 0.$$

2.5 Dynamic Equations

After substitution of the above relations into (5)-(9) one obtains the following dynamic system:

$$n_{t+1} = (1-\delta_n)\, n_t + v_t \mu_t\,(v)\,, \tag{20}$$

$$q_{t+1} = (1-\delta_i)\,(1 - Bzs_t^{\alpha})\, q_t + f\,(n_{t+1})\,, \tag{21}$$

$$\omega_t = E\left\{\frac{\beta}{\gamma_t}.\frac{q_{t+1}}{q_t}\left(\omega_{t+1} + zs_{t+1}^{\alpha}\left[U'\,(c_{t+1}) - \omega_{t+1}\right]\right)\right\}, \tag{22}$$

$$\omega_{it} = \beta E\left\{(1-\delta_i)\,\omega_{it+1} + Bzs_t^{\alpha}\,(\omega_{t+1} - (1-\delta_i)\,\omega_{it+1})\right\}, \tag{23}$$

$$k\,(v_t) = E\left\{\beta\,(1-\delta_n)\,k\,(v_{t+1}) + \sigma\left[\omega_{it} f'\,(n_{t+1}) - \beta\varphi\right]\right\}. \tag{24}$$

2.6 Note on the Timing of the Model

Throughout this paper I use the same timing of events as in the model of Shi, which will be explained below. The following discussion, however, is intended to make the reader aware that the timing matters.

Households receive a lump-sum monetary transfer at the end of each period. The gross rate of money growth between periods t and $t+1$ is

$$\gamma_t = M_{t+1}/M_t = (M_t + \tau_t)/M_t,$$

and follows an AR(1) process:

$$\gamma_t = (1-\rho_g)\gamma^* + \rho_g \gamma_{t-1} + \varepsilon_{gt}, \qquad E(\varepsilon_g) = 0, \quad E(\varepsilon_g^2) = \sigma_g^2.$$

The issue of the influence of the timing of a monetary injection in CIA models was put forward in Salyer[20]. The timing of the monetary injection in the search-theoretic model of Shi[22] differs, e.g., from the one in Lucas[15] where the monetary injection is at the beginning of the period. As pointed out by Salyer[20]: "with the transfer received at the end of the period, the realization of the monetary growth rate at time t provides agents with perfect information about the money stock in the goods market at times t and $t+1$. [...] ...it is the current, as opposed to next period's monetary growth rate that influences the inflation rate between periods t and $t+1$. (p. 770)"

An implication of this timing is that independently and identically distributed growth rates over time are not a sufficient condition for dynamic neutrality.[17] As a consequence impulse shocks do have real effects in this economy, as opposed to the case with monetary injections at the beginning of the period, where some persistence of the shocks is needed to generate effects of a monetary shock on employment, consumption and other real variables. With serially correlated shocks impulse responses have exactly the same shape under both timing assumptions, but the respective magnitudes differ. Assuming, for example, $\rho_g = 0.4$, the magnitude of the impulse responses is 2.5 to 3 times higher with an end-of-the-period shock than with a beginning-of-the-period shock, depending on which variable is considered. For $\rho_g = 0.85$, however, the difference in magnitude vanishes mostly. As stated above, I stick to the timing of Shi[22] for the sake of better comparison, but the reader should keep in mind that the timing matters.

3 Calibration and Sensitivity Analysis

3.1 Calibration to Intervals

The usual method of calibrating parameters to point estimates assumes implicitly that these values are known with certainty. Instead, I use here a procedure suggested by Canova[2],[3] that reflects better the uncertainty of a calibrator in choosing the proper values. The idea is to calibrate each parameter of the model to an interval, using the empirical information one obtains from the literature to construct a distribution over this interval. This means, for each parameter a range of economically reasonable values is specified. When different point estimates are found in the literature for some parameter, their frequency is used to form a distribution over this parameter range, otherwise a uniform distribution is assumed. Then the model is simulated drawing parameter vectors from the joint distribution. This procedure allows to perform an overall sensitivity analysis over the whole support of the parameter distributions by examination of the mean response and its 95% error bands. In order to achieve comparability I either center these parameter intervals around the calibrated values reported in Shi[22] when the range of uncertainty seems to be symmetric, or I choose economically reasonable ranges which contain Shi's values. The ranges of the intervals and the distributions used are shown in Table 1.

[17] Dynamic neutrality means that real balances are constant when the only source of uncertainty is a monetary shock. For more details see Salyer[20], p. 770f.

3.2 Global Sensitivity Analysis

After log-linearizing the dynamic equilibrium equations around the steady state I conduct 250 simulations.[18] In each iteration a new set of the exogenously chosen parameters is drawn from the distributions indicated in Table 1. The realized values and the values of the endogenously determined parameters are stored. Then the implied steady state values of the variables are calculated and the model is solved. Finally, the implied impulse responses to a monetary shock are calculated.

The mean and the interval containing 90% of the simulated parameter values are documented in Table 2. Simulated mean values are quite near to the values implied by the point calibration. Exceptions are the production elasticity e_f, the disutility of labor φ and the hiring cost parameter K_0. In each case the respective mean value is larger than the associated point estimate. In what follows I describe the simulated impulse responses of the model variables to an end-of-the-period shock in money growth.

Consider first the results shown in Figure 1. Each graph displays the mean response of a variable i, $\hat{\Theta}_i$, together with its 95% error bands. The latter are computed as one-dimensional error bands $\hat{\Theta}_i(t) \pm \Delta_i(t)$ for each quarter t after the shock. In other words these are pointwise error bands. Employment shows a significant positive response up to 15 quarters. The mean response is quite similar to the response reported in Shi[22]. It also peaks after 4 quarters and is slightly higher in magnitude. The error bands indicate uncertainty about the quarter in which the response peaks. With 97.5% probability the responses do not reach their maximum before the third quarter. The upper limit of this range is less clear. The waves in the upper error band indicate that there were some simulated responses with very high persistence. Figure 1b shows a significant decrease of the quantity exchanged up to 8 quarters. The uncertainty seems to be mostly with respect to the level of the response and only little with respect to the shape. This last feature holds also for the variables shown in Figures 1c to 1h. For these variables only two things need to be emphasized. First, there is a substantial uncertainty regarding the impact response of search intensity and therefore of sales. Second, the mean response - and similarly the response with Shi's[22] parametrization - of the search intensity stays 8 quarters above steady state, and the 95% error bands indicate that the search intensity stays significantly above steady state for 5 quarters.

[18] For this porpuse I use the solver of Uhlig[27].

However, the reader should be careful in interpreting the upper and lower bands as impulse responses. Sims and Zha[25] point out that pointwise error bands do not necessarily represent impulse response functions at the "boundaries of likely variation" in the distribution of impulse responses Θ_i. In other words, the upper and lower bands do not necessarily represent likely deviations of the impulse response function from the mean response. Interpreting these bands as draws from the distribution of impulse responses Θ_i is only plausible if there is a very high serially correlation in the uncertainty about Θ_i across time. In order to better characterize uncertainty regarding the magnitude and shape of impulse responses Sims and Zha[25] propose a change of the coordinate system were the impulse response functions are examined. The new coordinate system is formed by projections on the principal components of the covariance matrix ω of the jointly distributed $\{\Theta_i(t)\}_{t=0}^{H}$ vector, where H is the horizon considered for the impulse response. As Sims and Zha[25] claim, the resulting probability bands can convey more information about the kind of uncertainty there is regarding the shape and the magnitude of the impulse responses.[19]

Figure 2 shows the plots of the 95%probability bands for the first and second components of ω for the impulse responses of employment and quantity-exchanged. The first component accounts for 80% and 85%, respectively, of the sum of the eigenvalues, the second component for 19% and 13%, respectively. Graphs for the other variables are not presented since they do not convey additional information: the bands of the first component and the pointwise bands are nearly identical.

For employment and quantity-exchanged, however, these graphs give additional insights. Figures 2a and 2c together show that the shape of the impulse response of employment is quite uncertain. It is revealed more clearly

[19] To calculate this probability bands one has to accumulate a first and second moment matrix during the simulations from which one computes the estimated covariance matrix ω and its eigenvalue decomposition $\Psi\Lambda\Psi' = \omega$. Any Θ_i is now representable as $\Theta_i = \hat{\Theta}_i + \sum_{k=1}^{H} \xi_k \Psi_{\cdot k}$, where $\Psi_{\cdot k}$ is the kth column of Ψ, i.e. the kth eigenvector of ω. Then the 2.5% and 97.5% quantiles of the weights ξ_j corresponding to the largest eigenvalues λ_j (principal components) of ω can be calculated by making another pass through the saved draws of the impulse responses. $\xi = \{\xi_k\}_{k=0}^{H}$ can be calculated for a particular draw of Θ_i by $\xi = \Psi' \cdot (\Theta_i - \hat{\Theta}_i)$. For each component j the two functions $\hat{\Theta}_i + \xi_{j,0.975}\Psi_{\cdot j}$ and $\hat{\Theta}_i + \xi_{j,0.025}\Psi_{\cdot j}$ each show a likely direction of variation in Θ_i. Unlike the functions of t plotted in pointwise error bands the plots of these functions show impulse response functions that lie in the boundary of the Gaussian confidence ellipsoid when the distribution of Θ_i is Gaussian.

than in Figure 1 that the uncertainty does not only concern the magnitude of the impulse response but also the time it takes until the response peaks. The upper bound for likely peaks is 6 quarters for the first component, but is likely to be reduced by the second component, since the second component of those impulse responses which start above the mean response reaches its maximum between the 4th and 5th quarter. On the other hand, responses that start below the mean response are likely to have a higher persistence and to peak later. In contrast with the above conjecture that uncertainty regarding the response of the quantity exchanged is merely uncertainty regarding the level, Figures 2b and 2d reveal that there is substantial uncertainty regarding the shape of the response, too.

To summarize, one can say that, although the shape and the magnitude of the responses of employment vary considerably, Shi's claim to have found a model with highly persistent hump-shaped employment responses is shown to be quite insensitive to variations of the parameters within economically reasonable ranges.

4 Examination of the Propagation Mechanism

In this section the search-theoretic monetary model of Shi[22] is modified in several ways in order to isolate the role of its three key elements: goods-search (S), inventory holdings (I) and labor-search (L). For this purpose it suffices to consider 4 variations. The scheme below shows how only one feature varies in each pairwise comparison of subsequent models. In the "goods search and inventories" (SI) model the labor search friction is replaced by a Walrasian labor market. The "goods and labor-search" (S_L) model is another variation of Shi's[22] model where the produced goods can not be stored in inventories. The "CIA and labor-search" (C_L) model is set up similar to the "goods and labor-search" (S_L) model but using a CIA constraint in a Walrasian goods market, as opposed to costly search for goods.[20]

[20] When we replace costly search for goods by a CIA constraint in a Walrasian goods market in a model without inventories and without labor search, the disutility of labor parameter does not enter in the log-linearized dynamic system. So, the model is not rich enough to be sensitive to labor supply changes. This problem does not arise when labor search is introduced.

Scheme of Pairwise Model Comparison

Model	Symbol	goods search	inventory holdings	labor search	isolates role of:
Goods Search & Inventory	SI_	S	I	–	} L
Shi (1998)	SIL	S	I	L	} I
Goods- & Labor-Search	S_L	S	–	L	} S
CIA & Labor-Search	C_L	–	–	L	

In the following subsections I describe shortly the main differences of each of these models with respect to the model of Shi[22]. In particular, the derived FOCs and equilibrium conditions as well as the resulting system of dynamic equations are presented there. Proofs of existence of a steady state can be found in Appendix A, while modifications in the calibration procedure are presented in Appendix B. The log-linearized systems of equations were again solved with Uhlig's[27] toolkit. The impulse responses are analyzed at the end of this section.

4.1 Goods-Search and Inventory Model

In this model variant labor search is replaced by Walrasian labor market clearing. A symmetric equilibrium with Walrasian labor market would imply that each firm of each household employs the same amount of workers n_t and pays a market clearing wage rate. The optimality condition that determines n_t is that the marginal disutility of labor is equalized to the marginal benefit resulting from employing an additional worker. However, the same condition and the same allocation would arise if workers were employed in the own household's firms. Thus, for sake of simplicity I assume the latter setup where no equilibrium wages have to be determined. But I still refer to it as a Walrasian labor market because of the above stated equivalence.

4.1.1 The Households' Decisions

Households do not maintain vacancies in an economy without labor search. Instead, at the beginning of each period, they decide directly about the amount of labor they use in each of their firms in the next period $n_{t+1}(j)$.[21] As before, they decide about consumption c_t and the intensity of search for their desired goods s_t, the inventory level for period $t+1$, i_{t+1}, and the amount of money they want to distribute to the buyers in the next period, M_{t+1}. In the law of motion of money balances there are no wage receipts and wage payments to consider. The law of motion of employment vanishes since a new level of employment can be achieved immediately in the absence of a search friction.

The necessary conditions for an optimum are given by the following FOCs (with respect to M_{t+1}, i_{t+1}, s_t, and n_{t+1}):

$$\omega_{Mt} = \beta E\left[\omega_{Mt+1} + g_{bt+1} s_{t+1}(j)\, \Lambda_{t+1}(j)\right], \tag{25}$$

$$\omega_{it} = \beta E\left[(1-\delta_i)\,\omega_{it+1} + g_{st+1}\omega_{qt+1}(j)\right], \tag{26}$$

$$\Phi'(s_t(j)) = g_b(\hat{s}_t)\left[U'(c_t)\,\hat{q}_t(-j) - \omega_{Mt}\hat{m}_t(j)\right], \tag{27}$$

$$\varphi = E\left[\left(g_{st+1}\omega_{qt+1}(j) + (1-\delta_i)\,\omega_{it+1}\right)\right] f'(n_{t+1}(j)). \tag{28}$$

As stated already above, (28) determines the amount of employment by equating the marginal disutility of labor with the marginal benefit of an additional worker. The fact that this equation replaces (9) and (11) is the only difference in the FOCs with respect to the model with labor search.

The terms of trade are the same as before and equilibrium conditions can again be simplified. The latter can be expressed in terms of (ω_i, ω, n, q) by elimination of the variables $(i, \omega_q, \lambda, c, W, m, s)$.

[21] Households are assumed to decide n_{t+1}, as opposed to n_t, in order to have employment in period t predetermined as in the model of Shi [22]. Loosing this predeterminedness would lead to a negative impact reaction of employment in response to a non-persistent monetary shock. The difference in the effects of choosing n_t versus n_{t+1} is discussed in more detail in subsection 4.4. See also paragraph 4.1.3 for the model's solution when n_t is chosen.

Then (28) reads:

$$\varphi = E\left[Bzs_{t+1}^{\alpha}\omega_{t+1} + (1-\delta_i)\left(1 - Bzs_{t+1}^{\alpha}\right)\omega_{it+1}\right] f'(n_{t+1}). \tag{29}$$

Equation (26) can be written as:

$$\omega_{it} = \beta E\left[(1-\delta_i)(1 - Bzs_{t+1}^{\alpha})\omega_{it+1} + Bzs_{t+1}^{\alpha}\omega_{t+1}\right]. \tag{30}$$

Combining (29) and (30) we obtain:

$$\omega_{it} = \beta \frac{\varphi}{f'(n_{t+1})}.$$

Now, update the last expression and substitute into (29) to obtain:

$$n_{t+1}^{1-e_f} = E\left[\frac{e_f}{\varphi} Bzs_{t+1}^{\alpha}\omega_{t+1} + \beta(1-\delta_i)\left(1 - Bzs_{t+1}^{\alpha}\right) n_{t+2}^{1-e_f}\right]. \tag{31}$$

Equation (25) can be expressed as:

$$\omega_t = \frac{\beta}{\gamma_t q_t} E\left\{q_{t+1}\left(\omega_{t+1} + zs^{\alpha}\left[U'\left(a_p Bzs^{\alpha} q_{t+1}\right) - \omega_{t+1}\right]\right)\right\}. \tag{32}$$

Replacing i by $q - f(n)$, the equilibrium law of motion of inventories becomes:

$$q_{t+1} = (1-\delta_i)(1 - Bzs^{\alpha}) q_t + f(n_{t+1}). \tag{33}$$

4.1.2 Dynamic Equations

Plugging $s(\omega_t, q_t)$ into (31), (32) and (33) results in the following dynamic system:

$$n_{t+1}^{1-e_f} = E\left[\frac{e_f}{\varphi} Bzs(\omega_{t+1}, q_{t+1})^{\alpha}\omega_{t+1} + \beta(1-\delta_i)(1 - Bzs(\omega_{t+1}, q_{t+1})^{\alpha}) n_{t+2}^{1-e_f}\right], \tag{34}$$

$$\omega_t = \frac{\beta}{\gamma_t q_t} E\left\{q_{t+1}\left(\omega_{t+1} + zs(\omega_{t+1}, q_{t+1})^{\alpha}\left[U'\left(a_p Bzs(\omega_{t+1}, q_{t+1})^{\alpha} q_{t+1}\right) - \omega_{t+1}\right]\right)\right\}, \tag{35}$$

$$q_{t+1} = (1-\delta_i)(1 - Bzs(\omega_t, q_t)^{\alpha}) q_t + f(n_{t+1}). \tag{36}$$

Thus, the assumption of a Walrasian labor market instead of labor search reduces the dynamic system to 3 equations. The existence of a steady state is proven in Appendix A.1.

4.1.3 Modification: Choice of Current Employment

In the pairwise comparison I also consider a version of the above model where n_t is chosen in period t and hence employment is not predetermined anymore. In this case, the solution to the corresponding maximization problem is given by the following FOCs (with respect to $M_{t+1}, i_{t+1}, s_t, n_t$):

$$\omega_{Mt} = \beta E \left[\omega_{Mt+1} + g_{bt+1} s_{t+1} (j) \Lambda_{t+1} (j) \right], \tag{37}$$

$$\omega_{it} = \beta E \left[(1-\delta_i) \omega_{it+1} + g_{st+1} \omega_{qt+1} (j) \right], \tag{38}$$

$$\Phi' (s_t (j)) = g_b(\hat{s}_t) \left[U' (c_t) \hat{q}_t (-j) - \omega_{Mt} \hat{m}_t (j) \right], \tag{39}$$

$$\varphi = \left(g_{st} \omega_{qt} (j) + (1-\delta_i) \omega_{it} \right) f' (n_t (j)). \tag{40}$$

The terms of trade are the same as in the inventory model with predetermined employment. The optimality conditions can again be simplified as before. Substituting ω_{qt} into equation (40) yields:

$$\varphi = \left(Bzs(\omega_t, q_t)^{\alpha} \omega_t + (1-\delta_i) \left(1 - Bzs(\omega_t, q_t)^{\alpha}\right) \omega_{it} \right) f' (n_t). \tag{41}$$

Solving for ω_{it} we obtain:

$$\omega_{it} = \frac{\varphi n_t^{1-e_f}}{e_f (1-\delta_i) \left(1 - Bzs(\omega_t, q_t)^{\alpha}\right)} - \frac{Bzs(\omega_t, q_t)^{\alpha} \omega_t}{(1-\delta_i) \left(1 - Bzs(\omega_t, q_t)^{\alpha}\right)}. \tag{42}$$

Equation (38) can be written as:

$$\begin{aligned} \omega_{it} &= \beta E \left[(1-\delta_i) (1 - Bzs(\omega_{t+1}, q_{t+1})^{\alpha}) \omega_{it+1} + Bzs(\omega_{t+1}, q_{t+1})^{\alpha} \omega_{t+1} \right] \\ &= \beta E \left[\frac{\varphi {n_{t+1}}^{1-e_f}}{e_f} \right]. \end{aligned} \tag{43}$$

Thus, equalizing (43) and (42) one obtains:

$$\begin{aligned} \beta E \left[\frac{\varphi {n_{t+1}}^{1-e_f}}{e_f} \right] &= \frac{\varphi n_t^{1-e_f}}{e_f (1-\delta_i) (1 - Bzs_t^{\alpha})} - \frac{Bzs_t^{\alpha} \omega_t}{(1-\delta_i) (1 - Bzs_t^{\alpha})} \Leftrightarrow \\ E \left[{n_{t+1}}^{1-e_f} \right] &= \frac{n_t^{1-e_f} - \frac{e_f}{\varphi} Bzs_t^{\alpha} \omega_t}{\beta (1-\delta_i) (1 - Bzs_t^{\alpha})}. \end{aligned} \tag{44}$$

The remaining two dynamic equations are the same as in the model with predetermined employment. Both models have the same steady state.

4.2 Goods- and Labor-Search Model

4.2.1 The Household's Decisions

Here it is assumed that firms cannot store their non-sold goods for next period. Thus, the sellers can serve the demand of buyers only from current production. Hence, the sellers' constraint has to be modified to:

$$f\left(n_t\left(j\right)\right) \geq \hat{q}_t\left(j\right), \qquad \forall j \in A_{pt^*}.$$

Note that in the absence of a storage technology there is no law of motion of inventories in the optimization problem anymore.

The associated FOCs are:

$$\omega_{Mt} = \beta E\left\{g_b(\hat{s}_{t+1})s_{t+1}\left(j\right)\Lambda_{t+1}\left(j\right) + \omega_{Mt+1}\right\}, \tag{45}$$

$$\omega_{nt} = \beta E\left\{\left(1-\delta_n\right)\omega_{nt+1} - \omega_{Mt+1}\hat{P}_{t+1}\hat{W}_{t+1}\left(j\right) + g_s(\hat{s}_{t+1})\omega_{qt+1}\left(j\right)f'\left(n_{t+1}\left(j\right)\right)\right\}, \tag{46}$$

$$\Phi'\left(s_t\left(j\right)\right) = g_b(\hat{s}_t)\left[U'\left(c_t\right)\hat{q}_t\left(-j\right) - \omega_{Mt}\hat{m}_t\left(j\right)\right], \tag{47}$$

$$\omega_{nt} = K'\left(v((j))\right)/\mu\left(j\right). \tag{48}$$

Apart from the lack of the dynamic equation for the shadow value of inventories, ω_i, there is only one difference with the complete model: Comparing (46) with (9) one sees that ω_{it} is now replaced by $\beta E\left\{g_s(\hat{s}_{t+1})\omega_{qt+1}\left(j\right)\right\}$. Remember from expression (26) that $\omega_{it} = \beta E\{g_s(\hat{s}_{t+1})\omega_{qt+1}\left(j\right) + \left(1-\delta_i\right)\omega_{it+1}\}$. This means that the value of the units of consumption good produced by an additional worker tomorrow is now given solely by their value of being sold. In the complete model, instead, the facility to serve as inventory adds to their value.

4.2.2 Terms of Trade

The terms of trade are determined again by Nash bargaining. In the following I expose the implied modifications and the solution to the modified bargaining problem.

Goods Market The seller's surplus is unchanged while the buyer's surplus is now: $\omega_{Mt}\bar{m}_t\Delta - \omega_{qt}q_t\Delta$.

This leads to the following Nash bargaining solution:

$$\omega_{qt} = \omega_t, \tag{49}$$

$$\bar{\lambda}_t = U'(\bar{c}_t) - \bar{\omega}_t. \tag{50}$$

The shadow value of giving up one unit of consumption equals the shadow value of the received money. There is no compensation for the forgone value of storing the good as inventory, since it cannot be stored under the maintained assumptions.

Wage Bargaining The firm's surplus from hiring a new worker is:

$$[\omega_{nt} - \beta(1-\delta_n)\omega_{nt+1}]\Delta = \beta g_s(\hat{s}_{t+1})\omega_{qt+1}(f(n_{t+1}+\Delta) - f(n_{t+1})) - \beta\omega_{t+1}W_{t+1}\Delta \ .$$

The increase in the household's utility from another member being working is as in the complete model:

$$\beta(\bar{\omega}_{t+1}W_{t+1} - \varphi)\Delta \ .$$

The bargaining outcome with weight $\sigma \in (0,1)$ is:

$$W_{t+1} = \sigma\frac{\varphi}{\bar{\omega}_{t+1}} + (1-\sigma)\frac{\omega_{qt+1}g_s(\hat{s}_{t+1})f'(n_{t+1})}{\omega_{t+1}} = \sigma\frac{\varphi}{\bar{\omega}_{t+1}} + (1-\sigma)g_s(\hat{s}_{t+1})f'(n_{t+1}) \,. \tag{51}$$

Hence, in equilibrium $\left(\hat{m}_t, \hat{q}_t, \hat{W}_t\right)$ have to satisfy (49), (50) and (51).

4.2.3 Equilibrium

The conditions to ensure $\lambda > 0$ and $\omega_q > 0$ remain unchanged. But now, $q = f(n) = M/(a_b P)$. The gross inflation rate between periods t and $t+1$ reads:

$$P_{t+1}/P_t = \gamma_t q_t/q_{t+1} = \gamma_t f(n_t)/f(n_{t+1}) \,.$$

Equilibrium conditions can be expressed in terms of (n, ω, v) by elimination of the variables (ω_q, λ, c, W, and m). Equation (47) takes the following form after substituting the expressions for g_b and c_t :

$$s_t^{1-\alpha}\Phi'(s_t) = zf(n_t)\left[U'(a_p B z s_t^{\alpha} f(n_t)) - \omega_t\right]. \tag{52}$$

The search intensity s is thus a decreasing function of ω and n:

$$s_t = s(\omega_t, n_t), \quad s_\omega < 0, \quad s_n < 0.$$

4.2.4 Dynamic Equations

After substitution of these relationships into the laws of motion for money and employment and into (45)-(46) the economy is described by the following dynamic system of three variables (n, ω, v) in three equations:

$$n_{t+1} = (1 - \delta_n) n_t + v_t \mu_t (v_t), \tag{53}$$

$$\begin{aligned} \omega_t &= E\left\{ \frac{\beta}{\gamma_{t+1}} . \frac{f(n_{t+1})}{f(n_t)} \cdot [\omega_{t+1} - zs(\omega_{t+1}, n_{t+1})^\alpha U' \left(a_p B zs(\omega_{t+1}, n_{t+1})^\alpha f(n_{t+1})\right) \right. \\ & \left. -\omega_{t+1}] \right\}, \end{aligned} \tag{54}$$

$$k(v_t) = \beta E \left\{ (1 - \delta_n) k(v_{t+1}) + \sigma \left(\omega_{t+1} z B s(\omega_{t+1}, n_{t+1})^\alpha f'(n_{t+1}) - \varphi \right) \right\}. \tag{55}$$

The proof of existence of a steady state can be found in Appendix A.2.

4.3 CIA Model with Labor Search

The CIA model modifies the Lucas[15] model such that agents hold bonds instead of equity in their portfolio. As opposed to Lucas'[15] model, the money injection is at the end of the period in order to mimic the timing of the search-theoretic model. Following Hansen[9] a production technology with indivisible labor is introduced. In addition, costly labor search is added as in Shi[22]. Hence, the resulting model has the same features as the search model without inventories, except for the CIA constraint in a Walrasian goods market, as opposed to costly search for goods.[22] It can therefore be interpreted as the Walrasian counterpart of the goods- and labor search model.

Another way to think of this model is to consider it as a variant of the search-model with goods search at no cost. Then s always adjusts such that the probability for a suitable match is 1. This, in turn, is equivalent to a matching technology that yields a suitable match with probability 1, independently of the search intensity. With this technology the household does not need to decide about the level of search effort. The buyers/sellers

[22] Note, that the incorporation of positive inventory holdings into a standard CIA model is not possible without the incorporation of other frictions which break up the one-to-one relation between sales and output. This would introduce differences with respect to the search-theoretic models in other dimensions and would harm comparability.

ratio B is equal to 1 as in the "goods and labor search" model (See Appendix B.2). The matching technology is such that the expressions for the matching rate of buyers and sellers, zBs^{α} and zs^{α}, can be replaced by 1. There are no given terms of trade $\hat{m}$ and $\hat{q}$, but prices adjust such that a seller's production is sold for the money the buyer carries. Thus, there is no trading constraint for the seller and the buyers' money constraint results in a simple cash-in-advance constraint:

$$M_t \geq a_b P_t \frac{\int_{A_p} f\left(n_t(j)\right) dj}{a_p} = P_t \int_{A_p} f\left(n_t(j)\right) dj = P_t c_t.$$

4.3.1 The Households' Decisions

A household decides in period t about consumption c_t, the number of vacancies for each firm in period t, $v_t\left(j\right)$, the level of employment in period $t+1$, n_{t+1}, and the money balances hold at the beginning of period $t+1$, M_{t+1}. Thus, its decision problem is:

$$\max_{\{c_t, v_t, n_{t+1}, M_{t+1}\}_{t\geq 0}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U\left(c_t\right) - \int_{A_{n_t}} \varphi dj - \int_{A_p} K\left(v_t\left(j\right)\right) dj \right\} \quad \text{(PHCIA)}$$

subject to:

$$M_t \geq P_t c_t,$$

$$M_t - P_t c_t + P_t \int_{A_{n_t}} \hat{W}_t(-j) dj + P_t \int_{A_p} f\left(n_t(j)\right) dj - P_t \int_{A_p} \hat{W}_t(j) n_t(j) dj + \tau_t \geq M_{t+1},$$

$$\int_{A_p} \left[\left(1-\delta_n\right) n_t\left(j\right) + v_t\left(j\right) \mu_t\left(j\right) - n_{t+1}\left(j\right)\right] dj \geq 0.$$

The optimal choice is characterized by:

$$\omega_t = \beta E\left\{ \left(\frac{P_t}{P_{t+1}} U'(c_{t+1}) \right) \right\}, \tag{56}$$

$$\omega_{nt} = \beta E\left\{ \omega_{t+1} \left(f'\left(n_{t+1}(j)\right) - \hat{W}_{t+1}(j) \right) + (1-d_n)\omega_{nt+1} \right\}, \tag{57}$$

$$k\left(v_t(j)\right) \equiv \frac{K'\left(v_t(j)\right)}{\mu_t(j)} = \omega_{nt}. \tag{58}$$

Wage Bargaining

The firm's surplus from hiring a new worker is given by:

$$\left[\omega_{nt} - \beta\left(1-\delta_n\right)\omega_{nt+1}\right]\Delta = \beta\omega_{t+1}\left(f\left(n_{t+1}+\Delta\right) - f\left(n_{t+1}\right)\right) - \beta\omega_{t+1}W_{t+1}\Delta \ .$$

The increase in the household's utility from another member being working is:

$$\beta\left(\bar{\omega}_{t+1}W_{t+1} - \varphi\right)\Delta \ .$$

The bargaining outcome with weight $\sigma \in (0,1)$ is:

$$W_{t+1} = \sigma\frac{\varphi}{\bar{\omega}_{t+1}} + (1-\sigma)\, f'\left(n_{t+1}\right). \tag{59}$$

4.3.2 Equilibrium

In equilibrium: $c = a_p f(n)$, $M = \bar{M}$. Thus, $P_{t+1}/P_t = M_{t+1}c_t/M_t c_{t+1} = \gamma_t c_t/c_{t+1} = \gamma_t f(n_t)/f(n_{t+1})$. Furthermore, $\hat{W}_{t+1} = W_{t+1}$ which can be replaced by 59. Plugging these equilibrium expressions into (56), substituting (58) into (57), and using symmetry one receives:

$$\omega_t = \beta E\left\{\left(\frac{f(n_{t+1})}{\gamma_t f(n_t)}U'(a_p f(n_{t+1}))\right)\right\}, \tag{60}$$

$$\begin{aligned} k(v_t) &= \beta E\left\{\omega_{t+1}f'\left(n_{t+1}(j)\right) - \sigma\varphi - (1-\sigma)\,\omega_{t+1}f'\left(n_{t+1}\right) + (1-d_n)k(v_{t+1})\right\} \\ &= \beta E\left\{(1-\delta_n)k(v_{t+1}) + \sigma\left(\omega_{t+1}f'\left(n_{t+1}(j)\right) - \varphi\right)\right\}. \end{aligned} \tag{61}$$

4.3.3 Dynamic Equations

Combined with the law of motion for employment equations (60) and (61) result in a dynamic system consisting of three equations in n, ω, and v:

$$n_{t+1} = \left(1-\delta_n\right)n_t + v_t\mu\left(v_t\right), \tag{62}$$

$$\omega_t = \beta E\left\{\left(\frac{f(n_{t+1})}{\gamma_t f(n_t)}U'(a_p f(n_{t+1}))\right)\right\}, \tag{63}$$

$$k(v_t) = \beta E\left\{(1-\delta_n)k(v_{t+1}) + \sigma\left(\omega_{t+1}f'\left(n_{t+1}\right) - \varphi\right)\right\}. \tag{64}$$

As expected, these equations are equivalent to the ones in the goods search model when $zBs^{\alpha} = 1$ and when there is no decision about s. Existence of a steady state is proven in Appendix A.3.

4.4 Results of Pairwise Comparison

The different model versions are calibrated as documented in Appendix B. The log-linearized systems are solved with Uhlig's[27] toolkit. Then impulse responses after a monetary shock are calculated.

To isolate the effects of **Goods Search (S)** consider the comparison between the "CIA and labor-search model" (C_L) and the "goods- and labor-search model" (S_L). The only difference between the models lies in the goods market. In the first model we have a Walrasian goods market, while in the second there is a search friction. Neither model has inventory holdings and both models feature a labor market friction. Figure 3 represents the impulse responses of various variables of the C_L model (Figures 3a-3d) and the S_L model (Figures 3e-3h), respectively, to a positive monetary shock in period 0 with low persistence ($\rho = 0.2$).[23] In Figures 3a and 3e we see that the shadow value of money decreases by 1 percent in the CIA model and by 1.2% in the goods search model. Since in the goods search model the value of money depends also on the future value of money, there is an 'expected inflation effect' in the impact response of the value of money. Figures 3d and 3h show the employment reactions. The reason for why employment drops by less in the goods search model is the following: As can be seen in Figure 3e, a money shock enhances search intensity by 0.6% on impact. Figure 3f shows the related increase of sales of 0.5 % and the drop in sales revenues, that is sales multiplied by the value of money, by 0.7%. In subsequent periods the shadow value of money keeps staying below steady state because of the autocorrelation in the money growth shock, and this induces search intensity to decline steadily, too. In the CIA model sales are equal to consumption which as can be seen in Figure 3b keeps constant in period 0 since employment and hence production are predetermined.[24] Together with the decrease of the value of money this implies that sales revenues decrease by 1%. Hence employment is even less profitable in the CIA model than in the Goods Search model. Figures 3c and 3g show how the difference in sales revenues and profitability of employment across models

[23] A shock with no persistence would not affect employment at all. But for any $\rho = \varepsilon$, ε small and positive, e.g. $\varepsilon = 10^{-10}$, we find that the employment reaction of the CIA model is twice as high as the one of the search-theoretical counterpart.

[24] Note, that in the CIA model without labor search consumption would decline on impact. The reason is that agents reduce their real money balances when confronted with expected inflation. The CIA constraint implies then a reduction in consumption, too.

is reflected in the respective drop of vacancies. Vacancies drop less in the S_L model and hence employment in next period declines less in this model, too. Finally note, that a decline in employment reduces production. In the C_L model goods market clearing implies that consumption has to be equal to production. So the consumption response is proportional to the employment response, while in the S_L model consumption is proportional to sales. Qualitatively, the above statements hold for all monetary shocks with low persistence. For values of $\rho > 0.3$ the expected inflation effect starts getting stronger. It dominates the search enhancing effect for highly persistent shocks. Since there is no expected inflation effect with respect to the impact response of the value of money in the CIA model, at some value of ρ the employment responses of the two models have the same magnitude. For higher values of ρ the negative employment response in the CIA model becomes smaller than in the goods search model.

To understand the role of **Inventory Holdings (I)** we compare the "goods- and labor-search model" (S_L) with the complete model (SIL). Thus, the only difference is the possibility of storing goods as inventories. For this purpose consider Figure 4 that shows impulse responses after an impulse shock ($\rho = 0$). Let the period where the shock arises again be period 0. Figure 4a indicates that the search intensity rises and the shadow value of money falls on impact but they reach their respective steady state values already 1 period after the shock. So there is no persistent effect on search intensity. In other words the search-enhancing effect on impact is not reinforced by any feedback mechanism. Figure 4b shows the quantitative impact of the higher search intensity and the lower value of money on sales and sales revenues. The former increase substantially, but not enough to compensate the loss in the value of money. Thus, changes in sales revenues become negative on impact. But since sales revenues reach the steady state already in period 1, and only future sales revenues matter for hiring, there is no impact on vacancies and hence on employment, as can be seen in Figures 4c and 4d. In the complete model where firms can hold final goods inventories we have a different situation. Figure 4e shows that after a monetary shock there is an increase in search intensity that is slightly larger on impact than in the model without inventories. The implied increase in sales reduces inventories held at the beginning of period 1 as can be seen in Figure 4f.

Define inventory investment in period t as $\Delta_{it} \equiv i_{t+1} - (1 - \delta_i)\, i_{t.}$ Then $\Delta_{i0} = i_0 - (1 - \delta_i)\, i^*$. Figure 4f indicates that inventory investment drops in period 0 by 75%. That means that the unsold goods account for only one quarter of $\delta_i i^*$,[25] the necessary investment to maintain the steady state inventory level. So, if next period's production level were to be held constant there would be less supply of goods. In the model with inventories firms would like to increase production to refill their inventories. Because of costly labor search this is optimally done in a smooth way. Hence the supply of goods is below steady state in period 1. Other things being equal this would reduce sales. However, the reduced supply of goods induces buyers to search more intensively in period 1, too. The reason is that the consumption smoothing motive due to risk aversion implies that buyers' search intensity is a negative function of the quantity exchanged. The higher search intensity results in a further increase of the probability of successful matches and implies that sales drop by less than they would have dropped with steady state search intensity. This in turn reduces inventories for period 2. This feedback mechanism leads to the persistence in the response of search intensity and hence in sales. Sales revenues are affected furthermore by changes in the value of money shown in Figure 4e. The latter are caused by two effects. First, there is a negative effect of a monetary shock. Second, a higher search intensity in subsequent periods increases the value of relaxing the buyers' cash constraint by holding an additional unit of money. Since the former effect on the shadow value of money is not persistent, from period 1 onward the second effect dominates and the shadow value of money stays for several periods above steady state. As can be seen in Figure 4f, sales revenues are positive already in period 1 and hence make inventory investment profitable. Thus, although sales revenues drop in period 0, firms hire more workers in period 1 to fill up inventories. Now we can see the key difference between the two models: if goods are not storable as it is the case in the S_L model there is no effect on the goods supply in period 1, since unsold goods fully depreciate and the goods supply in period 1 is equal to production. Hence there is no need to search harder in subsequent periods and the search enhancing effect arises only on impact. We can conclude that the possibility of holding inventory is necessary for the persistent propagation of monetary shocks.

[25] To see this, note that the percentage deviation from steady state of Δi_0 is given by $\frac{\Delta i_0 - \Delta i^*}{\Delta i^*} = \frac{i_0 - i^* + \delta_i i^* - \delta_i i^*}{\delta i^*} = -0.75$. This implies $i_0 = i^* - 0.75\delta_i i^*$ and hence $\Delta i_0 \equiv i_0 - (1 - \delta_i i^*) = 0.25\delta_i i^*$.

To find the additional role of **Labor Search (L)** in the monetary propagation mechanism the complete model of Shi[22] (SIL) is compared to two versions of the "goods-search and inventory" (SI) model, again with $\rho = 0$. Remember that this model differs from Shi's[22] model only by the absence of the labor search assumption. Consider first the version where employment is predetermined. This assumption ensures that a negative impact on sales revenues due to a feedback of "search-enhancing effect" and "inventory effect" as contemplated above cannot lead to a reduction of employment and production in period 0. Figures 5a-5h show the respective impulse responses for the SI model with predetermined labor and the SIL model. Comparing Figures 5a and 5b with Figures 5e and 5f we see that the shapes of the impulse responses of the value of money, the search intensity, inventory investment, sales and sales revenues are qualitatively similar. Hence the above stated feedback effect is present also in the model without labor search. Quantitatively speaking, among these variables only the volatility of inventory investment differs substantially across models being larger in the SI model. In addition the persistence of the responses is smaller in the SI model: they reach their steady states about 4-5 periods earlier. So, labor search adds 1 year of persistence to the reaction of those variables. The reason is the following: In the paragraph above we saw that the feedback mechanism between a "search enhancing effect" and an "inventory effect" induces firms to hire more workers in period 1. However, if there is no friction in the labor market and employment can adjust immediately inventories can be refilled more quickly. Ss can be seen comparing Figures 5b and 5f, inventory investment in the SI model is larger in subsequent periods until the steady state is reached and the excess demand is smaller. This, in turn, implies that the search intensity from period 1 onward is smaller than in the model with labor search. Hence, the feedback is weaker and the economy shows less persistent responses.

With respect to employment we see a slightly different picture. Figure 5d shows that the model without labor search is able to generate an employment response that stays 8 quarters above steady state. Comparing this with Figure 5h we find that labor-search prolongs the persistence by more than 8 quarters and in addition has a smoothing effect on employment. The peak in the employment reaction is more than ten times lower in the model with labor search and is reached later. This results in a hump shape employment response. Note also that the overall effect on employment is half as large as in the model with Walrasian labor markets.

Finally, labor search also implies that employment is predetermined. To isolate the effect of this implication Figures 5i-5m show the corresponding responses of the version of the SI model where firms can decide their employment level in the current period. The only difference with respect to the model with predetermined employment is the negative impact reaction of employment and therefore a higher decrease of inventories and less sales. In the following periods the households optimal decisions are the same as in the model with predetermined labor except for inventory investment which adjusts endogenously to the implied optimal changes in production and quantity exchanged. Thus, except for inventory investment the responses from period 1 onward are identical in both model versions. Therefore, the predeterminedness of employment decreases volatility in inventories, sales revenue and employment by the elimination of negative impacts of a monetary shock to current employment, but increases the volatility of sales.

5 Discussion

The results show that good-search together with inventory holding alone can generate a substantial propagation mechanism of monetary policy and that both elements are essential. However, only by introducing labor search one receives a highly persistent and hump-shaped employment response. The above analysis suggests that if one is interested in having a model featuring high persistence, the complete model should be used. In order to see the direction of the economy's reaction after a money shock, however, it could be sufficient to use the model with goods-search and inventories and predetermined labor as an approximation of the more complex labor search model. Changing the timing of the model to a beginning-of-period shock and considering an autocorrelated monetary shock with an AR(1) parameter $\rho = 0.25$ the model without labor search features roughly the same overall impact on employment as the complete model. However, instead of being hump-shaped the response of employment shows a sharp peak after 3 quarters followed by a steady decline and the steady state is reached 4 quarters earlier. Unfortunately, there seems to be no way of getting a hump-shaped employment response without adding some kind of labor market frictions.

An interesting extension is to introduce capital accumulation to study the joint effects of those propagation mechanisms and to see the models implications for consumption, investment and output.[26] In a companion paper, Menner[16], I show that with moderate capital adjustment costs the search-theoretic feedback mechanism survives and the model‘s implications are very similar to the model without capital. Moreover, the model can replicate many US business cycle stylized facts quite well, especially with respect to sales and inventories. For future research I plan to estimate the search-theoretic model using Bayesian techniques. This will reveal in which dimensions the model's performance differs from the performance of standard CIA and limited participation models[27] and allows to judge whether it is an appealing alternative as a laboratory for monetary research and policy evaluation.

Unfortunately, in these search-theoretic models one cannot address questions of monetary policy as they are usually formulated, since there are no markets in which a central bank could conduct open-market operations. Up to now it is only possible to study the effect of helicopter dropping of money. As Wallace[28] pointed out, money and perfect credit markets cannot coexist in a model where money is essential since imperfect commitment and imperfect memory inhibit credit. And with the lack of perfect commitment and memory one cannot appeal to the usual equivalence of open-market operations and lump-sum monetary transfers when there are perfect credit markets. Hence, the explicit introduction of bank credit and markets for central bank money is a prerequisite for the examination of central bank policy. By extending this model to include a market where the central bank can conduct open-market operations, one could study the effects of monetary policy on the behavior of the agents in the economy with regard to the holding and use of fiat money or other means of transaction. Or in other words, the examination of monetary policy would have a micro-foundation where 'money' is the asset that actually serves as a means of transaction.[28]

[26] Shi[22] studies the effects of search in a neoclassical growth model but does not examine business cycle fluctuations, neither considers inventory holdings.

[27] See Nason and Cogley[17] for a comparison of predicted fluctuations of various equilibrium monetary business cycle models with sample fluctuations of the U.S. business cycle.

[28] See the recent work of Faig[6],[5] who advances in this direction using the village interpretation mentioned in footnote 11. His model has no inventories and he only does comparative statics analysis.

A. Existence Proofs

A.1. Goods Search and Inventory Model

Steady state consumption is given by: $c^* = a_b Bzs^{*\alpha} q^*$. With this, the FOC with respect to the search intensity and the dynamic equations 34-36 imply the following steady state relations:

$$c^* = a_b Bzs^{*\alpha} \frac{f(n^*)}{1-(1-d_i)(1-Bzs^{*\alpha})}, \tag{65}$$

$$s^* \Phi'(s^*) = \frac{c^*}{a_b B} \left[U'(c^*) - \omega^*\right], \tag{66}$$

$$\varphi = \frac{e_f}{{n^*}^{1-e_f}} \frac{Bzs^{*\alpha}\omega^*}{(1-\beta(1-\delta_i)(1-Bzs^{*\alpha}))}, \tag{67}$$

$$zs^{*\alpha} = \frac{\gamma-\beta}{\beta} \frac{\omega^*}{U'(c^*)-\omega^*}, \tag{68}$$

$$q^* = \frac{f(n^*)}{1-(1-d_i)(1-Bzs^{*\alpha})}, \tag{69}$$

$$\omega_i^* = \beta \frac{\varphi {n^*}^{1-e_f}}{e_f} = \frac{\beta Bzs^{*\alpha}\omega^*}{(1-\beta(1-\delta_i)(1-Bzs^{*\alpha}))}. \tag{70}$$

Equations (69) - (70) give (q^*, ω_i^*) as functions of $(c^*, s^*, n^*, \omega^*)$. Further, (65) - (68) involve only $(c^*, s^*, n^*, \omega^*)$ and can be solved in two blocks. First, substitute (65) into (66) to solve for $s^* = s(n^*, \omega^*)$. Under the assumption of a risk aversion parameter $RA \geq 1$, the partial derivatives denoted by subscripts are $s_\omega < 0$ and $s_n < 0$. This implies that c^* can be expressed as $c^* = c(n^*, \omega^*)$, with $c_\omega < 0$ and $c_n > 0$. Denote $a_b B s^* \Phi'(s^*)$ by ξ, then $\xi_\omega < 0$ and from (66) c_ω is found to be:

$$c_\omega = \frac{c^* U''(c^*) + U'(c^*) - \omega^*}{\xi_\omega + c^*} > \frac{c^* U''(c^*) + U'(c^*) - \omega^*}{c^*},$$

which in turn implies that $U'(c(n^*, \omega^*))/\omega^*$ is a decreasing function of ω^*. Substituting $s(n^*, \omega^*)$ and $c^* = c(n^*, \omega^*)$ into (67) and (68) gives:

$$zs(n^*, \omega^*)^\alpha \left(\frac{U'(c(n^*, \omega^*))}{\omega^*} - 1\right) = \frac{\gamma}{\beta} - 1, \tag{71}$$

$$0=\frac{Bzs\left(n^{*},\omega^{*}\right)^{\alpha}\omega^{*}e_{f}n^{*^{e_{f}-1}}}{\left(1-\beta\left(1-\delta_{i}\right)\left(1-Bzs\left(n^{*},\omega^{*}\right)^{\alpha}\right)\right)}-\varphi\equiv F\left(n^{*},\omega^{*}\right). \tag{72}$$

These two equations correspond to equations (3.4) and (3.5) in Shi[22], p. 329ff. The former is identical to (71), the latter equates $F\left(n^{*},\omega^{*}\right)$ as defined in (72) to $k(v(n^{*}))\frac{1-\beta(1-\delta_{n})}{\sigma\beta}$. Setting this expression to zero does not alter the proof in Shi[22]. It works exactly alike with this modification.

Finally it remains to check that these steady state values are consistent with the assumptions that $\lambda>0$ and $\omega_{q}>0:$ To see that $\omega_{q}^{*}>0$ remember that the bargaining solution determined ω_{q} as $\omega_{q}=\omega-\left(1-\delta_{i}\right)\omega_{i}$ and notice that (70) implies $\omega^{*}>\omega_{i}^{*}$. To verify $\lambda>0$ it again suffices to check that in the steady state the nominal interest rate R^{*} is positive: $R^{*}=\omega_{M}/\beta\omega_{M^{\prime}}-1=g_{b}(s^{*})s^{*}\lambda/\omega^{*}=g_{b}(s^{*})s^{*}(U^{\prime}\left(c^{*}\right)-\omega^{*})/\omega^{*}=\frac{\gamma-\beta}{\beta}>0$ iff. $\gamma>\beta$. Thus for all monetary growth rates higher than the 'Friedman rule' the considered steady state exists.

A.2 Goods- and Labor-Search Model

The steady state is given by the following relations:

$$v^{*}\mu\left(v^{*}\right)=\delta_{n}n^{*}, \tag{73}$$

$$q^{*}=f(n^{*})=n^{*^{e_{f}}}, \tag{74}$$

$$c^{*}=a_{b}Bzs^{*^{\alpha}}f(n^{*}), \tag{75}$$

$$s^{*}\Phi^{\prime}\left(s^{*}\right)=\frac{c^{*}}{a_{b}B}\left[U^{\prime}\left(c^{*}\right)-\omega^{*}\right], \tag{76}$$

$$zs^{*^{\alpha}}=\frac{\gamma-\beta}{\beta}\frac{\omega^{*}}{U^{\prime}\left(c^{*}\right)-\omega^{*}}, \tag{77}$$

$$k\left(v^{*}\right)\frac{\left(1-\beta\left(1-\delta_{n}\right)\right)}{\sigma\beta}=Bzs^{*^{\alpha}}\omega^{*}f^{\prime}\left(n^{*}\right)-\varphi. \tag{78}$$

Expressions (73) and (74) give v^{*} and q^{*}as increasing functions of n^{*}. Hence, (75)-(78) involve only $(c^{*},s^{*},\omega^{*},n^{*})$. Now it is possible to further reduce the system to one equation. First, substituting (75) into (76) gives:

$$s^{*^{1-\alpha}}\Phi^{\prime}\left(s^{*}\right)=zn^{*^{e_{f}}}\left[U^{\prime}\left(a_{p}Bzs^{*^{\alpha}}n^{*^{e_{f}}}\right)-\omega^{*}\right]. \tag{79}$$

This defines s as a function of n^* and ω^*: $s^* = s(\omega^*, n^*)$. Under the assumption $RA \geq 1$ this function is decreasing in both arguments: $s_{\omega^*} < 0$ and $s_{n^*} < 0$. With this one can show that $c^* = c(\omega^*, n^*)$, with $c_{\omega^*} < 0$ and $c_{n^*} > 0$. Equations (77) and (78) can then be rewritten as:

$$zs(\omega^*, n^*)^{\alpha} \left(\frac{U'\left(c(\omega^*, n^*)\right)}{\omega^*} - 1 \right) = \frac{\gamma}{\beta} - 1, \tag{80}$$

$$k\left(v^*\right) \frac{\left(1 - \beta\left(1 - \delta_n\right)\right)}{\sigma\beta} = Bzs(\omega^*, n^*)^{\alpha} \omega^* f'\left(n^*\right) - \varphi \equiv F_B\left(\omega^*, n^*\right). \tag{81}$$

Denote the relation between ω^* and n^* described in (80) by $n^* = n1(\omega^*)$, and the other given through (81) by $n^* = n2(\omega^*)$. The steady state value ω^* is then the one that solves $n1(\omega^*) = n2(\omega^*)$.

Similar to the existence proof in Appendix A.1, it is true that $U'\left(c(\omega^*, n^*)\right)/\omega^*$ is a decreasing function of ω^*. Now the proof of existence continues as in Shi [22], p.330, using (81) instead of (3.5) with one exception: to show that $n2(0) = 0$ we have to deviate from the first part of the proof of his LEMMA 3.3 which can be found in his Appendix A.2:
To show $n2(0) = 0$, note that since $s_{n^*} < 0$, it holds that

$$s(\omega, \bar{N}\left(\omega\right)) \leq s(\omega, n2\left(\omega\right)), \qquad \forall\ \bar{N}\left(\omega\right) \leq n2\left(\omega\right).$$

For any ω, fix a $\bar{N}\left(\omega\right) \leq n2\left(\omega\right)$. Since $s_{\omega^*} < 0$ it is also true that $s(\omega, \bar{N}\left(\omega\right)) \leq s(0, \bar{N}\left(\omega\right))$. Then

$$\begin{aligned} f'\left(n2\left(\omega\right)\right) &= \frac{\varphi + k\left(v\left(n2\left(\omega\right)\right)\right)\frac{(1-\beta(1-\delta_n))}{\sigma\beta}}{Bzs(\omega, n2\left(\omega\right))^{\alpha}\omega} \geq \frac{\varphi + k\left(v\left(\bar{N}\left(\omega\right)\right)\right)\frac{(1-\beta(1-\delta_n))}{\sigma\beta}}{Bzs(\omega, \bar{N}\left(\omega\right))^{\alpha}\omega} \\ &\geq \frac{\varphi + k\left(v\left(\bar{N}\left(\omega\right)\right)\right)\frac{(1-\beta(1-\delta_n))}{\sigma\beta}}{Bzs(0, \bar{N}\left(\omega\right))^{\alpha}\omega}. \end{aligned}$$

Taking the limit $\omega \to 0$, of both sides of the resulting inequality one gets:

$$\lim_{\omega \to 0} f'\left(n2\left(\omega\right)\right) = \lim_{\omega \to 0} \left\{ \frac{\varphi}{Bzs(0, \bar{N}\left(\omega\right))^{\alpha}} \frac{1}{\omega} \right\} = \frac{\varphi}{Bz} \cdot \frac{1}{0} \cdot \frac{1}{0} = \infty.$$

The characteristics of the production function thus imply $n2\left(0\right) = 0$. The rest of the proof of LEMMA 3.3 in Shi[22] applies also for this model.

To see that these steady state values are consistent with the assumptions that $\lambda > 0$ and $\omega_q > 0$ it suffices to check, that in the steady state the nominal interest rate R^* and the shadow value of money ω^* are positive: Expression (81) implies that $\omega^* > 0$. The nominal interest rate is determined as before as $\frac{\gamma-\beta}{\beta}$. Thus, again,for all monetary growth rates higher then the Friedman rule the considered steady state exists.

A.3 CIA Model with Labor-Search

The steady state is given by the following equations:

$$W^* = \frac{\sigma\varphi}{\omega^*} + (1-\sigma) f'(n^*), \tag{82}$$

$$v^* \mu(v^*) = \delta_n n^*, \tag{83}$$

$$c^* = a_p f(n^*), \tag{84}$$

$$\frac{U'(c(n^*))}{\omega^*} = \frac{\gamma}{\beta}, \tag{85}$$

$$\frac{k(v(n^*))(1-\beta(1-\delta_n))}{\sigma\beta} = \omega^* f'(n^*) - \varphi \equiv F_c(n^*, \omega^*). \tag{86}$$

Since c^* does not depend on ω^*, $\frac{U'(c(n^*))}{\omega^*}$ is a decreasing function in ω^*. Expressions (85) and (86) are the equivalents to equations (3.4) and (3.5) in the existence proof in Shi. They give again relations between $n^*$and ω^*, denoted in analogy to Shi as $n1_c(\omega^*)$ and $n2_c(\omega^*)$. We now examine the properties of the two curves. First, we proof that the properties shown in Lemma 3.2 in Shi[22] for $n1(\omega^*)$ hold as well for $n1_c(\omega^*)$: The left hand side of equation (85) is also a decreasing function in ω^*. Thus it also holds that $n1'_c(\omega^*) < 0$. Next it is to show that as well $n1_c(0) = \infty$ and $n1_c(\infty) = 0$. From (85) it follows that $U'(c(n1_c(0))) = \frac{\gamma}{\beta} \cdot 0 = 0$. The assumptions on the utility function, however, imply that $U'(c(n1_c(\omega^*))) \to 0$ as $\omega^* \to 0$ only if $c(n1_c(\omega^*)) \to \infty$, which in turn only happens if $n1_c(\omega^*) \to \infty$. A similar argument holds for $n1_c(\infty) = 0$.

In contrast to $n2(\omega^*)$, $n2_c(\omega^*)$ is monotonically increasing. To show $n2_c(0) = 0$, note that (86) implies

$$f'(n2(\omega^*)) = \frac{1}{\omega^*}\left[\varphi + \frac{k(v(n2(\omega^*)))(1-\beta(1-\delta_n))}{\sigma\beta}\right] > \frac{\varphi}{\omega^*}.$$

Hence, $f'(n2(0)) = \infty$ and therefore $n2(0) = 0$. These properties of $n1_c(\omega^*)$ and $n2_c(\omega^*)$ imply a unique intersection n^*.

Finally one has to check that this intersection is consistent with the assumption that the CIA is non-binding, that means $\lambda > 0$, which is implied by a money growth rate higher than the Friedman rule. Choose a arbitrarily small $\Delta > 0$. The solution must then satisfy $U'(c(n^*)) \geq \omega^* + \Delta$ in order to ensure $\lambda > 0$. That means n^* has to be smaller or equal to a number $N(\omega^*, \Delta)$ which is defined by $U'(c(N(\omega^*, \Delta))) = \omega^* + \Delta$.

Consider first the intersection between $n1_c(\omega^*)$ and $N(\omega^*, \Delta)$: By the definition of $N(\omega^*, \Delta)$ it follows $U'(c) = \omega + \Delta$ which leads to $c = U'^{-1}(\omega + \Delta)$. Expression (85) which defines $n1_c(\omega^*)$ implies:

$$\begin{aligned} \frac{\omega + \Delta}{\omega} &= \frac{\gamma}{\beta}, \\ \Delta &= \frac{\gamma - \beta}{\beta}\omega. \end{aligned}$$

Thus $n1_c(\omega^*)$ and $N(\omega^*, \Delta)$ intersect at:

$$\omega 1(\Delta) = \frac{\beta}{\gamma - \beta}\Delta, \qquad \omega 1_\Delta > 0.$$

For $\gamma > \beta$ it holds that:

$$\lim_{\Delta \to 0} \omega 1(\Delta) = 0.$$

Now look at the intersection between $n2_c(\omega^*)$ and $N(\omega^*, \Delta)$:
The latter implies $\omega = U'(c(n^*)) - \Delta$, and expression (84) gives:

$$n(c) \equiv f^{-1}\left(\frac{c}{a_p}\right), n_c > 0,$$

which can be substituted into (86):

$$\frac{1}{f'(n(c))}\left[\varphi + \frac{k(v(n(c)))(1 - \beta(1 - \delta_n))}{\sigma\beta}\right] = [U'(c)) - \Delta]. \qquad (87)$$

The left hand side is an increasing function of c, the right hand side is decreasing in c. Furthermore LHS(87)$|_{c=0}$= $0 < \infty =$ RHS(87)$|_{c=0}$ and LHS(87)$|_{c=\infty}$= $\infty > 0 =$ RHS(87)$|_{c=\infty}$. Thus the solution for c is unique and independent of Δ. Hence $\omega 2(\Delta) = U'(c) - \Delta$ and $\omega 2(0) = U'(c)$. That means that $n2_c(\omega)$ intersects $N(\omega, 0)$ at $\omega 2(0) = U'(c)$.

Putting things together we have that $\omega 1\left(0\right) = 0 < U'(a_p f(n^*)) = \omega 2\left(0\right).$ Since $\omega 1\left(\Delta\right)$ is increasing in Δ and $\omega 2\left(\Delta\right)$ is decreasing in Δ, there exists a $\Delta^* > 0$ such that $\omega 1\left(\Delta^*\right) = \omega 2\left(\Delta^*\right)$. Therefore, $n^* = N(\omega^*, \Delta^*)$ and $\lambda > 0$. Thus, the equilibrium is consistent with the assumed money growth rate.

B. Calibration

Since the calibration of the complete search-theoretic monetary model is documented in Shi's[22] I do not replicate it here. For the other models some modifications of his calibration strategy are necessary. In the following subsections I describe the choice of the parameter values and the implied steady state values of the variables. All exogenously chosen values are the same as in Shi[22].

B.1. Goods Search and Inventory Model

Interpreting the length of a period as a quarter, the value for the discount factor β of 0.99 implies an annual real interest rate of four percent. The steady state growth rate of money is set to $\gamma^* = 1.012$ to match the average quarterly inflation rate in postwar US data. See the text for the specification of the AR(1) coefficient of the shock to the money growth rate.

The steady state labor force is normalized to $n^* = 100$. To specify the cardinality of the set of producers a_p, Shi[22] sets the active labor force $a_p(n^*+1)$ to 0.7.[29] The unemployment rate $u/(a_p(n^*+1)+u)$ is set to 0.06. This leads to $u = 0.0447$ and implies a value for a_p of 0.0069.

The elasticity in the matching function of buyers and sellers α is arbitrarily set to 0.8. The buyers/sellers ratio B and the goods search matching parameter z are determined jointly with the inventory depreciation rate δ_i. The steady state inventory/output ratio and the inventory investment/output ratio are set to their quarterly averages in postwar US data, that is, $i^*/f\left(n^*\right) = 0.9$, $\delta_i i^*/f\left(n^*\right) = 0.0065$. Thus, $\delta_i = 0.0065/0.9 = 0.0072$. Setting the quarterly income velocity of money to 1, one gets a relation determining B:

$$\frac{a_p f\left(n^*\right) P}{M} = \frac{a_p f\left(n^*\right)}{a_b q^*} = \frac{a_p f\left(n^*\right)}{a_b (f\left(n^*\right) + i^*)} = \frac{a_p}{a_b}\frac{1}{1.9} = \frac{1}{1.9 \cdot B} = 1.$$

[29] Note, that in the paper Shi wrote that he set the labor participation rate $a_p(n^*+1)+u$ to 0.7, but inspection of his program revealed that he used the active labor force instead.

Thus, $B = 1/1.9 = 0.5263$. With these values we can calculate z and hence q^* :

$$z = \frac{B - \delta_i}{(1 - \delta_i) B s^{*^\alpha}},$$

$$q^* = \frac{f(n^*)}{1 - (1 - \delta_i)(1 - Bzs^{*^\alpha})}.$$

Shi[22] sets shopping time as 11.16% of working time and the latter as 30% of agents discretionary time. Hence, $s^* = 0.1116 * 0.3(n^* + 1)/B = 6.4248$ and z is determined.[30] Assuming a labor income share $LIS = W^* n^* / f(n^*)$ of 0.64, the value of the production parameter e_f can be computed from (67) and (28):

$$\varphi = W^* \omega^* = n^{*^{e_f - 1}} LIS \omega^* = \frac{e_f}{n^{*^{1-e_f}}} \frac{Bzs^{*\alpha}\omega^*}{(1 - \beta(1 - \delta_i)(1 - Bzs^{*\alpha}))},$$

$$e_f = LIS \frac{1 - \beta(1 - \delta i) * (1 - Bzs^{*\alpha})}{Bzs^{*\alpha}} = 0.64088 \ .$$

This differs slightly from the value of $e_f = 0.6804$ for the model with labor search.

In the absence of capital formation and investment the implied consumption/output ratio is not realistic. To incorporate expenditure on fixed investment, assume that fixed investment is a constant fraction, FI_k, of aggregate sales. Since the latter are exogenous to the households this is just a subtraction of a lump sum from each household's consumption. Thus the optimal choices remain the same and the equilibrium conditions continue to hold with the modification that $c_t = (1 - FI_k) a_b Bzs_t^\alpha q_t$. Shi[22] sets the steady state expenditure on fixed investment to 26.9% of output: $FI_k Bzs^{*^\alpha} q^* = 0.269 f(n^*)$, which defines FI_k.

The parameter φ is given by (67):

$$\varphi = \frac{e_f}{\beta n^{*1-e_f}} \frac{\beta Bzs^{*\alpha}\omega^*}{(1 - \beta(1 - \delta_i)(1 - Bzs^{*\alpha}))},$$

once ω^* is determined. Combining (68) and (69) leads to

$$(c^*)^{-2} zs^{*\alpha} = \left(zs^{*\alpha} + \frac{\gamma - \beta}{\beta} \right) \omega^*,$$

[30]Note, that in Shi[22] the fraction of shopping time to working time is misreported as 11.6%. Moreover, B times the value of z is incorrectly reported to be the value of z.

and thus:

$$\omega^* = \frac{zs^{*\alpha}}{c^{*2}\left(\frac{\gamma-\beta}{\beta} + zs^{*\alpha}\right)}.$$

Finally it remains to specify the disutility of search: Shi[22] uses $\Phi(s) = \varphi(\varphi_0 s)^{1+1/e_\Phi}$ where φ_0 is the efficiency of a buyer's search intensity relative to a worker's time. Shi[22] assumes that shopping has a higher elasticity than labor supply and sets $e_\Phi = 2$. Plugging (68) and (69) into (66) one gets:

$$\varphi_0 = \left(\frac{\gamma-\beta}{\beta} \frac{f(n^*)\omega^*}{(1-(1-d_i)(1-Bzs^{*\alpha}))\,\varphi s^{1+1/e_\Phi}(1+1/e_\Phi)}\right)^{\frac{e_\Phi}{e_\Phi+1}}.$$

B.2. Goods- and Labor Search Model

In addition to the parameters in the inventory model, the job separation rate δ_n and the exponent of the labor matching function A have to be specified. I use Shi's[22] value of 0.06 and 0.6, respectively. The values for u and a_p remain unchanged.

To determine B and z I have to depart once again from Shi's[22] calibration procedure, since there are no inventories. In line with Shi[22] the quarterly income velocity of money is set to 1:

$$\frac{a_p f^* P}{M} = \frac{a_p f^*}{a_b q^*} = \frac{a_p f^*}{a_b f^*} = \frac{a_p}{a_b} = 1.$$

Thus, $B = a_b/a_p = 1$. Again, shopping time is assumed to be 11.16% of working time and the latter to be 30% of agents discretionary time. Thus, $s^* = 0.1116 * 0.3(n^*+1)/B = 6.4248$. Setting the labor income share to 0.64, i.e. $W^* n^*/f = 0.64$, it follows: $W^* = 0.64 f/n^*$. In addition assume that the hiring cost is 2% of the labor cost: $K_0 v^{*^2}/(\omega^* W^* n^*) = 0.02$. Expression (51) implies:

$$W^* = \sigma\frac{\varphi}{\omega^*} + (1-\sigma)\, zBs^{*^\alpha} e_f n^{*^{(e_f-1)}},$$

which leads to a value for φ of

$$\varphi = \left(\frac{0.64}{e_f} - (1-\sigma)\, zBs^{*^\alpha}\right)\frac{f'(n^*)\omega^*}{\sigma}, \tag{88}$$

to be plugged in (78) :

$$2 * 0.02 \frac{0.64}{e_f} n^* \left(\frac{a_p}{u}\right)^{A-1} (v^*)^{A-2} \frac{(1-\beta(1-\delta_n))}{\beta} = Bz{s^*}^{\alpha} - \frac{0.64}{e_f}.$$

Hence, z is determined by

$$z = \left(0.04 n^* \left(\frac{a_p}{u}\right)^{A-1} (v^*)^{A-2} \frac{(1-\beta(1-\delta_n))}{\beta} + 1\right) \frac{0.64}{e_f B{s^*}^{\alpha}}, \tag{89}$$

once e_f and v^* are calibrated. Equation (73) implies with the choice of $A = 0.6$ a value for v^* of:

$$v^* = 6^{\frac{1}{0.6}} \left(\frac{0.7}{101 * 0.442}\right)^{\frac{0.4}{0.6}} = 1.2411 \ .$$

The coefficient e_f is now a free parameter. It is chosen in each model comparison to match the value of e_f in the compared model.

The fraction of fixed capital investment relative to sales is now: $FI_k = 0.269 f/(Bz{s^*}^{\alpha} q) = 0.269/Bz{s^*}^{\alpha}$ and this implies steady state consumption of $c^* = (1 - FI_k) a_b B z s_t^{\alpha} q_t$. The steady state value of ω can be calculated by (77) and assuming again a utility function of the CRRA-type with a relative risk aversion of $RA = 2$. Therefore $U'(c) = c^{-2}$, and one gets:

$$\omega^* = \frac{z{s^*}^{\alpha}}{(z{s^*}^{\alpha} + \frac{\gamma-\beta}{\beta})(a_p^2 f^{*2}(zB{s^*}^{\alpha} - 0.269)^2)}.$$

K_0 is given by

$$K_0 = 0.02 * 0.64 f(n^*)\, \omega^* {v^*}^{-2},$$

and φ can be calculated by (88):

$$\varphi = \left(\frac{0.64}{e_f} - (1-\sigma)\, zB{s^*}^{\alpha}\right) \frac{f'(n^*)\omega^*}{\sigma}. \tag{90}$$

Finally one receives a value for φ_0 setting $e_\Phi = 2$ and plugging (77) into (76):

$$\varphi_0 = \frac{\gamma - \beta}{\beta} \frac{f(n^*)\omega^*}{(\varphi s^{1+1/e_\Phi}(1+1/e_\Phi))^{\frac{e_\Phi}{e_\Phi+1}}} \ .$$

B.3. CIA and Labor Search Model

Expression (83) implies a value for v^* of:

$$v^* = 6^{\frac{1}{0.6}} \left(\frac{0.7}{101 * 0.442} \right)^{\frac{0.4}{0.6}} = 1.2411 \ .$$

Substituting the steady state expression for φ, (82), into (86) and using again the formulas corresponding to the labor income share and to hiring costs one gets:

$$k(v^*)(1-\beta(1-\delta_n)) = \sigma\omega^* f'(n^*) + (1-\sigma)\omega^* f'(n^*) - W^*\omega^*,$$

$$2*0.02*0.64\frac{f(n^*)}{n^*}\omega^* n^* \left(\frac{a_p}{u}\right)^{A-1} (v^*)^{A-2} (1-\beta(1-\delta_n)) = (e_f - 0.64)\frac{f(n^*)}{n^*}\omega^*.$$

This implies:

$$e_f = 0.64 \left(0.04 n^* \left(\frac{a_p}{u}\right)^{A-1} (v^*)^{A-2} (1-\beta(1-\delta_n)) + 1 \right).$$

Now, φ and K_0 are determined by:

$$\varphi = (0.64 - (1-\sigma) e_f)\,\omega^* \frac{f(n^*)}{n^*\sigma}, \qquad \text{and} \qquad K_0 = 0.02 * 0.64 f(n^*)\,\omega^* {v^*}^{-2}.$$

Finally, $c^*$and ω^* are given by

$$c^* = (1 - FI_k) a_p f(n^*), \text{ and } \omega^* = \frac{\beta U'(c^*)}{\gamma}.$$

C. Tables

Table 1 Calibration to Intervals

Parameter	Point Calibration	Empirical Density
ρ_g	$\{0, 0.4, 0.85\}$	
σ_g	0.1078	
β	0.99	truncated $Normal(0.9926, 1)$ $[0.9855 - 1.001]$
γ	1.012	truncated $Normal(1.0120, 1)$ $[1.002 - 1.022]$
RA	2	truncated $\chi^2(4)$ $[0.25 - 3.25]$, mode at 2
e_Φ	2	uniform $[1 - 3]$
α	0.8	uniform $[0.5 - 1]$
σ	0.7	uniform $[0.5 - 1]$ *)
δ_n	0.06	uniform $[0.04 - 0.08]$
A	0.6	uniform $[0.4 - 0.8]$
%. of unemployed	0.06	uniform $[0.04 - 0.08]$
Labor participation rate	0.6	uniform $[0.55 - 0.65]$
Labor income share	0.64	uniform $[0.5 - 0.75]$ *)
Shopping time / Working time	0.11166456	uniform $[0.05 - 0.15]$ *)
Fraction of Working time	0.3	uniform $[0.25 - 0.35]$
Fixed investment/output	0.269	uniform $[0.26 - 0.28]$ *)
Inventories/output	0.9	uniform $[0.8 - 1]$
Inventory investment/output	0.0065	uniform $[0.006 - 0.007]$

'Point Calibration' refers to the calibrated values used by Shi [22]. 'Empirical Density' refers to constructed parameter distributions using either existing estimates or a-priori intervals as in Canova [3].

*) indicates that the distribution is not centered around Shi's [22] value. See Section 1.3.1 for a discussion.

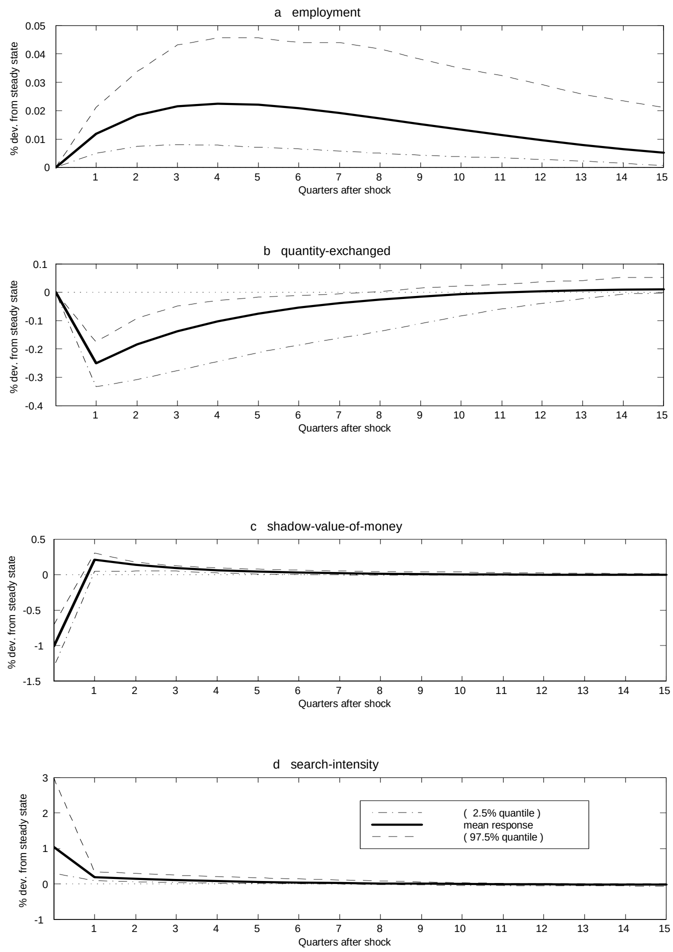
Table 2 Simulated Parameter Values

Parameter	Point Calibration	Mean	90% - Interval
β	0.99	0.9941	0.9870 – 1.0011
γ	1.012	1.0117	1.0032 – 1.0207
RA	2	1.9195	0.5262 – 3.1201
FI_k	0.2708	0.2716	0.2627 – 0.2808
B	0.5263	0.5279	0.5022 – 0.5535
z	0.2242	0.3017	0.1369 – 0.4579
δ_n	0.06	0.0599	0.0424 – 0.0786
A	0.6	0.6021	0.4255 – 0.7818
σ	0.7	0.7443	0.5318 – 0.9734
u	0.0447	0.0427	0.0310 – 0.0577
a_p	0.0069	0.0069	0.0065 – 0.0074
e_f	0.6804	0.7888	0.6698 – 0.9096
δ_i	0.0072	0.0073	0.0064 – 0.0083
φ	10.603	17.3748	0.7420 – 80.851
φ_0	0.4332	0.4033	0.1498 – 0.7889
e_Φ	2	1.9545	1.1247 – 2.7867
α	0.8	0.7501	0.5174 – 0.9720
K_0	0.6615	1.5431	0.0315 – 5.9362

Result of the calibration to intervals: Parameter values obtained by N = 250 random draws from the empirical distributions as specified in Table 1.

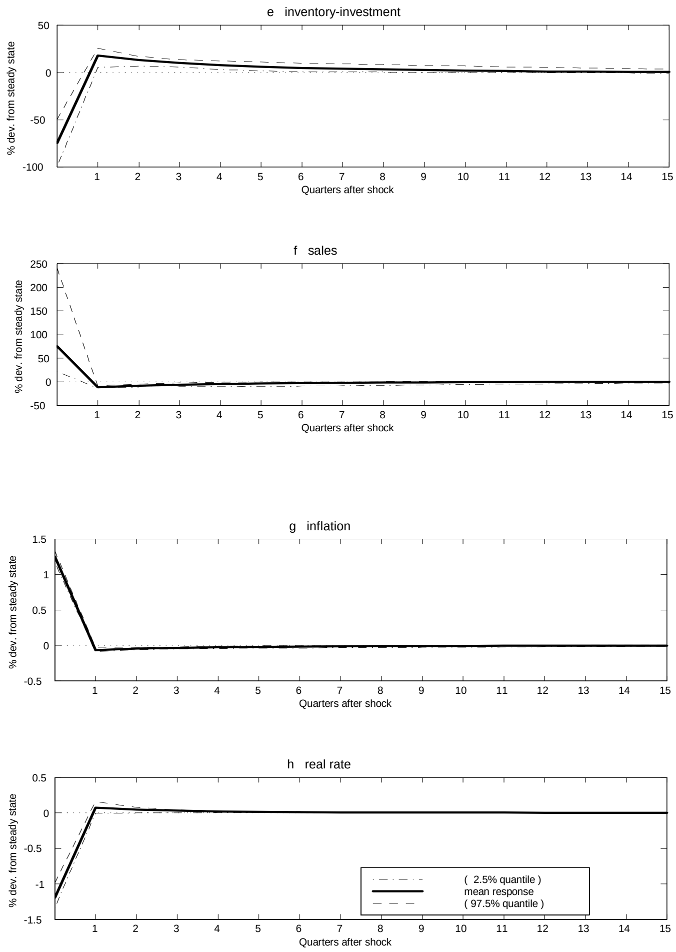
Figures Sensitivity Analysis

Figure 1 : Pointwise Error Bands: SIL Model



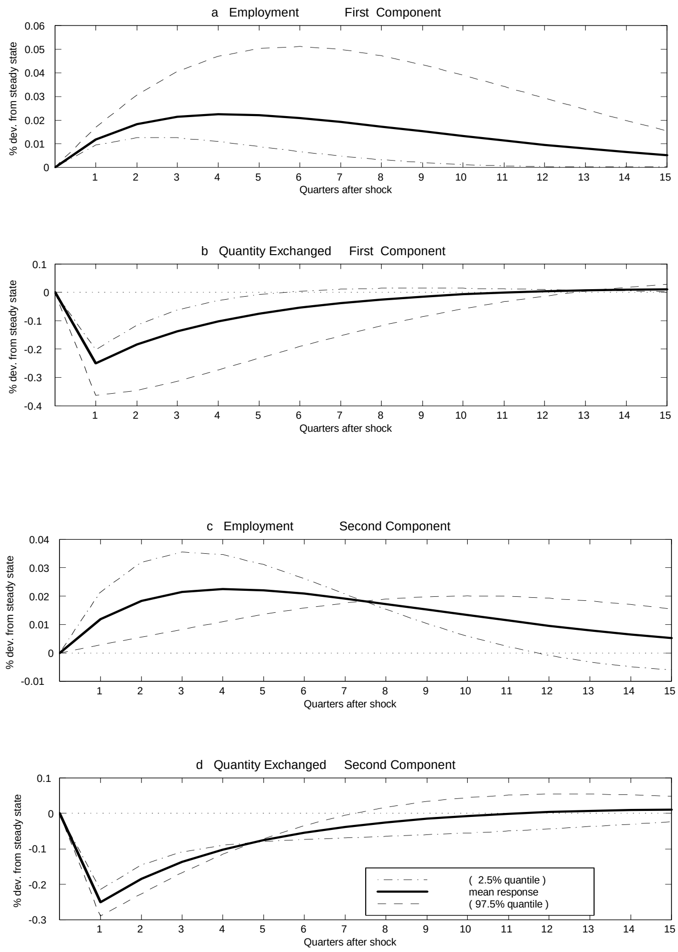
Pointwise error bands are the 95% error bands of the impulse responses for each point in time after the monetary shock. See Section 1.3.2 for a detailled explanation. SIL stands for the complete model with the elements Goods-Search (S), Inventories (I), and Labor-Search (L). The autocorrelation of the monetary shock is $\rho = 0$. Number of parameter draws: 250.

Figure 1 : continued



Pointwise error bands are the 95% error bands of the impulse responses for each point in time after the monetary shock. See Section 1.3.2 for a detailled explanation. SIL stands for the complete model with the elements Goods-Search (S), Inventories (I), and Labor-Search (L). The autocorrelation of the monetary shock is $\rho = 0$. Number of parameter draws: 250.

Figure 2 : Probabilitiy Bands of Principle Components: SIL Model

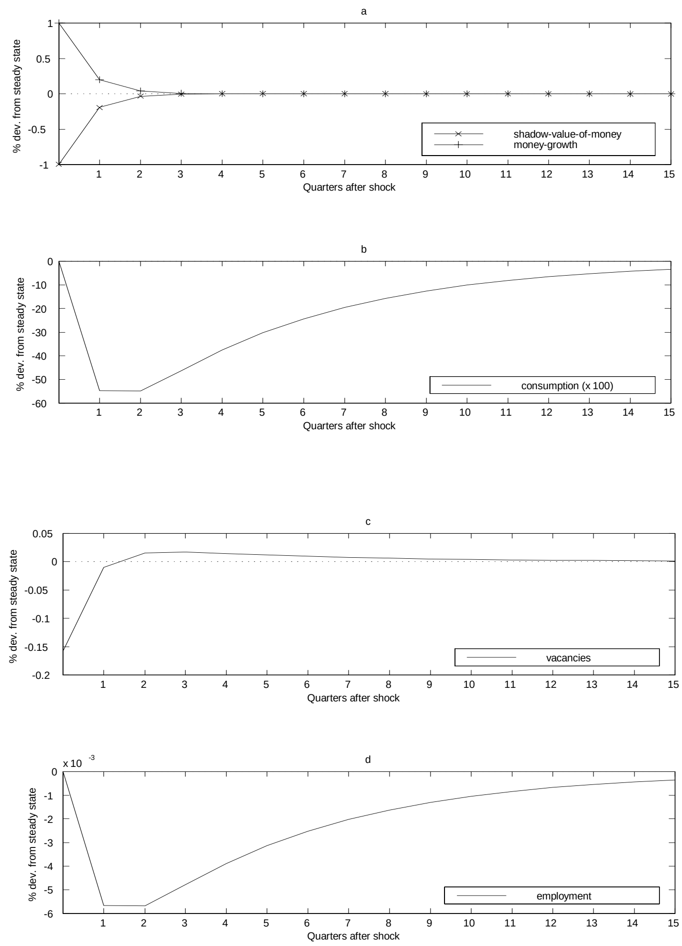


95% probability bands are constructed by the respective 2.5% and 97.5% quantiles of the projection on the principle components of the covariance matrix of the jointly distributed impulse response vector after a monetary shock. See Section 1.3.2 for a detailled explanation. SIL stands for the complete model with the elements Goods-Search (S), Inventories (I), and Labor-Search (L). The autocorrelation of the monetary shock has the value $\rho = 0$. Number of parameter draws: 250.

Figures

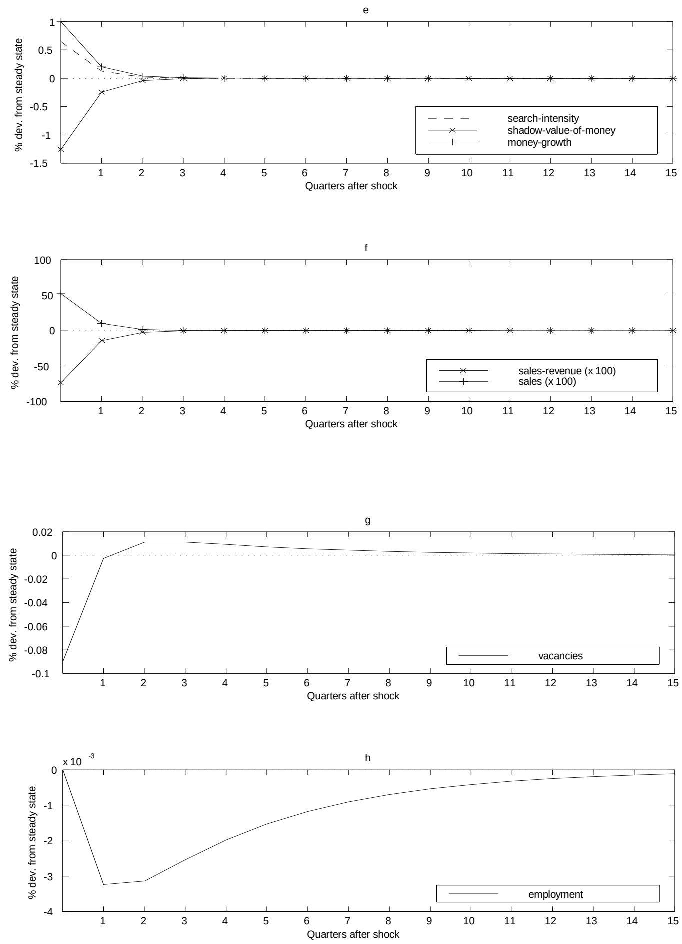
Role of Goods-Search

Figure 3 a) - d): Impulse Responses: C_L Model



Impulse responses after a monetary shock with autocorrelation $\rho = 0.2$. C_L stands for the Cash-in-Advance model (C) with Labor-Search (L). Note, in the following, the responses of consumption, sales and sales-revenue are multiplied by 100 for easier comparison with the response of inventory investment in the complete model.

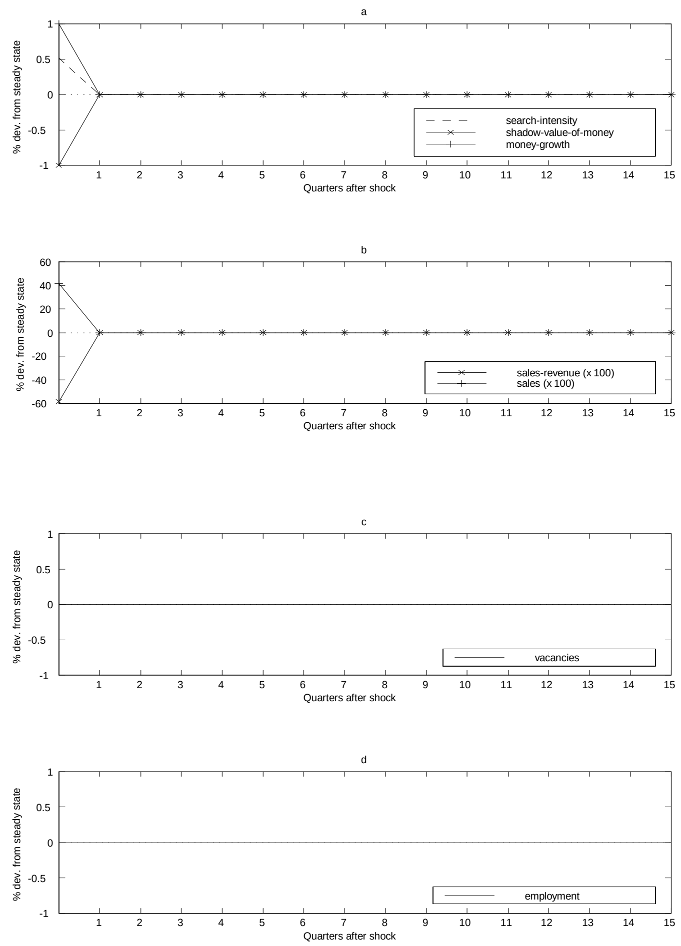
Figure 3 e) - h): Impulse Responses: S_L Model



Impulse responses after a monetary shock with autocorrelation $\rho = 0.2$. S_L stands for the Goods-Search model (S) with Labor-Search (L). Note, the responses of sales and sales-revenue are multiplied by 100 for easier comparison with the response of inventory investment in the complete model.

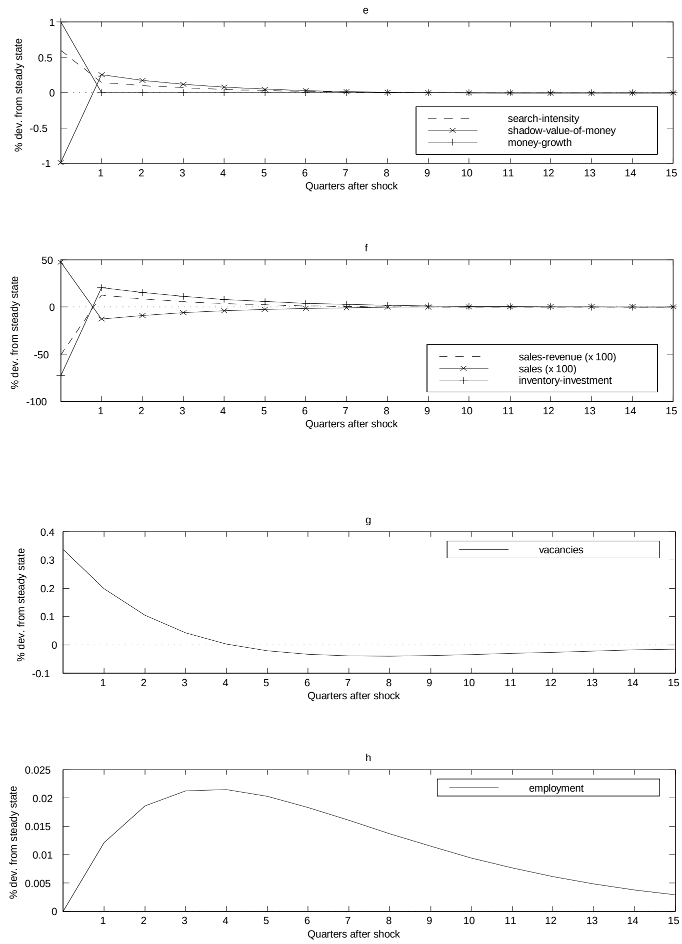
Figures   Role of Inventories

Figure 4 a) - d):   Impulse Responses: S_L Model



Impulse responses after a monetary shock with autocorrelation $\rho = 0$. S_L stands for the model with Goods-Search (S) and Labor-Search (L). Note, the responses of sales and sales-revenue are multiplied by 100 for easier comparison with the response of inventory investment in the complete model.

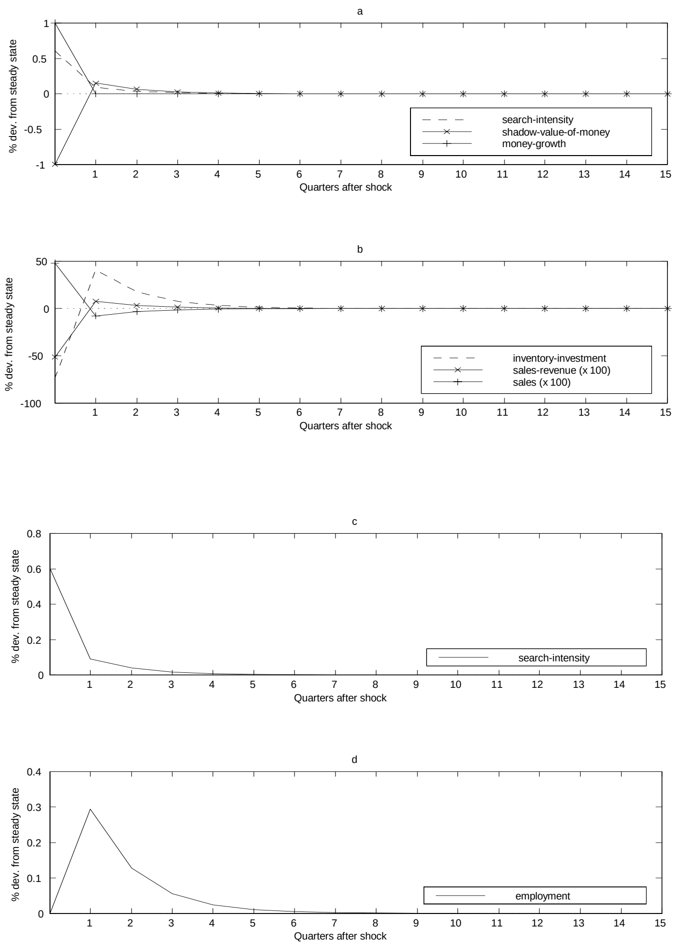
Figure 4 e) - h): Impulse Responses: SIL Model



Impulse responses after a monetary shock with autocorrelation $\rho = 0$. SIL stands for the complete model with the elements Goods-Search (S), Inventories (I) and Labor-Search (L). Note, the responses of sales and sales-revenue are multiplied by 100 for easier comparison with the response of inventory investment.

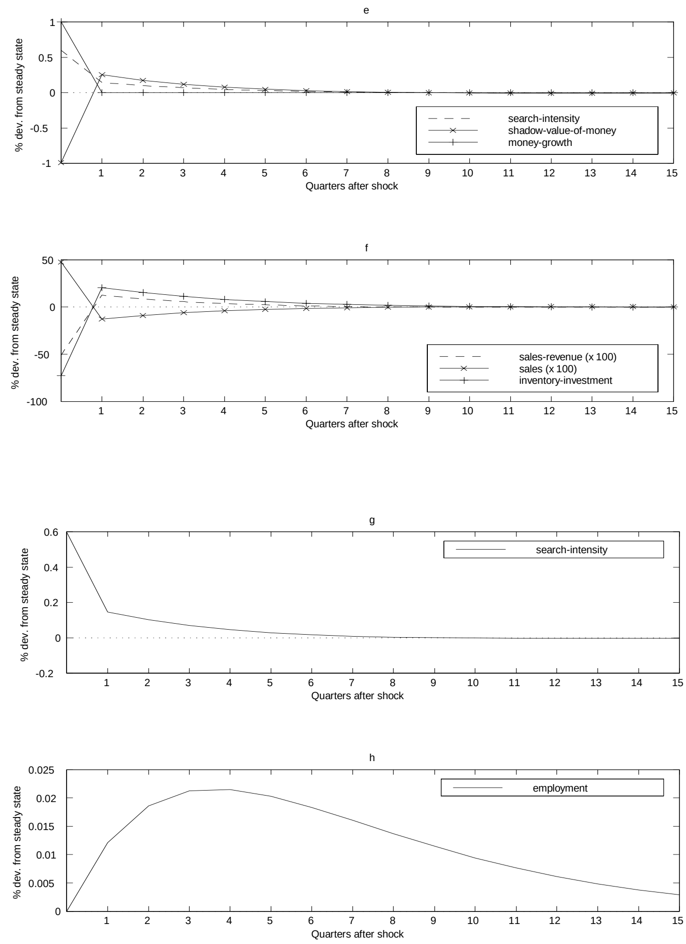
Figures Role of Labor-Search

Figure 5 a) - d): Impulse Responses: SI Model



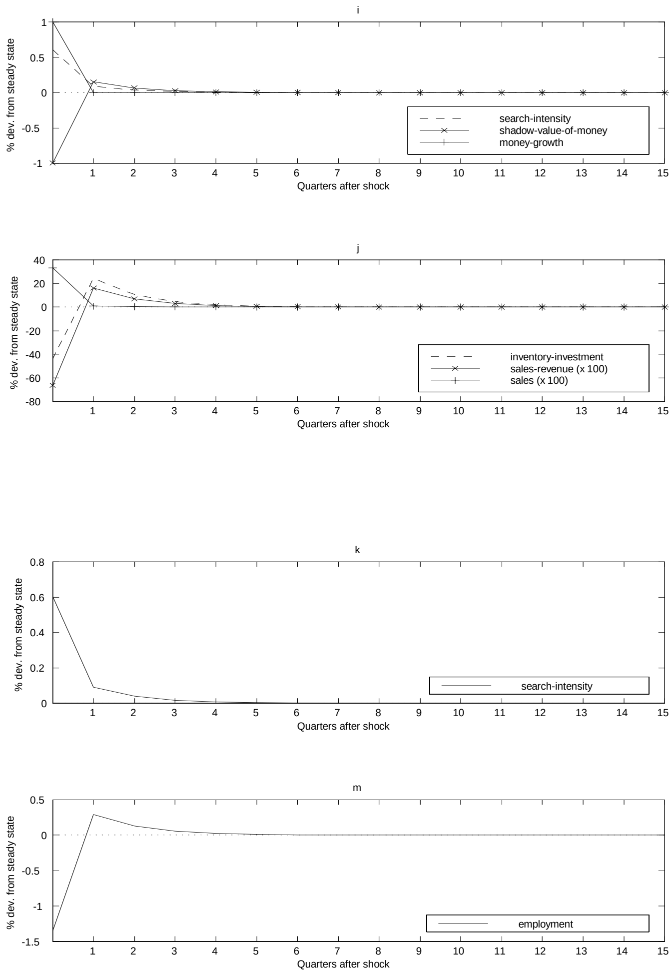
Impulse responses after a monetary shock with autocorrelation $\rho = 0$. SI stands for the model with Goods-Search (S) and Inventories (I). In period t agents choose employment in t + 1, n_{t+1}. Therefore n_t is predetermined. Note, the responses of sales and sales-revenue are multiplied by 100 for easier comparison with the response of inventory investment.

Figure 5 e) - h): Impulse Responses: SIL Model



Impulse responses after a monetary shock with autocorrelation $\rho = 0$. SIL stands for the complete model with the elements Goods-Search (S), Inventories (I) and Labor-Search (L). Note, the responses of sales and sales-revenue are multiplied by 100 for easier comparison with the response of inventory investment.

Figure 5 i) - m): SI Model (n_t not predetermined)



Impulse responses after a monetary shock with autocorrelation $\rho = 0$. SI stands for the model with Goods-Search (S) and Inventories (I). Here, employment of period t, n_t, is chosen in period t. Hence n_t is not predetermined. Note, the responses of sales and sales-revenue are multiplied by 100 for easier comparison with the response of inventory investment.

References

[1] J. Benhabib and R. E. A. Farmer. The monetary transmission mechanism. *Review of Economic Dynamics*, pages 523–550, 2000.

[2] F. Canova. Statistical inference in calibrated models. *Journal of Applied Econometrics 9*, pages S123–S144, 1994.

[3] F. Canova. Sensitivity analysis and model evaluation in simulated dynamic general equilibrium economies. *International Economic Review 36*, pages 477–501, 1995.

[4] D. Corbae, T. Temzelides, and R. Wright. Matching and money. *American Economic Review Papers and Proceedings 92*, pages 67–71, 2002.

[5] M. Faig. Money, credit and banking. *Manuscript*, 2004.

[6] M. Faig. Divisible money in an economy with villages. *Department of Economics Working Papers tecipa-216, University of Toronto*, page 34, 2006.

[7] T. S. Fuerst. Liquidity, loanable funds and real activity. *Journal of Monetary Economics 29(1)*, pages 3–24, 1992.

[8] T. S. Fuerst. Monetary and financial interactions in the business cycle. *Journal of Money, Credit and Banking 27*, pages 1321–1338, 1995.

[9] G. Hansen. Indivisible labor and the business cycle. *Journal of Monetary Economics 16(3)*, pages 309–27, 1985.

[10] W. S. Jevons. *Money and the Mechanism of Exchange.* London: Appleton, 1875.

[11] N. Kiyotaki and R. Wright. A contribution to the pure theory of money. *Journal of Economic Theory 53*, pages 215–253, 1991.

[12] N. Kiyotaki and R. Wright. A search-theoretic approach to monetary economics. *American Economic Review 83*, pages 63–77, 1993.

[13] N. Kocherlakota. Money is memory. *Journal of Economic Theory 81*, pages 231–50, 1998.

[14] D. K. Levine. Asset trading mechanisms and expansionary policy. *Journal of Economic Theory 54, June*, pages 148–64, 1991.

[15] R. E. Lucas, Jr. Interest rates and currency prices in a two-country world. *Journal of Monetary Economics 10(3)*, pages 335–60, 1982.

[16] M. Menner. A search-theoretic monetary business cycle model with capital formation. *Contributions to Macroeconomics, Vol. 6, No.1, Art. 11*, pages 1–36, 2006.

[17] J. M. Nason and T. Cogley. Testing the implications of long-run neutrality for monetary business cycle models. *Journal of Applied Econometrics 9*, pages S37–S70, 1994.

[18] J. Rotemberg and M. Woodford. The cyclical behaviour of prices and costs. *Handbook of Macroeconomics*, ed. J.B. Taylor and M. Woodford, Elsevier Science, North-Holland, 1999.

[19] P. Rupert, M. Schindler, A. Shevchenko, and R. Wright. The search-theoretic approach to monetary economics: A primer. *Federal Reserve Bank of Cleveland: Economic Review*, pages 10–28, 2000.

[20] K. D. Salyer. The timing of markets and monetary transfers in cash-in-advance economies. *Economic Inquiry 39*, pages 762–73, 1991.

[21] P. A. Samuelson. An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of Political Economy 66*, pages 467–82, 1958.

[22] S. Shi. Search for a monetary propagation mechanism. *Journal of Economic Theory 81*, pages 313–357, 1998.

[23] M. Shubik. Commodity money, oligopoly, credit and bankruptcy in a general equilibrium model. *Western Economic Journal 11, March*, pages 24–38, 1973.

[24] M. Shubik. The game theoretic approach to the theory of money and financial institutions. *Handbook of Monetary Economics*, pages 171–219, ed. Benjamin M. Friedman and Frank H. Hahn, Vol. 1, Elsevier Science, Amsterdam, 1990.

[25] C. A. Sims and T. Zha. Error bands for impulse responses. *Econometrica 67(5), September*, pages 1113–1155, 1999.

[26] R. M. Townsend. Models of money with spatially separated agents. *Models of Monetary Economies*, pages 265–303, ed. J.H. Kareken and N. Wallace, Federal Reserve Bank of Minneapolis Press, Minneapolis, 1980.

[27] H. Uhlig. A toolkit for analyzing nonlinear dynamic stochastic models easily. *Electronic: http:// www.wiwi.hu-berlin.de/wpol/html/toolkit.htm*, 1997.

[28] N. Wallace. Introduction to modeling money and studying monetary policy. *Journal of Economic Theory 81*, pages 223–31, 1998b.