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**CORRIGENDUM TO “EXISTENCE OF EQUILIBRIUM IN SINGLE AND
DOUBLE PRIVATE VALUE ACTIONS”**

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Abstract

This is a corrigendum to Theorem 15 of Jackson and Swinkels (2005) [Existence of Equilibrium in Single and Double Private Value Auctions, *Econometrica*, 73, 93-140], which proves the existence of equilibrium with positive probability of trade for private value auctions.

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Jackson and Swinkels (2005) proved the existence of equilibrium with positive probability of trade for private value auctions (Theorem 15). This important theorem was established with great ingenuity, but there is a slight error in the last part of its proof, on p. 137. In the penultimate inequality, a δ appears on the right hand side which is absent in the previous inequality. Thus, the term $\frac{\omega}{k}$ should be dropped from the right in the last inequality. This is not yet sufficient to break the argument, but the observation that ζ can be bounded above by $2M\frac{\omega}{k}$ is.

The following modification in the proof is sufficient. The definition of the modified auction \mathcal{A}^x , for $x \in \{3, 4, \dots\}$ is changed to the following: with probability $1/x$, a non-strategic player $n + 1$ has endowment $e_{n+1} = \ell$ and submits ℓ sell offers which are all equal to a random variable uniform on $[\underline{w}, \bar{w}]$; with probability $1/x$, $e_{n+1} = 0$ and $n + 1$ submits ℓ buy offers which are all equal to a random variable uniform in $[\underline{w}, \bar{w}]$. For such a game, most of the arguments given in the original proof work without changes.¹ The modification is in what follows.

Since player in $i^x \in H$ is (occasionally) a buyer, there is a probability $\zeta > 0$ that such a player has an endowment of at most $\ell - 1$ units.² Define E_1'' as the event where $Q_{B,n+1}^x > 0$ and i^x has endowment of at most $\ell - 1$ units. Define E_1' and E_1 as before. Again, we have $\Pr_x(E_1) \geq \zeta \hat{\mu}_x$.³

If $E_1 = E_1'$, i^x has no sell bids at or below $\bar{w} - 2\delta$ and there is at least one buy bid above $\bar{w} - 2\delta$. If $E_1 = E_1''$, i^x has at most $\ell - 1$ sell bids at or below $\bar{w} - 2\delta$ (because she has only $\ell - 1$ units), while there are at least ℓ buy bids above $\bar{w} - 2\delta$. Then, under $E_1 \cap E_2 \cap E_{3j}$, j sells at least one extra object by d_j .

The rest of the argument works.⁴

REFERENCE

JACKSON, M. O. AND J. M. SWINKELS (2005): "Existence of Equilibrium in Single and Double Private Value Auctions," *Econometrica*, 73, 93-140.

¹The limitation to the probability $\Pr_x(Q_B^x > \ell)$ was based on the argument that this event will occur only if more than two players bid above $\bar{w} - 2\delta$. This remains true.

²Such ζ cannot be limited by $2M\frac{\omega}{k}$ as before.

³This limitation is also sufficient to use the consequences of (6). I thank Prof. Swinkels for this observation.

⁴There is a typo in (5): the n in the right hand side should be $n + 1$. Similar replacements should be done in its consequences.