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Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624 98 75

Capacity precommitment and price competition yield Cournot outcomes

Diego Moreno y Luis Úbeda

Abstract

We study an industry of a homogeneous good where n firms with identical technology compete by first building capacity, and then, after observing the capacity decisions, choosing a “reservation price” at which they are willing to sell their entire capacities. We show that every pure strategy equilibrium yields the Cournot outcome, and that the Cournot outcome can be sustained by a pure strategy subgame perfect equilibrium.

Keywords: Capacity precommitment, Cournot, Bertrand, price competition.

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1 Introduction

We study an industry of a homogeneous good where n firms with identical technology compete by first building capacity, and then, after observing the capacity decisions, choosing a “reservation price” at which they are willing to sell their entire capacities. We show that every pure strategy equilibrium yields the Cournot outcome, and that the Cournot outcome can be sustained by a pure strategy subgame perfect equilibrium.

Unlike in Kreeps and Scheinkman’s model where price competition is *à la* Bertrand, in the present model firms compete by setting a sort of elementary supply function which, together with the market demand, determine the market clearing price and the output of each firm. Hence all firms sell their output at the same price. As a consequence, no rationing rule is necessary. (Only when several firms choose the same price and there is not enough demand to absorb their capacities, a “tie-breaking” rule is needed to allocate demand. The above result holds regardless of the particular tie-breaking rule used.)

This model of price competition describes more closely than the Bertrand model many markets. Further, when firms can build capacity instantaneously, i.e., when capacity decisions have no pre-commitment value, the model reduces to Bertrand competition. In addition, as in Kreeps and Scheinkman’s model, when capacity is costless, a case we rule out, outcomes other than the Cournot outcome can be sustained by pure strategy equilibria – see also Osborne and Pitchik (1985).

Interestingly, in this price competition game pure strategy equilibria always exist. In contrast, in the Bertrand model for some capacity choices the unique equilibrium is in mixed strategies; in these equilibria firms sometimes “regret” ex-post their pricing decisions, which questions the validity of the equilibrium prediction since firms can easily change their prices – see Maggi (1996). Pure strategy equilibria have the “no regret” property, and are therefore exempt from this critique.

It is also worth to note that for some capacity decisions there are multiple outcomes that can be sustained by pure strategy equilibria at the price competition stage. (Multiplicity of equilibria is pervasive in models of competition via supply functions when uncertainty is absent – see, e.g., Klemperer and Meyer (1989). Note that a reservation price implicitly defines a sort of elementary supply function.) When ca-

capacity is endogenized, however, this multiplicity disappears, and only the Cournot outcome can be sustained by pure strategy equilibria.

2 The model

The description of the industry, except for allowing more than two firms, is identical to that of Kreeps and Scheinkman (1983).

There are $n \geq 2$ firms in the industry. The market (inverse) demand function $P(x)$ is twice continuously differentiable, strictly decreasing and concave on a bounded interval $(0, X)$, where $X > 0$ satisfies $P(x) > 0$ for $x < X$, and $P(x) = 0$ for $x \geq X$. We write $D = P^{-1}$ for the market demand. All firms have access to the same technology. The cost to install capacity x is $b(x)$, where $b : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable, non-decreasing and convex on \mathbb{R}_+ , and satisfies $0 < b'(0) < P(0)$, and $b(0) = 0$. The marginal cost of production up to capacity is constant, and without loss of generality it is assumed to be zero.

Competition runs in two stages: at the first stage firms choose their capacities. After the first stage firms observe their opponents capacity decisions. At the second stage firms choose “reservation prices” at which to sell their entire capacities. Firms capacities and reservation prices are then used to form a supply function which, together with the market demand, determines the market clearing price, p , and firms outputs, (y_1, \dots, y_n) . A firm’s profit is the difference between its revenue, py_i , and its total cost, $b(x_i)$.

3 Price competition with capacity constraints

Let $x = (x_1, \dots, x_n) \in \mathbb{R}_+^n$ be a profile of firms’ capacities, and denote by $g(x)$ the subgame firms face in the price competition stage. In this game, each firm i chooses a reservation price $\rho_i \in \mathbb{R}_+$ at which to sell its entire capacity. A profile of reservation prices $\rho = (\rho_1, \dots, \rho_n) \in \mathbb{R}_+^n$ determines the aggregate supply, $S(\rho; \cdot)$, given for $p \in \mathbb{R}_+$ by $S(\rho; p) = [\sum_{j \in \{i \in N | \rho_i < p\}} x_j, \sum_{j \in \{i \in N | \rho_i \leq p\}} x_j]$, where $\sum_{j \in I} x_j = 0$ if $I = \emptyset$. The market clearing price, $p(\rho)$, is uniquely determined by the market clearing condition $D(p) \in S(\rho; p)$, and the profile of firms’ outputs $y(\rho) = (y_1(\rho), \dots, y_n(\rho))$ is then

readily calculated. (Note that in case of “ties”, i.e., when the reservation price of several firms are equal to the market clearing price and the demand allocated to them is less than their capacities, a tie breaking rule must operate to determine the allocation of output among the tying firms. Our results do not depend on the particular tie breaking rule used.) Firms’ payoffs (profits) are given for $i \in N$ by $\pi_i(\rho) = p_i(\rho)y_i(\rho) - b(x_i)$. Note that since firms’ capacity costs are sunk, in the game $g(x)$ firms maximize revenue.

Proposition A below describes the pure strategy equilibria of game $g(x)$. In order to describe these equilibria, the following notation will be useful. For $q \in \mathbb{R}_+$ let $r_0(q) = \arg \max_{s \in \mathbb{R}_+} P(q+s)s$; i.e., r_0 is the “reaction function” (in output) calculated ignoring capacity constraints and assuming that marginal cost is zero. For $i \in N$, write $x_{-i} = \sum_{j \in N \setminus \{i\}} x_j$, and denote by $I(x)$ the set $\{i \in N \mid x_i > r_0(x_{-i})\}$. Also let

$$M(x) = \{i \in I(x) \mid P(r_0(x_{-i}) + x_{-i})x_j \geq P(r_0(x_{-j}) + x_{-j})r_0(x_{-j}), \forall j \in I(x) \setminus \{i\}\}.$$

It is easy to see that $M(x) \neq \emptyset$ whenever $I(x) \neq \emptyset$: let $i \in I(x)$ be such that $x_i = \max_{j \in I(x)} x_j$; then

$$x_{-i} \leq x_{-j} + (x_i - x_j) = x_{-j}$$

for $j \in N$, and since $P(r_0(q) + q)$ is decreasing, we have

$$P(r_0(x_{-i}) + x_{-i}) = \max_{i \in N} P(r_0(x_{-i}) + x_{-i});$$

hence $i \in M(x)$. For $i \in M(x)$ define the set of pure strategy profiles

$$E(i) = \{\rho \in \mathbb{R}_+^n \mid \rho_i = P(r_0(x_{-i}) + x_{-i}), \rho_j \leq P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})/x_i, \forall j \in N \setminus \{i\}\}.$$

Note that if firms’ capacities are not too large (that any group of $n - 1$ firms have enough capacity to serve $X = D(0)$), then every strategy profile $\rho \in E(i)$ leads to the same outcome $(y(\rho), p(\rho))$, where all firms but Firm i produce at full capacity (i.e., $y_j(\rho) = x_j$, for $j \in N \setminus \{i\}$), and Firm i maximizes on the “residual demand,” determining the market clearing price $p(\rho) = P(r_0(x_{-i}) + x_{-i}) = \rho_i$, and setting its output to $y_i(\rho) = r_0(x_{-i})$. Thus, the set $E(i)$ contains strategies for which Firm i is, in a clear sense, the “marginal firm.” The set $M(x)$ contains the indices corresponding to the firms that can be marginal in a pure strategy equilibrium.

Proposition A. *Let $x \in \mathbb{R}_+^n$ be a vector of capacities.*

(A.1) If $I(x) = \emptyset$, then the set of pure strategy equilibria of $g(x)$ is $[0, P(\sum_{j \in N} x_j)]^n$. Moreover, the unique outcome that can be sustained by a pure strategy equilibrium is $y_i = x_i$ and $p = P^d(\sum_{i \in N} x_i)$.

(A.2) If $I(x) \neq \emptyset$, then every profile $\rho \in \bigcup_{i \in M(x)} E(i)$ is a pure strategy equilibrium of $g(x)$. Moreover, when $\#M(x) > 1$ there are multiple outcomes that can be sustained by pure strategy equilibria.

Proof: We prove A.1. Assume that $I(x) = \emptyset$. Then each Firm i 's residual demand is at least $D(p) - x_{-i}$, and since $r_0(x_{-i}) > x_i$ profit maximization requires that it produces at full capacity. Let $\rho \in [0, P(\sum_{j \in N} x_j)]^n$. We show that ρ is a pure strategy equilibrium. Since $\rho_k \leq P(\sum_{j \in N} x_j)$ for $k \in N \setminus \{i\}$, any reservation price $\rho_i > P(\sum_{j \in N} x_j)$ leads to an output for Firm i less than x_i and is therefore suboptimal, whereas $\rho_i \leq P(\sum_{j \in N} x_j)$ leads to an output for Firm i equal to x_i and hence is optimal. Thus, ρ is a pure strategy equilibrium. Now let $\rho \notin [0, P(\sum_{j \in N} x_j)]^n$. We show that ρ is not an equilibrium. Let $N(\rho) = \{i \in N \mid \rho_i > P(\sum_{j \in N} x_j)\}$. Thus $N(\rho) \neq \emptyset$ and $\sum_{i \in N(\rho)} y_i(\rho) < \sum_{i \in I(\rho)} x_i$. Hence $y_k(\rho) < x_k$ for some $k \in N$, and therefore ρ is not an equilibrium. Therefore the set of pure strategy equilibria is $[0, P(\sum_{j \in N} x_j)]^n$.

We prove A.2. Let $i \in M(x)$ and $\rho \in E(i)$. We show that ρ is a pure strategy equilibrium. We have $p(\rho) = \rho_i = P(r_0(x_{-i}) + x_{-i})$. By construction, Firm i cannot increase its revenue by setting $\rho'_i > P(r_0(x_{-i}) + x_{-i})$, whereas setting

$$P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})/x_i < \rho'_i < P(r_0(x_{-i}) + x_{-i})$$

does not change the outcome (recall that $\rho_j \leq P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})/x_i$ for $j \in N \setminus \{i\}$). Moreover, undercutting some other firm j (by setting $\rho'_i \leq \rho_j \leq P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})/x_i$), leads to a revenue no larger than $P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})$: if

$$D(P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})/x_i) \leq \sum_{j=1}^n x_j,$$

then Firm i 's revenue is no larger than $(P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})/x_i) x_i$, whereas if

$$D(P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})/x_i) > \sum_{j=1}^n x_j$$

then Firm i continues to be "marginal" and therefore its maximal revenue is $P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})$. Hence Firm i does not have an improving deviation. Now, a deviation

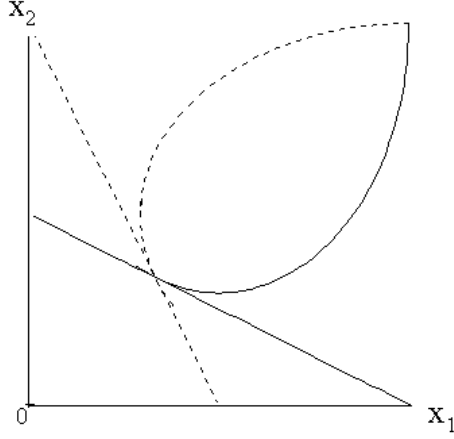


Figure 1: Equilibria at the price competition stage

by Firm $j \in N \setminus \{i\}$ to ρ'_j has no effect on its revenue unless $\rho'_j \geq \rho_i$. Moreover, setting $\rho'_j = \rho_i$ leads a revenue no larger than $\rho_i x_j$, and a deviation to $\rho'_j > \rho_i$ leads to a revenue no larger than

$$P(r_0(x_{-j}) + x_{-j})r_0(x_{-j}) \leq P(r_0(x_{-i}) + x_{-i})x_j = \rho_i x_j.$$

Hence no firm $j \in N \setminus \{i\}$ has an improving deviation either. Therefore $\rho \in E(i)$ is an equilibrium. \square

For some profiles of capacities there are multiple outcomes that can be sustained by pure strategy equilibria at the price competition stage. In Figure 1 we have plotted, for a linear duopoly, the functions $x_i = r_0(x_{-i})$ and $P(r_0(x_{-i}) + x_{-i})x_j = P(r_0(x_{-j}) + x_{-j})r_0(x_{-j})$ for $i \in \{1, 2\}$ – the continuous lines correspond to Firm 1 and the dotted ones to Firm 2. Multiplicity arises when the profile of capacities is in the area inside the “football.” (It also arises when the capacity of each firm is larger than the demand at price zero. In all these equilibria, however, the market price is zero and the entire demand is served.) Multiplicity is perhaps not surprising since in our model a reservation price implicitly defines an elementary supply function, and multiplicity of equilibria is common in models of competition via supply functions when uncertainty is absent; see Klemperer and Meyer (1989).

4 Equilibria in the full game

Interestingly, the multiplicity that arises in the price competition stage disappears when capacity is endogenized. As established in the Theorem B below, only the Cournot outcome (of the industry where firms' costs are the sum of the costs of capacity and production) can be sustained by pure strategy equilibria of the full game.

Theorem B. *Every pure strategy equilibrium yields the Cournot outcome. Moreover, the Cournot outcome can be sustained by a subgame perfect equilibrium in pure strategies.*

Proof: Denote by \bar{x} and \bar{y} the vectors of capacity choices and outputs at an arbitrary pure strategy equilibrium of the full game, and let \bar{p} be the resulting market price. Clearly $\bar{x}_i > 0$ for some $i \in N$ for if $\bar{x}_i = 0$ for every $i \in N$, then since $b'(0) < P(0)$ a firm benefits by installing a small but positive capacity. And since our assumptions on cost imply that a firm obtains zero profits by installing no capacity and producing zero units, we must have $\bar{p} \geq b(\bar{x}_i) > 0$ and $\bar{y}_i > 0$ whenever $\bar{x}_i > 0$. Also, it is easy to show that all but at most one firm must produce at full capacity: If firms i and j were producing less than their capacities, then $\rho_i = \rho_j = \bar{p} > 0$, and therefore either firm could undercut the other firm (by choosing a reservation price slightly below the market clearing price), and increase its profit. Assume, w.l.o.g., that firms 2 to n are producing at full capacity; i.e., $\bar{y}_i = \bar{x}_i$ for $i \in N \setminus \{1\}$. For $q \in \mathbb{R}_+$ let $r_b(q) = \arg \max_{s \in \mathbb{R}_+} P(q+s)s - b(s)$; i.e., r_b is the reaction function calculated taking into account both the cost of capacity and the cost of production. We show that $\bar{x}_1 = r_b(\bar{x}_{-1}) = \bar{y}_1$. In order for Firm 1 to maximize profits we must have $\bar{x}_1 \geq r_b(\bar{x}_{-1})$. Moreover, any $x_1 > r_b(\bar{x}_{-1})$ is suboptimal. Also producing $\bar{y}_1 < \bar{x}_1 = r_b(\bar{x}_{-1}) < r_0(\bar{x}_{-1}) = r_0(\sum_{j \in N \setminus \{1\}} y_j)$ is suboptimal (recall that production cost is zero). Hence $\bar{x}_1 = r_b(\bar{x}_{-1}) = \bar{y}_1$. Since Firm 1 is also producing at full capacity, the previous argument applies to firms 2 to n ; i.e., $\bar{x}_i = \bar{y}_i = r_b(\bar{x}_{-i})$ for $i \in \{2, \dots, n\}$. Hence $\bar{x} = \bar{y}$ form a Cournot equilibrium of the industry where firms' costs are the sum of the costs of capacity and production.

Now, every Cournot equilibrium \bar{x} can be sustained by subgame perfect equilibrium in pure strategies: simply let $x_i = \bar{x}_i$, for $i \in N$, and for $x \in \mathbb{R}_+^n$ let $\rho(x)$ be an

arbitrary pure strategy equilibrium of the game $g(x)$. \square

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