View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Research Papers in Economics

UNIVERSIDAD CARLOS III DE MADRID
working papers

Working Paper 05-70
Economics Series 36
November 2005

Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624 98 75

TECHNOLOGY TRANSFER IN OLIGOPOLISTIC MARKETS WITH HETEROGENEOUS GOODS*

Luis C. Corchón[1] and Clara Eugenia García[2]

Abstract

In this paper we study technology transfer (TT) in a duopoly model with heterogeneous goods under quantity and price competition. We prove that some but not all the properties of TT under homogeneous goods are preserved in our framework.

* **Acknowledgements.** This paper was presented in the V Economic Theory Meeting, Rhodos, 2003 and in Malaga University 2005. We thank specially to Carmen Beviá and also to Miguel Gonzalez-Maestre and Fernando Vega-Redondo for useful comments, to CICYT under grant BEC2002-02194 and to the Fundación of Banco Bilbao Vizcaya Argentaria for financial support.

[1] Univesidad Carlos III de Madrid, Calle Madrid, 126, 28903 Getafe (Madrid). Departamento de Economía. e-mail: lcorchon@eco.uc3m.es
[2] Univesidad Carlos III de Madrid, Calle Madrid, 126, 28903 Getafe (Madrid). Departamento de Economía. e-mail claragar@emp.uc3m.es

1. Introduction

Technology transfer (TT in the sequel) is a central issue within the economics of technological change and innovation. It has been analyzed from different perspectives (perfect competition, evolutionary models, etc.). In this paper we will be concerned with TT in static oligopolistic markets.

The basic model is as follows: TT is paid by a fixed fee.[1] Firm 1 makes a TT to Firm 2 in order to maximize joint profits. TT decreases the marginal cost of Firm 2. After TT has been made, both firms compete in the goods market.[2]

Economic intuition -borrowed from, say, the theory of international trade- suggests that with zero adoption costs the size of TT should be positively related with the technological distance between firms. However, Katz and Shapiro (1985) proved that the properties that characterized TT run counter to the this intuition:

I) TT always occurs between similar firms with similar costs.

II) TT never occurs if firms have very different costs.

III) Under linear demand, joint profits are U-shaped in the marginal cost of Firm 2 (see also Marjit (1991)).

The explanation of these paradoxical results is that profits of Firm 1 decrease

[1]If fees are charged on output, they can be used as a collusion device (Katz and Shapiro [1985] pp. 512-3).

[2]See Mukherjee (2001) for an extension of this model.

with the size of TT because the business stealing effect but profits of Firm 2 increase with the size of TT because its costs are reduced. When the marginal cost of 2 is high the business stealing effect is dominant because the substitution of output of 1 by output of 2 entails a large loss of profits of 1 but a small increase in profits for 2. When both firms are similar, business stealing is small and cost reduction dominates.

Katz and Shapiro used several simplifying assumptions, including that of product homogeneity. This assumption is a useful first approximation but restricts too tightly the scope of the marked under consideration: On the one hand, from the empirical point of view, product homogeneity does not allow experiences like the Silicon Valley -where a sizeable part of TT occurs among firms producing different products, the car industry -where six firms produce the blueprints for all engines or the consumer electronics industry -where the same invention (e.g. cordless appliances) has been applied to a variety of products. Moreover, product homogeneity implies that all firms face the same demand function and this discards the case in which each firm sells in a different country. On the other hand, from the theory point of view, product heterogeneity allows to consider Cournot and Bertrand competition and the comparison of both equilibria.

In this paper we study TT under the assumption that products are heterogeneous. By simplicity we assume that product heterogeneity is exogenous.[3]

In Section 2 we present some facts gathered from a specific market, that of a class of antidepressants. We see that under product heterogeneity TT takes a variety of forms that are hard to reconcile with the analysis of Katz and Shapiro.

Section 3 explain the model and the main concepts.

In Section 4 we consider two examples. Assuming demand is linear or isoelastic we show that Properties I-III above may not hold: The reason is that if Firm 2 faces a small demand joint profits are maximized if Firm 1 makes no TT. Similarly, TT may happen between firms with very different costs as long as Firm 2 faces a large demand relatively to that of Firm 1. These possibilities can not arise under good homogeneity because there, all firms face the same demand. This section prepares the ground for the more general findings of the next section.

In Section 5 we consider TT in general duopoly models of price and quantity competition. We show that, even under strong restrictions on the class of admissible demand functions, industry profits and the marginal cost of Firm 2 can be related in any arbitrary form (Proposition 5.1). This can not happen under product homogeneity where, for instance, joint profits are maximized when Firm

[3] The only paper with heterogeneous products that we are aware of is Mukherjee and Blasubramanian (1999). They focus attention on the impact of the threat of imitation on the optimal licencing contract.

2 produces zero output. The implication is that under product differentiation any TT can arise. We show that joint profits are increasing in the marginal cost of 2 when the output of 2 is close to zero, if and only if goods are substitutes (Proposition 5.2). This implies that a small TT to Firm 2 never pays off under these circumstances. Finally. we show that joint profits are increasing in the marginal cost of 2 when firms have a similar output, under symmetric demand and small cross price effects (Proposition 5.3). For the latter we need stronger conditions under price competition than under quantity competition.

Summing up, we obtain two basic conclusions that are robust to the consideration of both quantity and price competition. Firstly, under product heterogeneity, TT can take any form. Secondly, the properties found in the homogeneous case generalize to the heterogeneous case with various degrees of generality. In particular if demand is not symmetric we may find full TT between very different firms and that firms that are similar in technology do not engage in TT. The next section provides some evidence of that.

2. A Case Study: the Market for Antidepressants

Empirical evidence on the forms of TT is a difficult task. However, we have found useful the insights and lessons to be learn from TT practices in the pharmaceutical industry related to a specific subset of antidepressants, Selective Serotonine Reuptake Inhibitors (SSRI) for several reasons. Firstly, they involve a high level of innovation and rivalry as their market success have been increasing in the last fifteen years. Secondly, SSRI drugs have been the first antidepressants advertised directly to consumers and recognized by their trademark rather than by their molecules, leading to, possibly spurious, product differentiation.[4] And, finally the period covered allows us to compare the TT policies designed by companies before they faced the threat of generic substitutes and the marketing of new competing drugs.[5]

The set of products includes the so called N6A: Prozac®, Paxil®/Seroxat®; Celexa®/ Cipramil®, Zloft®, Effexor®/Efexor®/Vandral®/Dobupal® and Serzone®. The patented active principles behind each product are different but still close substitutes in terms of their treatment, prescriptions and effects,

[4] The results of several clinical trials show that fluoxetine, paroxetine, citalopram and sertraline are technologically homogeneous.

[5] The antidepressant drug market has been studied by Berndt et al. (2002) to determine wether marketing and advertising efforts made by pharmaceutical companies shape market shares.

specially for the case of Prozac® -fluoxetine-, Paxil®/Seroxat® -paroxetine-; Celexa®/Cipramil® -citalopram- and Zloft® -sertraline-. Here, the molecules are the basic technology while trademarks represent marketing differentiation.

N6A drug makers and technology holders have followed different technology strategies, ranging from exclusive exploitation granted by a series of patents and applications to full TT. In both situations, the products ranked among the top selling drugs in global sales figures and contributed to firms' growth dramatically. Several examples of this follow.

In 2001, Eli Lilly lost the exclusivity for marketing fluoxetine hydrochloride -Prozac®- and generic versions became available in the US. In December 2000, paroxetine's global market share was 24%. The threat of generic Prozac prompted to Eli Lilly to find alternative ways to keep ahead in the profitable SRRIs market -USD 17.1 billion in 2002- by licensing in fluoxetine R from Sepracor. Through that agreement the licensor -Sepracor- transferred to Eli Lilly all the rights to market the new active principle under the name of Prozac-R®, launched in 2001. However, Sepracor did not market fluoxetine-R in its own. This case illustrates that full TT may occur even when the creator of the technology does not sell it.

In contrast, full TT occurs in the case of Celexa®/Cipramil® in which the licensor -Lundbeck- a large Danish pharmaceutical company licensed out citalopram in 1995 to Forest Lab, a small US drug company, that largely contributed to their growth. Moreover, following the innovation path in this segment, Cipralex®/Lexapro® based on citalopram R, are also the outcome of full TT and co-marketing between these companies. Here we observe full TT from a large to a small firm.

The case concerning paroxetine -Paxil®/Seroxat®- is quite different from the two mentioned above. After the acquisition of the patent by Beecham Pharmaceuticals, paroxetine was marketed by SmithKline Beecham (SB) which was acquired by GlaxoSmithKline. The threat of generic paroxetine in 2001 pushed GlaxoSmithKline to search for SSRIs alternatives. Since 2002 GlaxoSmithKline markets Paxil-CR® in a formulation that uses SkyePharma's Geomatrix controlled-release technology. SkyePharma received an undisclosed payment from GSK in addition to royalties on net sales. The effects of TT on Skye Pharma' accounts have been bigger than expected, from 17,7 million sterling pounds in 1999, to 69,6 million in 2002. In this case there was partial TT from a small to a large firm.

The examples above show that TT under product differentiation may take forms that are not possible under product homogeneity and that TT does not always takes place among similar firms as under product homogeneity. We will see that the theoretical model backs these empirical findings.

3. The Model

There are two firms selling a product each. Firm 1 can sell to firm 2 a technology that influences its marginal cost. Marginal costs are constant. Let $\tilde{c}_2$ be the value of the marginal cost of firm 2 before transfer of technology (TT) takes place. Let c_2 be a generic value of the marginal cost of firm 2, which belongs to the interval $[\underline{c}_2, \tilde{c}_2]$ where $\underline{c}_2$ is the minimum marginal cost that can be achieved by TT. Let c_1 be the marginal cost of firm 1. Profits of firm i are $\pi_i \equiv (p_i - c_i)x_i$ where x_i is the output and p_i is the price charged by $i = 1, 2$.

Let y_i be the action of firm $i = 1, 2$ where $y_i \in [0, \bar{y}]$, some $\bar{y}$. We consider two cases: Price competition, where $y_i \equiv p_i$ and quantity competition where $y_i \equiv x_i$. Under price competition, payoffs are $(p_i - c_i)x_i(p_1, p_2)$ where $x_i = x_i(p_1, p_2)$ is the demand function for firm i, with $\frac{\partial x_i}{\partial p_i} < 0$.[6] We will say that good i is a substitute (resp. complement) of Good j if $\frac{\partial x_j}{\partial p_i} > 0$ (resp. < 0). Under quantity competition payoffs are $(p_i(x_1, x_2) - c_i)x_i$ where $p_i = p_i(x_1, x_2)$ is the inverse demand function for firm i with $\frac{\partial p_i}{\partial x_i} < 0$. In this case, goods are substitutes (resp. complements) if $\frac{\partial p_j}{\partial x_i} < 0$ (resp. > 0). Denote by $\pi_1(y_1, y_2)$ and $\pi_2(y_1, y_2, c_2)$ the payoff functions of firms 1 and 2. We will assume that they are twice continuously differentiable.

[6] For simplicity, when the context is clear, we will not write the arguments of the functions.

The game has two stages. In the first stage firms decide the value of c_2 co-operatively and in the second stage they set their actions non-cooperatively. We will refer to the second stage as *the competition stage* that we define below.

Definition 3.1. *A Nash Equilibrium (NE) in the competition stage is a pair* (y_1^*, y_2^*) *such that* y_1^* *maximizes* $\pi_1(y_1, y_2^*)$ *and* y_2^* *maximizes* $\pi_2(y_1^*, y_2, c_2)$.

Under price competition, NE is called a Bertrand equilibrium (BE) with first order conditions (FOC)

$$\frac{\partial x_i(p_1^*, p_2^*)}{\partial p_i}(p_i^* - c_i) + x_i(p_1^*, p_2^*) = 0,\ \ i = 1, 2. \tag{3.1}$$

Under quantity competition, NE is called a Cournot Equilibrium (CE) with FOC

$$\frac{\partial p_i(x_1^*, x_2^*)}{\partial x_i} x_i^* + p_i(x_1^*, x_2^*) - c_i = 0,\ \ i = 1, 2. \tag{3.2}$$

We assume that the system of FOC fulfills the Gale-Nikaido Property. Formally:

Assumption 1: a) $\dfrac{\partial^2 \pi_i(y_1, y_2)}{\partial y_i^2} < 0\ \forall (y_1, y_2) \in [0, \bar{y}]^2,\ \ i = 1, 2.$

b) $\dfrac{\partial^2 \pi_1(y_1, y_2)}{\partial y_1^2} \dfrac{\partial^2 \pi_2(y_1, y_2)}{\partial y_2^2} > \dfrac{\partial^2 \pi_1(y_1, y_2)}{\partial y_1 y_2} \dfrac{\partial^2 \pi_2(y_1, y_2)}{\partial y_1 y_2}\ \forall (y_1, y_2) \in [0, \bar{y}]^2$

Assumption 1 (A.1 in the sequel) has two parts: a) implies that the profit function of i is strictly concave in y_i. If part a) holds a sufficient condition for b) to hold is that the effect in absolute value of y_i on $\frac{\partial \pi_i}{\partial y_i}$ is larger than on $\frac{\partial \pi_i}{\partial y_j}$ $j \neq i$ i.e. that the matrix with typical element $\frac{\partial^2 \pi_i}{\partial y_i y_j}$ has a *Dominant Diagonal* (see Friedman [1977], Assumption 7, p. 71).[7] Under quantity competition, A1 reads

$$\textbf{a)}\ 2\frac{\partial p_i}{\partial x_i} + x_i \frac{\partial^2 p_i}{\partial x_i^2} < 0 \ i = 1, 2.$$

$$\textbf{b)}\ (2\frac{\partial p_1}{\partial x_1} + x_1 \frac{\partial^2 p_1}{\partial x_1^2})(2\frac{\partial p_2}{\partial x_2} + x_2 \frac{\partial^2 p_2}{\partial x_2^2}) > (\frac{\partial p_1}{\partial x_2} + x_1 \frac{\partial^2 p_1}{\partial x_1 x_2})(\frac{\partial p_2}{\partial x_1} + x_2 \frac{\partial^2 p_2}{\partial x_1 x_2})$$

Thus $A1a$) holds if inverse demand is not too convex. $A1b$) holds if the impact of the own price on marginal revenue is larger than the cross price impact. A similar condition applies under price competition. We now have the following.

Lemma 1. *Under A.1 there is a NE. If FOC hold with equality, NE is unique and y_i are continuously differentiable functions of c_2, $i = 1, 2$.*

Proof. The existence of a NE follows from Kakutani fixed point theorem (see, e.g. Friedman [1977]) since by A.1 $\pi_i(\ )$ is continuous and concave and actions belong to a compact set. Uniqueness and differentiability follows from applying

[7] If goods are homogeneous, A.1a) implies $\beta + 2 > 0$ and A1b) is equivalent to $\beta + 3 > 0$ where $\beta \equiv x \frac{d^2 p(x)}{dx^2} / \frac{dp(x)}{dx}$ where $x \equiv x_1 + x_2$. $\beta + 2 > 0$ is equivalent to marginal revenue decreasing which is the assumption made by Katz and Shapiro (1985).

Theorem 4 in Gale and Nikaido (1965) to FOC of a NE. ■

We will assume that Firm 1 is always active in any NE. Let $\bar{c}_2$ be the minimum value of c_2 for which the output of firm 2 is zero in a NE. Under our assumptions this value exists.

By Lemma 1 we can write y_1 and y_2 as functions of c_2, i.e. $y_i = y_i(c_2)$, $i = 1, 2$. Write $\pi_1(c_2) = \pi_1(y_1(c_2), y_2(c_2))$ and $\pi_2(c_2) = \pi_2(y_1(c_2), y_2(c_2)), c_2)$. Thus, industry profits can be written as $\Pi \equiv \sum_{i=1}^{2} \pi_i = \pi_1(c_2) + \pi_2(c_2) \equiv \Pi(c_2)$.

Finally we assume that in the first stage of the game c_2 is chosen from $[\underline{c}_2, \tilde{c}_2]$, to maximize Π. Formally,

Definition 3.2. *c_2^* solves the TT two stage game if* $\Pi(c_2^*) \geq \Pi(c_2')$, $\forall c_2' \in [\underline{c}_2, \tilde{c}_2]$

We finally remark that the model is sufficiently flexible to incorporate alternative interpretations. 1) TT may refer to a product that was previously not produced by Firm 2. In this case we set $\tilde{c}_2 = \bar{c}_2$. 2) Positive adoption costs. Suppose that these costs are a function of the pre-TT output and actual output. Since the former is given, write this as $F(x_2)$. Thus, under quantity competition, $\pi_2 \equiv (p_2(x_1, x_2) - c_2)x_2 - F(x_2) = (p_2(x_1, x_2) - F(x_2)/x_2 - c_2)x_2$. Now we can interprete $p_2(x_1, x_2) - F(x_2)/x_2$ as the inverse demand function of Firm 2 and carry on as before. These two interpretations do not fit well with product homogeneity:

The first because joint profits are maximized when Firm 2 does not produce at all so it is impossible to explain any positive TT. The second, because product homogeneity implies that the two firms face the same demand function.

4. Two Examples

In this section we explore what difference product differentiation makes to TT by considering two examples. The first deals with quantity competition and generalizes the linear inverse demand case worked out by Katz and Shapiro (1985) and Marjit (1991). The second example considers price competition with linear demand.

Example 1: Quantity competition. Inverse demand functions read

$$p_i = a_i - b_i x_i^{\alpha} - d_i x_j^{\alpha},\ \alpha,\ a_i,\ b_i > 0,\ i \neq j = 1, 2.$$

If d_1 and d_2 are positive (resp. negative), goods are substitutes (resp. complements). It is easy to check that $A1a)$ holds and $A1b)$ reads $(1+\alpha)^2 b_1 b_2 > d_1 d_2$.

We first calculate the CE for a given c_2. We have that,

$$\pi_i^* = \alpha b_i \left(\frac{(1+\alpha)b_j(a_i - c_i) - d_i(a_{ij} - c_j)}{(1+\alpha)^2 b_1 b_2 - d_1 d_2}\right)^{\frac{1}{\alpha}+1}\ i,\ j = 1, 2, i \neq j. \tag{4.1}$$

In order to guarantee that profits are non negative in the CE, we assume that $(1+\alpha)b_2(a_1 - c_1) > d_1(a_2 - c_2)$ and $(1+\alpha)b_1(a_2 - c_2) \geq d_2(a_1 - c_1)$ $\forall c_2 \in [c_1, \bar{c}_2]$.[8]

[8] If goods are substitutes these conditions imply A.1 b). If goods are complements these conditions always hold for $a_i > c_i$.

From (4.1) we get that the sign of $\frac{d\Pi}{dc_2}$ equals the sign of

$$d_1(b_2(a_1-c_1)(1+\alpha)-(a_2-c_2)d_1)^{\frac{1}{\alpha}}-(1+\alpha)b_2((a_2-c_2)b_1(1+\alpha)-d_2(a_1-c_1))^{\frac{1}{\alpha}}. \quad (4.2)$$

We now check for the properties I-II-III stated in the Introduction.

I : If a_2 is sufficiently small in relationship with a_1 we see from (4.2) that Π is increasing on c_2 so TT never occurs. The reason is that Firm 2 faces a demand curve that, relative to that of Firm 1, is small so is not profitable to transfer technology to a firm that is relatively inefficient selling the good. We will refer to the question of the relative size of demand as the *Marketing Effect*. Assuming $a_1 = a_2 \equiv a$, $b_1 = b_2 \equiv b$, $d_1 = d_2 \equiv d$, we see from (4.2) that $sign\frac{d\Pi}{dc_2}\mid_{c_1=c_2}= sign\{(a-c_1)^{\frac{1}{\alpha}}(b(1+\alpha)-d)^{\frac{1}{\alpha}}(d-(1+\alpha)b)\} < 0$. Thus I holds in our model under symmetric demand.

II : From (4.1) we see that $(1+\alpha)(a_2-\bar{c}_2)b_1 = d_2(a_1-c_1)$. Equation (4.2) implies that $sign\frac{d\Pi}{dc_2}\mid_{c_2=\bar{c}_2}= sign\ d_1$. So $\frac{d\Pi}{dc_2}\mid_{c_2=\bar{c}_2}> 0$ iff good 2 is substitute of good 1. Thus, II holds in our model as long as products are substitutes.

III : From (4.2) we get that $\frac{d^2\Pi}{dc_2^2} > 0$. Thus, $\Pi(\ )$ is strictly convex and $\Pi(\ )$ attains a minimum at $\frac{d\Pi}{da_2} = 0$. We now give sufficient conditions for $\Pi(\ )$ to be U-shaped with a minimum at $\hat{c}_2 \in (c_1, \bar{c}_2)$. Letting $Q \equiv (\frac{(1+\alpha)b_2}{d_1})^{\alpha}$ we get from

(4.2) that the value of c_2 which minimizes Π is

$$\hat{c}_2 = a_2 - \frac{(1+\alpha)(a_1-c_1)b_2 + (a_1-c_1)d_2Q}{d_1 + Qb_1(1+\alpha)} < \bar{c}_2 (= a_2 - \frac{d_2(a_1-c_1)}{(1+\alpha)b_1}).$$

Assume substitute goods (if goods are complements (4.2) implies $\frac{d\Pi}{da_2} > 0$ so full TT is optimal) and symmetric demand. Keep the notation introduced in I above. If $\hat{c}_2 \leq c_1$, we had that

$$1 \leq \frac{(1+\alpha)b + dQ}{d + Qb(1+\alpha)} \Leftrightarrow Q \leq 1 \Leftrightarrow (1+\alpha)b \leq d, \text{ contradiction. Thus } \hat{c}_2 > c_1.$$

Example 2: Price competition. Demand functions read

$$x_i = a_i - p_i + d_i p_j, i, j = 1, 2.$$

If d_1 and d_2 are positive (resp. negative), goods are substitutes (resp. complements).[9] It is easy to check that $A1a)$ holds an that $A1b)$ reads $4 > d_1 d_2$. In a BE,

$$\pi_i^* = (\frac{2(a_i - c_i) + d_i(a_j + c_j + c_i d_j)}{4 - d_1 d_2})^2, \tag{4.3}$$

[9] If $d_1 = d_2 = 1$ demand functions are identical to those in the circular model of Salop (1979).

In order to guarantee that BE prices never fall short of marginal costs, we assume $2(a_1 - c_1) + d_1(a_2 + c_2 + c_1 d_2) > 0$ and $2(a_2 - c_2) + d_2(a_1 + c_1 + c_2 d_1) \geq 0$.

From (4.3) we get that the sign of $\frac{d\Pi}{dc_2}$ equals the sign of

$$(2(a_1-c_1)+d_1(a_2+c_2+c_1d_2))d_1+(2(a_2-c_2)+d_2(a_1+c_1+c_2d_1))(d_1d_2-2)) \quad (4.4)$$

We now check for the properties I-II-III stated in the Introduction.

I) Again, it is easy to see that lack of symmetry may imply $\frac{d\Pi}{dc_2}\mid_{c_1=c_2} > 0$. But even under symmetric demand if $d_1 = d_2 > 0$ and $d_1 d_2 \geq 2$, $\frac{d\Pi}{dc_2}\mid_{c_1=c_2} > 0$. In this case cross substitution effects are very strong and reductions in c_2 exacerbate competition making TT unprofitable. However if $d_1, d_2 < 1$, $\frac{d\Pi}{dc_2}\mid_{c_1=c_2} > 0$. So in this case we need symmetry and small cross substitution effects for I to hold.

II) From (4.3) we get $\bar{c}_2 = \frac{2a_2+d_2(a_1+c_1)}{2-d_1d_2}$. If goods are substitutes, $\frac{d\Pi}{dc_2}\mid_{c_2=\bar{c}_2} > 0$.

III) From (4.4), we get that $\frac{d^2\Pi}{dc_2} > 0$. Thus $\Pi(\ )$ is strictly convex. Suppose that $\Pi(\ )$ achieves an interior minimum. Then,

$$\hat{c}_2 = \frac{(2 - d_1 d_2)(2a_2 + c_1 d_1 + d_2(a_1 + c_1)) - d_1(2a_1 + d_1 a_2)}{d_1^2 + (d_1 d_2 - 2)^2}.$$

Convexity of $\Pi(\ )$ and $\frac{d\Pi}{dc_2}\mid_{c_2=\bar{c}_2} > 0$ imply that $\hat{c}_2 < \bar{c}_2$. Let us prove that $\hat{c}_2 > c_1$.

Suppose otherwise. Then,

$$\hat{c}_2 = \frac{(2-d^2)(2a + c_1 d + d(a+c_1)) - d(2a+da)}{d^2 + (d^2-2)^2} \leq c_1 \Leftrightarrow$$

$(-d^3 - 3d^2 + 4)a \leq c_1(d^4 - d^3 - 4d + 4)$. Since $-d^3 - 3d^2 + 4 > 0$ for $1 < d < 0$ and $a > c_1$, we obtain $d^4 + 3d^2 > 4d$, which is impossible for $0 < d < 1$. Thus $\hat{c}_2 > c_1$. Since $\hat{c}_2 < \bar{c}_2$, $\Pi(\ )$ is U-shaped with a minimum at $\hat{c}_2 \in (c_1, \bar{c}_2)$.

It is now time to sum up our findings in this section:

1: The assumption of homogeneous product hides two important assumptions: that demand for both firm is the same and that goods are substitutes. If demand functions do not satisfy these assumptions the properties that characterize TT under homogeneous products do not hold. However, once demand is assumed to be symmetric and goods are substitutes these properties also hold in our framework.

2: Quantity competition and price competition yield the same conclusions under similar assumptions. The assumptions necessary for I) to hold are a little bit stronger in the case of price competition.

3: The marketing effect can explain that the owner of a patent does not exploit it, as in the Sepracor/Eli Lilly case in the previous section, or that a large firm licences to a small one, as in the Lundbeck/Forest Lab case. In the first case it may be argued that the success of Prozac® gave to Elli Lilly a large advantage in marketing the product. In the second case both firms had relative advantages in marketing the product in different locations, EU and USA.

4: However, as long as demand functions fulfill the properties assumed in this section, partial TT, as in the GlaxoSmithKline/SkyePharma case, can not be explained. Because joint profits are strictly convex on c_2 the TT policy can only be full transfer or no transfer.

Equipped with these findings we are prepared to study the form of $\Pi(\ )$ in a general framework.

5. General Properties of Technology Transfer

In this section we investigate the shape of $\Pi(\ )$ in order to derive the implications for the kind of TT that we may expect in general.

Our first result shows that the shape of $\Pi(\ )$ is arbitrary even under strong assumptions on the form of demand/inverse demand functions. All proofs are gather in the Appendix.

Proposition 5.1. *Let $\Psi : [\underline{c}_2, \bar{c}_2] \rightarrow \Re_{++}$ be a continuous function with $\bar{c}_2 > c_1$. There are inverse demand (resp. demand) functions $p_i(\ )$ (resp. $x_i(\ )$) $i = 1, 2$ that are linear on x_i (resp. p_i) yielding payoff functions that satisfy A.1 such that;*

a) There is a unique NE in which Firm 1 is active and Firm 2 inactive iff $c_2 = \bar{c}_2$.

b) $\Psi(c_2) = \Pi(c_2)\ \forall c_2 \in [\underline{c}_2, \bar{c}_2]$.

Proposition 5.1 implies that any TT can be rationalized as arising from the maximization of joint profits followed by a competition stage with payoff functions satisfying A.1 and demand/inverse demand that are linear in the action controlled by the firm. This result implies that any TT between two firms can be rationalized as arising from these firms playing the previous game with payoff functions satisfying A.1. Thus, we can not expect Properties I-II-III in the Introduction hold when the product is differentiated.

Next, we show that the properties of $\Pi(\ )$ in the boundaries, i.e. in $\underline{c}_2$ and $\bar{c}_2$ found in the case of homogeneous goods hold in our framework under the appropriate assumptions.

First, we study the form of $\Pi(\ )$ in $c_2 = \bar{c}_2$. In this case, the property found in the homogeneous good case holds under reasonable circumstances.

Proposition 5.2. *Under A.1,* $\frac{d\Pi}{dc_2}\mid_{c_2=\bar{c}_2} > 0 \Leftrightarrow$ *Good* 2 *substitute of good* 1.

Next, we study the form of $\Pi(\ )$ in $\underline{c}_2$. In this case, some extra conditions are needed in order to prove the result obtained under homogeneous goods. Examples 1 and 2 above give us some clues: Demand functions must are symmetric and cross effects must not be very large. Our next two assumptions deal with symmetry.

Assumption 2. **a)** $\underline{c}_2 = c_1$. **b)** $\pi_1(\ ) = \pi_2(\ ,\ c_1)$.

Assumption 3. a) $\frac{\partial \pi_1(0,\ 0)}{\partial y_1} > 0$. **b)** There is a y' such that $\frac{\partial \pi_1(y',\ y')}{\partial y_1} < 0$.

*A*2 is a symmetry assumption. *A*2*a*) says that the minimum marginal cost that can be achieved is that of Firm 1. *A*2*b*) says that demand/inverse demand functions are symmetric. Let us now concentrate on A.3. Consider quantity competition: From (2.2), $\frac{\partial \pi_1(0,0)}{\partial x_1} = p_1(0,0) - c_1$. Thus *A*3*a*) says that $p_1(0,0) > c_1$. If x_1 and x_2 are very large we expect that prices fall below marginal costs so

$\frac{\partial \pi_1}{\partial x_1} = \frac{\partial p_1}{\partial x_1} x_1 + p_1 - c_1 < 0$ and $A3b)$ holds. Under price competition from (2.1), $\frac{\partial \pi_1(0,0)}{\partial p_1} = -\frac{\partial x_1}{\partial p_1} c_1 + x_1 > 0$. If p_1 and p_2 are very large we expect $x_1 \simeq 0$ and $\frac{\partial \pi_1}{\partial y_1} \simeq \frac{\partial x_1}{\partial p_1}(p_1 - c_1) < 0$, so $A3b)$ holds. Thus, the behavior of $\frac{\partial \pi_1}{\partial y_1}$ assumed by A.3 seems reasonable.

Lemma 2. *Under A.1-2-3, NE is symmetric when* $c_1 = c_2$.

Proof. Consider $\frac{\partial \pi_1(y,\, y)}{\partial x_1}$ as a function of y. Let $f(y) \equiv \frac{\partial \pi_1(y,\, y)}{\partial x_1}$. By A. 3, $f(\ )$ changes sign when we go from $y = 0$ to $y = y'$. By the intermediate value theorem, there is a value of y, y^* such that $f(y^*) = 0$. By A. 2, say y^* also solves the FOC of profit maximization for Firm 2 so, under A.1a) y^* is a symmetric NE. By Lemma 1 NE is unique, so the symmetric NE is the only NE. ■

Now we make assumptions that guarantees that the impact of the action of the competitor on the marginal profit of a firm is smaller than the impact of the action taken by this firm. Let p^* (resp. x^*) be the action in the BE (CE) when $c_1 = c_2$.

Assumption 4.$-\frac{\partial x_i(p^*,p^*)}{\partial p_i} > \frac{\partial x_i(p^*,p^*)}{\partial p_j}$ and $-\frac{\partial x_i^2(p^*,p^*)}{\partial^2 p_i} \geq \frac{\partial^2 x_i(p^*,p^*)}{\partial p_i p_j}, i \neq j = 1, 2$

Assumption 4'. $\frac{\partial^2 p_i(x^*,x^*)}{\partial x_i x_j} x^* + \frac{\partial p_i(x^*,x^*)}{\partial x_i} < 0$ and $\frac{\partial^2 p_i(x^*,x^*)}{\partial x_i^2} x^* + \frac{\partial p_i(x^*,x^*)}{\partial x_i} \leq 0, i \neq j = 1, 2$

Under homogeneous products, A.4' is a local version of the so called strong concavity assumption (Corchón [1994]). Notice that, as hinted in Examples 1 and 2, the assumption dealing with price competition is, somehow, stronger than the assumption dealing with quantity competition. A. 2-3-4' are satisfied by the inverse demand functions used in Example 1 if $A_1 = A_2 > 0$, $b_1 = b_2$ and $d_1 = d_2$. A.2-3-4 are satisfied by the demand functions used in Example 2 if $a_1 = a_2$, and $d_1 = d_2 < 1$.[10] Now we are prepared for the result.

Proposition 5.3. *Assume A.1-2-3. Under price competition and A4, or under quantity competition and A4'* $\frac{d\Pi}{dc_2} \mid_{c_1=c_2} < 0$.

[10] We remark that there is no relationship between A.1 and A.4-4'. On the one hand A.1 is a global assumption, i.e. holds for all prices and quantities and A.4-4' only apply in points that are a NE when $c_1 = c_2$. On the other hand, Proposition 5.1 implies that demand/inverse demand functions satisfying A.1 can not yield a result like Proposition 5.3 below.

6. Conclusions

In this paper we have explored the consequences of assuming product heterogeneity by considering quantity and price competition. Both models yield identical results under similar assumptions. We proved that the three properties that characterized TT under product homogeneity hold in our framework under suitable assumptions:

- Under symmetric demands and small cross substitution effects, $\frac{d\Pi}{dc_2} \mid_{c_1=c_2} < 0$ (Proposition 5.3).

-If goods are substitutes $\frac{d\Pi}{dc_2} \mid_{c_2=\bar{c}_2} > 0$ (Proposition 5.2).

- If demands are linear and/or isoelastic, symmetric and cross effects are not very large $\Pi(\ )$ is U-shaped (Examples 1 and 2).

Our analysis has uncovered the importance of symmetry and small substitution effects for properties I and III to hold. Symmetry is specially important because, in our case, a small firm is not necessarily one with low productivity: It might be that the demand for this firm is small, i.e. the marketing effect is important here.

We also have found new features of TT, namely:

- Under asymmetric demand functions or with strong substitution effects $\frac{d\Pi}{dc_2} \mid_{c_1=c_2} >$ 0 is possible, even if goods are substitutes (Examples 1 and 2)

- Under general demand functions the shape of $\Pi(\ )$ is arbitrary (Proposition 5.1).

Summing up, the answer to the question what difference makes the consideration of heterogeneous products is that we expect here a richer set of interactions between firms regarding TT. In particular, we have to keep a close eye on the following possibilities.

i) If Firm 1 faces a small demand it might be off business after TT. This resembles the Sepracor/Eli Lilly case in Section 2 where Sepracor licenced the product but it did not market it.

ii) Even if $\tilde{c}_2$ is high, Firm 2 may receive large quantities of TT if its demand is large relative to that of Firm 1 or the markets in which both firms operate are independent. This resembles the Lundbeck/Forest Lab case in Section 2 where the two firms involved had very different sizes.

iii) TT may be partial like in the GlaxoSmithKline/SkyePharma case recorded in Section 2.

iv) Even if $\tilde{c}_2$ is close to c_1, firm 2 may not receive TT at all because its demand is small or because cross price effects are large. The latter could be a potential candidate to explain why TT seldom occurred among the big companies in the market for antidepressants.

7. References

Berndt, E. R., A. Bhattacharjya, D. N. Mishol, A. Arcelus and T. Lasky (2002): "An Analysis of the Diffusion of New Antidepressants: Variety, Quality, and Marketing Efforts". *Journal of Mental Health Policy Economics*, 5, 1, pp. 3-19.

Corchón, L. C. (1994): "Comparative Statics for Aggregative Games: The Strong Concavity Case". *Mathematical Social Sciences*, 28, pp. 151-165.

Dierker, E. and H. Dierker (1999). "Product Differentiation and Market Power". *Advances in Mathematical Economics,* 1, pp. 39-67.

Friedman, J. (1977): *Oligopoly and the Theory of Games.* North Holland.

Gale, D. and H. Nikaido (1965): "The Jacobian Matrix and the Global Univalence of Mappings". *Mathematische Annalen,* 159, pp. 81-93.

Katz, M. and C. Shapiro (1985): "On the Licencing of Innovations". *Rand Journal of Economics,* 16, 4, pp. 504-520.

Marjit, S. (1990): "On a non-Cooperative Theory of Technology Transfer". *Economics Letters,* 33, 3, pp. 293-298.

Mukherjee, A. (2001): "Technology Transfer with Commitment". *Economic Theory*, 17, 2, pp. 345-369.

Mukherjee, A. and N. Balasubramanian (1999): "Technology Transfer in a Differentiated Product Market". W. P., Technisque Universiteit Eindhoven.

Salop, S. (1979): "Monopolistic Competition with Outside Goods". *Bell Journal of Economics,* 10. pp. 141-156.

8. APPENDIX

Proof of Proposition 5.1

Let us tackle first the case of price competition. Let ϵ be such that $\epsilon < \Psi(c_2) \forall c_2 \in [\underline{c}_2, \bar{c}_2]$. Define the candidate demand functions: For good 2, $x_2 = \max(0, b(\bar{c}_2 - p_2))$ and for good 1, $x_1 = b(\bar{c}_2 - p_1)f(p_2)$ with

$$f(p_2) = \frac{4\Psi(\max(\underline{c}_2, \min(2p_2 - \bar{c}_2, \bar{c}_2))) - b(\bar{c}_2 - p_2)^2}{b(\bar{c}_2 - c_1)^2}.$$

Take b such that $4\Psi(c_2) - b(\bar{c}_2 - c_1)^2 \geq \epsilon \; \forall c_2 \in [\underline{c}_2, \bar{c}_2]$. Thus $f(p_2) > 0$.

The above demand functions fulfill the desired properties: They are linear and satisfy A1. Since demands are linear in their own price and marginal costs are constant, profit functions are strictly concave on their own price. Therefore, the

BE for the above demand functions is found by setting $\frac{\partial \pi_i}{\partial p_i} = 0$. This yields,

$$
\begin{aligned}
p_i^* &= \frac{\bar{c}_2 + c_i}{2} i = 1, 2, \quad x_1^* = \frac{b f(p_2^*)(\bar{c}_2 - c_1)}{2}, \quad \text{and} \quad x_2^* = \frac{b\bar{c}_2 - bc_2}{2}. \\
2p_2^* - \bar{c}_2 &= c_2 \le \bar{c}_2, \;\; \min(2p_2^* - \bar{c}_2, \bar{c}_2) = c_2 \text{ and } \max(\underline{c}_2, c_2) = c_2. \\
f(p_2^*) &= 4\frac{\Psi(c_2) - b(\bar{c}_2 - p_2^*)^2}{b(\bar{c}_2 - c_1)^2} = 4\frac{\Psi(c_2) - b(\frac{\bar{c}_2 - c_2}{2})^2}{b(\bar{c}_2 - c_1)^2} \\
\pi_1(c_2) &= \frac{f(p_2(c_2))(\bar{c}_2 - c_1)^2 b}{4} \text{ and } \pi_2(c_2) = \frac{b(\bar{c}_2 - c_2)^2}{4}. \\
\Pi(c_2) &= \Psi(c_2) - b(\frac{\bar{c}_2 - c_1}{2})^2 + b(\frac{\bar{c}_2 - c_1}{2})^2 = \Psi(c_2).
\end{aligned}
$$

Now consider the case of quantity competition. Let ϵ be such $0 < \epsilon < \Psi(c_2)$ $\forall c_2 \in [\underline{c}_2, \bar{c}_2]$. Under our conditions ϵ exists. Let $p_1 = \max(0, \bar{c}_2 - f(x_2)x_1)$, with

$$
f(x_2) = \frac{(\frac{\bar{c}_2 - c_1}{2})^2}{\max(\Psi(\max(\underline{c}_2, \bar{c}_2 - 2bx_2)) - bx_2^2, \epsilon)}
$$

and $p_2 = \max(0, \bar{c}_2 - bx_2)$ where b is chosen such that

$$
\epsilon < \Psi(c_2) - \frac{(\bar{c}_2 - c_2)^2}{4b}, \forall c_2 \in [c_1, \bar{c}_2].
$$

These demand functions fulfill the conditions stated in the result: They are linear, satisfy A.1 -because $\frac{\partial^2 \pi_2}{\partial x_1 x_2} = 0$- and profit functions are strictly concave on their own output Thus, the unique CE for the above inverse demand functions is found by setting $\frac{\partial \pi_i}{\partial x_i} = 0$. Then,

$$p_i^* = \frac{\bar{c}_2 + c_i}{2},\ i = 1, 2.\ x_1^* = \frac{\bar{c}_2 - c_1}{2f(x_2^*)},\ \pi_1^* = \frac{(\bar{c}_2 - c_1)^2}{4f(x_2^*)}\ x_2^* = \frac{\bar{c}_2 - c_2}{2b}\ \pi_2^* = \frac{(\bar{c}_2 - c_2)^2}{4b}.$$

Since $\bar{c}_2 > c_1$, $x_1^* > 0$. Also $x_2^* = 0 \Leftrightarrow c_2 = \bar{c}_2$, so part a) is proved. Then

$$\begin{aligned}
\max(\underline{c}_2, \bar{c}_2 - 2bx_2^*) &= \max(\underline{c}_2, c_2) = c_2. \text{ Thus } \Psi(c_2) - bx_2^{*2} = \Psi(c_2) - \frac{(\bar{c}_2 - c_2)^2}{4b} > \epsilon. \\
f(x_2(c_2)) &= \frac{b(\bar{c}_2 - c_1)^2}{4b\Psi(c_2) - (\bar{c}_2 - c_2)^2}.\ \pi_1(c_2) = \frac{(\bar{c}_2 - c_1)^2}{4f(x_2(c_2))} = \Psi(c_2) - \frac{(\bar{c}_2 - c_2)^2}{4b}. \\
\text{Finally, } \pi_2(c_2) &= \frac{(\bar{c}_2 - c_2)^2}{4b}. \text{ Therefore } \pi_1(c_2) + \pi_2(c_2) = \Psi(c_2). \blacksquare
\end{aligned}$$

Proof of Proposition 5.2

Taking into account that in a NE $\frac{\partial \pi_i}{\partial y_i} = 0$, $i = 1, 2$, we have that

$$\frac{d\Pi}{dc_2} = \frac{\partial \pi_1}{\partial y_2}\frac{dy_2}{dc_2} + \frac{\partial \pi_2}{\partial y_1}\frac{dy_1}{dc_2} + \frac{\partial \pi_2}{\partial c_2}. \tag{8.1}$$

At $c_2 = \bar{c}_2$, $x_2 = 0$. Thus, $\frac{\partial \pi_2}{\partial c_2} = -x_2 = 0$. Under CE, $\frac{\partial \pi_2}{\partial x_1} = \frac{\partial p_2}{\partial x_1} x_2 = 0$. Under BE

$\frac{\partial \pi_2}{\partial p_1} = (p_2 - \bar{c}_2)\frac{\partial x_1}{\partial p_1} = 0$. Differentiating FOC of NE we obtain that

$$\frac{dy_1}{dc_2} = \frac{\frac{\partial^2 \pi_1}{\partial y_1 y_2}\frac{\partial^2 \pi_2}{\partial c_2 y_2}}{\frac{\partial^2 \pi_1}{\partial y_1^2}\frac{\partial^2 \pi_2}{\partial y_2^2} - \frac{\partial^2 \pi_1}{\partial y_1 y_2}\frac{\partial^2 \pi_2}{\partial y_1 y_2}} \quad \text{and} \quad \frac{dy_2}{dc_2} = \frac{-\frac{\partial^2 \pi_1}{\partial y_1^2}\frac{\partial^2 \pi_2}{\partial c_2 y_2}}{\frac{\partial^2 \pi_1}{\partial y_1^2}\frac{\partial^2 \pi_2}{\partial y_2^2} - \frac{\partial^2 \pi_1}{\partial y_1 y_2}\frac{\partial^2 \pi_2}{\partial y_1 y_2}} \tag{8.2}$$

Plugging (8.2) in (8.1), and taking into account $A1b$)

$$\begin{aligned} \frac{d\Pi}{dc_2} \mid_{c_2=\bar{c}_2} &= \frac{\partial \pi_1}{\partial y_2}\frac{dy_2}{dc_2} = \frac{\partial \pi_1}{\partial y_2}\frac{-\frac{\partial^2 \pi_1}{\partial y_1^2}\frac{\partial^2 \pi_2}{\partial c_2 y_2}}{\frac{\partial^2 \pi_1}{\partial y_1^2}\frac{\partial^2 \pi_2}{\partial y_2^2} - \frac{\partial^2 \pi_1}{\partial y_1 y_2}\frac{\partial^2 \pi_2}{\partial y_1 y_2}}. \\ sign\frac{d\Pi}{dc_2} \mid_{c_2=\bar{c}_2} &= sign\frac{\partial \pi_1}{\partial y_2}\frac{\partial^2 \pi_2}{\partial c_2 y_2} \end{aligned}$$

In BE, $\frac{\partial \pi_1}{\partial y_2} = p_1\frac{\partial x_1}{\partial p_2}$ and $\frac{\partial^2 \pi_2}{\partial c_2 y_2} = -\frac{\partial x_2}{\partial p_2} > 0$ and in CE, $\frac{\partial \pi_1}{\partial y_2} = \frac{\partial p_1}{\partial x_2}$ and $\frac{\partial^2 \pi_2}{\partial c_2 y_2} = -1$. Thus, in both cases $\frac{d\Pi}{dc_2} \mid_{c_2=\bar{c}_2} > 0 \Leftrightarrow$ Good 2 is substitute of good 1. ■

Proof of Proposition 5.3:

Proof. From (8.1), $A.2$ and $\frac{\partial \pi_2}{\partial c_2} = -x^*$, we obtain that

$$\frac{d\Pi}{dc_2} = \frac{\partial \pi_1}{\partial y_2}[\frac{dy_2}{dc_2} + \frac{dy_1}{dc_2}] - x^*$$

Now from (8.2), and $A.2$ again

$$\frac{dy_2}{dc_2} + \frac{dy_1}{dc_2} = \frac{\frac{\partial^2 \pi_2}{\partial c_2 y_2}(\frac{\partial^2 \pi_1}{\partial y_1 y_2} - \frac{\partial^2 \pi_1}{\partial y_1^2})}{(\frac{\partial^2 \pi_1}{\partial y_1^2})^2 - (\frac{\partial^2 \pi_1}{\partial y_1 y_2})^2} = \frac{-\frac{\partial^2 \pi_2}{\partial c_2 y_2}}{\frac{\partial^2 \pi_1}{\partial y_1^2} + \frac{\partial^2 \pi_1}{\partial y_1 y_2}}$$

Let us tackle first the case of price competition. In this case, $-\frac{\partial^2\pi_2}{\partial c_2 y_2} = \frac{\partial x_2}{\partial p_2}$ and $\frac{\partial \pi_1}{\partial y_2} = (p^* - c_1)\frac{\partial x_1}{\partial p_2}$. From the FOC of a BE $(p^* - c_1) = \frac{-x^*}{\frac{\partial x_1}{\partial p1}}$, thus $\frac{\partial \pi_1}{\partial y_2} = \frac{-x^*}{\frac{\partial x_1}{\partial p1}}\frac{\partial x_1}{\partial p_2}$. Then, using symmetry again,

$$\frac{d\Pi}{dc_2} = \frac{-x^*}{\frac{\partial x_1}{\partial p1}}\frac{\partial x_1}{\partial p_2}\frac{\frac{\partial x_2}{\partial p_2}}{\frac{\partial^2\pi_1}{\partial p_1^2} + \frac{\partial^2\pi_1}{\partial p_1 p_2}} - x^* = \frac{-x^*\frac{\partial x_1}{\partial p_2}}{\frac{\partial^2\pi_1}{\partial p_1^2} + \frac{\partial^2\pi_1}{\partial p_1 p_2}} - x^* = x^*\frac{-\frac{\partial x_1}{\partial p_2} - \frac{\partial^2\pi_1}{\partial p_1^2} - \frac{\partial^2\pi_1}{\partial p_1 p_2}}{\frac{\partial^2\pi_1}{\partial p_1^2} + \frac{\partial^2\pi_1}{\partial p_1 p_2}}$$

By $A1b$) the denominator is negative so $sign\frac{d\Pi}{dc_2} = sign\ (\frac{\partial x_1}{\partial p_2} + \frac{\partial^2\pi_1}{\partial p_1^2} + \frac{\partial^2\pi_1}{\partial p_1 p_2})$

Computing, $\frac{\partial^2\pi_1}{\partial p_1^2} = \frac{\partial^2 x_1}{\partial p_1^2}(p^* - c_1) + 2\frac{\partial x_1}{\partial p_1}$ and $\frac{\partial^2\pi_1}{\partial p_1 p_2} = \frac{\partial^2 x_1}{\partial p_1 p_2}(p^* - c_1) + \frac{\partial x_1}{\partial p_2}$.
$\frac{\partial x_1}{\partial p_2} + \frac{\partial^2\pi_1}{\partial p_1^2} + \frac{\partial^2\pi_1}{\partial p_1 p_2} = (\frac{\partial^2 x_1}{\partial p_1^2} + \frac{\partial^2 x_1}{\partial p_1 p_2})(p^* - c_1) + 2(\frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial p_2}) < 0,$ by A.4.

Next consider the case of quantity competition. In this case, $\frac{\partial^2\pi_2}{\partial c_2 y_2} = -1$ and $\frac{\partial \pi_1}{\partial y_2} = x^*\frac{\partial p_1}{\partial x_2}$. Then, using symmetry again,

$$\frac{d\Pi}{dc_2} = \frac{x^*\frac{\partial p_1}{\partial x_2}}{\frac{\partial^2\pi_1}{\partial x_1^2} + \frac{\partial^2\pi_1}{\partial x_1 x_2}} - x^* = x^*\frac{\frac{\partial p_1}{\partial x_2} - \frac{\partial^2\pi_1}{\partial x_1^2} - \frac{\partial^2\pi_1}{\partial x_1 x_2}}{\frac{\partial^2\pi_1}{\partial x_1^2} + \frac{\partial^2\pi_1}{\partial x_1 x_2}}$$

By $A1b$) the denominator is negative so $sign\ \frac{d\Pi}{dc_2} = sign\ (\frac{\partial p_1}{\partial x_2} - \frac{\partial^2\pi_1}{\partial x_1^2} - \frac{\partial^2\pi_1}{\partial x_1 x_2})$

$$\begin{aligned} \text{Computing, } \frac{\partial^2 \pi_1}{\partial x_1^2} &= \frac{\partial^2 p_1}{\partial x_1^2} x_1 + 2\frac{\partial p_1}{\partial x_1} \text{ and } \frac{\partial^2 \pi_1}{\partial x_1 x_2} = \frac{\partial^2 p_1}{\partial x_1 x_2} x^* + \frac{\partial p_1}{\partial x_2}. \\ sign\ \frac{d\Pi}{dc_2} &= sign[-(\frac{\partial^2 p_1}{\partial x_1 x_2} + \frac{\partial^2 p_1}{\partial x_1^2})x^* - 2\frac{\partial p_1}{\partial x_1}] > 0, \text{ by A.4'. } \blacksquare \end{aligned}$$