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A MATCHING MODEL OF CROWDING-OUT AND ON-THE- JOB SEARCH (WITH AN APPLICATION TO SPAIN) *

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Abstract

This paper considers a matching model of heterogenous and jobs which includes on-the-job search. High-educated workers transitorily accept unskilled jobs and continue to search for skilled jobs. We study the implications of this model for the unemployment rates of high and low-educated workers, for the share of mismatched workers and wage nequality both within and between skill groups. The model is used to shed light on the Spanish experience following a large educational upgrading since the mid-eighties.

Keywords: crowding-out, matching, on-the-job search, unemployment.

JEL Clasification: J41, J62.

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INTRODUCTION

The aim of this paper is twofold. The first one is theoretical and is related to recent papers in the literature on matching models of the labour market which deal with skill differences across workers and skill requirements across jobs.[1] Our contribution in this area is to allow for on-the-job-search in a model close in spirit to Albrecht and Vroman (2002) (AV, henceforth) where low and high-educated workers can be hired for unskilled jobs while only high-educated workers can perform skilled jobs. In particular, the introduction of on-the-job search by high-educated workers who are employed in unskilled jobs gives rise to new theoretical results regarding both between and within wage inequality and, most importantly, crowding-out effects on the employability of low-educated workers in unskilled jobs. This outcome motivates the second goal of our paper which is an empirical one and draws upon the intense drive of upper education in Spain since the mid-eighties that, in turn, has given rise to some of the phenomena that we try to replicate with our theoretical model. Specifically, as documented in Dolado et al: (2000), starting from one of the lowest stocks of human capital in the OECD during the seventies (8% of the population aged 10-14 and 40% of those above 65 were illiterate), Spain has experienced a remarkable improvement in the relative supply of highly educated workers as a result of the extension of compulsory education to 16 years of age and the creation of many new (mostly) public and private universities, the former with very low enrollment fees financed through general taxation. Indeed, the number of undergraduates in Spanish universities is about 1.5 million in an total population of 41 million, almost the same number as in Germany with a total population which doubles the Spanish one.

This rapid educational upgrading has produced a number of effects which we try to understand by means of a matching model which incorporates two types of workers and two types of jobs where on-the-job-search is allowed to take place. In a market subject to search frictions, high-educated workers may end up in unskilled jobs which they are

[1] See, inter alia, McKenna (1996), Acemoglu (1999), Gautier (1999), Mortensen and Pissarides (1999), Albrecht and Vroman (2002) and Albrecht et al. (2002).

willing to accept transitorily, as long as they pay a larger wage than their reservation wage. This gives rise to an over-education phenomenon in the sense that workers' educational attainments surpass the skill requirements of jobs. At the same time, they engage in an on-the-job search for more adequate jobs to their educational attainments which a®ects both the creation of either type of vacancy and the prospects of less-educated workers in ¯nding an unskilled job. To the extent that this process of on-the-job search leads to a lower job-¯nding rate of less-skilled workers, despite the likely increase in the supply of unskilled jobs when the high-skilled workers are willing to take them, it will give rise to a crowding-out phenomenon.

Figure 1. Jobs' types, workers' types and °ows

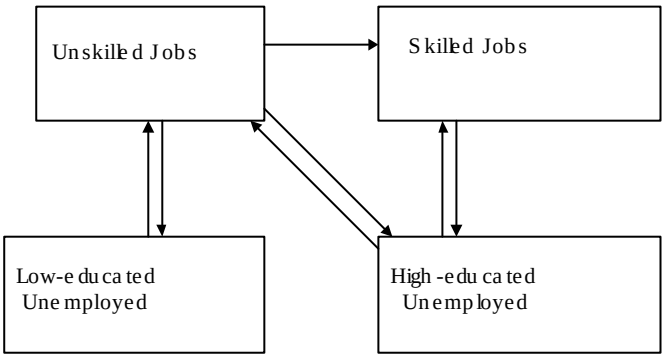


Figure 1 illustates the main di®erence between our model and AV with which it shares several features. In both models, high-educated workers are allowed to search for both skilled and unskilled jobs, low-educated workers can only be hired for unskilled jobs, search is undirected and the distribution of workers' skills is taken to be exogenously given[2]. Thus,

[2]In recent works by Albrecht et al: (2002) and Bl¶zquez (2002), the investment on higher education is endogeneously determined within a matching model with both heterogenous workers and jobs. However,

the mass of searchers in AV is formed by unemployed workers of either type. This implies that there is a direct interaction between both types of workers so that less-educated workers can bene¯t from an increase in the share of unskilled vacancies and viceversa. However, in contrast to AV, we relax the assumption whereby high-educated workers get stuck in unskilled jobs until they are destroyed. In particular, our alternative assumption is that these mismatched workers can search as e±ciently for skilled jobs as unemployed workers. This creates an additional mass of searchers that is depicted in the graph by the arrow from unskilled jobs to skilled jobs, re°ecting the high-educated workers who move from unskilled jobs to skilled jobs.

Our assumption of on-the-job search by mismatched workers can be seen as an alternative that lies somewhere in between of random search and perfectly directed search. Under random search, wages and job characteristics play no allocative role whereas, under directed search, workers can perfectly target jobs. In our set-up, high-educated workers will rationally accept unskilled jobs as long as the wage paid to them when performing those jobs is above their outside option value, and they will subsequently stream onwards for better jobs. As mentioned above, in AV's set-up high-educated workers either accept an unskilled job and remain there until the job is destroyed, or they refuse the o®er, leading to two kinds of equilibrium outcomes: one where both high and low-educated workers have unskilled jobs (cross-skill matching) and another where high-educated workers only take skilled jobs (ex-post segmentation). In this framework, one of the most interesting results in AV is that there is a threshold value of the productivity in skilled jobs beyond which there will be a shift from a cross-skill matching equilibrium to one with ex-post segmentation. By contrast, allowing for on-the-job search as we do, implies that there will be more mismatch between educational attainments and job requirements than in AV and that it will take much higher values of such a productivity before moving to an ex-post segmentation regime.

Some other contrasting results between both models relate to AV's ¯nding that the vacancy-unemplyoment ratio (tightness) is invariant to changes in the productivity of high-

insofar as the enrolment fees in Spanish universities are very low, we believe that the chosen assumption is not too restrictive.

educated workers in skilled jobs and in the proportion of high-educated individuals in the population (in the cross-skill matching equilibrium regime). This is no longer the case in our model once on-the-job search is allowed for. For example, skill-biased technical progress increases tightness, since a higher productivity of high-educated workers increases the supply of skilled jobs, and reduces the proportion of on-the-job search. Furthermore, the improved sorting of workers gives …rms with an unskilled job a higher probability of …nding a stable worker and given our assumption that both types of workers are equally productive on unskilled jobs, this increases the supply of unskilled vacancies as well. In contrast, in the case of an increase in the proportion of high-education in the population, we obtain a decrease in tightness as the increase in the number of skilled vacancies is more than o®set by a decrease in the number of unskilled vacancies which form the majority of jobs that now face more di±culties in attracting stable workers.

A second di®erence lies in the e®ect of an increase in the relative supply of high-educated workers on their wages. Whereas in AV's model, it leads to a reduction in those wages, in our model the demand e®ect induced by the larger supply of high-educated workers is even larger, so that those wages increase.

The rest of the paper is structured as follows. Section 2 presents the model and discusses the properties of the steady-state equilibrium in terms of its existence and uniqueness. Section 3 summarises a few relevant stylised facts about the interaction between the educational drive and the working of the labour market in Spain. In light of those stylised facts, Section 4 discusses a few simulations of the proposed model which serve to shed light on the the Spanish experience. Finally, Section 5 draws some concluding remarks. The algebraic details of the proof of existence and uniqueness are provided in an Appendix.

THE MODEL

Main assumptions

Consider an economy populated by a continuum of workers with measure normalised to one. A share $\mu \in (0;1)$ of these workers is high-educated (H) while the remaining portion

$1 - \iota$ is low-educated (L). Moreover, all workers are risk neutral and infinitely-lived and time is continuous.

There are two types of jobs: skilled jobs, denoted by S, and unskilled jobs denoted by U. An unskilled jobs can be performed by both types of workers, while a skilled job requires a high-educated worker. Furthermore, we assume that both types of workers have the same productivity on an unskilled job. Formally, let y_{ij} denote the flow output of a job of type $i \in I = (S;U)$ that is filled by a worker of type $j \in J = (H;L)$. Our assumptions on the production technology can then be summarised as follows:

$$y_{SH} > y_{UH} = y_{UL} > y_{SL} = 0:$$

For convenience, firms can open at most one job. The choice of the type of job is irreversible and the mass of each type of job is determined by a free entry condition.

Finally, job destruction is completely exogeneous. A filled job of type i is destroyed at the Poisson rate $\pm_i \in R^+$ and we assume that $\pm_U > \pm_S$: Whenever a job is destroyed the worker becomes unemployed while the job becomes vacant.

The labour market is characterised by matching frictions. Moreover, unlike AV we allow for on-the-job search. Specifically, the total number of matches between a worker and a firm is determined by a constant returns to scale matching function

$$m(v_U + v_S; u_L + u_H + e_{UH});$$

where u_j is the mass of unemployed workers of type j, v_i denotes the mass of vacancies of each type of job and e_{UH} is the mass of high educated workers in unskilled jobs. We assume that $m(:;:)$ is strictly increasing in both arguments and we denote the labour market tightness by $\mu = (v_U + v_S)/(u_L + u_H + e_{UH})$: Accordingly, the arrival rate of firms is equal to $q(\mu) = m(1;\frac{1}{\mu})$, but some skilled jobs will meet a worker who is not qualified. Formally, let a_H denote the share of searching workers who are high-educated. The effective matching rate of a skilled job is then $a_H q(\mu)$. Similarly, the matching rate of workers is $\mu q(\mu)$, but a low-educated worker cannot perform a skilled job. In what follows we denote the share of vacant jobs that require a high-educated worker by b_S: The effective matching rate of a

low-educated worker is thus equal to $(1 - b_S)\mu q(\mu)$: Finally, the properties of the matching function imply that the matching rate of workers (firms) is increasing (decreasing) in μ and we assume

$$\lim_{\mu \to 0} q(\mu) = \lim_{\mu \to \infty} \mu q(\mu) = 1 \text{ and } \lim_{\mu \to \infty} q(\mu) = \lim \mu q(\mu) = 0.$$

Below we concentrate on the steady state equilibrium in which high-educated workers accept both types of jobs and engage in on-the-job search. This cross-skill matching equilibrium can be summarised by a vector $\{\mu; b_S; u_L; u_H; e_{UH}\}$ and needs to satisfy the following conditions: (i) match formation is voluntary (ii) the expected profit of each type of job is equal to zero (iii) the state variables $u_L; u_H$ and e_{UH} satisfy the appropriate steady state conditions. These flow equations will be derived in Section 2.3, but first we will derive the payoffs of workers and firms.

Wages and asset values

In the equilibrium of our interest there are three types of matches: high-educated workers on skilled jobs, high-educated workers on unskilled jobs and low-educated workers on unskilled jobs. In each of these matches, the firm-worker pair divides the surplus of the match according to the asymmetric Nash bargaining solution. The exogenous surplus share of workers is denoted by $\beta \in (0; 1)$. Moreover, we adopt the following notation: U_j denotes the value of unemployment for a worker of type j, V_i denotes the value of a vacant job of type i; W_{ij} denotes the value of employment for a worker of type j on a job of type i and J_{ij} denotes the value to the firm of filling a job of type i with a worker of type j. Accordingly, the surplus of a match between a job of type i and a worker of type j can be expressed as $S_{ij} = W_{ij} + J_{ij} - V_i - U_j$ and the corresponding wage w_{ij} solves the Nash bargaining condition:[3]

[3]Notice that the threat point of a worker is equal to the value of unemployment. We therefore exclude the possibility that mismatched workers can use employment in an unskilled job to negotiate a higher wage in a skilled job.

$$W_{ij} - U_j = \beta(W_{ij} + J_{ij} - V_i - U_j). \tag{1}$$

We now continue with the derivation of the various asset value equations. Let $r \in \mathbb{R}^+$ denote the common discount rate of firms and workers and let z_j denote the flow income of an unemployed worker of type $j \in J$. The value of employment for a low-educated worker, denoted by W_{UL}, then satisfies:

$$rW_{UL} = w_{UL} - \delta_U [W_{UL} - U_L]. \tag{2}$$

Similarly, the expected lifetime income of a high-educated worker on a skilled job, W_{SH}, satisfies:

$$rW_{SH} = w_{SH} - \delta_S [W_{SH} - U_H] \tag{3}$$

The corresponding profits of the firm are also easily derived. These values satisfy the following standard asset value equations:

$$rJ_{UL} = y_{UL} - w_{UL} - \delta_U [J_{UL} - V_U] \tag{4}$$

$$rJ_{SH} = y_{SH} - w_{SH} - \delta_S [J_{SH} - V_S]. \tag{5}$$

The asset value of a mismatched worker, i.e. a high-educated worker in an unskilled job, is slightly more involved. Since these workers are engaged in on-the-job search, they will quit their job at the Poisson rate $\mu q(\mu) b_S$, incurring an income gain equal to $W_{SH} - W_{UH}$. Accordingly, the asset value of a mismatched worker, W_{UH}, satisfies

$$rW_{UH} = w_{UH} - \delta_U [W_{UH} - U_H] + \mu q(\mu) b_S [W_{SH} - W_{UH}], \tag{6}$$

while the associated profit of the firm, J_{UH}, satisfies:

$$rJ_{UH} = y_{UH} - w_{UH} - (\mu q(\mu) b_S + \delta_U) [J_{UH} - V_U]. \tag{7}$$

Finally, in our cross-skill matching equilibrium the values of unemployment satisfy:

$$rU_L = z_L + \mu q(\mu)(1 - b_S)[W_{UL} - U_L] \quad (8)$$

$$rU_H = z_H + \mu q(\mu)[(1 - b_S)W_{UH} + b_S W_{SH} - U_H]: \quad (9)$$

According to (9), the expected income of a high-educated unemployed is a weighted average of the expected income in skilled and unskilled jobs. Furthermore, in our economy, skilled jobs are both more productive and more stable than unskilled jobs. Other things equal, high-educated workers therefore bene¯t from an increase in b_S. However, such a shift in the composition of the pool of vacancies hurts low-educated workers as it lowers their exit rate out of unemployment.

A similar logic applies to jobs. Let c_i denotes the °ow cost of a vacant job of type $i \in I$. The value of a vacant job is then given by:

$$rV_U = -c_U + q(\mu)[(1 - a_H)J_{UL} + a_H J_{UH} - V_U] \quad (10)$$

$$rV_S = -c_S + q(\mu)a_H[J_{SH} - V_S]: \quad (11)$$

Thus, other things equal the pro¯ts of a skilled vacancy increase with a_H, while the response of V_U to changes in a_H will depend on the relative productivity of mismatched workers (see, e.g. Gautier 2002). Nonetheless, since high-educated workers exit these jobs at a higher rate than low-educated workers while both types of workers have the same productivity, ($y_{UH} = y_{UL}$); this relationship will be a negative one in our economy.

To conclude this section we also report the solutions for the three di®erent wages w_{UL}, w_{UH} and w_{SH}. Substituting eqs. (2)-(11) into (1) and imposing $V_i = 0$ for $\forall i \in I$ yields:

$$w_{UL} = rU_L + \beta(y_{UL} - rU_L) \quad (12)$$

$$w_{SH} = rU_H + \beta(y_{SH} - rU_H) \tag{13}$$

$$w_{UH} = rU_H + \beta(y_{UH} - rU_H) - (1-\beta)\mu q(\mu)b\frac{(y_{SH} - rU_H)}{r+\delta_S}. \tag{14}$$

According to (12), low-educated workers obtain a share β of the flow surplus of their job. The same is true for high educated workers who are matched to a skilled job. Mismatched workers, on the contrary, receive less than β of the flow surplus $y_{UH} - rU_H$ as the firm appropriates a share $(1-\beta)$ of the expected capital gain $\mu q(\mu)b_S(W_{SH} - W_{UH})$ from succesful on-the-job search.

EQUILIBRIUM

To solve for the cross-skill matching equilibrium with on-the-job search we need to find the equilibrium values of the five endogeneous variables μ, b_S, u_L, u_H and e_{UH}. These values are found using the free entry conditions for skilled and unskilled jobs plus the three flow equations for the state variables.

We start with a derivation of the free entry conditions. In a first step, we substitute the wage equations into the expressions for $J_{UL}; J_{UH}$ and J_{SH}:[4]

$$J_{UL} = (1-\beta)\left[\frac{y_{UL} - rU_L}{r+\delta_U}\right] \tag{15}$$

$$J_{SH} = (1-\beta)\left[\frac{y_{SH} - rU_H}{r+\delta_S}\right] \tag{16}$$

$$J_{UH} = (1-\beta)\left[\frac{y_{UH} - rU_H}{r+\delta_U+\mu q(\mu)} + \mu q(\mu)b_S\frac{(y_{SH} - rU_H)}{r+\delta_S}\right] \tag{17}$$

In each of these expressions the term between brackets denotes the value of the match

[4]The same solutions are obtained by substituting the asset values for J_{ij}, W_{ij} and U_j into the Nash bargaining condition imposing the zero profit condition $V_i = 0 \; \forall i \; 2 \; I$.

surplus. To obtain the reservation wage of workers we need to substitute these surplus expressions into the value functions for U_L and U_H: Taking into account that a worker's wage satisfies the Nash bargaining solution this yields

$$rU_L = \frac{(r + \delta_U)z_L + \mu q(\mu)(1 - b_S)\bar{y}_{UL}}{r + \delta_U + \mu q(\mu)(1 - b_S)}; \tag{18}$$

for the flow income of an unemployed worker with low education and

$$rU_H = \frac{(r + \delta_U + \mu q(\mu)b_S)z_H + \mu q(\mu)(1 - b_S)\bar{y}_{UH}}{r + \delta_U + \mu q(\mu)(b_S + (1 - b_S)\bar{})} + \frac{\bar{}}{1 - \bar{}}\left(\frac{b_S}{a_H}\right)c_S\mu; \tag{19}$$

for the flow income of unemployed workers with high education. When $b_S = 0$ both expressions take the same form. In contrast, when $b_S > 0$; high-educated workers may end up in a skilled job. The associated income gain $\mu q(\mu)b_S\bar{}S_{SH}$ is captured by the second term on the right hand side of (19).

Finally, substituting the reservation wages into (10) and (11), and imposing the condition that $V_U = V_S = 0$; yields the following free entry conditions:

$$\frac{c_U}{q(\mu)} = (1 - \bar{})\left(\frac{1 - a_H}{}\right)\left(\frac{\bar{y}_{UL} - z_L}{,_1}\right) + a_H\left(\frac{\bar{y}_{UH} - z_H}{,_2}\right) \tag{20}$$

$$\frac{c_S}{a_H q(\mu)} = (1 - \bar{})\left(\frac{\bar{y}_{SH} - z_H}{,_3}\right) - \left(\frac{\mu q(\mu)\bar{}(1 - b_S)(\bar{y}_{UH} - z_H)}{,_2,_3}\right) \tag{21}$$

where

$$\begin{aligned} ,_1 &\equiv r + \delta_U + \mu q(\mu)(1 - b_S)\bar{} \\ ,_2 &\equiv r + \delta_U + \mu q(\mu)[b_S + (1 - b_S)\bar{}] \\ ,_3 &\equiv r + \delta_S + \mu q(\mu)b_S\bar{}: \end{aligned}$$

Equations (20) and (21) are our first two equilibrium conditions. The remaining equilibrium conditions follow from the steady state conditions for u_L, u_H and e_{UH}. First of all,

recognising that the mass of employed workers with low education is $1 - \tau - u_L$, we can express the steady state condition for u_L as:

$$\mu q(\mu)(1 - b_S)u_L = \pi_U[1 - \tau - u_L] \tag{22}$$

On the left-hand side we have the mass of low-educated workers who find employment per unit of time, while the term on the right measures the flow into unemployment per unit of time.

Similarly, since the mass of high-educated workers on skilled jobs is equal to $\tau - u_H - e_{UH}$, the steady state conditions for u_H and e_{UH} reduce to:

$$\mu q(\mu)(1 - b_S)u_H = [\pi_U + \mu q(\mu)b_S]e_{UH} \tag{23}$$

$$\mu q(\mu)b_S[u_H + e_{UH}] = \pi_S[\tau - u_H - e_{UH}] \tag{24}$$

From (22) it follows immediately that u_L is equal to:

$$u_L = \frac{\pi_U}{\pi_U + \mu q(\mu)(1 - b_S)}(1 - \tau): \tag{25}$$

This equation shows that u_L is positively related to π_U and b_S, while the unemployment rate of low-educated workers tends to decrease with μ:

Similarly, from (24), it follows that $u_H + e_{UH}$ is given by $\tau\pi_S/(\pi_S + \mu q(\mu)b_S)$ and, together with (23), this implies that:

$$u_H = \frac{\mu q(\mu)b_S + \pi_U}{\mu q(\mu) + \pi_U}\left(\frac{\pi_S}{\pi_S + \mu q(\mu)b_S}\right)\tau \tag{26}$$

$$e_{UH} = \frac{\mu q(\mu)(1 - b_S)}{\mu q(\mu) + \pi_U}\left(\frac{\pi_S}{\pi_S + \mu q(\mu)b_S}\right)\tau: \tag{27}$$

This completes our derivation of the equilibrium. The procedure to find the vector (μ, b_S, u_L, u_H, e_{UH}) that solves equations (20),(21) and (25)-(27) is simple. In a first step we

guess a value for $a_H \acute{} (u_H + e_{UH})/(u_L + u_H + e_{UH})$ and solve (20) and (21) for (μ, b_S): Together with the °ow equations this yields a realisation for the remaining variables and we repeat this procedure until the realisation of a_H coincides with our initial guess. Finally, in a last step we need to verify that the match surplus S_{UH} is positive so that it is optimal for high-educated workers to accept an unskilled job.

Existence and Uniqueness

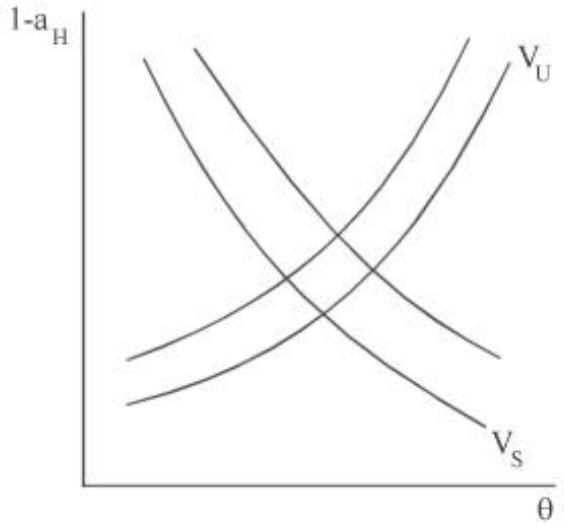
This section brie°y discusses the conditions for existence and uniqueness of a cross-skill matching equilibrium. The treatment is based on the analysis in AV (2002). As shown by AV once it is ensured that ¯rms are willing to create skilled jobs, three equilibrium con¯gurations may occur. First, if the fraction of low-educated workers, $1 ¡ ¹$; is large and the di®erence between y_{UH} and y_{SH} is small, it is worthwhile for an individual high-educated worker to accept an unskilled job, even if all other high-educated were to reject such jobs and there exists a unique cross-skill matching equilibrium. Conversely, if $¹$ is small and the di®erence between skill requirements is large, the opposite would happen and the unique equilibrium is the ex-post segmentation one. Finally, for an intermediate range of parameter values, the two pure-strategy equilibria may coexist.

The basic ¯nding of AV is that there are two loci representing the free-entry conditions in the croos-skill matching equilibrium, denotes by $V_U(y_{UH}) = 0$ and $Vs(y_{SH}) = 0$, respectively. The ¯rst locus is upward sloping in the $(\mu; 1 ¡ a_H)$ plane whilst the second locus is downward sloping in the same plane. Hence, the equilibrium is always unique. Furthermore, in the absence of on-the-job search μ does not depend on either $¹$ or y_{SH}; implying that an increase in y_{SH} shifts both loci upwards increasing $1 ¡ a_H$; yet leaving μ unchanged. Additionally, increases in y_{SH} eventually shift the cross-skill equilibrium to an ex-post segmentation one.

By contrast, once on-the-job search is allowed for, the possibilty of having an ex-post segmentation equlibrium decreases since now a higher value of y_{SH} is needed to shift equlibria. In the Appendix, we discuss in detail the circumstances under which the cross-skill equi-

librium remains as the valid one. In particular, we derive conditions under which V_U and V_S share the same slopes as in AV's analysis in the $(\mu; 1 - a_H)$ plane. The strategy is to prove that $V_U = V_U(\mu; a_H)$ with $\pm V_U/\pm\mu < 0$ and $\pm V_U/\pm a_H < 0$; and that $V_S = V_S(\mu; a_H)$ with $\pm V_S/\pm\mu < 0$ and $\pm V_S/\pm a_H > 0$: For that, a su±cient condition is that the elasticity of $\mu q(\mu)$ w.r.t. μ is larger than the elasticity of b_S w.r.t. μ .Nonetheless, it is no longer the case that $\pm\mu/\pm^1 = \pm\mu/\pm y_{SH} = 0$: As it will be checked in the simulations and is depicted in Figure 2, a rise of y_{SH} shifts the $V_S = 0$ locus to the right since, for a given value of a_H; μ needs to increase to restore zero pro¯ts, while it shifts the $V_U = 0$ locus to the left as the increase in the number of skilled vacancies reduces a_H for a given value of μ. In general, the net e®ect on μ is ambiguous. However, in our simulations, the shift of V_S is larger and the overall labour market tightness μ increases. Likewise, an increase of ^{1} moves the two loci in a similar way but this time the shift in V_U is larger and μ decreases.

Figure 2. Comparative statics of an increase in y_{hs}



As mentioned in the Introduction, one of the main motivations for our study is the evolution of the Spanish youth labour market since the mid-eighties, following a remarkable improvement in the supply of highly educated workers. An important characteristic of this educational drive is the strong shift towards university / tertiary degrees. This process has been documented in detail in other studies and is illustrated in Table 2, which presents a cross-country comparison of the educational attainment of the population aged 25-64 in the EU as of 1999.

Table 1. Educational Attainment by age groups in EU countries, 1999

	A. Upper secondary education				B. Tertiary Education			
	25-64	25-34/ 35-44	25-34/ 45-54	25-34/ 55-64	25-64	25-34/ 35-44	25-34/ 45-54	25-34/ 55-64
Austria	75	1.1	1.1	1.4	8	0.9	1.3	1.9
Belgium	57	1.2	1.5	2,1	27	1.2	1.4	2.2
Denmark	79	1,1	1.1	1.3	26	1.0	1.0	1.5
Finland	71	1.0	1.3	1.9	31	1.1	1.3	1.8
France	61	1.2	1.4	1.9	21	1.4	1.7	2.6
Germany	76	1.0	1.1	1.2	22	0.8	0.9	1.1
Greece	47	1.2	1.8	3.1	17	1.2	1.8	3.1
Italy	42	1.1	1.5	2.7	9	0.9	1.0	1.8
Luxembourg	59	1.0	1.1	1.5	17	1.2	1.0	1.8
Netherlands	65	1.1	1.3	1.5	23	1.0	1.2	1.5
Portugal	19	1.5	2.1	3.9	9	1.3	1.5	2.7
Spain	33	1.3	2.1	3.9	15	1.4	1.9	3.4
Sweden	77	1.1	1.2	1.4	28	1.1	1.1	1.5
UK	69	1.1	1.3	1.9	24	1.0	1.1	1.8
EU	63	1.1	1.2	1.5	20	1.0	1.2	1.6

Source: OECD (Education at the Glance, 2001)

As it can be observed, Spain has one of the largest shares of the population with at most upper secondary education (67%) and the fraction of the population with higher education is about 75% of the EU average. Nonetheless, the remaining columns in parts A and B of Table 1 indicate that, when comparing the relative educational attainments of cohorts aged 25-34 and 55-64, the proportion of people who just completed upper secondary (tertiary) education in the former cohort is 3.9 (3.4) times higher than in the second cohort and 1.3 (1.4) times higher than in the 35-44 age bracket. A comparison of those ratios with other EU countries shows that Spain has undergone a more intense educational drive than the remaining countries.

As Dolado et al: (2000) have pointed out, this striking evolution of the supply of high-educated workers has not seemingly been matched by an equal increase in the supply of skilled vacancies. Moreover, this seems to have given rise to an over-education phenomenon, in the sense that high-educated workers who work in unskilled jobs tend to have the following characteristics. First, over-educated workers tend to earn less than identically educated workers in skilled jobs and no more than low-educated workers in unskilled jobs. Second, they tend to have higher rates of ¯rm and occupational mobility tending to move to higher-level occupations as a result of exerting on-the-job search. And, third, there is some evidence in favour of crowding-out, in the sense that high-educated workers take jobs away from low-educated ones. The ¯rst two characteristics have been tested by Galindo-Rueda (2001) using a subsample of males with attained tertiary education, in the age bracket 20-65, extracted from the European Community Household Panel (ECHP) for the 1994-1996 period[5]. This dataset contains information on the individual¶s perception of mismatch between his educational attainments and the skills required in the particular job he or she performs. He ¯nds some favourable evidence for higher upward mobility by over-educated workers and a strong evidence, even after correcting for sample-selection problems, in favour

[5]Further evidence stating that returns to higher education has declined over the early nineties can be found in Alba-Ramirez (1993), where it is estimated that 17% of high-educated workers are over-educated. As for evidence in other countries, Muysken and Ter-Weel (1998) and Gautier et al. (2002) discuss the Dutch case.

of over-education leading to lower individual¶s earnings. Further indirect evidence can be found in Bover et al. (2002) where, in a study about the (log) earnings distribution of workers in Spain during the eighties, it is found that the ratio between the 75^{th} and 25^{th} percentiles of workers with tertiary education increased by 8%, whilst the corresponding ratio for lower-educated workers hardly changed.

The third characteristic is illustrated in Figure 3. This graph is borrowed from Dolado et al. (2000) and it depicts the occupational structure of the so-called \entry-jobs" by attained educational degree, that is, the type of job that youth workers take after completing a given educational degree. For that purpose, four age groups and four educational levels have been chosen so that we can analyse the evolution over time of the kind of job that young workers were occupying up to four years after they ¯nished a given degree. Thus, the 16-20 cohort corresponds to upper secondary education and 23-27 to a university degree. The entry-jobs have been classi¯ed in increasing order of \complexity": Professional/Technicians (P1), Teaching Professionals and Employees in Public Administration (P2), Clerical and Administratives (P3), Manual Craft and Operators (P4), and Sales Elementary and Hotel & Restaurants Occupations, Unskilled Services and Labourers (P5). The solid lines represent the proportion of wage earners with a given age and degree who work in a given entry-occupation whereas the dotted line plots the share of each occupation in employment. The lesson to be drawn from this Figure is that the more educated workers seem to be increasingly ¯lling both high-skill jobs (P1) and semi-skill ones (P3), at the expense of a drastic reduction in P2. At the same time, low-educated workers have been \crowded out" from their tradional entry jobs (P3) towards very unskilled jobs (P5).

Figure 3. Entry jobs by age and educational attainments

Men

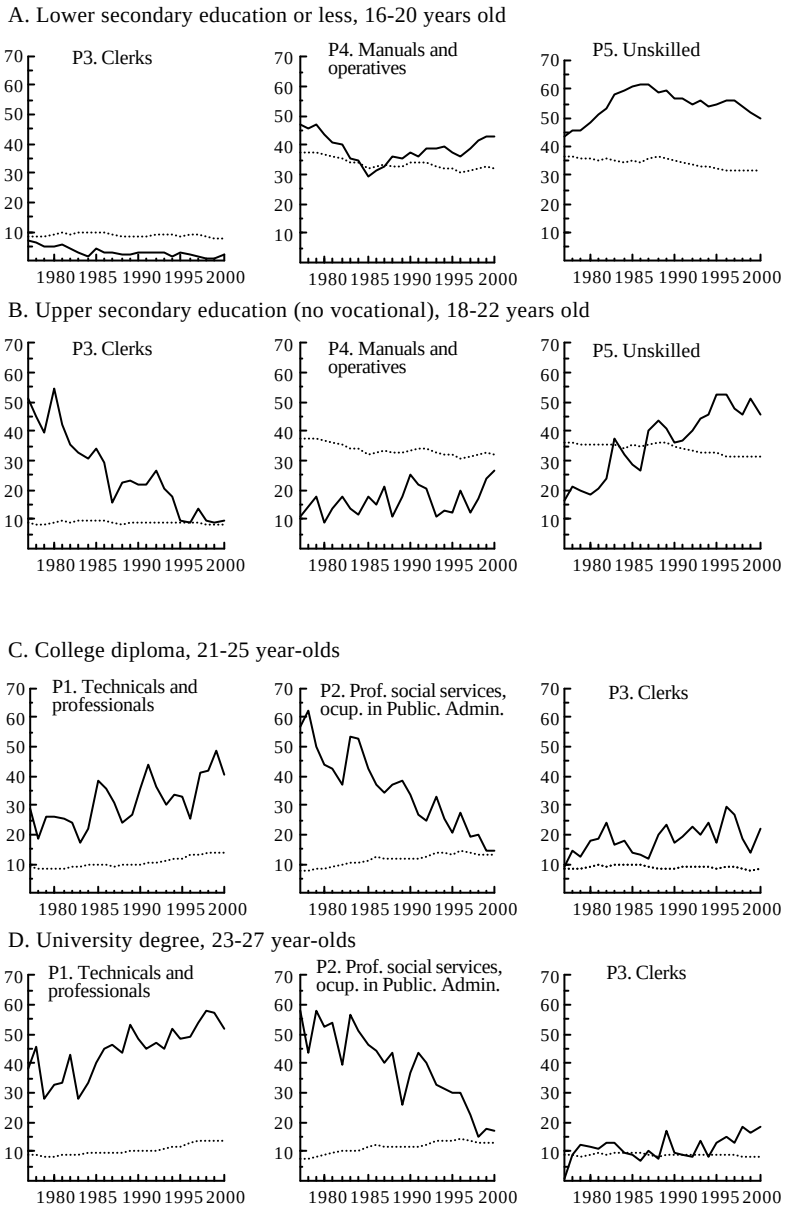
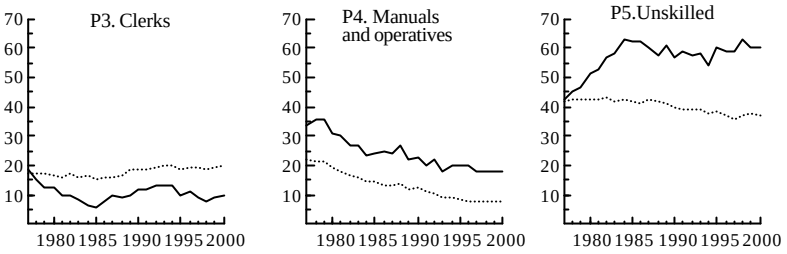


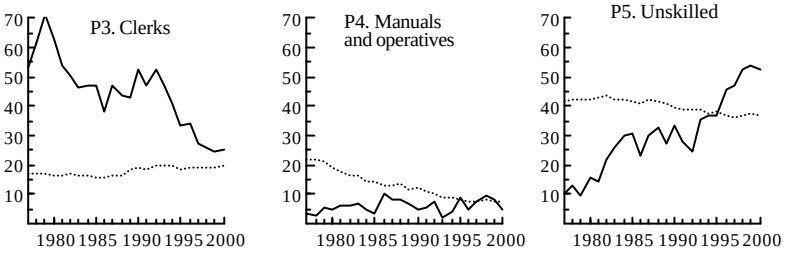
Figure 3 (continued)

Women

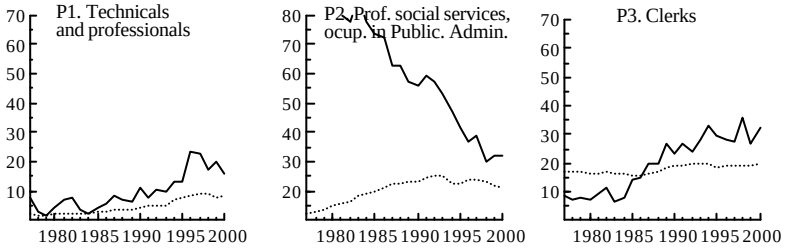
A. Lower secondary education or less, 16-20 years old



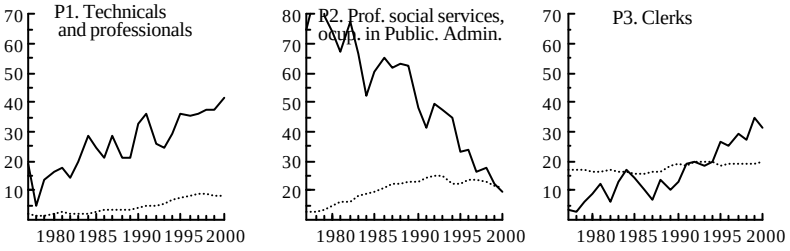
B. Upper secondary education (no vocational), 18-22 years old



C. College diploma, 21-25 year-olds

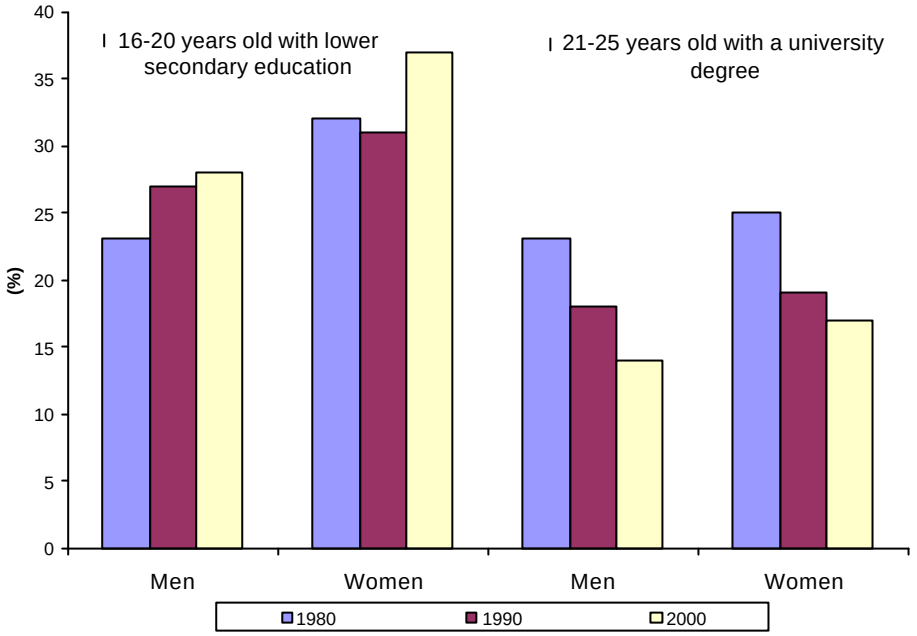


D. University degree, 23-27 year-olds



A further indication of the crowding-out of low-educated workers can be obtained by looking at the evolution of the unemployment rates for low and high-educated workers. As is well known, Spain has had since the mid-eighties one of the higher, if not the highest, unemployment rates in the OECD (varying from 20% in 1985 to 12% nowadays, reaching a peak of 24% in 1994). Moreover, both the unemployment rates of low and high-educated workers have been high (the former moved from 34% in 1985 to 21% in 2000 whereas the latter fell from 20% in 1985 to 13% in 2000). However, that raw comparison does not control for previous job experience. There is strong evidence that, besides education, job experience is a key screening device used by ¯rms when making hiring decisions and that the role of the latter has increased as the over-education phenomenon has spread out (see F¶ernández, 2002). Indeed, once job experience is controlled for, it is found that, while the unemployment rate of low-educated workers has increased since the early eighties, the unemployment rate of high-educated workers has decreased. This is illustrated in Figure 4 where the unemployment rates of low-educated workers (those with lower secondary education) in the age cohort 16-20 are compared with the unemployment rates of high-educated workers (those with a university 1st. degree) in the age cohort 21-25 in 1980, 1990 and 2000 for male and female workers. As it can be observed, there is an upward trend in the former rate whereas there is a downward trend in the latter.

Figure 4. Unemployment rates by age and educational attainment



The above evidence seems to suggest that the skill upgrading of the Spanish labour force and the increased willingness of high-educated workers to take unskilled jobs had a negative e®ect on the employment opportunities of low-skilled workers. Moreover, in the case of Spain, this process seems to aggravated by institutional factors, namely the widespread use of temporary jobs (mostly ¯xed-term contracts subject to low or no severance payments) since the mid-eighties. The rate of incidence of employment under those contracts (share in total salaried employment) reached 33% in the early nineties and has remained resilient eversince. An overall assessment of the implications of the generalised use of those contracts for the working of the Spanish labour market can be found in Dolado et al.(2002). Insofar, as most of those temporary jobs were of low quality, we can interpret their larger availability as a reduction in the cost of opening unskilled vacancies.[6] Accordingly, in section 5 we will simulate the joint e®ect of an increase in both the share of high-educated workers (ι) in

[6]Note that we miss the e®ect of the temporary-contract reform on job separation which, however, is taken to be three times larger for unskilled than for skilled jobs in the simulations. We choose this simpler roye because we wish to concentrate on on-the-job search.

the population and/or in their productivity (y_{SH}), and the increase in the use of temporary contracts, the latter being captured by a reduction in the °ow cost of keeping un¯lled an unskilled vacancy (c_U):

SIMULATIONS AND COMPARATIVE STATICS

In this section we perform a few simulations with the model. Our aim is to examine the comparative statics of the model following a change in some of the model¶s parameters which try to mimic the e®ects on the endogenous variables of: (i) an increase in the proportion of high-educated workers in the economy (^{1}); capturing the educational drive, (ii) an increase in the productivity of high-educated workers in skilled jobs (y_{SH}), representing the e®ects of skill-biased technical change, and (iii) a reduction in the °ow cost of keeping un¯lled an unskilled vacancy, re°ecting the lower cost of creating unskilled vacancies brought about by the introduction of temporary jobs.

The model is calibrated using a standard Cobb-Douglas matching function, $m = \frac{1}{2}(v_U + v_S)^{1=2}(u_L + u_H + e_{SH})^{1=2}$; together with the following parameter con¯guration: $^- = 0{:}5$ (i.e. the Hosios' value), $r = 0{:}05$, $c_S = 1{:}0$; $c_U = 0{:}5$, $y_{UL} = y_{UH} = 1$ (equal productivity in unskilled jobs), $y_{SH} = 2{:}0$; $\pm_S = 0{:}1$, $\pm_U = 0{:}3$; $z_L = 0{:}25$, $z_H = 0{:}25$ and $^1 = 0{:}1$: Thus, in the baseline version of the model, the proportion of educated workers is 10% of the (unit mass) population.

Table 2. Comparative Statics for separate changes in ^{1};y_{HS} and c_U

	Baseline	^{1} = 0:3	y_{HS} = 3	c_U = 0:35
μ	0.799	0.791	0.967	1.070
b_S	0.124	0.301	0.178	0.101
a_H	0.177	0.258	0.136	0.206
$\mathbf{e}_L$	0.161	0.194	0.156	0.139
$\mathbf{e}_H$	0.077	0.063	0.061	0.069
$\mathbf{e}_{UH}$	0.233	0.094	0.158	0.254
w_{SH}	1.621	1.741	2.458	1.622
w_{UH}	0.842	0.780	0.825	0.859
w_{UL}	0.884	0.865	0.886	0.897

The solution for the baseline model is reported in column 1 of Table 2, yielding unemployment rates for low and high-educated workers ($\mathbf{e}_L$ and $\mathbf{e}_H$) of 16.1% and 7.7%, respectively, and a proportion ($\mathbf{e}_{UH}$) of 23.3% of high-educated workers engaged in on-the-job search while working in unskilled jobs[7]. The share of skilled vacancies in total vacancies (b_S) is 12.4% and the proportion of high-educated workers in the pool of searchers (a_H) is 17.7%. Further the three wages are: 1.621 (w_{SH}) for high-educated workers in skilled jobs, 0.842 (w_{SH}) for the same workers in unskilled jobs, and 0.884 (w_{UL}) for low-educated workers in unskilled jobs. Note that, in contrast to the result in AV -where, under the assumption that low and high-educated workers are equally productive in unskilled jobs, the latter get a higher wage than the former due to their higher outside option value- we get an opposite result. The intuition for our result is that the negative effect of on-the-job search on the match surplus and on the wage of mismatched workers outweighs the positive effect on the reservation wage.[8] Had we allowed for $y_{UH} > y_{UL}$; then we could have obtained that $w_{SH} \gtrless w_{SL}$: However, we prefer to keep the chosen baseline specification to

[7]Note that $\mathbf{e}_L$ $(= u_L=(u_L + e_{UL}))$ and $\mathbf{e}_H$ $(= u_H=(u_H + e_{UH} + e_{SH}))$ are rates and not masses. The same applies to $\mathbf{e}_{UH}$ and $\mathbf{e}_{SH}$:

[8]By comparing equations (12) and (14) it can be checked that $w_{UL} > w_{UH}$;with $y_{UL} = y_{UH}$; if $\mu q(\mu) b_S (y_{SH} - rU_H)/(r+\pm_S) > r(U_H - U_L)$:

highlight the differences between our model and AV¶s. As for wages, we will pay attention in what follows to the so-called \within-group" wage inequality (henceforth, within) given by the difference between the wages of high-educated workers in skilled jobs (w_{SH}) and the one received by those workers in unskilled jobs (w_{UH}): This differential is a good proxy for the penalty to over-education. Finally, the equilibrium value of μ is 0:8 which implies an admittedly high unemployment duration of 21 months but this is not too far away from the average duration of unemployment in Spain in the mid-eighties, around 15 months, where, with an overall unemployment rate of about 20%, strong hysteresis has been present (see Dolado and Jimeno, 1998).

In column 2, results are reported for the case where the proportion of high-educated people in the population (^{1}) increases from 0.1 to 0.3. In constrast to AV, where μ is unaffected by changes in ^{1}, we find that μ falls slightly from 0.8 to 0.79. This reduction occurs despite the increase in the supply of skilled vacancies (b_S rises from 12.4% to 30.1%) and the decrease in the share of mismatched workers ($\mathbf{e}_{UH}$ falls from 23.3% to 9.4%) and is due to the increase in a_H from 17.7% to 25.8%. Under our assumption that $y_{UH} = y_{UL}$; this shift in the composition of the pool of searchers towards high-educated workers exerts a negative effect on V_U leading to a fall in the supply of unskilled jobs and to a reduction in the overall labour market tightness.[9] Finally, notice that w_{SH} increases whereas w_{UH} and w_{UL} decrease, with a particularly large reduction in the former wage. Thus, for the parameter configuration chosen here, an increase in the proportion of high-educated individuals gives rise to an increase in the unemployment rate of low-educated workers, yet it reduces both the unemployment rate of high-educated workers and the share of on-the-job searchers.[10] Moreover, we observe an increase in the \within" wage differential for high-educated workers.

Column 3 displays the results stemming from an increase in y_{SH} from 2.0 in the baseline

[9]Notice that the negative effect on μ cannot be explained by a pure compositional effect. In the baseline model, the costs of both types of vacant job are proportional to output. As a result, a shift in the job distribution towards skilled jobs cannot cause a fall in μ unless the supply of unskilled jobs falls.

[10]This result is somewhat similar to the one obtained by Shimer (2001) who observes that an increase in the cohort size improves labour market performance and reduces unemployment.

model to 3.0. This time, again in contrast to AV¶s results where μ is invariant to changes in y_{SH}, μ increases strongly to 0.967. Furthermore, the increase in b_S implies that the higher productivity of high-educated workers leads to a larger creation of skilled vacancies which are now more attractive to ¯rms. This of course leads to a lower θ_H (from 7.7% to 6.1%) and a to much smaller proportion of on-the-job searchers (from 23.3% to 15.8%). However, interestingly enough, the improved sorting of the two types of workers also gives rise to a decrease in θ_L as ¯rms with an unskilled job ¯nd it easier to locate a low-educated worker (i.e. a_H falls from 17.7% to 13.6%). With regard to wages, naturally w_{SH} increases whereas w_{UH} experiences a small reduction and w_{UL} remains basically the same. Summing up, an increase in the productivity of high-educated workers leads to a reduction in the unemployment rate of both types of workers, albeit much larger for high-educated workers, a fall in the proportion of on-the-job searchers and a larger within wage gap.

Column 4 presents the results from reducing c_U from 0.5 to 0.35. Since it is now cheaper to create unskilled vacancies, μ increases to 1.070 whilst b_S decreases. As a result, the proportion of on-the-job searchers increases from 23.3% to 25.4% and θ_H falls slightly from 7.7% to 6.9% while θ_L falls by more than 2 percentage points from 16.1% to 13.9 %. As regards wages, w_{SH} remains unaltered whereas w_{UH} and w_{UL} slightly increase so that the \within" wage wap decreases. Hence, a reduction in the °ow cost of keeping an unskilled vacancy un¯lled reduces both unemployment rates, increases the proportion of on-the-job searchers and reduces the \within" wage inequality.

Table 3. Comparative Statics for joint changes in τ; γ_{HS} and c_U

	Baseline	(γ_{HS} ^ τ)	(γ_{HS} ^ c_U)	(c_U ^ τ)	(γ_{HS} ^ c_U ^ τ)[a]	(γ_{HS} ^ c_U ^ τ)[b]
μ	0.799	1.019	1.322	1.003	1.321	1.093
b_S	0.124	0.366	0.148	0.259	0.317	0.131
a_H	0.177	0.212	0.159	0.292	0.243	0.211
θ_L	0.161	0.190	0.133	0.168	0.160	0.169
θ_H	0.077	0.053	0.056	0.057	0.048	0.064
θ_{UH}	0.233	0.066	0.171	0.104	0.073	0.242
w_{SH}	1.621	2.643	2.459	1.743	2.645	1.749
w_{SL}	0.842	0.764	0.845	0.798	0.783	0.847
w_{UL}	0.884	0.867	0.901	0.687	0.661	0.886

Note: (a) $\tau = 0.3, \gamma_{HS} = 3.0$ and $c_U = 0.35$; (b) $\tau = 0.15, \gamma_{HS} = 2.25$ and $c_U = 0.35$.

Next, Table 3 reports the results of combining the joint effects of : (i) an increase in γ_{SH} and τ (column 2);(ii) an increase in γ_{SH} and a reduction in c_U (column 3), and (iii) an increase in τ and a reduction in c_U (column 4), and (iv) an increase in γ_{SH} and in τ and a reduction in c_U (column 5): Column 1 reproduces for convenience the results for the baseline model and column 6 displays the results fron a different parameter configuration to be discussed at the end of this section. The joint effect of skill-biased tecnical progress ("γ_{SH}) and the educational drive ("τ) imply both higher values of μ and b_S; reflecting the larger availability of more productive high-educated workers. There is a decrease in θ_H since both effects separately work in the same direction whereas θ_L increases since the unfavourable effect of a rise in τ overcomes the favourable effect of the rise in γ_{SH}. The proportion of on-the-job searchers strongly decreases again as result of the compound separate effects. Within wage inequality increases since there is a strong rise of w_{SH} accompanied by a strong reduction in w_{UH} and a mild one in w_{UL}: The joint effect of skill-biased technical progress and the reform facilitating the use of temporary jobs (#c_U) leads to: a strong rise in μ and a mild rise in b_S; a reduction in both θ_H and θ_L;a fall in the proportion of on-the-job searchers and an increase in the within wage gap although both w_{UL} and w_{UH} raise.

The joint effect of the educational drive and the reform on temporary jobs gives rise to an increase in μ, a reduction in θ_H and an increase in θ_L reflecting a stronger unfavourable effect from the increase in τ than the favorable effect of the decease in c_U: Likewise, there is a strong reduction in θ_{HU} stemming from the stronger effect of the increase in τ: Within wage inequality raises with the increase in w_{SH} and a strong reduction in w_{UH} and, particularly, in w_{UL}: Finally, the joint effect of the three changes taking place simultaneously point out to results fairly similar to the discussed in column 2 of Table 3, except that now there is a large reduction in w_{UH} and w_{UL} reflecting the fact the negative effects of the joint increase in τ and the reduction in c_U seem to dominate the behaviour of wages in the unskilled jobs.

In sum, it can be concluded that in order to replicate the Spanish experience, the upper-education lift cannot explain the stylised facts by itself since, despite increasing the unemployment rate of low-educated workers and reducing the unemployment rate of high-educated workers, the proportion of on-the-job searchers falls. Likewise, skill-biased technical progress reduces both unemployment rates and the share of on-the-job searchers. The only change which increases that share is a reduction in the flow cost of opening unskilled vacancies (our proxy for the widespread use of temporary jobs). However, in a such a case, both unemployment rates decrease again. Thus, it seems that the joint effects of a raise in τ and a reduction in c_U are key ingredients (plus an increase in y_{HS} which undoubtedly has taken place) in explaining the Spanish experience. For illustrative purposes, we present in column 6 of Table 3 the results of a simulation where c_U is set at 0.35 whereas y_{HS} only increases to 2.25 and τ raises to 0.15[11]. As it can be observed, now both the unemployment rate of low-skilled workers and the proportion of on-the-job searchers increase whereas the unemployment rate of high-educated workers decreases so that the direction of the comparative-statics effects somewhat reproduces the Spanish experience.

[11] There is evidence that the increase in productivity in Spain during the nineties has been lower than in the average EU country, due to poor R&D policies, low adoption of IT technologies and the effect of some labour market institutions (see Estrada and López-Salido, 2001, and Dolado et al., 2002).

CONCLUSIONS

In this paper, we have analysed the properties of a matching model with two types of workers (low and high-educated) and two types of jobs (unskilled and skilled) where on-the-job searchby mismatched workers is allowed. We show that this model can account for some of the stylised facts of the Spanish labour market following a large tertiary edicational upgrading which has been taking place since the mid-eighties.

The model could be extended in a number of ways. One extention is to endogeneise the skill distribution by allowing workers to invest in education. A second extension would be to consider a model of directed search. In that environment workers can target their search to di®erent types of jobs but nonetheless high-educated may consciously decide to apply for unskilled jobs (with some probability). Finally, one could consider the possibility of allowing for multiple meetings so that the model can address issues of ranking of applicants by ¯rms.

APPENDIX

In this Appendix we discuss issues related to the existence and uniqueness of the cross-skill matching equilibrium. The treatment is based on the analysis of AV (2002).

For existence, we first need to rule out the corner solution in which firms only supply unskilled jobs. Next, we need to ensure that skilled workers accept unskilled vacancies, namely, that the the ex post segregation equilibrium does not exist.

Existence

Homogeneous jobs.

The corner solution with a homogeneous supply of unskilled jobs can be summarised by a pair $(\mu^{\alpha}; u^{\alpha})$ that solves:

$$\frac{c_U}{q(\mu^{\alpha})} = (1 - \beta)\left[\frac{(1 - \gamma)(y_{UL} - z_L) + \gamma(y_{UH} - z_H)}{r + \delta_U + \mu^{\alpha} q(\mu^{\alpha})\beta}\right] \tag{28}$$

$$\mu^{\alpha} q(\mu^{\alpha}) u^{\alpha} = \delta_U (1 - u^{\alpha}) \tag{29}$$

The above free-entry condition follows immediately from the substitution of the reservation wages

$$rU_j = \frac{(r + \delta_U) z_j + \mu q(\mu)\beta y_{Uj}}{r + \delta + \mu q(\mu)\beta} \tag{30}$$

into the job creation condition

$$\frac{c_U}{q(\mu)} = (1 - \beta)\left[(1 - \gamma)\left(\frac{y_{UL} - rU_L}{r + \delta_U}\right) + \gamma\left(\frac{y_{UH} - z_H}{r + \delta_U}\right)\right] \tag{31}$$

where we have used the result that $a_H = \gamma$ since both types of workers have the same exit rate out of unemployment.

Moreover, it is easy to show that the equilibrium with unskilled jobs is (i) always unique and (ii) invariant to changes in γ if $z_L = z_H$:

The second result follows from (28) by setting z_H equal to z_L: The uniqueness result is also straightforward. Inspection of (28) shows that the left-hand side de¯nes a strictly increasing and continuous function $F : (0; 1) \to (0; 1)$, while the right-hand side de¯nes a function $G : (0; 1) \to [0; (1 - \beta)(\frac{(1-\iota)(y_{UL}-z_L)+\iota(y_{UH}-z_H)}{r+\pm_U}]$ that is strictly decreasing on the entire domain as $\partial\mu q(\mu)/\partial\mu > 0$. The two curves associated with F and G therefore intersect exactly once.

What is more important for our purposes is that an equilibrium with unskilled jobs can be ruled out by imposing the condition that

$$\frac{c_S}{q(\mu^*)} < (1 - \beta)\left[\frac{y_{SH} - z_H}{r + \pm + \mu^* q(\mu^*)^{-}}\right]^{1} \tag{32}$$

Given (32), a ¯rm can pro¯tably open a skilled job at $(\mu; b_S) = (\mu^*; 0)$ even though it faces a lower arrival rate $\iota q(\mu^*)$: Henceforth we shall assume that condition (32) is satis¯ed. Inspection of (32) shows that this is more likely at higher values of y_{SH} and ι and lower values of z_H:

Ex-post segmentation. |

In an ex-post segmentation equilibrium skilled workers refuse unskilled jobs and continue to search until they ¯nd a skilled job. In order for this strategy to be optimal, it must be true that $y_{UH} - rU_H < 0$, where the reservation wage rU corresponds to the reservation value in the ex post segmentation case.

That is,

$$rU_H = z_H + \mu q(\mu) b_S^{-}\left[\frac{y_{SH} - rU_H}{r + \pm_S}\right]; \tag{33}$$

while the reservation wage of unskilled workers is unchanged at

$$rU_L = z_L + \mu q(\mu)(1 - b_S)^{-}\left[\frac{y_{UL} - rU_L}{r + \pm_U}\right]: \tag{34}$$

Moreover, with ex post segmentation the free entry conditions are given by:

$$\frac{c_S}{q(\mu)} = (1 - \beta)a_H \left[\frac{y_{SH} - rU_H}{r + \delta_S}\right]^{\alpha} \tag{35}$$

$$\frac{c_N}{q(\mu)} = (1 - \beta)(1 - a_H) \left[\frac{y_{UL} - rU_L}{r + \delta_U}\right]^{\alpha}: \tag{36}$$

Solving (35)-(36) for the outside option values of workers and substituting these values into the entry conditions gives the following pair of equilibrium conditions:

$$\frac{c_S}{q(\mu)} = (1 - \beta)a_H \left[\frac{y_{SH} - z_H}{r + s + \mu q(\mu) b_S}\right]^{\alpha} \tag{37}$$

$$\frac{c_U}{q(\mu)} = (1 - \beta)(1 - a_H) \left[\frac{y_{UL} - z_L}{r + s + \mu q(\mu)(1 - b_S)}\right]^{\alpha} \tag{38}$$

where

$$rU_H = \frac{(r + \delta_S)z_H + \mu q(\mu) b_S^{\alpha} y_{SH}}{r + \delta_S + \mu q(\mu) b_S^{\alpha}} \tag{39}$$

$$rU_H = \frac{(r + \delta_U)z_L + \mu q(\mu)(1 - b_S)^{\alpha} y_{UL}}{r + \delta_U + \mu q(\mu)(1 - b_S)^{\alpha}} \tag{40}$$

An ex post segmentation equilibrium can then be summarised by a vector $\{\mu; b_S; u_L; u_H\}$ that solves (37) and (38) and that satisfies the flow equations for u_L and u_H:

$$(1 - b_S)\mu q(\mu)(1 - a_H)u = \delta_U(1 - u - (1 - a_H)u); \tag{41}$$

$$b_S \mu q(\mu) a_H u = \delta_S(u - a_H u); \tag{42}$$

where $u = u_H + u_L$ is the total mass of unemployed workers.

Uniqueness. |

As in AV (2002) we can show that the ex post segmentation equilibrium is always unique.[12] The way to obtain this result is to reduce the entry equations into expressions in terms of $(\mu; 1 - a_H)$: In a first step solve (41) for the share of unskilled vacancies:

$$(1 - b_S) = \frac{\pm_U (1 - {}^{1} - (1 - a_H)u)}{\mu q(\mu)(1 - a_H)u}: \tag{43}$$

The value of u can be solved from (42).

$$u = \frac{\pm_S {}^{1}}{[b_S \mu q(\mu) + \pm_S] a_H}: \tag{44}$$

Substituting this solution into (43) and rearranging terms yields:

$$1 - b_S = \frac{\pm_U (1 - {}^{1})\mu q(\mu) a_H + \pm_U \pm_S (1 - {}^{1} - (1 - a_H))}{\mu q(\mu)[\pm_S {}^{1}(1 - a_H) + \pm_U a_H (1 - {}^{1})]}: \tag{45}$$

Equation (45) expresses $1 - b_S$ as a function of μ and a_H. Differentiating the above condition shows that

$$\frac{\pm b_S}{\pm a_H} < 0: \tag{46}$$

Moreover, by multiplying both sides of (45) we can also show that

$$\frac{\pm \mu q(\mu)(1 - b_S)}{\pm \mu} > 0 \tag{47}$$

$$\frac{\pm \mu q(\mu) b_S}{\pm \mu} > 0: \tag{48}$$

Given the above results, it is easy to show that the expected profits of a skilled job increase with a_H (as a higher value of a_H is associated with a lower value of b_S). Similarly, the expected profit of an unskilled job, V_U; increases with $1 - a_H$, the share of low-educated workers in the pool of unemployed. Since $@V_U = @\mu$ and $@V_S = @\mu$ are both negative, the free

[12] The only difference with their case is that we allow for values of z_j and $\pm_i$ that differ across workers and firms.

entry conditions for unskilled jobs is therefore associated with a curve that slopes upward in $(\mu; 1 - a_H)$ space, while the curve associated to (37), the free entry condition for skilled jobs, slopes downward in the space $(\mu; 1 - a_H)$:

Thus, whenever an ex post segmentation equilibrium exists it is unique. Furthermore, from the condition that $y_{UH} - rU_H < 0$ and our solution for rU_H, it follows that an ex post segmentation equilibrium is more likely at high values of y_{SH} and ^{1}. In other words, our cross-skill matching equilibrium corresponds to intermediate values of $(y_{SH}, {}^1)$ at which ¯rms ¯nd it pro¯table to create skilled jobs while high-educated workers do not ¯nd in their interest to refuse unskilled jobs.

Uniqueness in cross-skill matching. |

The results of the ex- post separation equilibrium can also be used to analyse the cross-skill matching equilibrium. Notice that the three °ow equations of the cross-skill matching eq. can be rewritten as:

$$(1 - b_S)\mu q(\mu)(1 - a_H)\bar{u} = \pm_U(1 - {}^1 - (1 - a_H)\bar{u})$$

$$b_S\mu q(\mu)a_H\bar{u} = \pm_S({}^1 - a_H\bar{u})$$

$$(1 - b_S)\mu q(\mu)a_H\gg\bar{u} = a_H(1 - \gg)\bar{u}[\pm_U + \mu q(\mu)b_S]$$

where $\bar{u} \acute{} u_L + u_H + e_{UH}$ denotes the total mass of searching workers while $\gg \acute{} u_H/(u_H + e_{UH})$:

A comparison with the ex-post separation equilibrium shows that the ¯rst two °ow equations are identical. As a result, we can derive the same solution for $1 - a_H$ plus the same relation between changes in a_H and b_S. The only di®erence is that we need to replace u by $\bar{u}$ to account for the mass of workers who search on-the-job.

Nonetheless, this is not su±cient to obtain the equivalent of Figure 1 in AV(2002). To check this, take for example equation (20) for V_U in the main text

$$\frac{c_U}{q(\mu)} = (1 - \gamma)\ (1 - a_H)\frac{y_{UL} - z_L}{\rho_1} + a_H \frac{y_{UH} - z_H}{\rho_2}, \tag{49}$$

$$\rho_1 = r + \pm_U + \mu q(\mu)(1 - b_S)^{\gamma} \tag{50}$$

$$\rho_2 = r + \pm_U + \mu q(\mu)[b_S + (1 - b_S)^{\gamma}];$$

According to this free entry condition an increase in a_H (at unchanged b_S) increases the expected pro¯ts of unskilled jobs because $\frac{y_{UL} - z_L}{r + \pm_U + \mu q(\mu)(1 - b_S)} > \frac{y_{UH} - z_H}{r + \pm_U + \mu q(\mu)[b_S + (1 - b_S)^{\gamma}]}$. Moreover, the denominator of the ¯rst term becomes smaller as $\pm(1 - b_S)/\pm(1 - a_H) < 0$: However, the denominator of the second term (ρ_2) decreases .The ¯rst two e®ects tend to increase V_U while the last e®ect tends to decrease V_U. Hence, in order to obtain the result of AV (2002) we need to compute the total derivative explicitly and derive its sign, to verify whether the last e®ect is always dominated by the ¯rst two e®ects.

The di®erence between our result and AV (2002)'s is that the equivalent of ρ_2 in their model contains $\mu q(\mu)^{\gamma}$ in the bracketed term and, hence, is invariant to changes in b_S: However, using some straightforward but tedious calculus, it can be proved that, by di®erentiating (A.22) with respect to μ; a su±cient condition for ρ_2 to increase when a_H raises is that the $\pm \log \mu q(\mu)/\pm \log \mu > \pm \log b_S/\pm \log \mu$, so that that the elasticity of $\mu q(\mu)$ w.r.t. μ is larger than the elasticity of b_Sw.r.t. μ:

REFERENCES

[1] ACEMOGLU, D., \ Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence, " American Economic Review 89 (1999), 1259-78.

[2] ALBA-RAMIREZ, A., \ Mismatch in the Spanish Labour Market, " Journal of Human Resources 28 (1993), 259-278.

[3] ALBRECHT, J., AND S. VROMAN, \ A Matching Model with Endogenous Skill Requirements, " International Economic Review 43 (2002), 283-305.

[4] ALBRECHT, J., VAN DEN BERG, G. A., AND S. VROMAN, \ The Equilibrium Labor Market E®ects of of Major Skill-Enhancing Adult Programs-Theory and Applications to the Swedish Knowledge Lift, " mimeo.Georgetown University, 2002.

[5] BLAZQUEZ, M.T.\ A Matching Model of Over-Education with Endogenous Human Capital Investment, " mimeo. Universidad Carlos III, 2002.

[6] BOVER, O., ARELLANO, M., AND S: BENTOLILA, \ The Distribution of Earnings in Spain During the 1980s: The E®ects of Skill, Unemployment, and Union Power, " forthcoming in: D. Cohen, T. Piketty and G. Saint-Paul (eds.): The New Economics of Rising Inequalities, CEPR and Oxford University Press (2002).

[7] DOLADO, J.J., AND J.F. JIMENO, \ The Causes of Spanish Unemployment: A Structural VAR Approach, " European Economic Review 41 (1997), 1281-1307.

[8] DOLADO, J. J., FELGUEROSO, F., AND J.F. JIMENO, \ Youth Labour Markets in Spain: Education, Training, and Crowding-Out, " European Economic Review 44 (2000), 943-56.

[9] DOLADO, J. J., GARCIA-SERRANO, C., F., AND J.F. JIMENO, \Drawing Lessons from the Boom of Temporary Jobs in Spain, " forthcoming in Economic Journal (june, 2002).

[10] ESTRADA, A., AND J.D. LÓPEZ-SALIDO, \ Sectoral and Aggregate Technological Progress in Spain, " Working Paper 01-16, Servicio de Estudios Banco de España (2001).

[11] FERNANDEZ, C., \ The Role of Education and Job Experience in Explaining the Transitions from Unemployment to Employment in the Spanish Youth Labour Market," mimeo. FEDEA, 2002.

[12] GALINDO-RUEDA, F., \ Graduate Overeducation in Spain, " mimeo. University College London, 2001.

[13] GAUTIER , P. A., \ Unemployment and Search Search Externalities in a Model with Heterogeneous Jobs and Heterogenous Workers, " mimeo, Tinbergen Institute, 1999.

[14] GAUTIER , P. A., VAN DEN BERG, G. A., VAN OURS, J.C. , AND G. RIDDER, \ Worker Turnover at the Firm Level and Crowding Out of Lower Educated Workers, " European Economic Review 46 (2002) 523-38.

[15] MCKENNA, C.J., \ Education and the Distribution of of Unemployment, " European Journal of Political Economy 12 (1996), 113-132.

[16] MORTENSEN, D. T., AND C. PISSARIDES, \ Unemployment Responses to `Skill- Biased Technology Shocks: The Role of Labour Market Policy, " Economic Journal 109 (1999), 242-65.

[17] MUYSKEN, J., AND B.J. TER-WEEL, \Overeducation and Crowding-Out of Low-Skilled Workers, " Discussion Paper 98-023, MERIT, Maastricht University (1998).

[18] SHIMER, R., \ The Impact of Young Workers on the Aggregate Labor Market, ", Quarterly Journal of Economics 116 (2001), 986-1007.