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ON THE (INTRADAILY) SEASONALITY AND DYNAMICS OF A FINANCIAL POINT PROCESS: A SEMIPARAMETRIC APPROACH.

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Abstract

A component model for the analysis of financial durations is proposed. The components are the long-run dynamics and the seasonality. The later is left unspecified and the former is assumed to fall within the class of certain family of parametric functions. The joint model is estimated by maximizing a (local) quasi-likelihood function, and the resulting nonparametric estimator of the seasonal curve has an explicit form that turns out to be a transformation of the Nadaraya-Watson estimator. The estimators of the parameters of interest are shown to be root-N consistent and asymptotically efficient. Furthermore, the seasonal curve is also estimated consistently. The methodology is applied to the trade duration process of Bankinter, a medium size Spanish bank traded in Bolsa de Madrid. We show that adjusting data by seasonality produces important misspecifications.

Keywords: Tick-by-tick; ACD model; Seasonal Analysis, Nonparametric method.

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1 INTRODUCTION

The issue of modelling financial duration processes is a fashionable area of research since Engle and Russell (1998) introduced the Autoregressive Conditional Duration (ACD) model. Their analysis is justified from an economic and a statistical point of view. On one hand, market microstructure theory shows that time between events in a stock exchange market conveys information and thus time has to be analyzed. On the other hand, since data are the "so called" tick-by-tick data, they are nothing else than a one dimensional point process (with time as space). Thus time is the random variable of the point process and in each point there is an associated vector of marks, and both time and the marks can be modelled.

Since the former model a plethora of modifications and alternatives have been proposed. Among others, Bauwens and Giot (2000) introduced the Log-ACD model, which is an exponential version of the ACD. Grammig and Mauer (2000) used a Burr distribution in the ACD model. Zhang et al. (1999) introduced a threshold ACD. Drost and Werker (2001) provide a method to obtain efficient estimators of the ACD model without need to specify the distribution. Camacho and Veredas (2001) consider the analysis of a bivariate duration process using random aggregation techniques. Alternative models are the the Stochastic Conditional Duration (SCD) model of Bauwens and Veredas (1999) and the Stochastic Volatility Duration (SVD) model of Ghysels et al. (1998) which are both based on latent factor models. Almost all these models are surveyed in Bauwens et al. (2000).

In most of the above studies, durations show a strong intradaily seasonality. In an explanatory graphic analysis the strong seasonal component is detected by the presence of the U (or inverted U) shape that ultra high frequency financial variables exhibit during the day. For example, in figure 5 it is shown the intradaily and intraweekly behaviour of trade durations for Bankinter, a medium size Spaninsh bank traded at Bolsa de Madrid during January-March 1998.

The way to study this feature is well known when dealing with regularly spaced variables, that is, when dealing with variables that are observed at equidistant periods of time. Moreover this analysis has focused mainly on the volatility's intradaily behaviour of either an stock exchange market or a foreign exchange (FX) market. Engle et al. (1990) analyze how the information flow is transmitted through world regions in the FX market using hourly data. Harris (1986) does a panel data analysis using 15 minute interval returns data of firms traded in NYSE. Baillie and Bollerslev (1990) studied the intradaily and inter-market FX volatility using a qualitative approach with hourly data. Bollerslev and Domowitz (1993) do a similar analysis but for returns and bid-ask spread of the deutsche mark-dollar exchange rate using data recorded at 5 minute intervals. Andersen and Bollerslev (1997) used a frequency domain approach for filtering the five minutes deutsche mark-dollar exchange rate and getting rid of the seasonal pattern. Andersen and Bollerslev (1998) used a diffent approach and analyzed the intradaily and intraweekly seasonality using spectral analysis and they took into account macroeconomic announcements. Finally, Beltratti and Morana (1999) used half hour deutsche

mark-dollar exchange rate and they modeled it following a structural approach "*à* la Harvey".

All these previous works have been done using regularly spaced data (hourly, half-hourly, 15 minutes or 5 minutes). For tick-by-tick data the most popular approach to deal with intradaily seasonality was introduced by Engle and Russell (1998). The method consists in estimating the intradaily seasonality by means of a piecewise cubic spline. Although Engle and Russell (1998) apparently succeed in the joint estimation of the parameters of the cubic spline and the ACD model, it is a hard task and the convergence towards a global maximun is not assured. For these reasons most of other studies have focused in a two step procedure, where in the first step, the inverted U shape is removed through some filter and, in a second step, the ACD model is estimated by using the deseasonalized variables. The filter basically consists in calculating the average durations every, say, 30 minutes and then smoothing this piecewise constant function through cubic splines. Alternatively Gouriéroux et al. (1999) analyzed the intraday market activity using kernels for the intraday intensity as well as for the survivor function, but they do not differentiate between seasonal pattern and long-run dynamics since their analysis is purely nonparametric. Gerhard and Haustch (2000) proposed a model for financial durations using a proportional hazard model where seasonality is modeled using a flexible Fourier transform.

The two step procedure presents some serious drawbacks. Mainly it performs accurately if both the seasonal and the non-seasonal components depend on some deterministic time index, and the non-seasonal dynamics of the duration process is linear in the parameters to be estimated. Otherwise, the two step estimation procedure can lead to serious misspecification errors.

In this paper we assume that tick-by-tick processes can be decomposed in two components that stand for the short-run and the long-run behaviours. The short-run refers to the intradaily seasonality while the long-run can be considered as the core dynamics of the process.

In the standard theory of time series, two approaches exist for dealing with these components. The first one considers that a time series can be analyzed by means of an ARMA model that, using different lags in the polynomials and exogenous variables, account for the components. The second approach assumes that the time series can be decomposed in latent components which are not observed but have some dynamics and/or some cyclical patterns.

In the framework of tick-by-tick data, the ARMA approach is not feasible since one of the main characteristics of these data is the lack of periodicity. Therefore we focus in the second approach, assuming the decomposition of the time series in components that are estimated separately but not independently. In order to do so, we rely on the assumption that the conditional expectation of the duration can be decomposed in the two mentioned terms. Under this assumption, they can be estimated simultaneously.

The short-run component is modeled nonparametrically and the long-run component is assumed to belong to the parametric ACD family, specifically a Log-ACD model. Both components are estimated simultaneously by maximizing alternatively a local and a global version of the likelihood function. Under the correct choice of the smoothing parameter, this estima-

tion method provides root-N consistent semiparametric estimators of the parameters of the Log-ACD model. Furthermore, if the conditional likelihood is correctly specified the estimators are efficient.

We also deal with durations equal to zero. These durations are often found in the trade process. Previous studies eliminate them using the microstructure argument that all the trades executed in the same second come from the same trader that has split a big order block in small blocks. We show that this is not always true and, indeed, most of the times the null durations are clustered around round prices due to the fact that the limit orders of the retail traders are set for being executed at the round prices and hence trades executed in the same second do not belong to the same trader but to many retail traders.

We apply the proposed methodology to Bankinter, a medium size spanish bank traded at Bolsa de Madrid, an order driven market and hence its trading mechanism is equivalent to the most important continental Europe exchanges (e.g. Brussels, Milan and Paris). For comparing the goodness of fit of the proposed model we focus on forecasting in a twofold exercise. On one hand the evaluation of the density forecast accuracy is done on the basis of the technique proposed by Diebold et al. (1998). We show that the joint estimation of seasonality and dynamics improves the density forecast. On the other hand we show as well that the forecasting errors of the models adjusting data and forgetting the existence of seasonality have some cyclical pattern that has not been captured by the model, whilst it is not the case for the model proposed here.

The plan of the paper is a as follows. Section two develops a general framework for analyzing tick-by-tick financial variables, decomposing the process in the two above mentioned terms and in the framework of Generalized Linear Models. Notice that even if notation and empirical application are done for duration processes, any other variable can be analyzed in the same way. Section three is twofold. First it is analyzed each component introducing a modelling strategy for them. Second the theoretical properties of the resulting estimators are studied. Section four is devoted to the empirical application focusing on the nonparametric estimates and the forecasting exercise. Section five concludes. Finally, the assumptions and proofs of the main results are relegated to the Appendix.

2 BASIC ECONOMETRIC MODEL

In order to introduce the main contribution of our paper, we need to establish a basic econometric framework. Following Engle and Rusell (1998) and Engle (2000), let t_i be the time at which the i-th trade occurs and let $d_i = t_i - t_{i-1}$ be the duration between trades. Let us consider also that we have observed k marks, denoted y_i, at the i-th event. For example, if d_i are trade durations, the marks could be the price and the volume of the trade. Then, we have available the following set of observations

$$\{(d_i, y_i)\}_{i=1,\cdots,n}.$$

Furthermore, assume that the i-th observation has the joint density conditional on the past filtration as

$$(d_i, y_i)|\, I_{i-1} \sim f\left(d_i, y_i|\bar{d}_{i-1}, \bar{y}_{i-1}; \delta\right),$$

where $(\bar{d}_{i-1}, \bar{y}_{i-1}) = \bar{z}_i = (z_i, z_{i-1}, \cdots, z_1)$ is the present and past information of the z stochastic process and δ is a set of parameters in some possibly infinite dimensional space. Within this statistical framework, our aim is to estimate this parameter vector δ (or any nonlinear combination of its components) by using maximum likelihood techniques. To this end, we construct the following likelihood function

$$L_n(d, y; \delta) = \sum_{i=1}^{n} \log\, f\left(d_i, y_i|\bar{d}_{i-1}, \bar{y}_{i-1}; \delta\right). \tag{1}$$

Following a reduction process we can considerably simplify the previous log-likelihood expression. Without loss of generality we can write

$$\log\, f\left(d_i, y_i|\bar{d}_{i-1}, \bar{y}_{i-1}; \delta\right) = \log\, p\left(d_i|\bar{d}_{i-1}, \bar{y}_{i-1}; \delta_1\right) + \log\, g\left(y_i|\bar{d}_i, \bar{y}_{i-1}; \delta_2\right),$$

where $\delta = (\delta_1, \delta_2)$. Moreover, if the parameters of interest are function of δ_1 only, and the marks, y, are weakly exogenous for these parameters, then its estimation can be based on the following likelihood function

$$L_n(d, y; \delta_1) = \sum_{i=1}^{n} \log\, p\left(d_i|\bar{d}_{i-1}, \bar{y}_{i-1}; \delta_1\right). \tag{2}$$

The exogeneity assumption is crucial and arguable. This relationship has been pointed out by Ghysels (2000), among others, and in terms of market microstructure it seems that a joint analysis of (d_i, y_i) is more adequate. However, this is out of the scope of this paper and we leave this issue for further research.

If the conditional density is correctly specified and standard regularity conditions apply, the maximum likelihood estimator of δ_1 is consistent and asymptotically normal. Alternatively, as pointed out in Engle and Rusell (1998) and Engle (2000) it is of interest to have available some estimation techniques that do not rely on the knowledge of the functional form of conditional density function. Two alternative approaches that allow for consistent estimation of the parameters of interest without specifying the conditional density are the Quasi Maximum Likelihood technique, QML, (see Gouriéroux, Monfort and Trognon, 1984) and Generalized Linear Models, GLM, (see McCullagh and Nelder, 1989). In both approaches, it is assumed that the duration variable d, conditonally on past values of d and y depends on a scalar parameter $\theta = h\left(\bar{d}_{i-1}, \bar{y}_{i-1}; \delta_1\right)$, and its distribution belongs to a one dimensional exponential family with conditional density

$$p\left(d_i|\bar{d}_{i-1}, \bar{y}_{i-1}; \theta\right) = \exp\left(d_i\theta - b(\theta) + c(d_i)\right),$$

where $b(\cdot)$ and $c(\cdot)$ are known functions. The main difference between the QML and the GLM approach is simply a different parametrization of this exponential family. Here in this paper we

will adopt for convenience the GLM approach. Then it is straightforward to see that the Maximum Likelihood estimator of θ solves the following first order conditions: $\sum_i \{d_i - b'(\theta)\} = 0$. Furthermore, since by the properties of the exponential family,

$$E\left[d_i|\bar{d}_{i-1}, \bar{y}_{i-1}\right] = b'(\theta) = \mu\left\{\bar{d}_{i-1}, \bar{y}_{i-1}; \delta_1\right\} \tag{3}$$

and

$$\text{Var}\left[d_i|\bar{d}_{i-1}, \bar{y}_{i-1}\right] = b''(\theta) = \sigma^2 V\left\{\mu(\bar{d}_{i-1}, \bar{y}_{i-1}; \delta_1)\right\}, \tag{4}$$

then the MLE estimator of θ can also be obtained from the solution of the following equation

$$\sum_{i=1}^{n} \frac{(d_i - \mu(\theta))\,\mu'(\theta)}{V(\mu(\theta))} = 0. \tag{5}$$

As it can be clearly realized from equations (3), (4) and (5) the estimation of the parameter of interest θ (the so called canonical parameter) can be performed without needing to specify the whole conditional distribution function. It is only necessary to specify the functional form of the conditional mean, $\mu(\cdot)$, and of the conditional variance $V(\cdot)$, but not the whole distribution. Engle and Rusell (1998) propose to specify the conditional mean function by using the ACD class of models that consists on parametrisations such as

$$E\left[d_i|\bar{d}_{i-1}, \bar{y}_{i-1}\right] = \mu\left(\bar{d}_{i-1}, \bar{y}_{i-1}; \delta_1\right) = \varphi\left(\omega + \sum_{j=1}^{J} \alpha_j g(d_{i-j}) + \sum_{k=1}^{K} \beta_k \mu_{i-k}\right), \tag{6}$$

where the parameters of interest are $\delta_1 = (\omega, \alpha_1, \cdots, \alpha_J, \beta_1, \cdots, \beta_K)$. The functions $\varphi(\cdot)$ and $g(\cdot)$ take the values $\varphi(s) = s$ and $g(s) = s$ for the ACD model and $\varphi(s) = \exp(s)$ and $g(s) = \ln(s)$ for the Log-ACD model. The relationship between the predictors in equation (6) and the canonical parameter is given by the so called *link function*. This function is going to depend on the member of the exponential family that we are going to use. For the exponential distribution the link function is

$$\theta = -\frac{1}{\varphi\left(\omega + \sum_{j=1}^{J} \alpha_j g(d_{i-j}) + \sum_{k=1}^{K} \beta_k \mu_{i-k}\right)}. \tag{7}$$

Noting that under this distribution $\mu(\theta) = -\theta^{-1}$ and $V(\mu(\theta)) = \mu^2$, then (5) are the first order conditions for the maximization of the log-likelihood function for exponentially distributed data.

As it has been pointed out in many recent studies, the ACD specification is sometimes too simple since the expected duration can vary over time, or can be subject to many different time effects. One way to extend the previous model is to decompose the conditional mean in different effects. In the standard time series literature any stochastic process can be decomposed in a combination (we adopt a multiplicative decomposition being the additive straightforward) of cycle and trend, seasonal pattern and noise, i.e. $X_t = X_t^{CT} \cdot S_t \cdot \varepsilon_t$. This decomposition, of long tradition in time series analysis, has been already used in volatility analysis (see for example Andersen and Bollerslev, 1998).

In this paper we propose the following nonlinear decomposition:

$$E[d_i|\bar{d}_{i-1}, \bar{y}_{i-1}] = \varphi\left(\psi(\bar{d}_{i-1}, \bar{y}_{i-1}; \vartheta_1), \phi(\bar{d}_{i-1}, \bar{y}_{i-1}; \vartheta_2)\right). \tag{8}$$

The function $\varphi(u, v)$ can nest a great variety of models. $\varphi(u, v) = (u \times v)$ stands for an ACD representation whereas $\varphi(u, v) = \exp(u + v)$ represents a Log-ACD representation. Then, following (8) the durations, volatility, trading intensity and volume (in a high frequency framework) can be modelled as a possibly nonlinear function of two components that represents the long-run, $\psi(\cdot; \vartheta_1)$ and the short-run, $\phi(\cdot; \vartheta_2)$, respectively. The long-run component can be considered as the core dynamics and on it the dynamics of the process are modelled. It can be done using autoregressive models (like GARCH or ACD), latent factor models (like SV and SCD) or any other alternative. The short-run component represents the seasonal pattern, that can be intradaily and intraweekly.

Note that in previous research on ultra high frequency data, the ACD model has been usually estimated using a duration time series that was already adjusted by seasonality. See, among others, Engle and Russell (1997), and Bauwens and Giot (2000). In order to estimate the parameters of interest in the ACD model, under the specification assumed in (8), the previous filtration is not sucessful. The reason is that a mere nonparametric regression of $\ln d_i$ into t_{i-1} does not identify separately both seasonal and long run components, and therefore the filtration would remove more than just the seasonal component.

Finally, a third component could be added to (8) accounting for the news effect. It would be the short-run component because since we are working with tick-by-tick data, short-run means some hours and usually the effect of a news in the stock remains for no more than a couple of hours, as documented by Payne (1996) and Almeida et al. (1996).

3 COMPONENTS' SPECIFICATION AND ESTIMATION

The following natural question is how to model each of the components. As a first guess, we should choose between a fully nonparametric approach, a semiparametric or a fully parametric. Since we have to specify two different components it would be sensible to specify parametrically those functions where a lot of information is available, whereas in the case of ignorance a fully nonparametric approach is much more feasible. For the long-run component we adopt some previous pre-specified parametric form. The seasonal component is much less investigated, and to our knowledge there is no accepted standard form for this type of models. On these grounds, we choose to leave it unspecified in the form of a nonparametric function. Furthermore, the interest of the analyst is to predict the process as a whole, that is predict the raw data and not the adjusted one. This is an additional reason for modeling parametrically the component that conveys the past information whilst the deterministic pattern is approached nonparametrically.

For the long-run Engle and Russell (1998) introduced the ACD model that accounts for these features. Since this model, more refined versions have appeared in the literature. See

Bauwens et al. (2000) for a survey of these kind of models. A version of particular interest is the Log-ACD model of Bauwens and Giot (2000). They model the expected duration exponentially, similarly to the EGARCH model for volatility. This model is useful because it avoids the positivity restrictions of the parameters of the dynamic equation and it permits the introduction of exogenous variables that are negatively correlated with the duration process. Hence the specification of the long-run component is done by means of the log of the conditional expectation of a Log-ACD model

$$\psi(\bar{d}_{i-1}, \bar{y}_{i-1}; \vartheta_1) = \omega + \alpha \ln d_{i-1} + \beta \psi_{i-1}, \tag{9}$$

where, for completeness of the model, we assume an exponential form for (8), i.e. $\varphi(u, v) = \exp(u + v)$.

With respect to the so called short-run component, $\phi(\cdot, \vartheta_2)$, several alternative approaches are available. When modelling seasonality, in this type of models, it is usually assumed that the seasonal term is somehow related to the time t_i at which the i-th transaction occurs through some smooth function on time. As we have already indicated we let this function unspecified and hence we estimate it nonparametrically by only assuming some smoothness conditions on it. More precisely, we use a local likelihood method that is carefully explained in the sequel. The choice of this method is justified both from theoretical and computational reasons.

Given the two proposed specifications for the components, (8) is adapted and then we have

$$E\left[d_i | \bar{d}_{i-1}, \bar{y}_{i-1}\right] = \varphi\left(\psi(\bar{d}_{i-1}, \bar{y}_{i-1}; \vartheta_1), \phi(t_i)\right),$$

where the function $\psi(\cdot, \vartheta_1)$ is known and the other quantities, ϑ_1 and the function $\phi(\cdot)$ evaluated at time points $t_1, \cdots, t_n$ need to be estimated. This estimation problem is semiparametric since a nonparametric component, ϕ, needs to be estimated jointly with a parametric one ϑ_1. Under this setting standard (quasi-)maximum likelihood techniques do not apply directly and some developments are needed. This extension is based on the so called conditionally parametric approach introduced in Severini and Wong (1992). The basic idea of this method is to estimate the nonparametric function $\phi(\cdot)$ by maximizing a local likelihood function (see Staniswalis, 1989) and simultaneously estimate the parameter vector ϑ_1 by maximizing the un-smoothed likelihood function. If we specify only the conditional mean and the underlying density is assumed to belong to the family of exponential densities, then maximum likelihood methods are available (Severini and Staniswalis, 1994 ; Fan, Heckman and Wand, 1995). Unfortunately, the statistical results from these papers do not apply directly in our case since they assume independent observations. Nevertheless at the end of the section equivalent statistical results are shown for the dependent case.

The (quasi-)log-likelihood function takes the form

$$Q_n(d, \varphi) = \sum_{i=1}^{n} Q\left(\varphi\left(\psi(\bar{d}_{i-1}, \bar{y}_{i-1}; \vartheta_1), \phi(t_i)\right); d_i\right), \tag{10}$$

where the (quasi-)log-likelihood $Q(\cdot)$ is obtained by integrating (5), i.e.

$$Q\left(d,g\right)=\int_{g}^{d}\frac{\left(s-d\right)}{V(s)}ds.$$

For fixed values of ϑ_1, let us define $\hat{\phi}_{\vartheta_1}(\tau)$ as the solution to the following optimization problem

$$\hat{\phi}_{\vartheta_1}(\tau)=\arg\max_{\eta}\frac{1}{nh}\sum_{i=1}^{n}K\left(\frac{\tau-t_i}{h}\right)Q\left(\varphi\left(\psi(\bar{d}_{i-1},\bar{y}_{i-1};\vartheta_1),\eta\right);d_i\right)$$

for $\tau\in[a,b]$. Then $\hat{\phi}_{\vartheta_1}(\tau)$ must fulfill the following first order conditions

$$\frac{1}{nh}\sum_{i=1}^{n}K\left(\frac{\tau-t_i}{h}\right)\frac{\partial}{\partial\eta}Q\left(\varphi\left(\psi(\bar{d}_{i-1},\bar{y}_{i-1};\vartheta_1),\eta\right);d_i\right)=0. \tag{11}$$

The estimator of ϑ_1, $\hat{\vartheta}_{1n}$, is obtained as the solution to the following (un-smoothed) optimization problem

$$\hat{\vartheta}_{1n}=\arg\max_{\vartheta_1}\sum_{i=1}^{n}Q\left(\varphi\left(\psi(\bar{d}_{i-1},\bar{y}_{i-1};\vartheta_1),\hat{\phi}_{\vartheta_1}(t_i)\right);d_i\right),$$

and $\hat{\vartheta}_{1n}$ must fulfill the following

$$\sum_{i=1}^{n}\frac{\partial}{\partial\vartheta_1}Q\left(\varphi\left(\psi(\bar{d}_{i-1},\bar{y}_{i-1};\vartheta_1),\hat{\phi}_{\vartheta_1}(t_i)\right);d_i\right)=0. \tag{12}$$

As an example, set $\varphi\left(u,v\right)=(u\times v)$, the ACD representation, and $\mu=-\theta^{-1}$ and $V(\mu)=\mu^2$ (the exponential distribution). Then (10) corresponds to the log-likelihood function from an exponential conditional distribution and an ACD representation, i.e.

$$-\sum_{i=1}^{n}\left[\log\left\{\psi(\bar{d}_{i-1},\bar{y}_{i-1};\vartheta_1)\times\phi(t_i)\right\}+\frac{d_i}{\psi(\bar{d}_{i-1},\bar{y}_{i-1};\vartheta_1)\times\phi(t_i)}\right] \tag{13}$$

and the first order condition (11), takes the explicit form

$$\hat{\phi}_{\vartheta_1}(\tau)=\frac{\sum_{i=1}^{N}K\left(\frac{\tau-t_i}{h}\right)\frac{d_i}{\psi(\bar{d}_{i-1},\bar{y}_{i-1};\vartheta_1)}}{\sum_{i=1}^{N}K\left(\frac{\tau-t_i}{h}\right)}. \tag{14}$$

Since a closed expression for the parametric part is not available, an iterative algorithm must be used. Now, instead assume that $\varphi(u,v)=\exp(u+v)$, then (10) corresponds to the log-likelihood function from an exponential distribution and a Log-ACD representation and hence the first order condition (11), takes the explicit form

$$\hat{\phi}_{\vartheta_1}(\tau)=\log\left\{\frac{\sum_{i=1}^{N}K\left(\frac{\tau-t_i}{h}\right)\frac{d_i}{\exp\left\{\psi(\bar{d}_{i-1},\bar{y}_{i-1};\vartheta_1)\right\}}}{\sum_{i=1}^{N}K\left(\frac{\tau-t_i}{h}\right)}\right\}. \tag{15}$$

In some situations, it might be also of interest to use a more flexible density function that does not belong to the exponential family. This could be the case when we are interested not only in the values of the estimated parameters but also in density forecast. In this case, it is possible to perform estimation under these distributions through the use of standard maximum likelihood techniques. The densities used here are the generalized gamma, the Weibull and the exponential. A brief summary of the definitions of the densities is gathered in the Appendix. It is straightforward to show that under correct specification of the density the results we show further hold. We now introduce the whole model with the error term

$$d_i = \varphi\left(\psi(\bar{d}_{i-1}, \bar{y}_{i-1}; \vartheta_1), \phi(t_i)\right) \mu_\varepsilon(\vartheta_3)^{-1} \varepsilon_i \tag{16}$$

where ε is an *i.i.d.* random variable with density function $p(\varepsilon_i; \vartheta_3)$. We introduce $\mu_\varepsilon(\vartheta_3)^{-1}$ due to identification reasons and to the fact that the conditional expectations of the durations is equal to $\varphi(\cdot)$. Clearly ϑ_3 is the set of parameters of the of the assumed distribution.

For example, for the generalized gamma with parameters $\vartheta_3 = (1, \gamma, \nu)$ by maximizing the corresponding (smoothed) log-likelihood function we obtain the following nonparametric estimator for the seasonal component in the Log-ACD representation

$$\hat{\phi}_{\vartheta_1}(\tau) = \frac{1}{\gamma} \log \left\{ \frac{\sum_{i=1}^N K\left(\frac{\tau - t_i}{h}\right) \left(\frac{d_i \mu_\varepsilon}{\exp\left\{\psi(\bar{d}_{i-1}, \bar{y}_{i-1}; \vartheta_1)\right\}} \right)^\gamma}{\sum_{i=1}^N K\left(\frac{\tau - t_i}{h}\right) \nu} \right\}. \tag{17}$$

Note that we attain the nonparametric seasonal estimator using the Weibull distribution when $\nu = 1$ (it coincides as well with the estimator in the Burr case). Finally, the previous expressions have been obtained by assuming an intradaily seasonal component. However, it is also possible to extend it to several seasonal effects. For example, if we consider intraweekly seasonal effects, then we might identify five different seasonal patterns corresponding to each day of the week. In this case, for $s = 1, \cdots, 5$, we have in the exponential and the Log-ACD representation

$$\hat{\phi}_{\vartheta_1}(\tau) = \log \left\{ \frac{\sum_{i=1}^N K\left(\frac{\tau - t_i}{h}\right) \frac{d_i \mu_\varepsilon}{\exp\left\{\psi(\bar{d}_{i-1}, \bar{y}_{i-1}; \vartheta_1)\right\}} I\left(t_i \in \Delta_s\right)}{\sum_{i=1}^N K\left(\frac{\tau - t_i}{h}\right) I\left(t_i \in \Delta_s\right)} \right\} \tag{18}$$

where Δ_s is a subset in $[a, b]$ that contains τ.

Hence, for any distribution we consider, the non parametric seasonal curve is estimated by nothing else that a transformation of the Nadaraya-Watson estimator of the non parametric regression of the duration adjusted by the long-run component on τ. This method is very flexible since very mild assumptions (see Appendix) are needed. As we have already said within this statistical framework, the results available in the literature are obtain under independent observations. Therefore, they do not hold for tick-by-tick data. Nevertheless in the following theorems we show the equivalent statistical results to make correct inference for the unknown parameters of the Log-ACD model (proofs are given in the Appendix):

Theorem 1 *Under the conditions stated in the Appendix, and if* $h \to 0$ *and* $nh \to \infty$ *then,*

$$\sup_{\vartheta_1} \sup_{\tau} |\hat{\phi}_{\vartheta_1}(\tau) - \phi(\tau)| = o_p\left(n^{-1/4}\right)$$

as n *tends to infinity.*

Theorem 2 *Under the conditions stated in Theorem 1 then,*

$$\sqrt{n}\left(\hat{\vartheta}_{1n} - \vartheta_1\right) \to_d \mathbf{N}\left(0, \Sigma_{\vartheta_1}^{-1}\right),$$

where

$$\Sigma_{\vartheta_1} = -E\left(\frac{\partial^2}{\partial\vartheta_1\partial\vartheta_1^T} Q\left(\varphi\left(\psi(\bar{d}, \bar{y}; \vartheta_1), \phi(t)\right); d\right)\right),$$

as n *tends to infinity*

4 APPLICATION TO THE TRADE DURATION PROCESS OF A STOCK IN AN ORDER DRIVEN MARKET

4.1 Data and transformations

In this section we apply the model proposed to a trade duration process. Data are trades during January-March 1998 of Bankinter, a medium size Spanish bank traded at Bolsa de Madrid. This stock exchange market is an order driven market and thus it works as some of the most important stock markets in continental Europe like Brussels, Milan or Paris. In a purely order driven market, there is no market maker and all the orders are entered in the order book. When a buy and a sell order match the order is executed. These orders can be either limit orders or market orders. A more detailed analysis on the functioning of an order driven market can be found in Bauwens and Giot (2001).

The database is a trade database and thus it is not possible to know whether an order comes from a bid or an ask, or from a limit or a market order. As we shall see later on, the difference between limit and market orders is relevant when explaining why there are durations equal to zero. From the original data base two transformations are required. The first has to do with the opening effect while the second one is a way to deal with null durations.

When a trading day begins, before opening there is an auction in order to fix the opening trading price. Once the auction price is fixed, all the remaining orders in the auction stay, not being possible to introduce new orders or cancel the existing ones. Then the market opens and all the orders from the auction are executed in the first minutes. Therefore these trades are not informative about the dynamics of the process and they can be eliminated. Recent studies have eliminated the first half hour of the day for avoiding the effects of the auction in the trading day. Since the moment of time in which the auction orders are traded varies every day, we believe that by adopting this approach we lose informative durations. Thus we adopt the

"second price" strategy, i.e. consider that the trading day begins from the second price since all the orders traded with the first price correspond to the orders of the preopening auction. This data transformation has an important effect on the number of null durations. Figure 1 shows this effect. It is the number of durations equal to zero every ten minutes from the opening to the closing including the first trading day price (left plot) and excluding it (right plot). In the case that we include the first price trades it is clear that we increase artificially the number of trades as well as the number of zeros in the sample. Moreover the amount of first price trades is important. In our sample they represent 9.32% of all the trades.

With respect to trades that occur at the same moment of time and are not due to the preopening auction, previous studies have assumed that they come from a trader that wants to buy or sell a big volume and hence the trader splits the order in small blocks that are sent to the order book producing quick execution of some or all of the split orders. Under this assumption, these studies eliminate these trades and thus no null durations remains in the sample. This trading phenomena can be true in some cases but not in all. Indeed another feasible, and certainly logic, explanation is that these null durations occur because retail traders post small limit orders at a round price. In order to verify this conjecture we take a look to Figure 2. It represents the number of durations equal to zero (y axis) for all observed prices (x axis). It seems that as a round price happens, for example 1000 pesetas (6.04 euros), the number of trades increases and thus the number of null durations also increases. This increasing of null durations does not only happens around "very round" prices. All the small pikes that can be seen in the figure correspond to prices which are multiples of 50 pesetas (0.3 euros), two times the tick (a tick is the minimum price variation). This confirms the hypothesis that almost all the null durations occur at round prices and thus they are caused by retail traders that post limit orders at these particular prices.

A drawback of the ACD and the Log-ACD models, as well as all financial duration models existing in the literature, is that they do not permit durations equal to zero since the distributions used for durations are not defined at zero (except the exponential distributions but as we will see it is not the best choice). In the exchange markets this a quite common event when dealing with transaction data, where several transactions occur at the same time. When dealing with this particular type of durations we are willing to replace the durations equal to zero by some quantity. This quantity can be either estimated or chosen ad hoc. We propose the following strategy:

$$d_i^* = \begin{cases} d_i & \text{if} \quad d_i > 0 \text{ and } d_{i-1} > 0 \\ c_i & \text{if} \quad d_i = 0 \\ d_i - \sum_{j=1}^{J} c_j & \text{if} \quad d_i > 0 \text{ and } d_{i-j} = 0,\ j = 1, \dots J, \end{cases} \tag{19}$$

where J is the number of immediately past successive null durations. This transformation is subject to the constraints $c_i > 0$ and $0 < \sum_{j=1}^{J} c_j \leq 1$. There is one special case when $d_i = 1$ and $d_{i-1} = 0$. Then d_i is also considered as a duration zero but then if next duration, d_{i+1}, is

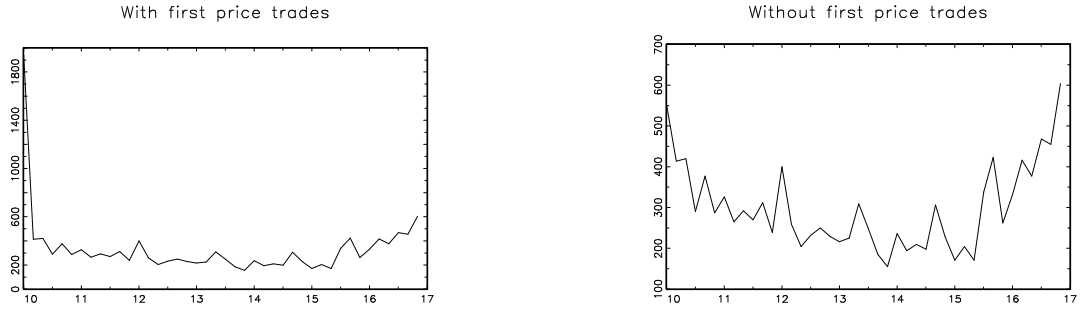


Figure 1: Intradaily seasonality of null durations. Left including first trading day price. Right excluding it.

strictly positive it is not transformed. Notice that in order to maintain that the sum of all the durations remains equal to the total time spell considered, the strictly positive duration that occurs after successive null durations is also modified. In terms of time deformation it means that null durations are enlarged while the next strictly positive duration is shrunk.

Thus, given this transformation, what is of interest is to set the values c_i. There exists several alternatives depending on the interest of the analysis. The first approach consists in replacing c_i by some ad hoc constant. The second approach is to estimate them. Estimation can be done considering the model as a left censored model where the censoring is that we do not observe values below one. Another possibility would be to consider that the DGP of null durations differs from the DGP of the strictly positive durations. Then we can use a similar technique to the hurdle models used in Tobit models and in count data (see Cragg, 1971, and Mullahy, 1986 respectively). The principal drawback of these models is that we are dealing with dynamical processes and hence either censoring or hurdle in these processes is not as easy as in the static case since we have to integrate with respect to past censoring and tractability is not assured (see, for example Wei, 1997, for a Bayesian approach to dynamic Tobit models). Since $0 < c_i \leq 1$ another possible functional form is by means of a logistic function whose value may depend on extra variables such as the number of successive zeros, past durations, prices, etc. These approaches are with no doubt cumbersome and they are themselves subject of a proper research.

Hence, in our framework, since we are mainly interesting in analyzing the intradaily seasonality but without despise the information content in null durations, we replace null durations by $c_j = 1/J$ where J is the number of successive null durations. The drawback of this approach is that null durations are considered to be regularly spaced within the second in which they occur. However, this transformation carries out the above scheme and the constraints are fulfilled. We adopt the easiest approach not expecting great results and letting this subject open for future research.

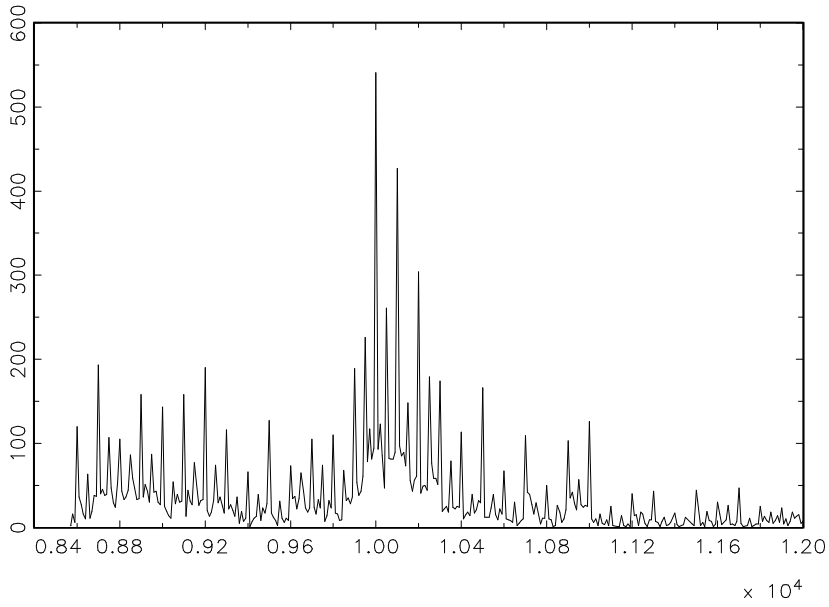
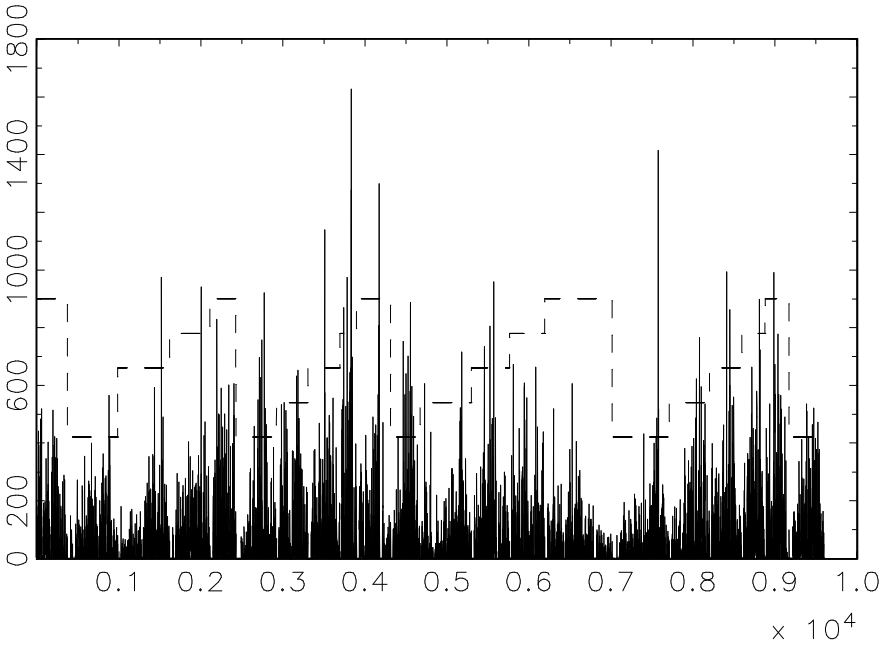


Figure 2: Number of null durations for every price

4.2 Descriptive analysis

Figures 3 and 4 are the observed durations, the autocorrelogram and a kernel estimate of the density. In Figure 3 there is also a piecewise constant line (the dashed line) indicating when a day begins and ends. The lowest steps correspond to Mondays and they are increasing until Friday. Obviously if one day is a holiday there is no piecewise line for that day. In order to see clearly the intradaily seasonal pattern we just show the first month, January 1998. From this figure we can see that for each day durations are generally small at the beginning and the end of the day, and large in between, indicating the intradaily seasonality. The autocorrelogram of hte durations (the left plot of Figure 4) confirms this feature. Even if we should use this plot only for illustrative purposes (when dealing with point processes it has not an exact meaning since data are irregularly spaced. A possible alternative is the variogram), one sees that there is a clear seasonal pattern. Finally the right plot of Figure 4 shows a kernel estimate of the density. It seems that the density has an asymptote at zero which is incompatible with the exponential distribution and the Burr distribution can be redundant (in the sense that the second parameter should not be significative and hence we attain the Weibull). Therefore a priori the correct distributions could be either Weibull or generalized gamma.

In Table one we provide a few descriptive statistics of the durations. Numbers in parenthesis are the same statistics but eliminating null durations. The basic insights that can be extracted from this table are: durations are overdispersed and highly autocorrelated as it was expected given that they are financial processes. The number of durations equal to zero is



Solid line are observed durations. Dashed line are the days of the week. Each piecewise is a day and it is increasing from Monday up to Friday. The scheme is repeated every week. Only January has been plotted

Figure 3: Observed duration and day of the week

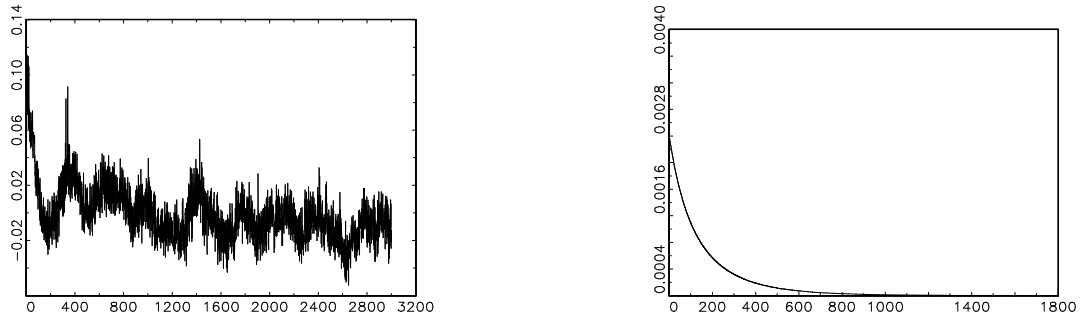
very significative, 26.5% percent of the total. Eliminating them implies that the dynamical properties of the process change. For example the Q-statistics are higher when only considering strictly positive durations.

Since the aim of the paper is about (intradaily) seasonality, it is worthwhile compute the diurnal component (i.e. the function $\hat{\phi}^a(\tau)$ used to seasonally adjust data). Up to now this function has been specified by means of cubic splines:

$$\hat{\phi}^a(\tau) = \sum_{j=1}^{J} \mathbf{1}_{[\Delta_j \leq \tau < \Delta_{j+1}]} \left[a_j + b_j \left(\tau - \Delta_j \right) + c_j (\tau - \Delta_j)^2 + d_j (\tau - \Delta_j)^3 \right], \tag{20}$$

where Δ_j are the knots and $\mathbf{1}_{[\Delta_j \leq \tau < \Delta_{j+1}]}$ is an indicator function for the $j+1th$ segment. We introduce a second estimator which is a standard Nadaraya-Watson estimator

$$\hat{\phi}^a(\tau) = \frac{\sum_{i=1}^{N} K\left(\frac{\tau - t_i}{h}\right) d_i}{\sum_{i=1}^{N} K\left(\frac{\tau - t_i}{h}\right)}. \tag{21}$$



Density estimated non parametrically with a Gamma Kernel. See Chen (2000). The bandwidth is $(0.9\sigma N^{-0.2})^2$ where σ is the standard deviation and N the number of observations.

Figure 4: Autocorrelogram and Marginal Density

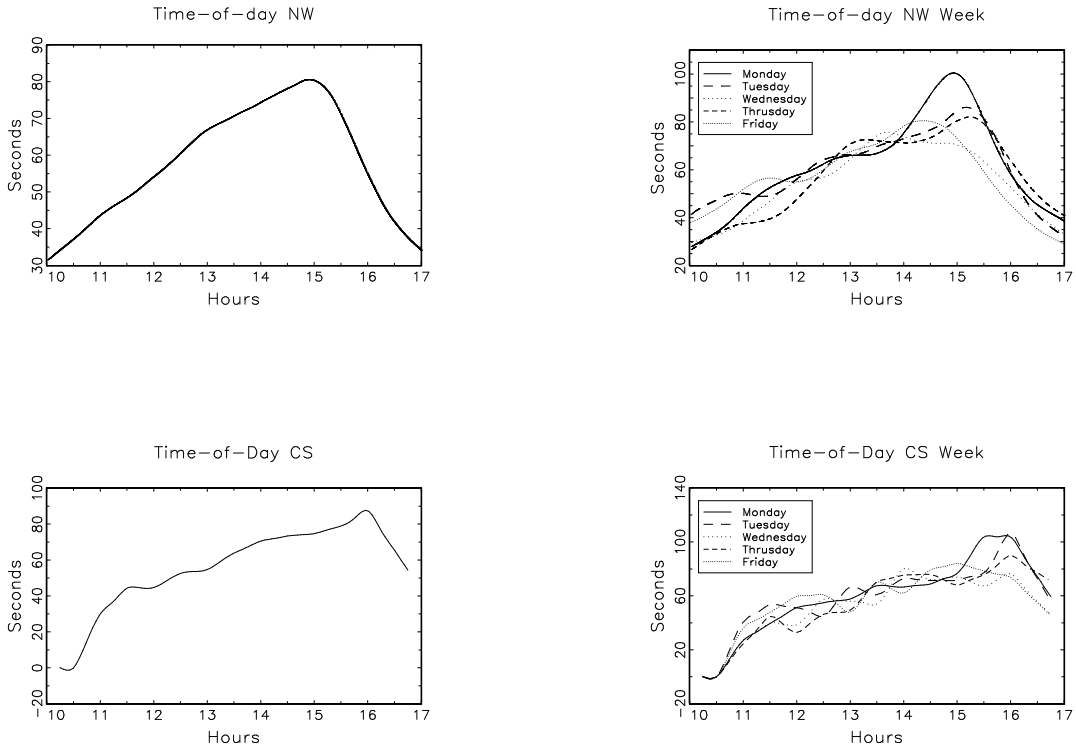
Table 1: Information on Duration Data

No. Days	No. Obs	No. Durations=0	% Durations=0
61 (61)	27298 (20067)	7231 (0)	26.5 (0)
Mean	S.d	Q(1)	Q(10)
55.82 (75.94)	102.28 (112.711)	364.57 (457.58)	2413.3 (2881.8)

Descriptive statistics for the trade durations of Bankinter during January - March 1998. Durations are measured in seconds. $Q(k)$ is the Ljung-Box statistic for autocorrelation of order k.

With respect to the latter estimator, the time variable is the number of cumulative seconds from midnight every day. The kernel chosen is the quartic and the bandwidth is $2.78\sigma N^{-1/5}$ where σ is the standard deviation of the data and N the number of observations. With respect to the former estimator, the nodes are set every hour, as used in previous studies. Figure 5 represents the two diurnally estimators for the mean day and for the five days of the week (excluding null durations).

From this figure it seems that the variation of the daily seasonal pattern is not clearly significative across days. The Nadaraya-Watson estimator is smoother than the piecewise cubic spline but the latter varies more within a day since it ranges from zero to approximately 90 while the Nadaraya-Watson ranges from approximately 30 to approx 90. The same exercise has been done including null durations and results are very similar. This detail is important since as it will be showed later there are remarkable differences between the estimated curves when including and excluding the null durations.



NW stands for Nadaraya-Watson and CS for Cubic Splines

Figure 5: Diurnal component

4.3 Estimation

We now proceed with estimation. Using the estimation method proposed in the former section, we estimate the parameters of a Log-ACD model and the seasonal component under three alternative specifications for the conditional distribution of the durations: exponential (QMLE), Weibull and generalized gamma. For comparative purposes, we perform the estimation on the raw data, the data adjusted for seasonality, both with and without null durations.

Results are in Tables 2 and 3. The first column is the estimation result when we do not consider seasonality. In the next two columns the intradaily and the intraweekly nonparametric estimators respectively are included. Last column is the result when we adjust durations by the Nadaraya-Watson estimator.

The mean equation parameter values, (ω), α and β are as expected according to the properties of a financial duration process. The model is stationary and in all cases models capture a strong persistence affect. Notice that ω is not present in the estimation with the seasonal component since the nonparametric curve plays the role of a varying parameter. However α and β are always present and their estimates are quite similar through model specifications.

This effect can be explained as follows: Under the multiplicative specification introduced in (16), and a pre-specified form for $p(\varepsilon;\vartheta_3)$ then the likelihood function can be decomposed into three components (see Appendix for further details)

$$L_n(d,\vartheta_1,\vartheta_3) = L_{1n}(\vartheta_3)L_{2n}(d,\vartheta_1,\vartheta_3)L_{3n}(d,\vartheta_1,\vartheta_3,\phi(t_i)) \tag{22}$$

Notice that the nonparametric curve estimator is only present in L_{3n}. The idea behind the similitude of the estimates of α and β is that L_{3n} conveys few information about ϑ_1 and ϑ_3. In order to verify this conjecture we use the Kullback discrepancy for measuring the information carried by L_{3n}. If the information is small means that $\phi(t_i)$ is not crucial on the estimation of ϑ_1 and ϑ_3. The discrepancy is

$$I = E_{\widehat{\vartheta}_1,\widehat{\vartheta}_3}\left[\log\frac{L_{1n}(\vartheta_3)L_{2n}(d,\vartheta_1,\vartheta_3)}{L_n(d,\varphi,\vartheta_3)}\right] \tag{23}$$

Note that the expectation is with respect to the estimates under $L_n(\cdot)$. The discrepancy is thus equal to

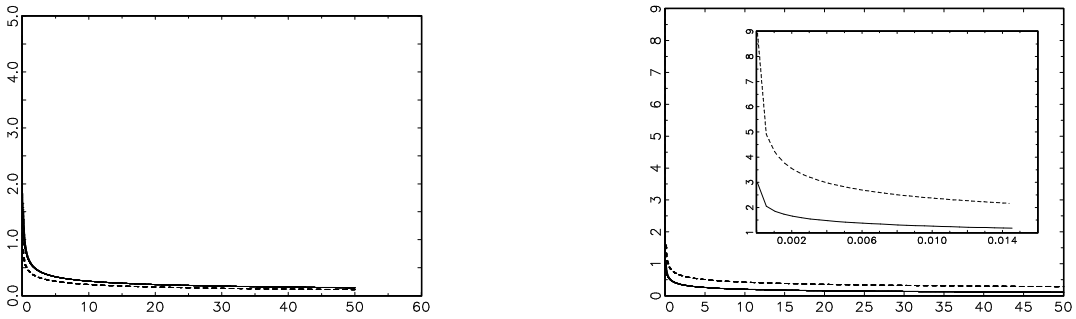
$$I = -E_{\widehat{\vartheta}_1,\widehat{\vartheta}_3}\left[\log L_{3n}(d,\varphi,\vartheta_3)\right] \tag{24}$$

As benchmark we use the generalized gamma since Weibull and exponential are nested on it. Under some calculations (23) is equal to

$$I = \phi(t_i)\hat{\gamma}\hat{\nu} + 1 \tag{25}$$

Applied to the trade duration process, the mean log-likelihood function is 18.9 while the mean Kullback discrepancy is 2.2, i.e. L_{3n} carries just 11.6% of the total information about ϑ_1 and ϑ_3 which makes $\hat{\vartheta}_1$ and $\hat{\vartheta}_3$ to be close through model specifications. A direct consequence of this result is that all the market microstructure's testing done using the ACD family models (Engle and Russell, 1998 and Bauwens and Giot, 1999 among others) are valid in this framework.

Nevertheless it is worthwhile to explain why when the seasonal component is considered the parameters γ and ν for the generalized gamma increases and decreases respectively. This change can be explained in terms of hazard functions since it is the most important function when dealing with durations. Left plot of Figure 6 shows the hazard functions for the generalized gamma distributions when considering the seasonal component (dashed line) and when ignoring it (solid). Although small, in this plot we can see the effect of including or not the seasonal term. The hazard function is shifted down (from the solid to the dashed line) when considering the seasonal component. This is due to the following: when getting rid of the seasonal component in the long-run term we are excluding a part of the high activity in the opening and the closing, related with the shorter durations. Equivalently for the lunch time: it is expected that a part of the low trading activity is captured by the seasonal component,



Left plot is the estimated hazard functions for the estimation without null durations and with and without intradaily seasonal component (dashed and solid lines respectively) for the generalized gamma distribution. Right plot is the estimated hazard functions for the Weibull and the generalized gamma distributions (solid and dashed lines respectively) without null durations. The inserted window shows a zoom of the graph close to the origin.

Figure 6: Estimated hazard functions

related with the longest durations. Therefore the seasonal term will capture a proportion of the lowest and the highest trading activities implying that the hazard function, or the instantaneous probability, will decrease and, because the construction of the hazard functions, they also decrease for medium durations.

The right plot can be used for looking at the differences between distributions. The solid line represents the Weibull hazard function while the dashed line is the generalized gamma. We estimate without zeros (the inserted window is a zoom of the area close to the origin). The hazard function of the generalized gamma is above the hazard function of the Weibull. It means that the generalized gamma distribution increases the instantaneous probability of a trade. Finally, remark that as the distribution function becomes more flexible, the changes in the hazard function when estimating with and without seasonal component increases. For example, for the exponential the hazard function is equal through any specification (since it is constant and equal to one), for the Weibull case it varies but very slightly while for the generalized gamma changes are relevant as already explained.

With respect to the seasonal curve, Figure 7 shows the intradaily and intraweekly seasonal patterns when using a Weibull distribution for the estimation with and without zeros (bottom and top plots respectively). The patterns are centered i.e. the line represents $\phi(\tau) - \bar{\phi}$ where $\bar{\phi}$ is the mean. The first thing that draws the attention is the different shape of the estimated curves by including and excluding null durations. Although they have the same inverted U shape, differences come from the intensity of the seasonality at different periods of the day. It is particularly remarkable at the beginning of the day. In the bottom plots the deterministic seasonality increases sharply at the beginning of the day while it is not the case for the top estimated curves. It means that at the beginning of the day there exist a certain dynamics that

Table 2: Estimation Results excluding null durations

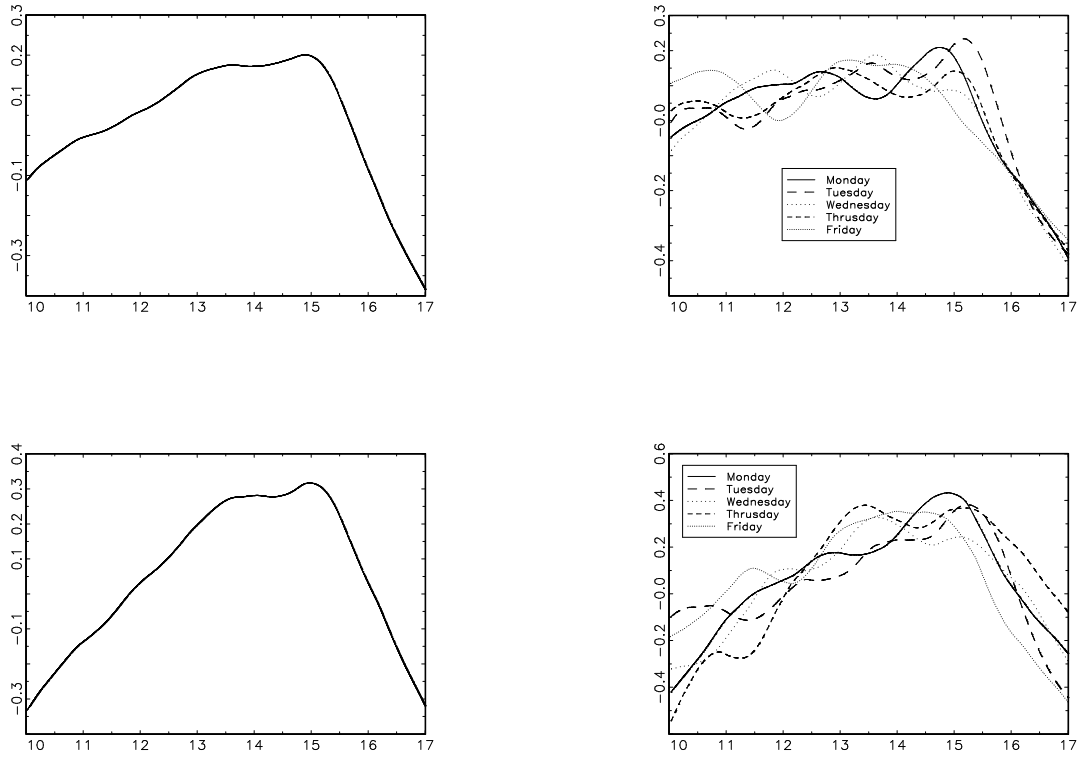
		No Seaso	Intraday	Intraweek	NW
Exp	ω	0.0809			0.0196
		[0.0056]			[0.0014]
	α	0.0391	0.0248	0.0235	0.0209
		[0.0016]	[0.0017]	[0.0018]	[0.0015]
	β	0.9506	0.9703	0.9724	0.9749
		[0.0024]	[0.0024]	[0.0024]	[0.0021]
Weibull	ω	0.0915			0.0234
		[0.0088]			[0.0023]
	α	0.0439	0.0285	0.0268	0.0249
		[0.0025]	[0.0027]	[0.0027]	[0.0025]
	β	0.9443	0.9655	0.9680	0.9694
		[0.0037]	[0.0038]	[0.0039]	[0.0035]
	γ	0.7357	0.7410	0.7421	0.7406
		[0.0047]	[0.0048]	[0.0048]	[0.0048]
GG	ω	0.1002			0.0261
		[0.0101]			[0.0027]
	α	0.0472	0.0308	0.0288	0.0276
		[0.0024]	[0.0030]	[0.0031]	[0.0029]
	β	0.9398	0.9622	0.9651	0.9658
		[0.0043]	[0.0044]	[0.0044]	[0.0042]
	γ	0.5937	0.6181	0.6252	0.6202
		[0.0213]	[0.0222]	[0.0225]	[0.0222]
	ν	1.4313	1.3517	1.3286	1.3429
		[0.0861]	[0.0804]	[0.0790]	[0.0796]

Estimation results ignoring the seasonal behaviour (termed No Seaso), with the nonparametric estimator proposed accounting for the intradaily and the intraweekly pattern (termed Intraday and Intraweek respectively) and using a preseasonal adjustment by means of the Nadaraya-Watson (NW) estimator. Exp, Weibull and GG stand for exponential, Weibull and generalized gamma distributions respectively. Numbers are the estimated parameters and in underneath between brackets are heterokedastic-consistent standard deviations.

Table 3: Estimation Results including null durations

		No Seaso	Intraday	Intraweek	NW
Exp	ω	0.4209 [0.0187]			0.1149 [0.0041]
	α	0.0696 [0.0023]	0.0731 [0.0025]	0.0751 [0.0025]	0.0734 [0.0025]
	β	0.8528 [0.0060]	0.8204 [0.0074]	0.8129 [0.0075]	0.8137 [0.0077]
Weibull	ω	0.5766 [0.0338]			0.2069 [0.0096]
	α	0.1259 [0.0053]	0.1279 [0.0055]	0.1292 [0.0055]	0.1276 [0.0055]
	β	0.7812 [0.0112]	0.7353 [0.0129]	0.7434 [0.0133]	0.7515 [0.0129]
	γ	0.5117 [0.0028]	0.5158 [0.0028]	0.5168 [0.0028]	0.5152 [0.0028]
GG	ω	-0.354 [0.3160]			-0.489 [0.8623]
	α	0.2096 [0.0066]	0.1537 [0.0034]	0.1496 [0.0032]	0.2060 [0.0066]
	β	0.6678 [0.0130]	0.7023 [0.0023]	0.7004 [0.0023]	0.6495 [0.0138]
	γ	0.0382 [0.0002]	0.7253 [0.0203]	0.7309 [0.0204]	0.0383 [0.0002]
	ν	146.24 [0.1948]	1.1235 [0.1259]	1.1137 [0.1262]	146.08 [0.4274]

For explanation see previous table



Top plots are the estimated seasonal intradaily and intraweekly centered components, i.e. $\phi(\tau) - \bar{\phi}$ where $\bar{\phi}$ is the mean. null durations are excluded. Bottom plots are the equivalent but including them. Weibull distribution is used

Figure 7: Estimated seasonal curves

is captured by the parametric part in the semiparametric estimation only when excluding null durations. A comparison can be done whith Figure 5. The ad hoc seasonal patterns (including and excluding null durations) are very similar to the estimated curve when including null durations. It means that when including null durations the ones produced in the first half of the day are not informative and hence they are captured by the seasonal curve while it is just the contrary for the null durations observed at the end of day since the second half day seasonal pattern in similar in any plot. This permit us to conjecture that information that occur during the period in which the market is closed is not informative of the stochastic part of the process while the flow of information, either exogenous or endogenous (i.e. either information generated in the market or outside the market), that arrives to the market, when it is open, matters. After 13:00 there are no remarkable differences. At this time traders go for lunch and just before they take positions, increasing again the trading intensity. Traders lunch and the market remains relatively constant up to a bit before 15:30 when NYSE and NASDAQ preopen and then the market becomes quickly very active as the trading activity increases till the closing at 17:00.

The fact that seasonal patterns when adjusting or estimating jointly are different is an important empirical evidence that the dynamic and the seasonal components are not orthogonal and hence they have to be estimated jointly. Moreover it verifies that a simple nonparametric regression does not identify separately both components as previously advanced.

Additionaly, there is not a significative intraweekly pattern since the intradaily seasonality across the days of the week is very similar. Finally, although it is not showed the seasonal curve is almost identical for any of the three distributions, meaning that it is "robust" with respect to the distribution in the parametric part of the model.

4.4 Diagnosis

For testing the specification of the model we use density forecast. This technique is based on the calculation of the probability integral transform and then test wether it is $i.i.d$ and uniformly distributed using histograms and autocorrelograms. It was introduced by Diebold et al. (1998) in the context of GARCH models and extensively used by Bauwens et al. (2000) for comparing different financial duration models. This technique is specially useful for evaluating the forecasting performance of different non nested models although it can be used as well for nested models.

Basically it works as follows: Let $\left\{f_i(d_i \mid \bar{d}_{i-1}, \bar{y}_{i-1})\right\}_{i=1}^m$ be a sequence of one-step-ahead density forecasts produced by the model and let $\left\{p_i(d_i \mid \bar{d}_{i-1}, \bar{y}_{i-1})\right\}_{i=1}^m$ be the sequence of densities defining the data generating process governing the duration series d_i. It can be showed that the correct density will be preferred by all forecast users regardless of their loss functions and hence it makes sense to test whether $\left\{f_i(d_i \mid \bar{d}_{i-1}, \bar{y}_{i-1})\right\}_{i=1}^m = \left\{p_i(d_i \mid \bar{d}_{i-1}, \bar{y}_{i-1})\right\}_{i=1}^m$.

This test is done using the probability integral transform

$$z_i = \int_{-\infty}^{d_i} f_i(u)du,$$

that must be $i.i.d.$ and uniformly distributed under the correct density. Hence when assuming some mean equation and some distribution both independence and uniformity of the estimated density can be checked.

Testing uniformity can be done using a histogram based on the computed z sequence. If the density is correctly specified the histogram should be statistically flat. For the independence checking, autocorrelation functions of various centered moments of the z sequence can reveal some dependency. For further details see the two above references.

A remark must be done on the way to compute z in the present model and when adjusting data. When estimation is joint z is computed on the raw data whilst in the adjusted case z is computed for the seasonally adjusted durations. Notice that density forecast evaluation on the adjusted data or on the raw data including the ad hoc seasonal component is done in the same way. For example, in the Weibull case and with adjusted data z is equal to

$$z = 1 - e^{(-\frac{d^a}{e^{\psi_i}})^\gamma} \tag{26}$$

where d^a denotes adjusted durations i.e. $d^a = d/\hat{\phi}^a(\tau)$ and $\hat{\phi}^a(\tau)$ denotes the seasonal filter (20) or (21). Of course if we replace in z d^a by $d^a = d/\hat{\phi}^a(\tau)$ a density forecast evaluation on the raw durations but including the ad hoc seasonal component is obtained.

Figures 8 and 9 show the out-of-sample histograms of z and autocorrelograms of $(z_i - \bar{z})$. Out-of-sample means that the estimation is performed on the first two-thirds of the sample, and then the forecast densities and z are computed on the last third of the sample using the estimates obtained on the first part. Figure 8 contains, from top to bottom, the density forecast results when estimating without null durations and the three distributions. Last row when considering null durations and the generalized gamma distribution. All estimations are done with the intradaily component. On the contrary in Figure 9 we give the results with the generalized gamma, the Nadaraya-Watson estimator and with and without null durations. We do not show the autocorrelograms for other centered moments and using the intraweekly component since results are similar in all cases.

From these figures some comments arise. Firstly in general the mean equation captures correctly the dynamics in all cases since most of the autocorrelations remain in the 90% cofindence bands. This result is also found in Bauwens et al. (2000) where they shown that the mean equation choice in not crucial for determining the accuracy of the model. There is some residual autocorrelation when null durations are included and when the seasonal curve is not estimated jointly. Secondly there is in general a huge difference between the results with and without the null durations. This is caused probably by the way in which null durations are dealt. As explained we did not expect good results and thus we let the improvement on the treatment of these data for future research. Nevertheless it is worthwhile to explain this shape. Indeed a similar shape (bottom histogram in figure 8) has been found in Bauwens et al. (2000) when dealing with price durations and previously adjusting data by means of a cubic spline. The considered distributions are not able to account for durations very close to zero which is probably due to their high proportion in the sample. This is represented in the histogram by a very small frequency for $0 < z < 0.05$ and hence this lack of values at this range provokes an over representation on the following bins. With respect to the distributional assumption, as expected the exponential distribution does not make a good job while there other two behave much better, especially the generalized gamma.

Related with the inclusion of the seasonal component differences are clear. When it is included in the estimation, forecasting results are much better and z is uniformly distributed (in the case of no null durations). When data are adjusted for seasonality the histogram is much worse. Hence we assert that when including in the estimation the seasonal component the forecasted probability integral transform is *i.i.d.* and uniformly distributed.

Remark that although the density forecast is much worse when data are adjusted, the estimates on table two are similar through model specifications. This is due to the fact that

even if L_{3n} does not carry important information about ϑ_1 and ϑ_3, it is crucial in prediction since z and L_{3n} are strongly related (see Appendix).

In order to analyze deeply the cause of the rejection of the null hypothesis we on the differences in forecasting errors, given by $\varepsilon_i = d_i - E[d_i|\bar{d}_{i-1}]$, for different model specifications. For example an interesting issue is to compare the differences in the forecasting errors of models without taking into account the seasonality (denoted by ε_i^{NS}), adjusting ad hoc (ε_i^a) and estimating jointly (ε_i). Notice that while ε_i^{NS} and ε_i are computed as the above difference between the observed duration and its conditional expectation, ε_i^a is computed multiplying the conditional expectation of the adjusted durations by the diurnally component $\hat{\phi}^a(\tau)$, i.e. $\varepsilon_i^a = d_i - E[d_i^a|\bar{d}_{i-1}]\hat{\phi}^a(\tau)$. In figure 10 there are the differences of different forecasting errors (due to representation purposes only the first 18 days are plotted), in all cases using the generalized gamma distribution and excluding null durations. Left plot represent $(\varepsilon_i - \varepsilon_i^{NS})$. When not taking into account seasonality there is a clear seasonal pattern not captured by the model and hence captured by ε_i^{NS}. It makes sense since the model for ε_i^{NS} just forgets the existence of seasonality. Right plot represents $(\varepsilon_i - \varepsilon_i^a)$. There is as well a clear cyclical pattern although not so remarkable. This again a proof that seasonally adjusting data is not the most efficient way of dealing with seasonality.

5 CONCLUSIONS

In this paper we have proposed a component model for the analysis of financial durations. The components are the long-run dynamics and the seasonality. The later is left unspecified and the former is assumed to fall within the class of ACD (log-ACD) models. Joint estimation of the parameters of interest and the smooth curve is performed through a local (quasi-)likelihood method. For alternative specifications of the conditional density the resulting nonparametric estimator of the seasonal component shows a closed form expression that is a function of the Naradaya-Watson estimator.

A further advantage of this semiparametric component model is that under this approach any other tick-by-tick variable can be analyzed. The only requirements are define properly the dynamical component and the distribution. For example the analysis of tick-by-tick volatility could be succesfully done using the above methodology.

The empirical application in on the trade duration process of Bankinter, a medium size spanish bank traded in Bolsa de Madrid. The results shows significant differences with respect to previous alternative approaches.

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APPENDIX

Definitions and assumptions

In order to prove the results claimed in Theorems 1 and 2 we need to establish some definitions and assumptions. The proofs follow the same lines as in Severini and Staniwallis (1994).

(A.1) The random variable t takes values in a compact set $\mathbf{T} \subset R$. The marks y take values in a compact set $\mathbf{Y} \subset R^p$.

(A.2) The observations $\{(d_i, y_i, t_i)\}_{i=1,\cdots}$ are a sequence of stationary and ergodic random vectors.

(A.3) ϑ_{10} takes the values in the interior of Θ, a compact subset in R^p and ϕ takes the values in the interior of Λ, a compact subset of R.

$$\Lambda = \left\{ f \in C^2[a,b] : f(t) \in \operatorname{int}(\Lambda) \quad \text{for } \forall t \in [a,b] \right\}.$$

(A.4) Let Ξ be a compact subset of R such that $\varphi\left(\psi(\bar{d}, \bar{y}; \vartheta_1), \phi(t)\right) \in \Xi$ for all $t \in \mathbf{T}$, $y \in \mathrm{T}$, $\vartheta_1 \in \Theta$ and $\phi \in \Lambda$.

(A.5) The matrix

$$\Sigma_{\vartheta_1} = E\left(\frac{\partial^2}{\partial \vartheta_1 \partial \vartheta_1^T} Q\left(\varphi\left(\psi(\bar{d}, \bar{y}; \vartheta_1), \phi(t)\right); d\right) \right)$$

is positive definite.

(B.1) The kernel function $K(\cdot)$ is of order $k > 3/2$ with support $[-1, 1]$ and it has bounded $k+2$ derivatives.

(B.2) For $r = 1, \cdots, 10 + k$ the functions $\partial^r \varphi(m)/\partial m^r$ and $\partial^r V(\mu)/\partial \mu^r$ exist and they are bounded in their respective supports.

(B.3) d is a strong mixing process where the mixing coefficients must satisfy for some $p > 2$ and r being a positive integer

$$\sum_{i=1}^{\infty} i^{r-1}\alpha(i)^{1-2/p} < \infty.$$

Furthermore, for some even integer q satisfying $\frac{(k+2)(3+2k)}{(2k-3)} \leq q \leq 2r$

$$E\left|d\right|^{q} < \nu,$$

where ν is a constant not depending on t.

(B.4) The conditional density of t, given the information set I_{i-1}, $f(t)$, and the conditional density of d given t and I_{i-1} has $k+2$ bounded derivatives uniformly in $t \in \mathbf{T}$, $y \in \mathbf{Y}$ and $d \in \mathbf{D}$.

(B.5) Let

$$M\left(\eta;\vartheta_1,t\right) = E\left\{\frac{\partial}{\partial\eta}Q\left(\varphi\left(\psi(\bar{d},\bar{y};\vartheta_1),\eta\right);d\right)\left|\bar{y},\bar{d}\right.\right\}.$$

For each fixed ϑ_1 and t, let $\phi_{\vartheta_1}(t)$ the unique solution to $M\left(\eta;\vartheta_1,t\right) = 0$. Then for any $\epsilon > 0$ there exists a $\delta > 0$ such that

$$\sup_{\vartheta_1\in\Theta}\sup_{t\in\mathbf{T}}\left|\phi_{\vartheta_1}(t) - \phi(t)\right| < \epsilon$$

whenever

$$\sup_{\vartheta_1\in\Theta}\sup_{t\in\mathbf{T}}\left|M\left(\phi(t);\vartheta_1,t\right)\right| < \delta.$$

(B.6) The sequence of bandwidths must satisfy $h = O(n^{-\alpha})$ where

$$\frac{1}{4k} < \alpha < \frac{1}{4}\frac{q-(2+p)}{q+(2+p)}.$$

Proof of Theorem 1

The proof of this theorem follows the same steps as in the proof of Lemma 5 from Severini and Wong (1992), p. 1784. The bias term must be treated in the same way as they do. With respect to the variance term an additional result must be included to account for the dependence. Consider the following expression

$$\frac{1}{nh}\sum_{i=1}^{n}\left[K\left(\frac{\tau-t_i}{h}\right)\varphi\left(\psi(\bar{d},\bar{y};\vartheta_1),\eta\right) - E\left\{K\left(\frac{\tau-t}{h}\right)\varphi\left(\psi(\bar{d},\bar{y};\vartheta_1),\eta\right)\right\}\right]$$

and define

$$W_i = \frac{1}{h}K\left(\frac{\tau-t_i}{h}\right)\varphi\left(\psi(\bar{d},\bar{y};\vartheta_1),\eta\right) - E\left\{K\left(\frac{\tau-t}{h}\right)\varphi\left(\psi(\bar{d},\bar{y};\vartheta_1),\eta\right)\right\}$$

Then, under assumptions (A.2) and (B.3) the process $W_1, \cdots, W_n$ is strong mixing and therefore theorem 1 from Cox and Kim (1995) applies and the following sequence of inequalities hold. For $\epsilon > 0$

$$P\left\{\left|\frac{1}{n}\sum W_j\right| > \epsilon\right\} \leq \frac{E\left[(\sum W_i)^q\right]}{n^q\epsilon^q} \leq$$

$$\frac{1}{n^q\epsilon^q}C\left\{n^{q/2}\sum_{i=P}^{\infty} i^{q/2-1}\alpha(i)^{1-2/p} + \sum_{j=1}^{q/2} n^j P^{q-j}\nu^j\right\}$$

for any integers n and P with $0 < P < n$. Then using assumptions (B.1) to (B.6) and proceeding as Severini and Wong (1992) in the proof of Lemma 8, the proof is closed.

Proof of Theorem 2

The proof of this theorem relies consists in verifying conditions I (Identification), S (Smoothness) and NP (Nuissance Parameter) fromm Severini and Wong (1992). Condition NP(a) is the result already shown in Theorem 1. Condition NP(b) (least favorable curve) is inmediate from Lemma 6 of Severini and Wong (1992). This is due to the fact that we assume that the conditional density function belongs to the exponential family. By assuming (A.1) to (A.4) the smoothness condition holds. Finally, assumption (A.5) implies I. Then, using both a Uniform Weak Law of Large Numbers and a Central limit theorem for a stationary and ergodic process (see for example Wooldridge, 1994) propositions 1 and 2 from Severini and Wong (1992) apply and the proof is done.

Distributions

The generalized gamma density function for $d > 0$ is

$$f_{GG}(d) = \frac{c^{-\gamma\nu}\gamma\epsilon^{\gamma\nu-1}}{\Gamma(\nu)}\exp\left(-\frac{\epsilon}{c}\right)^{\gamma}, \tag{27}$$

where $\nu > 0$, $\gamma > 0$, $c > 0$ and $\Gamma(.)$ denotes the gamma function. For the Log-ACD model the parameter c is equal to

$$c = \frac{\exp(\psi(\vartheta_1) + \phi(\tau))}{\mu_\varepsilon(\vartheta_3)}, \tag{28}$$

where $\vartheta_3 = (1, \gamma, \nu)$. Rearranging terms the density can be expressed as

$$f_{GG}(d) = f_1(\vartheta_3)f_2(\vartheta_3, \vartheta_1)f_3(\vartheta_3, \vartheta_1, \phi(\tau)), \tag{29}$$

where

$$\begin{aligned} f_1(\vartheta_3) &= \tfrac{\gamma}{\Gamma(\nu)}\mu(\vartheta_3)^{\gamma\nu} \\ f_2(\vartheta_3, \vartheta_1) &= \left(\tfrac{1}{e^{\psi(\theta)}}\right)^{\gamma\nu} d^{\gamma\nu-1} \\ f_3(\vartheta_3, \vartheta_1, \phi(\tau)) &= e^{-\phi(\tau)\gamma\nu - \left(\frac{d\mu(\vartheta_3)}{e^{\psi(\theta)}e^{\phi(\tau)}}\right)^{\gamma}}. \end{aligned} \tag{30}$$

The mean and the distribution function (cdf) are given by

$$\begin{aligned} \mu(c,\gamma,\nu) &= c\frac{\Gamma(\nu+\frac{1}{\gamma}),}{\Gamma(\nu)} \\ F_{GG}(d) &= \frac{\Gamma(\nu,(d/c)^{\gamma})}{\Gamma(\nu)} \end{aligned} \tag{31}$$

and for computing $\Gamma(\nu, x)$ numerical integration is needed.

The Weibull density and its mean are attained when $\nu = 1$. The cdf is

$$F_W(d) = 1 - \exp\left(-\frac{\epsilon}{c}\right)^{\gamma}. \tag{32}$$

Finally the exponential density and its mean are attained when γ and ν are equal to one. The cdf is derived from (32).

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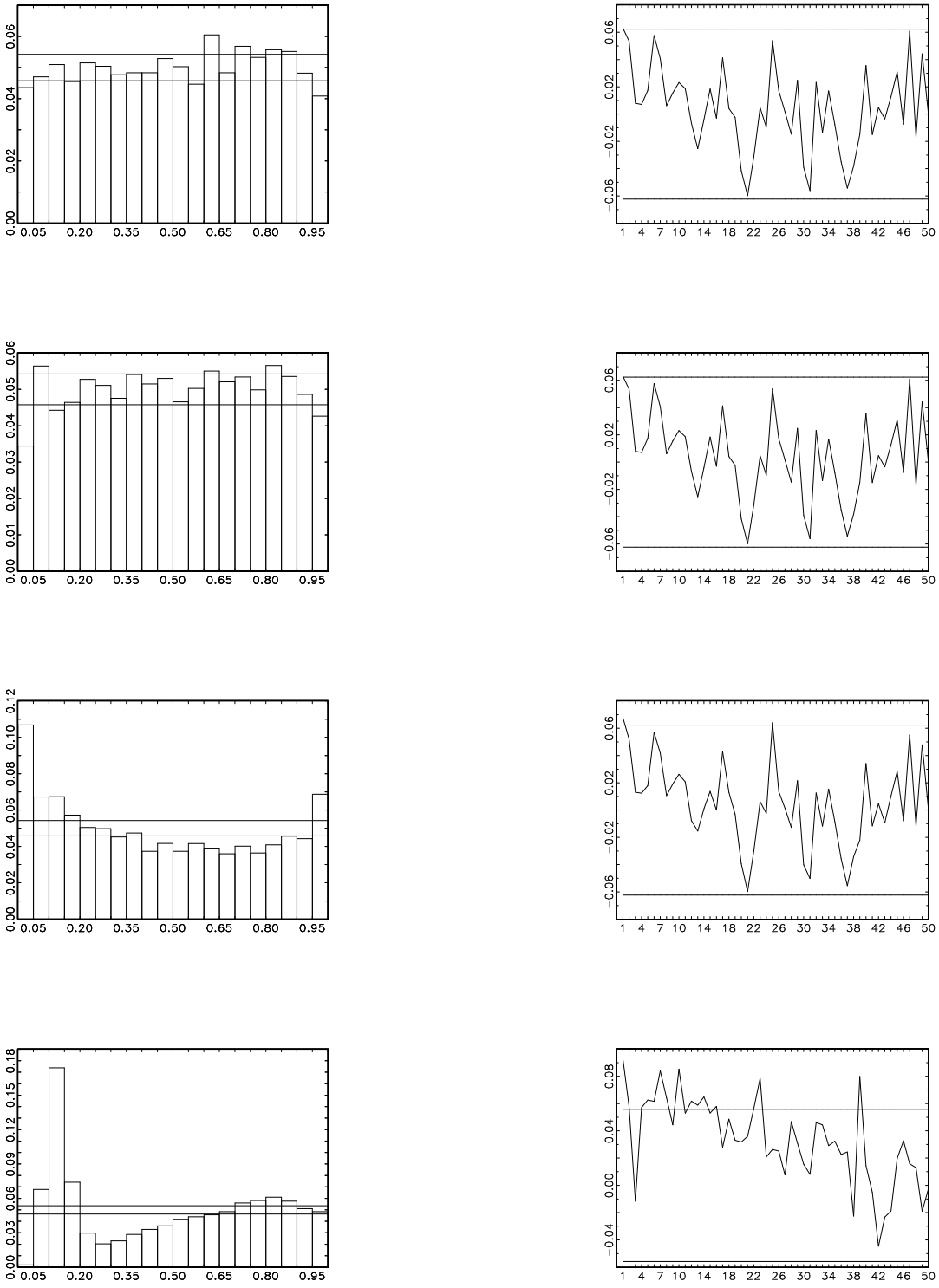
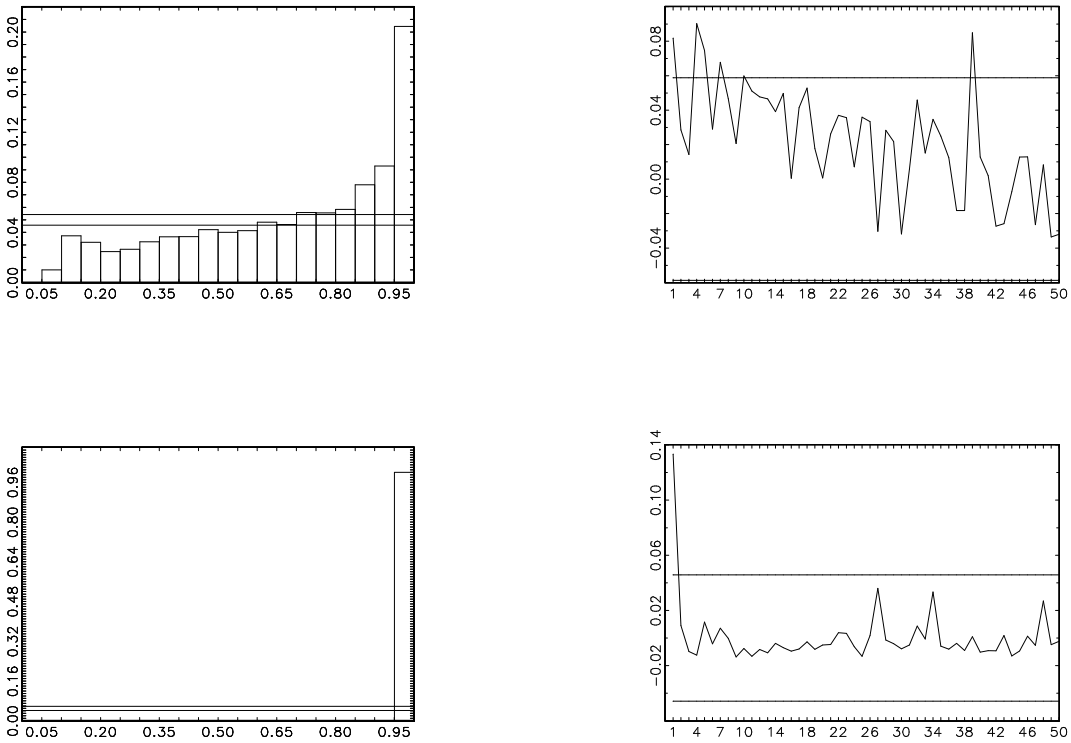


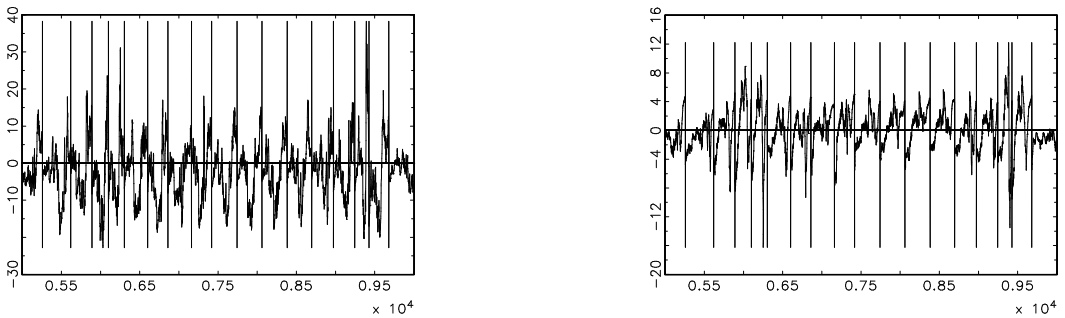
Histograms and autocorrelograms for z. Intradaily component used. Top three without null durations. Bottom one with null durations. Distributions from up to down: generalized gamma, Weibull, exponential and generalized gamma.

Figure 8: Density forecast evaluation for raw durations



Histograms and autocorrelograms for z. Adjusted for seasonality using the Nadaraya-Watson estimator and the generalized gamma distribution. Top without null durations. Bottom with.

Figure 9: Density forecast evaluation for seasonally adjusted durations



Left plot are differences in the forecasting errors of the model without seasonal component and without adjusting data and the model with seasonal component. Right plot are the differences in the forecasting errors of the model without seasonal component but adjusting data and the model with seasonal component. Vertical lines represent the moment of time in which a day begins. Only the first 18 days of the sample are plotted.

Figure 10: Differences on forecasting errors