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MARKET BASED COMPENSATION, TRADING AND LIQUIDITY*

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Abstract

This paper examines the role of trading and liquidity in a large competitive market with dispersed heterogenous information on market-based managerial compensation. The paper recognizes the endogenous nature of a firm's stock price - it is the outcome of self-interested speculative trading motivated by imperfect information about future firm value. Using the stock price as performance measure means bench-marking the manager's performance against the market's expectation of that performance. We obtain two main results: first, the degree of market-based compensation is proportional to the market depth, which is a measure of the ease of information trading. Secondly, using the dynamic trading model of Vives (1995) we show that if the investment horizon of informed traders decreases, at equilibrium the managerial e.ort reduces, and the optimal contract prescribes stock-compensation with longer vesting period.

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1 Introduction

Executive and managerial compensation makes ample use of market based performance measures, either directly via stock and options or indirectly via bonuses for stock appreciation (for a recent survey of the components and trends of executive compensation see Murphy (1999)). How market conditions, and in particular liquidity, impact on market based executive compensation is however still imperfectly understood.

The aim of this paper is to study how the investors' preferences and their private information affect the design of an optimal compensation scheme.

We provide two innovative results: the first is that market based compensation is directly proportional to the depth of the market, defined as the inverse of the Kyle (1985) 'lambda', a common measure of market liquidity in the microstructure literature. Secondly, we show that the time preferences of the market traders affect the effort decision of the managers through their effect on compensation. The intuition of this result relies on the fact that the presence of risk-averse informed investors with a short-term horizon drains liquidity from the market and reduces the amount of information conveyed by the price (Vives (1995)), thereby worsening the moral hazard problem and leading to lower effort provision (i.e. to a 'third best' solution).

The idea that market liquidity is beneficial for the aggregation of private information is widespread in the microstructure literature and dates back to Kyle (1985). A more liquid market accommodates more information-based trading and results in more informative prices. This effect was first used by Holmstrom and Tirole (1993) (HT from now on) to show that dispersed ownership is value-enhancing since it increases the benefits of market monitoring. A more dispersed ownership increases the volume of trade due to uninformed investors, and this in turn induces a single large inside-monitor to collect more private information which is useful for the compensation design. Thus, HT obtain that the degree of market-based compensation is directly proportional to the volume of uninformed trade. Notice however that their model does not imply, as we do, that the market-based power incentives increase with the depth of the market, as pointed out by Garvey and Swan (2002)[1].

This different empirical prediction of our model compared to HT is actually the result of a fundamentally different theoretical approach.

The argument in HT crucially relies on the costly collection of private information by a sole large risk-neutral insider. As in other microstructure models (e.g. Kyle (1985)), a higher volume of uninformed trade increases the value of private information, inducing the informed monopolist to collect a more precise signal at equilibrium. But if one considers the financial markets as a means of aggregating heterogeneous, dispersed information via self-interested, speculative trading, it is natural to assume that the market for information is open: then, a higher value of private information induces entry of other informed traders. If we allow entry of other speculators the market price converges quicker to its "full-information" value (Holden and Subrahmanyam (1992)) and the competition among informed speculators reduces the return on private information. Then, it is not obvious that at equilibrium traders acquire a more precise signal when the noise in the market increases. Moreover, Subrahmanyam (1992) shows that increasing the number of informed traders can actually decrease market liquidity when traders are risk-averse. Thus, if many informed traders operate on the market, the effect of the volume of uninformed trade on the optimal compensation is not as clear-cut as in HT.

Secondly, in our opinion, another flaw in the argument of HT is the following: the information in the market is mostly valuable for the owners of the firm, who then have the highest incentives to collect it. Thus, it seems unrealistic they leave this activity to a single external monitor. This critique does not apply to our model since it would be impossible or prohibitively costly for the inside owners to contract with a large number of traders each of whom has some information[2].

We overcome these two critiques of HT by assuming that on the market operates a continuum of risk-averse, informed speculators, each with a single piece of information, which we consider as the result of an "entry" game on the market for private information. These competitive traders receive an informative signal with different precisions on the future value of a share of the firm and they trade against liquidity traders in a semi-strong efficient market with risk-neutral market makers. The value of the firm is unknown and it is affected by the unobservable effort of the manager. The principal, acting in the interests of existing shareholders, offers before the trading starts an incentive contract to the manager, which is based on the observable future market price, and on the final cash flow of the firm. We find that the optimal degree of market-based compensation is directly proportional to the market depth, as measured by the Kyle's 'lambda', while the relation with the variance of uninformed trade is ambiguous.

A second contribution of our paper is to analyze how the investors' time preferences impact on the real decisions of managers through their effect on market liquidity. In order to do this we present a dynamic trading model in which investors have different investment horizons: either they maximize their final wealth (we call them long-termists), or they close their portfolio at every trading day, maximizing its value at the end of each period (short-termists). Due to the liquidity constraint of the manager, the principal can offer him a contract based only on the market prices, since the final realization of the firm cash-flows arise too far in the future. We derive then the equilibrium price process and the optimal compensation package.

To our knowledge, there are no existing papers studying how the liquidity dynamics affect the incentive design and its feedback on the managers' effort. We use Vives (1995) to show how the horizon of the market traders affects market liquidity: investors with short-term trading horizon drain liquidity since they suffer additional uncertainty over the realization of the next-period market price. This in turn reduces the information conveyed by the equilibrium price, making it less effective as an incentive device but, on the other hand, more useful for insurance purposes. We prove that the effect of short-termism on liquidity undoubtedly makes the moral hazard problem more severe, so that it becomes more expensive for the principal to induce effort. Short-term preferences by investors on the financial market then reduce the optimal effort choice by managers whose compensation is linked to the market price.

The financial literature shows other ways in which liquidity can affect the incentives of a large outside shareholder to monitor the firm. A liquid market can facilitate building up a controlling block but it can also hinder monitoring since it allows easy exit (see Bolton and von Thadden (1998)) for an analysis of the initial ownership structure of the firm and Maug (1998) for an analysis of secondary trading). The outside stock market can also provide incentives to the managers via take-overs (see Scharfstein (1988) and Stein (1988)).

Paul (1992) shows that information useful for incentive contracting may not necessarily be useful for stock valuation and vice versa. The complementarity of market and accounting information is also the theme in Diamond and Verrecchia (1981), Kim and Suh (1993) and Bushman and Indjejikan

(1993). But these papers do not recognize, as we do, that the value of the stock price as a performance measure is endogenous to the market model, and in particular depends on the liquidity of the stock, measured by the Kyle ‘lambda’.

Finally, we mention Bolton, Scheinkman and Xiong (2002) who depart from rational, efficient trading and analyze executive competition when traders in the market-place are overconfident. In their model, compensation induces a manager to invest in a worthless but risky project in order to increase the speculative value of the firm. Shares of such a "bubble" firm will be valuable as one group of overconfident investors speculates on the resale value to another, differently overconfident group. In that sense, they offer an alternative, equilibrium contracting view to the more common rent extraction view on the issue of lavish executive pay in recent times.

The plan of our paper is as follows: Section 2 introduces the two building blocks of the model: the moral-hazard problem and a market model of trading. Section 3 derives the conditions for an optimal incentive contract. The aim is to recognize that without an explicit model of trading and price formation, any conclusion about market based compensation remains ad-hoc. Section 4 solves the market model, defines liquidity and shows how a more liquid stock leads to more compensation based on the stock price. In section 5 we show that the horizon of informed investors affects market liquidity, and short-termism has impact on managerial effort and the optimal vesting period. Section 6 provides the relevant empirical evidence. In section 7 we informally discuss the case in which the principal can contract on the earnings reported by the manager but not on the true ones. This is the starting point to study how our results are robust to accounting manipulation. Section 8 concludes.

2 The static model

The model analyzes the moral-hazard problem between owners and management inside a publicly traded firm. Active trading of the firm's shares in a large competitive market where many traders have heterogenous, dispersed and imperfect information about the future value of the firm, results in a stock price that can be included in the managerial incentive contract. The moral-hazard problem is modelled as in Holmstrom and Milgrom (1991). The market model follows Vives (1995), which is a version of the standard large market noisy rational expectations model with the addition of a competitive risk-neutral market-making sector.

2.1 Agents

There are five types of agents. A publicly traded firm is run by a risk-averse manager (the agent) whose unobservable effort drives the expected value of the firm. The firm is owned by a risk-neutral collective of inside owners (the principal).[3] They are passive in the sense that they do not trade the firm's shares but hold them until the last period when the firm is liquidated. The key economic problem is that there is a conflict of interest between maximizing the shareholders' wealth and the private motives of the manager.

The company stock is traded by three different agents. First, there is a large number of informed risk-averse traders (rational speculators). Each trader possesses different imperfect information about the future value of the firm. They make use of all information available, i.e. they also take into account the information conveyed by prices. Second, there are noise traders who trade for reasons that are not related to information about the firm. And third, there is a competitive risk-neutral market making sector. It ensures that the price will be efficient in the sense that it reflects all available public information.

2.2 Technology, contracting and the sequence of events

The basic model has three dates: $t = 0, 1, 2$. It ends with the liquidation of the firm for a gross value v. At the beginning, $t = 0$, the inside owners (the principal) hire a manager (the agent) to run the firm and sign a management contract with him.

The contract drawn up at $t = 0$ specifies three payments to the manager at $t = 2$. A fixed payment and two payments that depend on the two observable (and verifiable) variables: the price of shares p at $t = 1$ and the *net* liquidation value of the firm at $t = 2$. We write managerial income I as:[4,5]

$$I = a_0 + a_p p + a_v(v - a_0 - a_p p) \tag{1}$$

After signing the contract (a_0, a_p, a_v) at time t=0, the manager exerts an unobservable effort e at a private cost $c(e)$ (as usual $c_e > 0$ and $c_{ee} \geq 0$). His effort drives the expected value of the firm v at time $t = 2$:

$$v = e + \theta + \eta \tag{2}$$

where θ and η are random variables, $\theta \sim N(0, \sigma_\theta^2)$ and $\eta \sim N(0, \sigma_\eta^2)$, that represent sources of noise outside the control of the manager.[6] The first-best level of effort, defined as the hypothetical effort that the risk-neutral principal would exert himself, is $e^{fb} = \frac{1}{c_e}$.

At time $t = 1$ competitive trading by informed traders, noise traders and the market making sector determines the market price p for the firm's shares (more on this in section 2.4).

At time $t = 2$, the gross value of the firm v is realized, the manager is paid income I according to his contract (a_0, a_p, a_v) and the model ends with the liquidation of the firm for a value $v - I$.

The sequence of events is summarized in figure 1.

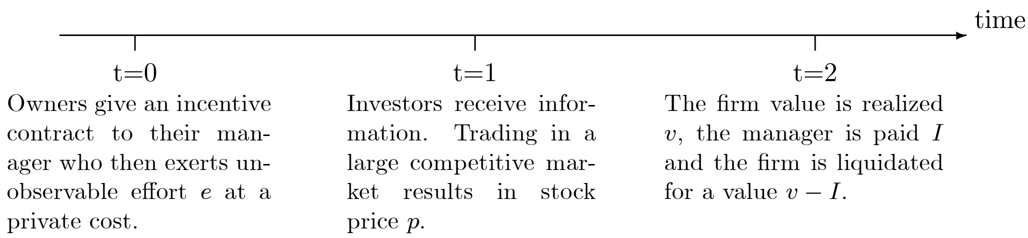

Figure 1: The timing of events

2.3 The moral-hazard problem

The manager's preferences are represented by a CARA utility function defined over income minus the (monetary) cost of effort: $U_m(I, e) = -\exp[-r_m I - c(e)]$, where r_m is the coefficient of constant absolute risk-aversion.

There is a conflict of interests between inside owners and the manager since managerial effort is not observable. Inside owners must choose the incentive contract (a_0, a_p, a_v) that maximizes the expected value of the firm net of managerial income,

$$\max_{a_0,a_p,a_v} E[v(e) - I(a_0, a_p, a_v)] \tag{3}$$

subject to the manager acting in his own interest,

$$e = \arg\max_{e'} E[U_m(I(a_0, a_p, a_v), e')] \tag{4}$$

and subject to the manager wanting to work for the owners,

$$E[U_m(I, e)] \geq 0 \tag{5}$$

where we have simplified the manager's outside opportunity to zero.

2.4 A competitive rational expectations market

At $t = 1$ the firm's shares are traded in a large competitive noisy rational expectations market. There is a continuum of risk-averse informed traders, indexed by $i \in [0, 1]$. At t=1, each informed trader sees a noisy signal s_i of future firm value v:

$$\begin{aligned} s_i &= e + \theta + \varepsilon_i \\ &= v + \xi_i \end{aligned} \tag{6}$$

where ε_i are i.i.d. random variables, $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$.[7]

At $t = 1$ an informed trader i buys (sells) x_i $(-x_i)$ shares of the company stock at a price p. One period later, at $t = 2$, he closes his position. The price at $t = 2$ will be equal to the liquidation value of the firm $p_2 = v - I$. An informed trader maximizes the expected CARA utility of the return on trading:

$$U_i(x_i) = -\exp[-rx_i((v - I) - p)] \tag{7}$$

where r is the coefficient of constant absolute risk aversion.

Informed traders have rational expectations, i.e. they use *all* information available to them. This means that they condition their trading not only on their private signal s_i but also on the publicly observable price p, which includes other traders' information. An informed trader's strategy therefore maps his private information s_i into a demand function $x_i(s_i, .)$.

In addition to informed traders, there are noise traders who trade the company stock for exogenous reasons. Their demand u is assumed to be random according to $u \sim N(0, \sigma_u^2)$ and independent of all other random variables in the model. The idea is that there are factors other than information that cause the price to vary, and that are imperfectly observed. Examples are stochastic life cycle motives for trade, margin calls or requirements for investors to hold certain assets in fixed proportions.

The stock price is determined by a competitive risk neutral market making sector. It observes the aggregate limit order book, i.e. the joint demand caused by information and non-information trading, $L(.) = \int_0^1 x_i(s_i, .)di + u$ and sets the price efficiently:

$$p = E[v - I|L(.)] \tag{8}$$

It is well known that a market model with CARA utility and normal random variables admits a linear equilibrium (which we will compute later in section 4.2). The aggregate order book L will therefore be a linear function of the price so that the price setting in (8) is equivalent to (semi-strong) efficient pricing, $p = E[p_2|p] = E[v - I|p]$.

3 Optimal incentive contracts

As usual, the analysis proceeds by backward induction. First, we need to solve for the stock price. Then we analyze the manager's effort choice and finally we characterize the incentive contract designed by the insider owners.

Before we start the analysis, we carry out a normalization that makes trading independent of incentive contracting. We can therefore analyze trading and incentive contracting separately which greatly simplifies the model. Also, we will initially conjecture a stock price instead of solving the market game explicitly. That way, it is more transparent how an optimal incentive contract combines two different but related performance measures to induce managerial effort. It also emphasizes the value added of recognizing the endogenous nature of the stock price. Without a model of trading, the role of the stock price as an performance measure remains opaque and the role of liquidity cannot be addressed.

3.1 A useful normalization

An informed trader's demand x_i depends on the terms of the incentive contract (a_0, a_p, a_v). To see this, we state the familiar condition that maximizes an informed traders expected CARA utility in (7) given that his wealth is normally distributed:

$$x_i(s_i, p) = \frac{E[(v - a_0 - a_p p) - p|s_i, p]}{rVar[(v - a_0 - a_p p) - p|s_i, p]}$$

It will simplify the analysis considerably if we make a trader's demand independent of the compensation contract so that we can analyze trading and incentive contracting separately. In order to achieve this, we define $\hat{p}$ as the price net of contracting:

$$\hat{p} = a_0 + (1 + a_p)p \tag{9}$$

An informed trader's demand can then be rewritten in terms of $\hat{p}$

$$\begin{aligned} x_i(\hat{p}) &= \frac{E[v - a_0 - a_p(\frac{\hat{p}-a_0}{1+a_p}) - (\frac{\hat{p}-a_0}{1+a_p})|s_i, \hat{p}]}{rVar[v - a_0 - a_p(\frac{\hat{p}-a_0}{1+a_p}) - (\frac{\hat{p}-a_0}{1+a_p})|s_i, \hat{p}]} \\ &= \frac{E[v|s_i, \hat{p}] - \hat{p}}{rVar[v|s_i, \hat{p}]} \end{aligned} \tag{10}$$

Note that we can change the conditioning in the expectation and the variance from p to $\hat{p}$ since they are informationally equivalent (and we have a linear framework). To construct $\hat{p}$ from p one only needs public information.

Managerial income using the normalized price is given by:

$$\begin{aligned} I &= a_0 + a_p\left(\frac{\hat{p} - a_0}{1 + a_p}\right) + a_v\left(v - a_0 - a_p\left(\frac{\hat{p} - a_0}{1 + a_p}\right)\right) \\ &= \frac{1 - a_v}{1 + a_p} a_0 + \frac{1 - a_v}{1 + a_p} a_p \hat{p} + a_v v \\ &= \hat{a}_0 + \hat{a}_{\hat{p}} \hat{p} + \hat{a}_v v \end{aligned}$$

The normalized contract $(\hat{a}_0, \hat{a}_{\hat{p}}, \hat{a}_v)$ is related to the original contract (a_0, a_p, a_v) according to:

$$\hat{a}_0 = \tfrac{1-a_v}{1+a_p} a_0; \quad \hat{a}_{\hat{p}} = \tfrac{1-a_v}{1+a_p} a_p; \quad \hat{a}_v = a_v \tag{11}$$

The incentive contract is again linear, but now in the gross liquidation value v and the normalized price $\hat{p}$. Furthermore, we can rewrite efficient pricing $p = E[v - I|p]$ using the normalized price as:

$$\hat{p} = E[v|\hat{p}]$$

which illustrates the consistency of the normalization.

3.2 A conjectured stock price

The stock price in our model is the outcome of trading in a large competitive market. The price therefore reflects an aggregate $e + \theta$ of the dispersed heterogenous information. The price will however also reflect non-information based trading u. Since our CARA-normal framework admits a linear equilibrium, we conjecture the price to be given by a linear function of information and non-information shocks:

$$\hat{p} = \alpha_0 + \alpha_I(e + \theta) + \alpha_{NI} u \tag{12}$$

Of course, the coefficients α_0, α_I and α_{NI} are endogenous and interdependent. For example, efficient pricing means that in equilibrium the expected price equals the expected value of the firm, $E[\hat{p}] = E[E[v|\hat{p}]] = E[v] = e^*$. This requires that $e^* = \alpha_0 + \alpha_I e^*$. Another issue is that α_I could (and in fact will) depend on α_{NI}.

Until we solve the market game explicitly in section 4.2, we use the conjectured price in (12).

3.3 Optimal minimum variance contracts

Given the incentive contract $\hat{a}_0, \hat{a}_{\hat{p}}, \hat{a}_v$, the manager chooses the unobservable effort e in order to maximize his expected utility of income minus the cost of effort. Since the manager has CARA utility and his income minus the cost of effort is normally distributed, the problem in equation (4) is equivalent to:

$$e = \arg\max_{e'} E[I] - \frac{r_m}{2} Var[I] - c(e') \tag{13}$$

The manager's expected income depends on his expectation of the price at $t = 1$ and on firm value at $t = 2$:

$$E[I] = \hat{a}_0 + \hat{a}_{\hat{p}} E[\hat{p}] + \hat{a}_v E[v]$$

Since the manager knows his own effort choice, he expects the firm value to be $E[v] = e$. His expectation of the price $E[\hat{p}]$ is more subtle since the market price will reflect the informed traders' inference process about the value of the firm v given their signals s_i and the information in the equilibrium price p. The price function that we conjectured in the previous section (equation 12) implies that the manager expects the price to be $E[\hat{p}] = \alpha_0 + \alpha_I e$. Since we will solve for the endogenous coefficients $(\alpha_0, \alpha_I, \alpha_{NI})$ later, we postpone the detailed discussion of how the manager's effort impacts on the share price and instead keep the general notation $E[\hat{p}]$ for the moment.

The variance of income

$$Var[I] = \hat{a}_{\hat{p}}^2 Var[\hat{p}] + \hat{a}_v^2 Var[v] + 2\hat{a}_{\hat{p}}\hat{a}_v Cov[\hat{p}, v]$$

is independent of effort. The manager can neither influence the risk of his company nor the volatility of the company's share price.

The first-order condition for (13) characterizing optimal managerial effort is then:

$$c_e = \hat{a}_{\hat{p}} \frac{\partial E[\hat{p}]}{\partial e} + \hat{a}_v \tag{14}$$

The condition shows that any appropriate linear combination of market based compensation $\hat{a}_{\hat{p}}$ and non-market based compensation $\hat{a}_v$ induces the same effort level. Inside owners' risk-neutrality however means that the cheapest way to induce such an effort level is to minimize the income risk borne by the risk-averse manager. An optimal contract must therefore solve:

$$\min_{a_p, a_v} Var[I]$$

subject to effort being optimal for manager, i.e. subject to (14).[8]

The optimal combination of market based compensation $\hat{a}_{\hat{p}}$ and non-market based compensation $\hat{a}_v$ that minimizes the manager's income risk and induces optimal managerial effort satisfies:

$$\hat{a}_v[Var[v]\frac{\partial E[\hat{p}]}{\partial e} - Cov[\hat{p}, v]] = \hat{a}_{\hat{p}}[Var[\hat{p}] - Cov[\hat{p}, v]\frac{\partial E[\hat{p}]}{\partial e}] \tag{15}$$

The condition admits already two intuitive conclusions. First, holding the responsiveness of price to effort, $\frac{\partial E[\hat{p}]}{\partial e}$, and the covariance between the price signal and the value signal, $Cov[\hat{p}, v]$, constant, the incentive contract places more weight on a performance measure if the measure is more precise, i.e. if its variance decreases. Second, holding the variances and the covariance constant, the incentive contract places more weight on the stock price if it is more responsive to effort.

3.4 Sensitivity-to-noise ratios

Condition (15) can be rewritten as

$$\frac{\hat{a}_{\hat{p}}}{\hat{a}_v} = \frac{\frac{\partial E[\hat{p}]}{\partial e} - \frac{Cov[\hat{p},v]}{Var[v]}}{Var[\hat{p}]} \frac{Var[v]}{1 - \frac{Cov[\hat{p},v]}{Var[\hat{p}]}\frac{\partial E[\hat{p}]}{\partial e}} \tag{16}$$

which is the "(adjusted) sensitivity-to-noise" ratio of Banker and Datar (1989) that is popular in the accounting literature on management compensation (see Lambert (2001) for a survey). It states that the relative weight on a performance measure depends on the ratio of its (adjusted) sensitivity to its precision. The precision of a signal is simply its variance.

The sensitivity of a performance measure is a more involved notion. It describes how responsive the measure is to changes in effort. The sensitivity is adjusted *downwards* when the signals are *positively* correlated. The sensitivity of the price measure (the numerator of the first fraction) is lower if the price co-varies more positively with value. The adjustment term for the price sensitivity, $\frac{Cov[\hat{p},v]}{Var[v]}$, is the coefficient of a regression of price on value. The intuition is that if value and price move together a lot, i.e. the regression coefficient is high, then there is "less room" for changes in effort to show up in the price.[9],[10]

An appealing feature of expression (16) is that the ratio of the weights given to performance measures is independent of the characteristics of the manager, e.g. his risk aversion or his private cost of effort. The reason is that these factors affect all variable incentive compensation in the same proportion. Due to the non-observability of managerial preferences for risk and effort, this property is especially valuable when designing empirical tests.

4 Stock price, market based compensation and liquidity

4.1 What information is in the price?

The price given in (12) affects the relative weight of market to non-market based compensation in (16) through three channels: the noise $Var[\hat{p}]$, the covariance $Cov[\hat{p}, v]$ and the sensitivity $\frac{\partial E[\hat{p}]}{\partial e}$.

The noisier the stock price is, the less compensation will be based on the price. Also, a higher variance of the price reduces the coefficient of a regression of value on price. This further decreases the weight on the price relative to value. The more responsive the price is to changes in effort, the more weight is given to it since this increases the sensitivity of the price measure and reduces the sensitivity of the value measure. Finally, a higher covariance has an ambiguous effect. It increases the downward adjustment of both, the price and the value measure.

An extra layer of complications is added by the fact that the price depends on both information and non-information shocks. If the price is more sensitive to either shock, then this increases its noise. But the crucial aspect for our result is the following: the price responsiveness to effort and its covariance with value depend *only on the information shock.* The more the information shock is impounded in the price, the higher will be the responsiveness of price to effort.

Substituting the price function (12) into the sensitivity to noise ratio (16) yields:

$$\frac{\hat{a}_{\hat{p}}}{\hat{a}_v} = \left(\frac{\alpha_I}{\alpha_{NI}^2}\right)\frac{\sigma_\eta^2}{\sigma_u^2} \tag{17}$$

The relative weight given to market and non-market performance measures is equal to the ratio of their idiosyncratic noises $\frac{\sigma_\eta^2}{\sigma_u^2}$ weighted by a market factor $\frac{\alpha_I}{\alpha_{NI}^2}$ The market factor depends on both information and non-information trading.

It is clear at this point that we cannot push the analysis further unless one recognizes the *endogenous* nature of the share price. Only if we bring in a model of trading and price formation, will we be able to interpret the ratio in (17) and to discuss the role of liquidity.

4.2 Bringing in the market model

This section solves the market model of trading and price formation. It is a version of Vives (1995) large market rational expectations model with a competitive market making sector. The aim is to obtain a characterization of the coefficients α_0, α_I and α_{NI} that determine the price in (12) and the relative weight of market to non-market based compensation in (16).

Recall that the price is set efficiently by a risk-neutral competitive market making sector upon seeing the aggregate limit order book $\hat{p} = E[v|L(.)]$. The aggregate limit order book is the sum of the aggregate informed demand $\int_0^1 x_i(s_i, .)di$ and noise traders' demand u. An individual informed trader's demand x_i was described in (10). Since we consider only linear, symmetric price functions, and since the conditional expectation of a normal variable is linear in the signals realization (and the conditional variance is not a random variable), we can write an informed trader's demand as:

$$x_i(s_i, .) = \beta s_i + f(.) \tag{18}$$

where β is the trading intensity of an informed trader on his private information and $f(.)$ is a linear function of the price. Note that β and $f(.)$ are common to all informed traders.

The aggregate limit order book can then be expressed as:

$$
\begin{aligned}
L(.) = \int_0^1 x_i(s_i, .)di + u &= \beta(e+\theta) + u + f(.) \\
&= z + f(.)
\end{aligned}
$$

where $z = \beta(e+\theta)+u$ is the part of the aggregate limit order book that is informative about the value of the firm v. This means that the price setting condition $\hat{p} = E[v|L(.)]$ can be written as $\hat{p} = E[v|z]$. The following proposition calculates this straight-forward conditional expectation:

Proposition 1 *The market price net of incentive contracting $\hat{p} = E[v|z]$ is:*

$$\hat{p} = (1-\lambda\beta)e^* + \lambda\beta(e+\theta) + \lambda u \tag{19}$$

where e^* is the hypothesized equilibrium effort, e is the actual effort and

$$\lambda = \frac{\beta\sigma_\theta^2}{\beta^2\sigma_\theta^2 + \sigma_u^2}$$

Proof: In the appendix. ■

In a rational expectations equilibrium, the actual effort e that determines firm value and the informed traders' imperfect information about value, and the hypothesized equilibrium effort e^* that determines traders' prior expectation of firm value, must coincide. The price is then $\hat{p} = e^* + \lambda\beta\theta + \lambda u = e^* + \lambda z$.[11]

The price is affected by two random shocks. One shock, u, is due to random non-information trading. The resilience of the price to the order flow, $1/\lambda$, describes the "depth" of the market as in Kyle (1985): a deep market reacts little to changes in the order flow z. The Kyle's lambda is an intuitive and widely used measure of liquidity.

As in Kyle, the measure of market liquidity is proportional to a ratio of the amount of noise trading to the amount of private information informed traders are expected to have. The market becomes more liquid if there is more non-information based trading (larger σ_u^2) and/or if the common shock to firm value and informed traders' signals is smaller (lower σ_θ^2). The trading model therefore builds on Bagehot (1971) classic intuition that a market will be more liquid if trading less plagued by adverse selection.

The other shock, $e+\theta$, is due to the private information of the informed traders. We see that the price aggregates the dispersed, heterogenous and private information of traders, i.e. demand is independent of a traders error ξ_i in measuring or anticipating future firm value. Informed demand is part of the order flow and its impact on the price depends on the depth/liquidity of the market, $1/\lambda$. The coefficient β measures an informed trader's trading intensity, i.e. it is a measure of how strongly his private information s_i affects his demand x_i.

A complete characterization of the market model requires solving for a trader's trading intensity β. The solution is obtained by computing the conditional expectation and variance in (10) using the price function of proposition 1 and comparing the expression with (18) in order to identify β. The result is presented in the next proposition:

Proposition 2 *A trader's trading intensity β is given by the solution to the following cubic equation:*

$$\frac{\sigma_\eta^2}{\sigma_u^2}\beta^3 + [1+\sigma_\eta^2(\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_\varepsilon^2})]\beta - \frac{1}{r\sigma_\varepsilon^2} = 0$$

Proof: In the appendix. ■

Unfortunately, a closed from solution for β depends of the roots of the cubic equation. Using the implicit function theorem however allows us to analyze the comparative statics of β with respect to the parameters of the model.

Corollary 1 *The trading intensity of an agent β increases if there is more noise trading (higher σ_u^2), if the trader is less risk averse (lower r), if the common shock to firm value and informed traders' signals is larger (higher σ_θ^2) and if the measurement error is smaller (higher $Var[\xi_i] = \sigma_\eta^2 + \sigma_\varepsilon^2$).*

Proof: In the appendix. ■

4.3 The role of liquidity

We are now ready to obtain the first main result of the paper by substituting $\alpha_I = \lambda\beta$ and $\alpha_{NI} = \lambda$ into (17):

Proposition 3 *The ratio of compensation based on the market price of the company stock at=1, $\hat{p}$, and compensation based on the final value of the firm at t=2, v, is:*

$$\frac{\hat{a}_{\hat{p}}}{\hat{a}_v} = \left(\frac{\beta}{\lambda}\right)\frac{\sigma_\eta^2}{\sigma_u^2} \tag{20}$$

The market factor that multiplies the ratio of the idiosyncratic risks is the given by the depth of the market λ^{-1} times the trading intensity of informed traders β. A large market factor means that the incentive contract should contain more market vs. non-market compensation. A liquid market directly favors market based compensation. The same holds for a high trading intensity directly and also indirectly via liquidity.

The intuition is that the stock price aggregates dispersed and heterogenous information about firm value and hence managerial effort. Since the price is the outcome of self-interested speculative trading, it overcomes the problem of communicating the information to the principal for incentive contracting. Although aggregated market information is valuable to the owners of the firm it is not possible for them to access it. Either because they would have to incur the cost of contracting with a large number of traders or because no individual trader has an incentive to reveal truthfully his information without receiving an adequate profit or because their information is "soft" in the sense that it cannot be credibly communicated. In other words, the inside owners of the firm free-ride on self-interested speculation. The information of the market however is not entirely costless. We saw that there is no information trading unless there is also some non-information trading. But non-information trading is random and independent of managerial effort. By contracting on the share price, the inside owners expose the manager to unnecessary fluctuations.

At this point a comparison with the result in Holmstrom and Tirole (their equation (24)) is instructive. Recall that HT use the variance of the uninformed trade σ_u^2 as a measure of liquidity. Quite surprisingly, their equation (24) shows that the ratio corresponding to our $\frac{\hat{a}_{\hat{p}}}{\hat{a}_v}$ is independent of σ_u^2: the uninformed trade affects the optimal weight of market-based compensation only indirectly through its effect on the endogenous collection of more precise information (a lower σ_ε^2 using our notation) by a single speculator. This effect is peculiar of a model with a monopolist of information, as in Kyle (1985), because the trading aggressiveness of the informed monopolist strategically depends on σ_u^2. For a constant σ_ε^2, higher σ_u^2 increases β (and reduces λ) but the net effect on $\frac{\hat{a}_{\hat{p}}}{\hat{a}_v}$ is nil.

If we rewrite our (20) as in HT we obtain:

$$\frac{\hat{a}_{\hat{p}}}{\hat{a}_v} = \left(\frac{\beta^2 \sigma_\theta^2 + \sigma_u^2}{\sigma_\theta^2} \right) \frac{\sigma_\eta^2}{\sigma_u^2} = \left(\beta^2 + \frac{\sigma_u^2}{\sigma_\theta^2} \right) \frac{\sigma_\eta^2}{\sigma_u^2}$$

which shows that the market-based compensation is higher when the moral hazard is higher (higher σ_η^2 and lower σ_θ^2), and when the private signals are more precise (by Corollary 1, β is increasing with the precision of the private signal). As in HT, σ_u^2 also increases the (ex-ante) variance of the price (hence reducing $\frac{\hat{a}_{\hat{p}}}{\hat{a}_v}$) but at the same time it increases β (Corollary 1). However, contrarily to HT, it affects also directly the ratio $\frac{\hat{a}_{\hat{p}}}{\hat{a}_v}$. What matters for the effect on $\frac{\hat{a}_{\hat{p}}}{\hat{a}_v}$ is the impact of σ_u^2 on the liquidity measure λ : in our model with many informed speculators, this is not univocal, hence we cannot a priori conclude that higher uninformed trade volatility leads to more market based compensation.

It is difficult to obtain explicit comparative statics on the relative weight of market based compensation with respect to the parameters of the model due to the problem of finding a closed form solution for the trading intensity β. Numerical simulations [12] however show that we can expect more market based compensation if there is *less* non-information (noise) trading (lower σ_u^2), if the common shock to firm value and informed traders' signals is smaller (lower σ_θ^2), if firm's idiosyncratic risk is larger (higher σ_η^2), if traders have better information (lower σ_ε^2) and if they are less risk averse (lower r).

5 The dynamic model: Market-based compensation and investors horizon

Proposition 3 states that one of the main determinants for the optimal weight of market-based compensation is the liquidity of the stock, measured by the Kyle's lambda λ. It seems natural then to study how the investors' behavior affect the optimal effort decision by the manager through its influence on λ. Up to now we have assumed that the true liquidation value of the firm v is known at a final stage, when the manager is due his compensation. This assumption is certainly too strong in reality: the value v, that is the value created directly by the management's actions (plus shocks outside his control), produces sometimes well after the end of the managerial contract. Or, alternatively, it is unrealistic to assume that it becomes publicly observable and verifiable before the managers need to be rewarded[13]. These observations in practice preclude the possibility that the principal can contract directly on v. Since in the present paper we focus on market-based contracts[14], when v is not contractible, it is natural to assume that the principal is obliged to compensate the manager according to the realization of the stock prices observed on the market: she can do this rewarding the manager with shares of the firm that he can vest at pre-specified periods during his contract.

To study how different market conditions affect the optimal contracting and the effort choices of the agent, we allow for multiple trading dates: the firm shares are exchanged during two trading rounds $t = 1, 2$. The true value of the firm v realizes at date $t = 3$. In order to simplify the equilibrium characterization, we assume that $v = e + \theta$.

As a short-cut that motivates why the liquidation value v is not contractible, we assume that the manager *does not stay* with the firm until $t = 3$. On the contrary, he must receive his payment at latest at $t = 2$[15].

The contract between the inside owners and the manager is signed before trading takes place, i.e. at $t = 0$. The contract specifies two payments (a_0, a_2) such that the manager's income is $I = a_0 + a_2 p_2$ where p_2 is the (end of) second-period equilibrium price[16]; denote with π the net liquidation value of the firm: $\pi = v - a_0 - a_2 p_2$, and a_2 represents the stock-appreciation rights.

Since the effort is assumed to be unobservable, the moral-hazard problem described before still holds, and this induces the principal to maximize her objective under the constraint of the manager acting in his own interest (IC) and accepting the contract (IR).

The manager has a negative exponential utility function with risk-aversion coefficient r_m, and the principal is risk-neutral and maximizes the ex-ante (i.e. $t = 0$) residual value per-share π in choosing the optimal contract.

As before informed speculators i and noise traders operate on the market.

We will consider two alternative cases according to the type of informed speculators:

(i) **Long termists**: they maximize their final wealth $W_{i3} = \sum_{t=1}^{3} \pi_{it}$:

$$W_{i3} = x_{i1}(p_2 - p_1) + x_{i2}(\pi - p_2)$$

where $x_{it} > 0$ denotes a long position of trader i at time t. Each agent i maximizes a CARA utility function $U^i(W_{i3}) = -\exp(-rW_{i3})$.

(ii) **Short termists**: they solve the following problems:

1) at date $t = 1$:

$$\underset{x_{i1}}{Max} EU^i(W_{i2}) = E\left[-\exp(-r(x_{i1}(p_2 - p_1)))\,|H_1\right]$$

- 2) at date $t = 2$:

$$\underset{x_{i2}}{Max} EU^i(W_{i3}) = E\left[-\exp(-r(x_{i2}(\pi - p_2)))\,|H_1, H_2\right]$$

where H_1 and H_2 are respectively their information sets at $t = 1$ and $t = 2$. For the interpretation of short-termists we refer to Vives (1995).

The noise traders demand at each trading date t the random quantity $u_t \sim N(0, \sigma_u^2)$, $t = 1, 2$. The two noise trading demands u_1 and u_2 are uncorrelated.

Before the first round of trade all speculators i receive an informative signal $s_i = e + \theta + \varepsilon_i$. Note that since $v = e + \theta$, under this information structure the market as a whole knows exactly the realization of v in the sense that $\int_i s_i di = v$. The shock θ is uncorrelated with ε_i and errors are also uncorrelated across agents. The precision of the signals is the same across agents (symmetry):

$$\varepsilon_i \; N(0, \sigma_\varepsilon^2)$$

If the investors have a short horizon, either we can assume that they are long-lived and myopic, or that a new generation is born at date 2 and inherits the private signal received by the previous generation.

Market equilibrium

The competitive market-making sector sets the equilibrium prices p_1 and p_2 as equal to the expected value of v conditional on the public information, represented here by the order flows.

We denote with

$$L_1 = \int_0^1 x_{i1} di + u_1$$

the order flow the market makers observe at date 1, where x_{i1} is the position taken by informed trader i at the same date.

Analogously,

$$L_2 = \int_0^1 x_{i2} di - \int_0^1 x_{i1} di + u_2$$

the order flow the market makers observe at date 2, where $\int_0^1 x_{i2} di - \int_0^1 x_{i1} di$ represents the net aggregate demand in period 2 by informed speculators.

As in Vives (1995), we will concentrate on linear symmetric equilibria, in which the position x_{it} is linear in the prices p_t and in the signal s_i: $x_{it} = \beta_t (s_i - p_t)$. Define then $z_1 = \beta_1 v + u_1$ and $z_2 = (\beta_2 - \beta_1) v + u_2$, it is easy to verify that z_t is informational equivalent to L_t.

In order to take into account for the dilution effect which affects the net position x_{i1}, the market makers compute the "adjusted" order flows $\widehat{L}_1 = \int_0^1 \widehat{x}_{i1} di + u_1$ where $\widehat{x}_{i1} = \widehat{\beta}_1 (s_i - p_1) = \frac{1}{1+a_2} \beta_1 (s_i - p_1)$, and $\widehat{L}_2 = \int_0^1 x_{i2} di - \int_0^1 \widehat{x}_{i1} di + u_2$. The order flows $\widehat{L}_1$, $\widehat{L}_2$ contain the informational trades with an aggressiveness net of the dilution effect due to the managerial compensation: since the contract a_2 is public information and at equilibrium market makers know β_1, the order flow $\widehat{L}_1$ does not require more information than L_1. Note that $\widehat{\beta}_1$ measures the information-related aggressiveness corrected for the dilution effect $1 + a_2$.

From the presence of competitive, risk-neutral market makers we have:

$$p_1 = E[\pi \,|\widehat{z}_1] = E[v \,|\widehat{z}_1] - a_0 - a_2 E[p_2 \,|\widehat{z}_1] \qquad (21)$$

$$p_2 = E[\pi \,|\widehat{z}_1, \widehat{z}_2] = E[v \,|\widehat{z}_1, \widehat{z}_2] - a_0 - a_2 p_2 \qquad (22)$$

From Vives (1995), $\widehat{z}_1 = \widehat{\beta}_1 v + u_1$ is informational equivalent to $\widehat{L}_1$ and $\widehat{z}_2 = \left(\beta_2 - \widehat{\beta}_1\right) v + u_2$ is i.o.

to $\widehat{L}_2$ Solving explicitly for p_2 :

$$p_2 = \frac{E[v\,|\widehat{z}_1, \widehat{z}_2] - a_0}{1 + a_2} \tag{23}$$

and taking the expectation $E[p_2\,|\widehat{z}_1]$ of (23) and substituting into (21) we obtain:

$$p_1 = \frac{E[v\,|\widehat{z}_1] - a_0}{1 + a_2} \tag{24}$$

Then we normalize the prices in the following way:

$$\begin{aligned} \widehat{p}_1 &= (1 + a_2)p_1 + a_0 = E[v\,|\widehat{z}_1] \\ \widehat{p}_2 &= (1 + a_2)p_2 + a_0 = E[v\,|\widehat{z}_1, \widehat{z}_2] \end{aligned}$$

We can then find the normalized equilibrium price irrespective from the investors horizon.

Proposition 4 *The (normalized) REE prices are:*

$$\widehat{p}_1 = e^*(1 - \widehat{\lambda}_1\widehat{\beta}_1) + \widehat{\lambda}_1\widehat{\beta}_1(e + \theta) + \widehat{\lambda}_1 u_1 \tag{25}$$

$$\widehat{p}_2 = e^*\left(1 - \widehat{\lambda}_1\widehat{\beta}_1\tfrac{\tau_1}{\tau_2} - \left(\beta_2 - \widehat{\beta}_1\right)\widehat{\lambda}_2\right) + \left(\widehat{\lambda}_1\widehat{\beta}_1\tfrac{\tau_1}{\tau_2} + \left(\beta_2 - \widehat{\beta}_1\right)\widehat{\lambda}_2\right)(e + \theta) + \widehat{\lambda}_1\tfrac{\tau_1}{\tau_2}u_1 + \widehat{\lambda}_2 u_2 \tag{26}$$

Proof: see the Appendix.

The aggressiveness of informational trade $\widehat{\beta}_t$, hence the market depth $\widehat{\lambda}_t$, depend on the horizon of the informed speculators. We now solve explicitly for the equilibria respectively in the case long-term speculators trade on the market and with short-term speculators.

Equilibrium with long-term speculators

Proposition 5 *When long-term investors operate on the market, the unique linear rational expectations equilibrium coincide with the static case:*

$$x_{i1} = \beta_{1l}(s_i - \widehat{p}_{1l}) = \widehat{\beta}_{1l}(1 + a_2)(s_i - \widehat{p}_{1l}) \tag{27}$$

$$\Rightarrow \widehat{\beta}_{1l} = \frac{\tau_\varepsilon}{r} \tag{28}$$

$$\widehat{\lambda}_{1l} = \frac{\widehat{\beta}_{1l}\tau_u}{\widehat{\beta}_{1l}^2\tau_u + \tau_\theta} \tag{29}$$

$$x_{i2} = \beta_{2l}(s_i - \widehat{p}_{2l}) \tag{30}$$

$$\Rightarrow \beta_{2l} = \widehat{\beta}_{2l} = \frac{\tau_\varepsilon}{r} \tag{31}$$

$$\widehat{\lambda}_{2l} = (\widehat{\beta}_{2l} - \widehat{\beta}_{1l})\frac{\tau_u}{\tau_{2l}} = 0 \tag{32}$$

and

$$\tau_{2l} = \widehat{\beta}_{1l}^2\tau_u + \tau_\theta + (\Delta\beta)^2\tau_u$$

Proof: see the Appendix.

Note that, for $a_2 = 0$, for the long-term investors it would be optimal to trade only once on their signal, at the first trading round, holding then their optimal position until the end (buy and hold strategy). For this reason the normalized order flow in the second period does not contain additional information (the net aggressiveness $\widehat{\beta}_{1l} = \beta_{2l}$), and hence, $\widehat{p}_{2l} = \widehat{p}_{1l}$. For $a_2 > 0$ the dilution term increases the total aggressiveness without anyway changing the depth $\widehat{\lambda}_1$ since the market makers correct the order flow for this effect which is not related to private information. As in (35), long-term competitive informed traders trade with the maximum aggressiveness on their private signal in the first trading period: waiting to reveal their signals in the second period is sub-optimal since it gives the incentive to the other informed speculators to trade according to their signal immediately.

Equilibrium with short-term speculators

Proposition 6 *When short-term investors operate on the market, the unique linear rational expectations equilibrium is*

$$x_{i1} = \frac{1}{r}(1+a_2)\frac{\tau_2\tau_\varepsilon}{\tau_2+\tau_\varepsilon}(s_i - \widehat{p}_{1s}) = \widehat{\beta}_{1s}(1+a_2)(s_i - \widehat{p}_{1s}) \quad (33)$$

$$\Rightarrow \widehat{\beta}_{1s} = \frac{1}{r}\frac{\tau_2\tau_\varepsilon}{\tau_2+\tau_\varepsilon} \quad (34)$$

$$\widehat{\lambda}_{1l} = \frac{\widehat{\beta}_{1s}\tau_u}{\widehat{\beta}_{1s}^2\tau_u + \tau_\theta} = \frac{\widehat{\beta}_{1s}\tau_u}{\tau_{1s}} \quad (35)$$

$$x_{i2}^s = \frac{\tau_\varepsilon}{r}(s_i - \widehat{p}_2^s) \quad (36)$$

$$\Rightarrow \beta_{2s} = \widehat{\beta}_{2s} = \frac{\tau_\varepsilon}{r} \quad (37)$$

$$\widehat{\lambda}_{2s} = \frac{(\beta_{2s} - \widehat{\beta}_{1s})\tau_u}{(\beta_{2s} - \widehat{\beta}_{1s})^2\tau_u + \widehat{\beta}_{1s}^2\tau_u + \tau_\theta} = (\beta_{2s} - \widehat{\beta}_{1s})\frac{\tau_u}{\tau_{2s}} > 0 \quad (38)$$

Proof: see the Appendix.

The trading intensity $\widehat{\beta}_t$ (net of dilution effect) increases across time, and the reason for that is that short-term traders in period 1 have to forecast p_2 in assessing their optimal trading strategy: p_2 depends not only on the fundamental value v (on which they receive an information), but also on the noise trade (that they cannot forecast). Their risk-aversion makes them less eager to play aggressively on their private signal, because of the additional uncertainty on p_2.

5.1 The effect of investors horizon on the optimal contract

In the dynamic setup the manager's expected income depends exclusively on the price p_2, which incorporates the market's expectations at $t = 2$ of the future realization of firm value:

$$\begin{aligned} I &= a_0 + a_2 p_2 \\ &= a_0 + a_2\left(\frac{\widehat{p}_2 - a_0}{1+a_2}\right) = \frac{a_0}{1+a_2} + \frac{a_2}{1+a_2}\widehat{p}_2 \\ &= \widehat{a}_0 + \widehat{a}_2\widehat{p}_2 \end{aligned}$$

and the choice of effort by the manager satisfies

$$\max_e \; E(I) - \frac{r_m}{2}Var(I) - \frac{k}{2}e^2$$

with $Var(I) = \widehat{a}_2^2 Var(\widehat{p}_2)$, independent of effort, and $\widehat{p}_2$ is given by (26).

The f.o.c. for the effort choice reads:

$$e = \frac{1}{k}\widehat{a}_2 \frac{\partial E(\widehat{p}_2)}{\partial e} = \frac{1}{k}\widehat{a}_2 \left(\widehat{\lambda}_1 \widehat{\beta}_1 \tfrac{\tau_1}{\tau_2} + \left(\beta_2 - \widehat{\beta}_1\right)\widehat{\lambda}_2\right) \tag{39}$$

This condition is different from (14) in two respects: first, only the market-based power-incentive $\widehat{a}_2$ matters for the manager's choice; second, the sensitivity of p_2 with respect to the effort depends on the *dynamic* trading strategy of the informed speculators. The more information is conveyed in the first market price, the higher this sensitivity (see Corollary 2 for a proof of this statement).

Since the principal is risk-neutral, at $t = 0$ she chooses the contract solving:

$$\begin{aligned} &\max_{e;\widehat{a}_2} \; E(v - I) \\ s.t. \; e &= \frac{1}{k}\widehat{a}_2 \left(\widehat{\lambda}_1 \widehat{\beta}_1 \tfrac{\tau_1}{\tau_2} + \left(\beta_2 - \widehat{\beta}_1\right)\widehat{\lambda}_2\right) \quad \text{(IC)} \\ 0 &= E(I) - \frac{r_m}{2} Var(I) - \frac{k}{2}e^2 \quad \text{(IR)} \end{aligned}$$

Obtaining $\widehat{a}_2$ necessary to implement any effort e from the incentive compatibility constraint, and substituting into the objective of the principal the expression for the expected income from the participation constraint, we get:

$$\begin{aligned} &\max_{e} \; E(v) - \frac{r_m}{2} Var(I) - \frac{k}{2}e^2 = e - \frac{r_m}{2}\widehat{a}_2^2 Var(\widehat{p}_2) - \frac{k}{2}e^2 \\ \Rightarrow \; &\max_{e} \; e - \frac{r_m}{2} \frac{k^2 e^2 Var(\widehat{p}_2)}{\left(\widehat{\lambda}_1 \widehat{\beta}_1 \tfrac{\tau_1}{\tau_2} + \left(\beta_2 - \widehat{\beta}_1\right)\widehat{\lambda}_2\right)^2} - \frac{k}{2}e^2 \end{aligned}$$

and the solution is characterized by the first-order condition:

$$e^* = \frac{1}{k} \frac{1}{1 + k r_m \frac{Var(\widehat{p}_2)}{\left(\widehat{\lambda}_1 \widehat{\beta}_1 \frac{\tau_1}{\tau_2} + \left(\beta_2 - \widehat{\beta}_1\right)\widehat{\lambda}_2\right)^2}} \tag{40}$$

The optimal effort, and then the optimal contract, depends on the investors' horizon, since this affects the expected volatility of the second period (normalized) price $\widehat{p}_2$, the aggressiveness of informed trade

$\widehat{\beta}_t$ and the depth of the market $\widehat{\lambda}_t$. The investors' preferences then produce real effects on the effort choices of the manager through the design of the optimal market-based compensation.

Corollary 2 *(i)* $Var(\widehat{p}_{2s}) < Var(\widehat{p}_{2l})$. *(ii)* $\widehat{\lambda}_{1s}\widehat{\beta}_{1s}\frac{\tau_{1s}}{\tau_{2s}} + \left(\beta_{2s} - \widehat{\beta}_{1s}\right)\widehat{\lambda}_{2s} < \widehat{\lambda}_{1l}\widehat{\beta}_{1l}$.

This proves that the $t = 2$ price in the case short-term investors operate on the market is ex-ante less volatile than with long-term traders, since the short-term investors are less willing to trade on their private signal. On the other hand, point (ii) shows that $E\left(\widehat{p}_{2s}\right)$ is less sensitive to effort than $E(\widehat{p}_{2l})$. Hence, from the contracting point of view, $\widehat{p}_{2s}$ is a less noisy (i.e. better), but less sensitive (i.e. worse) measure for the effort than $\widehat{p}_{2l}$. We can show that the second effect always prevails on the first, so that the moral hazard problem aggravates in the presence of short-term investors.

Proposition 7 *The presence of short-term informed traders on the market reduces the optimal managerial effort.*

We can explain this result as follows. Short-termism, even in an informationally efficient market, reduces the market depth λ_2^{-1} since it reduces the willingness by risk-averse informed traders to provide liquidity to the market. Indeed, with respect to the case of long-term traders, they face an additional uncertainty due to the realization of price $\widehat{p}_{2s}$, at which they close their position. This effect reduces the aggressiveness β_1 of their demands in the first period, but it increases β_2. In other terms, a less liquid market accommodates less informational trade at equilibrium; hence the price $\widehat{p}_{2s}$ conveys less information about e than $\widehat{p}_{2l}$, in the sense of being less sensitive to the effort. This aggravates the moral-hazard problem, so that it is more expensive for the principal to induce a high effort.

Optimal contract

Proposition 7 shows that in presence of short-termists it is more costly for the principal to give incentives to the manager, when he writes contracts on the second period price p_2. This raises the question whether it would be optimal to reward the manager with stock-appreciation rights based on the realization of the first price p_1. The following argument shows that this is not the case.

Proposition 8 *A contract with stock-appreciation rights based on price* p_2 *is preferred to a contract based on* p_1.

As explained above, the price p_{2s} contains more information (when short-termists operate) than

p_{1s}. It represents then a better statistics to infer the effort e. But on the other hand, it is ex-ante less volatile (Vives (1995), Proposition 5.1). Exactly with the same logic behind the proof of Proposition 7, the first effect prevails, because the market at $t = 2$ is more liquid than at $t = 1$ with short-term investors.

6 Empirical evidence

To the best of our knowledge, the empirical literature on executive compensation has not yet considered the role of liquidity for market based executive compensation (for recent surveys see Murphy (1999) and Core, Guay and Larcker (2003)).

The only exception is the recent work by Garvey and Swan (2002). Using stock turnover and the bid-ask spread as two major measures of liquidity, they find a strong positive effect of liquidity on the use of stock based pay. Their effect of liquidity on stock based pay is at least as great as that of size, risk, industry and other control variables. In order to justify their hypothesis about the effect of liquidity, they refer to the analysis of HT although they acknowledge that the Kyle 'lambda' has an ambiguous a priory relationship to the use of stock-based incentives. Moreover, they also recognize the importance of ownership concentration for their argument. The issue is that ownership can have its own independent effect on executive compensation via corporate governance channels (see Hartzell and Starks (2003)). Another problem of using the Holmstrom and Tirole argument for their analysis is that Garvey and Swan find that larger firms have a lower share price volatility and that share price volatility is strongly positively correlated with turnover. Both empirical findings are contrary to the results in Holmstrom and Tirole. None of these caveats applies to our framework.

Less direct evidence on the link between liquidity and executive compensation comes from the fact that larger firms typically have more stock based compensation (Baker and Hall (2002), Himmelberg, Hubbard and Palia (1999)). At the same time, it is well known that the stock of larger firms is more liquid (Amihud and Mendelsson (1986)). Although, it has been argued that larger firms need to pay more to attract more talented or wealthier managers, it seems that liquidity could also be a possible explanation.

There is also a debate to what extent incentives are positively or negatively related to risk. On the one hand, it is expected that managerial risk aversion causes a negative relationship (Aggarwal and Samwick (1999)). On the other hand, there are arguments that managerial effort is more important, or that it is more difficult to directly monitor the manager, in riskier environments (Prendergast (2002) and Core, Guay and Larcker (2003)). Our interpretation is that the debate has overlooked the potentially important impact of liquidity, a factor that also depends on risk, on management compensation.

There is also some evidence that following carve-outs, the management's equity compensation is based on the former subsidiaries' stock (Schipper and Smith (1986)). Our interpretation is that the carve-out increased liquidity and hence information trading. As a result, more valuable information is incorporated into stock prices.

Finally, our point of view on the recent wave of corporate scandals and lavish CEO pay in the late 1990s is that the situation represents an inefficient disequilibrium. The market collected little valuable information and a lot of non-information (noise) trading seem to have occurred, both of which *should* have lead to a decrease of market-based performance pay.

7 Discussion: the role of accounting manipulation

In this section we describe informally some results we obtained studying the case in which only market prices and reported earnings are contractible. During regular intervals, the firm is due to report her economic and financial results through accounting reports. There is clear empirical evidence that executives can "manage" earnings (to meet pre-specified targets (Degeorge et al. (1999)) or to meet analysts' forecasts (Jensen (2001), Fuller and Jensen (2002)). The reported earnings are clearly contractible, and most of the compensation packages used in practice include earning-based bonuses.

To check if the results obtained before are robust to this change of setup, we have assumed that the principal can write a contract on the market price and the reported cash flows, but not on the true ones.

The results we obtain from the analysis of this variation of the model confirm the main intuition that market-based compensation depends on market liquidity. But it also produces a new insight of the effect of liquidity on the degree of manipulation. More precisely, since the market is semi-strong efficient, we obtain the same result of the "manipulation game" obtained in Stein (1989), that is the existence of a perfectly separating equilibrium with inefficient manipulation. At equilibrium, the managers inflate earnings given that the market expects them to do so, and given this behavior the market rationally discounts the reported earnings. Observing the public report, everybody can infer the true earnings, but the managers pay a cost due to their manipulation.

If this manipulation activity does not produce real costs on the final cash-flows of the firm, and if the "manipulation game" has a unique, perfectly revealing equilibrium, the optimal compensation contract still puts higher weight on the price when the liquidity of the stock is higher. As a side effect, the lower the weight on the accounting report, the lower the degree of manipulation of earnings the manager will operate at optimum. Hence, in firms with very liquid stocks we should observe less frequently a complex "earnings management".

We leave for further research the analysis of a more detailed model of earnings manipulation in which possibly other equilibria (partially revealing) exist in the manipulation game (see Guttman et al. (2004)). We can conjecture that the different weights attributed in the optimal contract on market-based vs. accounting-based measures have interesting effects on the degree of earnings manipulation the managers choose.

8 Conclusion

This paper examined the role of trading and liquidity in a large competitive market with dispersed heterogenous information on market-based management compensation. An important feature of the model is that it recognizes the endogenous nature of a firm's stock price - it is the outcome of self-interested speculative trading motivated by imperfect information about future firm value. Using the stock price as performance measure means bench-marking the manager's performance against the market's expectation of that performance.

The intuition is that the stock price aggregates dispersed and heterogenous information about firm value and hence managerial effort. Since the price is the outcome of self-interested speculative trading, it overcomes the problem of communicating the information to the principal for incentive contracting. Although aggregated market information is valuable to the owners of the firm it is not possible for them to access it. Either because they would have to incur the cost of contracting with a large number of traders or because no individual trader has an incentive to reveal truthfully his information or because their information is "soft" in the sense that it cannot be credibly communicated. In other words, the inside owners of the firm free-ride on the result statistic of self-interested speculation, that is the market price. The information of the market however is not entirely costless. There is no information trading unless there is also some non-information trading. But non-information trading is random and independent of managerial effort. By contracting on the share price, the insider owners expose the manager to unnecessary fluctuations.

Liquidity essentially measures the ease of information based trading. Our first result is that the degree of market-based compensation is proportional to market liquidity, which is a measure of the ease of information trading. From this starting point, we can show that the horizon of the investors has real effects even in a semi-strong efficient market, since it affects the dynamic pattern of liquidity in the market.

Although the empirical compensation literature has not yet focused on the role of liquidity, our hope is that the present analysis adds to the debate on why large firms have more stock based compensation, whether risk is positively or negatively related to incentives and on how liquidity is an indirect channel through which market based compensation depends on systemic noise.

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Notes

[1] "The Kyle 'lambda' [...] in the HT model [...] has an ambiguous "*a priori*" relationship to the use of stock-based incentives" (cit. page 7, Garvey and Swan (2002)).

[2] The debate is a classic one. Already Hayek (1945) noted that:

> "the answer to our question will therefore largely turn on the relative importance of the different kinds of knowledge; those more likely to be at the disposal of particular individuals and those which we should with greater confidence expect to find in the possession of an authority made up of suitably chosen experts".

[3] The assumption is that inside owners can diversify away any firm specific risk while the manager cannot.

[4] Writing the contract on the net liquidation value $v - a_0 - a_p p$ instead of the gross liquidation value v is a modelling device that allows to abstract from dilution issues (see (20). We conform to the standard practice that the contract is linear in p and v, $I = a_0(1 - a_v) + a_p(1 - a_v)p + a_v v$, and that the manager is paid at the final period $t = 2$.

[5] (22) notes that linearity is a good first order approximation of reality. In practice, the convexity induced by option holdings appears to be negligible. (21) argues that linearity is desirable because it counters manager's incentives to game the capital budgeting process.

[6] One could view our information structure as a reduced form of the set-up in (22). Using the CAPM, he decomposes firm return into a systemic market return and an idiosyncratic firm return. Moreover, notice that this information structure is exactly as in (20), in order to facilitate the comparison between our and their results.

[7] Motivated by the law of large numbers, we will make the technical assumption that $\int_0^1 \varepsilon_i di = 0$ almost surely (see (35) for a discussion of this assumption). Note that $\xi_i \sim N(0, \sigma_\varepsilon^2 + \sigma_\eta^2)$.

[8] The second order condition is satisfied ($c_e e < 0$) and the maximum is a global one. Hence, we can substitute (14) for (13) in the principal's problem.

[9] There is another adjustment in addition to the covariance. A higher responsiveness of measure B to effort amplifies the downward correction of A's sensitivity caused by a positive covariance. To see this, consider the sensitivity of the value measure, i.e. the denominator of the second fraction in (16). Its downward adjustment due to a positive covariance with price is stronger if the *price* is sensitive to effort. Again, the intuition is that if effort affects price a lot and price and value move together a lot, then there it less possible to filter out effort from value.

[10] As an example, let us suppose that the price of the company stock is its future value plus noise, $\hat{p} = v+u = e+\theta+\eta+u$. We intuitively expect that no weight should be to the stock price. Since price is value plus noise and since the manager

is paid according to final firm value v anyway, the price is useless for figuring out whether good performance was due to effort/skill or pure luck.

And indeed, equation (16) results in:

$$\frac{\hat{a}_{\hat{p}}}{\hat{a}_v} = \frac{1 - \frac{\sigma_\theta^2 + \sigma_\theta^2}{\sigma_\theta^2 + \sigma_\theta^2}}{\sigma_\theta^2 + \sigma_\theta^2 + \sigma_u^2} \frac{\sigma_\theta^2 + \sigma_\theta^2}{1 - \frac{\sigma_\theta^2 + \sigma_\theta^2}{\sigma_\theta^2 + \sigma_\theta^2 + \sigma_u^2}} = 0$$

The responsiveness of price and value to effort are the same, i.e. one. Also, the coefficient of a regression of price on value is one. Together, this means that nothing new about effort can be learn from the price once we observe value. The example illustrates (18) informativeness principle. The price should not be include in the incentive contract since value is a sufficient statistic for the joint distribution of value and price with respect to effort.

[11] It is important to make the distinction between actual and equilibrium effort. Every actual effort by the manager creates a set of signals for the informed traders whose demand creates a signal for the market making sector. No one in the market knows the true effort. Formally speaking, the market game follows an information set that contains all possible effort levels. A set of beliefs, i.e. a probability distribution over effort, must be associated with that information set in order to compute the conditional probabilities according to Bayes' rule. The correct set of beliefs puts weight one on equilibrium play e^* since the information set is always on the equilibrium path.

[12] Available on request.

[13] See Section 7 for a discussion when the agent knows v privately and he reports it strategically to the market. Only the reported cash-flow is contractible then, while the true value realizes far in the future.

[14] In a previous version of the present paper we considered the case in which everybody can observe a public noisy signal π (i.e. an accounting statement) on the true value v, where $\pi = v + \varsigma$ and the noise ς cannot be affected by the manager choices. This public signal can then be used by the principal in the compensation contract. The result in (16) is robust to this generalization, hence market liquidity plays the same role described before. An interesting generalization considers the situation in which the manager can manipulate the signal π with a second effort that changes the noise ς.

[15] Alternatively, one can also think that the manager is liquidity constrained and he needs money at a date t before the true value v can be verified.

[16] In the following we will show that, if the principal has to choose among a contract written on p_1, a contract written on p_2, or a contract on both prices, she will (weakly) prefer the contract on p_2.

Appendix

Proofs

Proof of proposition 1

In order to calculate the conditional distribution, we use the following standard result for normally distributed variables:

Result 1 *Let Y_i be a $(n_i \times 1)$ vector with mean μ_i, i=1,2, and variance-covariance matrices Σ_{ij}, then*

$$Y_2|Y_1 = y_1 \sim N([\mu_2 + \Sigma_{21}\Sigma_{11}^{-1}(y_1 - \mu_1)], [\Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}])$$

Let $Y_1 = z$ with mean $\mu_1 = \beta e^*$ and $\Sigma_{11} = Var(z) = \beta^2\sigma_\theta^2 + \sigma_u^2$, $Y_2 = v$ with mean $\mu_2 = e^*$ and $\Sigma_{22} = \sigma_\theta^2 + \sigma_\eta^2$, and $\Sigma_{21} = \Sigma_{12} = Cov[v, z] = \beta\sigma_\theta^2$. Hence

$$E[v|z] = e^* + \frac{\beta\sigma_\theta^2}{\beta^2\sigma_\theta^2 + \sigma_u^2}(z - \beta e^*)$$

Letting $\lambda = \frac{\beta\sigma_\theta^2}{\beta^2\sigma_\theta^2+\sigma_u^2}$ and substituting for $z = \beta(e + \theta) + u$ gives the result for $\hat{p} = E[v|z]$.

Proof of proposition 2

It will be easier to calculate the conditional expectation if we use the following information equivalent of price $\hat{p}$,

$$\hat{\hat{p}} = \frac{\hat{p} - (1 - \lambda\beta e^*)}{\lambda\beta} = e + \theta + \frac{1}{\beta}u$$

which is constructed entirely from publicly available information (everybody "knows" e^* in equilibrium). We again use result 1. Let $Y_1 = (s_i, \hat{\hat{p}})$ with mean $\mu_1 = (e^*, e^*)$ and $Y_2 = v$ with mean $\mu_2 = e^*$. The covariance-variance matrices are:

$$\Sigma_{11} = \begin{pmatrix} \sigma_\theta^2 + \sigma_\varepsilon^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \frac{1}{\beta^2\sigma_u^2} \end{pmatrix}$$

and $\Sigma_{22} = \sigma_\theta^2 + \sigma_\eta^2$, and $\Sigma_{21} = \Sigma_{12} = (Cov[s_i, v], Cov[\hat{\hat{p}}, v]) = (\sigma_\theta^2, \sigma_\theta^2)$.

Hence

$$\begin{aligned} E[v|s_i,\hat{\hat{p}}] &= e^* + (\sigma_\theta^2, \sigma_\theta^2)\begin{pmatrix} \sigma_\theta^2+\sigma_\varepsilon^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2+\frac{1}{\beta^2\sigma_u^2}\end{pmatrix}^{-1}\left(\begin{pmatrix} s_i \\ \hat{\hat{p}}\end{pmatrix} - \begin{pmatrix} e^* \\ e^*\end{pmatrix}\right) \\ &= \frac{\beta^2\sigma_\varepsilon^2\sigma_\theta^2\hat{\hat{p}} + \sigma_u^2\sigma_\theta^2 s_i + \sigma_u^2\sigma_\varepsilon^2 e^*}{\beta^2\sigma_\varepsilon^2\sigma_\theta^2 + \sigma_u^2(\sigma_\varepsilon^2+\sigma_\theta^2)} \end{aligned}$$

Substituting $\hat{p}$ for $\hat{\hat{p}}$ and writing the expression in terms of precision $\tau_j = 1/\sigma_j^2$, we obtain:

$$E[v|s_i,\hat{p}] = \frac{\tau_\varepsilon s_i + (\beta^2\tau_u+\tau_\theta)\hat{p}}{\tau_\varepsilon + (\beta^2\tau_u+\tau_\theta)}$$

Next we need to calculate

$$\begin{aligned} Var[v|s_i,\hat{\hat{p}}] &= \sigma_\theta^2+\sigma_\eta^2 - (\sigma_\theta^2, \sigma_\theta^2)\begin{pmatrix} \sigma_\theta^2+\sigma_\varepsilon^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2+\frac{1}{\beta^2\sigma_u^2}\end{pmatrix}^{-1}\begin{pmatrix} \sigma_\theta^2 \\ \sigma_\theta^2\end{pmatrix} \\ &= \frac{\beta^2\sigma_\eta^2\tau_u + \sigma_\eta^2\tau_\varepsilon + \sigma_\eta^2\tau_\theta + 1}{\beta^2\tau_u+\tau_\theta+\tau_\varepsilon} \\ &= \sigma_\eta^2 + \frac{1}{\tau_\varepsilon + (\beta^2\tau_u+\tau_\theta)} \end{aligned}$$

Last, substituting $E[v|s_i,\hat{p}]$ and $Var[v|s_i,\hat{\hat{p}}] = Var[v|s_i,\hat{p}]$ into (10) yields

$$x_i(s_i,\hat{p}) = \frac{\tau_\varepsilon}{r[\sigma_\eta^2(\tau_\varepsilon+\beta^2\tau_u+\tau_\theta)+1]}(s_i-\hat{p})$$

and imposing $\beta = \frac{\tau_\varepsilon}{r[\sigma_\eta^2(\tau_\varepsilon+\beta^2\tau_u+\tau_\theta)+1]}$ means that β must satisfy the condition stated in the proposition.

Proof of corollary 1: The equation in proposition 2 is of the form $G(\beta, \sigma_u^2, \sigma_\varepsilon^2, \sigma_\theta^2, \sigma_\eta^2, r) = 0$ so that we can use the implicit function rule to calculate $\frac{\partial\beta}{\partial x} = -\frac{G_x}{G_\beta}$ where G_x is the partial derivative of G with respect to x.

$$\begin{aligned}
\frac{\partial \beta}{\partial r} &= -\frac{\sigma_u^2 \sigma_\theta^2}{r^2(3\beta^2 \sigma_\varepsilon^2 \sigma_\eta^2 \sigma_\theta^2 + (\sigma_\eta^2 \sigma_\theta^2 + \sigma_\varepsilon^2(\sigma_\eta^2 + \sigma_\theta^2))\sigma_u^2)} < 0 \\
\frac{\partial \beta}{\partial \sigma_u^2} &= \frac{\beta^3 \sigma_\varepsilon^2 \sigma_\eta^2 \sigma_\theta^2}{\sigma_u^2(3\beta^2 \sigma_\varepsilon^2 \sigma_\eta^2 \sigma_\theta^2 + (\sigma_\eta^2 \sigma_\theta^2 + \sigma_\varepsilon^2(\sigma_\eta^2 + \sigma_\theta^2))\sigma_u^2)} > 0 \\
\frac{\partial \beta}{\partial \sigma_\theta^2} &= \frac{\beta \sigma_\varepsilon^2 \sigma_\eta^2 \sigma_u^2}{\sigma_\theta^2(3\beta^2 \sigma_\varepsilon^2 \sigma_\eta^2 \sigma_\theta^2 + (\sigma_\eta^2 \sigma_\theta^2 + \sigma_\varepsilon^2(\sigma_\eta^2 + \sigma_\theta^2))\sigma_u^2)} > 0 \\
\frac{\partial \beta}{\partial \sigma_\eta^2} &= -\frac{\beta(\beta^2 \sigma_\varepsilon^2 \sigma_\theta^2 + (\sigma_\varepsilon^2 + \sigma_\theta^2)\sigma_u^2)}{3\beta^2 \sigma_\varepsilon^2 \sigma_\eta^2 \sigma_\theta^2 + (\sigma_\eta^2 \sigma_\theta^2 + \sigma_\varepsilon^2(\sigma_\eta^2 + \sigma_\theta^2))\sigma_u^2} < 0 \\
\frac{\partial \beta}{\partial \sigma_\varepsilon^2} &= \frac{(\beta r \sigma_\eta^2 - 1)\sigma_\theta^2 \sigma_u^2}{r\sigma_\varepsilon^2(3\beta^2 \sigma_\varepsilon^2 \sigma_\eta^2 \sigma_\theta^2 + (\sigma_\eta^2 \sigma_\theta^2 + \sigma_\varepsilon^2(\sigma_\eta^2 + \sigma_\theta^2))\sigma_u^2)}
\end{aligned}$$

where the sign of the last derivative is equal to the sign of $(\beta r \sigma_\eta^2 - 1)$. We checked numerically that this expression is negative for reasonable parameter estimates (in fact it was negative for all parameters that we tried). This coincides with out intuition that traders with less precise information place less weight on their private signal.

Proof of proposition 3: In choosing the effort level, the agent solves the following problem:

$$e = \arg\max_{e'} E[I] - \frac{r_m}{2} Var[I] - \frac{k}{2} e'^2$$

where

$$E[I] = \hat{a}_1 + \hat{a}_p E[\hat{p}] + \hat{a}_v E[v] = \hat{a}_1 + \hat{a}_p \left(e^* + \lambda\beta(e - e^*)\right) + \hat{a}_v e$$

and $Var[I] = \hat{a}_p^2 Var[\hat{p}] + \hat{a}_v^2 Var[v] + 2\hat{a}_p \hat{a}_v Cov[\hat{p}, v]$ is independent of the actual choice of effort (i.e. the manager can neither influence the fundamental risk of his environment nor the amount of noise trading). The f.o.c. of this problem reads then

$$\hat{a}_p \lambda\beta + \hat{a}_v = ke$$

that is anticipated by the principal and the investors given the contract. Hence, by rational expectations:

$$e^* : \hat{a}_p \lambda\beta + \hat{a}_v = ke^*$$

Since the principal is risk-neutral, the cheapest way to induce any equilibrium effort level is to minimize the income risk borne by the manager. An optimal contract must therefore solve:

$$\min_{a_p,a_v} Var[I]$$
$$s.t.\ \ \hat{a}_p\lambda\beta + \hat{a}_v = ke^*$$

and the f.o.c.'s of this problem can be reduced to

$$\hat{a}_v[Var[v] - Cov[\hat{p},v]] = \hat{a}_{\widehat{p}}[Var[\hat{p}] - Cov[\hat{p},v]]$$

and rewritten as

$$\begin{aligned}
\frac{\hat{a}_{\widehat{p}}}{\hat{a}_v} &= \frac{\frac{\partial E(\widehat{p})}{\partial e} - \frac{Cov[\hat{p},v]}{Var[v]}\frac{\partial E(v)}{\partial e}}{Var[\hat{p}]} \frac{Var[v]}{\frac{\partial E(v)}{\partial e} - \frac{Cov[\hat{p},v]}{Var[\hat{p}]}\frac{\partial E(\widehat{p})}{\partial e}} \\
&= \frac{\lambda\beta - \frac{\lambda\beta\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2}}{\lambda^2\beta^2\sigma_\theta^2 + \lambda^2\sigma_u^2} \frac{\sigma_\theta^2+\sigma_\eta^2}{1 - \frac{\lambda\beta\sigma_\theta^2}{\lambda^2\beta^2\sigma_\theta^2+\lambda^2\sigma_u^2}\lambda\beta} = \left(\frac{\beta}{\lambda}\right)\frac{\sigma_\eta^2}{\sigma_u^2}
\end{aligned}$$

Proof of Proposition 4: Applying the Projection Theorem:

$$\begin{aligned}
E[v\,|\widehat{z}_1] &= e^* + \frac{\widehat{\beta}_1\sigma_\theta^2}{\widehat{\beta}_1^2\sigma_\theta^2+\sigma_u^2}(\widehat{z}_1 - \widehat{\beta}_1 e^*) = e^* + \frac{\widehat{\beta}_1\tau_u}{\widehat{\beta}_1^2\tau_u+\tau_\theta}(\widehat{z}_1 - \widehat{\beta}_1 e^*) \\
&= e^*(1-\widehat{\lambda}_1\widehat{\beta}_1) + \widehat{\lambda}_1\widehat{z}_1 \\
&= e^*(1-\widehat{\lambda}_1\widehat{\beta}_1) + \widehat{\lambda}_1\widehat{\beta}_1(e+\theta) + \widehat{\lambda}_1 u_1
\end{aligned}$$

with $\widehat{\lambda}_1 = \frac{\widehat{\beta}_1\sigma_\theta^2}{\widehat{\beta}_1^2\sigma_\theta^2+\sigma_u^2}$ and substituting for $\widehat{z}_1 = \widehat{\beta}_1(e+\theta)+u_1$. By semi-strong efficiency, $\widehat{p}_1 = E[v\,|\widehat{z}_1]$. Analogously, calling $\tau_1 = \widehat{\beta}_1^2\tau_u + \tau_\theta$:

$$\begin{aligned}
E[v\,|\widehat{z}_1,\widehat{z}_2] &= e^*\left(1 - \frac{\widehat{\beta}_1^2\tau_u + \left(\beta_2-\widehat{\beta}_1\right)^2\tau_u}{\tau_\theta+\widehat{\beta}_1^2\tau_u+\left(\beta_2-\widehat{\beta}_1\right)^2\tau_u}\right) + \frac{\widehat{\beta}_1\tau_u}{\tau_\theta+\widehat{\beta}_1^2\tau_u+\left(\beta_2-\widehat{\beta}_1\right)^2\tau_u}\widehat{z}_1 + \frac{\left(\beta_2-\widehat{\beta}_1\right)\tau_u}{\tau_\theta+\widehat{\beta}_1^2\tau_u+\left(\beta_2-\widehat{\beta}_1\right)^2\tau_u}\widehat{z}_2 \\
&= e^*\left(1 - \frac{\widehat{\beta}_1^2\tau_u+\left(\beta_2-\widehat{\beta}_1\right)^2\tau_u}{\tau_2}\right) + \frac{\widehat{\beta}_1\tau_u}{\tau_2}\left(\widehat{\beta}_1(e+\theta)+u_1\right) + \frac{\left(\beta_2-\widehat{\beta}_1\right)\tau_u}{\tau_2}\left(\left(\beta_2-\widehat{\beta}_1\right)v + u_2\right) \\
&= e^*(1-\widehat{\lambda}_1\widehat{\beta}_1\tfrac{\tau_1}{\tau_2} - \left(\beta_2-\widehat{\beta}_1\right)\widehat{\lambda}_2) + \left(\widehat{\lambda}_1\widehat{\beta}_1\tfrac{\tau_1}{\tau_2} + \left(\beta_2-\widehat{\beta}_1\right)\widehat{\lambda}_2\right)(e+\theta) + \widehat{\lambda}_1\tfrac{\tau_1}{\tau_2}u_1 + \widehat{\lambda}_2 u_2
\end{aligned}$$

and again by semi-strong efficiency the REE normalized price in the second period is $\widehat{p}_1 = E[v\,|\widehat{z}_1,\widehat{z}_2]$.

Proof of Proposition 5: The final wealth of agent i receiving signal s_i before the first round of trade is:

$$\begin{aligned} W_{i3} &= x_{i1}(p_2 - p_1) + x_{i2}(\pi - p_2) \\ &= x_{i1}\left(\frac{\widehat{p}_2 - a_0}{(1+a_2)} - \frac{\widehat{p}_1 - a_0}{(1+a_2)}\right) + x_{i2}(v - a_0 - a_2 p_2 - p_2) \\ &= \tfrac{1}{1+a_2} x_{i1}\left(\widehat{p}_2 - \widehat{p}_1\right) + x_{i2}(v - \widehat{p}_2) \\ &= \widehat{x}_{i1}\left(\widehat{p}_2 - \widehat{p}_1\right) + x_{i2}(v - \widehat{p}_2) \end{aligned}$$

By backward induction, let us solve the problem at $t = 2$:

$$\max_{x_{i2}} - \exp(-rW_{i2}) E\left[\exp(-r x_{i2}(v - \widehat{p}_2)) \,|s_i, \widehat{p}_2\right] \tag{41}$$

where $W_{i2} = \frac{1}{1+a_2} x_{i1}\left(\widehat{p}_2 - \widehat{p}_1\right)$ is known at $t = 2$, given the observed $\widehat{p}_2$. Hence, the solution of (41) corresponds to the solution of the one-period trading case illustrated in Proposition 1 with $v = e + \theta$ and $s_i = v + \xi_i$: $x_{i2} = \beta_{2l}(s_i - \widehat{p}_2)$ with $\beta_{2l} = \frac{\tau_\varepsilon}{r}$.

At $t = 1$ the optimal trading strategy solves:

$$\max_{x_{i1}} E\left[\exp(-\tfrac{r}{1+a_2} x_{i1}\left(\widehat{p}_2 - \widehat{p}_1\right)) E\left[\exp(-r x_{i2}(v - \widehat{p}_2)) \,|s_i, \widehat{p}_2\right] |s_i, \widehat{p}_1\right]$$

which is equivalent to

$$\max_{\widehat{x}_{i1}} E\left[\exp(-r\widehat{x}_{i1}\left(\widehat{p}_2 - \widehat{p}_1\right)) E\left[\exp(-r x_{i2}(v - \widehat{p}_2)) \,|s_i, \widehat{p}_2\right] |s_i, \widehat{p}_1\right]$$

and substituting for $x_{i2} = \beta_{2l}(s_i - \widehat{p}_2)$ we can rewrite it as:

$$\max_{x_{i1}} E\left[\exp(-r\widehat{x}_{i1}\left(\widehat{p}_2 - \widehat{p}_1\right)) E\left[\exp(-r\beta_{2l}(s_i - \widehat{p}_2)(v - \widehat{p}_2)) \,|s_i, \widehat{p}_2\right] |s_i, \widehat{p}_1\right]$$

hence (see Vives (1995), proof of Proposition 4.1):

$$\begin{aligned} & \max_{\widehat{x}_{i1}} E\left[\exp\left(-r\widehat{x}_{i1}\left(\widehat{p}_2 - \widehat{p}_1\right)\right) \exp\left(-\tfrac{1}{2} r^2 \beta_{2l}^2 \frac{(s_i - \widehat{p}_2)^2}{\tau_{2l} + \tau_\varepsilon}\right) |s_i, \widehat{p}_1\right] \\ \Rightarrow\ & \max_{\widehat{x}_{i1}} E\left[\exp\left(-r\widehat{x}_{i1}\left(\widehat{p}_2 - \widehat{p}_1\right) - \tfrac{1}{2} r^2 \beta_{2l}^2 \frac{(s_i - \widehat{p}_2)^2}{\tau_{2l} + \tau_\varepsilon}\right) |s_i, \widehat{p}_1\right] \\ \Rightarrow\ & \max_{\widehat{x}_{i1}} E\left[\exp\left(-r\left(\widehat{x}_{i1}\left(\widehat{p}_2 - \widehat{p}_1\right) - \tfrac{1}{2} r \beta_{2l}^2 \frac{(s_i - \widehat{p}_2)^2}{\tau_{2l} + \tau_\varepsilon}\right)\right) |s_i, \widehat{p}_1\right] \end{aligned} \tag{42}$$

Vives (1995), shows that the solution to (42) is:

$$\widehat{x}_{i1} = \frac{\tau_\varepsilon}{r}(s_i - \hat{p}_1) = \widehat{\beta}_{1l}(s_i - \hat{p}_1)$$

hence the optimal trading strategy for the long-term investor i at $t = 1$ is:

$$x_{i1} = \frac{\tau_\varepsilon}{r}(1 + a_2)(s_i - \hat{p}_1) = \beta_{1l}(s_i - \widehat{p}_1)$$

Proof of Proposition 6: At time $t = 2$ the informed trader observes s_i and he solves:

$$\underset{x_{i2}}{Max} U^i(W_2^i) = E\left[-\exp(-r\left(x_{i2}(\pi - p_2)\right))\,|s_i, p_1, p_2\right]$$

where $\pi = v - a_0 - a_2 p_2$ is the liquidation value; hence:

$$x_{i2}^s = \frac{E\left[v - a_0 - a_2 p_2 - p_2\,|s_i, p_2\right]}{rVar\left[v - p_2\,|s_i, p_2\right]} = \frac{E\left[v - a_0 - (1 + a_2)p_2\,|s_i, p_2\right]}{rVar\left[v - a_0 - (1 + a_2)p_2\,|s_i, p_2\right]} = \frac{E\left[v - \widehat{p}_2\,|s_i, p_2\right]}{rVar\left[v - \widehat{p}_2\,|s_i, p_2\right]}$$

which corresponds to the demand with long-term investors. Hence, we can write $x_2^i = \beta_2 s_i + \zeta(\widehat{p}_1, \widehat{p}_2)$ and $\beta_2^s = \beta_{static}$:

$$\begin{aligned} x_{i2}^s &= \beta_{2s}(s_i - \widehat{p}_2) \\ &\beta_{2s} = \beta_{2l} = \frac{\tau_\varepsilon}{r} \end{aligned} \tag{43}$$

At time $t = 1$ the informed trader observes s_i and he solves:

$$\begin{aligned} \underset{x_{i1}}{Max} U^i(W_1^i) &= E\left[-\exp(-r\left(x_{i1}(p_2 - p_1)\right))\,|s_i, p_1\right] = \\ &= E\left[-\exp\left(-r\left(x_{i1}\left(\frac{\widehat{p}_2 - a_0}{1 + a_2} - \frac{\widehat{p}_1 - a_0}{1 + a_2}\right)\right)\right)|s_i, \widehat{p}_1\right] \\ &= E\left[-\exp\left(-r\left(\tfrac{x_{i1}}{1+a_2}\left((\widehat{p}_2 - \widehat{p}_1)\right)\right)\right)|s_i, \widehat{p}_1\right] \\ &= E\left[-\exp\left(-r\left(\widehat{x}_{i1}\left((\widehat{p}_2 - \widehat{p}_1)\right)\right)\right)|s_i, \widehat{p}_1\right] \end{aligned}$$

Let $\Delta\widehat{\beta} = \beta_2 - \widehat{\beta}_1$. Let us compute first (from Proposition 4):

$$\begin{aligned}
\widehat{p}_2 - \widehat{p}_1 &= -e^* \left(\widehat{\lambda}_1 \widehat{\beta}_1 \tfrac{\tau_1}{\tau_2} + \Delta\widehat{\beta}\widehat{\lambda}_2 \right) + e^* \widehat{\lambda}_1 \widehat{\beta}_1 + \widehat{\lambda}_1 \widehat{z}_1 \left(\tfrac{\tau_1}{\tau_2} - 1 \right) + \widehat{\lambda}_2 \widehat{z}_2 \\
&= e^* \left(\left(1 - \tfrac{\tau_1}{\tau_2} \right) \widehat{\lambda}_1 \widehat{\beta}_1 - \Delta\widehat{\beta}\widehat{\lambda}_2 \right) - \Delta\widehat{\beta}\widehat{\lambda}_2 \widehat{\lambda}_1 \widehat{z}_1 + \widehat{\lambda}_2 \widehat{z}_2 \\
&= \Delta\widehat{\beta}\widehat{\lambda}_2 \left(e^* \left(\widehat{\lambda}_1 \widehat{\beta}_1 - 1 \right) - \widehat{\lambda}_1 \widehat{z}_1 \right) + \widehat{\lambda}_2 \widehat{z}_2 \\
&= \widehat{\lambda}_2 \widehat{z}_2 - \Delta\widehat{\beta}\widehat{\lambda}_2 \widehat{p}_1 \\
&= \widehat{\lambda}_2 \left(\Delta\widehat{\beta} \left(v - \widehat{p}_1 \right) + u_2 \right)
\end{aligned}$$

so

$$\begin{aligned}
E\left[\left(\widehat{p}_2 - \widehat{p}_1 \right) | s_i, \widehat{p}_1 \right] &= \widehat{\lambda}_2 \Delta\widehat{\beta} E\left[\left(v - \widehat{p}_1 \right) | s_i, \widehat{p}_1 \right] \\
Var\left[\left(\widehat{p}_2 - \widehat{p}_1 \right) | s_i, p_1 \right] &= \widehat{\lambda}_2^2 \left(\Delta\widehat{\beta}^2 Var\left[\left(v - \widehat{p}_1 \right) | s_i, p_1 \right] + \sigma_u^2 \right)
\end{aligned}$$

Using

$$\begin{aligned}
E\left[\left(v - \widehat{p}_1 \right) | s_i, \widehat{p}_1 \right] &= \frac{\tau_\varepsilon s_i + \tau_1 \widehat{p}_1}{\tau_\varepsilon + \tau_1} \\
Var\left[\left(v - \widehat{p}_1 \right) | s_i, p_1 \right] &= \frac{1}{\tau_\varepsilon + \tau_1}
\end{aligned}$$

and substituting into the demand:

$$\begin{aligned}
\widehat{x}_{i1} &= \frac{E\left[\left(\widehat{p}_2 - \widehat{p}_1 \right) | s_i, \widehat{p}_1 \right]}{r Var\left[\left(\widehat{p}_2 - \widehat{p}_1 \right) | s_i, p_1 \right]} = \frac{\widehat{\lambda}_2 \Delta\widehat{\beta} \left(\frac{\tau_\varepsilon s_i + \tau_1 \widehat{p}_1}{\tau_\varepsilon + \tau_1} - \widehat{p}_1 \right)}{r \widehat{\lambda}_2^2 \left(\Delta\widehat{\beta}^2 \frac{1}{\tau_\varepsilon + \tau_1} + \sigma_u^2 \right)} \\
&= \frac{\widehat{\lambda}_2 \Delta\widehat{\beta} \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_1} \left(s_i - \widehat{p}_1 \right)}{r \widehat{\lambda}_2^2 \left(\Delta\widehat{\beta}^2 \frac{1}{\tau_\varepsilon + \tau_1} + \sigma_u^2 \right)} \\
&= \frac{\Delta\widehat{\beta} \tau_\varepsilon}{r \widehat{\lambda}_2 \left(\Delta\widehat{\beta}^2 + \left(\tau_\varepsilon + \tau_1 \right) \frac{1}{\tau_u} \right)} \left(s_i - \widehat{p}_1 \right) \\
&= \frac{\tau_\varepsilon}{r} \frac{\Delta\widehat{\beta} \tau_u}{\widehat{\lambda}_2 \left(\Delta\widehat{\beta}^2 \tau_u + \tau_1 + \tau_\varepsilon \right)} \left(s_i - \widehat{p}_1 \right) \\
&= \frac{\tau_\varepsilon}{r} \frac{\Delta\widehat{\beta} \tau_u}{\widehat{\lambda}_2 \left(\tau_2 + \tau_\varepsilon \right)} \left(s_i - \widehat{p}_1 \right) \\
&= \frac{\tau_\varepsilon}{r} \frac{\tau_2 \widehat{\lambda}_2}{\widehat{\lambda}_2 \left(\tau_2 + \tau_\varepsilon \right)} \left(s_i - \widehat{p}_1 \right) = \frac{1}{r} \frac{\tau_\varepsilon \tau_2}{\left(\tau_2 + \tau_\varepsilon \right)} \left(s_i - \widehat{p}_1 \right)
\end{aligned}$$

so that

$$\widehat{\beta}_{1s} = \frac{1}{r}\frac{\tau_{2s}\tau_\varepsilon}{\tau_{2s}+\tau_\varepsilon}$$

Proof of Corollary 2: (i) From the projection theorem, $Var[v\,|\widehat{z}_1,\widehat{z}_2] = \frac{1}{\tau_2} = \frac{1}{\tau_\theta+\widehat{\beta}_1^2\tau_u+\left(\beta_2-\widehat{\beta}_1\right)^2\tau_u}$ thus

$$\tau_2 = \tau_\theta + \widehat{\beta}_1^2\tau_u + \left(\beta_2-\widehat{\beta}_1\right)^2\tau_u$$

and computing this expression in the two cases with long and short termists:

$$\begin{aligned}\tau_{2l} &= \tau_\theta + \left(\frac{\tau_\varepsilon}{r}\right)^2\tau_u \\ \tau_{2s} &= \tau_\theta + \widehat{\beta}_{1s}^2\tau_u + \left(\frac{\tau_\varepsilon}{r}-\widehat{\beta}_{1s}\right)^2\tau_u\end{aligned}$$

so that

$$\tau_{2l} > \tau_{2s} \Longleftrightarrow \widehat{\beta}_{1s} < \frac{\tau_\varepsilon}{r} \Longleftrightarrow \frac{\tau_{2s}}{\tau_{2s}+\tau_\varepsilon} < 1$$

which is always verified. Hence, $Var[v\,|\widehat{z}_1,\widehat{z}_2]$ is lower with long-termists.

Since $Var[\widehat{p}_2] = Var(v) - Var[v\,|\widehat{z}_1,\widehat{z}_2]$ the unconditional variance of $\widehat{p}_2$ is higher with long-termists.

(ii) Rewriting explicitly $\widehat{\lambda}_1\widehat{\beta}_1\frac{\tau_1}{\tau_2} + \left(\beta_2-\widehat{\beta}_1\right)\widehat{\lambda}_2$ in the two cases with long and short termists:

$$\begin{aligned}\widehat{\lambda}_{1s}\widehat{\beta}_{1s}\tfrac{\tau_{1s}}{\tau_{2s}} + \left(\beta_{2s}-\widehat{\beta}_{1s}\right)\widehat{\lambda}_{2s} &= \frac{\widehat{\beta}_{1s}^2\tau_u + \left(\beta_{2s}-\widehat{\beta}_{1s}\right)\tau_u}{\tau_{2s}} = 1-\frac{\tau_\theta}{\tau_{2s}} \\ \widehat{\lambda}_{1l}\widehat{\beta}_{1l}\tfrac{\tau_{1l}}{\tau_{2l}} + \left(\beta_{2l}-\widehat{\beta}_{1l}\right)\widehat{\lambda}_{2l} &= \frac{\widehat{\beta}_{1l}^2\tau_u}{\tau_{1l}} = 1-\frac{\tau_\theta}{\tau_{2l}}\end{aligned}$$

since $\beta_{2l} = \widehat{\beta}_{1l}$ thus $\tau_{1l} = \tau_{2l}$. But then, from point (i), $1-\frac{\tau_\theta}{\tau_{2l}} > 1-\frac{\tau_\theta}{\tau_{2s}}$.

Proof of Proposition 7: Using the equality obtained above $\widehat{\lambda}_1\widehat{\beta}_1\frac{\tau_1}{\tau_2} + \left(\beta_2-\widehat{\beta}_1\right)\widehat{\lambda}_2 = 1-\frac{\tau_\theta}{\tau_2}$ in (40):

$$e^* = \frac{1}{k}\frac{1}{1+kr_m\frac{Var(\widehat{p}_2)}{\left(1-\frac{\tau_\theta}{\tau_2}\right)^2}} = \frac{1}{k}\frac{1}{1+kr_m\frac{\frac{1}{\tau_v}-\frac{1}{\tau_2}}{\left(1-\frac{\tau_\theta}{\tau_2}\right)^2}}$$

since $\widehat{p}_2 = E(v\,|\widehat{p}_2)$ and $\tau_v = \tau_\theta$ (recall that $v = e+\theta$). This gives us:

$$e^* = \frac{1}{k}\frac{1}{1+kr_m\frac{\frac{1}{\tau_\theta}-\frac{1}{\tau_2}}{\left(1-\frac{\tau_\theta}{\tau_2}\right)^2}} \tag{44}$$

and the term $\frac{\frac{1}{\tau_\theta}-\frac{1}{\tau_2}}{\left(1-\frac{\tau_\theta}{\tau_2}\right)^2}$ is decreasing in τ_2[17], so that e^* is increasing in τ_2. The result follows from Corollary 2 (i).

Proof of Proposition 8: Vives (1995), Proposition 5.1, shows that the precision of prices with short-term investors increases with time: hence $\tau_{2s} > \tau_{1s}$. Moreover,
$\widehat{\lambda}_{1s}\widehat{\beta}_{1s}\frac{\tau_{1s}}{\tau_{2s}} + \left(\beta_{2s} - \widehat{\beta}_{1s}\right)\widehat{\lambda}_{2s} = 1 - \frac{\tau_\theta}{\tau_{2s}} > \widehat{\lambda}_{1s}\widehat{\beta}_{1s} = 1 - \frac{\tau_\theta}{\tau_{1s}}$ since $\tau_{2s} > \tau_{1s}$. Writing the optimal effort in the two cases with contract written on $\widehat{p}_1$ and contract written on $\widehat{p}_2$, from (44) gives respectively[18]:

$$
\begin{aligned}
e^*(\widehat{p}_1) &= \frac{1}{k}\frac{1}{1+kr_m\frac{Var(\widehat{p}_{1s})}{\left(\widehat{\lambda}_{1s}\widehat{\beta}_{1s}\right)^2}} \\
e^*(\widehat{p}_2) &= \frac{1}{k}\frac{1}{1+kr_m\frac{Var(\widehat{p}_{2s})}{\left(1-\frac{\tau_\theta}{\tau_{2s}}\right)^2}}
\end{aligned}
$$

Simplifying:

$$
\begin{aligned}
\frac{Var(\widehat{p}_{1s})}{\left(\widehat{\lambda}_{1s}\widehat{\beta}_{1s}\right)^2} &= \frac{\frac{1}{\tau_\theta}-\frac{1}{\tau_{1s}}}{\left(1-\frac{\tau_\theta}{\tau_{1s}}\right)^2} = \frac{\frac{\tau_{1s}-\tau_\theta}{\tau_\theta\tau_{1s}}}{\left(\frac{\tau_{1s}-\tau_\theta}{\tau_{1s}}\right)^2} = \frac{1}{\tau_\theta}\frac{\tau_{1s}}{\tau_{1s}-\tau_\theta} = \frac{1}{\tau_\theta}\left(1+\frac{\tau_\theta}{\tau_{1s}-\tau_\theta}\right) \\
\frac{Var(\widehat{p}_{2s})}{\left(1-\frac{\tau_\theta}{\tau_{2s}}\right)^2} &= \frac{\frac{1}{\tau_\theta}-\frac{1}{\tau_{2s}}}{\left(1-\frac{\tau_\theta}{\tau_{2s}}\right)^2} = \frac{\frac{\tau_{2s}-\tau_\theta}{\tau_\theta\tau_{2s}}}{\left(\frac{\tau_{2s}-\tau_\theta}{\tau_{2s}}\right)^2} = \frac{1}{\tau_\theta}\frac{\tau_{2s}}{\tau_{2s}-\tau_\theta} = \frac{1}{\tau_\theta}\left(1+\frac{\tau_\theta}{\tau_{2s}-\tau_\theta}\right)
\end{aligned}
$$

Finally, since $\tau_{2s} > \tau_{1s}$, $\frac{Var(\widehat{p}_{2s})}{\left(1-\frac{\tau_\theta}{\tau_{2s}}\right)^2} < \frac{Var(\widehat{p}_{1s})}{\left(\widehat{\lambda}_{1s}\widehat{\beta}_{1s}\right)^2}$ and the principal can optimally induce a higher effort writing a contract with stock-appreciation rights based on $\widehat{p}_{2s}$.