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Departamento de Economía de la Empresa
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34-91) 6249608

OPTIMAL DEMAND FOR LONG-TERM BONDS WHEN RETURNS ARE PREDICTABLE[1]

Javier Gil-Bazo*

Abstract

This paper further explores the horizon effect in the optimal static and dynamic demand for risky assets under return predictability as documented by Barberis (2000). Contrary to the case of stocks, the optimal demand for long-term Government bonds of a buy-and-hold investor is not necessarily increasing in the investment horizon, and may in fact be decreasing for some initial levels of the predicting variable. The paper provides an analytical explanation based on the dependence of the mean variance ratio on the investor´s time horizon. Under stationarity of the predicting variable, unusually high or unusually low levels of the predictor tend to dissapear over time inducing the mean of cumulative returns to grow less or more than linearly as the investment horizon increases. If this effect dominates that on the variance, optimal demands can either be increasing or decresing in the investment horizon. On the other hand, the solution to the investor´s dynamic allocation problem in the presence of bonds indicates that long-term Government bonds do not provide a good hedge for adverse changes in the investor´s opportunity set: optimal dynamic demands for bonds do not differ from static portfolio choices at any horizon.

JEL Classification: G11, G12

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*Universidad Carlos III de Madrid. Departamento de Economía de la Empresa. C/ Madrid, 126, 28903 Getafe (Madrid), Spain. E-mail:jjgil@emp.uc3m.es. Tel.: +34-91-624.5844. Fax: +34-91-624.9608

1 Introduction

Optimal portfolio selection is an old problem in Financial Economics. In a single-period context, mean variance analysis predicts that the optimal portfolio composition depends on the first two moments of asset returns. In the presence of a risk free asset, investors will divide their investment between a portfolio of risky assets –identical for all investors– and the riskless asset. The fraction of the total portfolio that each investor will allocate to the risky asset will depend on his or her attitude towards risk. However, making recommendations regarding optimal portfolio choice is not an easy task.

First of all, mean variance analysis is consistent with expected utility maximization only under the assumption that the distribution of returns is multivariate normal or when the investor has quadratic preferences. But even under those circumstances, there do not exist analytic results that completely characterize the solution to the investor's asset allocation problem in every possible case.

Second, there is no direct relationship between predictability and optimal portfolio choice even under the most simple preference characterization (preferences defined over the mean and variance of the distribution of asset returns). Nevertheless, optimal portfolio policy must take predictability into account since there exists a vast amount of empirical evidence in the literature that favors the time-varying conditional moments hypothesis. See for instance Campbell (1987), Campbell and Shiller (1988a, 1988b), Fama (1984), Fama (1990), Fama and French (1988, 1989), or Hodrick (1992).

Third, closed-form solutions to the dynamic portfolio problem faced by an investor with a general class of preferences are unknown when the investment opportunity set changes over time.

Finally, in the real world investors never know exactly the true value of the moments of the distribution of future returns and must therefore estimate them from past realizations. An investor that recognizes estimation risk must take it into account when deciding the composition of his portfolio.

Luckily, some recent contributions have addressed these difficulties in a straightforward way.

Kandel and Stambaugh (1996) extend Klein and Bawa's (1976) optimal portfolio analysis under parameter uncertainty to the context in which expected moments are not independently and identically distributed but can be predicted by a set of variables. The investor in their model is aware of asset return predictability but is ignorant about the true value of the return generating model parameters. Kandel and Stambaugh are able to characterize the single-period optimal demand using a numerical method and in a Bayesian framework. However, return predictability does not only affect its conditional distribution over any given number of periods ahead, but it also potentially introduces a horizon effect on the optimal portfolio choice. Albeit different, that effect is closely related to the concept of intertemporal hedging demand, that is, the difference between the optimal demand for an asset by an investor with a long-term horizon and the demand by a myopic investor. Intertemporal hedging demand is different from zero whenever investment opportunities are time-varying[1]. If returns are however not identically and independently distributed (i.i.d.), part of an investor's position is explained by his desire to hedge future averse predictable changes (see Merton (1969, 1971, 1973) and Samuelson (1969)). Similarly, the static demand (the demand of an investor who follows a buy-and-hold strategy) depends on the investment horizon when the conditional distribution of returns is time-varying. In this case, however, the reason is di¤erent. When returns are not i.i.d., each period's return increases the mean and variance of the cumulative return up to that particular period in a di¤erent proportion. As a consequence of the di¤erent impact on each moment, the optimal portfolio choice that solves the investor's problem depends on his or her investment horizon.

The horizon e¤ect on the optimal portfolio has been studied by Barberis[2] (2000), who …nds that when stock returns are predictable, the optimal investment in stocks is increasing in the investment horizon both with and without portfolio rebalancing. Barberis argues that predictability in stock expected returns induces mean reversion which in turn makes the variance of cumulative returns grow less than linearly as the horizon increases. As a result, a risk averse long-term investor should allocate more to stocks than a short-term investor, independently of the initial value of the predicting variable. As for the case of an investor who periodically rebalances his portfolio Barberis' results suggest show that investors choose to hedge adverse changes in the investment opportunity set by investing more in stocks since realized returns and expected returns are negatively related. Nevertheless, when the investor takes estimation risk into account, the e¤ect of the investment horizon on the optimal allocation may decrease and even become negative.

This paper starts by exploring analytically the e¤ect of predictability on both the variance and the mean of cumulative returns. The main …nding is that mean reversion does not necessarily induce a positive horizon e¤ect on optimal asset allocation. Intuitively, when the predictive variable is stationary, its e¤ect on the mean of cumulative returns decays with time. If the predicting variable has a positive e¤ect on future expected returns, its dilution with time may o¤set the positive horizon e¤ect induced by mean reversion. The empirical analysis of the optimal static demand for long-term Government bonds con…rms the theoretical prediction: horizon e¤ects may actually be either positive or negative depending on the initial value of the predicting variable(s) and, in any case, disappear as the investment horizon grows to in…nity. This study suggests that Barberis' (2000) results could be speci…c to the choice of the investable asset (a stock

[1] Except for the case of an investor with logarithmic preferences.

[2] Barberis' paper is part of a long series of recent contributions to the asset allocation literature. Brennan, Schwartz and Lagnado (1997) …nd a numerical solution to the problem of a long-term investor who rebalances his portfolio frequently. They …nd that long-term demand for both bonds and stocks is higher than that of a single-period investor. Balduzzi and Lynch (1999) emply an alternative numerical method to solve the intertemporal consumption and portfolio choice in the presence of transaction costs. Brandt (1999) estimates nonparametrically the values of consumption and portfolio choice that solve Euler's intertemporal equation. Finally, Campbell and Viceira (1999) obtain a log-linear approximation to the solution of the investor's dynamic problem.

index) and the predicting variable (dividend yield) and not a consequence of return predictability or mean reversion.

Allowing for the presence of fixed income –and more specifically Government debt– in the investor's portfolio choice problem not only illustrates the theoretical analysis of the relationship between predictability and investment horizon, but it is also particularly relevant given the important role that this kind of assets play in the investment industry. In the case of Spain, for instance, investment institutions manage 111 billion Euros worth of fixed income assets –53 per cent of total assets under management as opposed to 11 per cent invested in stocks– most of which (83 per cent) corresponds to Government debt[3].

Next, the intertemporal hedging demand for long-term credit risk free bonds is studied for investors with different investment horizons. The demand for long-term bonds has been previously analyzed by Campbell and Viceira (1998) through an approximate analytical solution to the dynamic problem[4]. Taking inflation risk into account, they find that the demand for long-term bonds is explained especially by hedging demand. Campbell and Viceira's (1998) model assumes that the investor has an infinite horizon, however, when conditional moments are time-varying, the optimal portfolio choice of an investor with a finite horizon may in principle differ to a great extent from that of an infinitely lived investor. As opposed to Campbell and Viceira (1998), the framework used in this paper makes it possible to analyze the effect of the investment horizon and the initial value of the predicting variables. The results suggest that in the absence of inflation risk, the hedging demand for long-term bonds by investors with different finite horizons is zero. In other words, investing in bonds does not provide a good hedge for changes in the predicting variable.

Finally, the paper studies optimal portfolio choice when both risky assets are available for investment. Results show that:

1. changes in the initial values of the predictors alter the optimal portfolio choice for any given horizon,

2. the corresponding reallocation between different assets also depends on the investment horizon, and

3. estimation risk affects the investor's position in stocks to a greater extent than his position in bonds.

The rest of the paper is divided in the following sections: section 2 presents the portfolio choice framework employed by Barberis (2000); section 3 presents the dataset employed for this research; section 4 deals with the static demand for long-term bonds in the absence of stocks, section 5 shows the results for the dynamic problem, section 6 explores the horizon effect on the optimal asset mix between long-term bonds and stocks; and finally section 7 concludes.

[3] Figures have been taken from the Spanish Security Market Committee report for the first quarter of 1999 ("Informe trimestral de instituciones de inversión colectiva", Comisión Nacional del Mercado de Valores).

[4] The method was also used by Restoy (1992), Campbell (1993), and Campbell and Viceira (1999).

2 Optimal portfolio choice when returns are predictable

Consider the problem at time T of an investor who derives utility from his wealth at time $T+\hat{T}$. The investor can invest his wealth in three different assets: one-month Treasury Bills; a representative portfolio (index) of stocks; and a representative portfolio of Government Bonds with 10 years to maturity. The investor does not rebalance his portfolio. It will be assumed that the continuously compounded monthly real return on Treasury Bills, denoted by r_f is constant. In order to compare results to those obtained by Barberis (2000), r_f will also be taken to be 0.0036, or 4.32% in annual terms. Returns in excess of the riskless interest rate on the stock index and the bond portfolio between periods $t-1$ and t are denoted by r_{st} and r_{bt} respectively.

If initial wealth equals one, ω_s stands for the fraction of the investor's initial wealth allocated to stocks, and ω_b denotes the allocation to bonds, then the investor's terminal wealth is given by:

$$W_{T+\hat{T}} = (1-\omega_s-\omega_b)\exp(r_f\hat{T}) + \omega_s\exp(r_f\hat{T}+R_{s;T+\hat{T}}) + \omega_b\exp(r_f\hat{T}+R_{b;T+\hat{T}}), \tag{1}$$

where $R_{s;T+\hat{T}}$ and $R_{b;T+\hat{T}}$ are cumulative excess returns between T and $T+\hat{T}$:

$$R_{s;T+\hat{T}} = r_{s;T+1} + r_{s;T+2} + \cdots + r_{s;T+\hat{T}}, \tag{2}$$

$$R_{b;T+\hat{T}} = r_{b;T+1} + r_{b;T+2} + \cdots + r_{b;T+\hat{T}}. \tag{3}$$

Investor's preferences can be represented by utility functions with constant relative risk aversion:

$$u(W) = \frac{W^{1-A}}{1-A}, \quad \text{with } A \neq 1. \tag{4}$$

The problem can then be formally stated as follows:

$$\max_{\omega_s,\,\omega_b} E_T\left\{\frac{\left[(1-\omega_s-\omega_b)\exp(r_f\hat{T})+\omega_s\exp(r_f\hat{T}+R_{s;T+\hat{T}})+\omega_b\exp(r_f\hat{T}+R_{b;T+\hat{T}})\right]^{1-A}}{1-A}\right\}, \tag{5}$$

with portfolio weights constrained to be between zero and one.

As in Barberis (2000) the objective function is numerically evaluated as the integral:

$$\int u(W_{T+\hat{T}})p(R_{T+\hat{T}} \mid \Phi_T)dR_{T+\hat{T}}; \tag{6}$$

where R is the return vector $(R_s; R_b)$ and Φ_T is the investor's information set at T.

In order to compute the integral (6), first a large number of realizations of future returns are simulated from the distribution defined by $p(R_{T+\hat{T}} \mid \Phi_T)$. Next, for every simulated realization the value of the integral's argument is calculated and finally the arithmetic mean of all values is obtained. The solution to the optimization problem is the portfolio choice that maximizes the integral's numeric value.

Therefore, we first need to know the predictive distribution of future returns. Excess returns are assumed to be generated by the following VAR model:

$$z_t = a + Bx_{t-1} + \epsilon_t; \tag{7}$$

where $z_t' = (r_{st}; r_{bt}; x_t')$; $x_t = (x_{1t}; \ldots; x_{nt})'$ and ϵ_t is identically and independently distributed according to $N(0; \Sigma)$. Vector x_t is the vector of variables that predict expected returns. Prior to Barberis (2000), a similar VAR model has been used in the context of return prediction by Campbell (1991), and Hodrick (1992). It is useful to rewrite (7) as:

$$z_t = a + B_0 z_{t-1} + \epsilon_t; \tag{8}$$

where:

$$B_0 = \left[\begin{pmatrix} 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix} \; B\right]: \tag{9}$$

The sum $Z_{T+\hat{T}} = z_{T+1} + z_{T+2} + \cdots + z_{T+\hat{T}}$ is distributed as a multivariate normal with mean μ_{sum} and variance Σ_{sum} given by:

$$\begin{aligned} \mu_{sum} &= \hat{T}a + (\hat{T} - 1)B_0 a + (\hat{T} - 2)B_0^2 a + \cdots + B_0^{\hat{T}-1} a \\ &\quad + (B_0 a + B_0^2 a + \cdots + B_0^{\hat{T}})z_T; \end{aligned} \tag{10}$$

$$\begin{aligned} \Sigma_{sum} &= \Sigma \\ &\quad + (I + B_0)\Sigma(I + B_0)' \\ &\quad + (I + B_0 + B_0^2)\Sigma(I + B_0 + B_0^2)' \\ &\quad + (I + B_0 + \cdots + B_0^{\hat{T}-1})\Sigma(I + B_0 + \cdots + B_0^{\hat{T}-1})': \end{aligned} \tag{11}$$

The investor could therefore estimate the parameters in (7) and take as the predictive distribution of returns a multivariate normal distribution with mean and variance $\hat{\mu}_{sum}$ and $\hat{\Sigma}_{sum}$; respectively. Simulating a large sample of returns, the investor could evaluate (6). This strategy however does not take into account the fact that the investor does not know the true values of parameters in (7), denoted by θ. On the other hand, if the investor decides to take estimation risk into account, he will take as the predictive distribution that resulting from integrating the joint distribution of future returns and the parameters in (7). Namely, the investor will obtain the probability density function for returns as:

$$p(R_{T+\tau} \mid \Phi_T) \equiv p(R_{T+\tau} \mid z) \begin{aligned} &= \int p(R_{T+\tau}, \theta \mid z) d\theta \\ &= \int p(R_{T+\tau} \mid \theta, z) p(\theta \mid z) d\theta; \end{aligned} \tag{12}$$

where $z = z_1, z_2, \ldots, z_T$. The probability density $p(R_{T+\tau} \mid \theta, z)$ corresponds to a multivariate normal distribution with mean and variance equal to μ_{sum} and Σ_{sum}. The density $p(\theta \mid z)$ can be obtained in a Bayesian fashion:

$$p(\theta \mid z) \propto p(z \mid \theta) p(\theta). \tag{13}$$

Intuitively, after observing the data set z; the investor learns the likelihood function $p(z \mid \theta)$ and will thus update his prior beliefs about the parameters –which can be summarized as the prior density $p(\theta)$– giving rise to the posterior density $p(\theta \mid z)$.

An investor who takes parameter uncertainty into account can therefore evaluate the following integral:

$$\int u(W_{T+\tau}) p(R_{T+\tau} \mid \theta, z) p(\theta \mid z) d\theta dR_{T+\tau}. \tag{14}$$

In practical terms, a realization of the vector θ is first drawn from the posterior distribution and next a possible realization of $R_{T+\tau}$ is simulated using the distribution of future returns conditional on those parameter values.

The posterior distribution $p(\theta \mid z)$ when $z = z_1, z_2, \ldots, z_T$ is obtained as follows. First, the model can be written as:

$$\begin{pmatrix} z_2' \\ \vdots \\ z_T' \end{pmatrix} = \begin{pmatrix} 1 & z_1' \\ \vdots & \vdots \\ 1 & z_{T-1}' \end{pmatrix} \begin{pmatrix} a' \\ B' \end{pmatrix} + \begin{pmatrix} \varepsilon_2' \\ \vdots \\ \varepsilon_T' \end{pmatrix} \tag{15}$$

or equivalently:

$$Z = XC + E; \tag{16}$$

where Z is a $(T-1, n+2)$ matrix with vector $z_2', \ldots, z_T'$ as columns; X is a $(T-1, n+1)$ matrix with vectors $(1\ x_1'), \ldots, (1\ x_{T-1}')$ as rows, and E is a

$(T - 1, n + 2)$ matrix with vectors $\epsilon_2', \ldots, \epsilon_T'$ as rows. Matrix C has dimension $(n + 1, n + 2)$ and contains row a' and matrix B'.

Barberis (2000) uses Zellner's (1971) Bayesian analysis of the multivariate regression model with exogenous regressors, given that the form of the likelihood function is the same when the distribution is conditioned on the sample's first observation z_1. As in Barberis, it will be assumed that the investor has no previous information about the value of θ and hence a standard diffuse probability density is used:

$$p(C, \Sigma) \propto |\Sigma|^{-\frac{n+2}{2}}.$$

The resulting posterior distribution $p(C, \Sigma^{-1} | z)$ is given by:

$$\begin{aligned} \Sigma^{-1} \mid z &\sim \text{Wishart}(T - n - 2, S^{-1}) \\ \text{vec}(C) \mid \Sigma, z &\sim N(\text{vec}(\hat{C}), \Sigma \otimes (X'X)^{-1}) \end{aligned}$$

where $S = (Z - X\hat{C})'(Z - X\hat{C})$ with $\hat{C} = (X'X)^{-1}X'Z$. Every simulated realization of (C, Σ) can be obtained by first drawing from the marginal distribution of Σ^{-1} and then drawing from the distribution of C conditioned on Σ.

In particular, the size of the simulated sample is 100,000 in the case of no parameter uncertainty and 10,000 when estimation risk is taken into account.

3 Data

The stock market return is proxied by the return on the value-weighted index of the New York Stock Exchange, and the return on long-term bonds is proxied by the return on a representative constant maturity portfolio of 10-year US Government Bonds. Data were obtained with a monthly frequency for the period covering January 1959 to December 1998. In order to compute excess returns the series of returns on one-month Treasury Bills was also collected.

In order to predict expected returns on bonds and stocks, four variables are selected:

1. The default spread, proxied by the yield difference between Moody's Baa and Aaa rated corporate bonds.
2. The dividend yield on the stock index, computed as the sum of dividends payed on the index over the previous 12 months divided by the current index level. It is henceforth denoted by DP.
3. The term spread, proxied by the yield difference between 10-year and one-year US Government Bonds.
4. The trend of the stock index, defined as the difference between the logarithm of the index's current level and the average level over the previous 12 months.

The default spread and the term spread were obtained from the Internet site maintained by the US Federal Reserve, whereas the rest of data were collected from the CRSP (Center for Research in Security Prices) database.

These predictors have been previously used by Fama and French (1988,1989) and Keim and Stambaugh (1986). Recently, Aït-Sahalia and Brandt (2001) have used them to extend Brandt's (1999) nonparametric optimal portfolio estimation to the case of multiple predictors.

If the investor performed an OLS regression of monthly excess stock and bond returns on the predictive variables in the previous month, he would obtain the following estimated coefficients and their associated p-values (in parentheses):

$$\begin{aligned} r_{st+1} = \; & \underset{(0.2181)}{-0.0077} \underset{(0.1608)}{+0.0066} \; x_{1t} \underset{(0.4257)}{+0.0007} \; x_{2t} \\ & \underset{(0.0071)}{+0.0054} \; x_{3t} \underset{(0.2291)}{-0.0004} \; x_{4t} + \varepsilon_{st+1} \end{aligned} \tag{17}$$

$$\begin{aligned} r_{bt+1} = \; & \underset{(0.4814)}{0.0002} \underset{(0.1333)}{+0.0038} \; x_{1t} \underset{(0.3288)}{-0.0008} \; x_{2t} \\ & \underset{(0.0140)}{+0.0024} \; x_{3t} \underset{(0.0010)}{-0.0009} \; x_{4t} + \varepsilon_{bt+1} \end{aligned} \tag{18}$$

where r_{st} and r_{bt} stand for continuously compounded excess returns on the stock and the 10-year bond indices respectively, and x_{it}; $i = 1; 2; 3$; and 4 denote the value in month t of the default spread, DP, the term spread, and the stock index trend.

In regression (17) only the term spread is significantly different from zero, whereas in regression (18) the term spread and especially the trend have a significant predictive power over next month's expected return. On the other hand, the R^2 coefficient equals 0.0211 in the case of stocks and 0.0319 for bonds. These results show the magnitude of the investor's problem of deciding whether he should consider the empirical evidence on predictability or whether he should think that evidence is too weak to take the regression estimated coefficients as the true model parameter values. The Bayesian approach can thus be seen as an intermediate solution between both extreme attitudes.

Tables 1 and 2 summarize the main descriptive statistics of the data set, and figure 1 plots the predictors.

4 Portfolio problem with a single risky asset

Suppose that the investor's problem amounts to simply choosing the fraction of his portfolio to be invested in the riskless asset and in a mutual fund that replicates returns on a risky asset (a stock or a 10-year bond index), so that he maximizes his expected utility at a given time in the future. The investor's problem is next analyzed in the context of Barberis (2000), namely with a single predicting variable and both from an empirical and theoretical perspective. Results are then presented for the case of multiple predictors when a portfolio of long-term bonds is the only risky asset available both with and without parameter uncertainty.

4.1 Single predictor

Suppose that the investor believes that there exists a single variable with predictive power over future monthly excess returns. In the presence of a single risky asset, the return generating model can be written as:

$$\begin{aligned} r_{t+1} &= \alpha + \beta x_t + \varepsilon_{1t+1} \\ x_{t+1} &= \gamma + \phi x_t + \varepsilon_{2t+1} \\ \begin{pmatrix} \varepsilon_{1t+1} \\ \varepsilon_{2t+1} \end{pmatrix} &\sim N\left(0, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}\right), \end{aligned} \tag{19}$$

where $r_t \equiv r_{st}$, and $x_t \equiv DP_t$ in the case of stocks and $r_t \equiv r_{bt}$, $x_t \equiv Trend_t$ for bonds. The fixed values of the model parameters in both cases are shown on table 4.3 and are obtained from the OLS estimation of (19).

Figure 2 shows the investor's optimal allocation to stocks when parameter uncertainty is not taken into account. A utility maximizing investor should allocate a larger fraction of his wealth to stocks the higher the value of DP and the longer the investment horizon. Both effects can only be attributed to predictability. If stock returns where i.i.d., the distribution of future returns would be independent of the current value of the predicting variable. Furthermore, in the i.i.d. case, both the mean and the variance of cumulative excess returns would grow linearly in the horizon so the ratio of both would be constant across all investment horizons.

The interesting question is why the optimal demand for stocks is increasing in the investment horizon.

Brennan, Schwartz and Lagnado (1997) and Barberis (2000) provide the same explanation: predictability induces mean reversion in monthly excess stock returns which in turn makes the variance of cumulative excess returns grow less than linearly as the investment horizon increases. Intuitively, a sudden shock to the stock return at a given period is most likely to happen when the predictor experiences a shock of opposite sign because $\sigma_{12} < 0$. Since $\beta > 0$; the shock in DP_t translates to r_{t+1} resulting in negative serial correlation in returns. As a consequence, the risk of investing in the stock grows less than linearly as the horizon increases which makes stocks more attractive as a long-term investment.

The above reasoning seems to suggest that negative serial correlation in returns justifies by itself a positive horizon effect as that found by Barberis and shown on figure 2. Accordingly, it would be reasonable to expect the demand for long-term bonds to also increase in the investment horizon when variable trend is the only predictor, since in this case $\sigma_{12} > 0$ and $\beta < 0$. However none of the previous conclusions is true. First of all, the fact that σ_{12} and β have different signs is not a sufficient condition for the variance of cumulative returns to be concave in the horizon, as shown below. Furthermore, the fact that the variance grows less than linearly with the investment horizon does not imply that the investor prefers to allocate more to stocks the longer the horizon. The reason is that the evolution of optimal demand with the horizon depends upon the joint variation of both the variance and the mean of cumulative returns.

The forthcoming paragraphs deal with the above questions by analyzing the dependence of the mean variance ratio of cumulative returns on the horizon in the context of model (19) and trying to answer the following questions:

1. What are the conditions under which the variance and the mean of cumulative returns are concave or convex in the investment horizon?

2. How can the answer to the previous question be exploited in order to analytically characterize the dependence of the mean variance ratio on the horizon?

First, the variance of the cumulative return between periods t and $t+n$ is analyzed. Applying the general result (11) to model (19) it follows that:

$$\begin{aligned} v_n \equiv \; & \mathrm{var}_t(R_{t+n}) = v_{n-1} + \sigma_1^2 + (\beta + \beta\phi + \cdots + \beta\phi^{n-2})\sigma_{12} + \\ & + (\sigma_{12} + (\beta + \beta\phi + \cdots + \beta\phi^{n-2})\sigma_2^2)(\beta + \beta\phi + \cdots + \beta\phi^{n-2}) \\ = \; & v_{n-1} + \sigma_1^2 + 2\beta\frac{1-\phi^{n-1}}{1-\phi}\sigma_{12} + \beta^2\left(\frac{1-\phi^{n-1}}{1-\phi}\right)^2\sigma_2^2. \end{aligned} \tag{20}$$

which holds for $n = 1, 2, \cdots, \infty$; with $v_0 = 0$. If r_{t+n} were unpredictable, i.e. $\beta = 0$; then $v_n = n\sigma_1^2$. The variance in that case would be linear and first-order homogeneous in n.

In order to know if one-period changes in the variance of the cumulative return are increasing or decreasing in the horizon, it is necessary to calculate the first derivative of $v_n - v_{n-1}$ with respect to n:

$$\begin{aligned} \frac{\partial(v_n - v_{n-1})}{\partial n} = \; & \frac{\partial\left(\sigma_1^2 + 2\beta\sigma_{12}\frac{1-\phi^{n-1}}{1-\phi} + \beta^2\sigma_2^2\left(\frac{1-\phi^{n-1}}{1-\phi}\right)^2\right)}{\partial n} \\ = \; & 2\beta\sigma_{12}\phi^{n-1}\frac{\ln\phi}{-1+\phi} + 2\beta^2\sigma_2^2\frac{1-\phi^{n-1}}{1-\phi}\phi^{n-1}\frac{\ln\phi}{-1+\phi} \end{aligned} \tag{21}$$

Under the assumption that the predictive variable is stationary, i.e., $|\phi| < 1$; it follows that the variance grows more than linearly in n if the following condition holds:

$$\beta\sigma_{12} > -\beta^2\sigma_2^2\frac{1-\phi^{n-1}}{1-\phi}. \tag{22}$$

Given that the right hand side of (22) is always negative, when β and σ_{12} have the same sign the above condition is always met. However, when β and σ_{12} have opposite signs, it is not possible to assert whether their joint effect is enough to make the variance grow less than linearly. In other words, negative serial correlation does not imply that v_n is concave in n.

On the other hand since $|\phi| < 1$, as the horizon grows to infinity one-period increments in the cumulative variance become constant:

$$\lim_{n \to \infty} (v_n - v_{n-1}) = \sigma_1^2 + \frac{2\beta\sigma_{12}}{1-\phi} + \frac{\beta^2\sigma_2^2}{(1-\phi)^2}. \tag{23}$$

It follows that in the limit the variance of cumulative returns becomes linear in the investment horizon.

As for the mean of excess cumulative returns, its growth between $n-1$ and n is simply the mean of the monthly return between periods $t+n-1$ and $t+n$:

$$\begin{aligned} m_n &\equiv E_t(R_{t+n}) = E_t(R_{t+n-1}) + E_t(r_{t+n}) \\ &= m_{n-1} + \alpha + \beta\gamma + \beta\gamma\phi + \beta\gamma\phi^2 + \cdots + \beta\gamma\phi^{n-2} + \beta\phi^{n-1}x_t. \\ &= m_{n-1} + \alpha + \beta\gamma\frac{1-\phi^{n-1}}{1-\phi} + \beta\phi^{n-1}x_t, \end{aligned} \tag{24}$$

which holds for $n = 1, 2, \cdots, \infty$, with $m_0 = 0$. Again, unpredictability implies that $m_n = nm_1$. Differentiating $m_n - m_{n-1}$ with respect to n gives:

$$\frac{\partial(m_n - m_{n-1})}{\partial n} = \frac{\partial\left(\alpha + \beta\gamma\frac{1-\phi^{n-1}}{1-\phi} + \beta\phi^{n-1}x_t\right)}{\partial n} \tag{25}$$

$$= \beta\phi^{n-1}\frac{\ln\phi}{-1+\phi}(\gamma - x_t(1-\phi)). \tag{26}$$

So, when $\beta > 0$, the mean of cumulative returns m_n grows more than linearly with n if:

$$x_t < \frac{\gamma}{1-\phi}, \tag{27}$$

i.e., if the predictor's initial value is below its long-run mean, and less than linearly otherwise.

Note that the effect of the predictor's initial value on $\frac{\partial(m_n - m_{n-1})}{\partial n}$ has an opposite sign to that on $E_t(r_{t+n})$. As a consequence, when x_t affects $E_t(r_{t+n})$ positively, its increments decrease as the horizon becomes longer if the predictor is initially above its long-run mean. Intuitively, when the predicting variable is stationary, the influence of the predictor's initial value on each period's return decays with time. The larger the initial effect of the predicting variable, the larger the loss in the cumulative expected return as n increases.

Taking limits in $m_n - m_{n-1}$; it follows that:

$$\lim_{n \to \infty} (m_n - m_{n-1}) = \alpha + \beta \frac{\gamma}{1-\phi}\,; \tag{28}$$

so just as happens with the variance, as the horizon grows to infinity the mean of cumulative returns grows linearly in n.

Conditions (22) or (27), and (??) determine whether the mean and variance of the cumulative return are linear functions in the investment horizon or whether they exhibit concavity or convexity. Next question is what conclusions can be drawn from the previous analysis regarding the evolution of the mean variance ratio with the investment horizon.

Between two different horizons n_1 and n_2; the mean variance ratio grows if the mean increases more than the variance in relative terms. For the case in which $m_{n_1} > 0$:

$$\frac{m_{n_2}}{v_{n_2}} > \frac{m_{n_1}}{v_{n_1}} \iff \frac{m_{n_2}}{m_{n_1}} > \frac{v_{n_2}}{v_{n_1}}: \tag{29}$$

If m's homogeneity order in n is higher than one and v's is lower than one, then:

$$\frac{m_{kn}}{m_n} > k > \frac{v_{kn}}{v_n}; \tag{30}$$

in that case the mean variance ratio is always increasing in n.

On the other hand, if m is a convex function in n and v is concave, then:

$$\frac{m_{kn} - m_0}{m_n - m_0} > k > \frac{v_{kn} - v_0}{v_n - v_0}: \tag{31}$$

Since $m_0 = v_0 = 0$; it can be concluded that the mean variance ratio is increasing in n when the mean grows more than linearly in n and the variance grows less than linearly. The converse reasoning could be used to show that when m is concave and v is convex in n, the ratio is decreasing in the horizon. Of course, the final effect when both the mean and variance are either concave or convex in n is ambiguous.

Finally, note that in the limit the mean variance ratio is constant and the horizon effect on the optimal demand vanishes. Of course, this result is only true when $|\phi| < 1$:

Consider next the static portfolio choice problem when the long-term bond portfolio is the only risky investable asset. Suppose that the investor believes that the return on that portfolio evolves according to model (19) and thinks that the true parameter values are those from table 3. Optimal asset allocation must be higher for a lower initial value of the predictor since $\beta < 0$. According to condition (22) it is possible to verify that v grows less than linearly with the investment horizon. Additionally, condition (??) does not hold when the predictor's initial value equals ¡ 1 since ° > 0; so in that case the mean grows less than linearly. When the trend's initial value equals 2, the mean is almost linear in n; and …nally for an initial value of 5 condition (??) is met, so the mean of cumulative returns grows more than linearly.

Figure 3 shows graphically the results from solving the investor' problem of allocating his investment between the risk free asset and the bond portfolio . When trend initially equals ¡ 1; the investor's position in the 10-year bond portfolio is larger than when the trend's initial value equals 2 and double his optimal position when it equals 5. This is simply a consequence of ¯ being negative. More interestingly, the predictor's initial value not only a¤ects the level of the investor's optimal demand but it also determines the sign of the horizon e¤ect. In particular, optimal demand decreases when trend is initially ¡ 1; and increases when trend equals 5. This result is consistent with the theoretical analysis. Since ¯ < 0; increases in $E_t(r_{t+n})$ are growing in the investment horizon when the predictor's initial value is above its long-run mean, and decreasing otherwise. This e¤ect dominates that on the cumulative variance, so the horizon e¤ect can be either increasing or decreasing depending on the predictor's initial value. This is in striking contrast to what happens as the investment horizon increases when investment in stocks is considered. In that case, given the parameter estimates the horizon e¤ect on the variance is dominant and makes the optimal demand for stocks to be increasing in the investment horizon regardless of the predictor's initial value.

4.2 General Model

Next, a general model that includes both stocks and long-term bonds as well as the four predicting variables is estimated and the investor's portfolio problem is solved when the long-term bond portfolio is the only risky investable asset. When the investor does not recognize estimation risk, it will be assumed that he takes as the true parameter values the means of C and § from the posterior distribution given by p(C; § j z); as shown on tables 4.4 and 4.5.

Figures 4, 5, 6, and 7 display the solution the investor's problem when returns are assumed to be generated by a general model with four predicting variables with parameter values as shown on tables 4 and 5. Each graph plots the solution to the maximization problem for di¤erent investment horizons and three di¤erent cases that correspond to three di¤erent initial values of one particular predictor. The values considered are the 25th sample percentile, the sample, and the 75th sample percentile of the predicting variable chosen in each case. The initial values for the rest of variables are taken to be their sample means. This way, it is possible to study the isolated e¤ect of the initial value of each predictor on the evolution of the demand with the horizon. In each case a similar pattern is found: when the coe¢cient in the regression equation for a given variable is positive, optimal demand is larger but decreasing in the investment horizon for initial values above the predictor's sample mean, and smaller but increasing for lower initial values. On the other hand, when the regression coe¢cient is negative, the situation is inverted. Consistently with the theoretical analysis, the horizon e¤ect disappears at long horizons. As opposed to the case when only stocks are available, horizon e¤ects arise only when the initial values of the predicting variables depart from their historical average.

When the in‡uence of the trend alone is considered (…gure 7), an investor with a relative risk aversion coe¢cient of 5 doubles his allocation into bonds when his investment horizon increases from 2.5 years to 10 years if the trend's initial value is high. Interestingly, including multiple predictors in the model does not alter the conclusions drawn from the single predictor case as long as the rest of variables are initially about their historical means. It thus would seem that, at least for the case of Government Bonds, model risk is negligible as long as the investor considers the only predictor whose initial value departs from its sample average. On the other hand, …gure 5 shows that optimal demand lines converge less rapidly for di¤erent initial values of DP. This is explained by the high persistence in the dividend yield series, which makes the in‡uence of the predictor's initial value last for a long number of periods. As a consequence, important di¤erences in the investor's position due to the initial value persist even when the investment horizon is 10 years apart.

When the investor acknowledges that the true values of the model parameters may depart from those on tables 3 and 4, he must incorporate estimation risk into his portfolio decision problem. Figure 8 shows the investor's optimal position in long-term bonds when parameter uncertainty is taken into account. Just as happens with stocks (Barberis (2000)), introducing estimation risk increases dispersion in the possible realizations of the future cumulative return which makes the risky asset appear riskier to the long-term investor's eyes. However, the investor's position as well as its evolution with the horizon is affected as dramatically as in the case studied by Barberis. In fact, estimation risk is higher for stocks than for Government Bonds as can be seen from the R^2 coe¢cients obtained in the predictive regressions. Besides, it is consistent with Elton's (1999) argument that estimation of future expected returns from past realized returns are better for Government bonds than for other assets since Government Bonds contain little speci…c information that can a¤ect their price. Elton provides …ve reasons to support his argument: the factors that a¤ect Government Bond prices can be found in the form of aggregated economic information; there exists wide consensus on what economic variables should a¤ect bond prices; potentially important variables a¤ect all agent; the moment when information about these variables is known and …xed; and …nally, the impact of the unexpected component of these announcements is rapidly incorporated into prices.

5 Dynamic allocation

Barberis (2000) observes that as the predicting variable changes over time, it also changes the investment opportunity set faced by the investor, namely it changes the investor's expected future return. Since shocks to the dividend yield are highly negatively correlated with shocks to stock returns, then higher realized returns will occur when the dividend yield –and hence future expected return– is relatively low. Therefore, allocating an extra amount to stock returns can be a good way to hedge periods of poor investment opportunities. Solving the Bellman equation numerically by backward induction, Barberis is able to characterize the solution to the asset allocation problem of an investor who rebalances his portfolio every year when the stock index is the only investable risky asset. Barberis …nds that Merton's intertemporal hedging demand for stocks is positive and increasing in the investment horizon. Does this result extend to the case of long-term Government bonds?

Note from table 3, that shocks to bond returns most likely occur when the predicting variable –trend– is a¤ected by shocks of the same sign. Since a higher current level of the predictor implies a lower future expected return, long-term bonds in principle appear to be a good way to hedge adverse changes in the investment opportunity set. Figure 9 shows optimal portfolio weights of an investor who believes that bond returns are predicted by current trend levels and who rebalances his portfolio every month. It seems clear that contrary to the optimal hedging demand for stocks, in the absence of in‡ation risk, the investor's hedging demand for long-term bonds is zero for all investment horizons and initial values of the predictor[5]. Although not reported, results for the rest of predictors do not alter this conclusion.

6 Portfolio choice with two risky assets

This section considers the problem of an investor who can invest in the riskless asset as well as in the stock index and the 10-year bond portfolio. Figure 10 summarizes the results graphically for two cases: when the trend's initial value is at its 25th sample percentile and when it is at its 75th sample percentile. Graphs show that the horizon e¤ect on the investor's position in long-term bonds extends to the two risky asset case, namely, the investor increases his position in bonds with the horizon if the trend's initial value is unusually high and viceversa. However, comparing …gure 10 with …gure 7, it can be veri…ed that the horizon e¤ect on the bond demand in the presence of stocks does not exhibit the same degree of symmetry. Since the appeal of stocks increases so rapidly with the investment horizon, the investor's position in bonds decreases more when the trend's initial value is low and increases less when the trend's initial value is high than when only bonds are available. A similar observation can be made when estimation risk in‡uences the investor's decision (…gure 11). In this case, however, estimation risk severely penalizes the investor's position

[5] The approach employed for solving Bellman's equation recursively is basically that proposed by Barberis except for the fact that splines have been used to interpolate between consecutive elements of the discretized state space. The advantage is that fewer design points are needed and hence precious computing time can be used to increase the rebalancing frequency.

in stocks, so the negative in‡uence on the horizon e¤ect of his position in bonds is more likely a result of parameter uncertainty alone.

Changes in the trend's initial value a¤ect expected returns on stocks and bonds in the same direction. In the following paragraphs, changes in the dividend yield's initial value are considered since this predictor's e¤ect is opposite for each asset's future returns.

The upper graph of …gure 12 corresponds to the case when DP is initially at its 25th sample percentile, the central graph corresponds to a situation in which all predictors initially take on their mean values, and …nally the lower graph shows the evolution of the investor's portfolio with the horizon when DP is initially at its 75th sample percentile. Two main conclusions can be drawn from …gure 12.

First, as the investment horizon increases, the investor increases his position in stocks and decreases the portfolio weight in the riskless asset. Changes in the predictor's initial value do not substantially alter the horizon e¤ect on the investment in stocks or the levels of investment for every horizon. However, a higher initial value of DP decreases the investor's position in long-term bonds and decreases the negative horizon e¤ect.

Second, it is interesting to note that predictability makes asset substitution –as a response to changes in the predictor's initial value– depend on the investment horizon. In particular, when the investment horizon is short, a higher initial value of the DP induces the investor to decrease his position in bonds and increase his position in the riskless asset. On the other hand, when the investment horizon is long, when DP increases, the investor allocates less to bonds and more to stocks.

Figure 13 shows that under parameter uncertainty the investor's position in stocks is practically constant in the investment horizon for any initial value of the predicting variable. As for bonds, under parameter uncertainty there is a negative horizon e¤ect for any of the predictor's initial values. As a consequence, as the horizon increases, the investor always increases his position in the riskless asset at the expense of long-term bonds. Comparing with the previous …gure, for long horizons introducing estimation risk has a dramatic impact on the position in stocks: when predictors are initially at their mean levels, the investor's optimal position in stocks falls by half. Again, the impact of estimation risk on the weight in bonds is less evident.

Another interesting question is how optimal asset allocation varies with the investor's degree of risk aversion for a …xed investment horizon. Figure 14 shows results for this analysis. Just as in Campbell and Viceira (1988) the demand for long-term bonds arises when the degree of risk aversion surpasses a given level and from that level on it slowly decreases as the risk aversion coe¢ cient becomes larger. Besides, a more risk averse investor allocates less resources to stocks and more to the riskless asset. It must be noted that the investor's optimal position in bonds is sensibly higher than the "myopic" demand found by Campbell and Viceira (1998). Given the small horizon e¤ect detected for long-term bonds in the presence of stocks, this di¤erence cannot be attributed to the horizon e¤ect, but rather to the absence of in‡ation risk in the present analysis.

Consistently with mean variance analysis, the fraction of 10-year bonds in the risky asset portfolio is stable for different degrees of risk aversion (31.5 per cent approximately), except for, of course, when the short sale restriction is effective. When the investor takes estimation risk into account (lower graph) although returns are no longer normally distributed, the fraction of the risky portfolio invested in long-term bonds is constant for all risk aversion coefficients considered and equals 1=3. Since estimation risk increases investment in the riskless asset, in terms of proportion relative to the total portfolio, parameter uncertainty has a larger impact on the investor's position in stocks than on the investor's allocation to bonds. For instance, when the risk aversion coefficient equals 10, stocks capture 47.5 per cent of the total portfolio without estimation risk and 32.5 percent with estimation risk, whereas investment in bonds reduces from 22.5 per cent to 17.5 per cent of the total portfolio.

7 Conclusions

This study addresses for the first time the question of horizon effect on the demand for Government debt when returns are predictable. It is well known that just as happens with stocks, bond returns also exhibit negative serial correlation. If negative serial correlation were a sufficient condition for the positive horizon effect found in stocks, then it would be reasonable to expect the same effect on the demand for long-term bonds. Numerical results however do not support this hypothesis. Horizon effects can either be positive or negative depending on whether the predictor's initial value is lower (higher) or higher (lower) than its historical mean when the regression coefficient is positive (negative). In the very long term horizon effects disappear, just as happens when the predictors' initial values are around their sample means. These results are supported by a theoretical analysis of the mean variance ration of cumulative returns for the case of a single risky asset and a single predictor. Intuitively, the influence (positive or negative) of the predictor's initial value tends to decay with time when the explaining variable is stationary. Stationarity thus induces optimal demands for different initial values of the predictor to converge in the limit whenever the mean horizon effect dominates that on the variance. This may thus imply both positive and negative horizon effects.

Introducing parameter uncertainty in the investor's problem decreases both the level of investment for every horizon as well as its growth as the horizon increases. Estimation risk seems to have a larger impact on the demand for stocks than on that for Government Bonds.

The dynamic allocation problem shows that optimal hedging demands for long-term bonds are negligible at all investment horizons.

Finally, optimal asset allocation when both risky assets are available seems to be highly consistent with the single risky asset case.

The results of the analysis have consequences both in terms of further approximating to the effect of predictability on optimal portfolio choice, as well as in more practical terms given the weight of investment by mutual funds in

…xed income assets.

An interesting extension, already pointed out by Kandel and Stambaugh (1996) is the problem of incorporating conditional heteroscedasticity in the model, which is likely to a¤ect the way predictability a¤ects optimal portfolio choice.

On the other hand, and in the context of parameter uncertainty, the use of informative prior probability density functions would enable us to see how optimal asset allocation varies for di¤erent degrees of con…dence in di¤erent models. Related to this point, Avramov (1999) has studied model risk in portfolio choice with stock portfolios.

Finally, it should be remembered that the implications of optimal portfolio choice under predictability in a general equilibrium context are yet to be explored.

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Tables

Table 1. Returns and predictors.
This table shows descriptive statistics of the return on the New York Stock Exchange index, the return on the 10-year US Government Bonds, the return on the one-month Treasury Bills, the default spread, the dividend yield on the stock index, the term spread and the stock index trend. Returns and observation frequency are monthly. Data cover the period from January 1959 to December 1998. The sample size is 468 since 12 observations are lost.

	Mean	Median	St. Dev.	Assyimetry	Kurtosis	Min	Max
Stocks	0.009	0.012	0.043	-0.709	3.256	-0.246	0.152
10-yr. Bonds	0.006	0.003	0.022	0.382	1.386	-0.069	0.095
1-month Bill	0.005	0.004	0.002	1.295	2.324	0.000	0.015
Def. Spread	0.997	0.850	0.450	1.246	1.404	0.32	2.690
DP	3.480	3.361	0.881	0.368	-0.124	1.544	6.123
Term Spread	0.696	0.738	0.970	-0.117	0.198	-2.686	3.135
Trend	1.712	2.295	3.961	-0.810	1.106	-16.434	11.012

Table 2. Correlations.
This table shows correlation coe¢cients in the sample between the variables (panel A), as well as the serial autocorrelation coe¢cients for 1, 3, 6, and 12 lags (panel B).

Panel A

	Bonds	Def. Spread	DP	Term Spread	Trend.
Stocks	0.2803	0.0979	-0.1313	0.1515	0.4621
Bonds	1	0.1129	-0.0032	0.1336	0.0211
Def. Spread		1	0.6635	0.0932	0.0617
DP			1	-0.1800	-0.3314
Term Spread				1	0.2277
Trend					1

Panel B

	½$_1$	½$_3$	½$_6$	½$_{12}$
Stocks	0.0601	-0.0027	-0.0662	0.0217
10-year Bonds	0.0755	-0.0685	0.0670	0.0096
Def. Spread	0.9720	0.9055	0.8287	0.6813
DP	0.9812	0.9439	0.8812	0.7720
Term Spread	0.9609	0.8441	0.7189	0.5348
Trend	0.8783	0.6180	0.2483	-0.1530

Table 3. Single predictor VAR parameters.
Parameter estimates on this table correspond to model:

$$r_{t+1} = \alpha + \beta x_t + \varepsilon_{1t+1}$$
$$x_{t+1} = \gamma + \phi x_t + \varepsilon_{2t+1}$$
$$\begin{pmatrix} \varepsilon_{1t+1} \\ \varepsilon_{2t+1} \end{pmatrix} \sim N\left(0; \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}\right),$$

where r_t is the continuously compunded monthly excess return on the risky asset (stocks or 10-year bonds) and x_t is the predictive variable (DP or Trend, respectively).

Regression coefficients

	Constant	DP
Stocks	-0.0044	0.0025
DP	0.0477	0.9862

	Constant	Trend
10-year Bonds	0.0025	-0.0007
Trend	0.2121	0.8787

Covariance of residuals.

	Stocks.	DP
Stocks	0.0019	-0.9296*
DP		0.0290

	10-year Bonds	Trend
10-year Bonds	0.0005	0.2690*
Trend		3.5932

(Values marked with * are correlation coefficients).

Table 4.4. Posterior mean of C.

	Constant	Def. spread	DP	Term Spread	Trend
Stocks	-0.0076	0.0066	0.0007	0.0053	-0.0004
10-year Bonds	0.0003	0.0038	-0.0008	0.0024	-0.0009
Def. spread	-0.0039	0.9499	0.0208	-0.0142	-0.0047
DP	0.0475	-0.0462	1.0014	-0.0195	0.0022
Term spread	0.0452	0.1522	-0.0416	0.9581	-0.0126
Trend	-0.5764	0.3902	0.0791	0.1989	0.8705

Table 4.5. Posterior mean of matrix §.
Values above the main diagonal are correlation coe¢cients.

	Stocks	Bonds	Def. spread	DP	Term spread	Trend.
Stocks	0.0019	0.2695	0.0783	-0.9379	0.1399	0.9903
Bonds		0.0005	0.2485	-0.2827	0.1620	0.2624
Def. spread			0.0102	-0.0990	0.1634	0.0723
DP				0.0289	-0.1935	-0.9325
Term spread					0.0704	0.1318
Trend						3.5845

Figures

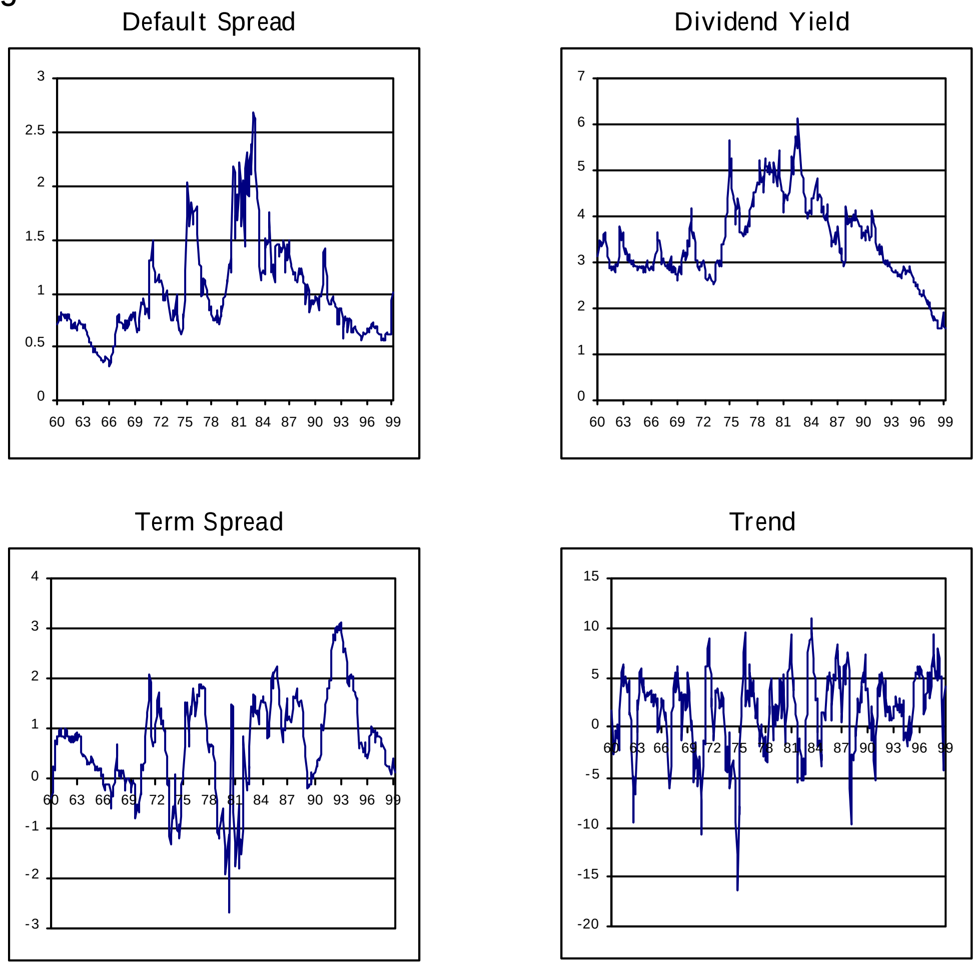


Figure 1. Predictors. This …gure shows the serial plot of the four variables chosen as predictors of the returns on the stock index and the 10-year bond index. The frequency of observation is monthly and data cover the period from January 1959 to December 1998. The sample size is 468 since 12 observations are lost.

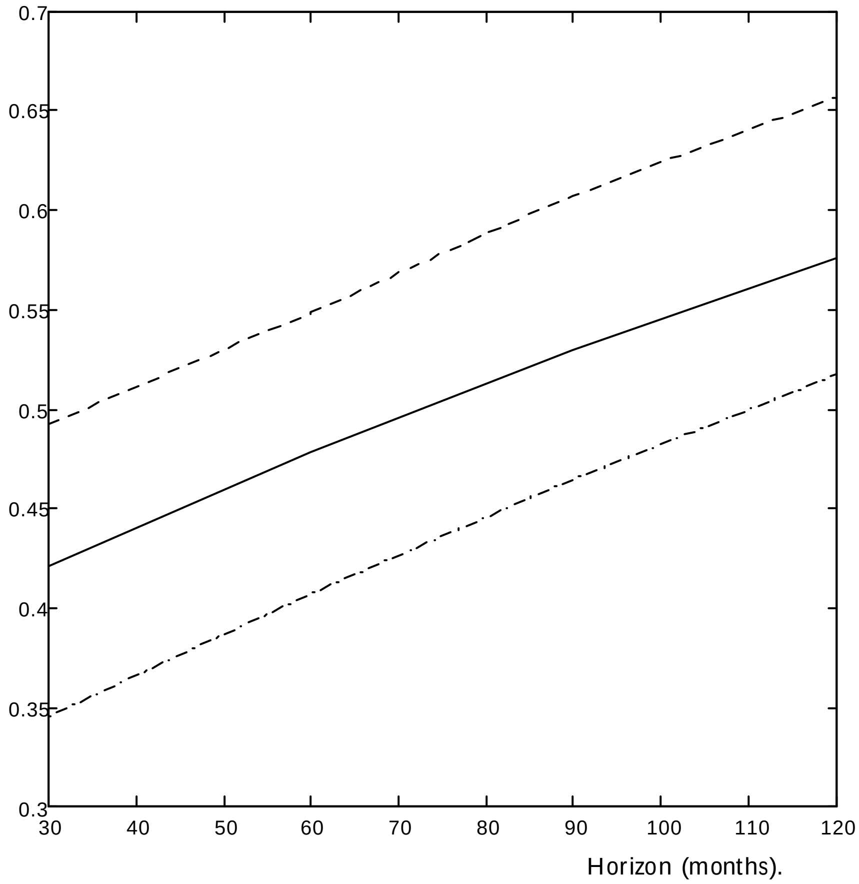


Figure 2. Allocation to stocks in the absence of Government Bonds as a function of the investment horizon when dividend yield is the only predictive variable. The graph plots the solution to the investor's static portfolio choice problem when the investment opportunity set contains the riskless asset and a stock index whose excess return is assumed to be generated by a VAR model that includes the dividend yield as the single predictor. The dash-dot line corresponds to an initial value of DP equal to 3.5, the solid line to 4, and the dash line 4.5.

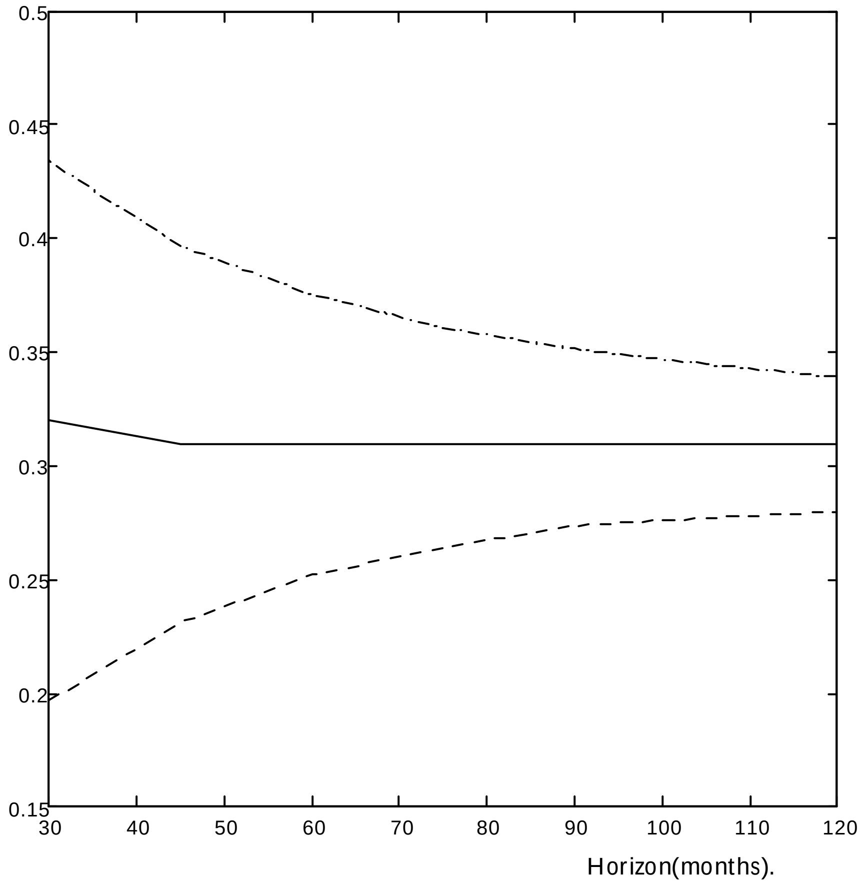


Figure 3. Allocation to long-term bonds in the absence of stocks as a function of the investment horizon when trend is the only predictor. The graph plots the solution to the investor's static portfolio choice when the investment opportunity set only contains the riskless asset and a 10-year bond portfolio whose excess return is assumed to be generated by a VAR model that includes trend as the single predictor. The dash-dot line corresponds to an initial value of trend equal to -1, the solid line corresponds to 2, and the dash line corresponds to 5. The risk aversion coe¢cient is 10.

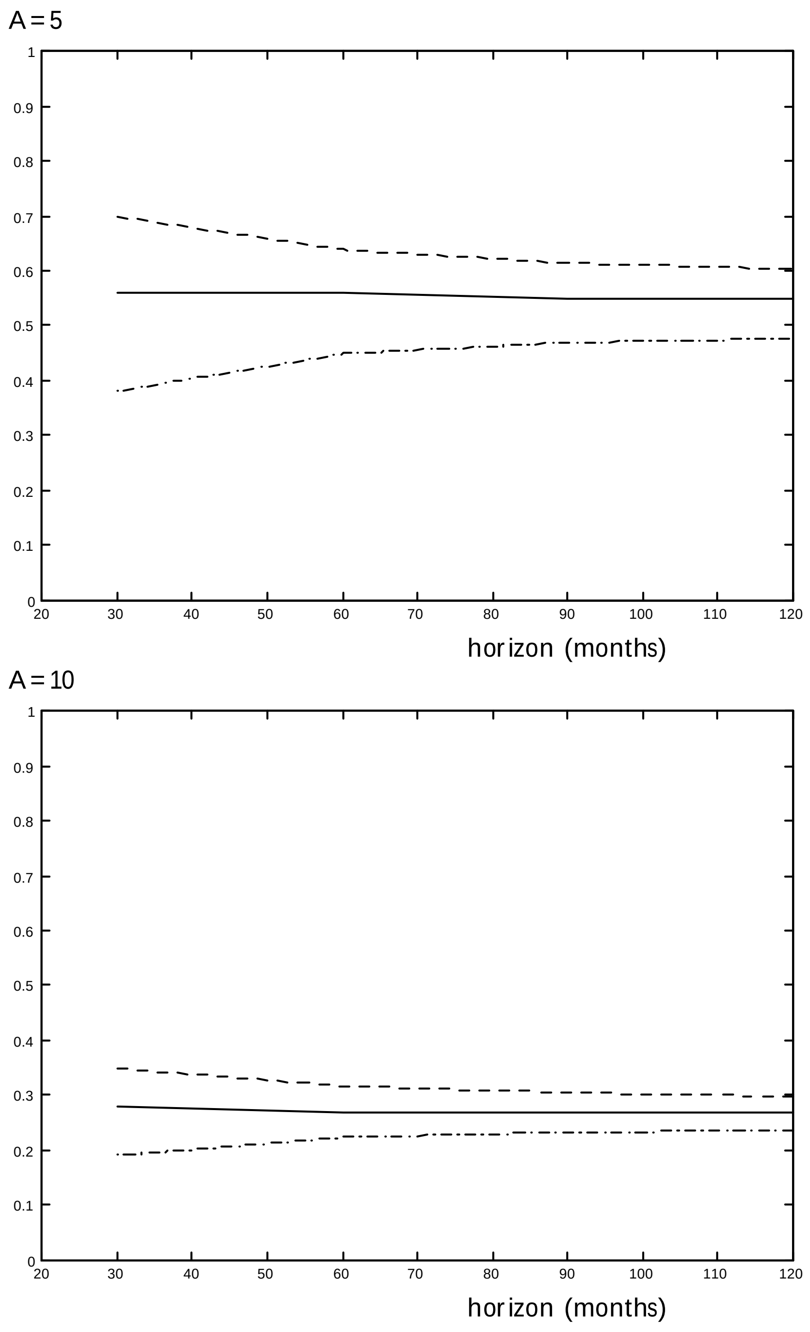


Figure 4. Allocation to bonds as a function of the investment horizon and the default spread. The graph plots for di¤erent horizons the allocation to 10-year bonds in the absence of stocks when returns are assumed to be generated by a VAR model with four predictors. The initial values of the default spread are 0.69 (dash-dot line), 0.99 (solid line), and 1.21 (dash line). In every graph, the initial values for the rest of variables are their sample means.

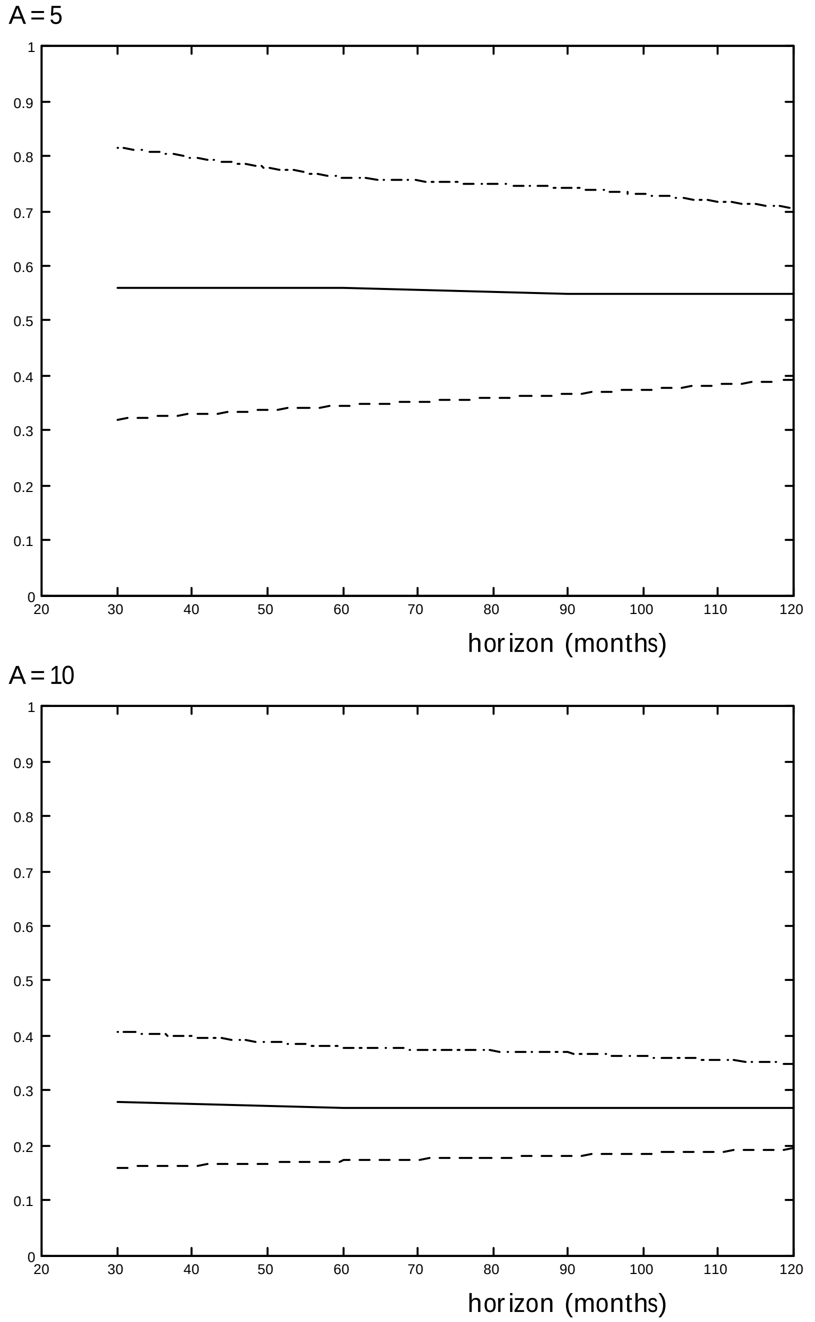


Figure 5. Allocation to bonds as a function of the investment horizon and the dividend yield. The graph plots for different horizons the allocation to 10-year bonds in the absence of stocks when returns are assumed to be generated by a VAR model with four predictors. The initial values of the dividend yield are 2.88 (dash-dot line), 3.48 (solid line), and 4.07 (dash line). In every graph, the initial values for the rest of variables are their sample means.

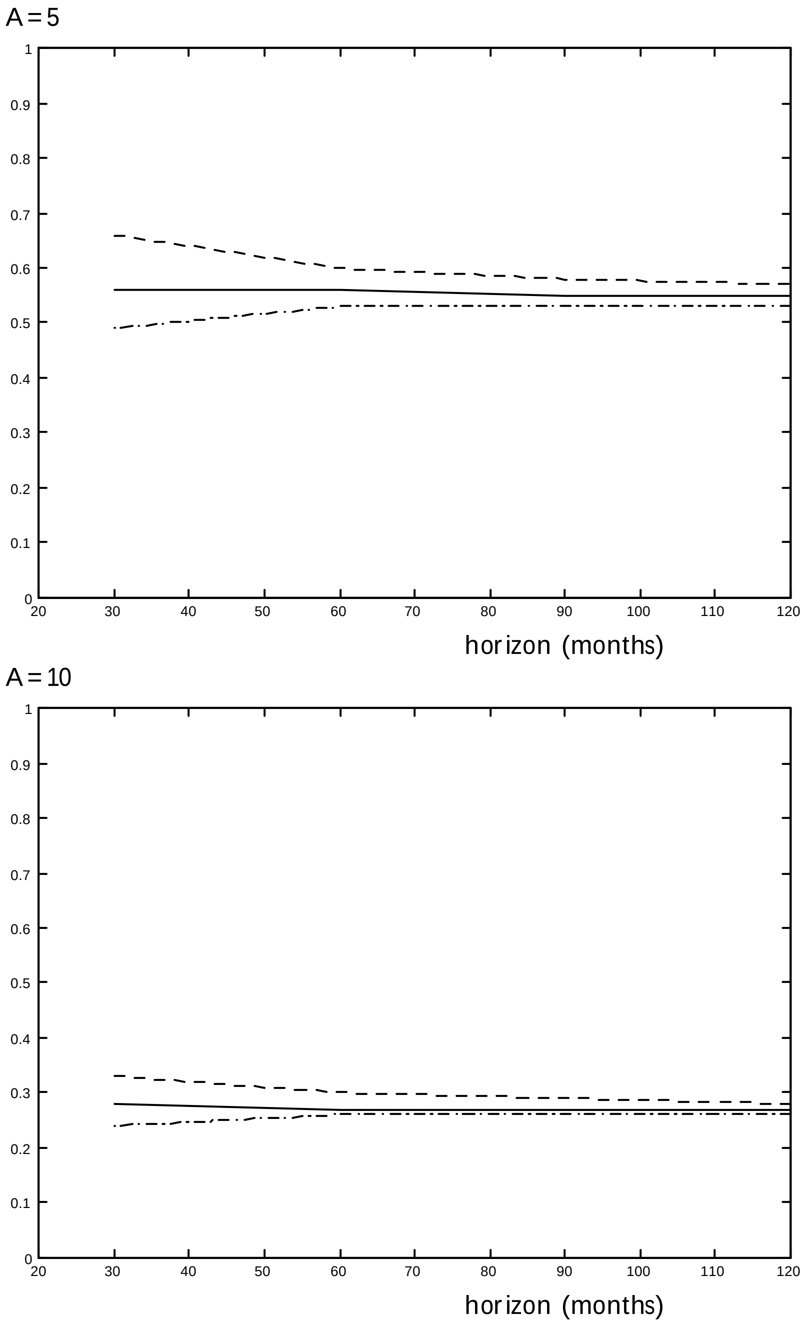
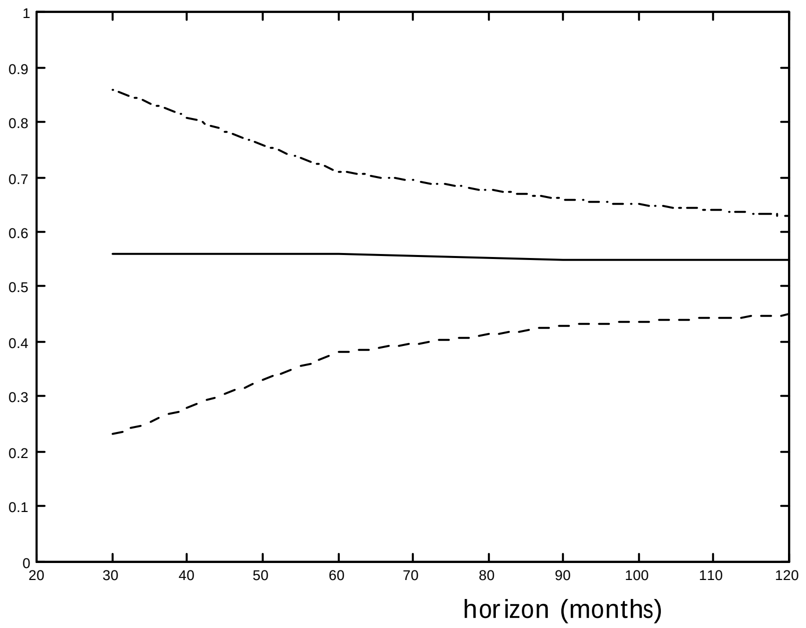


Figure 6. Allocation to bonds as a function of the investment horizon and the term spread. The graph plots for di¤erent horizons the allocation to 10-year bonds in the absence of stocks when returns are assumed to be generated by a VAR model with four predictors. The initial values of the term spread are 0.09 (dash-dot line), 0.69 (solid line), and 1.39 (dash line). In every graph, the initial values for the rest of variables are their sample means.

A = 5



A = 10

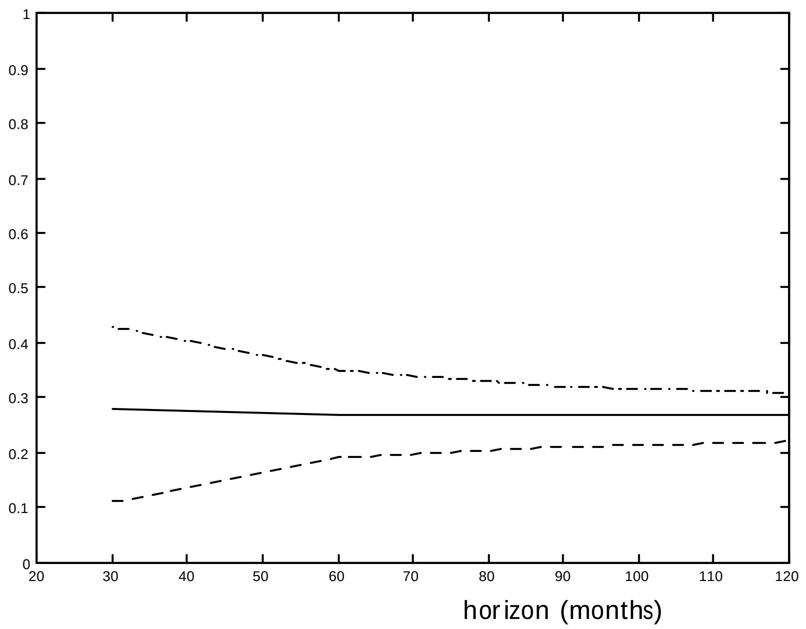


Figure 7. Allocation to bonds as a function of the investment horizon and the trend. The graph plots for di¤erent horizons the allocation to 10-year bonds in the absence of stocks when returns are assumed to be generated by a VAR model with four predictors. The initial values of the term spread are -0.7 (dash-dot line), 1.71 (solid line), and 4.5 (dash line). In every graph, the initial values for the rest of variables are their sample means.

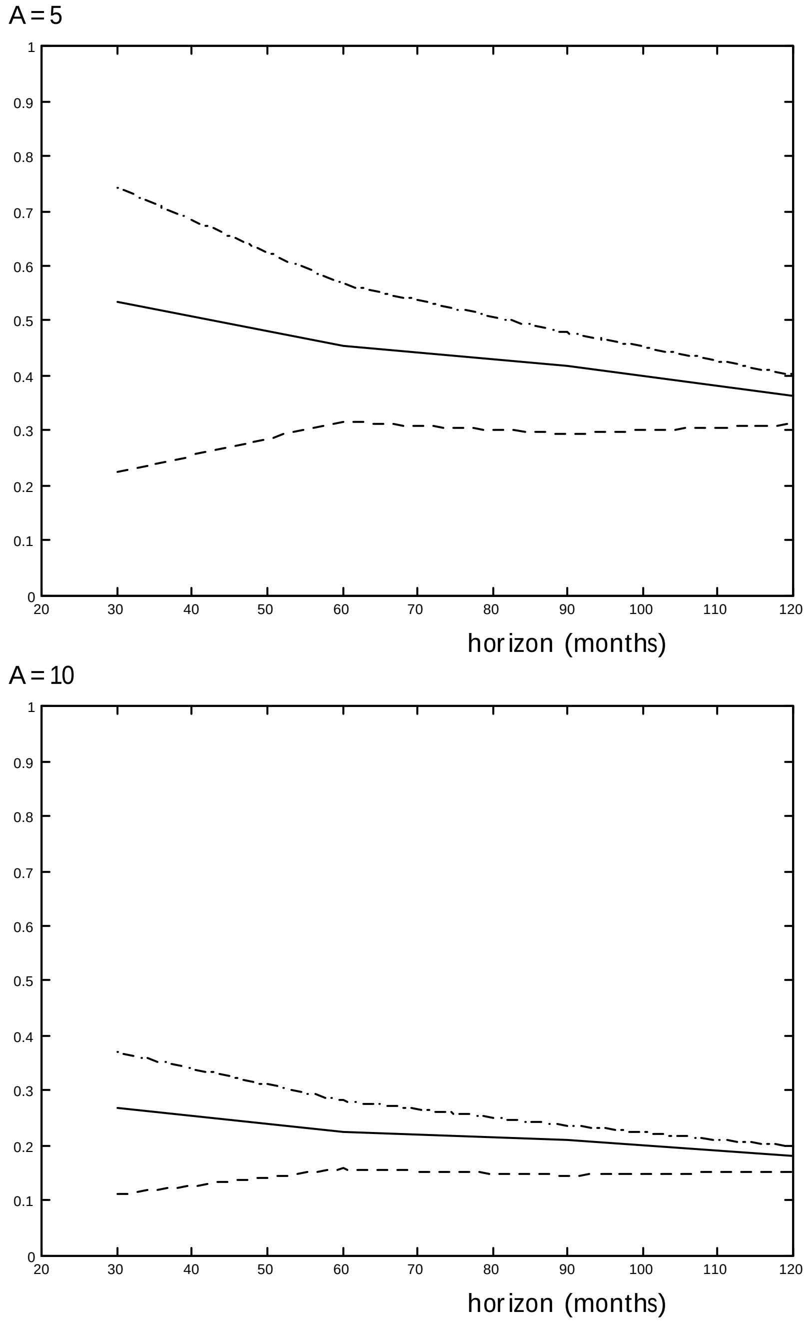
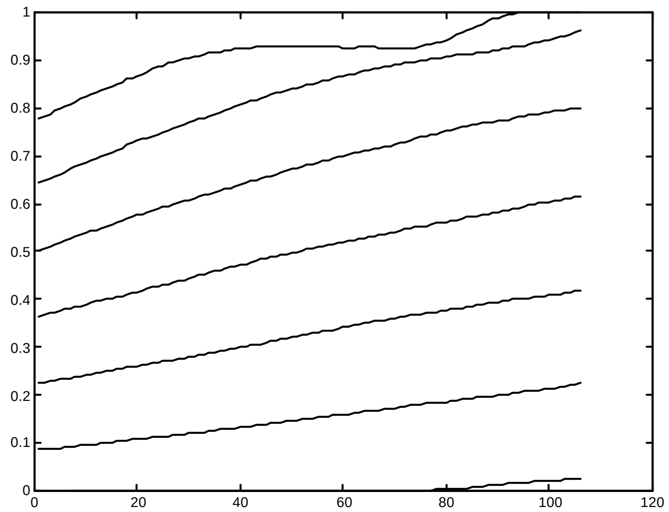


Figure 8. Allocation to bonds as a function of the investment horizon and the trend under parameter uncertainty. The graph plots for different horizons the allocation to 10-year bonds in the absence of stocks when returns are assumed to be generated by a VAR model with four predictors. The initial values of the term spread are -0.7 (dash-dot line), 1.71 (solid line), and 4.5 (dash line). In every graph, the initial values for the rest of variables are their sample means. The investor takes estimation risk into account.

Stocks



Long-Term Bonds

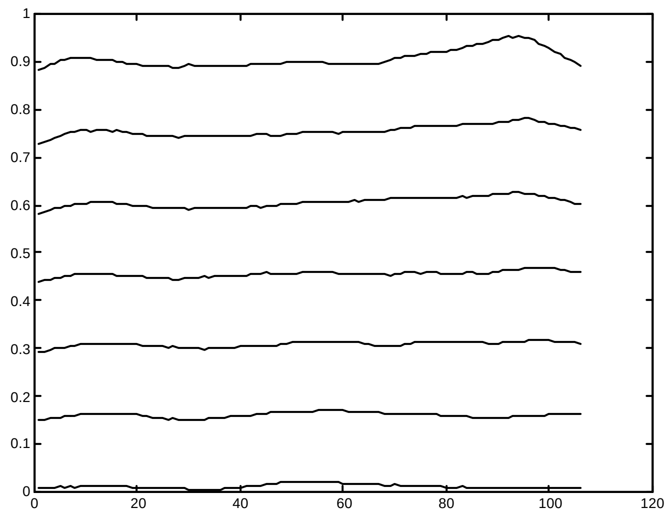


Figure 9. Dynamic allocation to stocks and bonds as a function of the investment horizon, and the dividend yield and trend respectively. The top graph plots for di¤erent horizons the allocation to stocks when returns are assumed to be predicted by the dividend yield. The initial values of the dividend yield equal 1, 2, ¢¢¢, 7 (from bottom to top). The bottom graph plots for di¤erent horizons the allocation to stocks when returns are assumed to be predicted by the trend. The initial values of the trend equal -2, -1, ¢¢¢, 3 (from top to bottom).

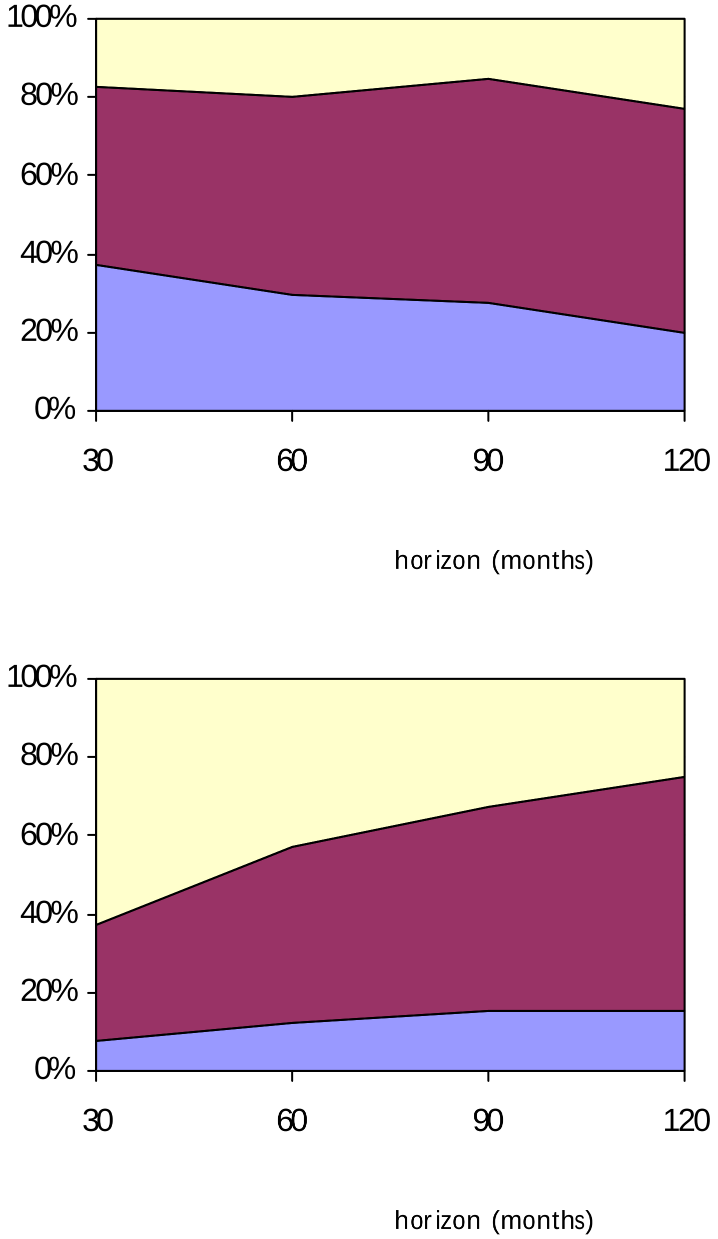


Figure 10. Allocation to bonds and stocks for different initial values of trend. The graph plots the static allocation to the riskless asset (light shade) 10-year bonds (medium shade) and stocks (dark shade) for different horizons when returns are assumed to be generated by a VAR model with four predictors. The initial values of trend are -0.7 (upper graph) and 4.5 (lower graph). The rest of initial variables are initially at their mean levels. The investor does not take parameter uncertainty into account. The risk aversion coefficient is 10.

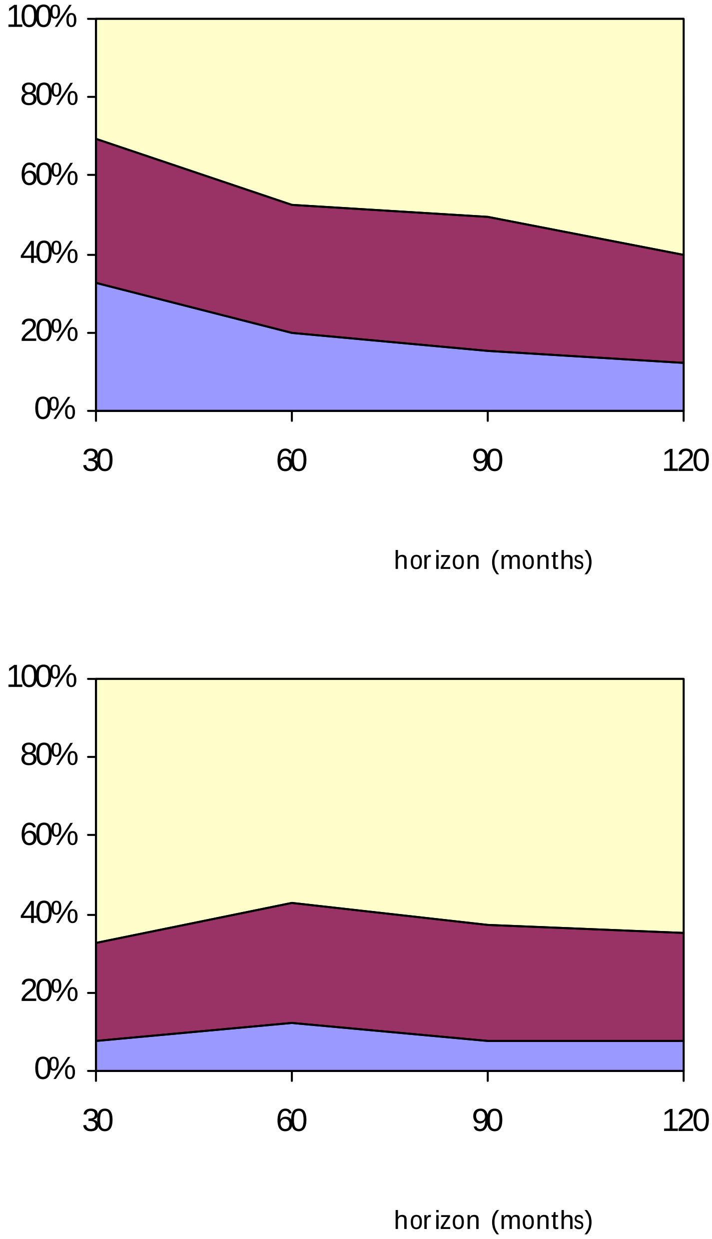


Figure 11. Allocation to bonds and stocks for different initial values of trend under parameter uncertainty. The graph plots the static allocation to the riskless asset (light shade) 10-year bonds (medium shade) and stocks (dark shade) for different horizons when returns are assumed to be generated by a VAR model with four predictors. The initial values of trend are -0.7 (upper graph) and 4.5 (lower graph). The rest of variables are initially at their mean levels. The investor takes parameter uncertainty into account. The risk aversion coefficient is 10.

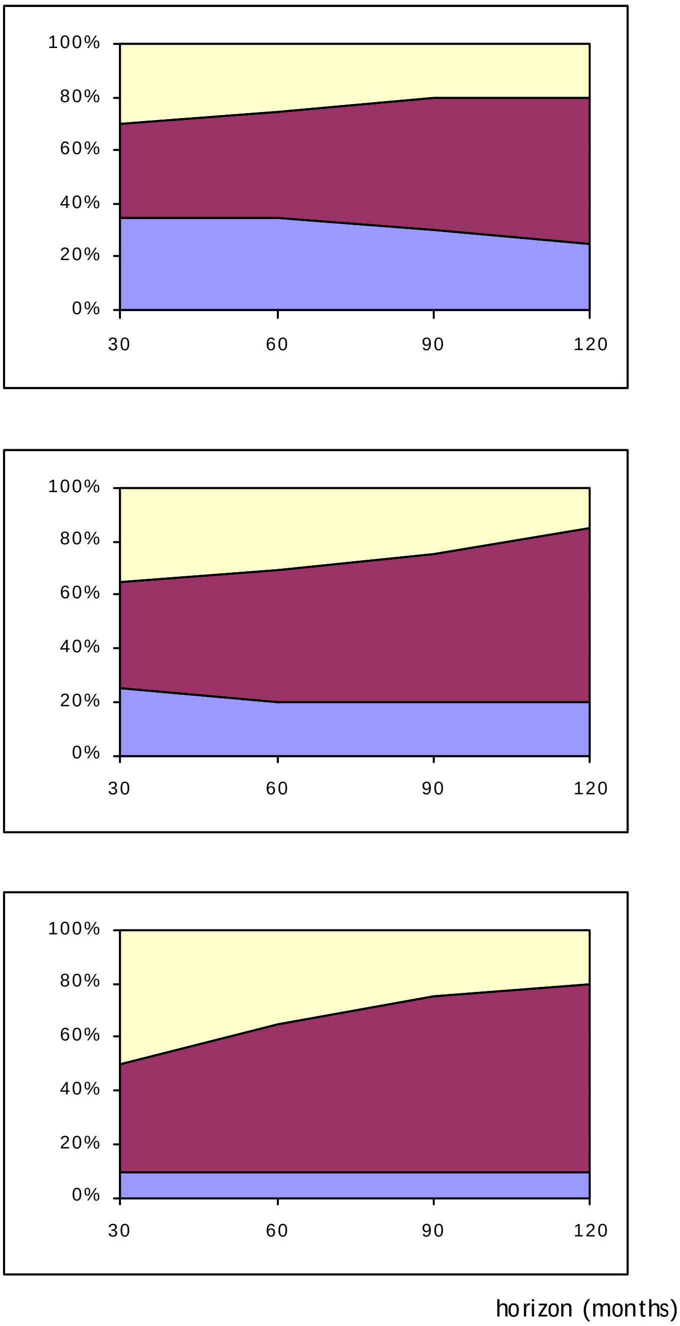


Figure 12. Allocation to bonds and stocks for di¤erent initial values of dividend yield. The graph plots the static allocation to the riskless asset (light shade) 10-year bonds (medium shade) and stocks (dark shade) for di¤erent horizons when returns are assumed to be generated by a VAR model with four predictors. The initial values of DP are 2.88 (upper graph) 3.44 (middle graph) and 4.07 (lower graph). The rest of variables are initially at their mean levels. The investor does not take parameter uncertainty into account. The risk aversion coe¢cient is 10.

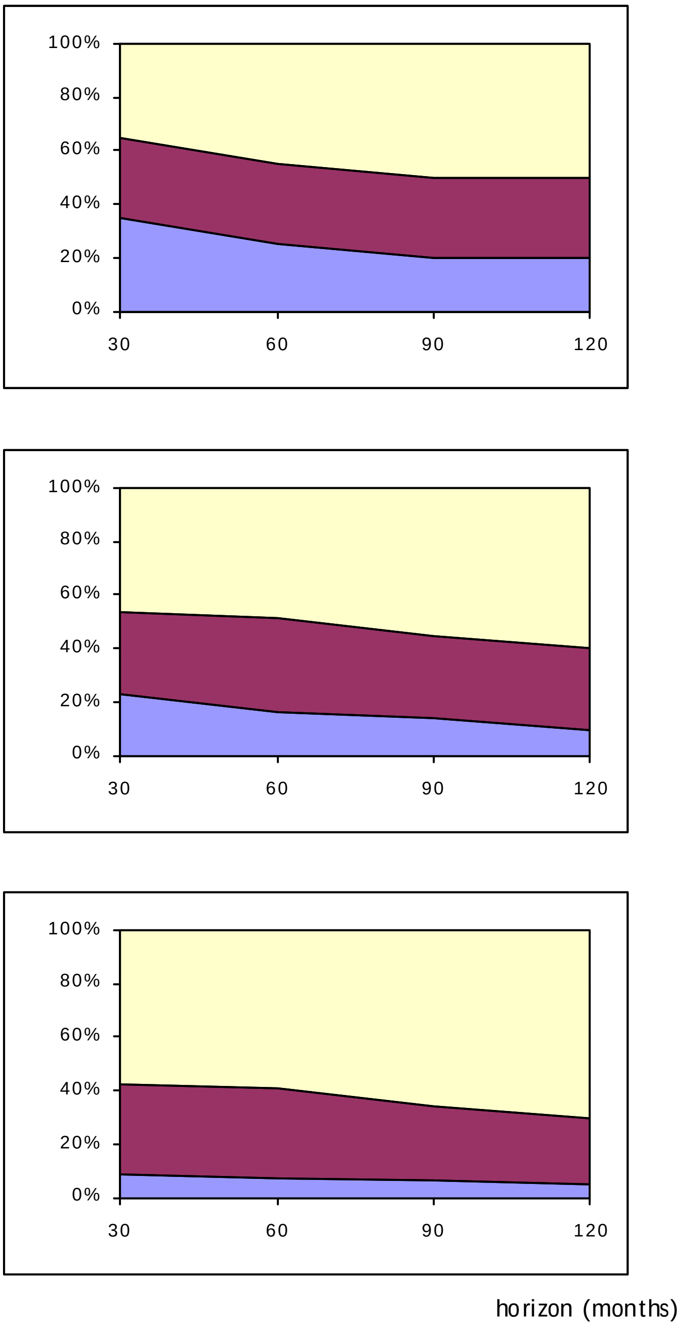
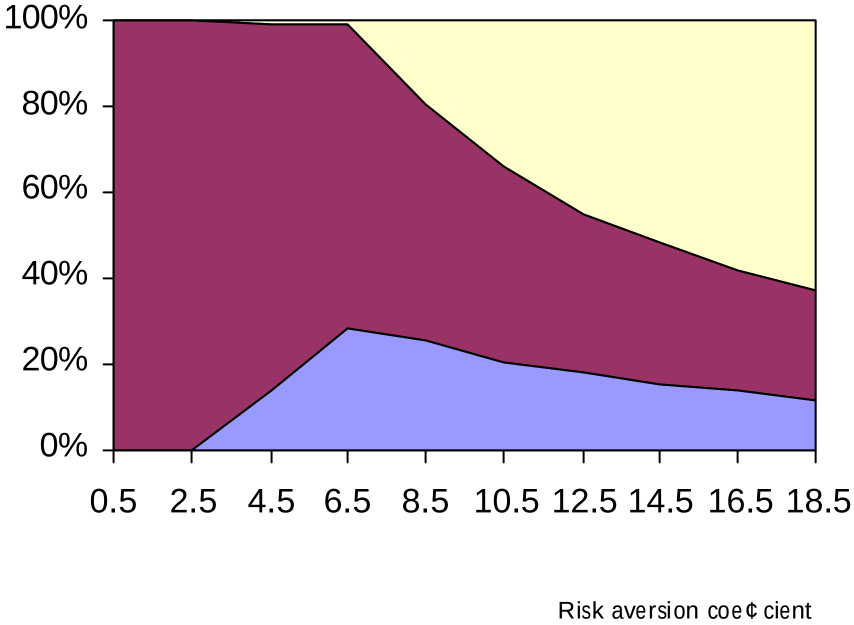


Figure 13. Allocation to bonds and stocks for different initial values of dividend yield under parameter uncertainty. The graph plots the optimal allocation to the riskless asset (light shade) 10-year bonds (medium shade) and stocks (dark shade) for different horizons when returns are assumed to be generated by a VAR model with four predictors. The initial values of DP are 2.88 (upper graph) 3.44 (middle graph) and 4.07 (lower graph). The rest of variables are initially at their mean levels. The investor takes parameter uncertainty into account. The risk aversion coefficient is 10.



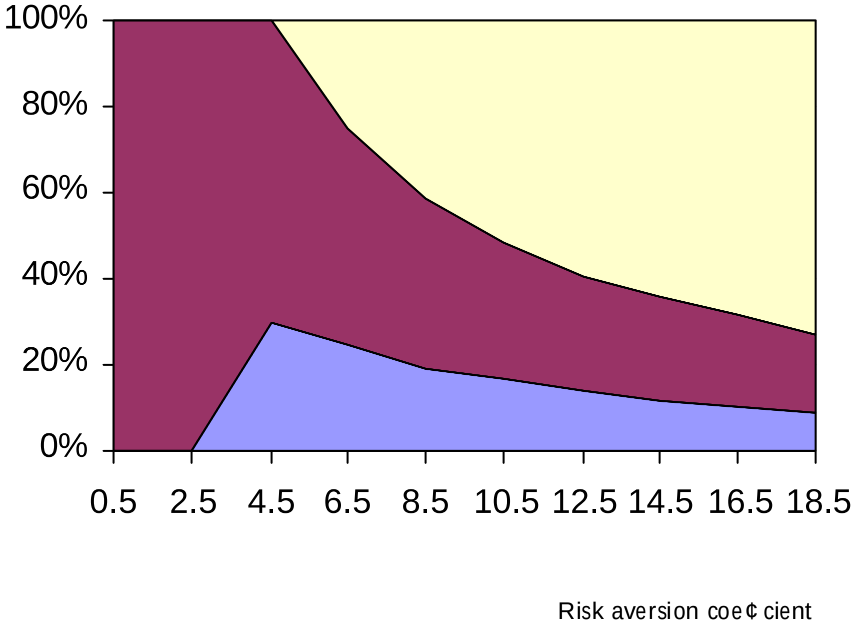


Figure 14. Portfolio choice and risk aversion. The graphs plot the optimal allocation to the riskless asset (light shade), 10-year bonds (medium shade) and stocks (dark shade) for a 5 year investment horizon and when predictors are initially at their sample means.